# Computer Algebra
# Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

## 4-Trig-functions/4.7-Miscellaneous/137-4.7.3-c+d-x-^m-trig^n-trig^p

Nasser M. Abbasi

December 8, 2023 Compiled on December 8, 2023 at 9:04pm

# Contents

**1 Introduction** **2**

**2 detailed summary tables of results** **20**

**3 Listing of integrals** **140**

**4 Appendix** **2820**

# CHAPTER 1

# Introduction


1.1 Listing of CAS systems tested . . . 3
1.2 Results . . . 4
1.3 Time and leaf size Performance . . . 7
1.4 Performance based on number of rules Rubi used . . . 9
1.5 Performance based on number of steps Rubi used . . . 10
1.6 Solved integrals histogram based on leaf size of result . . . 11
1.7 Solved integrals histogram based on CPU time used . . . 12
1.8 Leaf size vs. CPU time used . . . 13
1.9 list of integrals with no known antiderivative . . . 14
1.10 List of integrals solved by CAS but has no known antiderivative . . . 14
1.11 list of integrals solved by CAS but failed verification . . . 14
1.12 Timing . . . 15
1.13 Verification . . . 15
1.14 Important notes about some of the results . . . 16
1.15 Design of the test system . . . 19

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [ 397 ]. This is test number [ 137 ].

## 1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | % solved | % Failed |
|---|---|---|
| Mathematica | 100.00 ( 397 ) | 0.00 ( 0 ) |
| Rubi | 99.50 ( 395 ) | 0.50 ( 2 ) |
| Fricas | 91.18 ( 362 ) | 8.82 ( 35 ) |
| Maple | 90.43 ( 359 ) | 9.57 ( 38 ) |
| Maxima | 85.89 ( 341 ) | 14.11 ( 56 ) |
| Giac | 61.71 ( 245 ) | 38.29 ( 152 ) |
| Mupad | 39.04 ( 155 ) | 60.96 ( 242 ) |
| Sympy | 30.48 ( 121 ) | 69.52 ( 276 ) |

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|---|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons<br>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.<br>2. antiderivative contains a special function and the optimal antiderivative does not.<br>3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

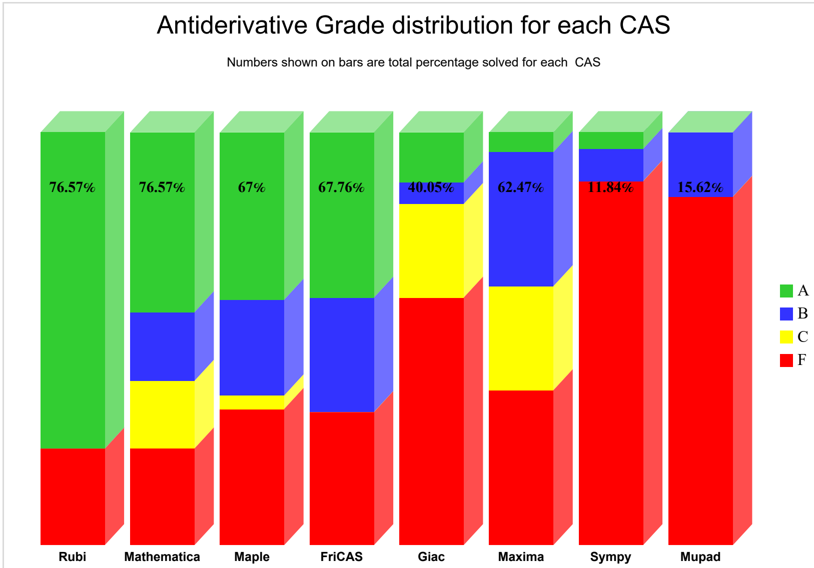
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

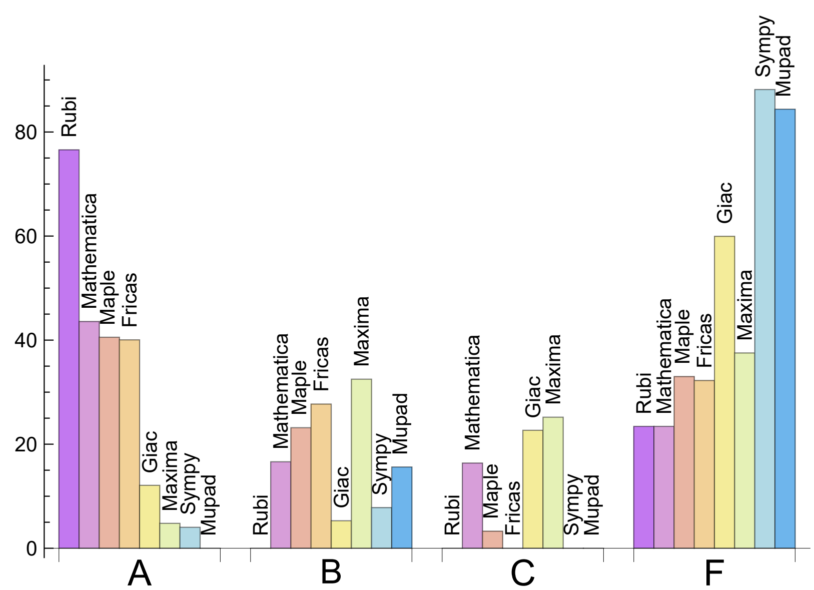
| System | % A grade | % B grade | % C grade | % F grade |
|---|---|---|---|---|
| Rubi | 76.071 | 0.000 | 0.000 | 23.929 |
| Mathematica | 43.577 | 16.625 | 16.373 | 23.426 |
| Maple | 40.554 | 23.174 | 3.275 | 32.997 |
| Fricas | 40.050 | 27.708 | 0.000 | 32.242 |
| Giac | 12.091 | 5.290 | 22.670 | 59.950 |
| Maxima | 4.786 | 32.494 | 25.189 | 37.531 |
| Sympy | 4.030 | 7.809 | 0.000 | 88.161 |
| Mupad | 0.000 | 15.617 | 0.000 | 84.383 |

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.




The figure below compares the grades of the CAS systems.




The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

| System | Number failed | Percentage normal failure | Percentage timeout failure | Percentage exception failure |
|---|---|---|---|---|
| Mathematica | 0 | 0.00 | 0.00 | 0.00 |
| Rubi | 2 | 100.00 | 0.00 | 0.00 |
| Fricas | 35 | 0.00 | 0.00 | 100.00 |
| Maple | 38 | 100.00 | 0.00 | 0.00 |
| Maxima | 56 | 94.64 | 0.00 | 5.36 |
| Giac | 152 | 95.39 | 3.95 | 0.66 |
| Mupad | 242 | 0.00 | 100.00 | 0.00 |
| Sympy | 276 | 73.19 | 20.65 | 6.16 |

Table 1.4: Failure statistics for each CAS

## 1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

| System | Mean time (sec) |
|---|---|
| Fricas | 0.27 |
| Rubi | 0.59 |
| Maxima | 0.85 |
| Maple | 1.61 |
| Mathematica | 5.10 |
| Giac | 5.23 |
| Sympy | 8.29 |
| Mupad | 22.43 |

Table 1.5: Time performance for each CAS

| System | Mean size | Normalized mean | Median size | Normalized median |
|---|---|---|---|---|
| Mupad | 100.03 | 1.40 | 26.00 | 1.18 |
| Rubi | 164.07 | 1.06 | 126.00 | 1.00 |
| Sympy | 168.71 | 1.75 | 22.00 | 1.00 |
| Mathematica | 289.95 | 1.57 | 127.00 | 1.09 |
| Maple | 294.75 | 1.75 | 187.00 | 1.17 |
| Fricas | 385.94 | 2.28 | 217.50 | 1.43 |
| Maxima | 1089.56 | 15.08 | 499.00 | 2.80 |
| Giac | 50599.61 | 160.79 | 209.00 | 1.57 |

Table 1.6: Leaf size performance for each CAS

## 1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the $y$ axis is the percentage solved which Rubi itself needed the number of rules given the $x$ axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

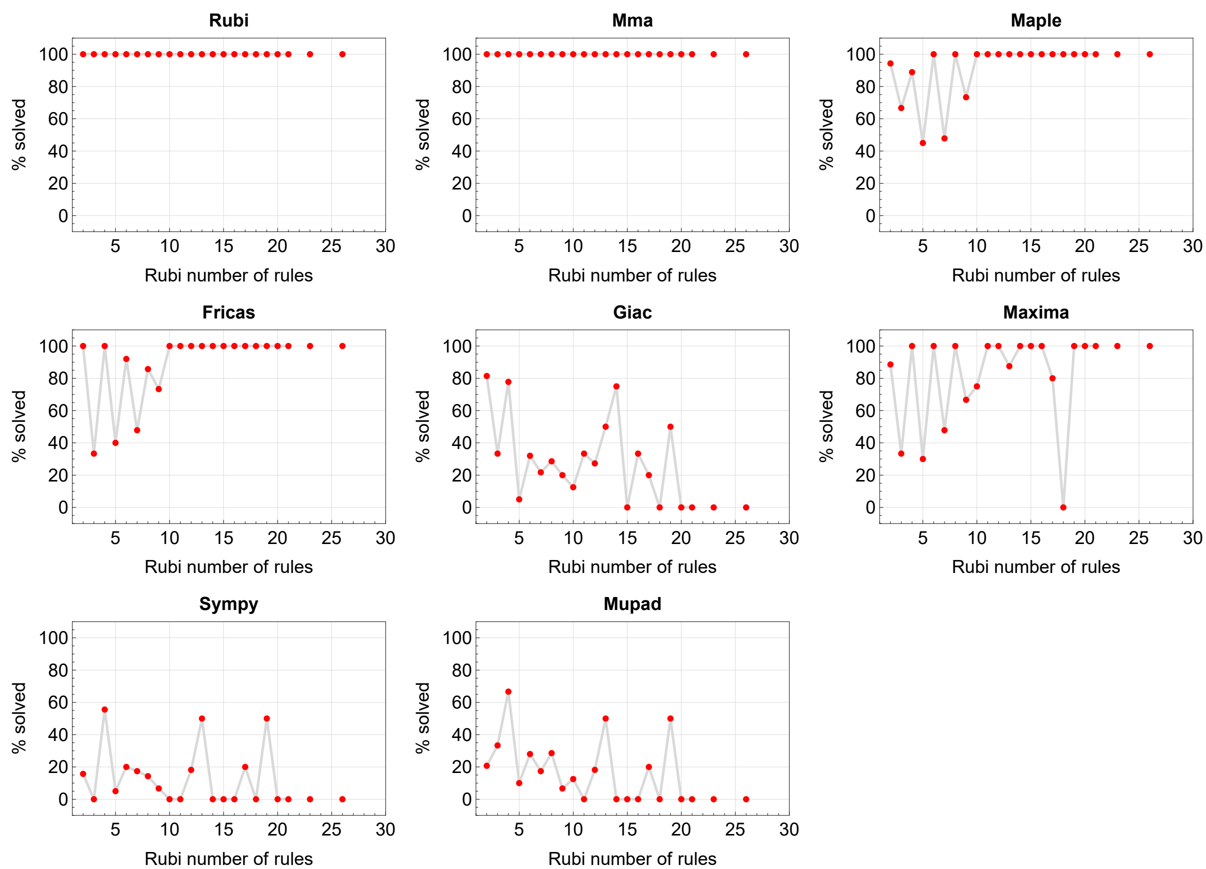



Figure 1.1: Solving statistics per number of Rubi rules used

## 1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

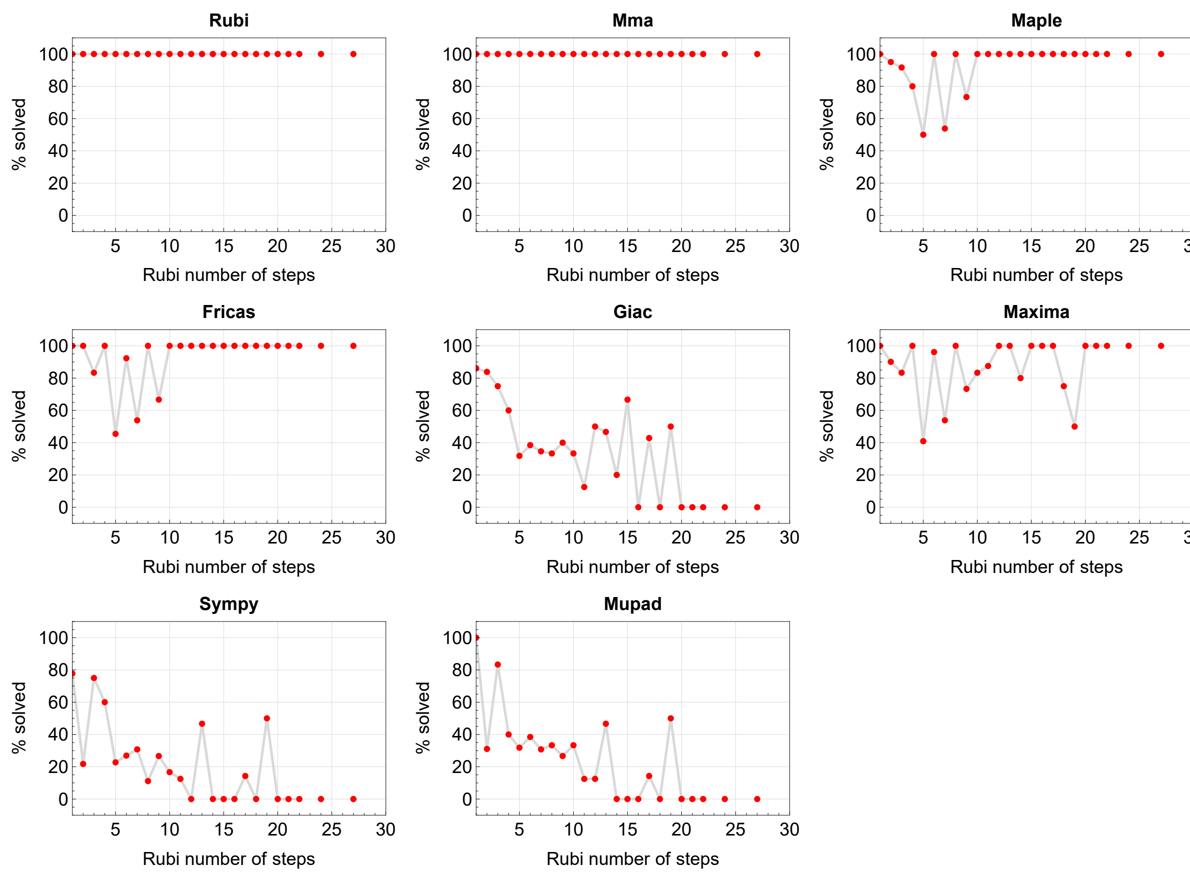



Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

## 1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

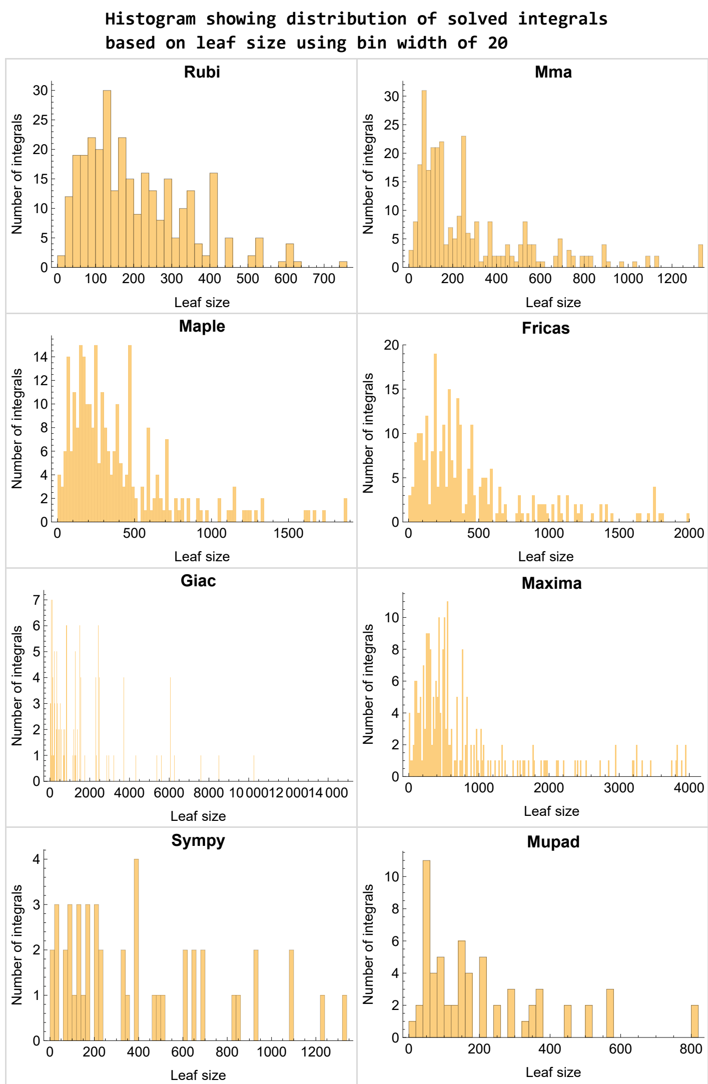



Figure 1.3: Solved integrals based on leaf size distribution

# 1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

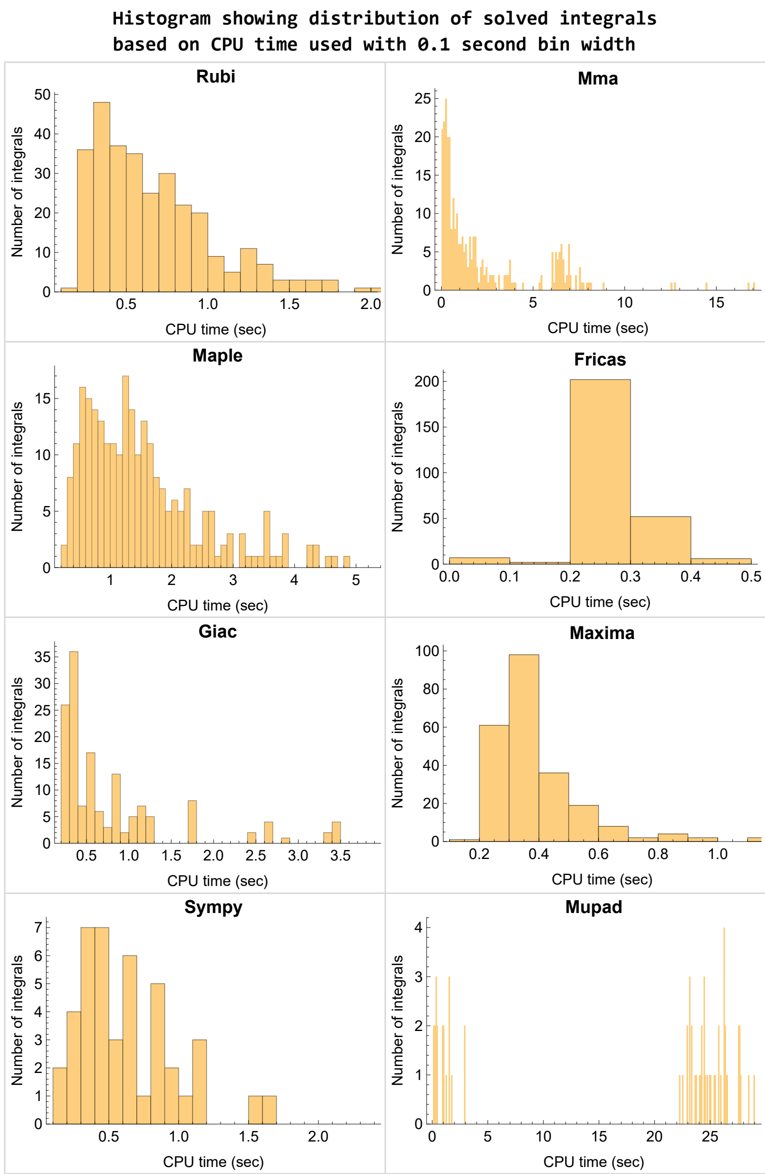



Figure 1.4: Solved integrals histogram based on CPU time used

## 1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

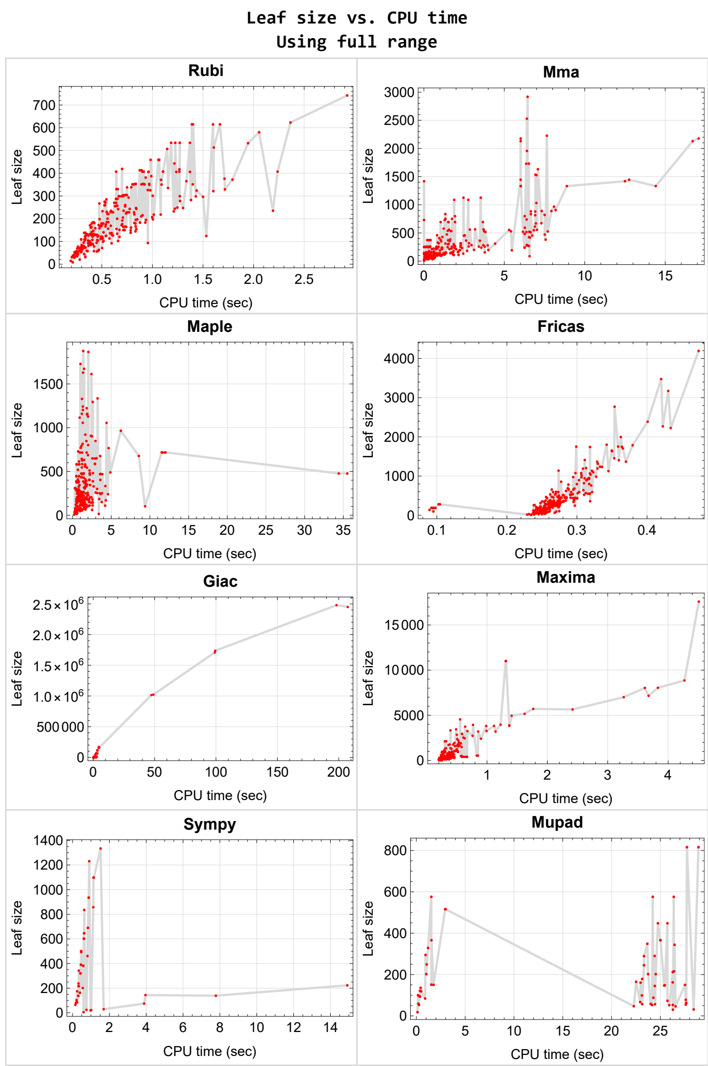



Figure 1.5: Leaf size vs. CPU time. Full range

## 1.9 list of integrals with no known antiderivative

{31, 36, 37, 38, 43, 44, 45, 50, 51, 97, 102, 103, 104, 109, 110, 111, 116, 117, 163, 168, 169, 170, 175, 176, 177, 182, 183, 208, 213, 214, 215, 219, 220, 221, 225, 226, 227, 232, 233, 234, 238, 239, 240, 244, 245, 246, 251, 252, 253, 257, 258, 259, 263, 264, 265, 270, 271, 272, 276, 277, 278, 282, 283, 284, 288, 289, 290, 295, 296, 297, 301, 302, 303, 307, 308, 309, 314, 315, 316, 320, 321, 322, 326, 327, 379, 380, 381, 386, 387, 388, 392, 393, 394}

## 1.10 List of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

## 1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

**Rubi** {}

**Mathematica** {178, 179, 222, 237, 241, 242, 261, 280, 300, 304, 310, 311, 312, 318, 319}

**Maple** {}

**Maxima** Verification phase not currently implemented.

**Fricas** Verification phase not currently implemented.

**Sympy** Verification phase not currently implemented.

**Giac** Verification phase not currently implemented.

**Mupad** Verification phase not currently implemented.

## 1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

## 1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

# 1.14 Important notes about some of the results

## 1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the **Timofeev** test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

### 1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

### 1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

### 1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

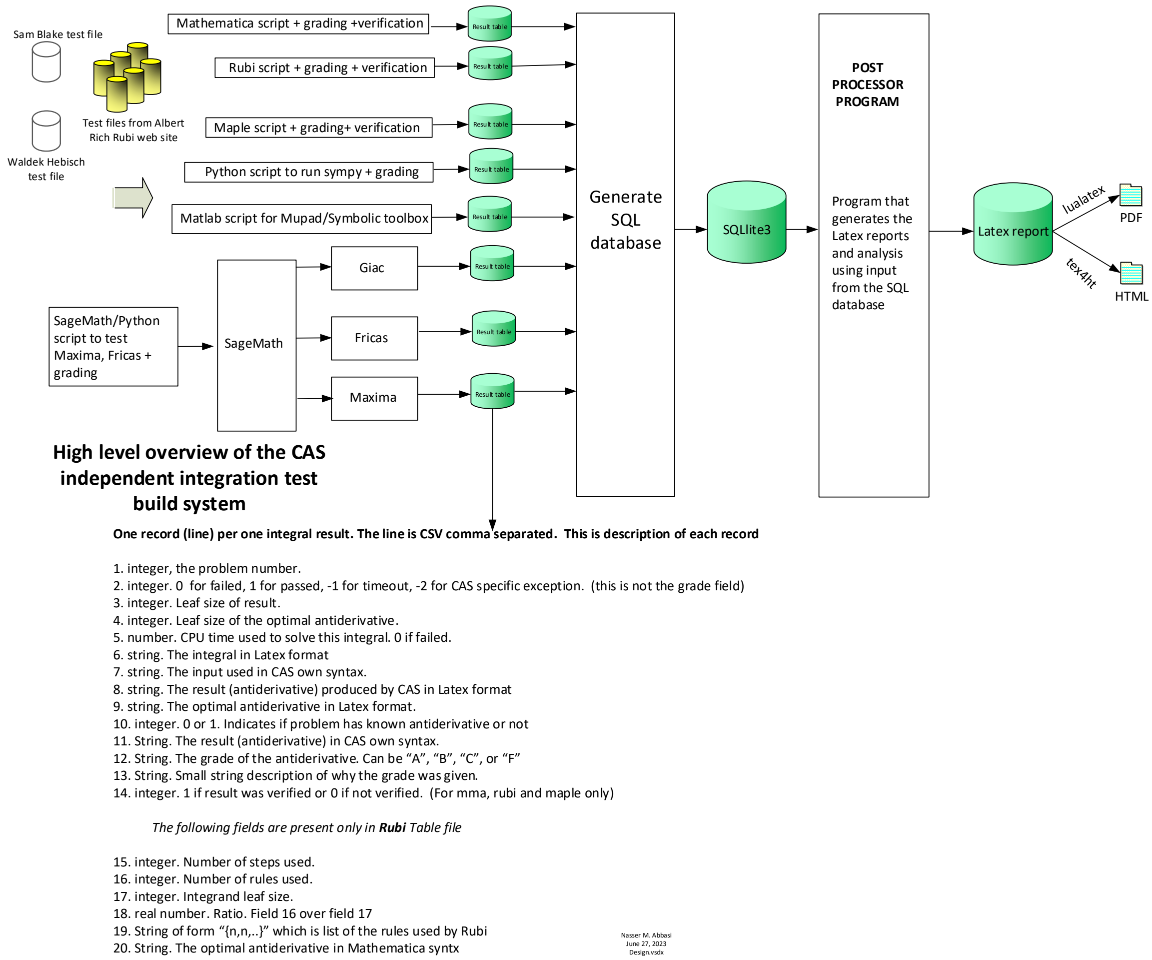
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

# 1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



# CHAPTER 2

## DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS . . . . . . . . . . . . . . . . . . . . . 21
2.2 Detailed conclusion table per each integral for all CAS systems . . . . . . . . . . 27
2.3 Detailed conclusion table specific for Rubi results . . . . . . . . . . . . . . . . . . . 127

## 2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi . . . . . . . . . . 21
2.1.2 Mma . . . . . . . . . . 22
2.1.3 Maple . . . . . . . . . . 22
2.1.4 Fricas . . . . . . . . . . 23
2.1.5 Maxima . . . . . . . . . . 23
2.1.6 Giac . . . . . . . . . . 24
2.1.7 Mupad . . . . . . . . . . 25
2.1.8 Sympy . . . . . . . . . . 25

### 2.1.1 Rubi

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 39, 40, 41, 42, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 105, 106, 107, 108, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 171, 172, 173, 174, 178, 179, 180, 181, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 207, 209, 210, 211, 212, 216, 217, 218, 222, 223, 224, 228, 229, 230, 231, 235, 236, 237, 241, 242, 243, 247, 248, 249, 250, 254, 255, 256, 260, 261, 262, 266, 267, 268, 269, 273, 274, 275, 279, 280, 281, 285, 286, 287, 291, 292, 293, 294, 298, 299, 300, 304, 305, 306, 310, 311, 312, 313, 317, 318, 319, 323, 324, 325, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 382, 383, 384, 385, 389, 390, 391, 395, 396, 397 }

**B grade** { }

**C grade** { }

**F normal fail** { 205, 206 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

### 2.1.2 Mma

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 39, 49, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 113, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 167, 174, 202, 203, 204, 205, 206, 207, 209, 210, 211, 212, 217, 224, 229, 230, 231, 243, 247, 248, 249, 256, 262, 266, 267, 268, 269, 275, 279, 285, 293, 294, 298, 313, 317, 323, 328, 329, 330, 332, 333, 334, 335, 336, 337, 338, 339, 341, 343, 344, 346, 348, 349, 350, 351, 352, 353, 354, 355, 357, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 391, 395, 396 }

**B grade** { 32, 33, 34, 35, 40, 41, 42, 46, 47, 98, 105, 106, 107, 112, 114, 115, 164, 165, 166, 171, 172, 173, 178, 179, 180, 181, 216, 218, 222, 223, 228, 235, 236, 241, 242, 250, 254, 255, 260, 261, 273, 274, 280, 281, 286, 287, 291, 292, 299, 300, 304, 305, 306, 310, 311, 312, 318, 324, 325, 382, 383, 384, 385, 389, 390, 397 }

**C grade** { 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 108, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 237, 319, 331, 340, 342, 345, 347, 356, 358 }

**F normal fail** { }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

### 2.1.3 Maple

**A grade** { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 42, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 108, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 218, 224, 237, 256, 269, 281, 294, 306, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 374, 375, 385, 391, 395, 396 }

**B grade** { 32, 33, 34, 35, 39, 40, 41, 46, 47, 48, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 164, 165, 166, 167, 171, 172, 173, 178, 179, 180, 181, 209, 210, 211, 212, 216, 217, 222, 223, 228, 229, 230, 231, 235, 236, 241, 242, 243, 247, 248, 249, 250, 254, 255, 260, 261, 266, 267, 268, 273, 274, 279, 280, 286, 287, 291, 292, 298, 299, 300, 304, 305, 310, 311, 312, 313, 317, 318, 319, 323, 324, 325, 376, 377, 378, 382, 383, 384, 389, 390, 397 }

**C grade** { 84, 85, 174, 204, 262, 275, 293, 331, 340, 347, 356, 372, 373 }

**F normal fail** { 1, 13, 22, 70, 79, 88, 136, 145, 154, 285, 328, 329, 330, 332, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 357, 358, 359 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

### 2.1.4 Fricas

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 157, 158, 159, 160, 161, 162, 174, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 204, 256, 262, 293, 294, 306, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 391, 396 }

**B grade** { 32, 33, 34, 35, 39, 40, 41, 42, 46, 47, 80, 81, 82, 87, 98, 99, 100, 101, 105, 106, 107, 108, 112, 113, 114, 115, 155, 156, 164, 165, 166, 167, 171, 172, 173, 178, 179, 180, 181, 202, 203, 205, 206, 207, 209, 210, 211, 212, 216, 217, 218, 222, 223, 224, 229, 230, 231, 235, 236, 237, 241, 242, 243, 247, 248, 249, 250, 254, 255, 260, 261, 267, 268, 269, 273, 274, 275, 279, 280, 281, 285, 286, 287, 291, 292, 298, 299, 300, 304, 305, 311, 312, 313, 317, 318, 319, 323, 324, 325, 376, 377, 378, 382, 383, 384, 385, 389, 390, 395, 397 }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { }

**F(-2) exception fail** { 228, 266, 310, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359 }

### 2.1.5 Maxima

**A grade** { 5, 17, 26, 74, 92, 140, 149, 158, 224, 360, 361, 362, 363, 364, 368, 369, 370, 371, 384 }

**B grade** { 2, 3, 4, 14, 15, 16, 23, 24, 25, 32, 33, 34, 35, 39, 40, 41, 42, 46, 47, 48, 49, 71, 72, 73, 80, 81, 82, 83, 89, 90, 91, 98, 99, 100, 101, 105, 106, 107, 108, 112, 113, 114, 115, 137, 138, 139, 146, 147, 148, 155, 156, 157, 164, 165, 166, 167, 171, 172, 173, 174, 178, 179, 180, 181, 205, 206, 207, 209, 210, 211, 212, 216, 217, 222, 223, 228, 229, 230, 231, 235, 236, 241, 242, 243, 247, 248, 250, 254, 255, 256, 260, 262, 266, 267, 268, 269, 273, 274, 275, 279, 280, 281, 285, 286, 287, 291, 292, 293, 294, 298, 299, 304, 305, 306, 310, 311, 312, 313, 317, 318, 323, 324, 325, 376, 377, 382, 383, 391, 396 }

**C grade** { 6, 7, 8, 9, 10, 11, 12, 18, 19, 20, 21, 27, 28, 29, 30, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 75, 76, 77, 78, 84, 85, 86, 87, 93, 94, 95, 96, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 365, 366, 367, 372, 373, 374, 375 }

**F normal fail** { 1, 13, 22, 70, 79, 88, 136, 145, 154, 218, 237, 249, 261, 300, 319, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 378, 385, 389, 390, 395, 397 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { 202, 203, 204 }

### 2.1.6 Giac

**A grade** { 2, 3, 4, 5, 10, 11, 12, 14, 15, 16, 17, 23, 24, 25, 26, 71, 72, 73, 74, 80, 81, 82, 83, 89, 90, 91, 92, 137, 138, 139, 140, 146, 147, 148, 149, 155, 156, 157, 158, 360, 361, 362, 363, 364, 365, 366, 371, 372 }

**B grade** { 42, 48, 49, 108, 174, 204, 250, 256, 262, 275, 293, 294, 367, 368, 369, 370, 373, 374, 375, 391, 396 }

**C grade** { 6, 7, 8, 9, 18, 19, 20, 21, 27, 28, 29, 30, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 75, 76, 77, 78, 84, 85, 86, 87, 93, 94, 95, 96, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201 }

**F normal fail** { 1, 13, 22, 32, 33, 34, 35, 39, 40, 41, 46, 47, 70, 79, 88, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 136, 145, 154, 164, 165, 166, 167, 171, 172, 173, 178, 179, 180, 181, 202, 203, 205, 206, 207, 209, 210, 211, 212, 216, 217, 218, 222, 223, 224, 228, 229, 230, 231, 235, 236, 237, 241, 242, 243, 247, 248, 249, 254, 255, 260, 261, 266, 267, 268, 269, 273, 274, 279, 280, 281, 285, 286, 287, 291, 292, 298, 299, 300, 304, 305, 306, 310, 311, 312, 313, 317, 318, 319, 323, 324, 325, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 376, 377, 378, 382, 383, 384, 385, 389, 390, 395, 397 }

**F(-1) timedout fail** { 51, 245, 283, 315, 321, 327 }

**F(-2) exception fail** { 117 }

### 2.1.7 Mupad

**A grade** { }

**B grade** { 2, 3, 4, 5, 14, 15, 16, 17, 23, 24, 25, 26, 42, 48, 49, 71, 72, 73, 74, 80, 81, 82, 83, 89, 90, 91, 92, 108, 137, 138, 139, 140, 146, 147, 148, 149, 155, 156, 157, 158, 174, 204, 212, 250, 256, 262, 275, 293, 294, 360, 361, 362, 363, 364, 368, 369, 370, 371, 391, 395, 396, 397 }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { 1, 6, 7, 8, 9, 10, 11, 12, 13, 18, 19, 20, 21, 22, 27, 28, 29, 30, 32, 33, 34, 35, 39, 40, 41, 46, 47, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 75, 76, 77, 78, 79, 84, 85, 86, 87, 88, 93, 94, 95, 96, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 141, 142, 143, 144, 145, 150, 151, 152, 153, 154, 159, 160, 161, 162, 164, 165, 166, 167, 171, 172, 173, 178, 179, 180, 181, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 209, 210, 211, 216, 217, 218, 222, 223, 224, 228, 229, 230, 231, 235, 236, 237, 241, 242, 243, 247, 248, 249, 254, 255, 260, 261, 266, 267, 268, 269, 273, 274, 279, 280, 281, 285, 286, 287, 291, 292, 298, 299, 300, 304, 305, 306, 310, 311, 312, 313, 317, 318, 319, 323, 324, 325, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 365, 366, 367, 372, 373, 374, 375, 376, 377, 378, 382, 383, 384, 385, 389, 390 }

**F(-2) exception fail** { }

### 2.1.8 Sympy

**A grade** { 5, 10, 11, 12, 17, 26, 74, 92, 140, 149, 256, 360, 361, 362, 363, 364 }

**B grade** { 2, 3, 4, 14, 15, 16, 23, 24, 25, 71, 72, 73, 80, 81, 82, 83, 89, 90, 91, 108, 137, 138, 139, 146, 147, 148, 155, 156, 157, 158, 396 }

**C grade** { }

**F normal fail** { 1, 6, 7, 8, 9, 13, 18, 19, 20, 21, 27, 28, 29, 30, 32, 33, 34, 35, 39, 40, 41, 42, 47, 48, 49, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 65, 66, 67, 68, 70, 75, 76, 77, 78, 84, 85, 86, 87, 93, 94, 95, 96, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 119, 120, 121, 122, 125, 126, 127, 128, 131, 132, 133, 134, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 164, 165, 166, 167, 171, 172, 173, 174, 178, 179, 180, 181, 185, 186, 187, 188, 191, 192, 193, 194, 197, 198, 199, 200, 202, 203, 204, 205, 206, 207, 209, 210, 211, 212, 216, 217, 218, 222, 223, 224, 228, 229, 230, 231, 235, 236, 237, 241, 242, 243, 247, 248, 249, 250, 254, 255, 260, 261, 262, 266, 267, 268, 269, 273, 274, 275, 280, 281, 285, 286, 287, 291, 292, 293, 294, 298, 299, 300, 304, 305, 306, 311, 312, 313, 318, 319, 325, 330, 331, 332, 340, 341, 342, 343, 345, 346, 347, 348, 349, 350, 356, 357, 358, 359, 365, 366, 367, 369, 370, 371, 372, 373, 384, 385, 395, 397 }

**F(-1) timedout fail** { 46, 58, 63, 64, 69, 118, 123, 124, 129, 130, 135, 162, 184, 189, 190, 195, 196, 201, 240, 272, 278, 279, 284, 309, 310, 316, 317, 322, 323, 324, 328, 329, 333, 334, 335, 336, 337, 338, 339, 344, 351, 352, 353, 354, 355, 368, 374, 375, 376, 377, 378, 380, 381, 382, 383, 388, 394 }

**F(-2) exception fail** { 22, 45, 79, 88, 136, 145, 154, 221, 225, 226, 386, 387, 389, 390, 391, 392, 393 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

| Problem 1 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 141 | 138 | 0 | 0 | 96 | 0 | 0 | 0 |
| N.S. | 1 | 1.03 | 1.01 | 0.00 | 0.00 | 0.70 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.365 | 0.414 | 0.000 | 0.000 | 0.095 | 0.000 | 0.000 | 0.000 |

| Problem 2 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 158 | 86 | 178 | 586 | 255 | 502 | 181 | 245 |
| N.S. | 1 | 1.01 | 0.55 | 1.14 | 3.76 | 1.63 | 3.22 | 1.16 | 1.57 |
| time (sec) | N/A | 0.433 | 0.450 | 0.637 | 0.236 | 0.238 | 0.467 | 0.286 | 23.322 |

| Problem 3 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 120 | 128 | 71 | 118 | 342 | 166 | 342 | 121 | 165 |
| N.S. | 1 | 1.07 | 0.59 | 0.98 | 2.85 | 1.38 | 2.85 | 1.01 | 1.38 |
| time (sec) | N/A | 0.379 | 0.278 | 0.437 | 0.238 | 0.242 | 0.343 | 0.287 | 22.514 |

| Problem 4 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 84 | 50 | 69 | 171 | 92 | 175 | 73 | 100 |
| N.S. | 1 | 0.94 | 0.56 | 0.78 | 1.92 | 1.03 | 1.97 | 0.82 | 1.12 |
| time (sec) | N/A | 0.268 | 0.218 | 0.347 | 0.234 | 0.252 | 0.249 | 0.281 | 0.167 |

| Problem 5 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 54 | 34 | 36 | 65 | 42 | 80 | 38 | 47 |
| N.S. | 1 | 1.08 | 0.68 | 0.72 | 1.30 | 0.84 | 1.60 | 0.76 | 0.94 |
| time (sec) | N/A | 0.224 | 0.189 | 0.258 | 0.206 | 0.238 | 0.183 | 0.287 | 22.279 |

| Problem 6 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 63 | 60 | 84 | 143 | 64 | 0 | 569 | 0 |
| N.S. | 1 | 0.97 | 0.92 | 1.29 | 2.20 | 0.98 | 0.00 | 8.75 | 0.00 |
| time (sec) | N/A | 0.432 | 0.109 | 0.371 | 0.267 | 0.243 | 0.000 | 0.312 | 0.000 |

| Problem 7 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 92 | 80 | 124 | 166 | 105 | 0 | 2870 | 0 |
| N.S. | 1 | 1.08 | 0.94 | 1.46 | 1.95 | 1.24 | 0.00 | 33.76 | 0.00 |
| time (sec) | N/A | 0.552 | 0.357 | 0.431 | 0.295 | 0.247 | 0.000 | 0.506 | 0.000 |

| Problem 8 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 114 | 120 | 102 | 162 | 201 | 187 | 0 | 5398 | 0 |
| N.S. | 1 | 1.05 | 0.89 | 1.42 | 1.76 | 1.64 | 0.00 | 47.35 | 0.00 |
| time (sec) | N/A | 0.660 | 1.102 | 0.582 | 0.339 | 0.244 | 0.000 | 0.465 | 0.000 |

| Problem 9 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 154 | 164 | 200 | 251 | 263 | 0 | 7592 | 0 |
| N.S. | 1 | 1.07 | 1.14 | 1.39 | 1.74 | 1.83 | 0.00 | 52.72 | 0.00 |
| time (sec) | N/A | 0.794 | 0.617 | 0.767 | 0.398 | 0.240 | 0.000 | 0.577 | 0.000 |

| Problem 10 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | A | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 8 | 8 | 8 | 7 | 13 | 6 | 5 | 6 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.88 | 1.62 | 0.75 | 0.62 | 0.75 | 0.00 |
| time (sec) | N/A | 0.197 | 0.010 | 0.210 | 0.247 | 0.235 | 0.600 | 0.269 | 0.000 |

| Problem 11 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | A | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 20 | 16 | 15 | 15 | 17 | 22 | 19 | 0 |
| N.S. | 1 | 1.25 | 1.00 | 0.94 | 0.94 | 1.06 | 1.38 | 1.19 | 0.00 |
| time (sec) | N/A | 0.262 | 0.011 | 0.536 | 0.246 | 0.229 | 1.012 | 0.268 | 0.000 |

| Problem 12 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | A | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 31 | 29 | 26 | 15 | 30 | 24 | 26 | 0 |
| N.S. | 1 | 1.07 | 1.00 | 0.90 | 0.52 | 1.03 | 0.83 | 0.90 | 0.00 |
| time (sec) | N/A | 0.328 | 0.012 | 0.616 | 0.253 | 0.232 | 0.756 | 0.283 | 0.000 |

| Problem 13 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 275 | 275 | 237 | 0 | 0 | 188 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.00 | 0.00 | 0.68 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.513 | 1.144 | 0.000 | 0.000 | 0.094 | 0.000 | 0.000 | 0.000 |

| Problem 14 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 205 | 249 | 385 | 173 | 880 | 352 | 646 | 350 | 448 |
| N.S. | 1 | 1.21 | 1.88 | 0.84 | 4.29 | 1.72 | 3.15 | 1.71 | 2.19 |
| time (sec) | N/A | 1.283 | 1.640 | 1.428 | 0.251 | 0.250 | 0.625 | 0.306 | 24.745 |

| Problem 15 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 164 | 121 | 224 | 499 | 227 | 391 | 231 | 289 |
| N.S. | 1 | 1.09 | 0.80 | 1.48 | 3.30 | 1.50 | 2.59 | 1.53 | 1.91 |
| time (sec) | N/A | 0.736 | 1.193 | 0.879 | 0.238 | 0.248 | 0.455 | 0.297 | 23.323 |

| Problem 16 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 106 | 93 | 128 | 240 | 130 | 216 | 137 | 161 |
| N.S. | 1 | 1.03 | 0.90 | 1.24 | 2.33 | 1.26 | 2.10 | 1.33 | 1.56 |
| time (sec) | N/A | 0.447 | 0.658 | 1.037 | 0.211 | 0.261 | 0.322 | 0.301 | 22.969 |

| Problem 17 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 48 | 44 | 56 | 85 | 59 | 85 | 69 | 59 |
| N.S. | 1 | 0.94 | 0.86 | 1.10 | 1.67 | 1.16 | 1.67 | 1.35 | 1.16 |
| time (sec) | N/A | 0.256 | 0.215 | 0.648 | 0.199 | 0.248 | 0.224 | 0.292 | 0.201 |

| Problem 18 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 121 | 121 | 102 | 171 | 276 | 122 | 0 | 6059 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 1.41 | 2.28 | 1.01 | 0.00 | 50.07 | 0.00 |
| time (sec) | N/A | 0.445 | 0.625 | 0.620 | 0.291 | 0.248 | 0.000 | 0.504 | 0.000 |

| Problem 19 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 168 | 139 | 247 | 305 | 186 | 0 | 67350 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 1.47 | 1.82 | 1.11 | 0.00 | 400.89 | 0.00 |
| time (sec) | N/A | 0.487 | 1.643 | 0.580 | 0.346 | 0.254 | 0.000 | 2.409 | 0.000 |

| Problem 20 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 221 | 221 | 183 | 316 | 340 | 312 | 0 | 114422 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 1.43 | 1.54 | 1.41 | 0.00 | 517.75 | 0.00 |
| time (sec) | N/A | 0.555 | 2.169 | 1.493 | 0.424 | 0.269 | 0.000 | 3.323 | 0.000 |

| Problem 21 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 270 | 270 | 298 | 389 | 390 | 450 | 0 | 168646 | 0 |
| N.S. | 1 | 1.00 | 1.10 | 1.44 | 1.44 | 1.67 | 0.00 | 624.61 | 0.00 |
| time (sec) | N/A | 0.635 | 1.508 | 1.769 | 0.584 | 0.277 | 0.000 | 4.616 | 0.000 |

| Problem 22 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 271 | 271 | 246 | 0 | 0 | 190 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 0.00 | 0.00 | 0.70 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.525 | 1.478 | 0.000 | 0.000 | 0.095 | 0.000 | 0.000 | 0.000 |

| Problem 23 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 260 | 326 | 158 | 187 | 967 | 434 | 935 | 361 | 576 |
| N.S. | 1 | 1.25 | 0.61 | 0.72 | 3.72 | 1.67 | 3.60 | 1.39 | 2.22 |
| time (sec) | N/A | 1.000 | 2.117 | 1.988 | 0.254 | 0.257 | 0.878 | 0.314 | 24.220 |

| Problem 24 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 196 | 243 | 135 | 148 | 549 | 283 | 602 | 241 | 366 |
| N.S. | 1 | 1.24 | 0.69 | 0.76 | 2.80 | 1.44 | 3.07 | 1.23 | 1.87 |
| time (sec) | N/A | 0.774 | 0.988 | 1.541 | 0.246 | 0.259 | 0.609 | 0.305 | 24.998 |

| Problem 25 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 133 | 91 | 111 | 263 | 159 | 320 | 145 | 202 |
| N.S. | 1 | 0.99 | 0.68 | 0.83 | 1.96 | 1.19 | 2.39 | 1.08 | 1.51 |
| time (sec) | N/A | 0.388 | 0.525 | 1.278 | 0.217 | 0.252 | 0.427 | 0.289 | 24.528 |

| Problem 26 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 80 | 75 | 69 | 92 | 76 | 138 | 75 | 94 |
| N.S. | 1 | 1.11 | 1.04 | 0.96 | 1.28 | 1.06 | 1.92 | 1.04 | 1.31 |
| time (sec) | N/A | 0.305 | 0.154 | 0.984 | 0.206 | 0.247 | 0.309 | 0.281 | 0.319 |

| Problem 27 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 110 | 178 | 281 | 126 | 0 | 6046 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 1.38 | 2.18 | 0.98 | 0.00 | 46.87 | 0.00 |
| time (sec) | N/A | 0.427 | 0.345 | 0.995 | 0.296 | 0.250 | 0.000 | 0.514 | 0.000 |

| Problem 28 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 179 | 151 | 256 | 308 | 194 | 0 | 63510 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 1.43 | 1.72 | 1.08 | 0.00 | 354.80 | 0.00 |
| time (sec) | N/A | 0.497 | 1.329 | 1.333 | 0.354 | 0.253 | 0.000 | 2.655 | 0.000 |

| Problem 29 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 229 | 229 | 199 | 329 | 343 | 334 | 0 | 111694 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 1.44 | 1.50 | 1.46 | 0.00 | 487.75 | 0.00 |
| time (sec) | N/A | 0.565 | 3.119 | 1.645 | 0.436 | 0.259 | 0.000 | 3.336 | 0.000 |

| Problem 30 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 287 | 287 | 316 | 404 | 393 | 474 | 0 | 157526 | 0 |
| N.S. | 1 | 1.00 | 1.10 | 1.41 | 1.37 | 1.65 | 0.00 | 548.87 | 0.00 |
| time (sec) | N/A | 0.644 | 2.281 | 2.125 | 0.614 | 0.272 | 0.000 | 4.529 | 0.000 |

| Problem 31 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 16 | 20 | 22 | 22 | 20 | 22 | 24 |
| N.S. | 1 | 1.00 | 1.14 | 1.43 | 1.57 | 1.57 | 1.43 | 1.57 | 1.71 |
| time (sec) | N/A | 0.199 | 3.546 | 0.286 | 0.380 | 0.233 | 18.240 | 0.288 | 23.551 |

| Problem 32 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 215 | 799 | 1159 | 1281 | 1204 | 0 | 0 | 0 |
| N.S. | 1 | 1.42 | 5.29 | 7.68 | 8.48 | 7.97 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.869 | 6.270 | 1.206 | 0.411 | 0.313 | 0.000 | 0.000 | 0.000 |

| Problem 33 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 127 | 171 | 560 | 792 | 759 | 818 | 0 | 0 | 0 |
| N.S. | 1 | 1.35 | 4.41 | 6.24 | 5.98 | 6.44 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.680 | 2.858 | 1.157 | 0.339 | 0.310 | 0.000 | 0.000 | 0.000 |

| Problem 34 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 125 | 356 | 477 | 411 | 502 | 0 | 0 | 0 |
| N.S. | 1 | 1.34 | 3.83 | 5.13 | 4.42 | 5.40 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.530 | 1.610 | 1.127 | 0.299 | 0.274 | 0.000 | 0.000 | 0.000 |

| Problem 35 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 81 | 192 | 215 | 190 | 250 | 0 | 0 | 0 |
| N.S. | 1 | 1.25 | 2.95 | 3.31 | 2.92 | 3.85 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.357 | 5.467 | 0.996 | 0.305 | 0.269 | 0.000 | 0.000 | 0.000 |

| Problem 36 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 16 | 20 | 22 | 22 | 19 | 22 | 24 |
| N.S. | 1 | 1.00 | 1.14 | 1.43 | 1.57 | 1.57 | 1.36 | 1.57 | 1.71 |
| time (sec) | N/A | 0.196 | 4.960 | 0.560 | 0.418 | 0.257 | 0.557 | 0.295 | 22.966 |

| Problem 37 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 16 | 20 | 22 | 33 | 20 | 22 | 24 |
| N.S. | 1 | 1.00 | 1.14 | 1.43 | 1.57 | 2.36 | 1.43 | 1.57 | 1.71 |
| time (sec) | N/A | 0.198 | 9.022 | 0.537 | 0.668 | 0.251 | 0.812 | 2.059 | 21.856 |

| Problem 38 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 22 | 24 | 24 | 22 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.10 | 1.10 | 1.20 | 1.20 | 1.10 | 1.20 | 1.20 |
| time (sec) | N/A | 0.318 | 3.159 | 0.303 | 0.712 | 0.251 | 66.692 | 0.304 | 21.340 |

| Problem 39 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 208 | 228 | 308 | 716 | 2948 | 1021 | 0 | 0 | 0 |
| N.S. | 1 | 1.10 | 1.48 | 3.44 | 14.17 | 4.91 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.802 | 1.534 | 1.232 | 0.585 | 0.307 | 0.000 | 0.000 | 0.000 |

| Problem 40 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 146 | 156 | 311 | 433 | 1770 | 669 | 0 | 0 | 0 |
| N.S. | 1 | 1.07 | 2.13 | 2.97 | 12.12 | 4.58 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.558 | 1.342 | 1.128 | 0.382 | 0.288 | 0.000 | 0.000 | 0.000 |

| Problem 41 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 90 | 92 | 234 | 212 | 553 | 375 | 0 | 0 | 0 |
| N.S. | 1 | 1.02 | 2.60 | 2.36 | 6.14 | 4.17 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.358 | 2.371 | 0.778 | 0.350 | 0.275 | 0.000 | 0.000 | 0.000 |

| Problem 42 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | B | B | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 131 | 46 | 259 | 62 | 0 | 697 | 88 |
| N.S. | 1 | 1.00 | 4.37 | 1.53 | 8.63 | 2.07 | 0.00 | 23.23 | 2.93 |
| time (sec) | N/A | 0.227 | 0.114 | 0.507 | 0.232 | 0.268 | 0.000 | 0.537 | 24.281 |

| Problem 43 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 22 | 508 | 24 | 20 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.10 | 1.10 | 25.40 | 1.20 | 1.00 | 1.20 | 1.20 |
| time (sec) | N/A | 0.256 | 19.170 | 0.527 | 0.583 | 0.257 | 0.953 | 0.890 | 23.853 |

| Problem 44 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 22 | 745 | 35 | 22 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.10 | 1.10 | 37.25 | 1.75 | 1.10 | 1.20 | 1.20 |
| time (sec) | N/A | 0.277 | 22.944 | 0.539 | 1.281 | 0.247 | 1.367 | 2.878 | 23.867 |

| Problem 45 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 24 | 0 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.09 | 0.00 | 1.09 | 1.09 |
| time (sec) | N/A | 0.331 | 7.834 | 0.288 | 0.747 | 0.250 | 0.000 | 0.296 | 23.118 |

| Problem 46 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 179 | 512 | 716 | 4551 | 1075 | 0 | 0 | 0 |
| N.S. | 1 | 1.31 | 3.74 | 5.23 | 33.22 | 7.85 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.818 | 6.640 | 1.822 | 0.555 | 0.298 | 0.000 | 0.000 | 0.000 |

| Problem 47 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 137 | 277 | 409 | 1044 | 591 | 0 | 0 | 0 |
| N.S. | 1 | 1.19 | 2.41 | 3.56 | 9.08 | 5.14 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.596 | 6.457 | 1.668 | 0.493 | 0.273 | 0.000 | 0.000 | 0.000 |

| Problem 48 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | A | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 59 | 94 | 136 | 1130 | 102 | 0 | 2978 | 147 |
| N.S. | 1 | 1.09 | 1.74 | 2.52 | 20.93 | 1.89 | 0.00 | 55.15 | 2.72 |
| time (sec) | N/A | 0.346 | 1.057 | 1.310 | 0.291 | 0.275 | 0.000 | 1.214 | 25.391 |

| Problem 49 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 35 | 48 | 61 | 287 | 44 | 0 | 526 | 53 |
| N.S. | 1 | 1.00 | 1.37 | 1.74 | 8.20 | 1.26 | 0.00 | 15.03 | 1.51 |
| time (sec) | N/A | 0.244 | 0.338 | 1.029 | 0.245 | 0.242 | 0.000 | 0.351 | 24.141 |

| Problem 50 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 1664 | 24 | 20 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 75.64 | 1.09 | 0.91 | 1.09 | 1.09 |
| time (sec) | N/A | 0.269 | 13.138 | 0.569 | 1.151 | 0.250 | 2.074 | 0.332 | 23.320 |

| Problem 51 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 2153 | 35 | 22 | 0 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 97.86 | 1.59 | 1.00 | 0.00 | 1.09 |
| time (sec) | N/A | 0.287 | 12.396 | 0.562 | 2.661 | 0.257 | 3.141 | 0.000 | 23.274 |

| Problem 52 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 196 | 218 | 144 | 234 | 277 | 222 | 0 | 1205 | 0 |
| N.S. | 1 | 1.11 | 0.73 | 1.19 | 1.41 | 1.13 | 0.00 | 6.15 | 0.00 |
| time (sec) | N/A | 1.078 | 0.682 | 0.889 | 0.403 | 0.259 | 0.000 | 0.597 | 0.000 |

| Problem 53 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 183 | 128 | 187 | 256 | 167 | 0 | 747 | 0 |
| N.S. | 1 | 1.09 | 0.76 | 1.11 | 1.52 | 0.99 | 0.00 | 4.45 | 0.00 |
| time (sec) | N/A | 0.866 | 0.601 | 0.618 | 0.372 | 0.247 | 0.000 | 0.454 | 0.000 |

| Problem 54 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 150 | 145 | 142 | 209 | 125 | 0 | 406 | 0 |
| N.S. | 1 | 1.06 | 1.02 | 1.00 | 1.47 | 0.88 | 0.00 | 2.86 | 0.00 |
| time (sec) | N/A | 0.694 | 0.402 | 0.611 | 0.398 | 0.240 | 0.000 | 0.384 | 0.000 |

| Problem 55 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 150 | 145 | 142 | 209 | 125 | 0 | 406 | 0 |
| N.S. | 1 | 1.06 | 1.02 | 1.00 | 1.47 | 0.88 | 0.00 | 2.86 | 0.00 |
| time (sec) | N/A | 0.687 | 0.009 | 0.556 | 0.389 | 0.244 | 0.000 | 0.382 | 0.000 |

| Problem 56 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 183 | 128 | 187 | 256 | 167 | 0 | 747 | 0 |
| N.S. | 1 | 1.09 | 0.76 | 1.11 | 1.52 | 0.99 | 0.00 | 4.45 | 0.00 |
| time (sec) | N/A | 0.833 | 0.024 | 0.556 | 0.383 | 0.256 | 0.000 | 0.471 | 0.000 |

| Problem 57 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 196 | 218 | 144 | 234 | 277 | 222 | 0 | 1205 | 0 |
| N.S. | 1 | 1.11 | 0.73 | 1.19 | 1.41 | 1.13 | 0.00 | 6.15 | 0.00 |
| time (sec) | N/A | 0.981 | 0.084 | 0.753 | 0.382 | 0.248 | 0.000 | 0.607 | 0.000 |

| Problem 58 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F(-1) | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 406 | 406 | 1418 | 474 | 547 | 370 | 0 | 2478 | 0 |
| N.S. | 1 | 1.00 | 3.49 | 1.17 | 1.35 | 0.91 | 0.00 | 6.10 | 0.00 |
| time (sec) | N/A | 1.365 | 12.503 | 1.444 | 0.409 | 0.273 | 0.000 | 1.002 | 0.000 |

| Problem 59 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 353 | 353 | 730 | 386 | 497 | 298 | 0 | 1547 | 0 |
| N.S. | 1 | 1.00 | 2.07 | 1.09 | 1.41 | 0.84 | 0.00 | 4.38 | 0.00 |
| time (sec) | N/A | 0.978 | 1.293 | 0.806 | 0.408 | 0.262 | 0.000 | 0.810 | 0.000 |

| Problem 60 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 304 | 304 | 251 | 294 | 424 | 245 | 0 | 848 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 0.97 | 1.39 | 0.81 | 0.00 | 2.79 | 0.00 |
| time (sec) | N/A | 0.757 | 0.244 | 0.694 | 0.365 | 0.263 | 0.000 | 0.550 | 0.000 |

| Problem 61 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 304 | 304 | 251 | 294 | 424 | 245 | 0 | 848 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 0.97 | 1.39 | 0.81 | 0.00 | 2.79 | 0.00 |
| time (sec) | N/A | 0.723 | 0.008 | 0.470 | 0.384 | 0.268 | 0.000 | 0.548 | 0.000 |

| Problem 62 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 353 | 353 | 730 | 386 | 497 | 298 | 0 | 1547 | 0 |
| N.S. | 1 | 1.00 | 2.07 | 1.09 | 1.41 | 0.84 | 0.00 | 4.38 | 0.00 |
| time (sec) | N/A | 0.842 | 0.011 | 0.482 | 0.354 | 0.271 | 0.000 | 0.819 | 0.000 |

| Problem 63 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F(-1) | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 406 | 406 | 1418 | 474 | 547 | 370 | 0 | 2478 | 0 |
| N.S. | 1 | 1.00 | 3.49 | 1.17 | 1.35 | 0.91 | 0.00 | 6.10 | 0.00 |
| time (sec) | N/A | 0.958 | 0.012 | 1.228 | 0.374 | 0.261 | 0.000 | 1.006 | 0.000 |

| Problem 64 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F(-1) | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 407 | 407 | 1332 | 470 | 551 | 406 | 0 | 2435 | 0 |
| N.S. | 1 | 1.00 | 3.27 | 1.15 | 1.35 | 1.00 | 0.00 | 5.98 | 0.00 |
| time (sec) | N/A | 1.238 | 14.414 | 3.852 | 0.386 | 0.276 | 0.000 | 1.164 | 0.000 |

| Problem 65 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 351 | 351 | 693 | 376 | 503 | 316 | 0 | 1515 | 0 |
| N.S. | 1 | 1.00 | 1.97 | 1.07 | 1.43 | 0.90 | 0.00 | 4.32 | 0.00 |
| time (sec) | N/A | 0.903 | 1.937 | 0.760 | 0.377 | 0.268 | 0.000 | 0.895 | 0.000 |

| Problem 66 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 299 | 252 | 286 | 429 | 244 | 0 | 826 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 0.96 | 1.43 | 0.82 | 0.00 | 2.76 | 0.00 |
| time (sec) | N/A | 0.675 | 0.380 | 0.762 | 0.363 | 0.267 | 0.000 | 0.622 | 0.000 |

| Problem 67 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 299 | 252 | 286 | 429 | 244 | 0 | 826 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 0.96 | 1.43 | 0.82 | 0.00 | 2.76 | 0.00 |
| time (sec) | N/A | 0.647 | 0.140 | 0.473 | 0.371 | 0.297 | 0.000 | 0.611 | 0.000 |

| Problem 68 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 351 | 351 | 693 | 376 | 503 | 316 | 0 | 1515 | 0 |
| N.S. | 1 | 1.00 | 1.97 | 1.07 | 1.43 | 0.90 | 0.00 | 4.32 | 0.00 |
| time (sec) | N/A | 0.770 | 1.536 | 0.531 | 0.390 | 0.269 | 0.000 | 0.854 | 0.000 |

| Problem 69 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F(-1) | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 407 | 407 | 1332 | 470 | 551 | 406 | 0 | 2435 | 0 |
| N.S. | 1 | 1.00 | 3.27 | 1.15 | 1.35 | 1.00 | 0.00 | 5.98 | 0.00 |
| time (sec) | N/A | 0.872 | 6.027 | 3.559 | 0.397 | 0.296 | 0.000 | 1.136 | 0.000 |

| Problem 70 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 267 | 267 | 250 | 0 | 0 | 186 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.94 | 0.00 | 0.00 | 0.70 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.483 | 0.770 | 0.000 | 0.000 | 0.098 | 0.000 | 0.000 | 0.000 |

| Problem 71 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 205 | 247 | 150 | 181 | 889 | 294 | 646 | 350 | 448 |
| N.S. | 1 | 1.20 | 0.73 | 0.88 | 4.34 | 1.43 | 3.15 | 1.71 | 2.19 |
| time (sec) | N/A | 1.221 | 1.782 | 1.773 | 0.301 | 0.254 | 0.638 | 0.308 | 25.722 |

| Problem 72 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 165 | 127 | 224 | 505 | 183 | 391 | 231 | 290 |
| N.S. | 1 | 1.09 | 0.84 | 1.48 | 3.34 | 1.21 | 2.59 | 1.53 | 1.92 |
| time (sec) | N/A | 0.718 | 0.839 | 1.439 | 0.266 | 0.245 | 0.453 | 0.293 | 24.406 |

| Problem 73 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 105 | 86 | 128 | 243 | 100 | 216 | 137 | 145 |
| N.S. | 1 | 1.02 | 0.83 | 1.24 | 2.36 | 0.97 | 2.10 | 1.33 | 1.41 |
| time (sec) | N/A | 0.451 | 0.486 | 1.315 | 0.256 | 0.248 | 0.338 | 0.295 | 24.426 |

| Problem 74 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 50 | 71 | 64 | 86 | 46 | 85 | 69 | 58 |
| N.S. | 1 | 0.98 | 1.39 | 1.25 | 1.69 | 0.90 | 1.67 | 1.35 | 1.14 |
| time (sec) | N/A | 0.244 | 0.138 | 1.070 | 0.225 | 0.237 | 0.250 | 0.277 | 24.002 |

| Problem 75 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 121 | 121 | 100 | 172 | 275 | 120 | 0 | 6279 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 1.42 | 2.27 | 0.99 | 0.00 | 51.89 | 0.00 |
| time (sec) | N/A | 0.401 | 0.332 | 0.780 | 0.329 | 0.238 | 0.000 | 0.508 | 0.000 |

| Problem 76 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 168 | 139 | 245 | 302 | 182 | 0 | 66726 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 1.46 | 1.80 | 1.08 | 0.00 | 397.18 | 0.00 |
| time (sec) | N/A | 0.469 | 1.220 | 1.020 | 0.367 | 0.254 | 0.000 | 2.454 | 0.000 |

| Problem 77 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 221 | 221 | 181 | 318 | 337 | 306 | 0 | 118262 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 1.44 | 1.52 | 1.38 | 0.00 | 535.12 | 0.00 |
| time (sec) | N/A | 0.538 | 2.686 | 1.514 | 0.446 | 0.267 | 0.000 | 3.479 | 0.000 |

| Problem 78 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 270 | 270 | 300 | 386 | 387 | 444 | 0 | 166374 | 0 |
| N.S. | 1 | 1.00 | 1.11 | 1.43 | 1.43 | 1.64 | 0.00 | 616.20 | 0.00 |
| time (sec) | N/A | 0.611 | 1.861 | 1.710 | 0.663 | 0.274 | 0.000 | 4.747 | 0.000 |

| Problem 79 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 162 | 162 | 155 | 0 | 0 | 136 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 0.00 | 0.00 | 0.84 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.391 | 3.785 | 0.000 | 0.000 | 0.090 | 0.000 | 0.000 | 0.000 |

| Problem 80 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 131 | 132 | 146 | 735 | 466 | 1231 | 224 | 349 |
| N.S. | 1 | 1.00 | 1.01 | 1.11 | 5.61 | 3.56 | 9.40 | 1.71 | 2.66 |
| time (sec) | N/A | 0.360 | 1.254 | 2.628 | 0.273 | 0.256 | 0.906 | 0.318 | 23.662 |

| Problem 81 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 105 | 106 | 121 | 442 | 308 | 835 | 153 | 329 |
| N.S. | 1 | 1.00 | 1.01 | 1.15 | 4.21 | 2.93 | 7.95 | 1.46 | 3.13 |
| time (sec) | N/A | 0.312 | 0.663 | 1.810 | 0.254 | 0.248 | 0.632 | 0.313 | 1.204 |

| Problem 82 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 79 | 77 | 79 | 232 | 180 | 493 | 94 | 179 |
| N.S. | 1 | 1.00 | 0.97 | 1.00 | 2.94 | 2.28 | 6.24 | 1.19 | 2.27 |
| time (sec) | N/A | 0.298 | 0.411 | 1.458 | 0.240 | 0.257 | 0.471 | 0.312 | 23.133 |

| Problem 83 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 54 | 46 | 96 | 85 | 238 | 48 | 57 |
| N.S. | 1 | 1.00 | 1.02 | 0.87 | 1.81 | 1.60 | 4.49 | 0.91 | 1.08 |
| time (sec) | N/A | 0.223 | 0.597 | 0.971 | 0.233 | 0.254 | 0.325 | 0.293 | 23.133 |

| Problem 84 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 65 | 107 | 162 | 73 | 0 | 669 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 1.37 | 2.08 | 0.94 | 0.00 | 8.58 | 0.00 |
| time (sec) | N/A | 0.318 | 0.158 | 0.921 | 0.294 | 0.252 | 0.000 | 0.329 | 0.000 |

| Problem 85 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 104 | 81 | 155 | 171 | 112 | 0 | 3218 | 0 |
| N.S. | 1 | 1.00 | 0.78 | 1.49 | 1.64 | 1.08 | 0.00 | 30.94 | 0.00 |
| time (sec) | N/A | 0.352 | 0.465 | 1.234 | 0.312 | 0.256 | 0.000 | 0.755 | 0.000 |

| Problem 86 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 127 | 127 | 105 | 193 | 206 | 213 | 0 | 5600 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 1.52 | 1.62 | 1.68 | 0.00 | 44.09 | 0.00 |
| time (sec) | N/A | 0.387 | 0.941 | 1.533 | 0.381 | 0.263 | 0.000 | 0.489 | 0.000 |

| Problem 87 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | B | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 158 | 123 | 230 | 258 | 349 | 0 | 8508 | 0 |
| N.S. | 1 | 1.00 | 0.78 | 1.46 | 1.63 | 2.21 | 0.00 | 53.85 | 0.00 |
| time (sec) | N/A | 0.427 | 1.795 | 2.017 | 0.456 | 0.267 | 0.000 | 0.568 | 0.000 |

| Problem 88 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 407 | 407 | 376 | 0 | 0 | 280 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 0.00 | 0.00 | 0.69 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.644 | 1.040 | 0.000 | 0.000 | 0.103 | 0.000 | 0.000 | 0.000 |

| Problem 89 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 330 | 330 | 238 | 259 | 1339 | 471 | 1098 | 531 | 816 |
| N.S. | 1 | 1.00 | 0.72 | 0.78 | 4.06 | 1.43 | 3.33 | 1.61 | 2.47 |
| time (sec) | N/A | 0.611 | 3.576 | 2.979 | 0.310 | 0.264 | 1.134 | 0.360 | 27.718 |

| Problem 90 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 259 | 259 | 369 | 205 | 766 | 296 | 690 | 351 | 516 |
| N.S. | 1 | 1.00 | 1.42 | 0.79 | 2.96 | 1.14 | 2.66 | 1.36 | 1.99 |
| time (sec) | N/A | 0.517 | 1.510 | 2.246 | 0.270 | 0.260 | 0.840 | 0.338 | 2.999 |

| Problem 91 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 184 | 184 | 127 | 152 | 375 | 166 | 382 | 209 | 249 |
| N.S. | 1 | 1.00 | 0.69 | 0.83 | 2.04 | 0.90 | 2.08 | 1.14 | 1.35 |
| time (sec) | N/A | 0.409 | 0.844 | 2.140 | 0.261 | 0.252 | 0.583 | 0.317 | 1.063 |

| Problem 92 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 109 | 94 | 92 | 139 | 76 | 163 | 106 | 99 |
| N.S. | 1 | 1.00 | 0.86 | 0.84 | 1.28 | 0.70 | 1.50 | 0.97 | 0.91 |
| time (sec) | N/A | 0.282 | 0.284 | 1.703 | 0.246 | 0.239 | 0.395 | 0.302 | 23.105 |

| Problem 93 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 185 | 185 | 154 | 258 | 414 | 182 | 0 | 46675 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 1.39 | 2.24 | 0.98 | 0.00 | 252.30 | 0.00 |
| time (sec) | N/A | 0.525 | 0.419 | 1.236 | 0.353 | 0.252 | 0.000 | 1.762 | 0.000 |

| Problem 94 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 257 | 257 | 213 | 370 | 445 | 272 | 0 | 1014406 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 1.44 | 1.73 | 1.06 | 0.00 | 3947.11 | 0.00 |
| time (sec) | N/A | 0.625 | 1.588 | 1.543 | 0.414 | 0.267 | 0.000 | 47.389 | 0.000 |

| Problem 95 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 338 | 338 | 279 | 480 | 480 | 456 | 0 | 1737414 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 1.42 | 1.42 | 1.35 | 0.00 | 5140.28 | 0.00 |
| time (sec) | N/A | 0.719 | 3.969 | 2.226 | 0.566 | 0.285 | 0.000 | 99.433 | 0.000 |

| Problem 96 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 413 | 413 | 457 | 585 | 530 | 653 | 0 | 2449286 | 0 |
| N.S. | 1 | 1.00 | 1.11 | 1.42 | 1.28 | 1.58 | 0.00 | 5930.47 | 0.00 |
| time (sec) | N/A | 0.859 | 2.545 | 2.894 | 0.850 | 0.295 | 0.000 | 207.311 | 0.000 |

| Problem 97 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 22 | 22 | 20 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 1.10 | 1.10 | 1.00 | 1.10 | 1.10 |
| time (sec) | N/A | 0.457 | 11.866 | 0.315 | 0.683 | 0.237 | 7.834 | 0.298 | 23.982 |

| Problem 98 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 333 | 376 | 837 | 1295 | 1548 | 1367 | 0 | 0 | 0 |
| N.S. | 1 | 1.13 | 2.51 | 3.89 | 4.65 | 4.11 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.669 | 1.334 | 2.605 | 0.525 | 0.329 | 0.000 | 0.000 | 0.000 |

| Problem 99 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 254 | 278 | 330 | 847 | 929 | 925 | 0 | 0 | 0 |
| N.S. | 1 | 1.09 | 1.30 | 3.33 | 3.66 | 3.64 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.233 | 0.867 | 1.793 | 0.375 | 0.317 | 0.000 | 0.000 | 0.000 |

| Problem 100 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 171 | 181 | 221 | 479 | 513 | 562 | 0 | 0 | 0 |
| N.S. | 1 | 1.06 | 1.29 | 2.80 | 3.00 | 3.29 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.811 | 0.838 | 1.432 | 0.297 | 0.301 | 0.000 | 0.000 | 0.000 |

| Problem 101 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 94 | 176 | 177 | 200 | 277 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.87 | 1.88 | 2.13 | 2.95 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.454 | 0.321 | 1.174 | 0.332 | 0.270 | 0.000 | 0.000 | 0.000 |

| Problem 102 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 228 | 22 | 19 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 11.40 | 1.10 | 0.95 | 1.10 | 1.10 |
| time (sec) | N/A | 0.527 | 9.746 | 0.776 | 0.489 | 0.252 | 0.464 | 0.516 | 23.173 |

| Problem 103 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 340 | 33 | 20 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 17.00 | 1.65 | 1.00 | 1.10 | 1.10 |
| time (sec) | N/A | 0.667 | 4.660 | 0.828 | 0.887 | 0.254 | 0.711 | 2.962 | 22.990 |

| Problem 104 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 18 | 16 | 18 | 18 | 15 | 18 | 18 |
| N.S. | 1 | 1.00 | 1.12 | 1.00 | 1.12 | 1.12 | 0.94 | 1.12 | 1.12 |
| time (sec) | N/A | 0.201 | 1.293 | 0.285 | 0.574 | 0.256 | 1.544 | 0.308 | 22.993 |

| Problem 105 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 155 | 210 | 833 | 921 | 3242 | 856 | 0 | 0 | 0 |
| N.S. | 1 | 1.35 | 5.37 | 5.94 | 20.92 | 5.52 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.876 | 6.651 | 1.648 | 0.675 | 0.277 | 0.000 | 0.000 | 0.000 |

| Problem 106 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 127 | 164 | 520 | 581 | 1953 | 599 | 0 | 0 | 0 |
| N.S. | 1 | 1.29 | 4.09 | 4.57 | 15.38 | 4.72 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.692 | 6.548 | 1.631 | 0.461 | 0.270 | 0.000 | 0.000 | 0.000 |

| Problem 107 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 120 | 268 | 305 | 645 | 384 | 0 | 0 | 0 |
| N.S. | 1 | 1.24 | 2.76 | 3.14 | 6.65 | 3.96 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.491 | 6.528 | 1.580 | 0.451 | 0.262 | 0.000 | 0.000 | 0.000 |

| Problem 108 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 45 | 82 | 64 | 292 | 97 | 104 | 1151 | 67 |
| N.S. | 1 | 1.10 | 2.00 | 1.56 | 7.12 | 2.37 | 2.54 | 28.07 | 1.63 |
| time (sec) | N/A | 0.255 | 0.587 | 1.260 | 0.361 | 0.250 | 0.218 | 0.552 | 22.928 |

| Problem 109 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 18 | 16 | 581 | 18 | 14 | 18 | 18 |
| N.S. | 1 | 1.00 | 1.12 | 1.00 | 36.31 | 1.12 | 0.88 | 1.12 | 1.12 |
| time (sec) | N/A | 0.203 | 6.458 | 0.655 | 0.573 | 0.243 | 0.412 | 0.361 | 22.934 |

| Problem 110 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 18 | 16 | 813 | 29 | 15 | 18 | 18 |
| N.S. | 1 | 1.00 | 1.12 | 1.00 | 50.81 | 1.81 | 0.94 | 1.12 | 1.12 |
| time (sec) | N/A | 0.207 | 2.788 | 0.651 | 1.299 | 0.236 | 0.643 | 2.334 | 22.766 |

| Problem 111 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 24 | 22 | 24 | 26 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.09 | 1.00 | 1.09 | 1.18 |
| time (sec) | N/A | 0.293 | 41.082 | 0.279 | 1.130 | 0.242 | 7.042 | 0.324 | 23.005 |

| Problem 112 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 416 | 742 | 966 | 1673 | 7004 | 2770 | 0 | 0 | 0 |
| N.S. | 1 | 1.78 | 2.32 | 4.02 | 16.84 | 6.66 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.954 | 8.092 | 1.494 | 3.264 | 0.354 | 0.000 | 0.000 | 0.000 |

| Problem 113 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 308 | 532 | 528 | 1056 | 3887 | 1742 | 0 | 0 | 0 |
| N.S. | 1 | 1.73 | 1.71 | 3.43 | 12.62 | 5.66 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.947 | 5.417 | 1.324 | 1.369 | 0.319 | 0.000 | 0.000 | 0.000 |

| Problem 114 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 324 | 471 | 546 | 1938 | 970 | 0 | 0 | 0 |
| N.S. | 1 | 1.81 | 2.63 | 3.05 | 10.83 | 5.42 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.347 | 7.512 | 1.200 | 0.532 | 0.306 | 0.000 | 0.000 | 0.000 |

| Problem 115 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 176 | 260 | 246 | 762 | 454 | 0 | 0 | 0 |
| N.S. | 1 | 1.63 | 2.41 | 2.28 | 7.06 | 4.20 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.685 | 1.944 | 0.879 | 0.353 | 0.266 | 0.000 | 0.000 | 0.000 |

| Problem 116 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 1793 | 24 | 20 | 24 | 26 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 81.50 | 1.09 | 0.91 | 1.09 | 1.18 |
| time (sec) | N/A | 0.296 | 40.315 | 0.519 | 2.819 | 0.251 | 1.034 | 4.946 | 24.045 |

| Problem 117 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 2289 | 35 | 22 | 0 | 26 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 104.05 | 1.59 | 1.00 | 0.00 | 1.18 |
| time (sec) | N/A | 0.291 | 45.766 | 0.500 | 9.880 | 0.259 | 3.860 | 0.000 | 24.381 |

| Problem 118 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F(-1) | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 406 | 406 | 1445 | 476 | 547 | 341 | 0 | 2468 | 0 |
| N.S. | 1 | 1.00 | 3.56 | 1.17 | 1.35 | 0.84 | 0.00 | 6.08 | 0.00 |
| time (sec) | N/A | 1.177 | 12.760 | 1.435 | 0.375 | 0.272 | 0.000 | 0.966 | 0.000 |

| Problem 119 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 353 | 353 | 751 | 384 | 499 | 280 | 0 | 1540 | 0 |
| N.S. | 1 | 1.00 | 2.13 | 1.09 | 1.41 | 0.79 | 0.00 | 4.36 | 0.00 |
| time (sec) | N/A | 0.885 | 1.747 | 0.720 | 0.376 | 0.269 | 0.000 | 0.812 | 0.000 |

| Problem 120 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 304 | 304 | 251 | 296 | 424 | 235 | 0 | 844 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 0.97 | 1.39 | 0.77 | 0.00 | 2.78 | 0.00 |
| time (sec) | N/A | 0.741 | 0.242 | 0.700 | 0.348 | 0.272 | 0.000 | 0.586 | 0.000 |

| Problem 121 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 304 | 304 | 251 | 296 | 424 | 235 | 0 | 844 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 0.97 | 1.39 | 0.77 | 0.00 | 2.78 | 0.00 |
| time (sec) | N/A | 0.716 | 0.049 | 0.494 | 0.386 | 0.262 | 0.000 | 0.610 | 0.000 |

| Problem 122 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 353 | 353 | 751 | 384 | 499 | 280 | 0 | 1540 | 0 |
| N.S. | 1 | 1.00 | 2.13 | 1.09 | 1.41 | 0.79 | 0.00 | 4.36 | 0.00 |
| time (sec) | N/A | 0.841 | 1.346 | 0.484 | 0.368 | 0.265 | 0.000 | 0.825 | 0.000 |

| Problem 123 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F(-1) | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 406 | 406 | 1445 | 476 | 547 | 341 | 0 | 2468 | 0 |
| N.S. | 1 | 1.00 | 3.56 | 1.17 | 1.35 | 0.84 | 0.00 | 6.08 | 0.00 |
| time (sec) | N/A | 0.971 | 6.015 | 1.187 | 0.395 | 0.295 | 0.000 | 0.975 | 0.000 |

| Problem 124 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F(-1) | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 228 | 228 | 140 | 251 | 285 | 347 | 0 | 1379 | 0 |
| N.S. | 1 | 1.00 | 0.61 | 1.10 | 1.25 | 1.52 | 0.00 | 6.05 | 0.00 |
| time (sec) | N/A | 0.732 | 1.384 | 3.809 | 0.339 | 0.265 | 0.000 | 1.049 | 0.000 |

| Problem 125 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 200 | 200 | 140 | 206 | 264 | 249 | 0 | 857 | 0 |
| N.S. | 1 | 1.00 | 0.70 | 1.03 | 1.32 | 1.24 | 0.00 | 4.28 | 0.00 |
| time (sec) | N/A | 0.615 | 0.993 | 0.691 | 0.332 | 0.274 | 0.000 | 0.842 | 0.000 |

| Problem 126 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 174 | 174 | 120 | 159 | 219 | 175 | 0 | 460 | 0 |
| N.S. | 1 | 1.00 | 0.69 | 0.91 | 1.26 | 1.01 | 0.00 | 2.64 | 0.00 |
| time (sec) | N/A | 0.506 | 0.868 | 0.697 | 0.335 | 0.260 | 0.000 | 0.553 | 0.000 |

| Problem 127 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 174 | 174 | 120 | 159 | 219 | 175 | 0 | 460 | 0 |
| N.S. | 1 | 1.00 | 0.69 | 0.91 | 1.26 | 1.01 | 0.00 | 2.64 | 0.00 |
| time (sec) | N/A | 0.488 | 0.019 | 0.474 | 0.320 | 0.261 | 0.000 | 0.569 | 0.000 |

| Problem 128 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 200 | 200 | 140 | 206 | 264 | 249 | 0 | 857 | 0 |
| N.S. | 1 | 1.00 | 0.70 | 1.03 | 1.32 | 1.24 | 0.00 | 4.28 | 0.00 |
| time (sec) | N/A | 0.548 | 0.047 | 0.570 | 0.336 | 0.274 | 0.000 | 0.791 | 0.000 |

| Problem 129 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F(-1) | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 228 | 228 | 140 | 251 | 285 | 347 | 0 | 1379 | 0 |
| N.S. | 1 | 1.00 | 0.61 | 1.10 | 1.25 | 1.52 | 0.00 | 6.05 | 0.00 |
| time (sec) | N/A | 0.615 | 0.141 | 3.529 | 0.340 | 0.281 | 0.000 | 1.068 | 0.000 |

| Problem 130 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F(-1) | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 615 | 615 | 2177 | 719 | 826 | 521 | 0 | 3706 | 0 |
| N.S. | 1 | 1.00 | 3.54 | 1.17 | 1.34 | 0.85 | 0.00 | 6.03 | 0.00 |
| time (sec) | N/A | 1.587 | 17.080 | 12.006 | 0.400 | 0.310 | 0.000 | 1.741 | 0.000 |

| Problem 131 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 534 | 534 | 1126 | 580 | 760 | 427 | 0 | 2314 | 0 |
| N.S. | 1 | 1.00 | 2.11 | 1.09 | 1.42 | 0.80 | 0.00 | 4.33 | 0.00 |
| time (sec) | N/A | 1.252 | 3.535 | 0.806 | 0.371 | 0.296 | 0.000 | 1.229 | 0.000 |

| Problem 132 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 459 | 459 | 372 | 447 | 680 | 356 | 0 | 1270 | 0 |
| N.S. | 1 | 1.00 | 0.81 | 0.97 | 1.48 | 0.78 | 0.00 | 2.77 | 0.00 |
| time (sec) | N/A | 1.019 | 0.353 | 0.846 | 0.379 | 0.319 | 0.000 | 0.821 | 0.000 |

| Problem 133 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 459 | 459 | 372 | 447 | 680 | 356 | 0 | 1270 | 0 |
| N.S. | 1 | 1.00 | 0.81 | 0.97 | 1.48 | 0.78 | 0.00 | 2.77 | 0.00 |
| time (sec) | N/A | 0.980 | 0.275 | 0.484 | 0.386 | 0.274 | 0.000 | 0.837 | 0.000 |

| Problem 134 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 534 | 534 | 1126 | 580 | 760 | 427 | 0 | 2314 | 0 |
| N.S. | 1 | 1.00 | 2.11 | 1.09 | 1.42 | 0.80 | 0.00 | 4.33 | 0.00 |
| time (sec) | N/A | 1.169 | 2.460 | 0.534 | 0.378 | 0.300 | 0.000 | 1.233 | 0.000 |

| Problem 135 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F(-1) | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 615 | 615 | 2177 | 719 | 826 | 521 | 0 | 3706 | 0 |
| N.S. | 1 | 1.00 | 3.54 | 1.17 | 1.34 | 0.85 | 0.00 | 6.03 | 0.00 |
| time (sec) | N/A | 1.373 | 6.019 | 11.522 | 0.385 | 0.292 | 0.000 | 1.708 | 0.000 |

| Problem 136 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 273 | 273 | 245 | 0 | 0 | 188 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.90 | 0.00 | 0.00 | 0.69 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.506 | 0.278 | 0.000 | 0.000 | 0.098 | 0.000 | 0.000 | 0.000 |

| Problem 137 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 260 | 325 | 158 | 187 | 967 | 378 | 935 | 361 | 576 |
| N.S. | 1 | 1.25 | 0.61 | 0.72 | 3.72 | 1.45 | 3.60 | 1.39 | 2.22 |
| time (sec) | N/A | 0.964 | 1.806 | 2.129 | 0.272 | 0.267 | 0.875 | 0.361 | 1.533 |

| Problem 138 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 196 | 243 | 135 | 148 | 549 | 238 | 602 | 241 | 366 |
| N.S. | 1 | 1.24 | 0.69 | 0.76 | 2.80 | 1.21 | 3.07 | 1.23 | 1.87 |
| time (sec) | N/A | 0.818 | 0.895 | 1.588 | 0.249 | 0.264 | 0.627 | 0.326 | 25.016 |

| Problem 139 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 133 | 89 | 109 | 263 | 130 | 320 | 145 | 202 |
| N.S. | 1 | 0.99 | 0.66 | 0.81 | 1.96 | 0.97 | 2.39 | 1.08 | 1.51 |
| time (sec) | N/A | 0.405 | 0.467 | 1.359 | 0.239 | 0.250 | 0.432 | 0.333 | 23.746 |

| Problem 140 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 80 | 75 | 68 | 92 | 58 | 138 | 75 | 94 |
| N.S. | 1 | 1.11 | 1.04 | 0.94 | 1.28 | 0.81 | 1.92 | 1.04 | 1.31 |
| time (sec) | N/A | 0.311 | 0.144 | 1.003 | 0.226 | 0.239 | 0.318 | 0.323 | 0.388 |

| Problem 141 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 110 | 178 | 281 | 124 | 0 | 6046 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 1.38 | 2.18 | 0.96 | 0.00 | 46.87 | 0.00 |
| time (sec) | N/A | 0.425 | 0.246 | 0.969 | 0.307 | 0.242 | 0.000 | 0.537 | 0.000 |

| Problem 142 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 179 | 151 | 256 | 308 | 183 | 0 | 63510 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 1.43 | 1.72 | 1.02 | 0.00 | 354.80 | 0.00 |
| time (sec) | N/A | 0.483 | 1.715 | 1.325 | 0.361 | 0.256 | 0.000 | 2.633 | 0.000 |

| Problem 143 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 231 | 231 | 197 | 329 | 343 | 309 | 0 | 111694 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 1.42 | 1.48 | 1.34 | 0.00 | 483.52 | 0.00 |
| time (sec) | N/A | 0.570 | 4.011 | 1.694 | 0.461 | 0.270 | 0.000 | 3.423 | 0.000 |

| Problem 144 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 287 | 287 | 316 | 404 | 393 | 455 | 0 | 157526 | 0 |
| N.S. | 1 | 1.00 | 1.10 | 1.41 | 1.37 | 1.59 | 0.00 | 548.87 | 0.00 |
| time (sec) | N/A | 0.622 | 2.303 | 2.200 | 0.645 | 0.283 | 0.000 | 4.641 | 0.000 |

| Problem 145 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 419 | 419 | 409 | 0 | 0 | 280 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 0.00 | 0.00 | 0.67 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.685 | 0.956 | 0.000 | 0.000 | 0.104 | 0.000 | 0.000 | 0.000 |

| Problem 146 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 330 | 330 | 563 | 253 | 1339 | 527 | 1098 | 531 | 816 |
| N.S. | 1 | 1.00 | 1.71 | 0.77 | 4.06 | 1.60 | 3.33 | 1.61 | 2.47 |
| time (sec) | N/A | 0.648 | 3.710 | 3.183 | 0.321 | 0.269 | 1.155 | 0.390 | 28.902 |

| Problem 147 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 259 | 259 | 195 | 205 | 766 | 342 | 690 | 351 | 516 |
| N.S. | 1 | 1.00 | 0.75 | 0.79 | 2.96 | 1.32 | 2.66 | 1.36 | 1.99 |
| time (sec) | N/A | 0.515 | 2.246 | 2.440 | 0.274 | 0.259 | 0.827 | 0.344 | 2.954 |

| Problem 148 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 184 | 184 | 252 | 144 | 375 | 193 | 382 | 209 | 295 |
| N.S. | 1 | 1.00 | 1.37 | 0.78 | 2.04 | 1.05 | 2.08 | 1.14 | 1.60 |
| time (sec) | N/A | 0.402 | 0.915 | 2.375 | 0.246 | 0.247 | 0.573 | 0.347 | 0.941 |

| Problem 149 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 109 | 110 | 91 | 139 | 91 | 163 | 106 | 119 |
| N.S. | 1 | 1.00 | 1.01 | 0.83 | 1.28 | 0.83 | 1.50 | 0.97 | 1.09 |
| time (sec) | N/A | 0.289 | 0.355 | 1.817 | 0.232 | 0.248 | 0.401 | 0.327 | 0.498 |

| Problem 150 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 185 | 185 | 154 | 257 | 413 | 182 | 0 | 44961 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 1.39 | 2.23 | 0.98 | 0.00 | 243.03 | 0.00 |
| time (sec) | N/A | 0.521 | 0.412 | 1.282 | 0.361 | 0.245 | 0.000 | 1.724 | 0.000 |

| Problem 151 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 257 | 257 | 212 | 372 | 446 | 264 | 0 | 1022022 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 1.45 | 1.74 | 1.03 | 0.00 | 3976.74 | 0.00 |
| time (sec) | N/A | 0.623 | 2.005 | 1.612 | 0.435 | 0.256 | 0.000 | 49.034 | 0.000 |

| Problem 152 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 338 | 338 | 283 | 478 | 481 | 438 | 0 | 1708998 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 1.41 | 1.42 | 1.30 | 0.00 | 5056.21 | 0.00 |
| time (sec) | N/A | 0.733 | 2.962 | 2.207 | 0.583 | 0.296 | 0.000 | 98.985 | 0.000 |

| Problem 153 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 413 | 413 | 451 | 588 | 531 | 640 | 0 | 2478918 | 0 |
| N.S. | 1 | 1.00 | 1.09 | 1.42 | 1.29 | 1.55 | 0.00 | 6002.22 | 0.00 |
| time (sec) | N/A | 0.861 | 2.488 | 2.901 | 0.830 | 0.309 | 0.000 | 198.186 | 0.000 |

| Problem 154 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 285 | 285 | 255 | 0 | 0 | 190 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 0.00 | 0.00 | 0.67 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.536 | 2.274 | 0.000 | 0.000 | 0.092 | 0.000 | 0.000 | 0.000 |

| Problem 155 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 233 | 233 | 153 | 185 | 1033 | 546 | 1334 | 359 | 576 |
| N.S. | 1 | 1.00 | 0.66 | 0.79 | 4.43 | 2.34 | 5.73 | 1.54 | 2.47 |
| time (sec) | N/A | 0.509 | 1.374 | 3.443 | 0.298 | 0.270 | 1.522 | 0.422 | 26.370 |

| Problem 156 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 181 | 132 | 148 | 602 | 349 | 857 | 241 | 366 |
| N.S. | 1 | 1.00 | 0.73 | 0.82 | 3.33 | 1.93 | 4.73 | 1.33 | 2.02 |
| time (sec) | N/A | 0.432 | 2.449 | 2.541 | 0.262 | 0.253 | 1.126 | 0.381 | 1.547 |

| Problem 157 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 91 | 111 | 303 | 194 | 461 | 145 | 202 |
| N.S. | 1 | 1.00 | 0.71 | 0.86 | 2.35 | 1.50 | 3.57 | 1.12 | 1.57 |
| time (sec) | N/A | 0.334 | 0.545 | 2.213 | 0.260 | 0.265 | 0.812 | 0.349 | 1.023 |

| Problem 158 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 63 | 69 | 119 | 87 | 201 | 75 | 84 |
| N.S. | 1 | 1.00 | 0.82 | 0.90 | 1.55 | 1.13 | 2.61 | 0.97 | 1.09 |
| time (sec) | N/A | 0.250 | 0.509 | 1.674 | 0.235 | 0.252 | 0.551 | 0.321 | 0.901 |

| Problem 159 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 110 | 178 | 281 | 126 | 0 | 6046 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 1.38 | 2.18 | 0.98 | 0.00 | 46.87 | 0.00 |
| time (sec) | N/A | 0.435 | 0.296 | 1.548 | 0.331 | 0.249 | 0.000 | 0.575 | 0.000 |

| Problem 160 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 179 | 189 | 256 | 308 | 198 | 0 | 63798 | 0 |
| N.S. | 1 | 1.00 | 1.06 | 1.43 | 1.72 | 1.11 | 0.00 | 356.41 | 0.00 |
| time (sec) | N/A | 0.503 | 1.001 | 1.960 | 0.369 | 0.255 | 0.000 | 3.428 | 0.000 |

| Problem 161 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 235 | 235 | 239 | 329 | 343 | 348 | 0 | 111694 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 1.40 | 1.46 | 1.48 | 0.00 | 475.29 | 0.00 |
| time (sec) | N/A | 0.576 | 1.109 | 2.724 | 0.459 | 0.266 | 0.000 | 3.469 | 0.000 |

| Problem 162 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F(-1) | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 287 | 287 | 554 | 404 | 393 | 524 | 0 | 157526 | 0 |
| N.S. | 1 | 1.00 | 1.93 | 1.41 | 1.37 | 1.83 | 0.00 | 548.87 | 0.00 |
| time (sec) | N/A | 0.653 | 5.301 | 3.589 | 0.670 | 0.290 | 0.000 | 4.836 | 0.000 |

| Problem 163 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 24 | 22 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.09 | 1.00 | 1.09 | 1.09 |
| time (sec) | N/A | 0.566 | 12.321 | 0.336 | 0.767 | 0.247 | 24.325 | 0.315 | 25.145 |

| Problem 164 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 307 | 372 | 2918 | 1335 | 1654 | 1453 | 0 | 0 | 0 |
| N.S. | 1 | 1.21 | 9.50 | 4.35 | 5.39 | 4.73 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.755 | 6.448 | 3.237 | 0.546 | 0.353 | 0.000 | 0.000 | 0.000 |

| Problem 165 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 246 | 298 | 1956 | 908 | 979 | 984 | 0 | 0 | 0 |
| N.S. | 1 | 1.21 | 7.95 | 3.69 | 3.98 | 4.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.346 | 6.407 | 2.372 | 0.422 | 0.327 | 0.000 | 0.000 | 0.000 |

| Problem 166 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 207 | 564 | 544 | 529 | 594 | 0 | 0 | 0 |
| N.S. | 1 | 1.14 | 3.12 | 3.01 | 2.92 | 3.28 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.990 | 3.184 | 1.872 | 0.358 | 0.292 | 0.000 | 0.000 | 0.000 |

| Problem 167 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 114 | 134 | 139 | 249 | 223 | 292 | 0 | 0 | 0 |
| N.S. | 1 | 1.18 | 1.22 | 2.18 | 1.96 | 2.56 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.623 | 0.795 | 1.680 | 0.297 | 0.274 | 0.000 | 0.000 | 0.000 |

| Problem 168 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 231 | 24 | 20 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 10.50 | 1.09 | 0.91 | 1.09 | 1.09 |
| time (sec) | N/A | 0.639 | 0.893 | 0.871 | 0.538 | 0.240 | 0.962 | 0.330 | 26.137 |

| Problem 169 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 343 | 35 | 22 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 15.59 | 1.59 | 1.00 | 1.09 | 1.09 |
| time (sec) | N/A | 0.755 | 2.878 | 1.000 | 0.884 | 0.250 | 1.261 | 2.356 | 26.525 |

| Problem 170 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 24 | 22 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.09 | 1.00 | 1.09 | 1.09 |
| time (sec) | N/A | 0.642 | 13.473 | 0.286 | 1.180 | 0.247 | 22.089 | 0.337 | 25.346 |

| Problem 171 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 329 | 798 | 1056 | 17589 | 1233 | 0 | 0 | 0 |
| N.S. | 1 | 1.10 | 2.67 | 3.53 | 58.83 | 4.12 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.707 | 1.935 | 4.395 | 4.509 | 0.336 | 0.000 | 0.000 | 0.000 |

| Problem 172 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 216 | 232 | 539 | 649 | 10994 | 797 | 0 | 0 | 0 |
| N.S. | 1 | 1.07 | 2.50 | 3.00 | 50.90 | 3.69 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.173 | 1.487 | 2.565 | 1.309 | 0.317 | 0.000 | 0.000 | 0.000 |

| Problem 173 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 144 | 310 | 332 | 3199 | 448 | 0 | 0 | 0 |
| N.S. | 1 | 1.04 | 2.23 | 2.39 | 23.01 | 3.22 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.772 | 4.431 | 1.780 | 1.141 | 0.278 | 0.000 | 0.000 | 0.000 |

| Problem 174 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | B | A | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 58 | 104 | 124 | 2110 | 95 | 0 | 1759 | 162 |
| N.S. | 1 | 1.00 | 1.79 | 2.14 | 36.38 | 1.64 | 0.00 | 30.33 | 2.79 |
| time (sec) | N/A | 0.440 | 1.089 | 1.309 | 0.301 | 0.261 | 0.000 | 0.862 | 26.264 |

| Problem 175 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 1438 | 24 | 20 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 65.36 | 1.09 | 0.91 | 1.09 | 1.09 |
| time (sec) | N/A | 0.680 | 3.793 | 0.409 | 0.658 | 0.264 | 0.629 | 1.332 | 26.129 |

| Problem 176 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 1678 | 35 | 22 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 76.27 | 1.59 | 1.00 | 1.09 | 1.09 |
| time (sec) | N/A | 0.841 | 4.046 | 0.436 | 1.431 | 0.249 | 0.960 | 6.955 | 25.787 |

| Problem 177 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 18 | 16 | 18 | 18 | 15 | 18 | 18 |
| N.S. | 1 | 1.00 | 1.12 | 1.00 | 1.12 | 1.12 | 0.94 | 1.12 | 1.12 |
| time (sec) | N/A | 0.212 | 14.811 | 0.299 | 0.764 | 0.235 | 2.200 | 0.343 | 25.137 |

| Problem 178 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 302 | 407 | 1632 | 1877 | 7158 | 1751 | 0 | 0 | 0 |
| N.S. | 1 | 1.35 | 5.40 | 6.22 | 23.70 | 5.80 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.195 | 7.087 | 1.376 | 3.676 | 0.299 | 0.000 | 0.000 | 0.000 |

| Problem 179 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 256 | 321 | 1032 | 1203 | 3958 | 1139 | 0 | 0 | 0 |
| N.S. | 1 | 1.25 | 4.03 | 4.70 | 15.46 | 4.45 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.576 | 6.996 | 1.334 | 1.224 | 0.274 | 0.000 | 0.000 | 0.000 |

| Problem 180 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 197 | 548 | 644 | 1966 | 657 | 0 | 0 | 0 |
| N.S. | 1 | 1.17 | 3.26 | 3.83 | 11.70 | 3.91 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.953 | 6.834 | 1.184 | 0.542 | 0.264 | 0.000 | 0.000 | 0.000 |

| Problem 181 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 124 | 240 | 281 | 830 | 339 | 0 | 0 | 0 |
| N.S. | 1 | 1.14 | 2.20 | 2.58 | 7.61 | 3.11 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.518 | 6.253 | 0.836 | 0.384 | 0.262 | 0.000 | 0.000 | 0.000 |

| Problem 182 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 18 | 16 | 1636 | 18 | 14 | 18 | 18 |
| N.S. | 1 | 1.00 | 1.12 | 1.00 | 102.25 | 1.12 | 0.88 | 1.12 | 1.12 |
| time (sec) | N/A | 0.208 | 9.055 | 0.464 | 2.644 | 0.249 | 0.441 | 0.390 | 24.340 |

| Problem 183 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 18 | 16 | 2124 | 29 | 15 | 18 | 18 |
| N.S. | 1 | 1.00 | 1.12 | 1.00 | 132.75 | 1.81 | 0.94 | 1.12 | 1.12 |
| time (sec) | N/A | 0.213 | 11.518 | 0.469 | 10.314 | 0.245 | 0.695 | 2.389 | 24.609 |

| Problem 184 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F(-1) | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 407 | 407 | 1332 | 470 | 551 | 376 | 0 | 2435 | 0 |
| N.S. | 1 | 1.00 | 3.27 | 1.15 | 1.35 | 0.92 | 0.00 | 5.98 | 0.00 |
| time (sec) | N/A | 1.044 | 8.890 | 3.804 | 0.357 | 0.284 | 0.000 | 1.093 | 0.000 |

| Problem 185 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 351 | 351 | 693 | 376 | 503 | 294 | 0 | 1515 | 0 |
| N.S. | 1 | 1.00 | 1.97 | 1.07 | 1.43 | 0.84 | 0.00 | 4.32 | 0.00 |
| time (sec) | N/A | 0.831 | 1.515 | 0.722 | 0.335 | 0.276 | 0.000 | 0.824 | 0.000 |

| Problem 186 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 299 | 252 | 286 | 429 | 233 | 0 | 826 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 0.96 | 1.43 | 0.78 | 0.00 | 2.76 | 0.00 |
| time (sec) | N/A | 0.654 | 0.385 | 0.730 | 0.344 | 0.253 | 0.000 | 0.575 | 0.000 |

| Problem 187 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 299 | 252 | 286 | 429 | 233 | 0 | 826 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 0.96 | 1.43 | 0.78 | 0.00 | 2.76 | 0.00 |
| time (sec) | N/A | 0.640 | 0.086 | 0.505 | 0.347 | 0.262 | 0.000 | 0.601 | 0.000 |

| Problem 188 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 351 | 351 | 693 | 376 | 503 | 294 | 0 | 1515 | 0 |
| N.S. | 1 | 1.00 | 1.97 | 1.07 | 1.43 | 0.84 | 0.00 | 4.32 | 0.00 |
| time (sec) | N/A | 0.773 | 1.044 | 0.553 | 0.366 | 0.272 | 0.000 | 0.828 | 0.000 |

| Problem 189 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F(-1) | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 407 | 407 | 1332 | 470 | 551 | 376 | 0 | 2435 | 0 |
| N.S. | 1 | 1.00 | 3.27 | 1.15 | 1.35 | 0.92 | 0.00 | 5.98 | 0.00 |
| time (sec) | N/A | 0.878 | 6.021 | 3.579 | 0.378 | 0.275 | 0.000 | 1.145 | 0.000 |

| Problem 190 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F(-1) | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 615 | 615 | 2130 | 716 | 826 | 548 | 0 | 3716 | 0 |
| N.S. | 1 | 1.00 | 3.46 | 1.16 | 1.34 | 0.89 | 0.00 | 6.04 | 0.00 |
| time (sec) | N/A | 1.665 | 16.707 | 11.953 | 0.405 | 0.318 | 0.000 | 1.711 | 0.000 |

| Problem 191 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 534 | 534 | 1088 | 583 | 760 | 446 | 0 | 2319 | 0 |
| N.S. | 1 | 1.00 | 2.04 | 1.09 | 1.42 | 0.84 | 0.00 | 4.34 | 0.00 |
| time (sec) | N/A | 1.370 | 2.760 | 0.885 | 0.378 | 0.285 | 0.000 | 1.210 | 0.000 |

| Problem 192 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 459 | 459 | 374 | 444 | 680 | 365 | 0 | 1270 | 0 |
| N.S. | 1 | 1.00 | 0.81 | 0.97 | 1.48 | 0.80 | 0.00 | 2.77 | 0.00 |
| time (sec) | N/A | 1.034 | 0.417 | 0.847 | 0.351 | 0.281 | 0.000 | 0.796 | 0.000 |

| Problem 193 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 459 | 459 | 374 | 444 | 680 | 365 | 0 | 1270 | 0 |
| N.S. | 1 | 1.00 | 0.81 | 0.97 | 1.48 | 0.80 | 0.00 | 2.77 | 0.00 |
| time (sec) | N/A | 1.006 | 0.208 | 0.505 | 0.351 | 0.273 | 0.000 | 0.804 | 0.000 |

| Problem 194 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 534 | 534 | 1088 | 583 | 760 | 446 | 0 | 2319 | 0 |
| N.S. | 1 | 1.00 | 2.04 | 1.09 | 1.42 | 0.84 | 0.00 | 4.34 | 0.00 |
| time (sec) | N/A | 1.187 | 1.888 | 0.519 | 0.375 | 0.283 | 0.000 | 1.236 | 0.000 |

| Problem 195 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F(-1) | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 615 | 615 | 2130 | 716 | 826 | 548 | 0 | 3716 | 0 |
| N.S. | 1 | 1.00 | 3.46 | 1.16 | 1.34 | 0.89 | 0.00 | 6.04 | 0.00 |
| time (sec) | N/A | 1.349 | 6.016 | 11.655 | 0.386 | 0.316 | 0.000 | 1.738 | 0.000 |

| Problem 196 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F(-1) | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 407 | 407 | 257 | 477 | 561 | 445 | 0 | 2434 | 0 |
| N.S. | 1 | 1.00 | 0.63 | 1.17 | 1.38 | 1.09 | 0.00 | 5.98 | 0.00 |
| time (sec) | N/A | 1.060 | 3.418 | 35.515 | 0.361 | 0.298 | 0.000 | 2.644 | 0.000 |

| Problem 197 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 351 | 351 | 259 | 383 | 511 | 326 | 0 | 1514 | 0 |
| N.S. | 1 | 1.00 | 0.74 | 1.09 | 1.46 | 0.93 | 0.00 | 4.31 | 0.00 |
| time (sec) | N/A | 0.871 | 2.787 | 0.811 | 0.343 | 0.276 | 0.000 | 1.787 | 0.000 |

| Problem 198 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 299 | 256 | 293 | 439 | 242 | 0 | 826 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.98 | 1.47 | 0.81 | 0.00 | 2.76 | 0.00 |
| time (sec) | N/A | 0.663 | 1.742 | 0.832 | 0.332 | 0.265 | 0.000 | 1.161 | 0.000 |

| Problem 199 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 299 | 256 | 293 | 439 | 242 | 0 | 826 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.98 | 1.47 | 0.81 | 0.00 | 2.76 | 0.00 |
| time (sec) | N/A | 0.641 | 0.179 | 0.534 | 0.347 | 0.269 | 0.000 | 1.168 | 0.000 |

| Problem 200 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 351 | 351 | 259 | 383 | 511 | 326 | 0 | 1514 | 0 |
| N.S. | 1 | 1.00 | 0.74 | 1.09 | 1.46 | 0.93 | 0.00 | 4.31 | 0.00 |
| time (sec) | N/A | 0.768 | 0.395 | 0.578 | 0.349 | 0.272 | 0.000 | 1.790 | 0.000 |

| Problem 201 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | C | A | F(-1) | C | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 407 | 407 | 257 | 477 | 561 | 445 | 0 | 2434 | 0 |
| N.S. | 1 | 1.00 | 0.63 | 1.17 | 1.38 | 1.09 | 0.00 | 5.98 | 0.00 |
| time (sec) | N/A | 0.869 | 0.307 | 34.412 | 0.377 | 0.285 | 0.000 | 2.667 | 0.000 |

| Problem 202 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 138 | 104 | 150 | 0 | 244 | 0 | 0 | 0 |
| N.S. | 1 | 1.23 | 0.93 | 1.34 | 0.00 | 2.18 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.751 | 0.796 | 3.778 | 0.000 | 0.275 | 0.000 | 0.000 | 0.000 |

| Problem 203 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 102 | 72 | 112 | 0 | 162 | 0 | 0 | 0 |
| N.S. | 1 | 1.23 | 0.87 | 1.35 | 0.00 | 1.95 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.616 | 0.138 | 2.604 | 0.000 | 0.259 | 0.000 | 0.000 | 0.000 |

| Problem 204 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F(-2) | A | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 33 | 60 | 0 | 45 | 0 | 206 | 56 |
| N.S. | 1 | 1.00 | 1.00 | 1.82 | 0.00 | 1.36 | 0.00 | 6.24 | 1.70 |
| time (sec) | N/A | 0.321 | 0.042 | 2.530 | 0.000 | 0.255 | 0.000 | 0.304 | 24.447 |

| Problem 205 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | A | A | B | B | F | F | F(-1) |
| verified | N/A | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 180 | 0 | 159 | 240 | 3740 | 508 | 0 | 0 | 0 |
| N.S. | 1 | 0.00 | 0.88 | 1.33 | 20.78 | 2.82 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.000 | 0.473 | 4.583 | 0.638 | 0.281 | 0.000 | 0.000 | 0.000 |

| Problem 206 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | A | A | B | B | F | F | F(-1) |
| verified | N/A | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 0 | 111 | 170 | 2842 | 370 | 0 | 0 | 0 |
| N.S. | 1 | 0.00 | 1.05 | 1.60 | 26.81 | 3.49 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.000 | 0.418 | 4.300 | 0.500 | 0.296 | 0.000 | 0.000 | 0.000 |

| Problem 207 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 93 | 62 | 109 | 1718 | 203 | 0 | 0 | 0 |
| N.S. | 1 | 1.27 | 0.85 | 1.49 | 23.53 | 2.78 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.928 | 0.191 | 4.210 | 0.363 | 0.274 | 0.000 | 0.000 | 0.000 |

| Problem 208 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 16 | 20 | 22 | 22 | 20 | 22 | 24 |
| N.S. | 1 | 1.00 | 1.14 | 1.43 | 1.57 | 1.57 | 1.43 | 1.57 | 1.71 |
| time (sec) | N/A | 0.183 | 3.724 | 0.340 | 0.341 | 0.264 | 30.374 | 0.288 | 24.671 |

| Problem 209 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 195 | 157 | 625 | 803 | 1410 | 0 | 0 | 0 |
| N.S. | 1 | 1.23 | 0.99 | 3.96 | 5.08 | 8.92 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.822 | 0.156 | 1.240 | 0.413 | 0.311 | 0.000 | 0.000 | 0.000 |

| Problem 210 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 132 | 155 | 126 | 432 | 497 | 974 | 0 | 0 | 0 |
| N.S. | 1 | 1.17 | 0.95 | 3.27 | 3.77 | 7.38 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.637 | 0.151 | 1.215 | 0.382 | 0.295 | 0.000 | 0.000 | 0.000 |

| Problem 211 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 113 | 100 | 266 | 284 | 594 | 0 | 0 | 0 |
| N.S. | 1 | 1.18 | 1.04 | 2.77 | 2.96 | 6.19 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.495 | 0.089 | 1.186 | 0.388 | 0.303 | 0.000 | 0.000 | 0.000 |

| Problem 212 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 73 | 70 | 123 | 115 | 310 | 0 | 0 | 148 |
| N.S. | 1 | 1.11 | 1.06 | 1.86 | 1.74 | 4.70 | 0.00 | 0.00 | 2.24 |
| time (sec) | N/A | 0.331 | 0.180 | 1.026 | 0.370 | 0.263 | 0.000 | 0.000 | 25.461 |

| Problem 213 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 16 | 20 | 22 | 22 | 19 | 22 | 24 |
| N.S. | 1 | 1.00 | 1.14 | 1.43 | 1.57 | 1.57 | 1.36 | 1.57 | 1.71 |
| time (sec) | N/A | 0.184 | 5.281 | 0.536 | 0.428 | 0.246 | 0.594 | 0.295 | 25.425 |

| Problem 214 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 16 | 20 | 22 | 33 | 20 | 22 | 24 |
| N.S. | 1 | 1.00 | 1.14 | 1.43 | 1.57 | 2.36 | 1.43 | 1.57 | 1.71 |
| time (sec) | N/A | 0.185 | 3.484 | 0.534 | 0.549 | 0.240 | 0.964 | 2.223 | 26.122 |

| Problem 215 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 22 | 24 | 27 | 22 | 24 | 26 |
| N.S. | 1 | 1.00 | 1.10 | 1.10 | 1.20 | 1.35 | 1.10 | 1.20 | 1.30 |
| time (sec) | N/A | 0.460 | 11.880 | 0.552 | 0.680 | 0.246 | 121.927 | 0.299 | 25.232 |

| Problem 216 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 275 | 299 | 557 | 901 | 934 | 1075 | 0 | 0 | 0 |
| N.S. | 1 | 1.09 | 2.03 | 3.28 | 3.40 | 3.91 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.236 | 1.573 | 2.507 | 0.517 | 0.306 | 0.000 | 0.000 | 0.000 |

| Problem 217 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 186 | 197 | 315 | 512 | 516 | 656 | 0 | 0 | 0 |
| N.S. | 1 | 1.06 | 1.69 | 2.75 | 2.77 | 3.53 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.811 | 0.926 | 2.193 | 0.394 | 0.288 | 0.000 | 0.000 | 0.000 |

| Problem 218 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | F | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 103 | 213 | 180 | 0 | 331 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.07 | 1.75 | 0.00 | 3.21 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.456 | 0.498 | 1.386 | 0.000 | 0.278 | 0.000 | 0.000 | 0.000 |

| Problem 219 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 22 | 210 | 27 | 20 | 24 | 26 |
| N.S. | 1 | 1.00 | 1.10 | 1.10 | 10.50 | 1.35 | 1.00 | 1.20 | 1.30 |
| time (sec) | N/A | 0.529 | 3.402 | 0.885 | 0.658 | 0.251 | 1.318 | 0.551 | 24.449 |

| Problem 220 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 22 | 270 | 38 | 22 | 24 | 26 |
| N.S. | 1 | 1.00 | 1.10 | 1.10 | 13.50 | 1.90 | 1.10 | 1.20 | 1.30 |
| time (sec) | N/A | 0.651 | 7.189 | 0.904 | 0.867 | 0.243 | 2.020 | 3.020 | 24.921 |

| Problem 221 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 33 | 0 | 24 | 26 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.50 | 0.00 | 1.09 | 1.18 |
| time (sec) | N/A | 0.549 | 13.664 | 0.677 | 0.685 | 0.234 | 0.000 | 0.314 | 25.789 |

| Problem 222 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 251 | 282 | 1731 | 650 | 692 | 1134 | 0 | 0 | 0 |
| N.S. | 1 | 1.12 | 6.90 | 2.59 | 2.76 | 4.52 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.282 | 6.528 | 3.112 | 0.438 | 0.321 | 0.000 | 0.000 | 0.000 |

| Problem 223 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 184 | 195 | 518 | 388 | 383 | 688 | 0 | 0 | 0 |
| N.S. | 1 | 1.06 | 2.82 | 2.11 | 2.08 | 3.74 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.912 | 6.579 | 2.634 | 0.428 | 0.287 | 0.000 | 0.000 | 0.000 |

| Problem 224 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 126 | 134 | 179 | 146 | 346 | 0 | 0 | 0 |
| N.S. | 1 | 1.10 | 1.17 | 1.56 | 1.27 | 3.01 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.595 | 0.713 | 2.572 | 0.336 | 0.280 | 0.000 | 0.000 | 0.000 |

| Problem 225 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 182 | 33 | 0 | 24 | 26 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 8.27 | 1.50 | 0.00 | 1.09 | 1.18 |
| time (sec) | N/A | 0.632 | 0.886 | 1.004 | 0.486 | 0.241 | 0.000 | 0.325 | 25.531 |

| Problem 226 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 242 | 44 | 0 | 24 | 26 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 11.00 | 2.00 | 0.00 | 1.09 | 1.18 |
| time (sec) | N/A | 0.726 | 2.961 | 1.030 | 0.772 | 0.245 | 0.000 | 2.803 | 25.569 |

| Problem 227 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 22 | 22 | 20 | 22 | 26 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 1.10 | 1.10 | 1.00 | 1.10 | 1.30 |
| time (sec) | N/A | 0.314 | 10.153 | 0.307 | 0.394 | 0.233 | 33.987 | 0.310 | 25.956 |

| Problem 228 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 247 | 301 | 578 | 1242 | 1794 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.22 | 2.34 | 5.03 | 7.26 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.966 | 1.231 | 1.332 | 0.521 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 229 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 197 | 223 | 350 | 816 | 1078 | 1786 | 0 | 0 | 0 |
| N.S. | 1 | 1.13 | 1.78 | 4.14 | 5.47 | 9.07 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.708 | 1.079 | 1.303 | 0.506 | 0.379 | 0.000 | 0.000 | 0.000 |

| Problem 230 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 127 | 140 | 213 | 469 | 599 | 1098 | 0 | 0 | 0 |
| N.S. | 1 | 1.10 | 1.68 | 3.69 | 4.72 | 8.65 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.498 | 0.744 | 1.208 | 0.422 | 0.323 | 0.000 | 0.000 | 0.000 |

| Problem 231 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 73 | 141 | 208 | 269 | 554 | 0 | 0 | 0 |
| N.S. | 1 | 1.03 | 1.99 | 2.93 | 3.79 | 7.80 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.317 | 0.210 | 1.010 | 0.411 | 0.318 | 0.000 | 0.000 | 0.000 |

| Problem 232 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 22 | 22 | 19 | 22 | 26 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 1.10 | 1.10 | 0.95 | 1.10 | 1.30 |
| time (sec) | N/A | 0.243 | 5.046 | 0.720 | 0.545 | 0.243 | 0.614 | 0.396 | 25.325 |

| Problem 233 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 22 | 33 | 20 | 22 | 26 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 1.10 | 1.65 | 1.00 | 1.10 | 1.30 |
| time (sec) | N/A | 0.241 | 6.558 | 0.727 | 0.985 | 0.243 | 1.003 | 4.083 | 24.844 |

| Problem 234 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 24 | 22 | 24 | 26 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.09 | 1.00 | 1.09 | 1.18 |
| time (sec) | N/A | 0.326 | 23.115 | 0.287 | 0.538 | 0.243 | 118.630 | 0.331 | 25.445 |

| Problem 235 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 350 | 387 | 739 | 1158 | 3257 | 1753 | 0 | 0 | 0 |
| N.S. | 1 | 1.11 | 2.11 | 3.31 | 9.31 | 5.01 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.950 | 6.389 | 1.922 | 0.992 | 0.359 | 0.000 | 0.000 | 0.000 |

| Problem 236 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 226 | 260 | 593 | 556 | 1638 | 1067 | 0 | 0 | 0 |
| N.S. | 1 | 1.15 | 2.62 | 2.46 | 7.25 | 4.72 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.705 | 6.277 | 1.608 | 0.502 | 0.318 | 0.000 | 0.000 | 0.000 |

| Problem 237 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | B | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 128 | 517 | 235 | 0 | 434 | 0 | 0 | 0 |
| N.S. | 1 | 0.98 | 3.95 | 1.79 | 0.00 | 3.31 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.319 | 3.826 | 1.200 | 0.000 | 0.294 | 0.000 | 0.000 | 0.000 |

| Problem 238 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 677 | 24 | 20 | 24 | 26 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 30.77 | 1.09 | 0.91 | 1.09 | 1.18 |
| time (sec) | N/A | 0.306 | 22.399 | 0.609 | 0.797 | 0.257 | 1.114 | 5.153 | 24.817 |

| Problem 239 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 1008 | 35 | 22 | 3 | 26 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 45.82 | 1.59 | 1.00 | 0.14 | 1.18 |
| time (sec) | N/A | 0.310 | 21.434 | 0.634 | 1.570 | 0.245 | 2.029 | 4.246 | 25.028 |

| Problem 240 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 24 | 0 | 24 | 26 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.09 | 0.00 | 1.09 | 1.18 |
| time (sec) | N/A | 0.368 | 26.593 | 0.316 | 0.535 | 0.241 | 0.000 | 0.327 | 24.774 |

| Problem 241 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 325 | 350 | 1526 | 1223 | 5165 | 3475 | 0 | 0 | 0 |
| N.S. | 1 | 1.08 | 4.70 | 3.76 | 15.89 | 10.69 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.043 | 6.997 | 1.829 | 1.623 | 0.420 | 0.000 | 0.000 | 0.000 |

| Problem 242 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 201 | 228 | 880 | 632 | 2528 | 1995 | 0 | 0 | 0 |
| N.S. | 1 | 1.13 | 4.38 | 3.14 | 12.58 | 9.93 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.730 | 6.925 | 1.500 | 0.608 | 0.363 | 0.000 | 0.000 | 0.000 |

| Problem 243 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 141 | 138 | 224 | 270 | 1028 | 942 | 0 | 0 | 0 |
| N.S. | 1 | 0.98 | 1.59 | 1.91 | 7.29 | 6.68 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.326 | 1.715 | 1.065 | 0.460 | 0.311 | 0.000 | 0.000 | 0.000 |

| Problem 244 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 2040 | 24 | 20 | 24 | 26 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 92.73 | 1.09 | 0.91 | 1.09 | 1.18 |
| time (sec) | N/A | 0.301 | 19.660 | 0.626 | 2.698 | 0.290 | 2.586 | 0.757 | 24.859 |

| Problem 245 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 2702 | 35 | 22 | 0 | 26 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 122.82 | 1.59 | 1.00 | 0.00 | 1.18 |
| time (sec) | N/A | 0.316 | 18.156 | 0.642 | 9.087 | 0.370 | 4.941 | 0.000 | 24.831 |

| Problem 246 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 22 | 22 | 20 | 22 | 24 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 1.10 | 1.10 | 1.00 | 1.10 | 1.20 |
| time (sec) | N/A | 0.291 | 3.089 | 0.293 | 0.639 | 0.247 | 3.972 | 0.342 | 23.762 |

| Problem 247 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 227 | 247 | 428 | 767 | 2944 | 1190 | 0 | 0 | 0 |
| N.S. | 1 | 1.09 | 1.89 | 3.38 | 12.97 | 5.24 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.811 | 1.153 | 4.669 | 0.588 | 0.309 | 0.000 | 0.000 | 0.000 |

| Problem 248 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 159 | 169 | 256 | 463 | 1770 | 783 | 0 | 0 | 0 |
| N.S. | 1 | 1.06 | 1.61 | 2.91 | 11.13 | 4.92 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.557 | 0.796 | 2.236 | 0.439 | 0.288 | 0.000 | 0.000 | 0.000 |

| Problem 249 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 99 | 174 | 227 | 0 | 446 | 0 | 0 | 0 |
| N.S. | 1 | 1.02 | 1.79 | 2.34 | 0.00 | 4.60 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.363 | 1.842 | 1.230 | 0.000 | 0.291 | 0.000 | 0.000 | 0.000 |

| Problem 250 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 29 | 29 | 93 | 67 | 259 | 60 | 0 | 1267 | 78 |
| N.S. | 1 | 1.00 | 3.21 | 2.31 | 8.93 | 2.07 | 0.00 | 43.69 | 2.69 |
| time (sec) | N/A | 0.229 | 0.132 | 0.615 | 0.330 | 0.253 | 0.000 | 0.610 | 27.600 |

| Problem 251 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 349 | 22 | 19 | 22 | 24 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 17.45 | 1.10 | 0.95 | 1.10 | 1.20 |
| time (sec) | N/A | 0.258 | 12.683 | 0.494 | 0.573 | 0.259 | 0.408 | 0.818 | 25.357 |

| Problem 252 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 494 | 33 | 20 | 22 | 24 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 24.70 | 1.65 | 1.00 | 1.10 | 1.20 |
| time (sec) | N/A | 0.267 | 19.628 | 0.538 | 0.851 | 0.252 | 0.630 | 9.173 | 25.429 |

| Problem 253 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 18 | 16 | 18 | 18 | 15 | 18 | 18 |
| N.S. | 1 | 1.00 | 1.12 | 1.00 | 1.12 | 1.12 | 0.94 | 1.12 | 1.12 |
| time (sec) | N/A | 0.203 | 3.147 | 0.297 | 0.539 | 0.233 | 0.980 | 0.567 | 24.711 |

| Problem 254 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 128 | 151 | 424 | 356 | 1363 | 373 | 0 | 0 | 0 |
| N.S. | 1 | 1.18 | 3.31 | 2.78 | 10.65 | 2.91 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.669 | 6.537 | 1.700 | 0.396 | 0.255 | 0.000 | 0.000 | 0.000 |

| Problem 255 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 111 | 276 | 199 | 418 | 210 | 0 | 0 | 0 |
| N.S. | 1 | 1.16 | 2.88 | 2.07 | 4.35 | 2.19 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.473 | 6.397 | 1.582 | 0.386 | 0.242 | 0.000 | 0.000 | 0.000 |

| Problem 256 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 42 | 76 | 52 | 237 | 53 | 65 | 203 | 52 |
| N.S. | 1 | 1.05 | 1.90 | 1.30 | 5.92 | 1.32 | 1.62 | 5.08 | 1.30 |
| time (sec) | N/A | 0.251 | 0.415 | 1.182 | 0.307 | 0.241 | 0.156 | 0.454 | 25.918 |

| Problem 257 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 18 | 16 | 388 | 18 | 14 | 18 | 18 |
| N.S. | 1 | 1.00 | 1.12 | 1.00 | 24.25 | 1.12 | 0.88 | 1.12 | 1.12 |
| time (sec) | N/A | 0.200 | 5.082 | 0.672 | 0.483 | 0.233 | 0.360 | 0.580 | 25.445 |

| Problem 258 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 18 | 16 | 523 | 29 | 15 | 18 | 18 |
| N.S. | 1 | 1.00 | 1.12 | 1.00 | 32.69 | 1.81 | 0.94 | 1.12 | 1.12 |
| time (sec) | N/A | 0.200 | 6.351 | 0.674 | 0.905 | 0.227 | 0.535 | 7.916 | 25.268 |

| Problem 259 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 24 | 22 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.09 | 1.00 | 1.09 | 1.09 |
| time (sec) | N/A | 0.570 | 29.048 | 0.294 | 1.057 | 0.239 | 32.349 | 0.608 | 26.245 |

| Problem 260 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 228 | 244 | 532 | 677 | 11010 | 896 | 0 | 0 | 0 |
| N.S. | 1 | 1.07 | 2.33 | 2.97 | 48.29 | 3.93 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.192 | 1.676 | 3.573 | 1.315 | 0.314 | 0.000 | 0.000 | 0.000 |

| Problem 261 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | F | B | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 149 | 362 | 345 | 0 | 511 | 0 | 0 | 0 |
| N.S. | 1 | 1.03 | 2.50 | 2.38 | 0.00 | 3.52 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.772 | 3.482 | 2.273 | 0.000 | 0.280 | 0.000 | 0.000 | 0.000 |

| Problem 262 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | B | A | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 107 | 123 | 2123 | 93 | 0 | 2330 | 151 |
| N.S. | 1 | 1.00 | 1.91 | 2.20 | 37.91 | 1.66 | 0.00 | 41.61 | 2.70 |
| time (sec) | N/A | 0.446 | 0.864 | 1.583 | 0.323 | 0.256 | 0.000 | 1.110 | 1.535 |

| Problem 263 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 1268 | 24 | 20 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 57.64 | 1.09 | 0.91 | 1.09 | 1.09 |
| time (sec) | N/A | 0.658 | 4.213 | 0.421 | 0.744 | 0.253 | 0.634 | 1.565 | 27.958 |

| Problem 264 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 1412 | 35 | 22 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 64.18 | 1.59 | 1.00 | 1.09 | 1.09 |
| time (sec) | N/A | 0.809 | 4.485 | 0.455 | 1.298 | 0.256 | 1.095 | 18.138 | 28.434 |

| Problem 265 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 24 | 22 | 24 | 26 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.09 | 1.00 | 1.09 | 1.18 |
| time (sec) | N/A | 0.357 | 24.666 | 0.299 | 0.656 | 0.240 | 118.021 | 0.308 | 26.275 |

| Problem 266 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 469 | 507 | 694 | 1866 | 5709 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.08 | 1.48 | 3.98 | 12.17 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.114 | 3.668 | 2.047 | 1.766 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 267 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 343 | 381 | 473 | 1152 | 3202 | 1705 | 0 | 0 | 0 |
| N.S. | 1 | 1.11 | 1.38 | 3.36 | 9.34 | 4.97 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.897 | 1.429 | 1.861 | 0.853 | 0.365 | 0.000 | 0.000 | 0.000 |

| Problem 268 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 219 | 254 | 317 | 568 | 1590 | 1035 | 0 | 0 | 0 |
| N.S. | 1 | 1.16 | 1.45 | 2.59 | 7.26 | 4.73 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.706 | 2.806 | 1.651 | 0.467 | 0.319 | 0.000 | 0.000 | 0.000 |

| Problem 269 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 121 | 212 | 160 | 800 | 366 | 0 | 0 | 0 |
| N.S. | 1 | 1.07 | 1.88 | 1.42 | 7.08 | 3.24 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.321 | 1.124 | 1.162 | 0.435 | 0.273 | 0.000 | 0.000 | 0.000 |

| Problem 270 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 603 | 24 | 20 | 24 | 26 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 27.41 | 1.09 | 0.91 | 1.09 | 1.18 |
| time (sec) | N/A | 0.292 | 19.488 | 0.617 | 0.747 | 0.254 | 1.189 | 4.947 | 25.476 |

| Problem 271 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 931 | 35 | 22 | 3 | 26 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 42.32 | 1.59 | 1.00 | 0.14 | 1.18 |
| time (sec) | N/A | 0.291 | 19.832 | 0.594 | 1.353 | 0.262 | 1.971 | 4.099 | 26.026 |

| Problem 272 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 26 | 26 | 0 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 1.08 | 1.08 | 0.00 | 1.08 | 1.08 |
| time (sec) | N/A | 0.345 | 3.211 | 0.289 | 0.481 | 0.243 | 0.000 | 0.338 | 25.709 |

| Problem 273 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 118 | 161 | 285 | 687 | 2360 | 1635 | 0 | 0 | 0 |
| N.S. | 1 | 1.36 | 2.42 | 5.82 | 20.00 | 13.86 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.751 | 2.291 | 1.994 | 0.506 | 0.350 | 0.000 | 0.000 | 0.000 |

| Problem 274 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 113 | 277 | 351 | 772 | 950 | 0 | 0 | 0 |
| N.S. | 1 | 1.28 | 3.15 | 3.99 | 8.77 | 10.80 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.565 | 1.876 | 1.898 | 0.410 | 0.317 | 0.000 | 0.000 | 0.000 |

| Problem 275 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | B | B | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 44 | 32 | 72 | 308 | 75 | 0 | 10271 | 55 |
| N.S. | 1 | 1.26 | 0.91 | 2.06 | 8.80 | 2.14 | 0.00 | 293.46 | 1.57 |
| time (sec) | N/A | 0.312 | 0.536 | 1.524 | 0.319 | 0.257 | 0.000 | 2.843 | 27.600 |

| Problem 276 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 687 | 26 | 22 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 28.62 | 1.08 | 0.92 | 1.08 | 1.08 |
| time (sec) | N/A | 0.282 | 9.216 | 0.591 | 0.781 | 0.243 | 2.587 | 0.308 | 26.544 |

| Problem 277 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 1024 | 37 | 24 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 42.67 | 1.54 | 1.00 | 1.08 | 1.08 |
| time (sec) | N/A | 0.277 | 9.968 | 0.635 | 1.731 | 0.251 | 4.804 | 1.527 | 26.685 |

| Problem 278 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 26 | 26 | 0 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 1.08 | 1.08 | 0.00 | 1.08 | 1.08 |
| time (sec) | N/A | 0.360 | 27.554 | 0.282 | 0.951 | 0.248 | 0.000 | 0.344 | 26.971 |

| Problem 279 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 601 | 623 | 907 | 1613 | 8032 | 3173 | 0 | 0 | 0 |
| N.S. | 1 | 1.04 | 1.51 | 2.68 | 13.36 | 5.28 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.280 | 8.188 | 2.444 | 3.615 | 0.430 | 0.000 | 0.000 | 0.000 |

| Problem 280 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 305 | 335 | 889 | 802 | 3814 | 1801 | 0 | 0 | 0 |
| N.S. | 1 | 1.10 | 2.91 | 2.63 | 12.50 | 5.90 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.136 | 7.929 | 2.014 | 0.987 | 0.342 | 0.000 | 0.000 | 0.000 |

| Problem 281 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 154 | 171 | 520 | 267 | 1481 | 621 | 0 | 0 | 0 |
| N.S. | 1 | 1.11 | 3.38 | 1.73 | 9.62 | 4.03 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.376 | 6.158 | 1.428 | 0.579 | 0.284 | 0.000 | 0.000 | 0.000 |

| Problem 282 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 3697 | 26 | 22 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 154.04 | 1.08 | 0.92 | 1.08 | 1.08 |
| time (sec) | N/A | 0.342 | 23.773 | 0.647 | 3.320 | 0.293 | 7.687 | 119.208 | 26.186 |

| Problem 283 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 4757 | 37 | 24 | 0 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 198.21 | 1.54 | 1.00 | 0.00 | 1.08 |
| time (sec) | N/A | 0.351 | 29.139 | 0.596 | 11.085 | 0.334 | 14.706 | 0.000 | 26.580 |

| Problem 284 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 22 | 22 | 0 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 1.10 | 1.10 | 0.00 | 1.10 | 1.10 |
| time (sec) | N/A | 0.843 | 46.748 | 0.183 | 0.815 | 0.249 | 0.000 | 0.338 | 26.526 |

| Problem 285 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 387 | 417 | 672 | 0 | 3940 | 1747 | 0 | 0 | 0 |
| N.S. | 1 | 1.08 | 1.74 | 0.00 | 10.18 | 4.51 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.218 | 7.307 | 0.000 | 0.768 | 0.364 | 0.000 | 0.000 | 0.000 |

| Problem 286 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 235 | 235 | 613 | 429 | 2205 | 1237 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.61 | 1.83 | 9.38 | 5.26 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.677 | 6.898 | 1.461 | 0.526 | 0.333 | 0.000 | 0.000 | 0.000 |

| Problem 287 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 126 | 282 | 519 | 1173 | 531 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.24 | 4.12 | 9.31 | 4.21 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.345 | 3.693 | 0.698 | 0.493 | 0.262 | 0.000 | 0.000 | 0.000 |

| Problem 288 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 1872 | 22 | 19 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 93.60 | 1.10 | 0.95 | 1.10 | 1.10 |
| time (sec) | N/A | 0.544 | 54.244 | 0.316 | 1.001 | 0.256 | 8.697 | 1.199 | 26.042 |

| Problem 289 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 1884 | 22 | 20 | 22 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 94.20 | 1.10 | 1.00 | 1.10 | 1.10 |
| time (sec) | N/A | 0.575 | 23.742 | 0.309 | 1.089 | 0.246 | 11.405 | 1.821 | 26.392 |

| Problem 290 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 24 | 22 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.09 | 1.00 | 1.09 | 1.09 |
| time (sec) | N/A | 0.326 | 6.619 | 0.168 | 0.634 | 0.242 | 9.714 | 0.365 | 25.501 |

| Problem 291 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 166 | 418 | 489 | 3446 | 892 | 0 | 0 | 0 |
| N.S. | 1 | 1.19 | 3.01 | 3.52 | 24.79 | 6.42 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.778 | 6.600 | 4.888 | 0.489 | 0.313 | 0.000 | 0.000 | 0.000 |

| Problem 292 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 128 | 286 | 301 | 668 | 540 | 0 | 0 | 0 |
| N.S. | 1 | 1.11 | 2.49 | 2.62 | 5.81 | 4.70 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.575 | 6.401 | 3.176 | 0.443 | 0.294 | 0.000 | 0.000 | 0.000 |

| Problem 293 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | B | A | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 57 | 66 | 149 | 988 | 86 | 0 | 4331 | 150 |
| N.S. | 1 | 1.04 | 1.20 | 2.71 | 17.96 | 1.56 | 0.00 | 78.75 | 2.73 |
| time (sec) | N/A | 0.341 | 0.651 | 1.529 | 0.307 | 0.259 | 0.000 | 1.104 | 27.525 |

| Problem 294 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 35 | 48 | 61 | 283 | 36 | 0 | 571 | 53 |
| N.S. | 1 | 1.00 | 1.37 | 1.74 | 8.09 | 1.03 | 0.00 | 16.31 | 1.51 |
| time (sec) | N/A | 0.243 | 0.641 | 0.878 | 0.209 | 0.254 | 0.000 | 0.384 | 26.279 |

| Problem 295 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 1090 | 24 | 20 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 49.55 | 1.09 | 0.91 | 1.09 | 1.09 |
| time (sec) | N/A | 0.266 | 8.444 | 0.389 | 0.769 | 0.254 | 0.460 | 0.439 | 24.834 |

| Problem 296 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 1396 | 35 | 22 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 63.45 | 1.59 | 1.00 | 1.09 | 1.09 |
| time (sec) | N/A | 0.290 | 12.786 | 0.349 | 1.705 | 0.244 | 0.712 | 6.735 | 26.933 |

| Problem 297 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 24 | 22 | 24 | 26 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.09 | 1.00 | 1.09 | 1.18 |
| time (sec) | N/A | 0.294 | 51.865 | 0.175 | 1.079 | 0.257 | 9.301 | 0.604 | 28.195 |

| Problem 298 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 337 | 580 | 530 | 1127 | 3831 | 1315 | 0 | 0 | 0 |
| N.S. | 1 | 1.72 | 1.57 | 3.34 | 11.37 | 3.90 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.068 | 3.749 | 2.008 | 1.370 | 0.330 | 0.000 | 0.000 | 0.000 |

| Problem 299 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 193 | 351 | 526 | 584 | 1891 | 795 | 0 | 0 | 0 |
| N.S. | 1 | 1.82 | 2.73 | 3.03 | 9.80 | 4.12 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.375 | 7.704 | 1.252 | 0.567 | 0.300 | 0.000 | 0.000 | 0.000 |

| Problem 300 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | F | B | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 117 | 192 | 555 | 267 | 0 | 435 | 0 | 0 | 0 |
| N.S. | 1 | 1.64 | 4.74 | 2.28 | 0.00 | 3.72 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.693 | 7.029 | 0.704 | 0.000 | 0.300 | 0.000 | 0.000 | 0.000 |

| Problem 301 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 1331 | 24 | 20 | 24 | 26 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 60.50 | 1.09 | 0.91 | 1.09 | 1.18 |
| time (sec) | N/A | 0.296 | 32.615 | 0.345 | 2.046 | 0.257 | 0.441 | 5.855 | 25.069 |

| Problem 302 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 1645 | 35 | 22 | 24 | 26 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 74.77 | 1.59 | 1.00 | 1.09 | 1.18 |
| time (sec) | N/A | 0.292 | 33.366 | 0.376 | 5.264 | 0.261 | 0.708 | 28.890 | 25.722 |

| Problem 303 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 18 | 16 | 18 | 18 | 15 | 18 | 18 |
| N.S. | 1 | 1.00 | 1.12 | 1.00 | 1.12 | 1.12 | 0.94 | 1.12 | 1.12 |
| time (sec) | N/A | 0.195 | 10.901 | 0.194 | 0.760 | 0.245 | 1.555 | 0.855 | 25.339 |

| Problem 304 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 259 | 296 | 814 | 729 | 2413 | 592 | 0 | 0 | 0 |
| N.S. | 1 | 1.14 | 3.14 | 2.81 | 9.32 | 2.29 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.467 | 6.923 | 0.725 | 0.899 | 0.266 | 0.000 | 0.000 | 0.000 |

| Problem 305 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 169 | 183 | 454 | 409 | 1228 | 354 | 0 | 0 | 0 |
| N.S. | 1 | 1.08 | 2.69 | 2.42 | 7.27 | 2.09 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.916 | 6.738 | 0.698 | 0.500 | 0.263 | 0.000 | 0.000 | 0.000 |

| Problem 306 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | B | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 115 | 240 | 183 | 517 | 168 | 0 | 0 | 0 |
| N.S. | 1 | 1.06 | 2.22 | 1.69 | 4.79 | 1.56 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.502 | 6.260 | 0.492 | 0.436 | 0.257 | 0.000 | 0.000 | 0.000 |

| Problem 307 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 18 | 16 | 1077 | 18 | 14 | 18 | 18 |
| N.S. | 1 | 1.00 | 1.12 | 1.00 | 67.31 | 1.12 | 0.88 | 1.12 | 1.12 |
| time (sec) | N/A | 0.201 | 9.480 | 0.280 | 1.605 | 0.236 | 0.366 | 0.837 | 25.567 |

| Problem 308 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 18 | 16 | 1386 | 29 | 15 | 18 | 18 |
| N.S. | 1 | 1.00 | 1.12 | 1.00 | 86.62 | 1.81 | 0.94 | 1.12 | 1.12 |
| time (sec) | N/A | 0.202 | 9.172 | 0.314 | 6.830 | 0.240 | 0.530 | 12.457 | 28.069 |

| Problem 309 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 24 | 0 | 24 | 26 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.09 | 0.00 | 1.09 | 1.18 |
| time (sec) | N/A | 0.375 | 21.030 | 0.192 | 0.551 | 0.272 | 0.000 | 0.326 | 25.794 |

| Problem 310 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 399 | 443 | 2227 | 1729 | 8853 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.11 | 5.58 | 4.33 | 22.19 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.203 | 7.646 | 1.026 | 4.271 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 311 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 325 | 349 | 1535 | 1115 | 4954 | 2268 | 0 | 0 | 0 |
| N.S. | 1 | 1.07 | 4.72 | 3.43 | 15.24 | 6.98 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.954 | 6.934 | 0.924 | 1.408 | 0.423 | 0.000 | 0.000 | 0.000 |

| Problem 312 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 201 | 227 | 883 | 614 | 2447 | 1404 | 0 | 0 | 0 |
| N.S. | 1 | 1.13 | 4.39 | 3.05 | 12.17 | 6.99 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.732 | 6.649 | 0.836 | 0.630 | 0.360 | 0.000 | 0.000 | 0.000 |

| Problem 313 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 138 | 224 | 270 | 1028 | 760 | 0 | 0 | 0 |
| N.S. | 1 | 0.99 | 1.61 | 1.94 | 7.40 | 5.47 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.319 | 1.111 | 0.612 | 0.465 | 0.320 | 0.000 | 0.000 | 0.000 |

| Problem 314 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 1865 | 24 | 20 | 24 | 26 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 84.77 | 1.09 | 0.91 | 1.09 | 1.18 |
| time (sec) | N/A | 0.304 | 7.899 | 0.418 | 1.712 | 0.326 | 2.522 | 0.681 | 25.311 |

| Problem 315 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 2516 | 35 | 22 | 0 | 26 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 114.36 | 1.59 | 1.00 | 0.00 | 1.18 |
| time (sec) | N/A | 0.307 | 5.651 | 0.398 | 5.429 | 0.354 | 4.866 | 0.000 | 25.463 |

| Problem 316 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 26 | 26 | 0 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 1.08 | 1.08 | 0.00 | 1.08 | 1.08 |
| time (sec) | N/A | 0.380 | 25.379 | 0.192 | 0.833 | 0.262 | 0.000 | 0.361 | 25.381 |

| Problem 317 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 486 | 513 | 819 | 1629 | 8046 | 2226 | 0 | 0 | 0 |
| N.S. | 1 | 1.06 | 1.69 | 3.35 | 16.56 | 4.58 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.535 | 7.510 | 1.356 | 3.832 | 0.434 | 0.000 | 0.000 | 0.000 |

| Problem 318 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 341 | 371 | 889 | 770 | 3828 | 1366 | 0 | 0 | 0 |
| N.S. | 1 | 1.09 | 2.61 | 2.26 | 11.23 | 4.01 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.017 | 7.326 | 1.050 | 1.116 | 0.370 | 0.000 | 0.000 | 0.000 |

| Problem 319 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | B | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 162 | 179 | 669 | 344 | 0 | 592 | 0 | 0 | 0 |
| N.S. | 1 | 1.10 | 4.13 | 2.12 | 0.00 | 3.65 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.380 | 6.962 | 0.900 | 0.000 | 0.322 | 0.000 | 0.000 | 0.000 |

| Problem 320 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 3695 | 26 | 22 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 153.96 | 1.08 | 0.92 | 1.08 | 1.08 |
| time (sec) | N/A | 0.323 | 14.975 | 0.401 | 2.620 | 0.286 | 7.450 | 117.094 | 25.835 |

| Problem 321 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 4747 | 37 | 24 | 0 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 197.79 | 1.54 | 1.00 | 0.00 | 1.08 |
| time (sec) | N/A | 0.341 | 18.717 | 0.364 | 7.398 | 0.314 | 14.814 | 0.000 | 25.670 |

| Problem 322 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 26 | 26 | 0 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 1.08 | 1.08 | 0.00 | 1.08 | 1.08 |
| time (sec) | N/A | 0.382 | 28.976 | 0.221 | 0.593 | 0.243 | 0.000 | 0.367 | 25.125 |

| Problem 323 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 318 | 365 | 483 | 1329 | 5646 | 4193 | 0 | 0 | 0 |
| N.S. | 1 | 1.15 | 1.52 | 4.18 | 17.75 | 13.19 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.286 | 6.266 | 1.224 | 2.417 | 0.474 | 0.000 | 0.000 | 0.000 |

| Problem 324 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 190 | 219 | 381 | 716 | 2728 | 2387 | 0 | 0 | 0 |
| N.S. | 1 | 1.15 | 2.01 | 3.77 | 14.36 | 12.56 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.794 | 7.567 | 0.964 | 0.761 | 0.401 | 0.000 | 0.000 | 0.000 |

| Problem 325 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 125 | 236 | 325 | 1070 | 1193 | 0 | 0 | 0 |
| N.S. | 1 | 1.14 | 2.15 | 2.95 | 9.73 | 10.85 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.465 | 1.894 | 0.799 | 0.512 | 0.329 | 0.000 | 0.000 | 0.000 |

| Problem 326 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 2410 | 26 | 22 | 26 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 100.42 | 1.08 | 0.92 | 1.08 | 1.08 |
| time (sec) | N/A | 0.283 | 21.758 | 0.475 | 3.931 | 0.281 | 20.257 | 3.106 | 26.024 |

| Problem 327 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 3083 | 37 | 24 | 0 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 128.46 | 1.54 | 1.00 | 0.00 | 1.08 |
| time (sec) | N/A | 0.280 | 24.774 | 0.358 | 13.684 | 0.323 | 41.092 | 0.000 | 26.574 |

| Problem 328 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 96 | 73 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.16 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.363 | 0.416 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 329 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 68 | 51 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.13 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.289 | 0.419 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 330 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 68 | 52 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.13 | 0.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.285 | 0.248 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 331 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 66 | 310 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.00 | 9.39 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.215 | 0.694 | 0.365 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 332 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 33 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.214 | 0.271 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 333 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 64 | 54 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.07 | 0.90 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.282 | 0.285 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 334 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 68 | 53 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.13 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.273 | 0.321 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 335 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 92 | 65 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.11 | 0.78 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.364 | 0.345 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 336 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 112 | 65 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.09 | 0.63 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.487 | 0.369 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 337 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 88 | 61 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.10 | 0.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.378 | 0.298 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 338 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 84 | 54 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.05 | 0.68 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.385 | 0.275 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 339 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 42 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.293 | 0.221 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 340 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 53 | 66 | 310 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.25 | 5.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.288 | 0.637 | 0.300 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 341 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 88 | 63 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.10 | 0.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.383 | 0.359 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 342 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 88 | 97 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.10 | 1.21 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.390 | 0.827 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 343 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 116 | 89 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.13 | 0.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.491 | 0.423 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 344 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 101 | 67 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.15 | 0.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.368 | 0.565 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 345 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 73 | 108 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.12 | 1.66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.294 | 2.158 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 346 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 73 | 56 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.12 | 0.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.292 | 0.263 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 347 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 86 | 308 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.26 | 8.11 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.215 | 6.569 | 0.368 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 348 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 38 | 37 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.212 | 0.312 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 349 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 69 | 56 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.06 | 0.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.286 | 0.271 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 350 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 73 | 57 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.12 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.284 | 0.312 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 351 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 97 | 73 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.10 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.362 | 0.345 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 352 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 117 | 73 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.08 | 0.68 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.480 | 0.395 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 353 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 93 | 65 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.09 | 0.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.378 | 0.405 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 354 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 89 | 56 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.05 | 0.66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.378 | 0.291 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 355 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 58 | 46 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.291 | 0.258 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 356 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 58 | 106 | 308 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.83 | 5.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.288 | 1.866 | 0.349 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 357 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 93 | 65 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.09 | 0.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.375 | 0.381 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 358 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 93 | 114 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.09 | 1.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.371 | 1.283 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 359 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 121 | 93 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.12 | 0.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.479 | 0.511 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 360 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 22 | 19 | 18 | 17 | 19 | 18 | 18 |
| N.S. | 1 | 1.00 | 0.71 | 0.61 | 0.58 | 0.55 | 0.61 | 0.58 | 0.58 |
| time (sec) | N/A | 0.197 | 0.014 | 0.316 | 0.208 | 0.239 | 0.981 | 0.261 | 0.120 |

| Problem 361 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 131 | 154 | 174 | 146 | 200 | 223 | 167 | 212 |
| N.S. | 1 | 1.00 | 1.18 | 1.33 | 1.11 | 1.53 | 1.70 | 1.27 | 1.62 |
| time (sec) | N/A | 0.383 | 0.143 | 2.125 | 0.244 | 0.249 | 14.916 | 0.274 | 26.263 |

| Problem 362 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 108 | 109 | 119 | 101 | 127 | 139 | 112 | 136 |
| N.S. | 1 | 0.94 | 0.95 | 1.03 | 0.88 | 1.10 | 1.21 | 0.97 | 1.18 |
| time (sec) | N/A | 0.325 | 0.113 | 1.126 | 0.204 | 0.247 | 7.780 | 0.279 | 0.437 |

| Problem 363 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 73 | 60 | 70 | 60 | 70 | 75 | 64 | 73 |
| N.S. | 1 | 1.00 | 0.82 | 0.96 | 0.82 | 0.96 | 1.03 | 0.88 | 1.00 |
| time (sec) | N/A | 0.283 | 0.077 | 0.791 | 0.228 | 0.246 | 3.887 | 0.265 | 25.723 |

| Problem 364 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 44 | 34 | 28 | 27 | 27 | 29 | 27 | 30 |
| N.S. | 1 | 1.07 | 0.83 | 0.68 | 0.66 | 0.66 | 0.71 | 0.66 | 0.73 |
| time (sec) | N/A | 0.220 | 0.052 | 0.437 | 0.216 | 0.245 | 1.689 | 0.260 | 26.281 |

| Problem 365 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 49 | 58 | 97 | 51 | 0 | 51 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 1.02 | 1.70 | 0.89 | 0.00 | 0.89 | 0.00 |
| time (sec) | N/A | 0.400 | 0.049 | 0.681 | 0.249 | 0.246 | 0.000 | 0.262 | 0.000 |

| Problem 366 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 78 | 61 | 82 | 330 | 76 | 0 | 111 | 0 |
| N.S. | 1 | 1.00 | 0.78 | 1.05 | 4.23 | 0.97 | 0.00 | 1.42 | 0.00 |
| time (sec) | N/A | 0.397 | 0.114 | 1.026 | 0.266 | 0.257 | 0.000 | 0.258 | 0.000 |

| Problem 367 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 99 | 99 | 77 | 104 | 365 | 128 | 0 | 201 | 0 |
| N.S. | 1 | 1.00 | 0.78 | 1.05 | 3.69 | 1.29 | 0.00 | 2.03 | 0.00 |
| time (sec) | N/A | 0.465 | 0.181 | 2.072 | 0.300 | 0.255 | 0.000 | 0.271 | 0.000 |

| Problem 368 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 198 | 198 | 128 | 226 | 244 | 283 | 0 | 1255 | 344 |
| N.S. | 1 | 1.00 | 0.65 | 1.14 | 1.23 | 1.43 | 0.00 | 6.34 | 1.74 |
| time (sec) | N/A | 0.442 | 0.427 | 1.913 | 0.276 | 0.257 | 0.000 | 0.440 | 26.454 |

| Problem 369 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 171 | 161 | 105 | 156 | 173 | 188 | 0 | 682 | 216 |
| N.S. | 1 | 0.94 | 0.61 | 0.91 | 1.01 | 1.10 | 0.00 | 3.99 | 1.26 |
| time (sec) | N/A | 0.353 | 0.258 | 0.974 | 0.257 | 0.254 | 0.000 | 0.368 | 26.394 |

| Problem 370 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 112 | 73 | 95 | 108 | 111 | 0 | 313 | 121 |
| N.S. | 1 | 1.00 | 0.65 | 0.85 | 0.96 | 0.99 | 0.00 | 2.79 | 1.08 |
| time (sec) | N/A | 0.314 | 0.271 | 0.692 | 0.236 | 0.242 | 0.000 | 0.317 | 0.372 |

| Problem 371 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 69 | 46 | 44 | 55 | 54 | 0 | 106 | 53 |
| N.S. | 1 | 1.05 | 0.70 | 0.67 | 0.83 | 0.82 | 0.00 | 1.61 | 0.80 |
| time (sec) | N/A | 0.243 | 0.134 | 0.302 | 0.232 | 0.254 | 0.000 | 0.283 | 0.271 |

| Problem 372 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | C | A | F | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 63 | 106 | 119 | 71 | 0 | 102 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 1.49 | 1.68 | 1.00 | 0.00 | 1.44 | 0.00 |
| time (sec) | N/A | 0.430 | 0.145 | 0.571 | 0.265 | 0.248 | 0.000 | 0.290 | 0.000 |

| Problem 373 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | C | A | F | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 102 | 81 | 155 | 120 | 106 | 0 | 308 | 0 |
| N.S. | 1 | 1.00 | 0.79 | 1.52 | 1.18 | 1.04 | 0.00 | 3.02 | 0.00 |
| time (sec) | N/A | 0.458 | 0.411 | 0.971 | 0.326 | 0.255 | 0.000 | 0.313 | 0.000 |

| Problem 374 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F(-1) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 136 | 104 | 207 | 132 | 182 | 0 | 704 | 0 |
| N.S. | 1 | 1.00 | 0.76 | 1.52 | 0.97 | 1.34 | 0.00 | 5.18 | 0.00 |
| time (sec) | N/A | 0.542 | 0.737 | 1.795 | 0.292 | 0.254 | 0.000 | 0.340 | 0.000 |

| Problem 375 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | C | A | F(-1) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 205 | 205 | 125 | 243 | 143 | 286 | 0 | 1305 | 0 |
| N.S. | 1 | 1.00 | 0.61 | 1.19 | 0.70 | 1.40 | 0.00 | 6.37 | 0.00 |
| time (sec) | N/A | 0.580 | 0.800 | 3.634 | 0.311 | 0.267 | 0.000 | 0.377 | 0.000 |

| Problem 376 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 255 | 255 | 459 | 849 | 606 | 929 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.80 | 3.33 | 2.38 | 3.64 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.556 | 0.823 | 2.968 | 0.420 | 0.315 | 0.000 | 0.000 | 0.000 |

| Problem 377 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 172 | 172 | 223 | 481 | 413 | 566 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.30 | 2.80 | 2.40 | 3.29 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.424 | 0.602 | 2.635 | 0.424 | 0.313 | 0.000 | 0.000 | 0.000 |

| Problem 378 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 95 | 95 | 171 | 205 | 0 | 281 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.80 | 2.16 | 0.00 | 2.96 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.295 | 0.621 | 1.230 | 0.000 | 0.273 | 0.000 | 0.000 | 0.000 |

| Problem 379 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 25 | 230 | 27 | 24 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 9.20 | 1.08 | 0.96 | 1.08 | 1.08 |
| time (sec) | N/A | 0.405 | 4.239 | 0.589 | 0.358 | 0.243 | 125.857 | 1.174 | 30.429 |

| Problem 380 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 25 | 342 | 38 | 0 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 13.68 | 1.52 | 0.00 | 1.08 | 1.08 |
| time (sec) | N/A | 0.455 | 4.563 | 0.676 | 0.393 | 0.248 | 0.000 | 1.991 | 33.087 |

| Problem 381 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 25 | 463 | 49 | 0 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 18.52 | 1.96 | 0.00 | 1.08 | 1.08 |
| time (sec) | N/A | 0.511 | 5.423 | 0.637 | 0.427 | 0.247 | 0.000 | 4.521 | 34.237 |

| Problem 382 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 293 | 2529 | 965 | 610 | 1652 | 0 | 0 | 0 |
| N.S. | 1 | 0.98 | 8.46 | 3.23 | 2.04 | 5.53 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.710 | 6.404 | 6.243 | 0.360 | 0.350 | 0.000 | 0.000 | 0.000 |

| Problem 383 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 242 | 242 | 1730 | 648 | 444 | 1126 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 7.15 | 2.68 | 1.83 | 4.65 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.606 | 6.364 | 2.876 | 0.336 | 0.345 | 0.000 | 0.000 | 0.000 |

| Problem 384 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | A | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 173 | 166 | 516 | 386 | 303 | 681 | 0 | 0 | 0 |
| N.S. | 1 | 0.96 | 2.98 | 2.23 | 1.75 | 3.94 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.500 | 6.375 | 1.389 | 0.322 | 0.296 | 0.000 | 0.000 | 0.000 |

| Problem 385 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | F | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 107 | 257 | 177 | 0 | 340 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.40 | 1.65 | 0.00 | 3.18 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.360 | 3.786 | 2.028 | 0.000 | 0.275 | 0.000 | 0.000 | 0.000 |

| Problem 386 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 23 | 182 | 25 | 0 | 25 | 27 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 7.91 | 1.09 | 0.00 | 1.09 | 1.17 |
| time (sec) | N/A | 0.442 | 2.892 | 0.545 | 0.331 | 0.262 | 0.000 | 0.308 | 29.129 |

| Problem 387 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 23 | 242 | 36 | 0 | 25 | 27 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 10.52 | 1.57 | 0.00 | 1.09 | 1.17 |
| time (sec) | N/A | 0.490 | 3.384 | 0.556 | 0.431 | 0.254 | 0.000 | 0.335 | 30.660 |

| Problem 388 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 23 | 308 | 47 | 0 | 25 | 27 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 13.39 | 2.04 | 0.00 | 1.09 | 1.17 |
| time (sec) | N/A | 0.574 | 4.682 | 0.502 | 0.429 | 0.261 | 0.000 | 0.401 | 30.449 |

| Problem 389 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | F | B | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 230 | 230 | 532 | 677 | 0 | 896 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.31 | 2.94 | 0.00 | 3.90 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.539 | 1.706 | 8.568 | 0.000 | 0.303 | 0.000 | 0.000 | 0.000 |

| Problem 390 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | F | B | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 147 | 364 | 334 | 0 | 513 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 2.48 | 2.27 | 0.00 | 3.49 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.412 | 2.624 | 4.242 | 0.000 | 0.281 | 0.000 | 0.000 | 0.000 |

| Problem 391 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | F(-2) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 105 | 96 | 3330 | 93 | 0 | 365 | 150 |
| N.S. | 1 | 1.00 | 1.84 | 1.68 | 58.42 | 1.63 | 0.00 | 6.40 | 2.63 |
| time (sec) | N/A | 0.272 | 0.810 | 1.595 | 0.395 | 0.271 | 0.000 | 0.372 | 1.761 |

| Problem 392 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 25 | 1275 | 27 | 0 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 51.00 | 1.08 | 0.00 | 1.08 | 1.08 |
| time (sec) | N/A | 0.489 | 11.006 | 0.613 | 0.413 | 0.252 | 0.000 | 1.901 | 29.934 |

| Problem 393 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 25 | 1419 | 38 | 0 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 56.76 | 1.52 | 0.00 | 1.08 | 1.08 |
| time (sec) | N/A | 0.525 | 15.403 | 0.733 | 0.474 | 0.255 | 0.000 | 4.137 | 32.112 |

| Problem 394 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 25 | 1563 | 49 | 0 | 27 | 27 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 62.52 | 1.96 | 0.00 | 1.08 | 1.08 |
| time (sec) | N/A | 0.593 | 17.357 | 0.756 | 0.574 | 0.250 | 0.000 | 7.781 | 34.205 |

| Problem 395 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | B | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 77 | 66 | 0 | 106 | 0 | 0 | 46 |
| N.S. | 1 | 1.00 | 1.35 | 1.16 | 0.00 | 1.86 | 0.00 | 0.00 | 0.81 |
| time (sec) | N/A | 0.240 | 0.035 | 1.681 | 0.000 | 0.256 | 0.000 | 0.000 | 26.551 |

| Problem 396 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 14 | 15 | 111 | 26 | 144 | 118 | 31 |
| N.S. | 1 | 1.00 | 1.00 | 1.07 | 7.93 | 1.86 | 10.29 | 8.43 | 2.21 |
| time (sec) | N/A | 0.190 | 0.029 | 3.395 | 0.301 | 0.250 | 3.962 | 0.270 | 28.408 |

| Problem 397 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | F | B | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 67 | 146 | 102 | 0 | 144 | 0 | 0 | 63 |
| N.S. | 1 | 1.00 | 2.18 | 1.52 | 0.00 | 2.15 | 0.00 | 0.00 | 0.94 |
| time (sec) | N/A | 0.299 | 0.235 | 9.372 | 0.000 | 0.254 | 0.000 | 0.000 | 27.652 |

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [207] had the largest ratio of [2.60000000000000009]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1 | A | 5 | 5 | 1.03 | 20 | 0.250 |
| 2 | A | 7 | 7 | 1.01 | 20 | 0.350 |
| 3 | A | 7 | 7 | 1.07 | 20 | 0.350 |
| 4 | A | 4 | 4 | 0.94 | 20 | 0.200 |
| 5 | A | 4 | 4 | 1.08 | 18 | 0.222 |
| 6 | A | 7 | 7 | 0.97 | 20 | 0.350 |
| 7 | A | 9 | 9 | 1.08 | 20 | 0.450 |
| 8 | A | 12 | 12 | 1.05 | 20 | 0.600 |
| 9 | A | 14 | 14 | 1.07 | 20 | 0.700 |
| 10 | A | 4 | 4 | 1.00 | 8 | 0.500 |
| 11 | A | 6 | 6 | 1.25 | 8 | 0.750 |
| 12 | A | 9 | 9 | 1.07 | 8 | 1.125 |
| 13 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 14 | A | 17 | 17 | 1.21 | 22 | 0.773 |
| 15 | A | 13 | 12 | 1.09 | 22 | 0.545 |
| 16 | A | 7 | 7 | 1.03 | 22 | 0.318 |
| 17 | A | 5 | 4 | 0.94 | 20 | 0.200 |
| 18 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 19 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 20 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 21 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 22 | A | 2 | 2 | 1.00 | 22 | 0.091 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 23 | A | 13 | 13 | 1.25 | 22 | 0.591 |
| 24 | A | 13 | 13 | 1.24 | 22 | 0.591 |
| 25 | A | 6 | 6 | 0.99 | 22 | 0.273 |
| 26 | A | 6 | 6 | 1.11 | 20 | 0.300 |
| 27 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 28 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 29 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 30 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 31 | N/A | 3 | 0 | 1.00 | 14 | 0.000 |
| 32 | A | 10 | 9 | 1.42 | 14 | 0.643 |
| 33 | A | 9 | 8 | 1.35 | 14 | 0.571 |
| 34 | A | 8 | 7 | 1.34 | 14 | 0.500 |
| 35 | A | 7 | 6 | 1.25 | 12 | 0.500 |
| 36 | N/A | 3 | 0 | 1.00 | 14 | 0.000 |
| 37 | N/A | 3 | 0 | 1.00 | 14 | 0.000 |
| 38 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 39 | A | 8 | 7 | 1.10 | 20 | 0.350 |
| 40 | A | 7 | 6 | 1.07 | 20 | 0.300 |
| 41 | A | 6 | 5 | 1.02 | 20 | 0.250 |
| 42 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 43 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 44 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 45 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 46 | A | 11 | 10 | 1.31 | 22 | 0.455 |
| 47 | A | 10 | 9 | 1.19 | 22 | 0.409 |
| 48 | A | 6 | 6 | 1.09 | 22 | 0.273 |
| 49 | A | 5 | 4 | 1.00 | 20 | 0.200 |
| 50 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 51 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 52 | A | 17 | 16 | 1.11 | 22 | 0.727 |
| 53 | A | 15 | 14 | 1.09 | 22 | 0.636 |
| 54 | A | 12 | 11 | 1.06 | 22 | 0.500 |
| 55 | A | 12 | 11 | 1.06 | 22 | 0.500 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 56 | A | 15 | 14 | 1.09 | 22 | 0.636 |
| 57 | A | 17 | 16 | 1.11 | 22 | 0.727 |
| 58 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 59 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 60 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 61 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 62 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 63 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 64 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 65 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 66 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 67 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 68 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 69 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 70 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 71 | A | 19 | 19 | 1.20 | 22 | 0.864 |
| 72 | A | 13 | 12 | 1.09 | 22 | 0.545 |
| 73 | A | 8 | 8 | 1.02 | 22 | 0.364 |
| 74 | A | 5 | 4 | 0.98 | 20 | 0.200 |
| 75 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 76 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 77 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 78 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 79 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 80 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 81 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 82 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 83 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 84 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 85 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 86 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 87 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 88 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 89 | A | 2 | 2 | 1.00 | 24 | 0.083 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 90 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 91 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 92 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 93 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 94 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 95 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 96 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 97 | N/A | 5 | 0 | 1.00 | 20 | 0.000 |
| 98 | A | 20 | 19 | 1.13 | 20 | 0.950 |
| 99 | A | 16 | 15 | 1.09 | 20 | 0.750 |
| 100 | A | 13 | 12 | 1.06 | 20 | 0.600 |
| 101 | A | 9 | 8 | 1.00 | 18 | 0.444 |
| 102 | N/A | 7 | 0 | 1.00 | 20 | 0.000 |
| 103 | N/A | 9 | 0 | 1.00 | 20 | 0.000 |
| 104 | N/A | 2 | 0 | 1.00 | 16 | 0.000 |
| 105 | A | 13 | 12 | 1.35 | 16 | 0.750 |
| 106 | A | 12 | 11 | 1.29 | 16 | 0.688 |
| 107 | A | 11 | 10 | 1.24 | 16 | 0.625 |
| 108 | A | 7 | 7 | 1.10 | 14 | 0.500 |
| 109 | N/A | 2 | 0 | 1.00 | 16 | 0.000 |
| 110 | N/A | 2 | 0 | 1.00 | 16 | 0.000 |
| 111 | N/A | 3 | 0 | 1.00 | 22 | 0.000 |
| 112 | A | 15 | 14 | 1.78 | 22 | 0.636 |
| 113 | A | 14 | 13 | 1.73 | 22 | 0.591 |
| 114 | A | 13 | 12 | 1.81 | 22 | 0.545 |
| 115 | A | 11 | 10 | 1.63 | 20 | 0.500 |
| 116 | N/A | 3 | 0 | 1.00 | 22 | 0.000 |
| 117 | N/A | 3 | 0 | 1.00 | 22 | 0.000 |
| 118 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 119 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 120 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 121 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 122 | A | 2 | 2 | 1.00 | 24 | 0.083 |

Continued on next page

## 2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 123 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 124 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 125 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 126 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 127 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 128 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 129 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 130 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 131 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 132 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 133 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 134 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 135 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 136 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 137 | A | 13 | 13 | 1.25 | 22 | 0.591 |
| 138 | A | 13 | 13 | 1.24 | 22 | 0.591 |
| 139 | A | 6 | 6 | 0.99 | 22 | 0.273 |
| 140 | A | 6 | 6 | 1.11 | 20 | 0.300 |
| 141 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 142 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 143 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 144 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 145 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 146 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 147 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 148 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 149 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 150 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 151 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 152 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 153 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 154 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 155 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 156 | A | 2 | 2 | 1.00 | 24 | 0.083 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 157 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 158 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 159 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 160 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 161 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 162 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 163 | N/A | 9 | 0 | 1.00 | 22 | 0.000 |
| 164 | A | 18 | 17 | 1.21 | 22 | 0.773 |
| 165 | A | 17 | 16 | 1.21 | 22 | 0.727 |
| 166 | A | 13 | 12 | 1.14 | 22 | 0.545 |
| 167 | A | 12 | 11 | 1.18 | 20 | 0.550 |
| 168 | N/A | 11 | 0 | 1.00 | 22 | 0.000 |
| 169 | N/A | 13 | 0 | 1.00 | 22 | 0.000 |
| 170 | N/A | 6 | 0 | 1.00 | 22 | 0.000 |
| 171 | A | 21 | 20 | 1.10 | 22 | 0.909 |
| 172 | A | 18 | 17 | 1.07 | 22 | 0.773 |
| 173 | A | 14 | 13 | 1.04 | 22 | 0.591 |
| 174 | A | 9 | 9 | 1.00 | 20 | 0.450 |
| 175 | N/A | 7 | 0 | 1.00 | 22 | 0.000 |
| 176 | N/A | 10 | 0 | 1.00 | 22 | 0.000 |
| 177 | N/A | 3 | 0 | 1.00 | 16 | 0.000 |
| 178 | A | 24 | 23 | 1.35 | 16 | 1.438 |
| 179 | A | 22 | 21 | 1.25 | 16 | 1.312 |
| 180 | A | 18 | 17 | 1.17 | 16 | 1.062 |
| 181 | A | 13 | 12 | 1.14 | 14 | 0.857 |
| 182 | N/A | 3 | 0 | 1.00 | 16 | 0.000 |
| 183 | N/A | 3 | 0 | 1.00 | 16 | 0.000 |
| 184 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 185 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 186 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 187 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 188 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 189 | A | 2 | 2 | 1.00 | 24 | 0.083 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 190 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 191 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 192 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 193 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 194 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 195 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 196 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 197 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 198 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 199 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 200 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 201 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 202 | A | 19 | 18 | 1.23 | 12 | 1.500 |
| 203 | A | 18 | 17 | 1.23 | 12 | 1.417 |
| 204 | A | 10 | 10 | 1.00 | 10 | 1.000 |
| 205 | F | 0 | 0 | N/A | 0.000 | N/A |
| 206 | F | 0 | 0 | N/A | 0.000 | N/A |
| 207 | A | 27 | 26 | 1.27 | 10 | 2.600 |
| 208 | N/A | 2 | 0 | 1.00 | 14 | 0.000 |
| 209 | A | 9 | 8 | 1.23 | 14 | 0.571 |
| 210 | A | 8 | 7 | 1.17 | 14 | 0.500 |
| 211 | A | 7 | 6 | 1.18 | 14 | 0.429 |
| 212 | A | 6 | 5 | 1.11 | 12 | 0.417 |
| 213 | N/A | 2 | 0 | 1.00 | 14 | 0.000 |
| 214 | N/A | 2 | 0 | 1.00 | 14 | 0.000 |
| 215 | N/A | 6 | 0 | 1.00 | 20 | 0.000 |
| 216 | A | 17 | 16 | 1.09 | 20 | 0.800 |
| 217 | A | 13 | 12 | 1.06 | 20 | 0.600 |
| 218 | A | 10 | 9 | 1.00 | 18 | 0.500 |
| 219 | N/A | 7 | 0 | 1.00 | 20 | 0.000 |
| 220 | N/A | 10 | 0 | 1.00 | 20 | 0.000 |
| 221 | N/A | 8 | 0 | 1.00 | 22 | 0.000 |
| 222 | A | 16 | 15 | 1.12 | 22 | 0.682 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 223 | A | 12 | 11 | 1.06 | 22 | 0.500 |
| 224 | A | 11 | 10 | 1.10 | 20 | 0.500 |
| 225 | N/A | 10 | 0 | 1.00 | 22 | 0.000 |
| 226 | N/A | 12 | 0 | 1.00 | 22 | 0.000 |
| 227 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 228 | A | 9 | 8 | 1.22 | 20 | 0.400 |
| 229 | A | 8 | 7 | 1.13 | 20 | 0.350 |
| 230 | A | 7 | 6 | 1.10 | 20 | 0.300 |
| 231 | A | 6 | 5 | 1.03 | 18 | 0.278 |
| 232 | N/A | 3 | 0 | 1.00 | 20 | 0.000 |
| 233 | N/A | 3 | 0 | 1.00 | 20 | 0.000 |
| 234 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 235 | A | 5 | 5 | 1.11 | 22 | 0.227 |
| 236 | A | 5 | 5 | 1.15 | 22 | 0.227 |
| 237 | A | 2 | 2 | 0.98 | 20 | 0.100 |
| 238 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 239 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 240 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 241 | A | 6 | 6 | 1.08 | 22 | 0.273 |
| 242 | A | 6 | 6 | 1.13 | 22 | 0.273 |
| 243 | A | 2 | 2 | 0.98 | 20 | 0.100 |
| 244 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 245 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 246 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 247 | A | 8 | 7 | 1.09 | 20 | 0.350 |
| 248 | A | 7 | 6 | 1.06 | 20 | 0.300 |
| 249 | A | 6 | 5 | 1.02 | 20 | 0.250 |
| 250 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 251 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 252 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 253 | N/A | 2 | 0 | 1.00 | 16 | 0.000 |
| 254 | A | 10 | 9 | 1.18 | 16 | 0.562 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 255 | A | 9 | 8 | 1.16 | 16 | 0.500 |
| 256 | A | 5 | 5 | 1.05 | 14 | 0.357 |
| 257 | N/A | 2 | 0 | 1.00 | 16 | 0.000 |
| 258 | N/A | 2 | 0 | 1.00 | 16 | 0.000 |
| 259 | N/A | 5 | 0 | 1.00 | 22 | 0.000 |
| 260 | A | 17 | 16 | 1.07 | 22 | 0.727 |
| 261 | A | 14 | 13 | 1.03 | 22 | 0.591 |
| 262 | A | 8 | 8 | 1.00 | 20 | 0.400 |
| 263 | N/A | 7 | 0 | 1.00 | 22 | 0.000 |
| 264 | N/A | 9 | 0 | 1.00 | 22 | 0.000 |
| 265 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 266 | A | 6 | 6 | 1.08 | 22 | 0.273 |
| 267 | A | 6 | 6 | 1.11 | 22 | 0.273 |
| 268 | A | 6 | 6 | 1.16 | 22 | 0.273 |
| 269 | A | 2 | 2 | 1.07 | 20 | 0.100 |
| 270 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 271 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 272 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 273 | A | 11 | 10 | 1.36 | 24 | 0.417 |
| 274 | A | 10 | 9 | 1.28 | 24 | 0.375 |
| 275 | A | 6 | 6 | 1.26 | 22 | 0.273 |
| 276 | N/A | 3 | 0 | 1.00 | 24 | 0.000 |
| 277 | N/A | 3 | 0 | 1.00 | 24 | 0.000 |
| 278 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 279 | A | 6 | 6 | 1.04 | 24 | 0.250 |
| 280 | A | 6 | 6 | 1.10 | 24 | 0.250 |
| 281 | A | 2 | 2 | 1.11 | 22 | 0.091 |
| 282 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 283 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 284 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 285 | A | 4 | 4 | 1.08 | 20 | 0.200 |
| 286 | A | 4 | 4 | 1.00 | 20 | 0.200 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 287 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 288 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 289 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 290 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 291 | A | 11 | 10 | 1.19 | 22 | 0.455 |
| 292 | A | 10 | 9 | 1.11 | 22 | 0.409 |
| 293 | A | 6 | 6 | 1.04 | 22 | 0.273 |
| 294 | A | 5 | 4 | 1.00 | 20 | 0.200 |
| 295 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 296 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 297 | N/A | 3 | 0 | 1.00 | 22 | 0.000 |
| 298 | A | 14 | 13 | 1.72 | 22 | 0.591 |
| 299 | A | 13 | 12 | 1.82 | 22 | 0.545 |
| 300 | A | 11 | 10 | 1.64 | 20 | 0.500 |
| 301 | N/A | 3 | 0 | 1.00 | 22 | 0.000 |
| 302 | N/A | 3 | 0 | 1.00 | 22 | 0.000 |
| 303 | N/A | 2 | 0 | 1.00 | 16 | 0.000 |
| 304 | A | 17 | 16 | 1.14 | 16 | 1.000 |
| 305 | A | 13 | 12 | 1.08 | 16 | 0.750 |
| 306 | A | 10 | 9 | 1.06 | 14 | 0.643 |
| 307 | N/A | 2 | 0 | 1.00 | 16 | 0.000 |
| 308 | N/A | 2 | 0 | 1.00 | 16 | 0.000 |
| 309 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 310 | A | 6 | 6 | 1.11 | 22 | 0.273 |
| 311 | A | 6 | 6 | 1.07 | 22 | 0.273 |
| 312 | A | 6 | 6 | 1.13 | 22 | 0.273 |
| 313 | A | 2 | 2 | 0.99 | 20 | 0.100 |
| 314 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 315 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 316 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 317 | A | 6 | 6 | 1.06 | 24 | 0.250 |
| 318 | A | 6 | 6 | 1.09 | 24 | 0.250 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 319 | A | 2 | 2 | 1.10 | 22 | 0.091 |
| 320 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 321 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 322 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 323 | A | 12 | 11 | 1.15 | 24 | 0.458 |
| 324 | A | 10 | 9 | 1.15 | 24 | 0.375 |
| 325 | A | 8 | 7 | 1.14 | 22 | 0.318 |
| 326 | N/A | 3 | 0 | 1.00 | 24 | 0.000 |
| 327 | N/A | 3 | 0 | 1.00 | 24 | 0.000 |
| 328 | A | 7 | 7 | 1.16 | 18 | 0.389 |
| 329 | A | 5 | 5 | 1.13 | 18 | 0.278 |
| 330 | A | 5 | 5 | 1.13 | 18 | 0.278 |
| 331 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 332 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 333 | A | 5 | 5 | 1.07 | 18 | 0.278 |
| 334 | A | 5 | 5 | 1.13 | 18 | 0.278 |
| 335 | A | 7 | 7 | 1.11 | 18 | 0.389 |
| 336 | A | 9 | 9 | 1.09 | 18 | 0.500 |
| 337 | A | 7 | 7 | 1.10 | 18 | 0.389 |
| 338 | A | 7 | 7 | 1.05 | 18 | 0.389 |
| 339 | A | 5 | 5 | 1.00 | 18 | 0.278 |
| 340 | A | 5 | 5 | 1.00 | 18 | 0.278 |
| 341 | A | 7 | 7 | 1.10 | 18 | 0.389 |
| 342 | A | 7 | 7 | 1.10 | 18 | 0.389 |
| 343 | A | 9 | 9 | 1.13 | 18 | 0.500 |
| 344 | A | 7 | 7 | 1.15 | 18 | 0.389 |
| 345 | A | 5 | 5 | 1.12 | 18 | 0.278 |
| 346 | A | 5 | 5 | 1.12 | 18 | 0.278 |
| 347 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 348 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 349 | A | 5 | 5 | 1.06 | 18 | 0.278 |
| 350 | A | 5 | 5 | 1.12 | 18 | 0.278 |
| 351 | A | 7 | 7 | 1.10 | 18 | 0.389 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 352 | A | 9 | 9 | 1.08 | 18 | 0.500 |
| 353 | A | 7 | 7 | 1.09 | 18 | 0.389 |
| 354 | A | 7 | 7 | 1.05 | 18 | 0.389 |
| 355 | A | 5 | 5 | 1.00 | 18 | 0.278 |
| 356 | A | 5 | 5 | 1.00 | 18 | 0.278 |
| 357 | A | 7 | 7 | 1.09 | 18 | 0.389 |
| 358 | A | 7 | 7 | 1.09 | 18 | 0.389 |
| 359 | A | 9 | 9 | 1.12 | 18 | 0.500 |
| 360 | A | 2 | 2 | 1.00 | 8 | 0.250 |
| 361 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 362 | A | 2 | 2 | 0.94 | 14 | 0.143 |
| 363 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 364 | A | 2 | 2 | 1.07 | 12 | 0.167 |
| 365 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 366 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 367 | A | 2 | 2 | 1.00 | 14 | 0.143 |
| 368 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 369 | A | 2 | 2 | 0.94 | 23 | 0.087 |
| 370 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 371 | A | 2 | 2 | 1.05 | 21 | 0.095 |
| 372 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 373 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 374 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 375 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 376 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 377 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 378 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 379 | N/A | 2 | 0 | 1.00 | 25 | 0.000 |
| 380 | N/A | 2 | 0 | 1.00 | 25 | 0.000 |
| 381 | N/A | 2 | 0 | 1.00 | 25 | 0.000 |
| 382 | A | 2 | 2 | 0.98 | 23 | 0.087 |
| 383 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 384 | A | 2 | 2 | 0.96 | 23 | 0.087 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 385 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 386 | N/A | 2 | 0 | 1.00 | 23 | 0.000 |
| 387 | N/A | 2 | 0 | 1.00 | 23 | 0.000 |
| 388 | N/A | 2 | 0 | 1.00 | 23 | 0.000 |
| 389 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 390 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 391 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 392 | N/A | 2 | 0 | 1.00 | 25 | 0.000 |
| 393 | N/A | 2 | 0 | 1.00 | 25 | 0.000 |
| 394 | N/A | 2 | 0 | 1.00 | 25 | 0.000 |
| 395 | A | 2 | 2 | 1.00 | 8 | 0.250 |
| 396 | A | 2 | 2 | 1.00 | 10 | 0.200 |
| 397 | A | 2 | 2 | 1.00 | 10 | 0.200 |

# CHAPTER 3

## LISTING OF INTEGRALS


3.1 $\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx$ . . . 151
3.2 $\int (c+dx)^4 \cos(a+bx)\sin(a+bx)\,dx$ . . . 156
3.3 $\int (c+dx)^3 \cos(a+bx)\sin(a+bx)\,dx$ . . . 164
3.4 $\int (c+dx)^2 \cos(a+bx)\sin(a+bx)\,dx$ . . . 171
3.5 $\int (c+dx) \cos(a+bx)\sin(a+bx)\,dx$ . . . 177
3.6 $\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx$ . . . 182
3.7 $\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx$ . . . 188
3.8 $\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$ . . . 195
3.9 $\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$ . . . 203
3.10 $\int \frac{\cos(x)\sin(x)}{x}\,dx$ . . . 212
3.11 $\int \frac{\cos(x)\sin(x)}{x^2}\,dx$ . . . 217
3.12 $\int \frac{\cos(x)\sin(x)}{x^3}\,dx$ . . . 222
3.13 $\int (c+dx)^m \cos(a+bx)\sin^2(a+bx)\,dx$ . . . 227
3.14 $\int (c+dx)^4 \cos(a+bx)\sin^2(a+bx)\,dx$ . . . 232
3.15 $\int (c+dx)^3 \cos(a+bx)\sin^2(a+bx)\,dx$ . . . 244
3.16 $\int (c+dx)^2 \cos(a+bx)\sin^2(a+bx)\,dx$ . . . 253
3.17 $\int (c+dx) \cos(a+bx)\sin^2(a+bx)\,dx$ . . . 259
3.18 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx$ . . . 264
3.19 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$ . . . 269
3.20 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$ . . . 275
3.21 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx$ . . . 281
3.22 $\int (c+dx)^m \cos(a+bx)\sin^3(a+bx)\,dx$ . . . 288
3.23 $\int (c+dx)^4 \cos(a+bx)\sin^3(a+bx)\,dx$ . . . 293
3.24 $\int (c+dx)^3 \cos(a+bx)\sin^3(a+bx)\,dx$ . . . 302
3.25 $\int (c+dx)^2 \cos(a+bx)\sin^3(a+bx)\,dx$ . . . 311
3.26 $\int (c+dx) \cos(a+bx)\sin^3(a+bx)\,dx$ . . . 318
3.27 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx$ . . . 324

3.28 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$ . . . 330
3.29 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$ . . . 336
3.30 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$ . . . 342
3.31 $\int (c+dx)^m\cot(a+bx)\,dx$ . . . 349
3.32 $\int (c+dx)^4\cot(a+bx)\,dx$ . . . 354
3.33 $\int (c+dx)^3\cot(a+bx)\,dx$ . . . 366
3.34 $\int (c+dx)^2\cot(a+bx)\,dx$ . . . 375
3.35 $\int (c+dx)\cot(a+bx)\,dx$ . . . 382
3.36 $\int \frac{\cot(a+bx)}{c+dx}\,dx$ . . . 388
3.37 $\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx$ . . . 393
3.38 $\int (c+dx)^m\cot(a+bx)\csc(a+bx)\,dx$ . . . 398
3.39 $\int (c+dx)^4\cot(a+bx)\csc(a+bx)\,dx$ . . . 402
3.40 $\int (c+dx)^3\cot(a+bx)\csc(a+bx)\,dx$ . . . 411
3.41 $\int (c+dx)^2\cot(a+bx)\csc(a+bx)\,dx$ . . . 419
3.42 $\int (c+dx)\cot(a+bx)\csc(a+bx)\,dx$ . . . 425
3.43 $\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$ . . . 431
3.44 $\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$ . . . 435
3.45 $\int (c+dx)^m\cot(a+bx)\csc^2(a+bx)\,dx$ . . . 440
3.46 $\int (c+dx)^4\cot(a+bx)\csc^2(a+bx)\,dx$ . . . 444
3.47 $\int (c+dx)^3\cot(a+bx)\csc^2(a+bx)\,dx$ . . . 454
3.48 $\int (c+dx)^2\cot(a+bx)\csc^2(a+bx)\,dx$ . . . 462
3.49 $\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx$ . . . 469
3.50 $\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx$ . . . 475
3.51 $\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx$ . . . 480
3.52 $\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx$ . . . 485
3.53 $\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx$ . . . 496
3.54 $\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx$ . . . 505
3.55 $\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx$ . . . 513
3.56 $\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx$ . . . 521
3.57 $\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx$ . . . 530
3.58 $\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx$ . . . 541
3.59 $\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx$ . . . 549
3.60 $\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx$ . . . 557
3.61 $\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx$ . . . 564
3.62 $\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx$ . . . 571
3.63 $\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx$ . . . 579
3.64 $\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx$ . . . 587
3.65 $\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx$ . . . 595
3.66 $\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx$ . . . 602

3.67 $\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx$ . . . 609
3.68 $\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx$ . . . 616
3.69 $\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx$ . . . 623
3.70 $\int (c+dx)^m\cos^2(a+bx)\sin(a+bx)\,dx$ . . . 631
3.71 $\int (c+dx)^4\cos^2(a+bx)\sin(a+bx)\,dx$ . . . 636
3.72 $\int (c+dx)^3\cos^2(a+bx)\sin(a+bx)\,dx$ . . . 649
3.73 $\int (c+dx)^2\cos^2(a+bx)\sin(a+bx)\,dx$ . . . 658
3.74 $\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx$ . . . 665
3.75 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx$ . . . 670
3.76 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx$ . . . 675
3.77 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$ . . . 681
3.78 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$ . . . 687
3.79 $\int (c+dx)^m\cos^2(a+bx)\sin^2(a+bx)\,dx$ . . . 694
3.80 $\int (c+dx)^4\cos^2(a+bx)\sin^2(a+bx)\,dx$ . . . 699
3.81 $\int (c+dx)^3\cos^2(a+bx)\sin^2(a+bx)\,dx$ . . . 706
3.82 $\int (c+dx)^2\cos^2(a+bx)\sin^2(a+bx)\,dx$ . . . 713
3.83 $\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx$ . . . 719
3.84 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx$ . . . 724
3.85 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$ . . . 729
3.86 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$ . . . 735
3.87 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx$ . . . 741
3.88 $\int (c+dx)^m\cos^2(a+bx)\sin^3(a+bx)\,dx$ . . . 747
3.89 $\int (c+dx)^4\cos^2(a+bx)\sin^3(a+bx)\,dx$ . . . 753
3.90 $\int (c+dx)^3\cos^2(a+bx)\sin^3(a+bx)\,dx$ . . . 762
3.91 $\int (c+dx)^2\cos^2(a+bx)\sin^3(a+bx)\,dx$ . . . 769
3.92 $\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx$ . . . 775
3.93 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx$ . . . 781
3.94 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$ . . . 787
3.95 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$ . . . 794
3.96 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$ . . . 801
3.97 $\int (c+dx)^m\cos(a+bx)\cot(a+bx)\,dx$ . . . 809
3.98 $\int (c+dx)^4\cos(a+bx)\cot(a+bx)\,dx$ . . . 814
3.99 $\int (c+dx)^3\cos(a+bx)\cot(a+bx)\,dx$ . . . 828
3.100 $\int (c+dx)^2\cos(a+bx)\cot(a+bx)\,dx$ . . . 839
3.101 $\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx$ . . . 847
3.102 $\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx$ . . . 853
3.103 $\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$ . . . 859
3.104 $\int (c+dx)^m\cot^2(a+bx)\,dx$ . . . 865

3.105 $\int (c+dx)^4 \cot^2(a+bx)\,dx$ . . . 869
3.106 $\int (c+dx)^3 \cot^2(a+bx)\,dx$ . . . 879
3.107 $\int (c+dx)^2 \cot^2(a+bx)\,dx$ . . . 888
3.108 $\int (c+dx) \cot^2(a+bx)\,dx$ . . . 895
3.109 $\int \frac{\cot^2(a+bx)}{c+dx}\,dx$ . . . 901
3.110 $\int \frac{\cot^2(a+bx)}{(c+dx)^2}\,dx$ . . . 906
3.111 $\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx$ . . . 911
3.112 $\int (c+dx)^4 \cot^2(a+bx)\csc(a+bx)\,dx$ . . . 916
3.113 $\int (c+dx)^3 \cot^2(a+bx)\csc(a+bx)\,dx$ . . . 932
3.114 $\int (c+dx)^2 \cot^2(a+bx)\csc(a+bx)\,dx$ . . . 944
3.115 $\int (c+dx) \cot^2(a+bx)\csc(a+bx)\,dx$ . . . 955
3.116 $\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$ . . . 963
3.117 $\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$ . . . 969
3.118 $\int (c+dx)^{5/2} \cos^2(a+bx)\sin(a+bx)\,dx$ . . . 975
3.119 $\int (c+dx)^{3/2} \cos^2(a+bx)\sin(a+bx)\,dx$ . . . 983
3.120 $\int \sqrt{c+dx} \cos^2(a+bx)\sin(a+bx)\,dx$ . . . 991
3.121 $\int \sqrt{c+dx} \cos^2(a+bx)\sin(a+bx)\,dx$ . . . 998
3.122 $\int (c+dx)^{3/2} \cos^2(a+bx)\sin(a+bx)\,dx$ . . . 1005
3.123 $\int (c+dx)^{5/2} \cos^2(a+bx)\sin(a+bx)\,dx$ . . . 1013
3.124 $\int (c+dx)^{5/2} \cos^2(a+bx)\sin^2(a+bx)\,dx$ . . . 1021
3.125 $\int (c+dx)^{3/2} \cos^2(a+bx)\sin^2(a+bx)\,dx$ . . . 1028
3.126 $\int \sqrt{c+dx} \cos^2(a+bx)\sin^2(a+bx)\,dx$ . . . 1034
3.127 $\int \sqrt{c+dx} \cos^2(a+bx)\sin^2(a+bx)\,dx$ . . . 1040
3.128 $\int (c+dx)^{3/2} \cos^2(a+bx)\sin^2(a+bx)\,dx$ . . . 1046
3.129 $\int (c+dx)^{5/2} \cos^2(a+bx)\sin^2(a+bx)\,dx$ . . . 1052
3.130 $\int (c+dx)^{5/2} \cos^2(a+bx)\sin^3(a+bx)\,dx$ . . . 1059
3.131 $\int (c+dx)^{3/2} \cos^2(a+bx)\sin^3(a+bx)\,dx$ . . . 1069
3.132 $\int \sqrt{c+dx} \cos^2(a+bx)\sin^3(a+bx)\,dx$ . . . 1077
3.133 $\int \sqrt{c+dx} \cos^2(a+bx)\sin^3(a+bx)\,dx$ . . . 1085
3.134 $\int (c+dx)^{3/2} \cos^2(a+bx)\sin^3(a+bx)\,dx$ . . . 1093
3.135 $\int (c+dx)^{5/2} \cos^2(a+bx)\sin^3(a+bx)\,dx$ . . . 1101
3.136 $\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\,dx$ . . . 1111
3.137 $\int (c+dx)^4 \cos^3(a+bx)\sin(a+bx)\,dx$ . . . 1116
3.138 $\int (c+dx)^3 \cos^3(a+bx)\sin(a+bx)\,dx$ . . . 1126
3.139 $\int (c+dx)^2 \cos^3(a+bx)\sin(a+bx)\,dx$ . . . 1135
3.140 $\int (c+dx) \cos^3(a+bx)\sin(a+bx)\,dx$ . . . 1142
3.141 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx$ . . . 1148
3.142 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx$ . . . 1154
3.143 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$ . . . 1160

3.144 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$ . . . . . 1166
3.145 $\int (c+dx)^m \cos^3(a+bx)\sin^2(a+bx)\,dx$ . . . . . 1173
3.146 $\int (c+dx)^4 \cos^3(a+bx)\sin^2(a+bx)\,dx$ . . . . . 1179
3.147 $\int (c+dx)^3 \cos^3(a+bx)\sin^2(a+bx)\,dx$ . . . . . 1188
3.148 $\int (c+dx)^2 \cos^3(a+bx)\sin^2(a+bx)\,dx$ . . . . . 1195
3.149 $\int (c+dx) \cos^3(a+bx)\sin^2(a+bx)\,dx$ . . . . . 1202
3.150 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx$ . . . . . 1208
3.151 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$ . . . . . 1214
3.152 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$ . . . . . 1220
3.153 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx$ . . . . . 1227
3.154 $\int (c+dx)^m \cos^3(a+bx)\sin^3(a+bx)\,dx$ . . . . . 1235
3.155 $\int (c+dx)^4 \cos^3(a+bx)\sin^3(a+bx)\,dx$ . . . . . 1240
3.156 $\int (c+dx)^3 \cos^3(a+bx)\sin^3(a+bx)\,dx$ . . . . . 1247
3.157 $\int (c+dx)^2 \cos^3(a+bx)\sin^3(a+bx)\,dx$ . . . . . 1254
3.158 $\int (c+dx) \cos^3(a+bx)\sin^3(a+bx)\,dx$ . . . . . 1260
3.159 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx$ . . . . . 1265
3.160 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$ . . . . . 1271
3.161 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$ . . . . . 1277
3.162 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$ . . . . . 1284
3.163 $\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx$ . . . . . 1291
3.164 $\int (c+dx)^4 \cos^2(a+bx)\cot(a+bx)\,dx$ . . . . . 1297
3.165 $\int (c+dx)^3 \cos^2(a+bx)\cot(a+bx)\,dx$ . . . . . 1310
3.166 $\int (c+dx)^2 \cos^2(a+bx)\cot(a+bx)\,dx$ . . . . . 1322
3.167 $\int (c+dx) \cos^2(a+bx)\cot(a+bx)\,dx$ . . . . . 1331
3.168 $\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx$ . . . . . 1339
3.169 $\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$ . . . . . 1346
3.170 $\int (c+dx)^m \cos(a+bx)\cot^2(a+bx)\,dx$ . . . . . 1353
3.171 $\int (c+dx)^4 \cos(a+bx)\cot^2(a+bx)\,dx$ . . . . . 1358
3.172 $\int (c+dx)^3 \cos(a+bx)\cot^2(a+bx)\,dx$ . . . . . 1371
3.173 $\int (c+dx)^2 \cos(a+bx)\cot^2(a+bx)\,dx$ . . . . . 1381
3.174 $\int (c+dx) \cos(a+bx)\cot^2(a+bx)\,dx$ . . . . . 1389
3.175 $\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx$ . . . . . 1396
3.176 $\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx$ . . . . . 1402
3.177 $\int (c+dx)^m \cot^3(a+bx)\,dx$ . . . . . 1409
3.178 $\int (c+dx)^4 \cot^3(a+bx)\,dx$ . . . . . 1414
3.179 $\int (c+dx)^3 \cot^3(a+bx)\,dx$ . . . . . 1429
3.180 $\int (c+dx)^2 \cot^3(a+bx)\,dx$ . . . . . 1443
3.181 $\int (c+dx) \cot^3(a+bx)\,dx$ . . . . . 1454

3.182 $\int \frac{\cot^3(a+bx)}{c+dx}\,dx$ . . . . 1462
3.183 $\int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx$ . . . . 1468
3.184 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx$ . . . . 1474
3.185 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx$ . . . . 1482
3.186 $\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx$ . . . . 1489
3.187 $\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx$ . . . . 1495
3.188 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx$ . . . . 1501
3.189 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx$ . . . . 1508
3.190 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$ . . . . 1516
3.191 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$ . . . . 1526
3.192 $\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx$ . . . . 1534
3.193 $\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx$ . . . . 1541
3.194 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$ . . . . 1548
3.195 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$ . . . . 1556
3.196 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$ . . . . 1566
3.197 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$ . . . . 1574
3.198 $\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx$ . . . . 1581
3.199 $\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx$ . . . . 1588
3.200 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$ . . . . 1595
3.201 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$ . . . . 1602
3.202 $\int x^3\cos^2(x)\cot^2(x)\,dx$ . . . . 1610
3.203 $\int x^2\cos^2(x)\cot^2(x)\,dx$ . . . . 1618
3.204 $\int x\cos^2(x)\cot^2(x)\,dx$ . . . . 1626
3.205 $\int x^3\cos^2(x)\cot^3(x)\,dx$ . . . . 1632
3.206 $\int x^2\cos^2(x)\cot^3(x)\,dx$ . . . . 1644
3.207 $\int x\cos^2(x)\cot^3(x)\,dx$ . . . . 1655
3.208 $\int (c+dx)^m\tan(a+bx)\,dx$ . . . . 1664
3.209 $\int (c+dx)^4\tan(a+bx)\,dx$ . . . . 1668
3.210 $\int (c+dx)^3\tan(a+bx)\,dx$ . . . . 1677
3.211 $\int (c+dx)^2\tan(a+bx)\,dx$ . . . . 1685
3.212 $\int (c+dx)\tan(a+bx)\,dx$ . . . . 1691
3.213 $\int \frac{\tan(a+bx)}{c+dx}\,dx$ . . . . 1697
3.214 $\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx$ . . . . 1701
3.215 $\int (c+dx)^m\sin(a+bx)\tan(a+bx)\,dx$ . . . . 1705
3.216 $\int (c+dx)^3\sin(a+bx)\tan(a+bx)\,dx$ . . . . 1710
3.217 $\int (c+dx)^2\sin(a+bx)\tan(a+bx)\,dx$ . . . . 1721
3.218 $\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx$ . . . . 1729
3.219 $\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx$ . . . . 1735
3.220 $\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$ . . . . 1741
3.221 $\int (c+dx)^m\sin^2(a+bx)\tan(a+bx)\,dx$ . . . . 1747

3.222 $\int (c+dx)^3 \sin^2(a+bx) \tan(a+bx) \, dx$ . . . 1753
3.223 $\int (c+dx)^2 \sin^2(a+bx) \tan(a+bx) \, dx$ . . . 1765
3.224 $\int (c+dx) \sin^2(a+bx) \tan(a+bx) \, dx$ . . . 1774
3.225 $\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx} \, dx$ . . . 1781
3.226 $\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2} \, dx$ . . . 1787
3.227 $\int (c+dx)^m \csc(a+bx) \sec(a+bx) \, dx$ . . . 1794
3.228 $\int (c+dx)^4 \csc(a+bx) \sec(a+bx) \, dx$ . . . 1798
3.229 $\int (c+dx)^3 \csc(a+bx) \sec(a+bx) \, dx$ . . . 1807
3.230 $\int (c+dx)^2 \csc(a+bx) \sec(a+bx) \, dx$ . . . 1816
3.231 $\int (c+dx) \csc(a+bx) \sec(a+bx) \, dx$ . . . 1824
3.232 $\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx} \, dx$ . . . 1830
3.233 $\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2} \, dx$ . . . 1835
3.234 $\int (c+dx)^m \csc^2(a+bx) \sec(a+bx) \, dx$ . . . 1840
3.235 $\int (c+dx)^3 \csc^2(a+bx) \sec(a+bx) \, dx$ . . . 1844
3.236 $\int (c+dx)^2 \csc^2(a+bx) \sec(a+bx) \, dx$ . . . 1852
3.237 $\int (c+dx) \csc^2(a+bx) \sec(a+bx) \, dx$ . . . 1860
3.238 $\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx} \, dx$ . . . 1866
3.239 $\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2} \, dx$ . . . 1871
3.240 $\int (c+dx)^m \csc^3(a+bx) \sec(a+bx) \, dx$ . . . 1876
3.241 $\int (c+dx)^3 \csc^3(a+bx) \sec(a+bx) \, dx$ . . . 1880
3.242 $\int (c+dx)^2 \csc^3(a+bx) \sec(a+bx) \, dx$ . . . 1888
3.243 $\int (c+dx) \csc^3(a+bx) \sec(a+bx) \, dx$ . . . 1897
3.244 $\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx} \, dx$ . . . 1903
3.245 $\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2} \, dx$ . . . 1908
3.246 $\int (c+dx)^m \sec(a+bx) \tan(a+bx) \, dx$ . . . 1913
3.247 $\int (c+dx)^4 \sec(a+bx) \tan(a+bx) \, dx$ . . . 1917
3.248 $\int (c+dx)^3 \sec(a+bx) \tan(a+bx) \, dx$ . . . 1926
3.249 $\int (c+dx)^2 \sec(a+bx) \tan(a+bx) \, dx$ . . . 1934
3.250 $\int (c+dx) \sec(a+bx) \tan(a+bx) \, dx$ . . . 1940
3.251 $\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx} \, dx$ . . . 1946
3.252 $\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2} \, dx$ . . . 1950
3.253 $\int (c+dx)^m \tan^2(a+bx) \, dx$ . . . 1954
3.254 $\int (c+dx)^3 \tan^2(a+bx) \, dx$ . . . 1958
3.255 $\int (c+dx)^2 \tan^2(a+bx) \, dx$ . . . 1967
3.256 $\int (c+dx) \tan^2(a+bx) \, dx$ . . . 1974
3.257 $\int \frac{\tan^2(a+bx)}{c+dx} \, dx$ . . . 1980
3.258 $\int \frac{\tan^2(a+bx)}{(c+dx)^2} \, dx$ . . . 1985
3.259 $\int (c+dx)^m \sin(a+bx) \tan^2(a+bx) \, dx$ . . . 1990
3.260 $\int (c+dx)^3 \sin(a+bx) \tan^2(a+bx) \, dx$ . . . 1995

3.261 $\int (c+dx)^2 \sin(a+bx)\tan^2(a+bx)\,dx$ . . . . . 2005
3.262 $\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx$ . . . . . 2013
3.263 $\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$ . . . . . 2020
3.264 $\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$ . . . . . 2026
3.265 $\int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx$ . . . . . 2033
3.266 $\int (c+dx)^4 \csc(a+bx)\sec^2(a+bx)\,dx$ . . . . . 2037
3.267 $\int (c+dx)^3 \csc(a+bx)\sec^2(a+bx)\,dx$ . . . . . 2046
3.268 $\int (c+dx)^2 \csc(a+bx)\sec^2(a+bx)\,dx$ . . . . . 2055
3.269 $\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx$ . . . . . 2063
3.270 $\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx$ . . . . . 2069
3.271 $\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$ . . . . . 2074
3.272 $\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx$ . . . . . 2079
3.273 $\int (c+dx)^3 \csc^2(a+bx)\sec^2(a+bx)\,dx$ . . . . . 2083
3.274 $\int (c+dx)^2 \csc^2(a+bx)\sec^2(a+bx)\,dx$ . . . . . 2092
3.275 $\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx$ . . . . . 2100
3.276 $\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$ . . . . . 2106
3.277 $\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$ . . . . . 2111
3.278 $\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx$ . . . . . 2116
3.279 $\int (c+dx)^3 \csc^3(a+bx)\sec^2(a+bx)\,dx$ . . . . . 2120
3.280 $\int (c+dx)^2 \csc^3(a+bx)\sec^2(a+bx)\,dx$ . . . . . 2130
3.281 $\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx$ . . . . . 2139
3.282 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx$ . . . . . 2146
3.283 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$ . . . . . 2151
3.284 $\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx$ . . . . . 2156
3.285 $\int x^3 \csc^3(a+bx)\sec^2(a+bx)\,dx$ . . . . . 2160
3.286 $\int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx$ . . . . . 2168
3.287 $\int x \csc^3(a+bx)\sec^2(a+bx)\,dx$ . . . . . 2176
3.288 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx$ . . . . . 2182
3.289 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$ . . . . . 2187
3.290 $\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx$ . . . . . 2192
3.291 $\int (c+dx)^4 \sec^2(a+bx)\tan(a+bx)\,dx$ . . . . . 2196
3.292 $\int (c+dx)^3 \sec^2(a+bx)\tan(a+bx)\,dx$ . . . . . 2205
3.293 $\int (c+dx)^2 \sec^2(a+bx)\tan(a+bx)\,dx$ . . . . . 2213
3.294 $\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx$ . . . . . 2220
3.295 $\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx$ . . . . . 2226
3.296 $\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$ . . . . . 2231
3.297 $\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx$ . . . . . 2236
3.298 $\int (c+dx)^3 \sec(a+bx)\tan^2(a+bx)\,dx$ . . . . . 2241
3.299 $\int (c+dx)^2 \sec(a+bx)\tan^2(a+bx)\,dx$ . . . . . 2253

3.300 $\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx$ . . . 2264
3.301 $\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx$ . . . 2272
3.302 $\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$ . . . 2278
3.303 $\int (c+dx)^m\tan^3(a+bx)\,dx$ . . . 2284
3.304 $\int (c+dx)^3\tan^3(a+bx)\,dx$ . . . 2288
3.305 $\int (c+dx)^2\tan^3(a+bx)\,dx$ . . . 2301
3.306 $\int (c+dx)\tan^3(a+bx)\,dx$ . . . 2310
3.307 $\int \frac{\tan^3(a+bx)}{c+dx}\,dx$ . . . 2317
3.308 $\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx$ . . . 2322
3.309 $\int (c+dx)^m\csc(a+bx)\sec^3(a+bx)\,dx$ . . . 2327
3.310 $\int (c+dx)^4\csc(a+bx)\sec^3(a+bx)\,dx$ . . . 2331
3.311 $\int (c+dx)^3\csc(a+bx)\sec^3(a+bx)\,dx$ . . . 2341
3.312 $\int (c+dx)^2\csc(a+bx)\sec^3(a+bx)\,dx$ . . . 2349
3.313 $\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx$ . . . 2358
3.314 $\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$ . . . 2364
3.315 $\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$ . . . 2369
3.316 $\int (c+dx)^m\csc^2(a+bx)\sec^3(a+bx)\,dx$ . . . 2374
3.317 $\int (c+dx)^3\csc^2(a+bx)\sec^3(a+bx)\,dx$ . . . 2378
3.318 $\int (c+dx)^2\csc^2(a+bx)\sec^3(a+bx)\,dx$ . . . 2387
3.319 $\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx$ . . . 2397
3.320 $\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$ . . . 2404
3.321 $\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$ . . . 2409
3.322 $\int (c+dx)^m\csc^3(a+bx)\sec^3(a+bx)\,dx$ . . . 2414
3.323 $\int (c+dx)^3\csc^3(a+bx)\sec^3(a+bx)\,dx$ . . . 2418
3.324 $\int (c+dx)^2\csc^3(a+bx)\sec^3(a+bx)\,dx$ . . . 2428
3.325 $\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx$ . . . 2437
3.326 $\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$ . . . 2444
3.327 $\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$ . . . 2449
3.328 $\int x\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx$ . . . 2454
3.329 $\int x\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx$ . . . 2459
3.330 $\int x\sqrt{\cos(a+bx)}\sin(a+bx)\,dx$ . . . 2464
3.331 $\int \frac{x\sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx$ . . . 2469
3.332 $\int \frac{x\sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx$ . . . 2474
3.333 $\int \frac{x\sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx$ . . . 2478
3.334 $\int \frac{x\sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx$ . . . 2483
3.335 $\int \frac{x\sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\,dx$ . . . 2488
3.336 $\int x\sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx$ . . . 2493

3.337 $\int x \sec^{\frac{7}{2}}(a+bx)\sin(a+bx)\,dx$ . . . 2499
3.338 $\int x \sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx$ . . . 2504
3.339 $\int x \sec^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx$ . . . 2509
3.340 $\int x\sqrt{\sec(a+bx)}\sin(a+bx)\,dx$ . . . 2514
3.341 $\int \frac{x\sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx$ . . . 2519
3.342 $\int \frac{x\sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx$ . . . 2524
3.343 $\int \frac{x\sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx$ . . . 2529
3.344 $\int x\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)\,dx$ . . . 2535
3.345 $\int x\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)\,dx$ . . . 2540
3.346 $\int x\cos(a+bx)\sqrt{\sin(a+bx)}\,dx$ . . . 2545
3.347 $\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx$ . . . 2550
3.348 $\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx$ . . . 2555
3.349 $\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx$ . . . 2559
3.350 $\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx$ . . . 2564
3.351 $\int \frac{x\cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx$ . . . 2569
3.352 $\int x\cos(a+bx)\csc^{\frac{9}{2}}(a+bx)\,dx$ . . . 2574
3.353 $\int x\cos(a+bx)\csc^{\frac{7}{2}}(a+bx)\,dx$ . . . 2580
3.354 $\int x\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)\,dx$ . . . 2585
3.355 $\int x\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)\,dx$ . . . 2590
3.356 $\int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx$ . . . 2595
3.357 $\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$ . . . 2600
3.358 $\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx$ . . . 2605
3.359 $\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx$ . . . 2610
3.360 $\int x\csc(x)\sin(3x)\,dx$ . . . 2616
3.361 $\int (c+dx)^4\csc(x)\sin(3x)\,dx$ . . . 2620
3.362 $\int (c+dx)^3\csc(x)\sin(3x)\,dx$ . . . 2626
3.363 $\int (c+dx)^2\csc(x)\sin(3x)\,dx$ . . . 2631
3.364 $\int (c+dx)\csc(x)\sin(3x)\,dx$ . . . 2636
3.365 $\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx$ . . . 2640
3.366 $\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx$ . . . 2644
3.367 $\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx$ . . . 2649
3.368 $\int (c+dx)^4\csc(a+bx)\sin(3a+3bx)\,dx$ . . . 2654
3.369 $\int (c+dx)^3\csc(a+bx)\sin(3a+3bx)\,dx$ . . . 2660
3.370 $\int (c+dx)^2\csc(a+bx)\sin(3a+3bx)\,dx$ . . . 2666
3.371 $\int (c+dx)\csc(a+bx)\sin(3a+3bx)\,dx$ . . . 2671
3.372 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx$ . . . 2676

3.373 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$ . . . . . . . . . . 2681
3.374 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$ . . . . . . . . . . 2686
3.375 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx$ . . . . . . . . . . 2692
3.376 $\int (c+dx)^3\csc^2(a+bx)\sin(3a+3bx)\,dx$ . . . . . . . . . . 2698
3.377 $\int (c+dx)^2\csc^2(a+bx)\sin(3a+3bx)\,dx$ . . . . . . . . . . 2705
3.378 $\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx$ . . . . . . . . . . 2711
3.379 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$ . . . . . . . . . . 2716
3.380 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$ . . . . . . . . . . 2721
3.381 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$ . . . . . . . . . . 2726
3.382 $\int (c+dx)^4\sec(a+bx)\sin(3a+3bx)\,dx$ . . . . . . . . . . 2731
3.383 $\int (c+dx)^3\sec(a+bx)\sin(3a+3bx)\,dx$ . . . . . . . . . . 2739
3.384 $\int (c+dx)^2\sec(a+bx)\sin(3a+3bx)\,dx$ . . . . . . . . . . 2746
3.385 $\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx$ . . . . . . . . . . 2752
3.386 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx$ . . . . . . . . . . 2757
3.387 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$ . . . . . . . . . . 2762
3.388 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$ . . . . . . . . . . 2767
3.389 $\int (c+dx)^3\sec^2(a+bx)\sin(3a+3bx)\,dx$ . . . . . . . . . . 2772
3.390 $\int (c+dx)^2\sec^2(a+bx)\sin(3a+3bx)\,dx$ . . . . . . . . . . 2779
3.391 $\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx$ . . . . . . . . . . 2785
3.392 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$ . . . . . . . . . . 2790
3.393 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$ . . . . . . . . . . 2795
3.394 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$ . . . . . . . . . . 2800
3.395 $\int x\cos(2x)\sec(x)\,dx$ . . . . . . . . . . 2805
3.396 $\int x\cos(2x)\sec^2(x)\,dx$ . . . . . . . . . . 2810
3.397 $\int x\cos(2x)\sec^3(x)\,dx$ . . . . . . . . . . 2815

# 3.1 $\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx$


3.1.1 Optimal result . . . 151
3.1.2 Mathematica [A] (verified) . . . 151
3.1.3 Rubi [A] (verified) . . . 152
3.1.4 Maple [F] . . . 154
3.1.5 Fricas [A] (verification not implemented) . . . 154
3.1.6 Sympy [F] . . . 154
3.1.7 Maxima [F] . . . 155
3.1.8 Giac [F] . . . 155
3.1.9 Mupad [F(-1)] . . . 155


## 3.1.1 Optimal result

Integrand size = 20, antiderivative size = 137

$$\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx = -\frac{2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} - \frac{2^{-3-m}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2ib(c+dx)}{d}\right)}{b}$$

output
```
-2^(-3-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*
b*(d*x+c)/d)^m)-2^(-3-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a
-b*c/d))/((I*b*(d*x+c)/d)^m)
```

## 3.1.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01

$$\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx = -\frac{2^{-3-m}e^{-\frac{2i(bc+ad)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(e^{4ia}\left(\frac{ib(c+dx)}{d}\right)^m\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)+e^{\frac{4ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m\Gamma\left(\right.\right.}{b}$$

input
```
Integrate[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
-((2^(-3 - m)*(c + d*x)^m*(E^((4*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m,
((-2*I)*b*(c + d*x))/d] + E^(((4*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma
[1 + m, ((2*I)*b*(c + d*x))/d]))/(b*E^(((2*I)*(b*c + a*d))/d)*((b^2*(c + d
*x)^2)/d^2)^m))
```

### 3.1.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4906, 27, 3042, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cos(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{ 4906}$$

$$\int \frac{1}{2}\sin(2a+2bx)(c+dx)^m dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int (c+dx)^m\sin(2a+2bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int (c+dx)^m\sin(2a+2bx)dx$$

$$\Big\downarrow \text{ 3789}$$

$$\frac{1}{2}\left(\frac{1}{2}i\int e^{-2i(a+bx)}(c+dx)^m dx-\frac{1}{2}i\int e^{2i(a+bx)}(c+dx)^m dx\right)$$

$$\Big\downarrow \text{ 2612}$$

$$\frac{1}{2}\left(-\frac{2^{-m-2}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-m-2}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}}{b}\right.$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
(-((2^(-2 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*
(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)) - (2^(-2 - m)*(c + d*x)^m*Gam
ma[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*
x))/d)^m))/2
```

### 3.1.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.1.4 Maple [F]

$$\int (dx+c)^m \cos(xb+a)\sin(xb+a)\,dx$$

input
```
int((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x)
```

output
```
int((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x)
```

### 3.1.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.70

$$\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx = -\frac{e^{\left(-\frac{dm\log\left(-\frac{2ib}{d}\right)+2i\,bc-2i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{2\,(i\,bdx+i\,bc)}{d}\right)+e^{\left(-\frac{dm\log\left(\frac{2ib}{d}\right)-2i\,bc+2i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{2\,(-i\,bdx-i\,bc)}{d}\right)}{8\,b}$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/8*(e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)/d)*gamma(m + 1, -2*(I*b*
d*x + I*b*c)/d) + e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a*d)/d)*gamma(m +
1, -2*(-I*b*d*x - I*b*c)/d))/b
```

### 3.1.6 Sympy [F]

$$\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx = \int (c+dx)^m \sin(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*cos(b*x+a)*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**m*sin(a + b*x)*cos(a + b*x), x)
```

### 3.1.7 Maxima [F]

$$\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\sin(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a), x)
```

### 3.1.8 Giac [F]

$$\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\sin(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a), x)
```

### 3.1.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)\,(c+d\,x)^m\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^m, x)
```

# 3.2 $\int (c+dx)^4 \cos(a+bx) \sin(a+bx) \, dx$


3.2.1 Optimal result . . . . . . 156
3.2.2 Mathematica [A] (verified) . . . . . . 157
3.2.3 Rubi [A] (verified) . . . . . . 157
3.2.4 Maple [A] (verified) . . . . . . 160
3.2.5 Fricas [A] (verification not implemented) . . . . . . 160
3.2.6 Sympy [B] (verification not implemented) . . . . . . 161
3.2.7 Maxima [B] (verification not implemented) . . . . . . 161
3.2.8 Giac [A] (verification not implemented) . . . . . . 162
3.2.9 Mupad [B] (verification not implemented) . . . . . . 163


## 3.2.1 Optimal result

Integrand size = 20, antiderivative size = 156

$$\begin{aligned}\int (c+dx)^4 \cos(a+bx) \sin(a+bx) \, dx = {} & \frac{3cd^3x}{2b^3} + \frac{3d^4x^2}{4b^3} - \frac{(c+dx)^4}{4b} \\ & - \frac{3d^3(c+dx)\cos(a+bx)\sin(a+bx)}{2b^4} \\ & + \frac{d(c+dx)^3\cos(a+bx)\sin(a+bx)}{b^2} \\ & + \frac{3d^4\sin^2(a+bx)}{4b^5} - \frac{3d^2(c+dx)^2\sin^2(a+bx)}{2b^3} \\ & + \frac{(c+dx)^4\sin^2(a+bx)}{2b}\end{aligned}$$

output
```
3/2*c*d^3*x/b^3+3/4*d^4*x^2/b^3-1/4*(d*x+c)^4/b-3/2*d^3*(d*x+c)*cos(b*x+a)
*sin(b*x+a)/b^4+d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2+3/4*d^4*sin(b*x+a)^2
/b^5-3/2*d^2*(d*x+c)^2*sin(b*x+a)^2/b^3+1/2*(d*x+c)^4*sin(b*x+a)^2/b
```

### 3.2.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.55

$$\int (c+dx)^4 \cos(a+bx)\sin(a+bx)\,dx$$
$$= \frac{-2(3d^4-6b^2d^2(c+dx)^2+2b^4(c+dx)^4)\cos(2(a+bx))+4bd(c+dx)\left(-3d^2+2b^2(c+dx)^2\right)\sin(2(a+b}{16b^5}$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
(-2*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a + b*x)] +
 4*b*d*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(16*b^5)
```

### 3.2.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {4904, 3042, 3792, 17, 3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sin(a+bx)\cos(a+bx)\,dx$$
$$\Big\downarrow \ 4904$$
$$\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{2d\int (c+dx)^3\sin^2(a+bx)dx}{b}$$
$$\Big\downarrow \ 3042$$
$$\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{2d\int (c+dx)^3\sin(a+bx)^2dx}{b}$$
$$\Big\downarrow \ 3792$$
$$\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{2d\left(-\frac{3d^2\int (c+dx)\sin^2(a+bx)dx}{2b^2}+\frac{1}{2}\int (c+dx)^3dx+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}\right)}{b}$$
$$\Big\downarrow \ 17$$

$$\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{2d\left(-\frac{3d^2\int(c+dx)\sin^2(a+bx)dx}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)}{b}$$

$\downarrow$ 3042

$$\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{2d\left(-\frac{3d^2\int(c+dx)\sin(a+bx)^2dx}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)}{b}$$

$\downarrow$ 3791

$$\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{2d\left(-\frac{3d^2\left(\frac{1}{2}\int(c+dx)dx+\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)}{b}$$

$\downarrow$ 17

$$\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{2d\left(-\frac{3d^2\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)}{b}$$

input
```
Int[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
((c + d*x)^4*Sin[a + b*x]^2)/(2*b) - (2*d*((c + d*x)^4/(8*d) - ((c + d*x)^
3*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (3*d*(c + d*x)^2*Sin[a + b*x]^2)/(4*b
^2) - (3*d^2*((c + d*x)^2/(4*d) - ((c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(2
*b) + (d*Sin[a + b*x]^2)/(4*b^2)))/(2*b^2)))/b
```

## 3.2. $\int(c+dx)^4\cos(a+bx)\sin(a+bx)\,dx$

### 3.2.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

### 3.2.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.14

| method | result |
|---|---|
| risch | $-\frac{(2d^4x^4b^4+8b^4c\,d^3x^3+12b^4c^2d^2x^2+8b^4c^3dx+2b^4c^4-6b^2d^4x^2-12b^2c\,d^3x-6b^2c^2d^2+3d^4)\cos(2xb+2a)}{8b^5}+\frac{d(2b^2d^3x^3\cdots}{}$ |
| parallelrisch | $\frac{-4\left(\left(\frac{1}{2}x^2d^2+cdx+c^2\right)b^2-\frac{3d^2}{2}\right)b^2xd\left(\frac{dx}{2}+c\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^4-8bd\left((dx+c)^2b^2-\frac{3d^2}{2}\right)(dx+c)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3+\left(\left(6d^4x^4+24\cdots\right.\right.}{}$ |
| norman | $\frac{\frac{\left(2b^4c^4-6b^2c^2d^2+3d^4\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}{b^5}+\frac{cd\left(2b^2c^2-3d^2\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{b^4}-\frac{d^4x^4}{4b}-\frac{c\,d^3x^3}{b}-\frac{3d^2\left(2b^2c^2-d^2\right)x^2}{4b^3}+\frac{2d^4x^3\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{b^2}-\cdots}{}$ |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |

input
```
int((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(2*b^4*d^4*x^4+8*b^4*c*d^3*x^3+12*b^4*c^2*d^2*x^2+8*b^4*c^3*d*x+2*b^4
*c^4-6*b^2*d^4*x^2-12*b^2*c*d^3*x-6*b^2*c^2*d^2+3*d^4)/b^5*cos(2*b*x+2*a)+
1/4/b^4*d*(2*b^2*d^3*x^3+6*b^2*c*d^2*x^2+6*b^2*c^2*d*x+2*b^2*c^3-3*d^3*x-3
*c*d^2)*sin(2*b*x+2*a)
```

### 3.2.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.63

$$\int (c+dx)^4\cos(a+bx)\sin(a+bx)\,dx$$
$$=\frac{b^4d^4x^4+4\,b^4cd^3x^3+3\,(2\,b^4c^2d^2-b^2d^4)x^2-(2\,b^4d^4x^4+8\,b^4cd^3x^3+2\,b^4c^4-6\,b^2c^2d^2+3\,d^4+6\,(2\,b^4c^2d^2\cdots}{}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 3*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 - (2*
b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b
^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^
2 + 2*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^
3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)*sin(b*x + a) + 2*(2*b^4*c^3*d - 3*b^2*c
*d^3)*x)/b^5
```

### 3.2.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(153) = 306$.

Time = 0.47 (sec) , antiderivative size = 502, normalized size of antiderivative = 3.22

$$\int (c+dx)^4 \cos(a+bx)\sin(a+bx)\,dx$$
$$= \begin{cases} \frac{c^4\sin^2(a+bx)}{2b} + \frac{c^3dx\sin^2(a+bx)}{b} - \frac{c^3dx\cos^2(a+bx)}{b} + \frac{3c^2d^2x^2\sin^2(a+bx)}{2b} - \frac{3c^2d^2x^2\cos^2(a+bx)}{2b} + \frac{cd^3x^3\sin^2(a+bx)}{b} - \frac{cd^3x^3\cos \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\sin(a)\cos(a) \end{cases}$$

input
```
integrate((d*x+c)**4*cos(b*x+a)*sin(b*x+a),x)
```

output
```
Piecewise((c**4*sin(a + b*x)**2/(2*b) + c**3*d*x*sin(a + b*x)**2/b - c**3*
d*x*cos(a + b*x)**2/b + 3*c**2*d**2*x**2*sin(a + b*x)**2/(2*b) - 3*c**2*d*
*2*x**2*cos(a + b*x)**2/(2*b) + c*d**3*x**3*sin(a + b*x)**2/b - c*d**3*x**
3*cos(a + b*x)**2/b + d**4*x**4*sin(a + b*x)**2/(4*b) - d**4*x**4*cos(a +
b*x)**2/(4*b) + c**3*d*sin(a + b*x)*cos(a + b*x)/b**2 + 3*c**2*d**2*x*sin(
a + b*x)*cos(a + b*x)/b**2 + 3*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)/b**2
+ d**4*x**3*sin(a + b*x)*cos(a + b*x)/b**2 - 3*c**2*d**2*sin(a + b*x)**2/(
2*b**3) - 3*c*d**3*x*sin(a + b*x)**2/(2*b**3) + 3*c*d**3*x*cos(a + b*x)**2
/(2*b**3) - 3*d**4*x**2*sin(a + b*x)**2/(4*b**3) + 3*d**4*x**2*cos(a + b*x
)**2/(4*b**3) - 3*c*d**3*sin(a + b*x)*cos(a + b*x)/(2*b**4) - 3*d**4*x*sin
(a + b*x)*cos(a + b*x)/(2*b**4) + 3*d**4*sin(a + b*x)**2/(4*b**5), Ne(b, 0
)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/
5)*sin(a)*cos(a), True))
```

### 3.2.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(142) = 284$.

Time = 0.24 (sec) , antiderivative size = 586, normalized size of antiderivative = 3.76

$$\int (c+dx)^4 \cos(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{4\,c^4\cos(bx+a)^2 - \frac{16\,ac^3d\cos(bx+a)^2}{b} + \frac{24\,a^2c^2d^2\cos(bx+a)^2}{b^2} - \frac{16\,a^3cd^3\cos(bx+a)^2}{b^3} + \frac{4\,a^4d^4\cos(bx+a)^2}{b^4} + \frac{4\,(2\,(bx+a)\cos(}{}}{}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/8*(4*c^4*cos(b*x + a)^2 - 16*a*c^3*d*cos(b*x + a)^2/b + 24*a^2*c^2*d^2*
cos(b*x + a)^2/b^2 - 16*a^3*c*d^3*cos(b*x + a)^2/b^3 + 4*a^4*d^4*cos(b*x +
 a)^2/b^4 + 4*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*c^3*d/b -
12*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a*c^2*d^2/b^2 + 12*(2
*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a^2*c*d^3/b^3 - 4*(2*(b*x
+ a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a^3*d^4/b^4 + 6*((2*(b*x + a)^2
- 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 - 12*((2
*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3
/b^3 + 6*((2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2
*a))*a^2*d^4/b^4 + 2*(2*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3
*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*c*d^3/b^3 - 2*(2*(2*(b*x + a)^3 - 3
*b*x - 3*a)*cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a*d
^4/b^4 + ((2*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*cos(2*b*x + 2*a) - 2*(2*(b*x
 + a)^3 - 3*b*x - 3*a)*sin(2*b*x + 2*a))*d^4/b^4)/b
```

### 3.2.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.16

$$\int (c+dx)^4\cos(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{(2\,b^4d^4x^4+8\,b^4cd^3x^3+12\,b^4c^2d^2x^2+8\,b^4c^3dx+2\,b^4c^4-6\,b^2d^4x^2-12\,b^2cd^3x-6\,b^2c^2d^2+3\,d^4)\cos(2\,b}{8\,b^5}$$
$$+\frac{(2\,b^3d^4x^3+6\,b^3cd^3x^2+6\,b^3c^2d^2x+2\,b^3c^3d-3\,bd^4x-3\,bcd^3)\sin(2\,bx+2\,a)}{4\,b^5}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/8*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x
 + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3*x - 6*b^2*c^2*d^2 + 3*d^4)*cos
(2*b*x + 2*a)/b^5 + 1/4*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x
 + 2*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

### 3.2.9 Mupad [B] (verification not implemented)

Time = 23.32 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.57

$$\int (c+dx)^4\cos(a+bx)\sin(a+bx)\,dx = \frac{3\,x^2\,\cos\left(2\,a+2\,b\,x\right)\,\left(d^4-2\,b^2\,c^2\,d^2\right)}{4\,b^3} - \frac{\cos\left(2\,a+2\,b\,x\right)\,\left(\frac{b^4\,c^4}{2}-\frac{3\,b^2\,c^2\,d^2}{2}+\frac{3\,d^4}{4}\right)}{2\,b^5} - \frac{3\,x\,\sin\left(2\,a+2\,b\,x\right)\,\left(d^4-2\,b^2\,c^2\,d^2\right)}{4\,b^4} - \frac{d^4\,x^4\,\cos\left(2\,a+2\,b\,x\right)}{4\,b} - \frac{\sin\left(2\,a+2\,b\,x\right)\,\left(3\,c\,d^3-2\,b^2\,c^3\,d\right)}{4\,b^4} + \frac{x\,\cos\left(2\,a+2\,b\,x\right)\,\left(3\,c\,d^3-2\,b^2\,c^3\,d\right)}{2\,b^3} + \frac{d^4\,x^3\,\sin\left(2\,a+2\,b\,x\right)}{2\,b^2} - \frac{c\,d^3\,x^3\,\cos\left(2\,a+2\,b\,x\right)}{b} + \frac{3\,c\,d^3\,x^2\,\sin\left(2\,a+2\,b\,x\right)}{2\,b^2}$$

input
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^4,x)
```

output
```
(3*x^2*cos(2*a + 2*b*x)*(d^4 - 2*b^2*c^2*d^2))/(4*b^3) - (cos(2*a + 2*b*x)
*((3*d^4)/4 + (b^4*c^4)/2 - (3*b^2*c^2*d^2)/2))/(2*b^5) - (3*x*sin(2*a + 2
*b*x)*(d^4 - 2*b^2*c^2*d^2))/(4*b^4) - (d^4*x^4*cos(2*a + 2*b*x))/(4*b) -
(sin(2*a + 2*b*x)*(3*c*d^3 - 2*b^2*c^3*d))/(4*b^4) + (x*cos(2*a + 2*b*x)*(
3*c*d^3 - 2*b^2*c^3*d))/(2*b^3) + (d^4*x^3*sin(2*a + 2*b*x))/(2*b^2) - (c*
d^3*x^3*cos(2*a + 2*b*x))/b + (3*c*d^3*x^2*sin(2*a + 2*b*x))/(2*b^2)
```

# 3.3 $\int (c+dx)^3 \cos(a+bx)\sin(a+bx)\,dx$


3.3.1 Optimal result . . . 164
3.3.2 Mathematica [A] (verified) . . . 164
3.3.3 Rubi [A] (verified) . . . 165
3.3.4 Maple [A] (verified) . . . 167
3.3.5 Fricas [A] (verification not implemented) . . . 168
3.3.6 Sympy [B] (verification not implemented) . . . 168
3.3.7 Maxima [B] (verification not implemented) . . . 169
3.3.8 Giac [A] (verification not implemented) . . . 169
3.3.9 Mupad [B] (verification not implemented) . . . 170


## 3.3.1 Optimal result

Integrand size = 20, antiderivative size = 120

$$\int (c+dx)^3\cos(a+bx)\sin(a+bx)\,dx=\frac{3d^3x}{8b^3}-\frac{(c+dx)^3}{4b}-\frac{3d^3\cos(a+bx)\sin(a+bx)}{8b^4}+\frac{3d(c+dx)^2\cos(a+bx)\sin(a+bx)}{4b^2}-\frac{3d^2(c+dx)\sin^2(a+bx)}{4b^3}+\frac{(c+dx)^3\sin^2(a+bx)}{2b}$$

output
```
3/8*d^3*x/b^3-1/4*(d*x+c)^3/b-3/8*d^3*cos(b*x+a)*sin(b*x+a)/b^4+3/4*d*(d*x
+c)^2*cos(b*x+a)*sin(b*x+a)/b^2-3/4*d^2*(d*x+c)*sin(b*x+a)^2/b^3+1/2*(d*x+
c)^3*sin(b*x+a)^2/b
```

## 3.3.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.59

$$\int (c+dx)^3\cos(a+bx)\sin(a+bx)\,dx=\frac{-2b(c+dx)\left(-3d^2+2b^2(c+dx)^2\right)\cos(2(a+bx))+3d\left(-d^2+2b^2(c+dx)^2\right)\sin(2(a+bx))}{16b^4}$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
(-2*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 3*d*(-d^2
+ 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(16*b^4)
```

### 3.3.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {4904, 3042, 3792, 17, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\sin(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{4904}$$

$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{3d\int (c+dx)^2\sin^2(a+bx)dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{3d\int (c+dx)^2\sin(a+bx)^2dx}{2b}$$

$$\Big\downarrow \text{3792}$$

$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{3d\left(-\frac{d^2\int \sin^2(a+bx)dx}{2b^2}+\frac{1}{2}\int (c+dx)^2dx+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b}$$

$$\Big\downarrow \text{17}$$

$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{3d\left(-\frac{d^2\int \sin^2(a+bx)dx}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{3d\left(-\frac{d^2\int \sin(a+bx)^2dx}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{3d\left(-\frac{d^2\left(\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}$$

$$\Big\downarrow \; 24$$

$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{3d\left(\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
((c + d*x)^3*Sin[a + b*x]^2)/(2*b) - (3*d*((c + d*x)^3/(6*d) - ((c + d*x)^
2*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (d*(c + d*x)*Sin[a + b*x]^2)/(2*b^2)
- (d^2*(x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)))/(2*b^2)))/(2*b)
```

### 3.3.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

### 3.3.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98

| method | result |
|---|---|
| risch | $-\frac{(2b^2d^3x^3+6b^2c\,d^2x^2+6b^2c^2dx+2b^2c^3-3d^3x-3c\,d^2)\cos(2xb+2a)}{8b^3}+\frac{3d(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2)\sin(2xb+2a)}{16b^4}$ |
| parallelrisch | $\frac{-3bxd\left(\left(\frac{1}{3}x^2d^2+cdx+c^2\right)b^2-\frac{d^2}{2}\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^4-6d\left((dx+c)^2b^2-\frac{d^2}{2}\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3+8b\left(\left(\frac{3}{4}d^3x^3+\frac{9}{4}c\,d^2x^2+\frac{9}{4}c^2dx+c\right.\right.}{4b^4\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right.\right.}$ |
| norman | $\frac{\frac{(2b^2c^3-3c\,d^2)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}{b^3}-\frac{d^3x^3}{4b}-\frac{3c\,d^2x^2}{4b}+\frac{3d(2b^2c^2-d^2)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{4b^4}-\frac{3d(2b^2c^2-d^2)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3}{4b^4}-\frac{3d(2b^2c^2-d^2)x}{8b^3}+}{}$ |
| derivativedivides | $\frac{\frac{a^3d^3\cos(xb+a)^2}{2b^3}-\frac{3a^2c\,d^2\cos(xb+a)^2}{2b^2}+\frac{3a^2d^3\left(-\frac{(xb+a)\cos(xb+a)^2}{2}+\frac{\cos(xb+a)\sin(xb+a)}{4}+\frac{xb}{4}+\frac{a}{4}\right)}{b^3}+\frac{3a\,c^2d\cos(xb+a)^2}{2b}-\frac{6ac\,c}{}}{}$ |
| default | $\frac{\frac{a^3d^3\cos(xb+a)^2}{2b^3}-\frac{3a^2c\,d^2\cos(xb+a)^2}{2b^2}+\frac{3a^2d^3\left(-\frac{(xb+a)\cos(xb+a)^2}{2}+\frac{\cos(xb+a)\sin(xb+a)}{4}+\frac{xb}{4}+\frac{a}{4}\right)}{b^3}+\frac{3a\,c^2d\cos(xb+a)^2}{2b}-\frac{6ac\,c}{}}{}$ |

input
```
int((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/8/b^3*(2*b^2*d^3*x^3+6*b^2*c*d^2*x^2+6*b^2*c^2*d*x+2*b^2*c^3-3*d^3*x-3*
c*d^2)*cos(2*b*x+2*a)+3/16*d*(2*b^2*d^2*x^2+4*b^2*c*d*x+2*b^2*c^2-d^2)/b^4
*sin(2*b*x+2*a)
```

### 3.3.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.38

$$\int (c+dx)^3\cos(a+bx)\sin(a+bx)\,dx$$
$$= \frac{2\,b^3d^3x^3+6\,b^3cd^2x^2-2\,(2\,b^3d^3x^3+6\,b^3cd^2x^2+2\,b^3c^3-3\,bcd^2+3\,(2\,b^3c^2d-bd^3)x)\cos{(bx+a)}^2+3\,(2\,b}{8\,b^4}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2
+ 2*b^3*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2
*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a
) + 3*(2*b^3*c^2*d - b*d^3)*x)/b^4
```

### 3.3.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(116) = 232$.

Time = 0.34 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.85

$$\int (c+dx)^3\cos(a+bx)\sin(a+bx)\,dx$$
$$= \begin{cases} \frac{c^3\sin^2{(a+bx)}}{2b}+\frac{3c^2dx\sin^2{(a+bx)}}{4b}-\frac{3c^2dx\cos^2{(a+bx)}}{4b}+\frac{3cd^2x^2\sin^2{(a+bx)}}{4b}-\frac{3cd^2x^2\cos^2{(a+bx)}}{4b}+\frac{d^3x^3\sin^2{(a+bx)}}{4b}-\frac{d^3x^3\cos}{} \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\sin{(a)}\cos{(a)} \end{cases}$$

input
```
integrate((d*x+c)**3*cos(b*x+a)*sin(b*x+a),x)
```

output
```
Piecewise((c**3*sin(a + b*x)**2/(2*b) + 3*c**2*d*x*sin(a + b*x)**2/(4*b) -
 3*c**2*d*x*cos(a + b*x)**2/(4*b) + 3*c*d**2*x**2*sin(a + b*x)**2/(4*b) -
3*c*d**2*x**2*cos(a + b*x)**2/(4*b) + d**3*x**3*sin(a + b*x)**2/(4*b) - d*
*3*x**3*cos(a + b*x)**2/(4*b) + 3*c**2*d*sin(a + b*x)*cos(a + b*x)/(4*b**2
) + 3*c*d**2*x*sin(a + b*x)*cos(a + b*x)/(2*b**2) + 3*d**3*x**2*sin(a + b*
x)*cos(a + b*x)/(4*b**2) - 3*c*d**2*sin(a + b*x)**2/(4*b**3) - 3*d**3*x*si
n(a + b*x)**2/(8*b**3) + 3*d**3*x*cos(a + b*x)**2/(8*b**3) - 3*d**3*sin(a
+ b*x)*cos(a + b*x)/(8*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d*
*2*x**3 + d**3*x**4/4)*sin(a)*cos(a), True))
```

### 3.3.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(108) = 216$.

Time = 0.24 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.85

$$\int (c+dx)^3 \cos(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{8\,c^3\cos\left(bx+a\right)^2 - \frac{24\,ac^2d\cos(bx+a)^2}{b} + \frac{24\,a^2cd^2\cos(bx+a)^2}{b^2} - \frac{8\,a^3d^3\cos(bx+a)^2}{b^3} + \frac{6\,(2\,(bx+a)\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a))c}{b}}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/16*(8*c^3*cos(b*x + a)^2 - 24*a*c^2*d*cos(b*x + a)^2/b + 24*a^2*c*d^2*c
os(b*x + a)^2/b^2 - 8*a^3*d^3*cos(b*x + a)^2/b^3 + 6*(2*(b*x + a)*cos(2*b*
x + 2*a) - sin(2*b*x + 2*a))*c^2*d/b - 12*(2*(b*x + a)*cos(2*b*x + 2*a) -
sin(2*b*x + 2*a))*a*c*d^2/b^2 + 6*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*
x + 2*a))*a^2*d^3/b^3 + 6*((2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b*x +
 a)*sin(2*b*x + 2*a))*c*d^2/b^2 - 6*((2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a)
- 2*(b*x + a)*sin(2*b*x + 2*a))*a*d^3/b^3 + (2*(2*(b*x + a)^3 - 3*b*x - 3*
a)*cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*d^3/b^3)/b
```

### 3.3.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.01

$$\int (c+dx)^3 \cos(a+bx)\sin(a+bx)\,dx = -\frac{(2\,b^3d^3x^3 + 6\,b^3cd^2x^2 + 6\,b^3c^2dx + 2\,b^3c^3 - 3\,bd^3x - 3\,bcd^2)\cos\left(2\,bx+2\,a\right)}{8\,b^4} + \frac{3\,(2\,b^2d^3x^2 + 4\,b^2cd^2x + 2\,b^2c^2d - d^3)\sin\left(2\,bx+2\,a\right)}{16\,b^4}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^
3*x - 3*b*c*d^2)*cos(2*b*x + 2*a)/b^4 + 3/16*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*
x + 2*b^2*c^2*d - d^3)*sin(2*b*x + 2*a)/b^4
```

### 3.3.9 Mupad [B] (verification not implemented)

Time = 22.51 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.38

$$\int (c+dx)^3\cos(a+bx)\sin(a+bx)\,dx = \frac{\cos(2a+2bx)\,\left(\frac{3\,c\,d^2}{4}-\frac{b^2\,c^3}{2}\right)}{2\,b^3} - \frac{3\,\sin(2a+2bx)\,\left(d^3-2\,b^2\,c^2\,d\right)}{16\,b^4} - \frac{d^3\,x^3\,\cos(2a+2bx)}{4\,b} + \frac{3\,d^3\,x^2\,\sin(2a+2bx)}{8\,b^2} + \frac{3\,x\,\cos(2a+2bx)\,\left(d^3-2\,b^2\,c^2\,d\right)}{8\,b^3} + \frac{3\,c\,d^2\,x\,\sin(2a+2bx)}{4\,b^2} - \frac{3\,c\,d^2\,x^2\,\cos(2a+2bx)}{4\,b}$$

input
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^3,x)
```

output
```
(cos(2*a + 2*b*x)*((3*c*d^2)/4 - (b^2*c^3)/2))/(2*b^3) - (3*sin(2*a + 2*b*
x)*(d^3 - 2*b^2*c^2*d))/(16*b^4) - (d^3*x^3*cos(2*a + 2*b*x))/(4*b) + (3*d
^3*x^2*sin(2*a + 2*b*x))/(8*b^2) + (3*x*cos(2*a + 2*b*x)*(d^3 - 2*b^2*c^2*
d))/(8*b^3) + (3*c*d^2*x*sin(2*a + 2*b*x))/(4*b^2) - (3*c*d^2*x^2*cos(2*a
+ 2*b*x))/(4*b)
```

# 3.4 $\int (c+dx)^2 \cos(a+bx)\sin(a+bx)\,dx$

3.4.1 Optimal result . . . . . 171
3.4.2 Mathematica [A] (verified) . . . . . 171
3.4.3 Rubi [A] (verified) . . . . . 172
3.4.4 Maple [A] (verified) . . . . . 173
3.4.5 Fricas [A] (verification not implemented) . . . . . 174
3.4.6 Sympy [B] (verification not implemented) . . . . . 174
3.4.7 Maxima [B] (verification not implemented) . . . . . 175
3.4.8 Giac [A] (verification not implemented) . . . . . 175
3.4.9 Mupad [B] (verification not implemented) . . . . . 176

## 3.4.1 Optimal result

Integrand size = 20, antiderivative size = 89

$$\int (c+dx)^2\cos(a+bx)\sin(a+bx)\,dx = -\frac{cdx}{2b}-\frac{d^2x^2}{4b}+\frac{d(c+dx)\cos(a+bx)\sin(a+bx)}{2b^2} -\frac{d^2\sin^2(a+bx)}{4b^3}+\frac{(c+dx)^2\sin^2(a+bx)}{2b}$$

output
```
-1/2*c*d*x/b-1/4*d^2*x^2/b+1/2*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2-1/4*d^2
*sin(b*x+a)^2/b^3+1/2*(d*x+c)^2*sin(b*x+a)^2/b
```

## 3.4.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.56

$$\int (c+dx)^2\cos(a+bx)\sin(a+bx)\,dx = \frac{(d^2-2b^2(c+dx)^2)\cos(2(a+bx))+2bd(c+dx)\sin(2(a+bx))}{8b^3}$$

input
```
Integrate[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
((d^2 - 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 2*b*d*(c + d*x)*Sin[2*(a + b
*x)])/(8*b^3)
```

### 3.4.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4904, 3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\sin(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{ 4904}$$

$$\frac{(c+dx)^2\sin^2(a+bx)}{2b}-\frac{d\int(c+dx)\sin^2(a+bx)dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(c+dx)^2\sin^2(a+bx)}{2b}-\frac{d\int(c+dx)\sin(a+bx)^2dx}{b}$$

$$\Big\downarrow \text{ 3791}$$

$$\frac{(c+dx)^2\sin^2(a+bx)}{2b}-\frac{d\left(\frac{1}{2}\int(c+dx)dx+\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}\right)}{b}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(c+dx)^2\sin^2(a+bx)}{2b}-\frac{d\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{b}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
((c + d*x)^2*Sin[a + b*x]^2)/(2*b) - (d*((c + d*x)^2/(4*d) - ((c + d*x)*Co
s[a + b*x]*Sin[a + b*x])/(2*b) + (d*Sin[a + b*x]^2)/(4*b^2)))/b
```

### 3.4.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

## 3.4.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

| method | result |
|---|---|
| risch | $-\frac{\left(2x^{2}d^{2}b^{2}+4b^{2}cdx+2b^{2}c^{2}-d^{2}\right)\cos\left(2xb+2a\right)}{8b^{3}}+\frac{d\left(dx+c\right)\sin\left(2xb+2a\right)}{4b^{2}}$ |
| parallelrisch | $\frac{-b^{2}xd\left(\frac{dx}{2}+c\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{4}-2bd\left(dx+c\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{3}+\left(\left(3x^{2}d^{2}+6cdx+4c^{2}\right)b^{2}-2d^{2}\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}+2bd\left(dx+c\right)\tan\left(\ldots\right.}{2b^{3}\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)^{2}}$ |
| derivativedivides | $\frac{-\frac{a^{2}d^{2}\cos\left(xb+a\right)^{2}}{2b^{2}}+\frac{acd\cos\left(xb+a\right)^{2}}{b}-\frac{2a\,d^{2}\left(-\frac{\left(xb+a\right)\cos\left(xb+a\right)^{2}}{2}+\frac{\cos\left(xb+a\right)\sin\left(xb+a\right)}{4}+\frac{xb}{4}+\frac{a}{4}\right)}{b^{2}}-\frac{c^{2}\cos\left(xb+a\right)^{2}}{2}+\frac{2cd\left(-\frac{\left(xb+\ldots\right.}{}\right.}{}}{}$ |
| default | $\frac{-\frac{a^{2}d^{2}\cos\left(xb+a\right)^{2}}{2b^{2}}+\frac{acd\cos\left(xb+a\right)^{2}}{b}-\frac{2a\,d^{2}\left(-\frac{\left(xb+a\right)\cos\left(xb+a\right)^{2}}{2}+\frac{\cos\left(xb+a\right)\sin\left(xb+a\right)}{4}+\frac{xb}{4}+\frac{a}{4}\right)}{b^{2}}-\frac{c^{2}\cos\left(xb+a\right)^{2}}{2}+\frac{2cd\left(-\frac{\left(xb+\ldots\right.}{}\right.}{}}{}$ |
| norman | $\frac{\frac{\left(2b^{2}c^{2}-d^{2}\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}}{b^{3}}+\frac{cd\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{b^{2}}+\frac{d^{2}x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{b^{2}}-\frac{d^{2}x^{2}}{4b}-\frac{cd\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{3}}{b^{2}}-\frac{cdx}{2b}-\frac{d^{2}x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{3}}{b^{2}}+\frac{3d^{2}x^{2}\tan\left(\ldots\right.}{2\ldots}}{\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)^{2}}$ |

input
```
int((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(2*b^2*d^2*x^2+4*b^2*c*d*x+2*b^2*c^2-d^2)/b^3*cos(2*b*x+2*a)+1/4*d*(d
*x+c)*sin(2*b*x+2*a)/b^2
```

### 3.4.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int (c+dx)^2\cos(a+bx)\sin(a+bx)\,dx$$
$$=\frac{b^2d^2x^2+2\,b^2cdx-(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2)\cos{(bx+a)}^2+2\,(bd^2x+bcd)\cos{(bx+a)}\sin{(bx+}}{4\,b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2
- d^2)*cos(b*x + a)^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/b^3
```

### 3.4.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(78) = 156.

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.97

$$\int (c+dx)^2\cos(a+bx)\sin(a+bx)\,dx$$
$$=\begin{cases}\frac{c^2\sin^2{(a+bx)}}{2b}+\frac{cdx\sin^2{(a+bx)}}{2b}-\frac{cdx\cos^2{(a+bx)}}{2b}+\frac{d^2x^2\sin^2{(a+bx)}}{4b}-\frac{d^2x^2\cos^2{(a+bx)}}{4b}+\frac{cd\sin{(a+bx)}\cos{(a+bx)}}{2b^2}+\frac{d^2x\sin{(a}}{} \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sin{(a)}\cos{(a)}\end{cases}$$

input
```
integrate((d*x+c)**2*cos(b*x+a)*sin(b*x+a),x)
```

output
```
Piecewise(((c**2*sin(a + b*x)**2/(2*b) + c*d*x*sin(a + b*x)**2/(2*b) - c*d*
x*cos(a + b*x)**2/(2*b) + d**2*x**2*sin(a + b*x)**2/(4*b) - d**2*x**2*cos(
a + b*x)**2/(4*b) + c*d*sin(a + b*x)*cos(a + b*x)/(2*b**2) + d**2*x*sin(a
+ b*x)*cos(a + b*x)/(2*b**2) - d**2*sin(a + b*x)**2/(4*b**3), Ne(b, 0)), (
(c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)*cos(a), True))
```

### 3.4.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(79) = 158.

Time = 0.23 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.92

$$\int (c+dx)^2 \cos(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{4\,c^2\cos\left(bx+a\right)^2 - \frac{8\,acd\cos(bx+a)^2}{b} + \frac{4\,a^2d^2\cos(bx+a)^2}{b^2} + \frac{2\,(2\,(bx+a)\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a))cd}{b} - \frac{2\,(2\,(bx+a)\cos(2\,bx+}{}}{8\,b}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/8*(4*c^2*cos(b*x + a)^2 - 8*a*c*d*cos(b*x + a)^2/b + 4*a^2*d^2*cos(b*x
+ a)^2/b^2 + 2*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*c*d/b - 2
*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a*d^2/b^2 + ((2*(b*x +
a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*d^2/b^2)/b
```

### 3.4.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

$$\int (c+dx)^2 \cos(a+bx)\sin(a+bx)\,dx = -\frac{\left(2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 - d^2\right)\cos\left(2\,bx+2\,a\right)}{8\,b^3} + \frac{\left(bd^2x + bcd\right)\sin\left(2\,bx+2\,a\right)}{4\,b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/8*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2*a)/b^3
+ 1/4*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a)/b^3
```

### 3.4.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.12

$$\int (c+dx)^2\cos(a+bx)\sin(a+bx)\,dx = \frac{\cos\left(2\,a+2\,b\,x\right)\,\left(\frac{d^2}{4}-\frac{b^2\,c^2}{2}\right)}{2\,b^3} + \frac{d^2\,x\,\sin\left(2\,a+2\,b\,x\right)}{4\,b^2} - \frac{d^2\,x^2\,\cos\left(2\,a+2\,b\,x\right)}{4\,b} + \frac{c\,d\,\sin\left(2\,a+2\,b\,x\right)}{4\,b^2} - \frac{c\,d\,x\,\cos\left(2\,a+2\,b\,x\right)}{2\,b}$$

input
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^2,x)
```

output
```
(cos(2*a + 2*b*x)*(d^2/4 - (b^2*c^2)/2))/(2*b^3) + (d^2*x*sin(2*a + 2*b*x)
)/(4*b^2) - (d^2*x^2*cos(2*a + 2*b*x))/(4*b) + (c*d*sin(2*a + 2*b*x))/(4*b
^2) - (c*d*x*cos(2*a + 2*b*x))/(2*b)
```

# 3.5 $\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx$

3.5.1 Optimal result . . . . . . . . . . 177
3.5.2 Mathematica [A] (verified) . . . . . . . . . . 177
3.5.3 Rubi [A] (verified) . . . . . . . . . . 178
3.5.4 Maple [A] (verified) . . . . . . . . . . 179
3.5.5 Fricas [A] (verification not implemented) . . . . . . . . . . 180
3.5.6 Sympy [A] (verification not implemented) . . . . . . . . . . 180
3.5.7 Maxima [A] (verification not implemented) . . . . . . . . . . 180
3.5.8 Giac [A] (verification not implemented) . . . . . . . . . . 181
3.5.9 Mupad [B] (verification not implemented) . . . . . . . . . . 181

## 3.5.1 Optimal result

Integrand size = 18, antiderivative size = 50

$$\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx = -\frac{dx}{4b}+\frac{d\cos(a+bx)\sin(a+bx)}{4b^2}+\frac{(c+dx)\sin^2(a+bx)}{2b}$$

output
```
-1/4*d*x/b+1/4*d*cos(b*x+a)*sin(b*x+a)/b^2+1/2*(d*x+c)*sin(b*x+a)^2/b
```

## 3.5.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx = \frac{-2b(c+dx)\cos(2(a+bx))+d\sin(2(a+bx))}{8b^2}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
(-2*b*(c + d*x)*Cos[2*(a + b*x)] + d*Sin[2*(a + b*x)])/(8*b^2)
```

### 3.5.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4904, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{4904}$$

$$\frac{(c+dx)\sin^2(a+bx)}{2b}-\frac{d\int \sin^2(a+bx)dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)\sin^2(a+bx)}{2b}-\frac{d\int \sin(a+bx)^2dx}{2b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{(c+dx)\sin^2(a+bx)}{2b}-\frac{d\left(\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b}$$

$$\Big\downarrow \text{24}$$

$$\frac{(c+dx)\sin^2(a+bx)}{2b}-\frac{d\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b}$$

input
```
Int[(c + d*x)*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
((c + d*x)*Sin[a + b*x]^2)/(2*b) - (d*(x/2 - (Cos[a + b*x]*Sin[a + b*x])/(
2*b)))/(2*b)
```

### 3.5.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

## 3.5.4 Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.72

| method | result | size |
|---|---|---|
| risch | $-\frac{(dx+c)\cos(2xb+2a)}{4b}+\frac{d\sin(2xb+2a)}{8b^2}$ | 36 |
| parallelrisch | $\frac{-2b(dx+c)\cos(2xb+2a)+2cb+d\sin(2xb+2a)}{8b^2}$ | 39 |
| derivativedivides | $\frac{\frac{da\cos(xb+a)^2}{2b}-\frac{c\cos(xb+a)^2}{2}+\frac{d\left(-\frac{(xb+a)\cos(xb+a)^2}{2}+\frac{\cos(xb+a)\sin(xb+a)}{4}+\frac{xb}{4}+\frac{a}{4}\right)}{b}}{b}$ | 74 |
| default | $\frac{\frac{da\cos(xb+a)^2}{2b}-\frac{c\cos(xb+a)^2}{2}+\frac{d\left(-\frac{(xb+a)\cos(xb+a)^2}{2}+\frac{\cos(xb+a)\sin(xb+a)}{4}+\frac{xb}{4}+\frac{a}{4}\right)}{b}}{b}$ | 74 |
| norman | $\frac{\frac{d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{2b^2}-\frac{d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3}{2b^2}-\frac{dx}{4b}+\frac{2c\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}{b}+\frac{3dx\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}{2b}-\frac{dx\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^4}{4b}}{\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)^2}$ | 110 |

input
```
int((d*x+c)*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(d*x+c)*cos(2*b*x+2*a)/b+1/8*d*sin(2*b*x+2*a)/b^2
```

### 3.5.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.84

$$\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx = \frac{bdx - 2\,(bdx+bc)\cos{(bx+a)}^2 + d\cos{(bx+a)}\sin{(bx+a)}}{4\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*x + a))
/b^2
```

### 3.5.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.60

$$\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx = \begin{cases} \frac{c\sin^2{(a+bx)}}{2b} + \frac{dx\sin^2{(a+bx)}}{4b} - \frac{dx\cos^2{(a+bx)}}{4b} + \frac{d\sin{(a+bx)}\cos{(a+bx)}}{4b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\sin{(a)}\cos{(a)} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x)
```

output
```
Piecewise((c*sin(a + b*x)**2/(2*b) + d*x*sin(a + b*x)**2/(4*b) - d*x*cos(a
 + b*x)**2/(4*b) + d*sin(a + b*x)*cos(a + b*x)/(4*b**2), Ne(b, 0)), ((c*x
+ d*x**2/2)*sin(a)*cos(a), True))
```

### 3.5.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.30

$$\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx = -\frac{4\,c\cos{(bx+a)}^2 - \frac{4\,ad\cos{(bx+a)}^2}{b} + \frac{(2\,(bx+a)\cos{(2\,bx+2\,a)}-\sin{(2\,bx+2\,a)})d}{b}}{8\,b}$$

input
```
integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/8*(4*c*cos(b*x + a)^2 - 4*a*d*cos(b*x + a)^2/b + (2*(b*x + a)*cos(2*b*x
 + 2*a) - sin(2*b*x + 2*a))*d/b)/b
```

### 3.5.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.76

$$\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx=-\frac{(bdx+bc)\cos(2\,bx+2\,a)}{4\,b^2}+\frac{d\sin(2\,bx+2\,a)}{8\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/4*(b*d*x + b*c)*cos(2*b*x + 2*a)/b^2 + 1/8*d*sin(2*b*x + 2*a)/b^2
```

### 3.5.9 Mupad [B] (verification not implemented)

Time = 22.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx=\frac{d\,\sin(2\,a+2\,b\,x)}{8\,b^2}-\frac{c\,\cos(2\,a+2\,b\,x)}{4\,b}-\frac{d\,x\,\cos(2\,a+2\,b\,x)}{4\,b}$$

input
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x),x)
```

output
```
(d*sin(2*a + 2*b*x))/(8*b^2) - (c*cos(2*a + 2*b*x))/(4*b) - (d*x*cos(2*a +
 2*b*x))/(4*b)
```

# 3.6 $\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx$


3.6.1 Optimal result . . . . . 182
3.6.2 Mathematica [A] (verified) . . . . . 182
3.6.3 Rubi [A] (verified) . . . . . 183
3.6.4 Maple [A] (verified) . . . . . 185
3.6.5 Fricas [A] (verification not implemented) . . . . . 185
3.6.6 Sympy [F] . . . . . 186
3.6.7 Maxima [C] (verification not implemented) . . . . . 186
3.6.8 Giac [C] (verification not implemented) . . . . . 186
3.6.9 Mupad [F(-1)] . . . . . 187


## 3.6.1 Optimal result

Integrand size = 20, antiderivative size = 65

$$\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx = \frac{\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d} + \frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

output
```
1/2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d+1/2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b
*c/d)/d
```

## 3.6.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx = \frac{\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)+\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

input
```
Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x),x]
```

output
```
(CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d] + Cos[2*a - (2*b*c)/d
]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

### 3.6.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {4906, 27, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\cos(a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{4906}$$

$$\int \frac{\sin(2a+2bx)}{2(c+dx)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}dx$$

$$\Big\downarrow \text{3784}$$

$$\frac{1}{2}\left(\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx+\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx+\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)$$

$$\Big\downarrow \text{3780}$$

$$\frac{1}{2}\left(\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)$$

$$\Big\downarrow \text{3783}$$

$$\frac{1}{2}\left(\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)$$

input
```
Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x),x]
```

output
```
((CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/d + (Cos[2*a - (2*b
*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d)/2
```

### 3.6.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.6.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.29

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\operatorname{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{2d}-\frac{\operatorname{Ci}\left(2xb+2a+\frac{-2ad+2cb}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{2d}$ | 84 |
| default | $-\frac{\operatorname{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{2d}-\frac{\operatorname{Ci}\left(2xb+2a+\frac{-2ad+2cb}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{2d}$ | 84 |
| risch | $-\frac{i e^{-\frac{2i(ad-cb)}{d}} \operatorname{Ei}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{4d}+\frac{i e^{\frac{2i(ad-cb)}{d}} \operatorname{Ei}_1\left(-2ibx-2ia-\frac{2(-iad+icb)}{d}\right)}{4d}$ | 98 |

input
```
int(cos(b*x+a)*sin(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*Si(-2*x*b-2*a-2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-1/2*Ci(2*x*b+2*a+
2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d
```

### 3.6.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx = \frac{\operatorname{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+\cos\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{2\,(bdx+bc)}{d}\right)}{2\,d}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(cos_integral(2*(b*d*x + b*c)/d)*sin(-2*(b*c - a*d)/d) + cos(-2*(b*c -
 a*d)/d)*sin_integral(2*(b*d*x + b*c)/d))/d
```

### 3.6.6 Sympy [F]

$$\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx = \int \frac{\sin(a+bx)\cos(a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x), x)
```

### 3.6.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.20

$$\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx = -\frac{b\left(-i\,E_1\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_1\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b\left(E_1\left(\frac{2\,(-i\,bc-i\,(bx+a)d+}{d}\right.\right.}{4\,bd}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
-1/4*(b*(-I*exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*ex
p_integral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d)
/d) + b*(exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_int
egral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d))/
(b*d)
```

### 3.6.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.31 (sec) , antiderivative size = 569, normalized size of antiderivative = 8.75

$$\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx = \frac{\Im\left(\mathrm{Ci}\left(2\,bx+\frac{2\,bc}{d}\right)\right)\tan(a)^2\tan\left(\frac{bc}{d}\right)^2-\Im\left(\mathrm{Ci}\left(-2\,bx-\frac{2\,bc}{d}\right)\right)\tan(a)^2\tan\left(\frac{bc}{d}\right)^2+2\,\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)\tan(a)^2}{}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
1/4*(imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - imag
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*sin_integr
al(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(2*b
*x + 2*b*c/d))*tan(a)^2*tan(b*c/d) + 2*real_part(cos_integral(-2*b*x - 2*b
*c/d))*tan(a)^2*tan(b*c/d) - 2*real_part(cos_integral(2*b*x + 2*b*c/d))*ta
n(a)*tan(b*c/d)^2 - 2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan
(b*c/d)^2 - imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 + imag_part(
cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - 2*sin_integral(2*(b*d*x + b*c)/
d)*tan(a)^2 + 4*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)
 - 4*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 8*sin_i
ntegral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d) - imag_part(cos_integral(2*b*
x + 2*b*c/d))*tan(b*c/d)^2 + imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(b*c/d)^2 - 2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 + 2*real_part(c
os_integral(2*b*x + 2*b*c/d))*tan(a) + 2*real_part(cos_integral(-2*b*x - 2
*b*c/d))*tan(a) - 2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) -
2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) + imag_part(cos_int
egral(2*b*x + 2*b*c/d)) - imag_part(cos_integral(-2*b*x - 2*b*c/d)) + 2*si
n_integral(2*(b*d*x + b*c)/d))/(d*tan(a)^2*tan(b*c/d)^2 + d*tan(a)^2 + d*t
an(b*c/d)^2 + d)
```

### 3.6.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx = \int \frac{\cos(a+b\,x)\,\sin(a+b\,x)}{c+d\,x}\,dx$$

input
```
int((cos(a + b*x)*sin(a + b*x))/(c + d*x),x)
```

output
```
int((cos(a + b*x)*sin(a + b*x))/(c + d*x), x)
```

# 3.7 $\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx$

3.7.1 Optimal result . . . 188
3.7.2 Mathematica [A] (verified) . . . 188
3.7.3 Rubi [A] (verified) . . . 189
3.7.4 Maple [A] (verified) . . . 191
3.7.5 Fricas [A] (verification not implemented) . . . 192
3.7.6 Sympy [F] . . . 192
3.7.7 Maxima [C] (verification not implemented) . . . 193
3.7.8 Giac [C] (verification not implemented) . . . 193
3.7.9 Mupad [F(-1)] . . . 194

## 3.7.1 Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \frac{b\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2} - \frac{\sin(2a+2bx)}{2d(c+dx)} - \frac{b\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}$$

output
```
b*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^2-b*Si(2*b*c/d+2*b*x)*sin(2*a-2*b*c
/d)/d^2-1/2*sin(2*b*x+2*a)/d/(d*x+c)
```

## 3.7.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \frac{2b\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) - \frac{d\sin(2(a+bx))}{c+dx} - 2b\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2b(c+dx)}{d}\right)}{2d^2}$$

input
```
Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^2,x]
```

output
```
(2*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - (d*Sin[2*(a + b
*x)])/(c + d*x) - 2*b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])
/(2*d^2)
```

### 3.7.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {4906, 27, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\cos(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{ 4906}$$

$$\int \frac{\sin(2a+2bx)}{2(c+dx)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{1}{2}\left(\frac{2b\int \frac{\cos(2a+2bx)}{c+dx}dx}{d}-\frac{\sin(2a+2bx)}{d(c+dx)}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\left(\frac{2b\int \frac{\sin\left(2a+2bx+\frac{\pi}{2}\right)}{c+dx}dx}{d}-\frac{\sin(2a+2bx)}{d(c+dx)}\right)$$

$$\Big\downarrow \text{ 3784}$$

$$\frac{1}{2}\left(\frac{2b\left(\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx-\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}-\frac{\sin(2a+2bx)}{d(c+dx)}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\left(\frac{2b\left(\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx-\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}-\frac{\sin(2a+2bx)}{d(c+dx)}\right)$$

$$\Big\downarrow \; 3780$$

$$\frac{1}{2}\left(\frac{2b\left(\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx-\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}-\frac{\sin(2a+2bx)}{d(c+dx)}\right)$$

$$\Big\downarrow \; 3783$$

$$\frac{1}{2}\left(\frac{2b\left(\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}-\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}-\frac{\sin(2a+2bx)}{d(c+dx)}\right)$$

input
```
Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^2,x]
```

output
```
(-(Sin[2*a + 2*b*x]/(d*(c + d*x))) + (2*b*((Cos[2*a - (2*b*c)/d]*CosIntegr
al[(2*b*c)/d + 2*b*x])/d - (Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2
*b*x])/d))/d)/2
```

### 3.7.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.7.4 Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.46

| method | result |
|---|---|
| derivativedivides | $\frac{b\left(-\frac{2\sin(2xb+2a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{4\,\mathrm{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(2xb+2a+\frac{-2ad+2cb}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}}{d}\right)}{4}$ |
| default | $\frac{b\left(-\frac{2\sin(2xb+2a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{4\,\mathrm{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(2xb+2a+\frac{-2ad+2cb}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}}{d}\right)}{4}$ |
| risch | $-\frac{b\,\mathrm{e}^{-\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{2d^2}-\frac{b\,\mathrm{e}^{\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(-2ibx-2ia-\frac{2(-iad+icb)}{d}\right)}{2d^2}-\frac{(-2dxb-2cb)\sin(2xb-}{4d(-dxb-cb)(dx+}$ |

input
```
int(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*b*(-2*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d+2*(-2*Si(-2*x*b-2*a-2*(-a*
d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*x*b+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d
+b*c)/d)/d)/d)
```

### 3.7.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.24

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \frac{(bdx+bc)\cos\left(-\frac{2(bc-ad)}{d}\right)\text{Ci}\left(\frac{2(bdx+bc)}{d}\right) - d\cos(bx+a)\sin(bx+a) - (bdx+bc)\sin\left(-\frac{2(bc-ad)}{d}\right)\text{Si}\left(\frac{2(}{}\right.}{d^3x+cd^2}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
((b*d*x + b*c)*cos(-2*(b*c - a*d)/d)*cos_integral(2*(b*d*x + b*c)/d) - d*c
os(b*x + a)*sin(b*x + a) - (b*d*x + b*c)*sin(-2*(b*c - a*d)/d)*sin_integra
l(2*(b*d*x + b*c)/d))/(d^3*x + c*d^2)
```

### 3.7.6 Sympy [F]

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(a+bx)\cos(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x)**2, x)
```

### 3.7.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.95

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx =$$

$$-\frac{b^2\left(-i\,E_2\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_2\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^2\left(E_2\left(\frac{2\,(-i\,bc-i\,(bx+a)}{d}\right.\right.}{4\,(bcd+(bx+a)d^2-ad^2)b}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/4*(b^2*(-I*exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*
exp_integral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*
d)/d) + b^2*(exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp
_integral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/
d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

### 3.7.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.51 (sec) , antiderivative size = 2870, normalized size of antiderivative = 33.76

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/2*(b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*ta
n(b*c/d)^2 + b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*ta
n(a)^2*tan(b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan
(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 4*b*d*x*sin_integral(2*(b*d*x + b*c
)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b*d*x*imag_part(cos_integral(2*b*x
 + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 2*b*d*x*imag_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 4*b*d*x*sin_integra
l(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b*c*real_part(cos_in
tegral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + b*c*real_part(
cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b*d*x*r
eal_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - b*d*x*real_p
art(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 4*b*d*x*real_par
t(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 4*b*d*x*re
al_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 2*b
*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)
 + 2*b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan
(b*c/d) - 4*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*
c/d) - b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d
)^2 - b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*...
```

### 3.7.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)}{{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^2,x)
```

output
```
int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^2, x)
```

# 3.8 $\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$


3.8.1 Optimal result . . . . . . . . . . 195
3.8.2 Mathematica [A] (verified) . . . . . . . . . . 195
3.8.3 Rubi [A] (verified) . . . . . . . . . . 196
3.8.4 Maple [A] (verified) . . . . . . . . . . 199
3.8.5 Fricas [A] (verification not implemented) . . . . . . . . . . 200
3.8.6 Sympy [F] . . . . . . . . . . 200
3.8.7 Maxima [C] (verification not implemented) . . . . . . . . . . 200
3.8.8 Giac [C] (verification not implemented) . . . . . . . . . . 201
3.8.9 Mupad [F(-1)] . . . . . . . . . . 202


## 3.8.1 Optimal result

Integrand size = 20, antiderivative size = 114

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = -\frac{b\cos(2a+2bx)}{2d^2(c+dx)} - \frac{b^2\,\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^3} - \frac{\sin(2a+2bx)}{4d(c+dx)^2} - \frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}$$

output
```
-1/2*b*cos(2*b*x+2*a)/d^2/(d*x+c)-b^2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d
^3-b^2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^3-1/4*sin(2*b*x+2*a)/d/(d*x+c)
^2
```

## 3.8.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = -\frac{4b^2\,\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right)+\frac{d(2b(c+dx)\cos(2(a+bx))+d\sin(2(a+bx)))}{(c+dx)^2}+4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c-}{}\right.}{4d^3}$$

input
```
Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^3,x]
```

output
```
-1/4*(4*b^2*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (d*(2*b*
(c + d*x)*Cos[2*(a + b*x)] + d*Sin[2*(a + b*x)]))/(c + d*x)^2 + 4*b^2*Cos[
2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d^3
```

### 3.8.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4906, 27, 3042, 3778, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\cos(a+bx)}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \frac{\sin(2a+2bx)}{2(c+dx)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{2}\left(\frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^2}dx}{d}-\frac{\sin(2a+2bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{b\int \frac{\sin\left(2a+2bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx}{d}-\frac{\sin(2a+2bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{2}\left(\frac{b\left(\frac{2b\int -\frac{\sin(2a+2bx)}{c+dx}dx}{d}-\frac{\cos(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\sin(2a+2bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{b\left(-\frac{2b\int\frac{\sin(2a+2bx)}{c+dx}dx}{d}-\frac{\cos(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\sin(2a+2bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{b\left(-\frac{2b\int\frac{\sin(2a+2bx)}{c+dx}dx}{d}-\frac{\cos(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\sin(2a+2bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{3784}$$

$$\frac{1}{2}\left(\frac{b\left(-\frac{2b\left(\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx+\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}-\frac{\cos(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\sin(2a+2bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{b\left(-\frac{2b\left(\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx+\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}-\frac{\cos(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\sin(2a+2bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{3780}$$

$$\frac{1}{2}\left(\frac{b\left(-\frac{2b\left(\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}-\frac{\cos(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\sin(2a+2bx)}{2d(c+dx)^2}\right)$$

$$\Big\downarrow \text{3783}$$

3.8. $\int\frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}dx$

$$\frac{1}{2}\left(\frac{b\left(-\frac{2b\left(\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}-\frac{\cos(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\sin(2a+2bx)}{2d(c+dx)^2}\right)$$

input
```
Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^3,x]
```

output
```
(-1/2*Sin[2*a + 2*b*x]/(d*(c + d*x)^2) + (b*(-(Cos[2*a + 2*b*x]/(d*(c + d*
x))) - (2*b*((CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/d + (Co
s[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d))/d))/d)/2
```

### 3.8.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.8.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.42

| method | result |
|---|---|
| derivativedivides | $\frac{b^2\left(-\frac{\sin(2xb+2a)}{(-ad+cb+d(xb+a))^2d}+\frac{-\frac{2\cos(2xb+2a)}{(-ad+cb+d(xb+a))d}-\frac{2\left(-\frac{2\,\mathrm{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2xb+2a+\frac{-2ad}{\cdots}\right.}{\cdots}\right.}{d}}{d}\right.}{4}$ |
| default | $\frac{b^2\left(-\frac{\sin(2xb+2a)}{(-ad+cb+d(xb+a))^2d}+\frac{-\frac{2\cos(2xb+2a)}{(-ad+cb+d(xb+a))d}-\frac{2\left(-\frac{2\,\mathrm{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2xb+2a+\frac{-2ad}{\cdots}\right.}{\cdots}\right.}{d}}{d}\right.}{4}$ |
| risch | $\frac{ib^2\mathrm{e}^{-\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{2d^3}-\frac{ib^2\mathrm{e}^{\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(-2ibx-2ia-\frac{2(-iad+icb)}{d}\right)}{2d^3}+\frac{i\left(4ib^3d^3x^3+12ib^3c\,d^2\cdots\right.}{8d^2(x^2d^2\cdots}$ |

input
```
int(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*b^2*(-sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^2/d+(-2*cos(2*b*x+2*a)/(-a*d
+c*b+d*(b*x+a))/d-2*(-2*Si(-2*x*b-2*a-2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/
d-2*Ci(2*x*b+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d)
```

### 3.8.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.64

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{bd^2x - d^2\cos(bx+a)\sin(bx+a) + bcd - 2(bd^2x+bcd)\cos(bx+a)^2 - 2(b^2d^2x^2+2b^2cdx+b^2c^2)\operatorname{Ci}\left(\right.}{2(d^5x^2+2cd^4x+c^2d}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(b*d^2*x - d^2*cos(b*x + a)*sin(b*x + a) + b*c*d - 2*(b*d^2*x + b*c*d)
*cos(b*x + a)^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(2*(
b*d*x + b*c)/d)*sin(-2*(b*c - a*d)/d) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d))/(d^5*x^2 + 2*
c*d^4*x + c^2*d^3)
```

### 3.8.6 Sympy [F]

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin(a+bx)\cos(a+bx)}{(c+dx)^3}\,dx$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)**3,x)
```

output
```
Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x)**3, x)
```

### 3.8.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.76

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx =$$
$$-\frac{b^3\left(-i\,E_3\left(\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)+i\,E_3\left(-\frac{2(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+b^3\left(E_3\left(\frac{2(-ibc-i(bx+a)}{d}\right.\right.}{4\left(b^2c^2d-2abcd^2+(bx+a)^2d^3+a^2d^3+2(bcd^2-ad^3)(\right.}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4*(b^3*(-I*exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*
exp_integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*
d)/d) + b^3*(exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp
_integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/
d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a
*d^3)*(b*x + a))*b)
```

### 3.8.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.46 (sec) , antiderivative size = 5398, normalized size of antiderivative = 47.35

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/2*(b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(
a)^2*tan(b*c/d)^2 - b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*
tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*sin_integral(2*(b*d*x + b
*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*real_part(cos_inte
gral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*d^2*x^2*real
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b
^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(
b*c/d)^2 - 2*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x
)^2*tan(a)*tan(b*c/d)^2 + 2*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c
/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 2*b^2*c*d*x*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 4*b^2*c*d*x*sin_int
egral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b^2*d^2*x^2*im
ag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 + b^2*d^2*x^2*i
mag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 2*b^2*d^2*x
^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2 + 4*b^2*d^2*x^2*ima
g_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 4*b^2
*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b
*c/d) + 8*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*ta
n(b*c/d) + 4*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2
*tan(a)^2*tan(b*c/d) + 4*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*...
```

### 3.8.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos(a+b\,x)\,\sin(a+b\,x)}{(c+d\,x)^3}\,dx$$

input
```
int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^3,x)
```

output
```
int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^3, x)
```

# 3.9 $\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$


3.9.1 Optimal result . . . 203
3.9.2 Mathematica [A] (verified) . . . 203
3.9.3 Rubi [A] (verified) . . . 204
3.9.4 Maple [A] (verified) . . . 208
3.9.5 Fricas [A] (verification not implemented) . . . 209
3.9.6 Sympy [F] . . . 209
3.9.7 Maxima [C] (verification not implemented) . . . 210
3.9.8 Giac [C] (verification not implemented) . . . 210
3.9.9 Mupad [F(-1)] . . . 211


## 3.9.1 Optimal result

Integrand size = 20, antiderivative size = 144

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = -\frac{b\cos(2a+2bx)}{6d^2(c+dx)^2} - \frac{2b^3\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4} - \frac{\sin(2a+2bx)}{6d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{3d^3(c+dx)} + \frac{2b^3\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}$$

output
```
-2/3*b^3*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^4-1/6*b*cos(2*b*x+2*a)/d^2/(
d*x+c)^2+2/3*b^3*Si(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^4-1/6*sin(2*b*x+2*a)
/d/(d*x+c)^3+1/3*b^2*sin(2*b*x+2*a)/d^3/(d*x+c)
```

## 3.9.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.14

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \frac{-d\cos(2bx)\left(bd(c+dx)\cos(2a)+\left(d^2-2b^2(c+dx)^2\right)\sin(2a)\right)+d\left(\left(-d^2+2b^2(c+dx)^2\right)\cos(2a)+bd(c-\right.}{6d^4(c}$$

input
```
Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^4,x]
```

output
```
(-(d*Cos[2*b*x]*(b*d*(c + d*x)*Cos[2*a] + (d^2 - 2*b^2*(c + d*x)^2)*Sin[2*
a])) + d*((-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*a] + b*d*(c + d*x)*Sin[2*a])*Si
n[2*b*x] - 4*b^3*(c + d*x)^3*(Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d
*x))/d] - Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d]))/(6*d^4*(c
+ d*x)^3)
```

### 3.9.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {4906, 27, 3042, 3778, 3042, 3778, 25, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\cos(a+bx)}{(c+dx)^4}dx$$

$$\Big\downarrow \text{4906}$$

$$\int \frac{\sin(2a+2bx)}{2(c+dx)^4}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^4}dx$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{2}\left(\frac{2b\int \frac{\cos(2a+2bx)}{(c+dx)^3}dx}{3d}-\frac{\sin(2a+2bx)}{3d(c+dx)^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{2b\int \frac{\sin\left(2a+2bx+\frac{\pi}{2}\right)}{(c+dx)^3}dx}{3d}-\frac{\sin(2a+2bx)}{3d(c+dx)^3}\right)$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{2}\left(\frac{2b\left(\frac{b\int -\frac{\sin(2a+2bx)}{(c+dx)^2}dx}{d}-\frac{\cos(2a+2bx)}{2d(c+dx)^2}\right)}{3d}-\frac{\sin(2a+2bx)}{3d(c+dx)^3}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{2b\left(-\frac{b\int \frac{\sin(2a+2bx)}{(c+dx)^2}dx}{d}-\frac{\cos(2a+2bx)}{2d(c+dx)^2}\right)}{3d}-\frac{\sin(2a+2bx)}{3d(c+dx)^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{2b\left(-\frac{b\int \frac{\sin(2a+2bx)}{(c+dx)^2}dx}{d}-\frac{\cos(2a+2bx)}{2d(c+dx)^2}\right)}{3d}-\frac{\sin(2a+2bx)}{3d(c+dx)^3}\right)$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{2}\left(\frac{2b\left(-\frac{b\left(\frac{2b\int \frac{\cos(2a+2bx)}{c+dx}dx}{d}-\frac{\sin(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\cos(2a+2bx)}{2d(c+dx)^2}\right)}{3d}-\frac{\sin(2a+2bx)}{3d(c+dx)^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{2b\left(-\frac{b\left(\frac{2b\int \frac{\sin\left(2a+2bx+\frac{\pi}{2}\right)}{c+dx}dx}{d}-\frac{\sin(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\cos(2a+2bx)}{2d(c+dx)^2}\right)}{3d}-\frac{\sin(2a+2bx)}{3d(c+dx)^3}\right)$$

$$\Big\downarrow \text{3784}$$

3.9. $\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$

$$\frac{1}{2}\left(\frac{2b\left(-\frac{b\left(\frac{2b\left(\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx-\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}-\frac{\sin(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\cos(2a+2bx)}{2d(c+dx)^2}\right)}{3d}-\frac{\sin(2a+2bx)}{3d(c+dx)^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{2b\left(-\frac{b\left(\frac{2b\left(\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx-\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}-\frac{\sin(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\cos(2a+2bx)}{2d(c+dx)^2}\right)}{3d}-\frac{\sin(2a+2bx)}{3d(c+dx)^3}\right)$$

$$\Big\downarrow \text{3780}$$

$$\frac{1}{2}\left(\frac{2b\left(-\frac{b\left(\frac{2b\left(\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx-\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}-\frac{\sin(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\cos(2a+2bx)}{2d(c+dx)^2}\right)}{3d}-\frac{\sin(2a+2bx)}{3d(c+dx)^3}\right)$$

3.9. $\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$

$\Big\downarrow$ 3783

$$\frac{1}{2}\left(\frac{2b\left(-\frac{b\left(\frac{2b\left(\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}-\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}-\frac{\sin(2a+2bx)}{d(c+dx)}\right)}{d}-\frac{\cos(2a+2bx)}{2d(c+dx)^2}\right)}{3d}-\frac{\sin(2a+2bx)}{3d(c+dx)^3}\right)$$

input
```
Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^4,x]
```

output
```
(-1/3*Sin[2*a + 2*b*x]/(d*(c + d*x)^3) + (2*b*(-1/2*Cos[2*a + 2*b*x]/(d*(c
 + d*x)^2) - (b*(-(Sin[2*a + 2*b*x]/(d*(c + d*x))) + (2*b*((Cos[2*a - (2*b
*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d - (Sin[2*a - (2*b*c)/d]*SinIntegr
al[(2*b*c)/d + 2*b*x])/d))/d))/d))/(3*d))/2
```

### 3.9.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.9.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.39

| method | result |
|---|---|
| derivativedivides | $\frac{b^3\left(-\frac{2\sin(2xb+2a)}{3(-ad+cb+d(xb+a))^3 d}+\frac{-\frac{2\cos(2xb+2a)}{3(-ad+cb+d(xb+a))^2 d}-\frac{2\left(-\frac{2\sin(2xb+2a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{4\operatorname{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\sin\left(\frac{-2ad}{d}\right.}{d}}{}\right.}{3d}}{d}\right.}{4}$ |
| default | $\frac{b^3\left(-\frac{2\sin(2xb+2a)}{3(-ad+cb+d(xb+a))^3 d}+\frac{-\frac{2\cos(2xb+2a)}{3(-ad+cb+d(xb+a))^2 d}-\frac{2\left(-\frac{2\sin(2xb+2a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{4\operatorname{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\sin\left(\frac{-2ad}{d}\right.}{d}}{}\right.}{3d}}{d}\right.}{4}$ |
| risch | $\frac{b^3 e^{-\frac{2i(ad-cb)}{d}}\operatorname{Ei}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{3d^4}+\frac{b^3 e^{\frac{2i(ad-cb)}{d}}\operatorname{Ei}_1\left(-2ibx-2ia-\frac{2(-iad+icb)}{d}\right)}{3d^4}+\frac{i(2ib^4d^5x^4+8ib^4cd^4x^3}{12d^3(d^3x^3b}$ |

input
```
int(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/4*b^3*(-2/3*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^3/d+2/3*(-cos(2*b*x+2*a)
/(-a*d+c*b+d*(b*x+a))^2/d-(-2*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d+2*(-2*
Si(-2*x*b-2*a-2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*x*b+2*a+2*(-a*d
+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)/d)/d)
```

### 3.9.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.83

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$$
$$= \frac{bd^3x + bcd^2 - 2\,(bd^3x + bcd^2)\cos\left(bx+a\right)^2 - 4\,(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Ci}}{}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/6*(b*d^3*x + b*c*d^2 - 2*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2 - 4*(b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-2*(b*c - a*d)/d)*co
s_integral(2*(b*d*x + b*c)/d) + 2*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c
^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
 3*b^3*c^2*d*x + b^3*c^3)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*
c)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

### 3.9.6 Sympy [F]

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sin\left(a+bx\right)\cos\left(a+bx\right)}{\left(c+dx\right)^4}\,dx$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)**4,x)
```

output
```
Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x)**4, x)
```

### 3.9.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.74

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx =$$

$$-\frac{b^4\left(-i\,E_4\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_4\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^4\left(E_4\left(\frac{2\,(-i\,bc-i\,(bx+a)}{d}\right.\right.}{4\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3+(bx+a)^3d^4-a^3d^4+3\,(bcd^3-ad^4)(bx+a)^2+3\,(\right.}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/4*(b^4*(-I*exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*
exp_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*
d)/d) + b^4*(exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp
_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/
d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*
d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2
*d^4)*(b*x + a))*b)
```

### 3.9.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.58 (sec) , antiderivative size = 7592, normalized size of antiderivative = 52.72

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

output
```
-1/6*(2*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*ta
n(a)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 4*b^3*d^3*x^3*imag_part(cos_integra
l(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 4*b^3*d^3*x^3*imag_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 8*b^3*
d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 4
*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*ta
n(b*c/d)^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b
*x)^2*tan(a)*tan(b*c/d)^2 + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*
tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(2*
b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 2*b^
3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 2
*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2
 + 8*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a
)*tan(b*c/d) + 8*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(b*x)^2*tan(a)*tan(b*c/d) - 12*b^3*c*d^2*x^2*imag_part(cos_integral(2*b*x
+ 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 12*b^3*c*d^2*x^2*imag_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 24*b^3*c*d^
2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - ...
```

### 3.9.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)}{{\left(c+d\,x\right)}^4}\,dx$$

input
```
int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^4,x)
```

output
```
int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^4, x)
```

# 3.10 $\int \frac{\cos(x)\sin(x)}{x}\,dx$

3.10.1 Optimal result . . . . . 212
3.10.2 Mathematica [A] (verified) . . . . . 212
3.10.3 Rubi [A] (verified) . . . . . 213
3.10.4 Maple [A] (verified) . . . . . 214
3.10.5 Fricas [A] (verification not implemented) . . . . . 214
3.10.6 Sympy [A] (verification not implemented) . . . . . 215
3.10.7 Maxima [C] (verification not implemented) . . . . . 215
3.10.8 Giac [A] (verification not implemented) . . . . . 215
3.10.9 Mupad [F(-1)] . . . . . 216

## 3.10.1 Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \frac{\cos(x)\sin(x)}{x}\,dx=\frac{\mathrm{Si}(2x)}{2}$$

output
```
1/2*Si(2*x)
```

## 3.10.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)\sin(x)}{x}\,dx=\frac{\mathrm{Si}(2x)}{2}$$

input
```
Integrate[(Cos[x]*Sin[x])/x,x]
```

output
```
SinIntegral[2*x]/2
```

## 3.10.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4906, 27, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)\cos(x)}{x} \, dx$$
$$\Big\downarrow \; 4906$$
$$\int \frac{\sin(2x)}{2x} dx$$
$$\Big\downarrow \; 27$$
$$\frac{1}{2} \int \frac{\sin(2x)}{x} dx$$
$$\Big\downarrow \; 3042$$
$$\frac{1}{2} \int \frac{\sin(2x)}{x} dx$$
$$\Big\downarrow \; 3780$$
$$\frac{\text{Si}(2x)}{2}$$

input
```
Int[(Cos[x]*Sin[x])/x,x]
```

output
```
SinIntegral[2*x]/2
```

### 3.10.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.10.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| default | $\frac{\operatorname{Si}(2x)}{2}$ | 7 |
| meijerg | $\frac{\operatorname{Si}(2x)}{2}$ | 7 |
| risch | $-\frac{\pi\ \operatorname{csgn}(x)}{4}+\frac{\operatorname{Si}(2x)}{2}$ | 13 |

input
```
int(cos(x)*sin(x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*Si(2*x)
```

### 3.10.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cos(x)\sin(x)}{x}\,dx = \frac{1}{2}\operatorname{Si}\left(2\,x\right)$$

input
```
integrate(cos(x)*sin(x)/x,x, algorithm="fricas")
```

output
```
1/2*sin_integral(2*x)
```

### 3.10.6 Sympy [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int \frac{\cos(x)\sin(x)}{x}\,dx = \frac{\operatorname{Si}(2x)}{2}$$

input
```
integrate(cos(x)*sin(x)/x,x)
```

output
```
Si(2*x)/2
```

### 3.10.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.62

$$\int \frac{\cos(x)\sin(x)}{x}\,dx = -\frac{1}{4}i\operatorname{Ei}(2i\,x) + \frac{1}{4}i\operatorname{Ei}(-2i\,x)$$

input
```
integrate(cos(x)*sin(x)/x,x, algorithm="maxima")
```

output
```
-1/4*I*Ei(2*I*x) + 1/4*I*Ei(-2*I*x)
```

### 3.10.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cos(x)\sin(x)}{x}\,dx = \frac{1}{2}\operatorname{Si}(2\,x)$$

input
```
integrate(cos(x)*sin(x)/x,x, algorithm="giac")
```

output
```
1/2*sin_integral(2*x)
```

### 3.10.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)\sin(x)}{x}\,dx = \int \frac{\cos(x)\,\sin(x)}{x}\,dx$$

input
```
int((cos(x)*sin(x))/x,x)
```

output
```
int((cos(x)*sin(x))/x, x)
```

# 3.11 $\int \frac{\cos(x)\sin(x)}{x^2}\,dx$


3.11.1 Optimal result . . . . . 217
3.11.2 Mathematica [A] (verified) . . . . . 217
3.11.3 Rubi [A] (verified) . . . . . 218
3.11.4 Maple [A] (verified) . . . . . 219
3.11.5 Fricas [A] (verification not implemented) . . . . . 220
3.11.6 Sympy [A] (verification not implemented) . . . . . 220
3.11.7 Maxima [C] (verification not implemented) . . . . . 220
3.11.8 Giac [A] (verification not implemented) . . . . . 221
3.11.9 Mupad [F(-1)] . . . . . 221


## 3.11.1 Optimal result

Integrand size = 8, antiderivative size = 16

$$\int \frac{\cos(x)\sin(x)}{x^2}\,dx = \text{CosIntegral}(2x) - \frac{\sin(2x)}{2x}$$

output
```
Ci(2*x)-1/2*sin(2*x)/x
```

## 3.11.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)\sin(x)}{x^2}\,dx = \text{CosIntegral}(2x) - \frac{\sin(2x)}{2x}$$

input
```
Integrate[(Cos[x]*Sin[x])/x^2,x]
```

output
```
CosIntegral[2*x] - Sin[2*x]/(2*x)
```

### 3.11.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {4906, 27, 3042, 3778, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)\cos(x)}{x^2}dx$$
$$\Big\downarrow \text{4906}$$
$$\int \frac{\sin(2x)}{2x^2}dx$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{2}\int \frac{\sin(2x)}{x^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{2}\int \frac{\sin(2x)}{x^2}dx$$
$$\Big\downarrow \text{3778}$$
$$\frac{1}{2}\left(2\int \frac{\cos(2x)}{x}dx-\frac{\sin(2x)}{x}\right)$$
$$\Big\downarrow \text{3042}$$
$$\frac{1}{2}\left(2\int \frac{\sin\left(2x+\frac{\pi}{2}\right)}{x}dx-\frac{\sin(2x)}{x}\right)$$
$$\Big\downarrow \text{3783}$$
$$\frac{1}{2}\left(2\,\text{CosIntegral}(2x)-\frac{\sin(2x)}{x}\right)$$

input
```
Int[(Cos[x]*Sin[x])/x^2,x]
```

output
```
(2*CosIntegral[2*x] - Sin[2*x]/x)/2
```

#### 3.11.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.11.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

| method | result | size |
|---|---|---|
| default | $\operatorname{Ci}(2x)-\frac{\sin(2x)}{2x}$ | 15 |
| risch | $\operatorname{Ci}(2x)-\frac{i\pi\ \operatorname{csgn}(ix)\operatorname{csgn}(x)}{2}+\frac{i\operatorname{csgn}(ix)\pi}{2}-\frac{\sin(2x)}{2x}$ | 35 |
| meijerg | $\frac{\sqrt{\pi}\left(\frac{4}{\sqrt{\pi}}-\frac{4\gamma}{\sqrt{\pi}}-\frac{4\ln(2)}{\sqrt{\pi}}-\frac{4\ln(x)}{\sqrt{\pi}}-\frac{2\sin(2x)}{\sqrt{\pi}\,x}+\frac{4\ \operatorname{Ci}(2x)}{\sqrt{\pi}}+\frac{4\gamma-4+4\ln(2)+4\ln(x)}{\sqrt{\pi}}\right)}{4}$ | 71 |

input
```
int(cos(x)*sin(x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
Ci(2*x)-1/2*sin(2*x)/x
```

### 3.11.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\cos(x)\sin(x)}{x^2}\,dx = \frac{x\operatorname{Ci}(2x) - \cos(x)\sin(x)}{x}$$

input
```
integrate(cos(x)*sin(x)/x^2,x, algorithm="fricas")
```

output
```
(x*cos_integral(2*x) - cos(x)*sin(x))/x
```

### 3.11.6 Sympy [A] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.38

$$\int \frac{\cos(x)\sin(x)}{x^2}\,dx = -\log(x) + \frac{\log(x^2)}{2} + \operatorname{Ci}(2x) - \frac{\sin(2x)}{2x}$$

input
```
integrate(cos(x)*sin(x)/x**2,x)
```

output
```
-log(x) + log(x**2)/2 + Ci(2*x) - sin(2*x)/(2*x)
```

### 3.11.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\cos(x)\sin(x)}{x^2}\,dx = \frac{1}{2}\Gamma(-1, 2i x) + \frac{1}{2}\Gamma(-1, -2i x)$$

input
```
integrate(cos(x)*sin(x)/x^2,x, algorithm="maxima")
```

output
```
1/2*gamma(-1, 2*I*x) + 1/2*gamma(-1, -2*I*x)
```

### 3.11.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{\cos(x)\sin(x)}{x^2}\,dx = \frac{2\,x\,\mathrm{Ci}\,(2\,x) - \sin\,(2\,x)}{2\,x}$$

input
```
integrate(cos(x)*sin(x)/x^2,x, algorithm="giac")
```

output
```
1/2*(2*x*cos_integral(2*x) - sin(2*x))/x
```

### 3.11.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)\sin(x)}{x^2}\,dx = \int \frac{\cos\,(x)\,\sin\,(x)}{x^2}\,dx$$

input
```
int((cos(x)*sin(x))/x^2,x)
```

output
```
int((cos(x)*sin(x))/x^2, x)
```

# 3.12 $\int \frac{\cos(x)\sin(x)}{x^3}\,dx$

3.12.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 222
3.12.2 Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 222
3.12.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 223
3.12.4 Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 225
3.12.5 Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 225
3.12.6 Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 225
3.12.7 Maxima [C] (verification not implemented) . . . . . . . . . . . . . . . . . 226
3.12.8 Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 226
3.12.9 Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 226

## 3.12.1 Optimal result

Integrand size = 8, antiderivative size = 29

$$\int \frac{\cos(x)\sin(x)}{x^3}\,dx = -\frac{\cos(2x)}{2x} - \frac{\sin(2x)}{4x^2} - \text{Si}(2x)$$

output
```
-1/2*cos(2*x)/x-Si(2*x)-1/4*sin(2*x)/x^2
```

## 3.12.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\cos(x)\sin(x)}{x^3}\,dx = -\frac{\cos(2x)}{2x} - \frac{\sin(2x)}{4x^2} - \text{Si}(2x)$$

input
```
Integrate[(Cos[x]*Sin[x])/x^3,x]
```

output
```
-1/2*Cos[2*x]/x - Sin[2*x]/(4*x^2) - SinIntegral[2*x]
```

### 3.12.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {4906, 27, 3042, 3778, 3042, 3778, 25, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(x)\cos(x)}{x^3}dx$$

$$\Big\downarrow \text{ 4906}$$

$$\int \frac{\sin(2x)}{2x^3}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int \frac{\sin(2x)}{x^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int \frac{\sin(2x)}{x^3}dx$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{1}{2}\left(\int \frac{\cos(2x)}{x^2}dx-\frac{\sin(2x)}{2x^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\left(\int \frac{\sin\left(2x+\frac{\pi}{2}\right)}{x^2}dx-\frac{\sin(2x)}{2x^2}\right)$$

$$\Big\downarrow \text{ 3778}$$

$$\frac{1}{2}\left(2\int -\frac{\sin(2x)}{x}dx-\frac{\sin(2x)}{2x^2}-\frac{\cos(2x)}{x}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(-2\int \frac{\sin(2x)}{x}dx-\frac{\sin(2x)}{2x^2}-\frac{\cos(2x)}{x}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\left(-2\int \frac{\sin(2x)}{x}dx-\frac{\sin(2x)}{2x^2}-\frac{\cos(2x)}{x}\right)$$

$$\Big\downarrow \text{ 3780}$$

$$\frac{1}{2}\left(-2\text{Si}(2x)-\frac{\sin(2x)}{2x^2}-\frac{\cos(2x)}{x}\right)$$

input
```
Int[(Cos[x]*Sin[x])/x^3,x]
```

output
```
(-(Cos[2*x]/x) - Sin[2*x]/(2*x^2) - 2*SinIntegral[2*x])/2
```

### 3.12.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.12.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

| method | result | size |
|---|---|---|
| default | $-\frac{\cos(2x)}{2x}-\operatorname{Si}(2x)-\frac{\sin(2x)}{4x^2}$ | 26 |
| risch | $\frac{\pi\ \operatorname{csgn}(x)}{2}-\operatorname{Si}(2x)-\frac{\cos(2x)}{2x}-\frac{\sin(2x)}{4x^2}$ | 31 |
| meijerg | $\frac{\sqrt{\pi}\left(-\frac{2\cos(2x)}{x\sqrt{\pi}}-\frac{\sin(2x)}{x^2\sqrt{\pi}}-\frac{4\ \operatorname{Si}(2x)}{\sqrt{\pi}}\right)}{4}$ | 40 |

input
```
int(cos(x)*sin(x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*cos(2*x)/x-Si(2*x)-1/4*sin(2*x)/x^2
```

### 3.12.5 Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

$$\int\frac{\cos(x)\sin(x)}{x^3}\,dx=-\frac{2\,x\cos\left(x\right)^2+2\,x^2\operatorname{Si}\left(2\,x\right)+\cos\left(x\right)\sin\left(x\right)-x}{2\,x^2}$$

input
```
integrate(cos(x)*sin(x)/x^3,x, algorithm="fricas")
```

output
```
-1/2*(2*x*cos(x)^2 + 2*x^2*sin_integral(2*x) + cos(x)*sin(x) - x)/x^2
```

### 3.12.6 Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int\frac{\cos(x)\sin(x)}{x^3}\,dx=-\operatorname{Si}\left(2x\right)-\frac{\cos\left(2x\right)}{2x}-\frac{\sin\left(2x\right)}{4x^2}$$

input
```
integrate(cos(x)*sin(x)/x**3,x)
```

output
```
-Si(2*x) - cos(2*x)/(2*x) - sin(2*x)/(4*x**2)
```

### 3.12.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.52

$$\int \frac{\cos(x)\sin(x)}{x^3}\,dx = i\,\Gamma(-2, 2i\,x) - i\,\Gamma(-2, -2i\,x)$$

input
```
integrate(cos(x)*sin(x)/x^3,x, algorithm="maxima")
```

output
```
I*gamma(-2, 2*I*x) - I*gamma(-2, -2*I*x)
```

### 3.12.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.90

$$\int \frac{\cos(x)\sin(x)}{x^3}\,dx = -\frac{4\,x^2\,\mathrm{Si}\,(2\,x) + 2\,x\cos\,(2\,x) + \sin\,(2\,x)}{4\,x^2}$$

input
```
integrate(cos(x)*sin(x)/x^3,x, algorithm="giac")
```

output
```
-1/4*(4*x^2*sin_integral(2*x) + 2*x*cos(2*x) + sin(2*x))/x^2
```

### 3.12.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(x)\sin(x)}{x^3}\,dx = \int \frac{\cos\,(x)\,\sin\,(x)}{x^3}\,dx$$

input
```
int((cos(x)*sin(x))/x^3,x)
```

output
```
int((cos(x)*sin(x))/x^3, x)
```

# 3.13 $\int (c+dx)^m \cos(a+bx)\sin^2(a+bx)\,dx$


3.13.1 Optimal result . . . 227
3.13.2 Mathematica [A] (verified) . . . 228
3.13.3 Rubi [A] (verified) . . . 228
3.13.4 Maple [F] . . . 230
3.13.5 Fricas [A] (verification not implemented) . . . 230
3.13.6 Sympy [F] . . . 230
3.13.7 Maxima [F] . . . 231
3.13.8 Giac [F] . . . 231
3.13.9 Mupad [F(-1)] . . . 231


## 3.13.1 Optimal result

Integrand size = 22, antiderivative size = 275

$$\begin{aligned}\int (c+dx)^m \cos(a+bx)\sin^2(a+bx)\,dx &= -\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{8b}\\
&+\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)}{8b}\\
&+\frac{i3^{-1-m}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3ib(c+dx)}{d}\right)}{8b}\\
&-\frac{i3^{-1-m}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3ib(c+dx)}{d}\right)}{8b}\end{aligned}$$

output
```
-1/8*I*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+
c)/d)^m)+1/8*I*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b
*(d*x+c)/d)^m)+1/8*I*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3*I*
b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)-1/8*I*3^(-1-m)*(d*x+c)^m*GAMMA(1+m,3*I
*b*(d*x+c)/d)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

### 3.13.2 Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.86

$$\int (c+dx)^m \cos(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{ie^{-\frac{3i(bc+ad)}{d}}(c+dx)^m\left(3e^{2i\left(2a+\frac{bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)+\left(\frac{ib(c+dx)}{d}\right)^{-m}\left(-3e^{2ia+\frac{4ibc}{d}}\Gamma\left(1\right.\right.\right.}{}$$

input
```
Integrate[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
((-1/24*I)*(c + d*x)^m*((3*E^((2*I)*(2*a + (b*c)/d))*Gamma[1 + m, ((-I)*b*
(c + d*x))/d])/(((-I)*b*(c + d*x))/d)^m + (-3*E^((2*I)*a + ((4*I)*b*c)/d)*
Gamma[1 + m, (I*b*(c + d*x))/d] + (-((E^((6*I)*a)*((I*b*(c + d*x))/d)^(2*m
)*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/((b^2*(c + d*x)^2)/d^2)^m) + E^(((
6*I)*b*c)/d)*Gamma[1 + m, ((3*I)*b*(c + d*x))/d])/3^m)/((I*b*(c + d*x))/d)
^m))/(b*E^(((3*I)*(b*c + a*d))/d))
```

### 3.13.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cos(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \; 4906$$

$$\int \left(\frac{1}{4}\cos(a+bx)(c+dx)^m-\frac{1}{4}\cos(3a+3bx)(c+dx)^m\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}+\frac{i3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{8b}+\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{8b}-\frac{i3^{-m-1}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{3ib(c+dx)}{d}\right)}{8b}$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
((-1/8*I)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/
d])/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/8)*(c + d*x)^m*Gamma[1 + m, (I*b*(c
 + d*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + ((I/8)*3^(-1
- m)*E^((3*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))
/d])/(b*(((-I)*b*(c + d*x))/d)^m) - ((I/8)*3^(-1 - m)*(c + d*x)^m*Gamma[1
+ m, ((3*I)*b*(c + d*x))/d])/(b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d
)^m)
```

### 3.13.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.13.4 Maple [F]

$$\int (dx+c)^m \cos(xb+a)\sin(xb+a)^2\,dx$$

input
```
int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x)
```

### 3.13.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.68

$$\int (c+dx)^m \cos(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{3i\,e^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-ibc+iad}{d}\right)}\Gamma\left(m+1,\frac{ibdx+ibc}{d}\right)+i\,e^{\left(-\frac{dm\log\left(-\frac{3ib}{d}\right)+3ibc-3iad}{d}\right)}\Gamma\left(m+1,-\frac{3(ibdx+ibc)}{d}\right)-3i\,e^{\left(-\frac{dm\,\mathrm{lo}}{}\right.}}{24b}$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/24*(3*I*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x +
I*b*c)/d) + I*e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d)*gamma(m + 1,
-3*(I*b*d*x + I*b*c)/d) - 3*I*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gam
ma(m + 1, (-I*b*d*x - I*b*c)/d) - I*e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*
a*d)/d)*gamma(m + 1, -3*(-I*b*d*x - I*b*c)/d))/b
```

### 3.13.6 Sympy [F]

$$\int (c+dx)^m \cos(a+bx)\sin^2(a+bx)\,dx = \int (c+dx)^m \sin^2(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*cos(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*sin(a + b*x)**2*cos(a + b*x), x)
```

### 3.13.7 Maxima [F]

$$\int (c+dx)^m \cos(a+bx)\sin^2(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\sin(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^2, x)
```

### 3.13.8 Giac [F]

$$\int (c+dx)^m \cos(a+bx)\sin^2(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\sin(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^2, x)
```

### 3.13.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cos(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)^2\,(c+d\,x)^m\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^m, x)
```

# 3.14 $\int (c+dx)^4 \cos(a+bx)\sin^2(a+bx)\,dx$

3.14.1 Optimal result . . . . . 232
3.14.2 Mathematica [A] (verified) . . . . . 233
3.14.3 Rubi [A] (verified) . . . . . 233
3.14.4 Maple [A] (verified) . . . . . 238
3.14.5 Fricas [A] (verification not implemented) . . . . . 238
3.14.6 Sympy [B] (verification not implemented) . . . . . 239
3.14.7 Maxima [B] (verification not implemented) . . . . . 240
3.14.8 Giac [A] (verification not implemented) . . . . . 241
3.14.9 Mupad [B] (verification not implemented) . . . . . 242

## 3.14.1 Optimal result

Integrand size = 22, antiderivative size = 205

$$\int (c+dx)^4\cos(a+bx)\sin^2(a+bx)\,dx = -\frac{160d^3(c+dx)\cos(a+bx)}{27b^4} + \frac{8d(c+dx)^3\cos(a+bx)}{9b^2} + \frac{160d^4\sin(a+bx)}{27b^5} - \frac{8d^2(c+dx)^2\sin(a+bx)}{3b^3} - \frac{8d^3(c+dx)\cos(a+bx)\sin^2(a+bx)}{27b^4} + \frac{4d(c+dx)^3\cos(a+bx)\sin^2(a+bx)}{9b^2} + \frac{8d^4\sin^3(a+bx)}{81b^5} - \frac{4d^2(c+dx)^2\sin^3(a+bx)}{9b^3} + \frac{(c+dx)^4\sin^3(a+bx)}{3b}$$

output
```
-160/27*d^3*(d*x+c)*cos(b*x+a)/b^4+8/9*d*(d*x+c)^3*cos(b*x+a)/b^2+160/27*d
^4*sin(b*x+a)/b^5-8/3*d^2*(d*x+c)^2*sin(b*x+a)/b^3-8/27*d^3*(d*x+c)*cos(b*
x+a)*sin(b*x+a)^2/b^4+4/9*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2/b^2+8/81*d^4
*sin(b*x+a)^3/b^5-4/9*d^2*(d*x+c)^2*sin(b*x+a)^3/b^3+1/3*(d*x+c)^4*sin(b*x
+a)^3/b
```

### 3.14.2 Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.88

$$\int (c+dx)^4\cos(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{324bd(c+dx)\left(-6d^2+b^2(c+dx)^2\right)\cos(a+bx)-12bd(c+dx)\left(-2d^2+3b^2(c+dx)^2\right)\cos(3(a+bx))+8}{}$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
(324*b*d*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 12*b*d*(c + d
*x)*(-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 81*b^4*c^4*Sin[a + b*x
] - 972*b^2*c^2*d^2*Sin[a + b*x] + 1944*d^4*Sin[a + b*x] + 324*b^4*c^3*d*x
*Sin[a + b*x] - 1944*b^2*c*d^3*x*Sin[a + b*x] + 486*b^4*c^2*d^2*x^2*Sin[a
+ b*x] - 972*b^2*d^4*x^2*Sin[a + b*x] + 324*b^4*c*d^3*x^3*Sin[a + b*x] + 8
1*b^4*d^4*x^4*Sin[a + b*x] - 27*b^4*c^4*Sin[3*(a + b*x)] + 36*b^2*c^2*d^2*
Sin[3*(a + b*x)] - 8*d^4*Sin[3*(a + b*x)] - 108*b^4*c^3*d*x*Sin[3*(a + b*x
)] + 72*b^2*c*d^3*x*Sin[3*(a + b*x)] - 162*b^4*c^2*d^2*x^2*Sin[3*(a + b*x)
] + 36*b^2*d^4*x^2*Sin[3*(a + b*x)] - 108*b^4*c*d^3*x^3*Sin[3*(a + b*x)] -
 27*b^4*d^4*x^4*Sin[3*(a + b*x)])/(324*b^5)
```

### 3.14.3 Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.21, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {4904, 3042, 3792, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117, 3791, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4\sin^2(a+bx)\cos(a+bx)\,dx$$
$$\Big\downarrow \; 4904$$
$$\frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{4d\int (c+dx)^3\sin^3(a+bx)dx}{3b}$$
$$\Big\downarrow \; 3042$$
$$\frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{4d\int (c+dx)^3\sin(a+bx)^3dx}{3b}$$

$\downarrow$ 3792

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{4d\left(-\frac{2d^2\int (c+dx)\sin^3(a+bx)dx}{3b^2}+\frac{2}{3}\int (c+dx)^3\sin(a+bx)dx+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}-\frac{(c+dx)^3\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b}$$

$\downarrow$ 3042

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{4d\left(-\frac{2d^2\int (c+dx)\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\int (c+dx)^3\sin(a+bx)dx+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}-\frac{(c+dx)^3\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b}$$

$\downarrow$ 3777

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{4d\left(-\frac{2d^2\int (c+dx)\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{3d\int (c+dx)^2\cos(a+bx)dx}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}-\frac{(c+dx)^3\sin^2(a+bx)\,\mathrm{co}}{3b}\right.}{3b}$$

$\downarrow$ 3042

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{4d\left(-\frac{2d^2\int (c+dx)\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{3d\int (c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}-\frac{(c+dx)^3\sin^2(a+b}{3b}\right.}{3b}$$

$\downarrow$ 3777

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{4d\left(-\frac{2d^2\int (c+dx)\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{2d\int -((c+dx)\sin(a+bx))dx}{b}+\frac{(c+dx)^2\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)+\frac{d(c+dx)^2\sin^3(a+bx}{3b^2}\right.}{3b}$$

$\downarrow$ 25

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{4d\left(-\frac{2d^2\int (c+dx)\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int (c+dx)\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}-\right.}{3b}$$

$\downarrow$ 3042

---

3.14. $\int (c+dx)^4\cos(a+bx)\sin^2(a+bx)\,dx$

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{4d\left(-\frac{2d^2\int(c+dx)\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int(c+dx)\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}-}{3b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{4d\left(-\frac{2d^2\int(c+dx)\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\cos(a+bx)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)+}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{4d\left(-\frac{2d^2\int(c+dx)\sin(a+bx)^3dx}{3b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)+}{3b}$$

$$\Big\downarrow \text{3117}$$

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{4d\left(-\frac{2d^2\int(c+dx)\sin(a+bx)^3dx}{3b^2}+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos}{b}\right.}{3b}$$

$$\Big\downarrow \text{3791}$$

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{4d\left(-\frac{2d^2\left(\frac{2}{3}\int(c+dx)\sin(a+bx)dx+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\right.}{}\right.}{}\right.\right.}{3b}$$

$$\Big\downarrow \text{3042}$$

3.14. $\int(c+dx)^4\cos(a+bx)\sin^2(a+bx)\,dx$

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{4d\left(-\frac{2d^2\left(\frac{2}{3}\int(c+dx)\sin(a+bx)dx+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d s}{\cdots}\right.}{}\right.}{}\right.\right.}{3b}$$

$\downarrow$ 3777

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{4d\left(-\frac{2d^2\left(\frac{2}{3}\left(\frac{d\int\cos(a+bx)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)^2\sin(}{b}\right.}{}\right.\right.}{3b}$$

$\downarrow$ 3042

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{4d\left(-\frac{2d^2\left(\frac{2}{3}\left(\frac{d\int\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)}{}\right.}{}\right.\right.}{3b}$$

$\downarrow$ 3117

$$\frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{4d\left(-\frac{2d^2\left(\frac{2}{3}\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\sin^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+b}{b}\right.}{}\right.\right.}{3b}$$

input
```
Int[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

## 3.14. $\int(c+dx)^4\cos(a+bx)\sin^2(a+bx)\,dx$

output
```
((c + d*x)^4*Sin[a + b*x]^3)/(3*b) - (4*d*(-1/3*((c + d*x)^3*Cos[a + b*x]*
Sin[a + b*x]^2)/b + (d*(c + d*x)^2*Sin[a + b*x]^3)/(3*b^2) - (2*d^2*(-1/3*
((c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2)/b + (d*Sin[a + b*x]^3)/(9*b^2) + (
2*(-(((c + d*x)*Cos[a + b*x])/b) + (d*Sin[a + b*x])/b^2))/3))/(3*b^2) + (2
*(-(((c + d*x)^3*Cos[a + b*x])/b) + (3*d*(((c + d*x)^2*Sin[a + b*x])/b - (
2*d*(-(((c + d*x)*Cos[a + b*x])/b) + (d*Sin[a + b*x])/b^2))/b))/b))/3))/(3
*b)
```

### 3.14.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

## 3.14.4 Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.84

| method | result |
|---|---|
| parallelrisch | $\frac{\left(-27b^4(dx+c)^4+36d^2(dx+c)^2b^2-8d^4\right)\sin(3xb+3a)-36b\left((dx+c)^2b^2-\frac{2d^2}{3}\right)d(dx+c)\cos(3xb+3a)+81\left(b^4(dx+c)^4-12\right.}{324b^5}$ |
| risch | $\frac{d(b^2d^3x^3+3b^2c\,d^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6c\,d^2)\cos(xb+a)}{b^4}+\frac{(d^4x^4b^4+4b^4c\,d^3x^3+6b^4c^2d^2x^2+4b^4c^3dx+b^4c^4-12}{4b^5}$ |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |

input
```
int((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/324*((-27*b^4*(d*x+c)^4+36*d^2*(d*x+c)^2*b^2-8*d^4)*sin(3*b*x+3*a)-36*b*
((d*x+c)^2*b^2-2/3*d^2)*d*(d*x+c)*cos(3*b*x+3*a)+81*(b^4*(d*x+c)^4-12*d^2*
(d*x+c)^2*b^2+24*d^4)*sin(b*x+a)+324*b*(((d*x+c)^2*b^2-6*d^2)*(d*x+c)*cos(
b*x+a)+8/9*b^2*c^3-160/27*c*d^2)*d)/b^5
```

## 3.14.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.72

$$\int (c+dx)^4\cos(a+bx)\sin^2(a+bx)\,dx =$$
$$-\frac{12\left(3\,b^3d^4x^3+9\,b^3cd^3x^2+3\,b^3c^3d-2\,bcd^3+(9\,b^3c^2d^2-2\,bd^4)x\right)\cos\left(bx+a\right)^3-36\left(3\,b^3d^4x^3+9\,b^3cd^3x\right.}{}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/81*(12*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d - 2*b*c*d^3 + (9*
b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^3 - 36*(3*b^3*d^4*x^3 + 9*b^3*c*d^3
*x^2 + 3*b^3*c^3*d - 14*b*c*d^3 + (9*b^3*c^2*d^2 - 14*b*d^4)*x)*cos(b*x +
a) - (27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 - 252*b^2*c^2*d^2 +
488*d^4 + 18*(9*b^4*c^2*d^2 - 14*b^2*d^4)*x^2 - (27*b^4*d^4*x^4 + 108*b^4*
c*d^3*x^3 + 27*b^4*c^4 - 36*b^2*c^2*d^2 + 8*d^4 + 18*(9*b^4*c^2*d^2 - 2*b^
2*d^4)*x^2 + 36*(3*b^4*c^3*d - 2*b^2*c*d^3)*x)*cos(b*x + a)^2 + 36*(3*b^4*
c^3*d - 14*b^2*c*d^3)*x)*sin(b*x + a))/b^5
```

### 3.14.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 646 vs. $2(207) = 414$.

Time = 0.62 (sec) , antiderivative size = 646, normalized size of antiderivative = 3.15

$$\int (c+dx)^4\cos(a+bx)\sin^2(a+bx)\,dx$$
$$= \begin{cases} \frac{c^4\sin^3(a+bx)}{3b} + \frac{4c^3dx\sin^3(a+bx)}{3b} + \frac{2c^2d^2x^2\sin^3(a+bx)}{b} + \frac{4cd^3x^3\sin^3(a+bx)}{3b} + \frac{d^4x^4\sin^3(a+bx)}{3b} + \frac{4c^3d\sin^2(a+bx)\cos(a+bx)}{3b^2} - \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\sin^2(a)\cos(a) \end{cases}$$

input
```
integrate((d*x+c)**4*cos(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Piecewise((c**4*sin(a + b*x)**3/(3*b) + 4*c**3*d*x*sin(a + b*x)**3/(3*b) +
 2*c**2*d**2*x**2*sin(a + b*x)**3/b + 4*c*d**3*x**3*sin(a + b*x)**3/(3*b)
+ d**4*x**4*sin(a + b*x)**3/(3*b) + 4*c**3*d*sin(a + b*x)**2*cos(a + b*x)/
(3*b**2) + 8*c**3*d*cos(a + b*x)**3/(9*b**2) + 4*c**2*d**2*x*sin(a + b*x)*
*2*cos(a + b*x)/b**2 + 8*c**2*d**2*x*cos(a + b*x)**3/(3*b**2) + 4*c*d**3*x
**2*sin(a + b*x)**2*cos(a + b*x)/b**2 + 8*c*d**3*x**2*cos(a + b*x)**3/(3*b
**2) + 4*d**4*x**3*sin(a + b*x)**2*cos(a + b*x)/(3*b**2) + 8*d**4*x**3*cos
(a + b*x)**3/(9*b**2) - 28*c**2*d**2*sin(a + b*x)**3/(9*b**3) - 8*c**2*d**
2*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 56*c*d**3*x*sin(a + b*x)**3/(9*b
**3) - 16*c*d**3*x*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 28*d**4*x**2*si
n(a + b*x)**3/(9*b**3) - 8*d**4*x**2*sin(a + b*x)*cos(a + b*x)**2/(3*b**3)
 - 56*c*d**3*sin(a + b*x)**2*cos(a + b*x)/(9*b**4) - 160*c*d**3*cos(a + b*
x)**3/(27*b**4) - 56*d**4*x*sin(a + b*x)**2*cos(a + b*x)/(9*b**4) - 160*d*
*4*x*cos(a + b*x)**3/(27*b**4) + 488*d**4*sin(a + b*x)**3/(81*b**5) + 160*
d**4*sin(a + b*x)*cos(a + b*x)**2/(27*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*
d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)**2*cos(a), T
rue))
```

### 3.14.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 880 vs. $2(187) = 374$.

Time = 0.25 (sec) , antiderivative size = 880, normalized size of antiderivative = 4.29

$$\int (c+dx)^4 \cos(a+bx) \sin^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/324*(108*c^4*sin(b*x + a)^3 - 432*a*c^3*d*sin(b*x + a)^3/b + 648*a^2*c^2
*d^2*sin(b*x + a)^3/b^2 - 432*a^3*c*d^3*sin(b*x + a)^3/b^3 + 108*a^4*d^4*s
in(b*x + a)^3/b^4 - 36*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x
 + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*c^3*d/b + 108*(3*(b*x + a)*sin(
3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a
))*a*c^2*d^2/b^2 - 108*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x
 + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*a^2*c*d^3/b^3 + 36*(3*(b*x + a)
*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*
x + a))*a^3*d^4/b^4 - 18*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(
b*x + a) + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin
(b*x + a))*c^2*d^2/b^2 + 36*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*c
os(b*x + a) + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*
sin(b*x + a))*a*c*d^3/b^3 - 18*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a
)*cos(b*x + a) + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 -
2)*sin(b*x + a))*a^2*d^4/b^4 - 12*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) -
81*((b*x + a)^2 - 2)*cos(b*x + a) + 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*
b*x + 3*a) - 27*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*c*d^3/b^3 + 12*(
(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*cos(b*x + a) +
 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) - 27*((b*x + a)^3 - 6*b*
x - 6*a)*sin(b*x + a))*a*d^4/b^4 - (12*(3*(b*x + a)^3 - 2*b*x - 2*a)*co...
```

### 3.14.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.71

$$\int (c+dx)^4\cos(a+bx)\sin^2(a+bx)\,dx$$
$$= -\frac{(3\,b^3d^4x^3+9\,b^3cd^3x^2+9\,b^3c^2d^2x+3\,b^3c^3d-2\,bd^4x-2\,bcd^3)\cos(3\,bx+3\,a)}{27\,b^5}$$
$$+\frac{(b^3d^4x^3+3\,b^3cd^3x^2+3\,b^3c^2d^2x+b^3c^3d-6\,bd^4x-6\,bcd^3)\cos(bx+a)}{b^5}$$
$$-\frac{(27\,b^4d^4x^4+108\,b^4cd^3x^3+162\,b^4c^2d^2x^2+108\,b^4c^3dx+27\,b^4c^4-36\,b^2d^4x^2-72\,b^2cd^3x-36\,b^2c^2d^2+8}{324\,b^5}$$
$$+\frac{(b^4d^4x^4+4\,b^4cd^3x^3+6\,b^4c^2d^2x^2+4\,b^4c^3dx+b^4c^4-12\,b^2d^4x^2-24\,b^2cd^3x-12\,b^2c^2d^2+24\,d^4)\sin(bx}{4\,b^5}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/27*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2
*b*d^4*x - 2*b*c*d^3)*cos(3*b*x + 3*a)/b^5 + (b^3*d^4*x^3 + 3*b^3*c*d^3*x^
2 + 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*cos(b*x + a)/b^5
- 1/324*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^
4*c^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2
+ 8*d^4)*sin(3*b*x + 3*a)/b^5 + 1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4
*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x -
 12*b^2*c^2*d^2 + 24*d^4)*sin(b*x + a)/b^5
```

### 3.14.9 Mupad [B] (verification not implemented)

Time = 24.75 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.19

$$\int (c+dx)^4 \cos(a+bx)\sin^2(a+bx)\,dx = \frac{\sin\left(a+b\,x\right)^3\,\left(27\,b^4\,c^4-252\,b^2\,c^2\,d^2+488\,d^4\right)}{81\,b^5} - \frac{8\cos\left(a+b\,x\right)^3\,\left(20\,c\,d^3-3\,b^2\,c^3\,d\right)}{27\,b^4} + \frac{8\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\left(20\,d^4-9\,b^2\,c^2\,d^2\right)}{27\,b^5} - \frac{4\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2\,\left(14\,c\,d^3-3\,b^2\,c^3\,d\right)}{9\,b^4} + \frac{8\,d^4\,x^3\cos\left(a+b\,x\right)^3}{9\,b^2} - \frac{8\,x\cos\left(a+b\,x\right)^3\,\left(20\,d^4-9\,b^2\,c^2\,d^2\right)}{27\,b^4} + \frac{d^4\,x^4\sin\left(a+b\,x\right)^3}{3\,b} - \frac{4\,x\sin\left(a+b\,x\right)^3\,\left(14\,c\,d^3-3\,b^2\,c^3\,d\right)}{9\,b^3} - \frac{2\,x^2\sin\left(a+b\,x\right)^3\,\left(14\,d^4-9\,b^2\,c^2\,d^2\right)}{9\,b^3} + \frac{8\,c\,d^3\,x^2\cos\left(a+b\,x\right)^3}{3\,b^2} + \frac{4\,d^4\,x^3\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2}{3\,b^2} - \frac{8\,d^4\,x^2\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{3\,b^3} + \frac{4\,c\,d^3\,x^3\sin\left(a+b\,x\right)^3}{3\,b} - \frac{4\,x\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2\,\left(14\,d^4-9\,b^2\,c^2\,d^2\right)}{9\,b^4} + \frac{4\,c\,d^3\,x^2\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2}{b^2} - \frac{16\,c\,d^3\,x\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{3\,b^3}$$

input
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^4,x)
```

output
```
(sin(a + b*x)^3*(488*d^4 + 27*b^4*c^4 - 252*b^2*c^2*d^2))/(81*b^5) - (8*co
s(a + b*x)^3*(20*c*d^3 - 3*b^2*c^3*d))/(27*b^4) + (8*cos(a + b*x)^2*sin(a
+ b*x)*(20*d^4 - 9*b^2*c^2*d^2))/(27*b^5) - (4*cos(a + b*x)*sin(a + b*x)^2
*(14*c*d^3 - 3*b^2*c^3*d))/(9*b^4) + (8*d^4*x^3*cos(a + b*x)^3)/(9*b^2) -
(8*x*cos(a + b*x)^3*(20*d^4 - 9*b^2*c^2*d^2))/(27*b^4) + (d^4*x^4*sin(a +
b*x)^3)/(3*b) - (4*x*sin(a + b*x)^3*(14*c*d^3 - 3*b^2*c^3*d))/(9*b^3) - (2
*x^2*sin(a + b*x)^3*(14*d^4 - 9*b^2*c^2*d^2))/(9*b^3) + (8*c*d^3*x^2*cos(a
 + b*x)^3)/(3*b^2) + (4*d^4*x^3*cos(a + b*x)*sin(a + b*x)^2)/(3*b^2) - (8*
d^4*x^2*cos(a + b*x)^2*sin(a + b*x))/(3*b^3) + (4*c*d^3*x^3*sin(a + b*x)^3
)/(3*b) - (4*x*cos(a + b*x)*sin(a + b*x)^2*(14*d^4 - 9*b^2*c^2*d^2))/(9*b^
4) + (4*c*d^3*x^2*cos(a + b*x)*sin(a + b*x)^2)/b^2 - (16*c*d^3*x*cos(a + b
*x)^2*sin(a + b*x))/(3*b^3)
```

## 3.14. $\int (c+dx)^4 \cos(a+bx) \sin^2(a+bx)\, dx$

# 3.15 $\int (c+dx)^3 \cos(a+bx) \sin^2(a+bx) \, dx$

3.15.1 Optimal result . . . . . 244
3.15.2 Mathematica [A] (verified) . . . . . 245
3.15.3 Rubi [A] (verified) . . . . . 245
3.15.4 Maple [A] (verified) . . . . . 248
3.15.5 Fricas [A] (verification not implemented) . . . . . 249
3.15.6 Sympy [B] (verification not implemented) . . . . . 249
3.15.7 Maxima [B] (verification not implemented) . . . . . 250
3.15.8 Giac [A] (verification not implemented) . . . . . 251
3.15.9 Mupad [B] (verification not implemented) . . . . . 251

## 3.15.1 Optimal result

Integrand size = 22, antiderivative size = 151

$$\int (c+dx)^3 \cos(a+bx) \sin^2(a+bx) \, dx = -\frac{14d^3\cos(a+bx)}{9b^4} + \frac{2d(c+dx)^2\cos(a+bx)}{3b^2} + \frac{2d^3\cos^3(a+bx)}{27b^4} - \frac{4d^2(c+dx)\sin(a+bx)}{3b^3} + \frac{d(c+dx)^2\cos(a+bx)\sin^2(a+bx)}{3b^2} - \frac{2d^2(c+dx)\sin^3(a+bx)}{9b^3} + \frac{(c+dx)^3\sin^3(a+bx)}{3b}$$

output
```
-14/9*d^3*cos(b*x+a)/b^4+2/3*d*(d*x+c)^2*cos(b*x+a)/b^2+2/27*d^3*cos(b*x+a
)^3/b^4-4/3*d^2*(d*x+c)*sin(b*x+a)/b^3+1/3*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+
a)^2/b^2-2/9*d^2*(d*x+c)*sin(b*x+a)^3/b^3+1/3*(d*x+c)^3*sin(b*x+a)^3/b
```

### 3.15.2 Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.80

$$\int (c+dx)^3 \cos(a+bx)\sin^2(a+bx)\,dx =$$
$$-\frac{-81d(-2d^2+b^2(c+dx)^2)\cos(a+bx)+d(-2d^2+9b^2(c+dx)^2)\cos(3(a+bx))+6b(c+dx)\,(26d^2-3}{108b^4}$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
-1/108*(-81*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + d*(-2*d^2 + 9*b^2*
(c + d*x)^2)*Cos[3*(a + b*x)] + 6*b*(c + d*x)*(26*d^2 - 3*b^2*(c + d*x)^2
+ (-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/b^4
```

### 3.15.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {4904, 3042, 3792, 3042, 3113, 2009, 3777, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\sin^2(a+bx)\cos(a+bx)\,dx$$
$$\Big\downarrow \text{4904}$$
$$\frac{(c+dx)^3\sin^3(a+bx)}{3b}-\frac{d\int (c+dx)^2\sin^3(a+bx)dx}{b}$$
$$\Big\downarrow \text{3042}$$
$$\frac{(c+dx)^3\sin^3(a+bx)}{3b}-\frac{d\int (c+dx)^2\sin(a+bx)^3dx}{b}$$
$$\Big\downarrow \text{3792}$$
$$\frac{(c+dx)^3\sin^3(a+bx)}{3b}-\frac{d\left(-\frac{2d^2\int \sin^3(a+bx)dx}{9b^2}+\frac{2}{3}\int (c+dx)^2\sin(a+bx)dx+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{b}$$
$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^3\sin^3(a+bx)}{3b}-\frac{d\left(-\frac{2d^2\int\sin(a+bx)^3dx}{9b^2}+\frac{2}{3}\int(c+dx)^2\sin(a+bx)dx+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{b}$$

$$\Big\downarrow \ 3113$$

$$\frac{(c+dx)^3\sin^3(a+bx)}{3b}-\frac{d\left(\frac{2d^2\int(1-\cos^2(a+bx))d\cos(a+bx)}{9b^3}+\frac{2}{3}\int(c+dx)^2\sin(a+bx)dx+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{b}$$

$$\Big\downarrow \ 2009$$

$$\frac{(c+dx)^3\sin^3(a+bx)}{3b}-\frac{d\left(\frac{2}{3}\int(c+dx)^2\sin(a+bx)dx+\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{b}$$

$$\Big\downarrow \ 3777$$

$$\frac{(c+dx)^3\sin^3(a+bx)}{3b}-\frac{d\left(\frac{2}{3}\left(\frac{2d\int(c+dx)\cos(a+bx)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)+\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+bx)\,\mathrm{co}}{3b}\right.}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{(c+dx)^3\sin^3(a+bx)}{3b}-\frac{d\left(\frac{2}{3}\left(\frac{2d\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)+\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2\sin^2(a+ba}{3b}\right.}{b}$$

$$\Big\downarrow \ 3777$$

$$\frac{(c+dx)^3\sin^3(a+bx)}{3b}-\frac{d\left(\frac{2}{3}\left(\frac{2d\left(\frac{d\int-\sin(a+bx)dx}{b}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)+\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-(c+da\right.}{b}$$

$$\Big\downarrow \ 25$$

$$\frac{(c+dx)^3\sin^3(a+bx)}{3b}-\frac{d\left(\frac{2}{3}\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)+\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-(c+dx)^2\right.}{b}$$

$$\Big\downarrow \ 3042$$

---

3.15. $\int(c+dx)^3\cos(a+bx)\sin^2(a+bx)\,dx$

$$\frac{(c+dx)^3\sin^3(a+bx)}{3b} - \frac{d\left(\frac{2}{3}\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)+\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}-\frac{(c+dx)^2}{}\right.}{b}$$

$$\Big\downarrow \text{3118}$$

$$\frac{(c+dx)^3\sin^3(a+bx)}{3b} - \frac{d\left(\frac{2d^2\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\sin^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)-\frac{(c+dx)^2\sin}{}\right.}{b}$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
((c + d*x)^3*Sin[a + b*x]^3)/(3*b) - (d*((2*d^2*(Cos[a + b*x] - Cos[a + b*
x]^3/3))/(9*b^3) - ((c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2)/(3*b) + (2*d*
(c + d*x)*Sin[a + b*x]^3)/(9*b^2) + (2*(-(((c + d*x)^2*Cos[a + b*x])/b) +
(2*d*((d*Cos[a + b*x])/b^2 + ((c + d*x)*Sin[a + b*x])/b))/b))/3))/b
```

### 3.15.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

## 3.15.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.48

| method | result |
|---|---|
| risch | $\frac{3d(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2)\cos(xb+a)}{4b^4}+\frac{(b^2d^3x^3+3b^2c\,d^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6c\,d^2)\sin(xb+a)}{4b^3}-\frac{d(9x^2\cdots}{\cdots}$ |
| parallelrisch | $\frac{-36b^2x\,d^2\left(\frac{dx}{2}+c\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^6-72b\,d^2(dx+c)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^5+((-54x^2b^2-72)d^3-108b^2c\,d^2x)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^4+72b(dx\cdots}{\cdots}$ |
| norman | $\frac{\frac{36b^2c^2d-80d^3}{27b^4}+\frac{2d^3x^2}{3b^2}-\frac{8d^3\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^4}{3b^4}+\frac{(36b^2c^2d-56d^3)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}{9b^4}-\frac{8c\,d^2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{3b^3}-\frac{8c\,d^2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^5}{3b^3}+\frac{4c\,d^2x}{3b^2}+\cdots}{\cdots}$ |
| derivativedivides | $\frac{-\frac{a^3d^3\sin(xb+a)^3}{3b^3}+\frac{a^2c\,d^2\sin(xb+a)^3}{b^2}+\frac{3a^2d^3\left(\frac{(xb+a)\sin(xb+a)^3}{3}+\frac{(2+\sin(xb+a)^2)\cos(xb+a)}{9}\right)}{b^3}-\frac{a\,c^2d\sin(xb+a)^3}{b}-\frac{6ac\,d^2(\cdots}{\cdots}}{\cdots}$ |
| default | $\frac{-\frac{a^3d^3\sin(xb+a)^3}{3b^3}+\frac{a^2c\,d^2\sin(xb+a)^3}{b^2}+\frac{3a^2d^3\left(\frac{(xb+a)\sin(xb+a)^3}{3}+\frac{(2+\sin(xb+a)^2)\cos(xb+a)}{9}\right)}{b^3}-\frac{a\,c^2d\sin(xb+a)^3}{b}-\frac{6ac\,d^2(\cdots}{\cdots}}{\cdots}$ |

input
```
int((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
3/4*d*(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2-2*d^2)/b^4*cos(b*x+a)+1/4/b^3*(b^2*
d^3*x^3+3*b^2*c*d^2*x^2+3*b^2*c^2*d*x+b^2*c^3-6*d^3*x-6*c*d^2)*sin(b*x+a)-
1/108*d*(9*b^2*d^2*x^2+18*b^2*c*d*x+9*b^2*c^2-2*d^2)/b^4*cos(3*b*x+3*a)-1/
36/b^3*(3*b^2*d^3*x^3+9*b^2*c*d^2*x^2+9*b^2*c^2*d*x+3*b^2*c^3-2*d^3*x-2*c*
d^2)*sin(3*b*x+3*a)
```

### 3.15.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.50

$$\int (c+dx)^3\cos(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3)\cos{(bx+a)}^3-3\,(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-14\,d^3)\cos{(bx}}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/27*((9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a)
^3 - 3*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 14*d^3)*cos(b*x + a
) - 3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 14*b*c*d^2 - (3*b^3*d
^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 2*b*c*d^2 + (9*b^3*c^2*d - 2*b*d^3)
*x)*cos(b*x + a)^2 + (9*b^3*c^2*d - 14*b*d^3)*x)*sin(b*x + a))/b^4
```

### 3.15.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. 2(150) = 300.

Time = 0.46 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.59

$$\int (c+dx)^3\cos(a+bx)\sin^2(a+bx)\,dx$$

$$=\begin{cases}\frac{c^3\sin^3{(a+bx)}}{3b}+\frac{c^2dx\sin^3{(a+bx)}}{b}+\frac{cd^2x^2\sin^3{(a+bx)}}{b}+\frac{d^3x^3\sin^3{(a+bx)}}{3b}+\frac{c^2d\sin^2{(a+bx)}\cos{(a+bx)}}{b^2}+\frac{2c^2d\cos^3{(a+bx)}}{3b^2}+\frac{2cd^2}{} \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\sin^2{(a)}\cos{(a)}\end{cases}$$

input
```
integrate((d*x+c)**3*cos(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Piecewise((c**3*sin(a + b*x)**3/(3*b) + c**2*d*x*sin(a + b*x)**3/b + c*d**
2*x**2*sin(a + b*x)**3/b + d**3*x**3*sin(a + b*x)**3/(3*b) + c**2*d*sin(a
+ b*x)**2*cos(a + b*x)/b**2 + 2*c**2*d*cos(a + b*x)**3/(3*b**2) + 2*c*d**2
*x*sin(a + b*x)**2*cos(a + b*x)/b**2 + 4*c*d**2*x*cos(a + b*x)**3/(3*b**2)
 + d**3*x**2*sin(a + b*x)**2*cos(a + b*x)/b**2 + 2*d**3*x**2*cos(a + b*x)*
*3/(3*b**2) - 14*c*d**2*sin(a + b*x)**3/(9*b**3) - 4*c*d**2*sin(a + b*x)*c
os(a + b*x)**2/(3*b**3) - 14*d**3*x*sin(a + b*x)**3/(9*b**3) - 4*d**3*x*si
n(a + b*x)*cos(a + b*x)**2/(3*b**3) - 14*d**3*sin(a + b*x)**2*cos(a + b*x)
/(9*b**4) - 40*d**3*cos(a + b*x)**3/(27*b**4), Ne(b, 0)), ((c**3*x + 3*c**
2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**2*cos(a), True))
```

### 3.15.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 499 vs. $2(137) = 274$.

Time = 0.24 (sec) , antiderivative size = 499, normalized size of antiderivative = 3.30

$$\int (c+dx)^3 \cos(a+bx)\sin^2(a+bx)\,dx$$

$$= \frac{36\,c^3 \sin(bx+a)^3 - \frac{108\,ac^2d\sin(bx+a)^3}{b} + \frac{108\,a^2cd^2\sin(bx+a)^3}{b^2} - \frac{36\,a^3d^3\sin(bx+a)^3}{b^3} - \frac{9\,(3\,(bx+a)\sin(3\,bx+3\,a)-9\,(bx+a)\sin(b}{b}}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/108*(36*c^3*sin(b*x + a)^3 - 108*a*c^2*d*sin(b*x + a)^3/b + 108*a^2*c*d^
2*sin(b*x + a)^3/b^2 - 36*a^3*d^3*sin(b*x + a)^3/b^3 - 9*(3*(b*x + a)*sin(
3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a
))*c^2*d/b + 18*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) +
 cos(3*b*x + 3*a) - 9*cos(b*x + a))*a*c*d^2/b^2 - 9*(3*(b*x + a)*sin(3*b*x
 + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*a^
2*d^3/b^3 - 3*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a) +
(9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))*
c*d^2/b^2 + 3*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a) +
(9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))*
a*d^3/b^3 - ((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*c
os(b*x + a) + 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) - 27*((b*x
+ a)^3 - 6*b*x - 6*a)*sin(b*x + a))*d^3/b^3)/b
```

### 3.15.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.53

$$\int (c+dx)^3\cos(a+bx)\sin^2(a+bx)\,dx$$
$$= -\frac{(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3)\cos(3\,bx+3\,a)}{108\,b^4} + \frac{3\,(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d-2\,d^3)\cos(bx+a)}{4\,b^4} - \frac{(3\,b^3d^3x^3+9\,b^3cd^2x^2+9\,b^3c^2dx+3\,b^3c^3-2\,bd^3x-2\,bcd^2)\sin(3\,bx+3\,a)}{36\,b^4} + \frac{(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3-6\,bd^3x-6\,bcd^2)\sin(bx+a)}{4\,b^4}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/108*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(3*b*x +
3*a)/b^4 + 3/4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x +
 a)/b^4 - 1/36*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^
3 - 2*b*d^3*x - 2*b*c*d^2)*sin(3*b*x + 3*a)/b^4 + 1/4*(b^3*d^3*x^3 + 3*b^3
*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(b*x + a)
/b^4
```

### 3.15.9 Mupad [B] (verification not implemented)

Time = 23.32 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.91

$$\int (c+dx)^3\cos(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{2\,d^3\,x^2\cos(a+b\,x)^3}{3\,b^2} - \frac{\sin(a+b\,x)^3\,(14\,c\,d^2-3\,b^2\,c^3)}{9\,b^3} - \frac{\cos(a+b\,x)\,\sin(a+b\,x)^2\,(14\,d^3-9\,b^2\,c^2\,d)}{9\,b^4} - \frac{x\sin(a+b\,x)^3\,(14\,d^3-9\,b^2\,c^2\,d)}{9\,b^3} - \frac{2\cos(a+b\,x)^3\,(20\,d^3-9\,b^2\,c^2\,d)}{27\,b^4} + \frac{d^3\,x^3\sin(a+b\,x)^3}{3\,b} - \frac{4\,c\,d^2\cos(a+b\,x)^2\,\sin(a+b\,x)}{3\,b^3} + \frac{4\,c\,d^2\,x\cos(a+b\,x)^3}{3\,b^2} - \frac{4\,d^3\,x\cos(a+b\,x)^2\,\sin(a+b\,x)}{3\,b^3} + \frac{d^3\,x^2\,\cos(a+b\,x)\,\sin(a+b\,x)^2}{b^2} + \frac{c\,d^2\,x^2\sin(a+b\,x)^3}{b} + \frac{2\,c\,d^2\,x\,\cos(a+b\,x)\,\sin(a+b\,x)^2}{b^2}$$

input
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^3,x)
```

output
```
(2*d^3*x^2*cos(a + b*x)^3)/(3*b^2) - (sin(a + b*x)^3*(14*c*d^2 - 3*b^2*c^3
))/(9*b^3) - (cos(a + b*x)*sin(a + b*x)^2*(14*d^3 - 9*b^2*c^2*d))/(9*b^4)
- (x*sin(a + b*x)^3*(14*d^3 - 9*b^2*c^2*d))/(9*b^3) - (2*cos(a + b*x)^3*(2
0*d^3 - 9*b^2*c^2*d))/(27*b^4) + (d^3*x^3*sin(a + b*x)^3)/(3*b) - (4*c*d^2
*cos(a + b*x)^2*sin(a + b*x))/(3*b^3) + (4*c*d^2*x*cos(a + b*x)^3)/(3*b^2)
 - (4*d^3*x*cos(a + b*x)^2*sin(a + b*x))/(3*b^3) + (d^3*x^2*cos(a + b*x)*s
in(a + b*x)^2)/b^2 + (c*d^2*x^2*sin(a + b*x)^3)/b + (2*c*d^2*x*cos(a + b*x
)*sin(a + b*x)^2)/b^2
```

## 3.15. $\int (c+dx)^3 \cos(a+bx) \sin^2(a+bx) \, dx$

# 3.16 $\int (c+dx)^2 \cos(a+bx) \sin^2(a+bx)\,dx$


3.16.1 Optimal result . . . . . 253
3.16.2 Mathematica [A] (verified) . . . . . 253
3.16.3 Rubi [A] (verified) . . . . . 254
3.16.4 Maple [A] (verified) . . . . . 256
3.16.5 Fricas [A] (verification not implemented) . . . . . 256
3.16.6 Sympy [B] (verification not implemented) . . . . . 257
3.16.7 Maxima [B] (verification not implemented) . . . . . 257
3.16.8 Giac [A] (verification not implemented) . . . . . 258
3.16.9 Mupad [B] (verification not implemented) . . . . . 258


## 3.16.1 Optimal result

Integrand size = 22, antiderivative size = 103

$$\int (c+dx)^2 \cos(a+bx) \sin^2(a+bx)\,dx = \frac{4d(c+dx)\cos(a+bx)}{9b^2} - \frac{4d^2\sin(a+bx)}{9b^3} + \frac{2d(c+dx)\cos(a+bx)\sin^2(a+bx)}{9b^2} - \frac{2d^2\sin^3(a+bx)}{27b^3} + \frac{(c+dx)^2\sin^3(a+bx)}{3b}$$

output
```
4/9*d*(d*x+c)*cos(b*x+a)/b^2-4/9*d^2*sin(b*x+a)/b^3+2/9*d*(d*x+c)*cos(b*x+
a)*sin(b*x+a)^2/b^2-2/27*d^2*sin(b*x+a)^3/b^3+1/3*(d*x+c)^2*sin(b*x+a)^3/b
```

## 3.16.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.90

$$\int (c+dx)^2 \cos(a+bx) \sin^2(a+bx)\,dx$$
$$= \frac{54bd(c+dx)\cos(a+bx) - 6bd(c+dx)\cos(3(a+bx)) - 2(26d^2 - 9b^2(c+dx)^2 + (-2d^2 + 9b^2(c+dx)^2)}{108b^3}$$

input
```
Integrate[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
(54*b*d*(c + d*x)*Cos[a + b*x] - 6*b*d*(c + d*x)*Cos[3*(a + b*x)] - 2*(26*
d^2 - 9*b^2*(c + d*x)^2 + (-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*S
in[a + b*x])/(108*b^3)
```

### 3.16.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4904, 3042, 3791, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\sin^2(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{ 4904}$$

$$\frac{(c+dx)^2\sin^3(a+bx)}{3b}-\frac{2d\int (c+dx)\sin^3(a+bx)dx}{3b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(c+dx)^2\sin^3(a+bx)}{3b}-\frac{2d\int (c+dx)\sin(a+bx)^3dx}{3b}$$

$$\Big\downarrow \text{ 3791}$$

$$\frac{(c+dx)^2\sin^3(a+bx)}{3b}-\frac{2d\left(\frac{2}{3}\int (c+dx)\sin(a+bx)dx+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(c+dx)^2\sin^3(a+bx)}{3b}-\frac{2d\left(\frac{2}{3}\int (c+dx)\sin(a+bx)dx+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{(c+dx)^2\sin^3(a+bx)}{3b}-\frac{2d\left(\frac{2}{3}\left(\frac{d\int \cos(a+bx)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(c+dx)^2\sin^3(a+bx)}{3b}-\frac{2d\left(\frac{2}{3}\left(\frac{d\int \sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b}$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{(c+dx)^2\sin^3(a+bx)}{3b}-\frac{2d\left(\frac{2}{3}\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)+\frac{d\sin^3(a+bx)}{9b^2}-\frac{(c+dx)\sin^2(a+bx)\cos(a+bx)}{3b}\right)}{3b}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
((c + d*x)^2*Sin[a + b*x]^3)/(3*b) - (2*d*(-1/3*((c + d*x)*Cos[a + b*x]*Si
n[a + b*x]^2)/b + (d*Sin[a + b*x]^3)/(9*b^2) + (2*(-(((c + d*x)*Cos[a + b*
x])/b) + (d*Sin[a + b*x])/b^2))/3))/(3*b)
```

### 3.16.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

### 3.16.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.24

| method | result |
|---|---|
| risch | $\frac{d(dx+c)\cos(xb+a)}{2b^2}+\frac{(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2)\sin(xb+a)}{4b^3}-\frac{d(dx+c)\cos(3xb+3a)}{18b^2}-\frac{(9x^2d^2b^2+18b^2cdx+9b^2}{108b}$ |
| parallelrisch | $\frac{-12d^2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^6xb-24d^2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^5-36d^2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^4xb+((72x^2b^2-64)d^2+144b^2cdx+72b^2c^2)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{27b^3\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)^3}$ |
| derivativedivides | $\frac{\frac{a^2d^2\sin(xb+a)^3}{3b^2}-\frac{2acd\sin(xb+a)^3}{3b}-\frac{2a\,d^2\left(\frac{(xb+a)\sin(xb+a)^3}{3}+\frac{\left(2+\sin(xb+a)^2\right)\cos(xb+a)}{9}\right)}{b^2}+\frac{c^2\sin(xb+a)^3}{3}+\frac{2cd\left(\frac{(xb+a)\sin(}{3}\right.}{}}{}$ |
| default | $\frac{\frac{a^2d^2\sin(xb+a)^3}{3b^2}-\frac{2acd\sin(xb+a)^3}{3b}-\frac{2a\,d^2\left(\frac{(xb+a)\sin(xb+a)^3}{3}+\frac{\left(2+\sin(xb+a)^2\right)\cos(xb+a)}{9}\right)}{b^2}+\frac{c^2\sin(xb+a)^3}{3}+\frac{2cd\left(\frac{(xb+a)\sin(}{3}\right.}{}}{}$ |
| norman | $\frac{\frac{8cd}{9b^2}-\frac{8d^2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{9b^3}-\frac{8d^2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^5}{9b^3}+\frac{4d^2x}{9b^2}+\frac{8\left(9b^2c^2-8d^2\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3}{27b^3}+\frac{8cd\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}{3b^2}+\frac{4d^2x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}{3b^2}-\frac{4d^2}{}}{\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)^3}$ |

input
```
int((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*d*(d*x+c)*cos(b*x+a)/b^2+1/4*(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2-2*d^2)/b
^3*sin(b*x+a)-1/18*d*(d*x+c)*cos(3*b*x+3*a)/b^2-1/108*(9*b^2*d^2*x^2+18*b^
2*c*d*x+9*b^2*c^2-2*d^2)/b^3*sin(3*b*x+3*a)
```

### 3.16.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.26

$$\int (c+dx)^2\cos(a+bx)\sin^2(a+bx)\,dx =$$
$$-\frac{6\left(bd^2x+bcd\right)\cos\left(bx+a\right)^3-18\left(bd^2x+bcd\right)\cos\left(bx+a\right)-\left(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-\left(9\,b^2d^2x^2\right.\right.}{27\,b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/27*(6*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - 18*(b*d^2*x + b*c*d)*cos(b*x +
 a) - (9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - (9*b^2*d^2*x^2 + 18*b^2*
c*d*x + 9*b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - 14*d^2)*sin(b*x + a))/b^3
```

### 3.16.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(102) = 204$.

Time = 0.32 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.10

$$\int (c+dx)^2\cos(a+bx)\sin^2(a+bx)\,dx$$
$$= \begin{cases} \frac{c^2\sin^3(a+bx)}{3b} + \frac{2cdx\sin^3(a+bx)}{3b} + \frac{d^2x^2\sin^3(a+bx)}{3b} + \frac{2cd\sin^2(a+bx)\cos(a+bx)}{3b^2} + \frac{4cd\cos^3(a+bx)}{9b^2} + \frac{2d^2x\sin^2(a+bx)\cos(a+bx)}{3b^2} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin^2(a)\cos(a) \end{cases}$$

input
```
integrate((d*x+c)**2*cos(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Piecewise((c**2*sin(a + b*x)**3/(3*b) + 2*c*d*x*sin(a + b*x)**3/(3*b) + d*
*2*x**2*sin(a + b*x)**3/(3*b) + 2*c*d*sin(a + b*x)**2*cos(a + b*x)/(3*b**2
) + 4*c*d*cos(a + b*x)**3/(9*b**2) + 2*d**2*x*sin(a + b*x)**2*cos(a + b*x)
/(3*b**2) + 4*d**2*x*cos(a + b*x)**3/(9*b**2) - 14*d**2*sin(a + b*x)**3/(2
7*b**3) - 4*d**2*sin(a + b*x)*cos(a + b*x)**2/(9*b**3), Ne(b, 0)), ((c**2*
x + c*d*x**2 + d**2*x**3/3)*sin(a)**2*cos(a), True))
```

### 3.16.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(93) = 186$.

Time = 0.21 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.33

$$\int (c+dx)^2\cos(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{36\,c^2\sin(bx+a)^3 - \frac{72\,acd\sin(bx+a)^3}{b} + \frac{36\,a^2d^2\sin(bx+a)^3}{b^2} - \frac{6\,(3\,(bx+a)\sin(3\,bx+3\,a)-9\,(bx+a)\sin(bx+a)+\cos(3\,bx+3\,a)-9\,\cos(}{b}}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/108*(36*c^2*sin(b*x + a)^3 - 72*a*c*d*sin(b*x + a)^3/b + 36*a^2*d^2*sin(
b*x + a)^3/b^2 - 6*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a
) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*c*d/b + 6*(3*(b*x + a)*sin(3*b*x +
3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*a*d^2
/b^2 - (6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a) + (9*(b*x
 + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))*d^2/b^2
)/b
```

### 3.16.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.33

$$\int (c+dx)^2\cos(a+bx)\sin^2(a+bx)\,dx = -\frac{(bd^2x+bcd)\cos(3\,bx+3\,a)}{18\,b^3}+\frac{(bd^2x+bcd)\cos(bx+a)}{2\,b^3} - \frac{(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2)\sin(3\,bx+3\,a)}{108\,b^3} + \frac{(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,d^2)\sin(bx+a)}{4\,b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/18*(b*d^2*x + b*c*d)*cos(3*b*x + 3*a)/b^3 + 1/2*(b*d^2*x + b*c*d)*cos(b
*x + a)/b^3 - 1/108*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*sin
(3*b*x + 3*a)/b^3 + 1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(
b*x + a)/b^3
```

### 3.16.9 Mupad [B] (verification not implemented)

Time = 22.97 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.56

$$\int (c+dx)^2\cos(a+bx)\sin^2(a+bx)\,dx = \frac{4\,d^2\,x\cos(a+b\,x)^3}{9\,b^2}-\frac{4\,d^2\cos(a+b\,x)^2\,\sin(a+b\,x)}{9\,b^3}-\frac{\sin(a+b\,x)^3\,(14\,d^2-9\,b^2\,c^2)}{27\,b^3} + \frac{d^2\,x^2\sin(a+b\,x)^3}{3\,b}+\frac{4\,c\,d\cos(a+b\,x)^3}{9\,b^2}+\frac{2\,c\,d\,\cos(a+b\,x)\,\sin(a+b\,x)^2}{3\,b^2} + \frac{2\,c\,d\,x\sin(a+b\,x)^3}{3\,b}+\frac{2\,d^2\,x\,\cos(a+b\,x)\,\sin(a+b\,x)^2}{3\,b^2}$$

input
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^2,x)
```

output
```
(4*d^2*x*cos(a + b*x)^3)/(9*b^2) - (4*d^2*cos(a + b*x)^2*sin(a + b*x))/(9*
b^3) - (sin(a + b*x)^3*(14*d^2 - 9*b^2*c^2))/(27*b^3) + (d^2*x^2*sin(a + b
*x)^3)/(3*b) + (4*c*d*cos(a + b*x)^3)/(9*b^2) + (2*c*d*cos(a + b*x)*sin(a
+ b*x)^2)/(3*b^2) + (2*c*d*x*sin(a + b*x)^3)/(3*b) + (2*d^2*x*cos(a + b*x)
*sin(a + b*x)^2)/(3*b^2)
```

# 3.17 $\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx$

3.17.1 Optimal result . . . . . 259
3.17.2 Mathematica [A] (verified) . . . . . 259
3.17.3 Rubi [A] (verified) . . . . . 260
3.17.4 Maple [A] (verified) . . . . . 261
3.17.5 Fricas [A] (verification not implemented) . . . . . 262
3.17.6 Sympy [A] (verification not implemented) . . . . . 262
3.17.7 Maxima [A] (verification not implemented) . . . . . 262
3.17.8 Giac [A] (verification not implemented) . . . . . 263
3.17.9 Mupad [B] (verification not implemented) . . . . . 263

## 3.17.1 Optimal result

Integrand size = 20, antiderivative size = 51

$$\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx=\frac{d\cos(a+bx)}{3b^2}-\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin^3(a+bx)}{3b}$$

output
```
1/3*d*cos(b*x+a)/b^2-1/9*d*cos(b*x+a)^3/b^2+1/3*(d*x+c)*sin(b*x+a)^3/b
```

## 3.17.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.86

$$\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx =\frac{9d\cos(a+bx)-d\cos(3(a+bx))+12b(c+dx)\sin^3(a+bx)}{36b^2}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
(9*d*Cos[a + b*x] - d*Cos[3*(a + b*x)] + 12*b*(c + d*x)*Sin[a + b*x]^3)/(3
6*b^2)
```

### 3.17.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4904, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin^2(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{ 4904}$$

$$\frac{(c+dx)\sin^3(a+bx)}{3b}-\frac{d\int \sin^3(a+bx)dx}{3b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(c+dx)\sin^3(a+bx)}{3b}-\frac{d\int \sin(a+bx)^3dx}{3b}$$

$$\Big\downarrow \text{ 3113}$$

$$\frac{d\int \left(1-\cos^2(a+bx)\right)d\cos(a+bx)}{3b^2}+\frac{(c+dx)\sin^3(a+bx)}{3b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d\left(\cos(a+bx)-\frac{1}{3}\cos^3(a+bx)\right)}{3b^2}+\frac{(c+dx)\sin^3(a+bx)}{3b}$$

input
```
Int[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
(d*(Cos[a + b*x] - Cos[a + b*x]^3/3))/(3*b^2) + ((c + d*x)*Sin[a + b*x]^3)
/(3*b)
```

#### 3.17.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

### 3.17.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{\frac{8b(dx+c)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{3}}{3}+\frac{4\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}d}{3}+\frac{4d}{9}}{b^{2}\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)^{3}}$ | 56 |
| risch | $\frac{d\cos(xb+a)}{4b^{2}}+\frac{(dx+c)\sin(xb+a)}{4b}-\frac{d\cos(3xb+3a)}{36b^{2}}-\frac{(dx+c)\sin(3xb+3a)}{12b}$ | 64 |
| derivativedivides | $\frac{-\frac{da\sin(xb+a)^{3}}{3b}+\frac{c\sin(xb+a)^{3}}{3}+\frac{d\left(\frac{(xb+a)\sin(xb+a)^{3}}{3}+\frac{\left(2+\sin(xb+a)^{2}\right)\cos(xb+a)}{9}\right)}{b}}{b}$ | 71 |
| default | $\frac{-\frac{da\sin(xb+a)^{3}}{3b}+\frac{c\sin(xb+a)^{3}}{3}+\frac{d\left(\frac{(xb+a)\sin(xb+a)^{3}}{3}+\frac{\left(2+\sin(xb+a)^{2}\right)\cos(xb+a)}{9}\right)}{b}}{b}$ | 71 |
| norman | $\frac{\frac{4d}{9b^{2}}+\frac{8c\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{3}}{3b}+\frac{4d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}}{3b^{2}}+\frac{8dx\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{3}}{3b}}{\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)^{3}}$ | 76 |

input
```
int((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
4/9*(6*b*(d*x+c)*tan(1/2*a+1/2*x*b)^3+3*tan(1/2*a+1/2*x*b)^2*d+d)/b^2/(1+t
an(1/2*a+1/2*x*b)^2)^3
```

### 3.17.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx$$
$$= -\frac{d\cos{(bx+a)}^3 - 3\,d\cos{(bx+a)} - 3\left(bdx - (bdx+bc)\cos{(bx+a)}^2 + bc\right)\sin{(bx+a)}}{9\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/9*(d*cos(b*x + a)^3 - 3*d*cos(b*x + a) - 3*(b*d*x - (b*d*x + b*c)*cos(b
*x + a)^2 + b*c)*sin(b*x + a))/b^2
```

### 3.17.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.67

$$\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx$$
$$= \begin{cases} \frac{c\sin^3{(a+bx)}}{3b} + \frac{dx\sin^3{(a+bx)}}{3b} + \frac{d\sin^2{(a+bx)}\cos{(a+bx)}}{3b^2} + \frac{2d\cos^3{(a+bx)}}{9b^2} & \text{for}\ b\neq 0 \\ \left(cx+\frac{dx^2}{2}\right)\sin^2{(a)}\cos{(a)} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Piecewise((c*sin(a + b*x)**3/(3*b) + d*x*sin(a + b*x)**3/(3*b) + d*sin(a +
 b*x)**2*cos(a + b*x)/(3*b**2) + 2*d*cos(a + b*x)**3/(9*b**2), Ne(b, 0)),
((c*x + d*x**2/2)*sin(a)**2*cos(a), True))
```

### 3.17.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.67

$$\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{12\,c\sin{(bx+a)}^3 - \frac{12\,ad\sin(bx+a)^3}{b} - \frac{(3\,(bx+a)\sin(3\,bx+3\,a) - 9\,(bx+a)\sin(bx+a) + \cos(3\,bx+3\,a) - 9\,\cos(bx+a))d}{b}}{36\,b}$$

input
```
integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/36*(12*c*sin(b*x + a)^3 - 12*a*d*sin(b*x + a)^3/b - (3*(b*x + a)*sin(3*b
*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*
d/b)/b
```

### 3.17.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.35

$$\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx = -\frac{d\cos(3\,bx+3\,a)}{36\,b^2}+\frac{d\cos(bx+a)}{4\,b^2}-\frac{(bdx+bc)\sin(3\,bx+3\,a)}{12\,b^2}+\frac{(bdx+bc)\sin(bx+a)}{4\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/36*d*cos(3*b*x + 3*a)/b^2 + 1/4*d*cos(b*x + a)/b^2 - 1/12*(b*d*x + b*c)
*sin(3*b*x + 3*a)/b^2 + 1/4*(b*d*x + b*c)*sin(b*x + a)/b^2
```

### 3.17.9 Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx = \frac{\frac{2\,d\cos(a+b\,x)^3}{9}+b\left(\frac{c\sin(a+b\,x)^3}{3}+\frac{d\,x\sin(a+b\,x)^3}{3}\right)+\frac{d\,\cos(a+b\,x)\sin(a+b\,x)^2}{3}}{b^2}$$

input
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x),x)
```

output
```
((2*d*cos(a + b*x)^3)/9 + b*((c*sin(a + b*x)^3)/3 + (d*x*sin(a + b*x)^3)/3
) + (d*cos(a + b*x)*sin(a + b*x)^2)/3)/b^2
```

# 3.18 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx$

3.18.1 Optimal result . . . . . . . . . . 264
3.18.2 Mathematica [A] (verified) . . . . . . . . . . 264
3.18.3 Rubi [A] (verified) . . . . . . . . . . 265
3.18.4 Maple [A] (verified) . . . . . . . . . . 266
3.18.5 Fricas [A] (verification not implemented) . . . . . . . . . . 266
3.18.6 Sympy [F] . . . . . . . . . . 267
3.18.7 Maxima [C] (verification not implemented) . . . . . . . . . . 267
3.18.8 Giac [C] (verification not implemented) . . . . . . . . . . 268
3.18.9 Mupad [F(-1)] . . . . . . . . . . 268

## 3.18.1 Optimal result

Integrand size = 22, antiderivative size = 121

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d} - \frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d} - \frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d} + \frac{\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

output
```
-1/4*Ci(3*b*c/d+3*b*x)*cos(3*a-3*b*c/d)/d+1/4*Ci(b*c/d+b*x)*cos(a-b*c/d)/d
+1/4*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d-1/4*Si(b*c/d+b*x)*sin(a-b*c/d)/d
```

## 3.18.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right) - \cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right) - \sin\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right) +}{4d}$$

input
```
Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x),x]
```

output
```
(Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] - Cos[3*a - (3*b*c)/d]*CosInteg
ral[(3*b*(c + d*x))/d] - Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Sin[3
*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d)
```

### 3.18.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\cos(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{\cos(a+bx)}{4(c+dx)}-\frac{\cos(3a+3bx)}{4(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

input
```
Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x),x]
```

output
```
(Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d) - (Cos[3*a - (3*b*c)/d
]*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d) - (Sin[a - (b*c)/d]*SinIntegral[(b
*c)/d + b*x])/(4*d) + (Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x]
)/(4*d)
```

#### 3.18.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.18.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.41

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{b\left(-\frac{\mathrm{Si}\left(-xb-a-\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}+\frac{\mathrm{Ci}\left(xb+a+\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}\right)}{4}-\frac{b\left(-\frac{3\,\mathrm{Si}\left(-3xb-3a-\frac{3(-ad+cb)}{d}\right)\sin\left(\frac{-3ad+3}{d}\right.}{d}\right.}{}}{b}$ |
| default | $\frac{\frac{b\left(-\frac{\mathrm{Si}\left(-xb-a-\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}+\frac{\mathrm{Ci}\left(xb+a+\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}\right)}{4}-\frac{b\left(-\frac{3\,\mathrm{Si}\left(-3xb-3a-\frac{3(-ad+cb)}{d}\right)\sin\left(\frac{-3ad+3}{d}\right.}{d}\right.}{}}{b}$ |
| risch | $\frac{e^{-\frac{3i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{8d}-\frac{e^{-\frac{i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(ibx+ia-\frac{i(ad-cb)}{d}\right)}{8d}-\frac{e^{\frac{i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(-ibx-ia-\frac{-iad+ic}{d}\right.}{8d}$ |

input
```
int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4*b*(-Si(-x*b-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(x*b+a+(-a*d+b*
c)/d)*cos((-a*d+b*c)/d)/d)-1/12*b*(-3*Si(-3*x*b-3*a-3*(-a*d+b*c)/d)*sin(3*
(-a*d+b*c)/d)/d+3*Ci(3*x*b+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d))
```

### 3.18.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.01

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx =$$
$$-\frac{\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{3(bdx+bc)}{d}\right)-\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)-\sin\left(-\frac{3(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3(bdx+bc)}{d}\right)+\sin\left(-\frac{bc-a}{d}\right.}{4d}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
-1/4*(cos(-3*(b*c - a*d)/d)*cos_integral(3*(b*d*x + b*c)/d) - cos(-(b*c -
a*d)/d)*cos_integral((b*d*x + b*c)/d) - sin(-3*(b*c - a*d)/d)*sin_integral
(3*(b*d*x + b*c)/d) + sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

## 3.18.6 Sympy [F]

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \int \frac{\sin^2(a+bx)\cos(a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x), x)
```

## 3.18.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.28

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx = -\frac{b\left(E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b\left(E_1\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_1\left(-3\right.\right.}{}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
-1/8*(b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integr
al_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b*(exp_
integral_e(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(1, -3
*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) + b*(-I*exp_in
tegral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_e(1, -(I*b
*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b*(-I*exp_integral_e
(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integral_e(1, -3*(-I*b*c
 - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/(b*d)
```

### 3.18.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.50 (sec) , antiderivative size = 6059, normalized size of antiderivative = 50.07

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
-1/8*(real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(b*x + b*c/d))*ta
n(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_
integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2
*b*c/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2
*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(b*x + b
*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_
part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^
2*tan(1/2*b*c/d) + 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)
^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(cos_integral(3*b*x + 3*b*
c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*imag_p
art(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/
d)*tan(1/2*b*c/d)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1
/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(b*x + b
*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*imag_
part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*
tan(1/2*b*c/d)^2 - 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(3*b*x + 3*b*
c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*imag_p
art(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c...
```

### 3.18.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \int \frac{\cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^2}{c+d\,x}\,dx$$

input
```
int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x),x)
```

output
```
int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x), x)
```

# 3.19 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$

3.19.1 Optimal result . . . . . 269
3.19.2 Mathematica [A] (verified) . . . . . 270
3.19.3 Rubi [A] (verified) . . . . . 270
3.19.4 Maple [A] (verified) . . . . . 271
3.19.5 Fricas [A] (verification not implemented) . . . . . 272
3.19.6 Sympy [F] . . . . . 272
3.19.7 Maxima [C] (verification not implemented) . . . . . 273
3.19.8 Giac [C] (verification not implemented) . . . . . 273
3.19.9 Mupad [F(-1)] . . . . . 274

## 3.19.1 Optimal result

Integrand size = 22, antiderivative size = 168

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = -\frac{\cos(a+bx)}{4d(c+dx)}+\frac{\cos(3a+3bx)}{4d(c+dx)} + \frac{3b\,\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{4d^2} - \frac{b\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{4d^2} - \frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2} + \frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d^2}$$

output
```
-1/4*cos(b*x+a)/d/(d*x+c)+1/4*cos(3*b*x+3*a)/d/(d*x+c)-1/4*b*cos(a-b*c/d)*
Si(b*c/d+b*x)/d^2+3/4*b*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^2+3/4*b*Ci(3*
b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^2-1/4*b*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^2
```

### 3.19.2 Mathematica [A] (verified)

Time = 1.64 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.83

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{\frac{d\cos(a+bx)}{c+dx}-\frac{d\cos(3(a+bx))}{c+dx}-3b\,\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right)+b\,\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\right.}{4d^2}$$

input
```
Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
-1/4*((d*Cos[a + b*x])/(c + d*x) - (d*Cos[3*(a + b*x)])/(c + d*x) - 3*b*Co
sIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + b*CosIntegral[b*(c/d +
 x)]*Sin[a - (b*c)/d] + b*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - 3*b*
Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/d^2
```

### 3.19.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\cos(a+bx)}{(c+dx)^2}\,dx$$
$$\Big\downarrow \; 4906$$
$$\int \left(\frac{\cos(a+bx)}{4(c+dx)^2}-\frac{\cos(3a+3bx)}{4(c+dx)^2}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{\cos(a+bx)}{4d(c+dx)}+\frac{\cos(3a+3bx)}{4d(c+dx)}$$

input
```
Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
-1/4*Cos[a + b*x]/(d*(c + d*x)) + Cos[3*a + 3*b*x]/(4*d*(c + d*x)) + (3*b*
CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(4*d^2) - (b*CosInteg
ral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(4*d^2) - (b*Cos[a - (b*c)/d]*SinInte
gral[(b*c)/d + b*x])/(4*d^2) + (3*b*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*
c)/d + 3*b*x])/(4*d^2)
```

### 3.19.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.19.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.47

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{b^2\left(-\frac{\cos(xb+a)}{(-ad+cb+d(xb+a))d}-\frac{-\frac{\mathrm{Si}\left(-xb-a-\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}-\frac{\mathrm{Ci}\left(xb+a+\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}}{d}\right)}{4}-\frac{b^2\left(-\frac{3\cos(3xb+\cdots}{(-ad+cb+d(x\cdots}\right.}{\cdots}}{b}$ |
| default | $\frac{\frac{b^2\left(-\frac{\cos(xb+a)}{(-ad+cb+d(xb+a))d}-\frac{-\frac{\mathrm{Si}\left(-xb-a-\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}-\frac{\mathrm{Ci}\left(xb+a+\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}}{d}\right)}{4}-\frac{b^2\left(-\frac{3\cos(3xb+\cdots}{(-ad+cb+d(x\cdots}\right.}{\cdots}}{b}$ |
| risch | $-\frac{3ib\,e^{-\frac{3i(ad-cb)}{d}}\mathrm{Ei}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{8d^2}+\frac{ib\,e^{-\frac{i(ad-cb)}{d}}\mathrm{Ei}_1\left(ibx+ia-\frac{i(ad-cb)}{d}\right)}{8d^2}-\frac{ib\,e^{\frac{i(ad-cb)}{d}}\mathrm{Ei}_1\left(-ibx-ia\cdots\right.}{8d^2}$ |

input
```
int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4*b^2*(-cos(b*x+a)/(-a*d+c*b+d*(b*x+a))/d-(-Si(-x*b-a-(-a*d+b*c)/d)
*cos((-a*d+b*c)/d)/d-Ci(x*b+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)-1/12*b
^2*(-3*cos(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d-3*(-3*Si(-3*x*b-3*a-3*(-a*d+b
*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*x*b+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*
c)/d)/d)/d))
```

### 3.19.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.11

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{4\,d\cos\left(bx+a\right)^3-\left(bdx+bc\right)\operatorname{Ci}\left(\frac{bdx+bc}{d}\right)\sin\left(-\frac{bc-ad}{d}\right)+3\left(bdx+bc\right)\operatorname{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)+3\,(b}{4\,(d^3x+cd^2}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/4*(4*d*cos(b*x + a)^3 - (b*d*x + b*c)*cos_integral((b*d*x + b*c)/d)*sin(
-(b*c - a*d)/d) + 3*(b*d*x + b*c)*cos_integral(3*(b*d*x + b*c)/d)*sin(-3*(
b*c - a*d)/d) + 3*(b*d*x + b*c)*cos(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*
x + b*c)/d) - (b*d*x + b*c)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)
/d) - 4*d*cos(b*x + a))/(d^3*x + c*d^2)
```

### 3.19.6 Sympy [F]

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin^2\left(a+bx\right)\cos\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x)**2, x)
```

### 3.19.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.82

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{b^2\left(E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^2\left(E_2\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_2\left(-\right.\right.}{}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/8*(b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_inte
gral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b^2*(
exp_integral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(2
, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b^2*(I*e
xp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(2,
-(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b^2*(I*exp_inte
gral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(2, -3*(
-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b*c*d + (b*x
+ a)*d^2 - a*d^2)*b)
```

### 3.19.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.41 (sec) , antiderivative size = 67350, normalized size of antiderivative = 400.89

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/8*(3*b*d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - b*
d*x*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan
(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + b*d*x*imag_part
(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b*d*x*imag_part(cos_integ
ral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*
a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*b*d*x*sin_integral(3*(b*d*x + b
*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c
/d)^2*tan(1/2*b*c/d)^2 - 2*b*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x
)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 - 2*b*d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2
*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*b*d*
x*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(
3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 6*b*d*x*real_part(
cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*
tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 6*b*d*x*real_part(cos_integ
ral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*
a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*b*d*x*real_part(cos_integral(b*x
+ b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2...
```

### 3.19.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\,dx$$

input
```
int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^2,x)
```

output
```
int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^2, x)
```

# 3.20 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$

3.20.1 Optimal result . . . 275
3.20.2 Mathematica [A] (verified) . . . 276
3.20.3 Rubi [A] (verified) . . . 276
3.20.4 Maple [A] (verified) . . . 277
3.20.5 Fricas [A] (verification not implemented) . . . 278
3.20.6 Sympy [F] . . . 278
3.20.7 Maxima [C] (verification not implemented) . . . 279
3.20.8 Giac [C] (verification not implemented) . . . 279
3.20.9 Mupad [F(-1)] . . . 280

## 3.20.1 Optimal result

Integrand size = 22, antiderivative size = 221

$$\begin{aligned}\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx &= -\frac{\cos(a+bx)}{8d(c+dx)^2}+\frac{\cos(3a+3bx)}{8d(c+dx)^2} \\ &\quad -\frac{b^2\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3} \\ &\quad +\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3}+\frac{b\sin(a+bx)}{8d^2(c+dx)} \\ &\quad -\frac{3b\sin(3a+3bx)}{8d^2(c+dx)}+\frac{b^2\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{8d^3} \\ &\quad -\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^3}\end{aligned}$$

output
```
9/8*b^2*Ci(3*b*c/d+3*b*x)*cos(3*a-3*b*c/d)/d^3-1/8*b^2*Ci(b*c/d+b*x)*cos(a
-b*c/d)/d^3-1/8*cos(b*x+a)/d/(d*x+c)^2+1/8*cos(3*b*x+3*a)/d/(d*x+c)^2-9/8*
b^2*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^3+1/8*b^2*Si(b*c/d+b*x)*sin(a-b*c
/d)/d^3+1/8*b*sin(b*x+a)/d^2/(d*x+c)-3/8*b*sin(3*b*x+3*a)/d^2/(d*x+c)
```

### 3.20.2 Mathematica [A] (verified)

Time = 2.17 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.83

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{-b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)+\frac{d(-d\cos(a+bx)+b(c+dx)}{(c+dx)^2}}{8}$$

input
```
Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

output
```
(-(b^2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)]) + 9*b^2*Cos[3*a - (3*b*c
)/d]*CosIntegral[(3*b*(c + d*x))/d] + (d*(-(d*Cos[a + b*x]) + b*(c + d*x)*
Sin[a + b*x]))/(c + d*x)^2 + (d*(d*Cos[3*(a + b*x)] - 3*b*(c + d*x)*Sin[3*
(a + b*x)]))/(c + d*x)^2 + b^2*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] -
 9*b^2*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(8*d^3)
```

### 3.20.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\cos(a+bx)}{(c+dx)^3}\,dx$$
$$\Big\downarrow \ 4906$$
$$\int \left(\frac{\cos(a+bx)}{4(c+dx)^3}-\frac{\cos(3a+3bx)}{4(c+dx)^3}\right)dx$$
$$\Big\downarrow \ 2009$$
$$-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3}+\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3}+\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^3}+\frac{b\sin(a+bx)}{8d^2(c+dx)}-\frac{3b\sin(3a+3bx)}{8d^2(c+dx)}-\frac{\cos(a+bx)}{8d(c+dx)^2}+\frac{\cos(3a+3bx)}{8d(c+dx)^2}$$

input
```
Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

output
```
-1/8*Cos[a + b*x]/(d*(c + d*x)^2) + Cos[3*a + 3*b*x]/(8*d*(c + d*x)^2) - (
b^2*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d^3) + (9*b^2*Cos[3*a
- (3*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(8*d^3) + (b*Sin[a + b*x])/(8
*d^2*(c + d*x)) - (3*b*Sin[3*a + 3*b*x])/(8*d^2*(c + d*x)) + (b^2*Sin[a -
(b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^3) - (9*b^2*Sin[3*a - (3*b*c)/d]
*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^3)
```

### 3.20.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.20.4 Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.43

| method | result |
|---|---|
| derivativedivides | $\frac{b^3\left(-\frac{\cos(xb+a)}{2(-ad+cb+d(xb+a))^2 d}-\frac{-\frac{\sin(xb+a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{\mathrm{Si}\left(-xb-a-\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}+\frac{\mathrm{Ci}\left(xb+a+\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+c}{d}\right)}{d}}{d}}{2d}\right.}{4}$ |
| default | $\frac{b^3\left(-\frac{\cos(xb+a)}{2(-ad+cb+d(xb+a))^2 d}-\frac{-\frac{\sin(xb+a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{\mathrm{Si}\left(-xb-a-\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}+\frac{\mathrm{Ci}\left(xb+a+\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+c}{d}\right)}{d}}{d}}{2d}\right.}{4}$ |
| risch | $-\frac{9b^2 e^{-\frac{3i(ad-cb)}{d}}\mathrm{Ei}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{16d^3}+\frac{b^2 e^{-\frac{i(ad-cb)}{d}}\mathrm{Ei}_1\left(ibx+ia-\frac{i(ad-cb)}{d}\right)}{16d^3}+\frac{b^2 e^{\frac{i(ad-cb)}{d}}\mathrm{Ei}_1\left(-ibx-ia\right.}{16d^3}$ |

input
```
int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4*b^3*(-1/2*cos(b*x+a)/(-a*d+c*b+d*(b*x+a))^2/d-1/2*(-sin(b*x+a)/(-
a*d+c*b+d*(b*x+a))/d+(-Si(-x*b-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(x*b+
a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d)-1/12*b^3*(-3/2*cos(3*b*x+3*a)/(
-a*d+c*b+d*(b*x+a))^2/d-3/2*(-3*sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d+3*(-
3*Si(-3*x*b-3*a-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*x*b+3*a+3*(-a
*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d)/d)/d))
```

### 3.20.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.41

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \frac{4\,d^2\cos{(bx+a)}^3 - 4\,d^2\cos{(bx+a)} + 9\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right) - (b^2d^2x$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/8*(4*d^2*cos(b*x + a)^3 - 4*d^2*cos(b*x + a) + 9*(b^2*d^2*x^2 + 2*b^2*c*
d*x + b^2*c^2)*cos(-3*(b*c - a*d)/d)*cos_integral(3*(b*d*x + b*c)/d) - (b^
2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-(b*c - a*d)/d)*cos_integral((b*d*x
 + b*c)/d) - 9*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-3*(b*c - a*d)/d)
*sin_integral(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*s
in(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 4*(b*d^2*x + b*c*d - 3*
(b*d^2*x + b*c*d)*cos(b*x + a)^2)*sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2
*d^3)
```

### 3.20.6 Sympy [F]

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin^2{(a+bx)}\cos{(a+bx)}}{(c+dx)^3}\,dx$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c)**3,x)
```

output
```
Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x)**3, x)
```

### 3.20.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.54

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx =$$

$$-\frac{b^3\left(E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^3\left(E_3\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_3\left(\cdots\right.\right.}{\cdots}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/8*(b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_inte
gral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b^3*(
exp_integral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(3
, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b^3*(I*e
xp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(3,
-(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b^3*(I*exp_inte
gral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(3, -3*(
-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b^2*c^2*d - 2
*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

### 3.20.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.32 (sec) , antiderivative size = 114422, normalized size of antiderivative = 517.75

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/16*(9*b^2*d^2*x^2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^
2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
^2 - b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - b^
2*d^2*x^2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x
)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^
2*x^2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x
)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^
2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*t
an(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*b^2*d^2*x^2*i
mag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2
*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 4*b^2*d^2*x^2*sin_int
egral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*
a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 18*b^2*d^2*x^2*imag_part(cos_integr
al(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)
^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*imag_part(cos_integral
(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^
2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 36*b^2*d^2*x^2*sin_integral(3*(b*d*x +
 b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b
*c/d)*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c...
```

### 3.20.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^3}\,dx$$

input
```
int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^3,x)
```

output
```
int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^3, x)
```

# 3.21 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx$


3.21.1 Optimal result . . . 281
3.21.2 Mathematica [A] (verified) . . . 282
3.21.3 Rubi [A] (verified) . . . 282
3.21.4 Maple [A] (verified) . . . 284
3.21.5 Fricas [A] (verification not implemented) . . . 284
3.21.6 Sympy [F] . . . 285
3.21.7 Maxima [C] (verification not implemented) . . . 285
3.21.8 Giac [C] (verification not implemented) . . . 286
3.21.9 Mupad [F(-1)] . . . 287


## 3.21.1 Optimal result

Integrand size = 22, antiderivative size = 270

$$\begin{aligned}\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx =& -\frac{\cos(a+bx)}{12d(c+dx)^3}+\frac{b^2\cos(a+bx)}{24d^3(c+dx)}\\ &+\frac{\cos(3a+3bx)}{12d(c+dx)^3}-\frac{3b^2\cos(3a+3bx)}{8d^3(c+dx)}\\ &-\frac{9b^3\operatorname{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{8d^4}\\ &+\frac{b^3\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{24d^4}+\frac{b\sin(a+bx)}{24d^2(c+dx)^2}\\ &-\frac{b\sin(3a+3bx)}{8d^2(c+dx)^2}+\frac{b^3\cos\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{24d^4}\\ &-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^4}\end{aligned}$$

output
```
-1/12*cos(b*x+a)/d/(d*x+c)^3+1/24*b^2*cos(b*x+a)/d^3/(d*x+c)+1/12*cos(3*b*
x+3*a)/d/(d*x+c)^3-3/8*b^2*cos(3*b*x+3*a)/d^3/(d*x+c)+1/24*b^3*cos(a-b*c/d
)*Si(b*c/d+b*x)/d^4-9/8*b^3*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^4-9/8*b^3
*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^4+1/24*b^3*Ci(b*c/d+b*x)*sin(a-b*c/d
)/d^4+1/24*b*sin(b*x+a)/d^2/(d*x+c)^2-1/8*b*sin(3*b*x+3*a)/d^2/(d*x+c)^2
```

### 3.21.2 Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.10

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx$$
$$= \frac{d\cos(bx)\left((-2d^2+b^2(c+dx)^2)\cos(a)+bd(c+dx)\sin(a)\right)-d\cos(3bx)\left((-2d^2+9b^2(c+dx)^2)\cos(3a)+\right.}{}$$

input
```
Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

output
```
(d*Cos[b*x]*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*d*(c + d*x)*Sin[a]) - d
*Cos[3*b*x]*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*x)*Sin[3
*a]) + d*(b*d*(c + d*x)*Cos[a] - (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*
x] - d*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*x)^2)*Sin[3*a])*
Sin[3*b*x] + b^3*(c + d*x)^3*(CosIntegral[b*(c/d + x)]*Sin[a - (b*c)/d] +
Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)]) - 27*b^3*(c + d*x)^3*(CosIntegr
al[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + Cos[3*a - (3*b*c)/d]*SinInteg
ral[(3*b*(c + d*x))/d]))/(24*d^4*(c + d*x)^3)
```

### 3.21.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\cos(a+bx)}{(c+dx)^4}\,dx$$
$$\Big\downarrow \; 4906$$
$$\int \left(\frac{\cos(a+bx)}{4(c+dx)^4}-\frac{\cos(3a+3bx)}{4(c+dx)^4}\right)dx$$
$$\Big\downarrow \; 2009$$

$$-\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{24d^4}+\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{24d^4}-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^4}+\frac{b^2\cos(a+bx)}{24d^3(c+dx)}-\frac{3b^2\cos(3a+3bx)}{8d^3(c+dx)}+\frac{b\sin(a+bx)}{24d^2(c+dx)^2}-\frac{b\sin(3a+3bx)}{8d^2(c+dx)^2}-\frac{\cos(a+bx)}{12d(c+dx)^3}+\frac{\cos(3a+3bx)}{12d(c+dx)^3}$$

input
```
Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

output
```
-1/12*Cos[a + b*x]/(d*(c + d*x)^3) + (b^2*Cos[a + b*x])/(24*d^3*(c + d*x))
 + Cos[3*a + 3*b*x]/(12*d*(c + d*x)^3) - (3*b^2*Cos[3*a + 3*b*x])/(8*d^3*(
c + d*x)) - (9*b^3*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(8
*d^4) + (b^3*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(24*d^4) + (b*Si
n[a + b*x])/(24*d^2*(c + d*x)^2) - (b*Sin[3*a + 3*b*x])/(8*d^2*(c + d*x)^2
) + (b^3*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(24*d^4) - (9*b^3*Co
s[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^4)
```

### 3.21.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.21.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.44

| method | result |
|---|---|
| derivativedivides | $\frac{b^4\left(-\frac{\cos(xb+a)}{3(-ad+cb+d(xb+a))^3 d}-\frac{-\frac{\sin(xb+a)}{2(-ad+cb+d(xb+a))^2 d}+\frac{-\frac{\cos(xb+a)}{(-ad+cb+d(xb+a))d}-\frac{-\frac{\mathrm{Si}\left(-xb-a-\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}-\mathrm{Ci}\left(x\right.}{d}}{2d}}{3d}\right.}{4}$ |
| default | $\frac{b^4\left(-\frac{\cos(xb+a)}{3(-ad+cb+d(xb+a))^3 d}-\frac{-\frac{\sin(xb+a)}{2(-ad+cb+d(xb+a))^2 d}+\frac{-\frac{\cos(xb+a)}{(-ad+cb+d(xb+a))d}-\frac{-\frac{\mathrm{Si}\left(-xb-a-\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}-\mathrm{Ci}\left(x\right.}{d}}{2d}}{3d}\right.}{4}$ |
| risch | $\frac{9ib^3 e^{-\frac{3i(ad-cb)}{d}}\mathrm{Ei}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{16d^4}-\frac{ib^3 e^{-\frac{i(ad-cb)}{d}}\mathrm{Ei}_1\left(ibx+ia-\frac{i(ad-cb)}{d}\right)}{48d^4}+\frac{ib^3 e^{\frac{i(ad-cb)}{d}}\mathrm{Ei}_1\left(-ibx-ia\right.}{48d^4}$ |

input
```
int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/4*b^4*(-1/3*cos(b*x+a)/(-a*d+c*b+d*(b*x+a))^3/d-1/3*(-1/2*sin(b*x+a
)/(-a*d+c*b+d*(b*x+a))^2/d+1/2*(-cos(b*x+a)/(-a*d+c*b+d*(b*x+a))/d-(-Si(-x
*b-a-(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(x*b+a+(-a*d+b*c)/d)*sin((-a*d+b*
c)/d)/d)/d)/d)/d)-1/12*b^4*(-cos(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))^3/d-(-3/2
*sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))^2/d+3/2*(-3*cos(3*b*x+3*a)/(-a*d+c*b+
d*(b*x+a))/d-3*(-3*Si(-3*x*b-3*a-3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*C
i(3*x*b+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)/d)/d)/d))
```

### 3.21.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.67

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx =$$

$$-\frac{4\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)^3-\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\mathrm{Ci}\left(\frac{bdx+}{d}\right.}{}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/24*(4*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x +
a)^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integ
ral((b*d*x + b*c)/d)*sin(-(b*c - a*d)/d) + 27*(b^3*d^3*x^3 + 3*b^3*c*d^2*x
^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(3*(b*d*x + b*c)/d)*sin(-3*(b*c
- a*d)/d) + 27*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*c
os(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) - (b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-(b*c - a*d)/d)*sin_integral((b
*d*x + b*c)/d) - 4*(7*b^2*d^3*x^2 + 14*b^2*c*d^2*x + 7*b^2*c^2*d - 2*d^3)*
cos(b*x + a) - 4*(b*d^3*x + b*c*d^2 - 3*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2
)*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

### 3.21.6 Sympy [F]

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sin^2(a+bx)\cos(a+bx)}{(c+dx)^4}\,dx$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c)**4,x)
```

output
```
Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x)**4, x)
```

### 3.21.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.44

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = -\frac{b^4\left(E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^4\left(E_4\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_4\left(\cdots\right.\right.}{8\,\left(b^3c^3d-3\,ab^2c\cdots\right.}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/8*(b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_inte
gral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b^4*(
exp_integral_e(4, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(4
, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b^4*(I*e
xp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(4,
-(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b^4*(I*exp_inte
gral_e(4, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(4, -3*(
-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b^3*c^3*d - 3
*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 -
a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

### 3.21.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.62 (sec) , antiderivative size = 168646, normalized size of antiderivative = 624.61

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

output
```
-1/48*(27*b^3*d^3*x^3*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x
)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 - b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan
(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 +
b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b
*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 27*b^3
*d^3*x^3*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 54*b^
3*d^3*x^3*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*ta
n(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3*
sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*t
an(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3*real_part(co
s_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/
2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*b^3*d^3*x^3*real_part(cos_integ
ral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 54*b^3*d^3*x^3*real_part(cos_integral(3
*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*t
an(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 54*b^3*d^3*x^3*real_part(cos_integral(-3*
b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(b*...
```

### 3.21.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^4}\,dx$$

input
```
int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^4,x)
```

output
```
int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^4, x)
```

# 3.22 $\int (c+dx)^m \cos(a+bx)\sin^3(a+bx)\,dx$


3.22.1 Optimal result . . . 288
3.22.2 Mathematica [A] (verified) . . . 289
3.22.3 Rubi [A] (verified) . . . 289
3.22.4 Maple [F] . . . 291
3.22.5 Fricas [A] (verification not implemented) . . . 291
3.22.6 Sympy [F(-2)] . . . 291
3.22.7 Maxima [F] . . . 292
3.22.8 Giac [F] . . . 292
3.22.9 Mupad [F(-1)] . . . 292


## 3.22.1 Optimal result

Integrand size = 22, antiderivative size = 271

$$\begin{aligned}\int (c+dx)^m \cos(a+bx)\sin^3(a+bx)\,dx &= -\frac{2^{-4-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} \\ &\quad -\frac{2^{-4-m}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2ib(c+dx)}{d}\right)}{b} \\ &\quad +\frac{2^{-2(3+m)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{4ib(c+dx)}{d}\right)}{b} \\ &\quad +\frac{2^{-2(3+m)}e^{-4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{4ib(c+dx)}{d}\right)}{b}\end{aligned}$$

output
```
-2^(-4-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*
b*(d*x+c)/d)^m)-2^(-4-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a
-b*c/d))/((I*b*(d*x+c)/d)^m)+exp(4*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-4*I*b
*(d*x+c)/d)/(2^(6+2*m))/b/((-I*b*(d*x+c)/d)^m)+(d*x+c)^m*GAMMA(1+m,4*I*b*(
d*x+c)/d)/(2^(6+2*m))/b/exp(4*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

## 3.22.2 Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.91

$$\int (c+dx)^m \cos(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{4^{-3-m}e^{-\frac{4i(bc+ad)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(-2^{2+m}e^{2i\left(3a+\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^m\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)-2^{2+m}e^{2i(a+\cdots}}{}$$

input
```
Integrate[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
(4^(-3 - m)*(c + d*x)^m*(-(2^(2 + m)*E^((2*I)*(3*a + (b*c)/d))*((I*b*(c +
d*x))/d)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d]) - 2^(2 + m)*E^((2*I)*(a +
 (3*b*c)/d))*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d]
+ E^((8*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-4*I)*b*(c + d*x))/d] +
 E^(((8*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((4*I)*b*(c + d*x
))/d]))/(b*E^(((4*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^m)
```

## 3.22.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\cos(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{4}\sin(2a+2bx)(c+dx)^m-\frac{1}{8}\sin(4a+4bx)(c+dx)^m\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2^{-m-4}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{4ib(c+dx)}{d}\right)}{b}-\frac{2^{-m-4}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-2(m+3)}e^{-4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{4ib(c+dx)}{d}\right)}{b}$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
-((2^(-4 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(
c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)) - (2^(-4 - m)*(c + d*x)^m*Gamm
a[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x
))/d)^m) + (E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-4*I)*b*(c
+ d*x))/d])/(2^(2*(3 + m))*b*(((-I)*b*(c + d*x))/d)^m) + ((c + d*x)^m*Gamm
a[1 + m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*E^((4*I)*(a - (b*c)/d))*
((I*b*(c + d*x))/d)^m)
```

### 3.22.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.22.4 Maple [F]

$$\int (dx+c)^m \cos(xb+a)\sin(xb+a)^3\,dx$$

input
```
int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x)
```

output
```
int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x)
```

## 3.22.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.70

$$\int (c+dx)^m \cos(a+bx)\sin^3(a+bx)\,dx = -\frac{4e^{\left(-\frac{dm\log\left(-\frac{2ib}{d}\right)+2ibc-2iad}{d}\right)}\Gamma\left(m+1,-\frac{2(ibdx+ibc)}{d}\right)-e^{\left(-\frac{dm\log\left(-\frac{4ib}{d}\right)+4ibc-4iad}{d}\right)}\Gamma\left(m+1,-\frac{4(ibdx+ibc)}{d}\right)+4\cdots}{64b}$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/64*(4*e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)/d)*gamma(m + 1, -2*(I
*b*d*x + I*b*c)/d) - e^(-(d*m*log(-4*I*b/d) + 4*I*b*c - 4*I*a*d)/d)*gamma(
m + 1, -4*(I*b*d*x + I*b*c)/d) + 4*e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a
*d)/d)*gamma(m + 1, -2*(-I*b*d*x - I*b*c)/d) - e^(-(d*m*log(4*I*b/d) - 4*I
*b*c + 4*I*a*d)/d)*gamma(m + 1, -4*(-I*b*d*x - I*b*c)/d))/b
```

## 3.22.6 Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m \cos(a+bx)\sin^3(a+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d*x+c)**m*cos(b*x+a)*sin(b*x+a)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.22.7 Maxima [F]

$$\int (c+dx)^m \cos(a+bx)\sin^3(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\sin(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^3, x)
```

### 3.22.8 Giac [F]

$$\int (c+dx)^m \cos(a+bx)\sin^3(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\sin(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^3, x)
```

### 3.22.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cos(a+bx)\sin^3(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)^3\,(c+d\,x)^m\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^m, x)
```

# 3.23 $\int (c+dx)^4 \cos(a+bx)\sin^3(a+bx)\,dx$

3.23.1 Optimal result . . . . . 293
3.23.2 Mathematica [A] (verified) . . . . . 294
3.23.3 Rubi [A] (verified) . . . . . 294
3.23.4 Maple [A] (verified) . . . . . 297
3.23.5 Fricas [A] (verification not implemented) . . . . . 298
3.23.6 Sympy [B] (verification not implemented) . . . . . 298
3.23.7 Maxima [B] (verification not implemented) . . . . . 299
3.23.8 Giac [A] (verification not implemented) . . . . . 300
3.23.9 Mupad [B] (verification not implemented) . . . . . 301

## 3.23.1 Optimal result

Integrand size = 22, antiderivative size = 260

$$\begin{aligned}\int (c+dx)^4\cos(a+bx)\sin^3(a+bx)\,dx =& \frac{45cd^3x}{64b^3}+\frac{45d^4x^2}{128b^3}-\frac{3(c+dx)^4}{32b}\\ &-\frac{45d^3(c+dx)\cos(a+bx)\sin(a+bx)}{64b^4}\\ &+\frac{3d(c+dx)^3\cos(a+bx)\sin(a+bx)}{8b^2}\\ &+\frac{45d^4\sin^2(a+bx)}{128b^5}-\frac{9d^2(c+dx)^2\sin^2(a+bx)}{16b^3}\\ &-\frac{3d^3(c+dx)\cos(a+bx)\sin^3(a+bx)}{32b^4}\\ &+\frac{d(c+dx)^3\cos(a+bx)\sin^3(a+bx)}{4b^2}\\ &+\frac{3d^4\sin^4(a+bx)}{128b^5}-\frac{3d^2(c+dx)^2\sin^4(a+bx)}{16b^3}\\ &+\frac{(c+dx)^4\sin^4(a+bx)}{4b}\end{aligned}$$

output
```
45/64*c*d^3*x/b^3+45/128*d^4*x^2/b^3-3/32*(d*x+c)^4/b-45/64*d^3*(d*x+c)*co
s(b*x+a)*sin(b*x+a)/b^4+3/8*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2+45/128*d
^4*sin(b*x+a)^2/b^5-9/16*d^2*(d*x+c)^2*sin(b*x+a)^2/b^3-3/32*d^3*(d*x+c)*c
os(b*x+a)*sin(b*x+a)^3/b^4+1/4*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3/b^2+3/1
28*d^4*sin(b*x+a)^4/b^5-3/16*d^2*(d*x+c)^2*sin(b*x+a)^4/b^3+1/4*(d*x+c)^4*
sin(b*x+a)^4/b
```

### 3.23.2 Mathematica [A] (verified)

Time = 2.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.61

$$\int (c+dx)^4\cos(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{-64(3d^4-6b^2d^2(c+dx)^2+2b^4(c+dx)^4)\cos(2(a+bx))+(3d^4-24b^2d^2(c+dx)^2+32b^4(c+dx)^4)\cos(\ldots}{1\ldots}$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
(-64*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a + b*x)]
+ (3*d^4 - 24*b^2*d^2*(c + d*x)^2 + 32*b^4*(c + d*x)^4)*Cos[4*(a + b*x)] -
 8*b*d*(c + d*x)*(-16*(-3*d^2 + 2*b^2*(c + d*x)^2) + (-3*d^2 + 8*b^2*(c +
d*x)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*x)])/(1024*b^5)
```

### 3.23.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {4904, 3042, 3792, 3042, 3791, 3042, 3791, 17, 3792, 17, 3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4\sin^3(a+bx)\cos(a+bx)\,dx$$
$$\Big\downarrow \; 4904$$
$$\frac{(c+dx)^4\sin^4(a+bx)}{4b}-\frac{d\int (c+dx)^3\sin^4(a+bx)dx}{b}$$
$$\Big\downarrow \; 3042$$
$$\frac{(c+dx)^4\sin^4(a+bx)}{4b}-\frac{d\int (c+dx)^3\sin(a+bx)^4dx}{b}$$
$$\Big\downarrow \; 3792$$
$$\frac{(c+dx)^4\sin^4(a+bx)}{4b}-\frac{d\left(-\frac{3d^2\int (c+dx)\sin^4(a+bx)dx}{8b^2}+\frac{3}{4}\int (c+dx)^3\sin^2(a+bx)dx+\frac{3d(c+dx)^2\sin^4(a+bx)}{16b^2}-\frac{(c+dx)^3\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{b}$$

$\downarrow$ 3042

$$\frac{(c+dx)^4\sin^4(a+bx)}{4b}-\frac{d\left(-\frac{3d^2\int(c+dx)\sin(a+bx)^4dx}{8b^2}+\frac{3}{4}\int(c+dx)^3\sin(a+bx)^2dx+\frac{3d(c+dx)^2\sin^4(a+bx)}{16b^2}-\frac{(c+dx)^3\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{b}$$

$\downarrow$ 3791

$$\frac{(c+dx)^4\sin^4(a+bx)}{4b}-\frac{d\left(-\frac{3d^2\left(\frac{3}{4}\int(c+dx)\sin^2(a+bx)dx+\frac{d\sin^4(a+bx)}{16b^2}-\frac{(c+dx)\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int(c+dx)^3\sin(a+bx)^2dx+\frac{3d(c+dx)^2\sin^4(a+b}{16b^2}\right.}{b}$$

$\downarrow$ 3042

$$\frac{(c+dx)^4\sin^4(a+bx)}{4b}-\frac{d\left(-\frac{3d^2\left(\frac{3}{4}\int(c+dx)\sin(a+bx)^2dx+\frac{d\sin^4(a+bx)}{16b^2}-\frac{(c+dx)\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int(c+dx)^3\sin(a+bx)^2dx+\frac{3d(c+dx)^2\sin^4(a+b}{16b^2}\right.}{b}$$

$\downarrow$ 3791

$$\frac{(c+dx)^4\sin^4(a+bx)}{4b}-\frac{d\left(-\frac{3d^2\left(\frac{3}{4}\left(\frac{1}{2}\int(c+dx)dx+\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{d\sin^4(a+bx)}{16b^2}-\frac{(c+dx)\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int(c+dx)^3\sin(a+\right.}{b}$$

$\downarrow$ 17

$$\frac{(c+dx)^4\sin^4(a+bx)}{4b}-\frac{d\left(\frac{3}{4}\int(c+dx)^3\sin(a+bx)^2dx-\frac{3d^2\left(\frac{3}{4}\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)+\frac{d\sin^4(a+bx)}{16b^2}-\frac{(c+dx)\sin^3(a+bx)\cos(a+bx)}{4b}\right.}{8b^2}\right.}{b}$$

$\downarrow$ 3792

$$\frac{(c+dx)^4\sin^4(a+bx)}{4b}-\frac{d\left(\frac{3}{4}\left(-\frac{3d^2\int(c+dx)\sin^2(a+bx)dx}{2b^2}+\frac{1}{2}\int(c+dx)^3dx+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}\right)-\frac{3d^2\left(\frac{3}{4}\left(\frac{d\sin^2(a+b}{4b^2}\right.\right.}{}\right.}{}$$

3.23. $\int(c+dx)^4\cos(a+bx)\sin^3(a+bx)\,dx$

$$\Big\downarrow \ 17$$

$$\frac{(c+dx)^4\sin^4(a+bx)}{4b} - \frac{d\left(\frac{3}{4}\left(-\frac{3d^2\int(c+dx)\sin^2(a+bx)dx}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)-\frac{3d^2\left(\frac{3}{4}\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)}{}\right.\right.}{}\right.}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{(c+dx)^4\sin^4(a+bx)}{4b} - \frac{d\left(\frac{3}{4}\left(-\frac{3d^2\int(c+dx)\sin(a+bx)^2dx}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)-\frac{3d^2\left(\frac{3}{4}\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)}{}\right.\right.}{}\right.}{b}$$

$$\Big\downarrow \ 3791$$

$$\frac{(c+dx)^4\sin^4(a+bx)}{4b} - \frac{d\left(\frac{3}{4}\left(-\frac{3d^2\left(\frac{1}{2}\int(c+dx)dx+\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right.\right.}{}$$

$$\Big\downarrow \ 17$$

$$\frac{(c+dx)^4\sin^4(a+bx)}{4b} - \frac{d\left(\frac{3}{4}\left(-\frac{3d^2\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)-\right.}{}$$

input
```
Int[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
((c + d*x)^4*Sin[a + b*x]^4)/(4*b) - (d*(-1/4*((c + d*x)^3*Cos[a + b*x]*Si
n[a + b*x]^3)/b + (3*d*(c + d*x)^2*Sin[a + b*x]^4)/(16*b^2) - (3*d^2*(-1/4
*((c + d*x)*Cos[a + b*x]*Sin[a + b*x]^3)/b + (d*Sin[a + b*x]^4)/(16*b^2) +
 (3*((c + d*x)^2/(4*d) - ((c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (d*
Sin[a + b*x]^2)/(4*b^2)))/4))/(8*b^2) + (3*((c + d*x)^4/(8*d) - ((c + d*x)
^3*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (3*d*(c + d*x)^2*Sin[a + b*x]^2)/(4*
b^2) - (3*d^2*((c + d*x)^2/(4*d) - ((c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(
2*b) + (d*Sin[a + b*x]^2)/(4*b^2)))/(2*b^2)))/4))/b
```

## 3.23. $\int (c+dx)^4\cos(a+bx)\sin^3(a+bx)\,dx$

### 3.23.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

## 3.23.4 Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.72

| method | result |
|---|---|
| parallelrisch | $\frac{\left(-128b^4(dx+c)^4+384d^2(dx+c)^2b^2-192d^4\right)\cos(2xb+2a)+\left(32b^4(dx+c)^4-24d^2(dx+c)^2b^2+3d^4\right)\cos(4xb+4a)+256bd}{1024b^5}$ |
| risch | $\frac{\left(32d^4x^4b^4+128b^4c\,d^3x^3+192b^4c^2d^2x^2+128b^4c^3dx+32b^4c^4-24b^2d^4x^2-48b^2c\,d^3x-24b^2c^2d^2+3d^4\right)\cos(4xb+4a)}{1024b^5}-\frac{d}{}$ |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |

input
```
int((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/1024*((-128*b^4*(d*x+c)^4+384*d^2*(d*x+c)^2*b^2-192*d^4)*cos(2*b*x+2*a)+
(32*b^4*(d*x+c)^4-24*d^2*(d*x+c)^2*b^2+3*d^4)*cos(4*b*x+4*a)+256*b*d*((d*x
+c)^2*b^2-3/2*d^2)*(d*x+c)*sin(2*b*x+2*a)-32*b*d*((d*x+c)^2*b^2-3/8*d^2)*(
d*x+c)*sin(4*b*x+4*a)+96*b^4*c^4-360*b^2*c^2*d^2+189*d^4)/b^5
```

### 3.23.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.67

$$\int (c+dx)^4\cos(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{20\,b^4d^4x^4 + 80\,b^4cd^3x^3 + (32\,b^4d^4x^4 + 128\,b^4cd^3x^3 + 32\,b^4c^4 - 24\,b^2c^2d^2 + 3\,d^4 + 24\,(8\,b^4c^2d^2 - b^2d^4)x^2 +}{}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/128*(20*b^4*d^4*x^4 + 80*b^4*c*d^3*x^3 + (32*b^4*d^4*x^4 + 128*b^4*c*d^3
*x^3 + 32*b^4*c^4 - 24*b^2*c^2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*
x^2 + 16*(8*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^4 + 3*(40*b^4*c^2*d^2
 - 17*b^2*d^4)*x^2 - (64*b^4*d^4*x^4 + 256*b^4*c*d^3*x^3 + 64*b^4*c^4 - 12
0*b^2*c^2*d^2 + 51*d^4 + 24*(16*b^4*c^2*d^2 - 5*b^2*d^4)*x^2 + 16*(16*b^4*
c^3*d - 15*b^2*c*d^3)*x)*cos(b*x + a)^2 + 2*(40*b^4*c^3*d - 51*b^2*c*d^3)*
x - 2*(2*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(
8*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^3 - (40*b^3*d^4*x^3 + 120*b^3*c*d^3
*x^2 + 40*b^3*c^3*d - 51*b*c*d^3 + 3*(40*b^3*c^2*d^2 - 17*b*d^4)*x)*cos(b*
x + a))*sin(b*x + a))/b^5
```

### 3.23.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 935 vs. 2(262) = 524.

Time = 0.88 (sec) , antiderivative size = 935, normalized size of antiderivative = 3.60

$$\int (c+dx)^4\cos(a+bx)\sin^3(a+bx)\,dx$$
$$= \begin{cases} \frac{c^4\sin^4(a+bx)}{4b} + \frac{5c^3dx\sin^4(a+bx)}{8b} - \frac{3c^3dx\sin^2(a+bx)\cos^2(a+bx)}{4b} - \frac{3c^3dx\cos^4(a+bx)}{8b} + \frac{15c^2d^2x^2\sin^4(a+bx)}{16b} - \frac{9c^2d^2x^2\sin^2(a+ \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\sin^3(a)\cos(a) \end{cases}$$

input
```
integrate((d*x+c)**4*cos(b*x+a)*sin(b*x+a)**3,x)
```

output
```
Piecewise((c**4*sin(a + b*x)**4/(4*b) + 5*c**3*d*x*sin(a + b*x)**4/(8*b) -
 3*c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(4*b) - 3*c**3*d*x*cos(a + b*x
)**4/(8*b) + 15*c**2*d**2*x**2*sin(a + b*x)**4/(16*b) - 9*c**2*d**2*x**2*s
in(a + b*x)**2*cos(a + b*x)**2/(8*b) - 9*c**2*d**2*x**2*cos(a + b*x)**4/(1
6*b) + 5*c*d**3*x**3*sin(a + b*x)**4/(8*b) - 3*c*d**3*x**3*sin(a + b*x)**2
*cos(a + b*x)**2/(4*b) - 3*c*d**3*x**3*cos(a + b*x)**4/(8*b) + 5*d**4*x**4
*sin(a + b*x)**4/(32*b) - 3*d**4*x**4*sin(a + b*x)**2*cos(a + b*x)**2/(16*
b) - 3*d**4*x**4*cos(a + b*x)**4/(32*b) + 5*c**3*d*sin(a + b*x)**3*cos(a +
 b*x)/(8*b**2) + 3*c**3*d*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 15*c**2*
d**2*x*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 9*c**2*d**2*x*sin(a + b*x)*
cos(a + b*x)**3/(8*b**2) + 15*c*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)/(8*
b**2) + 9*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 5*d**4*x**3*
sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 3*d**4*x**3*sin(a + b*x)*cos(a + b
*x)**3/(8*b**2) - 15*c**2*d**2*sin(a + b*x)**4/(32*b**3) + 9*c**2*d**2*cos
(a + b*x)**4/(32*b**3) - 51*c*d**3*x*sin(a + b*x)**4/(64*b**3) + 9*c*d**3*
x*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**3) + 45*c*d**3*x*cos(a + b*x)**4/
(64*b**3) - 51*d**4*x**2*sin(a + b*x)**4/(128*b**3) + 9*d**4*x**2*sin(a +
b*x)**2*cos(a + b*x)**2/(64*b**3) + 45*d**4*x**2*cos(a + b*x)**4/(128*b**3
) - 51*c*d**3*sin(a + b*x)**3*cos(a + b*x)/(64*b**4) - 45*c*d**3*sin(a + b
*x)*cos(a + b*x)**3/(64*b**4) - 51*d**4*x*sin(a + b*x)**3*cos(a + b*x)/...
```

### 3.23.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 967 vs. $2(236) = 472$.

Time = 0.25 (sec) , antiderivative size = 967, normalized size of antiderivative = 3.72

$$\int (c+dx)^4 \cos(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/1024*(256*c^4*sin(b*x + a)^4 - 1024*a*c^3*d*sin(b*x + a)^4/b + 1536*a^2*
c^2*d^2*sin(b*x + a)^4/b^2 - 1024*a^3*c*d^3*sin(b*x + a)^4/b^3 + 256*a^4*d
^4*sin(b*x + a)^4/b^4 + 32*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*co
s(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*c^3*d/b - 96*(4*(b
*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a
) + 8*sin(2*b*x + 2*a))*a*c^2*d^2/b^2 + 96*(4*(b*x + a)*cos(4*b*x + 4*a) -
 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*a^
2*c*d^3/b^3 - 32*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x +
2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*a^3*d^4/b^4 + 24*((8*(b*x +
a)^2 - 1)*cos(4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(
b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 - 4
8*((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x
 + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*a*
c*d^3/b^3 + 24*((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 16*(2*(b*x + a)^2 -
 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b
*x + 2*a))*a^2*d^4/b^4 + 4*(4*(8*(b*x + a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*
a) - 64*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3*(8*(b*x + a)^2
- 1)*sin(4*b*x + 4*a) + 96*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*c*d^3/b^3
 - 4*(4*(8*(b*x + a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) - 64*(2*(b*x + a)^3
 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*...
```

### 3.23.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.39

$$\int (c+dx)^4\cos(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{(32\,b^4d^4x^4 + 128\,b^4cd^3x^3 + 192\,b^4c^2d^2x^2 + 128\,b^4c^3dx + 32\,b^4c^4 - 24\,b^2d^4x^2 - 48\,b^2cd^3x - 24\,b^2c^2d^2 + 3\,d^4 \cdots}{1024\,b^5}$$
$$- \frac{(2\,b^4d^4x^4 + 8\,b^4cd^3x^3 + 12\,b^4c^2d^2x^2 + 8\,b^4c^3dx + 2\,b^4c^4 - 6\,b^2d^4x^2 - 12\,b^2cd^3x - 6\,b^2c^2d^2 + 3\,d^4)\cos\left(2\,b \cdots\right.}{16\,b^5}$$
$$- \frac{(8\,b^3d^4x^3 + 24\,b^3cd^3x^2 + 24\,b^3c^2d^2x + 8\,b^3c^3d - 3\,bd^4x - 3\,bcd^3)\sin\left(4\,bx + 4\,a\right)}{256\,b^5}$$
$$+ \frac{(2\,b^3d^4x^3 + 6\,b^3cd^3x^2 + 6\,b^3c^2d^2x + 2\,b^3c^3d - 3\,bd^4x - 3\,bcd^3)\sin\left(2\,bx + 2\,a\right)}{8\,b^5}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/1024*(32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 192*b^4*c^2*d^2*x^2 + 128*b^4
*c^3*d*x + 32*b^4*c^4 - 24*b^2*d^4*x^2 - 48*b^2*c*d^3*x - 24*b^2*c^2*d^2 +
 3*d^4)*cos(4*b*x + 4*a)/b^5 - 1/16*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*
b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3
*x - 6*b^2*c^2*d^2 + 3*d^4)*cos(2*b*x + 2*a)/b^5 - 1/256*(8*b^3*d^4*x^3 +
24*b^3*c*d^3*x^2 + 24*b^3*c^2*d^2*x + 8*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)
*sin(4*b*x + 4*a)/b^5 + 1/8*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d
^2*x + 2*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

### 3.23.9 Mupad [B] (verification not implemented)

Time = 24.22 (sec) , antiderivative size = 576, normalized size of antiderivative = 2.22

$$\int (c+dx)^4\cos(a+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{192\,d^4\,\cos\left(2\,a+2\,b\,x\right)-3\,d^4\,\cos\left(4\,a+4\,b\,x\right)+128\,b^4\,c^4\,\cos\left(2\,a+2\,b\,x\right)-32\,b^4\,c^4\,\cos\left(4\,a+4\,b\,x\right)-2}{}$$

input
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^4,x)
```

output
```
-(192*d^4*cos(2*a + 2*b*x) - 3*d^4*cos(4*a + 4*b*x) + 128*b^4*c^4*cos(2*a
+ 2*b*x) - 32*b^4*c^4*cos(4*a + 4*b*x) - 256*b^3*c^3*d*sin(2*a + 2*b*x) +
32*b^3*c^3*d*sin(4*a + 4*b*x) - 384*b^2*c^2*d^2*cos(2*a + 2*b*x) + 24*b^2*
c^2*d^2*cos(4*a + 4*b*x) - 384*b^2*d^4*x^2*cos(2*a + 2*b*x) + 24*b^2*d^4*x
^2*cos(4*a + 4*b*x) + 128*b^4*d^4*x^4*cos(2*a + 2*b*x) - 32*b^4*d^4*x^4*co
s(4*a + 4*b*x) - 256*b^3*d^4*x^3*sin(2*a + 2*b*x) + 32*b^3*d^4*x^3*sin(4*a
 + 4*b*x) + 384*b*c*d^3*sin(2*a + 2*b*x) - 12*b*c*d^3*sin(4*a + 4*b*x) + 3
84*b*d^4*x*sin(2*a + 2*b*x) - 12*b*d^4*x*sin(4*a + 4*b*x) + 768*b^4*c^2*d^
2*x^2*cos(2*a + 2*b*x) - 192*b^4*c^2*d^2*x^2*cos(4*a + 4*b*x) - 768*b^2*c*
d^3*x*cos(2*a + 2*b*x) + 512*b^4*c^3*d*x*cos(2*a + 2*b*x) + 48*b^2*c*d^3*x
*cos(4*a + 4*b*x) - 128*b^4*c^3*d*x*cos(4*a + 4*b*x) + 512*b^4*c*d^3*x^3*c
os(2*a + 2*b*x) - 128*b^4*c*d^3*x^3*cos(4*a + 4*b*x) - 768*b^3*c^2*d^2*x*s
in(2*a + 2*b*x) - 768*b^3*c*d^3*x^2*sin(2*a + 2*b*x) + 96*b^3*c^2*d^2*x*si
n(4*a + 4*b*x) + 96*b^3*c*d^3*x^2*sin(4*a + 4*b*x))/(1024*b^5)
```

# 3.24 $\int (c+dx)^3 \cos(a+bx) \sin^3(a+bx)\,dx$


3.24.1 Optimal result . . . . . . . . . . 302
3.24.2 Mathematica [A] (verified) . . . . . . . . . . 303
3.24.3 Rubi [A] (verified) . . . . . . . . . . 303
3.24.4 Maple [A] (verified) . . . . . . . . . . 307
3.24.5 Fricas [A] (verification not implemented) . . . . . . . . . . 307
3.24.6 Sympy [B] (verification not implemented) . . . . . . . . . . 308
3.24.7 Maxima [B] (verification not implemented) . . . . . . . . . . 309
3.24.8 Giac [A] (verification not implemented) . . . . . . . . . . 309
3.24.9 Mupad [B] (verification not implemented) . . . . . . . . . . 310


## 3.24.1 Optimal result

Integrand size = 22, antiderivative size = 196

$$\int (c+dx)^3 \cos(a+bx) \sin^3(a+bx)\,dx = \frac{45d^3x}{256b^3} - \frac{3(c+dx)^3}{32b} - \frac{45d^3\cos(a+bx)\sin(a+bx)}{256b^4} + \frac{9d(c+dx)^2\cos(a+bx)\sin(a+bx)}{32b^2} - \frac{9d^2(c+dx)\sin^2(a+bx)}{32b^3} - \frac{3d^3\cos(a+bx)\sin^3(a+bx)}{128b^4} + \frac{3d(c+dx)^2\cos(a+bx)\sin^3(a+bx)}{16b^2} - \frac{3d^2(c+dx)\sin^4(a+bx)}{32b^3} + \frac{(c+dx)^3\sin^4(a+bx)}{4b}$$

output
```
45/256*d^3*x/b^3-3/32*(d*x+c)^3/b-45/256*d^3*cos(b*x+a)*sin(b*x+a)/b^4+9/3
2*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^2-9/32*d^2*(d*x+c)*sin(b*x+a)^2/b^3-
3/128*d^3*cos(b*x+a)*sin(b*x+a)^3/b^4+3/16*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+
a)^3/b^2-3/32*d^2*(d*x+c)*sin(b*x+a)^4/b^3+1/4*(d*x+c)^3*sin(b*x+a)^4/b
```

### 3.24.2 Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.69

$$\int (c+dx)^3\cos(a+bx)\sin^3(a+bx)\,dx$$
$$=\frac{-64b(c+dx)\left(-3d^2+2b^2(c+dx)^2\right)\cos(2(a+bx))+4b(c+dx)\left(-3d^2+8b^2(c+dx)^2\right)\cos(4(a+bx))-}{1024b^4}$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
(-64*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 4*b*(c +
d*x)*(-3*d^2 + 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] - 6*d*(-16*(-d^2 + 2*b^
2*(c + d*x)^2) + (-d^2 + 8*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b
*x)])/(1024*b^4)
```

### 3.24.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.24, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {4904, 3042, 3792, 3042, 3115, 3042, 3115, 24, 3792, 17, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\sin^3(a+bx)\cos(a+bx)\,dx$$
$$\Big\downarrow \text{4904}$$
$$\frac{(c+dx)^3\sin^4(a+bx)}{4b}-\frac{3d\int (c+dx)^2\sin^4(a+bx)dx}{4b}$$
$$\Big\downarrow \text{3042}$$
$$\frac{(c+dx)^3\sin^4(a+bx)}{4b}-\frac{3d\int (c+dx)^2\sin(a+bx)^4dx}{4b}$$
$$\Big\downarrow \text{3792}$$
$$\frac{(c+dx)^3\sin^4(a+bx)}{4b}-\frac{3d\left(-\frac{d^2\int \sin^4(a+bx)dx}{8b^2}+\frac{3}{4}\int (c+dx)^2\sin^2(a+bx)dx+\frac{d(c+dx)\sin^4(a+bx)}{8b^2}-\frac{(c+dx)^2\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{4b}$$

$\downarrow$ 3042

$$\frac{(c+dx)^3\sin^4(a+bx)}{4b}-\frac{3d\left(-\frac{d^2\int \sin(a+bx)^4dx}{8b^2}+\frac{3}{4}\int (c+dx)^2\sin(a+bx)^2dx+\frac{d(c+dx)\sin^4(a+bx)}{8b^2}-\frac{(c+dx)^2\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{4b}$$

$\downarrow$ 3115

$$\frac{(c+dx)^3\sin^4(a+bx)}{4b}-\frac{3d\left(-\frac{d^2\left(\frac{3}{4}\int \sin^2(a+bx)dx-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int (c+dx)^2\sin(a+bx)^2dx+\frac{d(c+dx)\sin^4(a+bx)}{8b^2}-\frac{(c+dx)^2\sin^3(a+bx)\cos}{4b}\right.}{4b}$$

$\downarrow$ 3042

$$\frac{(c+dx)^3\sin^4(a+bx)}{4b}-\frac{3d\left(-\frac{d^2\left(\frac{3}{4}\int \sin(a+bx)^2dx-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int (c+dx)^2\sin(a+bx)^2dx+\frac{d(c+dx)\sin^4(a+bx)}{8b^2}-\frac{(c+dx)^2\sin^3(a+bx)\cos}{4b}\right.}{4b}$$

$\downarrow$ 3115

$$\frac{(c+dx)^3\sin^4(a+bx)}{4b}-\frac{3d\left(-\frac{d^2\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int (c+dx)^2\sin(a+bx)^2dx+\frac{d(c+dx)\sin^4(a+bx)}{8b^2}-(c+dx)^2\,\mathrm{si}\right.}{4b}$$

$\downarrow$ 24

$$\frac{(c+dx)^3\sin^4(a+bx)}{4b}-\frac{3d\left(\frac{3}{4}\int (c+dx)^2\sin(a+bx)^2dx+\frac{d(c+dx)\sin^4(a+bx)}{8b^2}-\frac{d^2\left(\frac{3}{4}\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{8b^2}-\frac{(c+dx)^2\sin^3(a+}{4b}\right.}{4b}$$

$\downarrow$ 3792

$$\frac{(c+dx)^3\sin^4(a+bx)}{4b}-\frac{3d\left(\frac{3}{4}\left(-\frac{d^2\int \sin^2(a+bx)dx}{2b^2}+\frac{1}{2}\int (c+dx)^2dx+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{d(c+dx)\sin^4(a+bx)}{8b^2}-\frac{d^2(}{}\right.}{4b}$$

3.24. $\int (c+dx)^3\cos(a+bx)\sin^3(a+bx)\,dx$

$$\Big\downarrow \; 17$$

$$\frac{(c+dx)^3\sin^4(a+bx)}{4b}-\frac{3d\left(\frac{3}{4}\left(-\frac{d^2\int\sin^2(a+bx)dx}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)+\frac{d(c+dx)\sin^4(a+bx)}{8b^2}-\frac{d^2\left(\frac{3}{4}\left(\frac{x}{2}-\frac{\sin}{}\right.\right.}{}\right.}{4b}$$

$$\Big\downarrow \; 3042$$

$$\frac{(c+dx)^3\sin^4(a+bx)}{4b}-\frac{3d\left(\frac{3}{4}\left(-\frac{d^2\int\sin(a+bx)^2dx}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)+\frac{d(c+dx)\sin^4(a+bx)}{8b^2}-\frac{d^2\left(\frac{3}{4}\left(\frac{x}{2}-\frac{\sin}{}\right.\right.}{}\right.}{4b}$$

$$\Big\downarrow \; 3115$$

$$\frac{(c+dx)^3\sin^4(a+bx)}{4b}-\frac{3d\left(\frac{3}{4}\left(-\frac{d^2\left(\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)+\frac{d(c+dx)\sin^4(a+bx)}{8b^2}-\right.}{4b}$$

$$\Big\downarrow \; 24$$

$$\frac{(c+dx)^3\sin^4(a+bx)}{4b}-\frac{3d\left(\frac{3}{4}\left(\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)+\frac{d(c+dx)\sin^4(a+bx)}{8b^2}-\frac{d^2\left(\frac{3}{4}\right.}{}\right.}{4b}$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
((c + d*x)^3*Sin[a + b*x]^4)/(4*b) - (3*d*(-1/4*((c + d*x)^2*Cos[a + b*x]*
Sin[a + b*x]^3)/b + (d*(c + d*x)*Sin[a + b*x]^4)/(8*b^2) - (d^2*(-1/4*(Cos
[a + b*x]*Sin[a + b*x]^3)/b + (3*(x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b))
)/4))/(8*b^2) + (3*((c + d*x)^3/(6*d) - ((c + d*x)^2*Cos[a + b*x]*Sin[a +
b*x])/(2*b) + (d*(c + d*x)*Sin[a + b*x]^2)/(2*b^2) - (d^2*(x/2 - (Cos[a +
b*x]*Sin[a + b*x])/(2*b)))/(2*b^2)))/4))/(4*b)
```

## 3.24. $\int (c+dx)^3\cos(a+bx)\sin^3(a+bx)\,dx$

### 3.24.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

### 3.24.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.76

| method | result |
|---|---|
| parallelrisch | $\frac{-32b\left((dx+c)^2b^2-\frac{3d^2}{2}\right)(dx+c)\cos(2xb+2a)+8b\left((dx+c)^2b^2-\frac{3d^2}{8}\right)(dx+c)\cos(4xb+4a)+48d\left((dx+c)^2b^2-\frac{d^2}{2}\right)\sin(2xb}{256b^4}$ |
| risch | $\frac{(8b^2d^3x^3+24b^2c\,d^2x^2+24b^2c^2dx+8b^2c^3-3d^3x-3c\,d^2)\cos(4xb+4a)}{256b^3}-\frac{3d(8x^2d^2b^2+16b^2cdx+8b^2c^2-d^2)\sin(4xb+4a}{1024b^4}$ |
| derivativedivides | $-\frac{a^3d^3\sin(xb+a)^4}{4b^3}+\frac{3a^2c\,d^2\sin(xb+a)^4}{4b^2}+\frac{3a^2d^3\left(\frac{(xb+a)\sin(xb+a)^4}{4}+\frac{\left(\sin(xb+a)^3+\frac{3\sin(xb+a)}{2}\right)\cos(xb+a)}{16}-\frac{3xb}{32}-\frac{3a}{32}\right)}{b^3}-\frac{3a\,c^2}{}$ |
| default | $-\frac{a^3d^3\sin(xb+a)^4}{4b^3}+\frac{3a^2c\,d^2\sin(xb+a)^4}{4b^2}+\frac{3a^2d^3\left(\frac{(xb+a)\sin(xb+a)^4}{4}+\frac{\left(\sin(xb+a)^3+\frac{3\sin(xb+a)}{2}\right)\cos(xb+a)}{16}-\frac{3xb}{32}-\frac{3a}{32}\right)}{b^3}-\frac{3a\,c^2}{}$ |

input
```
int((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/256*(-32*b*((d*x+c)^2*b^2-3/2*d^2)*(d*x+c)*cos(2*b*x+2*a)+8*b*((d*x+c)^2
*b^2-3/8*d^2)*(d*x+c)*cos(4*b*x+4*a)+48*d*((d*x+c)^2*b^2-1/2*d^2)*sin(2*b*
x+2*a)-6*((d*x+c)^2*b^2-1/8*d^2)*d*sin(4*b*x+4*a)+24*b^3*c^3-45*c*d^2*b)/b
^4
```

### 3.24.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.44

$$\int (c+dx)^3\cos(a+bx)\sin^3(a+bx)\,dx$$
$$=\frac{40\,b^3d^3x^3+120\,b^3cd^2x^2+8\left(8\,b^3d^3x^3+24\,b^3cd^2x^2+8\,b^3c^3-3\,bcd^2+3\left(8\,b^3c^2d-bd^3\right)x\right)\cos\left(bx+a\right)^4-}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/256*(40*b^3*d^3*x^3 + 120*b^3*c*d^2*x^2 + 8*(8*b^3*d^3*x^3 + 24*b^3*c*d^
2*x^2 + 8*b^3*c^3 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^4
- 8*(16*b^3*d^3*x^3 + 48*b^3*c*d^2*x^2 + 16*b^3*c^3 - 15*b*c*d^2 + 3*(16*b
^3*c^2*d - 5*b*d^3)*x)*cos(b*x + a)^2 + 3*(40*b^3*c^2*d - 17*b*d^3)*x - 3*
(2*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^3 - (
40*b^2*d^3*x^2 + 80*b^2*c*d^2*x + 40*b^2*c^2*d - 17*d^3)*cos(b*x + a))*sin
(b*x + a))/b^4
```

### 3.24.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(197) = 394$.

Time = 0.61 (sec) , antiderivative size = 602, normalized size of antiderivative = 3.07

$$\int (c+dx)^3\cos(a+bx)\sin^3(a+bx)\,dx$$

$$= \begin{cases} \frac{c^3\sin^4{(a+bx)}}{4b} + \frac{15c^2dx\sin^4{(a+bx)}}{32b} - \frac{9c^2dx\sin^2{(a+bx)}\cos^2{(a+bx)}}{16b} - \frac{9c^2dx\cos^4{(a+bx)}}{32b} + \frac{15cd^2x^2\sin^4{(a+bx)}}{32b} - \frac{9cd^2x^2\sin^2{(a+}}{16} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin^3{(a)}\cos{(a)} \end{cases}$$

input
```
integrate((d*x+c)**3*cos(b*x+a)*sin(b*x+a)**3,x)
```

output
```
Piecewise((c**3*sin(a + b*x)**4/(4*b) + 15*c**2*d*x*sin(a + b*x)**4/(32*b)
 - 9*c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 9*c**2*d*x*cos(a +
b*x)**4/(32*b) + 15*c*d**2*x**2*sin(a + b*x)**4/(32*b) - 9*c*d**2*x**2*sin
(a + b*x)**2*cos(a + b*x)**2/(16*b) - 9*c*d**2*x**2*cos(a + b*x)**4/(32*b)
 + 5*d**3*x**3*sin(a + b*x)**4/(32*b) - 3*d**3*x**3*sin(a + b*x)**2*cos(a
+ b*x)**2/(16*b) - 3*d**3*x**3*cos(a + b*x)**4/(32*b) + 15*c**2*d*sin(a +
b*x)**3*cos(a + b*x)/(32*b**2) + 9*c**2*d*sin(a + b*x)*cos(a + b*x)**3/(32
*b**2) + 15*c*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 9*c*d**2*x*s
in(a + b*x)*cos(a + b*x)**3/(16*b**2) + 15*d**3*x**2*sin(a + b*x)**3*cos(a
 + b*x)/(32*b**2) + 9*d**3*x**2*sin(a + b*x)*cos(a + b*x)**3/(32*b**2) - 1
5*c*d**2*sin(a + b*x)**4/(64*b**3) + 9*c*d**2*cos(a + b*x)**4/(64*b**3) -
51*d**3*x*sin(a + b*x)**4/(256*b**3) + 9*d**3*x*sin(a + b*x)**2*cos(a + b*
x)**2/(128*b**3) + 45*d**3*x*cos(a + b*x)**4/(256*b**3) - 51*d**3*sin(a +
b*x)**3*cos(a + b*x)/(256*b**4) - 45*d**3*sin(a + b*x)*cos(a + b*x)**3/(25
6*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4
)*sin(a)**3*cos(a), True))
```

### 3.24.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(178) = 356$.

Time = 0.25 (sec) , antiderivative size = 549, normalized size of antiderivative = 2.80

$$\int (c+dx)^3 \cos(a+bx) \sin^3(a+bx)\, dx$$
$$= \frac{256\, c^3 \sin{(bx+a)}^4 - \frac{768\, ac^2 d \sin(bx+a)^4}{b} + \frac{768\, a^2 cd^2 \sin(bx+a)^4}{b^2} - \frac{256\, a^3 d^3 \sin(bx+a)^4}{b^3} + \frac{24\, (4\, (bx+a) \cos(4\, bx+4\, a) - 16\, (bx+a)}{}}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/1024*(256*c^3*sin(b*x + a)^4 - 768*a*c^2*d*sin(b*x + a)^4/b + 768*a^2*c*
d^2*sin(b*x + a)^4/b^2 - 256*a^3*d^3*sin(b*x + a)^4/b^3 + 24*(4*(b*x + a)*
cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*si
n(2*b*x + 2*a))*c^2*d/b - 48*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*
cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*a*c*d^2/b^2 + 24
*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x
 + 4*a) + 8*sin(2*b*x + 2*a))*a^2*d^3/b^3 + 12*((8*(b*x + a)^2 - 1)*cos(4*
b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b
*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*c*d^2/b^2 - 12*((8*(b*x + a)^2
- 1)*cos(4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x +
 a)*sin(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*a*d^3/b^3 + (4*(8*(b
*x + a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) - 64*(2*(b*x + a)^3 - 3*b*x - 3*
a)*cos(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) + 96*(2*(b*x
+ a)^2 - 1)*sin(2*b*x + 2*a))*d^3/b^3)/b
```

### 3.24.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.23

$$\int (c+dx)^3 \cos(a+bx) \sin^3(a+bx)\, dx$$
$$= \frac{(8\, b^3 d^3 x^3 + 24\, b^3 c d^2 x^2 + 24\, b^3 c^2 d x + 8\, b^3 c^3 - 3\, b d^3 x - 3\, b c d^2) \cos{(4\, bx + 4\, a)}}{256\, b^4}$$
$$- \frac{(2\, b^3 d^3 x^3 + 6\, b^3 c d^2 x^2 + 6\, b^3 c^2 d x + 2\, b^3 c^3 - 3\, b d^3 x - 3\, b c d^2) \cos{(2\, bx + 2\, a)}}{16\, b^4}$$
$$- \frac{3\, (8\, b^2 d^3 x^2 + 16\, b^2 c d^2 x + 8\, b^2 c^2 d - d^3) \sin{(4\, bx + 4\, a)}}{1024\, b^4}$$
$$+ \frac{3\, (2\, b^2 d^3 x^2 + 4\, b^2 c d^2 x + 2\, b^2 c^2 d - d^3) \sin{(2\, bx + 2\, a)}}{32\, b^4}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/256*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 - 3*b
*d^3*x - 3*b*c*d^2)*cos(4*b*x + 4*a)/b^4 - 1/16*(2*b^3*d^3*x^3 + 6*b^3*c*d
^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^2)*cos(2*b*x + 2*
a)/b^4 - 3/1024*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*sin(4
*b*x + 4*a)/b^4 + 3/32*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)
*sin(2*b*x + 2*a)/b^4
```

### 3.24.9 Mupad [B] (verification not implemented)

Time = 25.00 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.87

$$\int (c+dx)^3 \cos(a+bx) \sin^3(a+bx) \, dx =$$

$$-\frac{24\,d^3\,\sin\left(2\,a+2\,b\,x\right)-\frac{3\,d^3\,\sin\left(4\,a+4\,b\,x\right)}{4}+32\,b^3\,c^3\,\cos\left(2\,a+2\,b\,x\right)-8\,b^3\,c^3\,\cos\left(4\,a+4\,b\,x\right)-48\,b^2\,c^2\,d\,\mathrm{s}}{}$$

input
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^3,x)
```

output
```
-(24*d^3*sin(2*a + 2*b*x) - (3*d^3*sin(4*a + 4*b*x))/4 + 32*b^3*c^3*cos(2*
a + 2*b*x) - 8*b^3*c^3*cos(4*a + 4*b*x) - 48*b^2*c^2*d*sin(2*a + 2*b*x) +
6*b^2*c^2*d*sin(4*a + 4*b*x) + 32*b^3*d^3*x^3*cos(2*a + 2*b*x) - 8*b^3*d^3
*x^3*cos(4*a + 4*b*x) - 48*b^2*d^3*x^2*sin(2*a + 2*b*x) + 6*b^2*d^3*x^2*si
n(4*a + 4*b*x) - 48*b*c*d^2*cos(2*a + 2*b*x) + 3*b*c*d^2*cos(4*a + 4*b*x)
- 48*b*d^3*x*cos(2*a + 2*b*x) + 3*b*d^3*x*cos(4*a + 4*b*x) + 96*b^3*c^2*d*
x*cos(2*a + 2*b*x) - 24*b^3*c^2*d*x*cos(4*a + 4*b*x) - 96*b^2*c*d^2*x*sin(
2*a + 2*b*x) + 12*b^2*c*d^2*x*sin(4*a + 4*b*x) + 96*b^3*c*d^2*x^2*cos(2*a
+ 2*b*x) - 24*b^3*c*d^2*x^2*cos(4*a + 4*b*x))/(256*b^4)
```

# 3.25 $\int (c+dx)^2 \cos(a+bx) \sin^3(a+bx) \, dx$

3.25.1 Optimal result . . . . . . . . . . 311
3.25.2 Mathematica [A] (verified) . . . . . . . . . . 312
3.25.3 Rubi [A] (verified) . . . . . . . . . . 312
3.25.4 Maple [A] (verified) . . . . . . . . . . 314
3.25.5 Fricas [A] (verification not implemented) . . . . . . . . . . 315
3.25.6 Sympy [B] (verification not implemented) . . . . . . . . . . 315
3.25.7 Maxima [B] (verification not implemented) . . . . . . . . . . 316
3.25.8 Giac [A] (verification not implemented) . . . . . . . . . . 316
3.25.9 Mupad [B] (verification not implemented) . . . . . . . . . . 317

## 3.25.1 Optimal result

Integrand size = 22, antiderivative size = 134

$$\int (c+dx)^2 \cos(a+bx) \sin^3(a+bx) \, dx = -\frac{3cdx}{16b} - \frac{3d^2x^2}{32b} + \frac{3d(c+dx)\cos(a+bx)\sin(a+bx)}{16b^2} - \frac{3d^2\sin^2(a+bx)}{32b^3} + \frac{d(c+dx)\cos(a+bx)\sin^3(a+bx)}{8b^2} - \frac{d^2\sin^4(a+bx)}{32b^3} + \frac{(c+dx)^2\sin^4(a+bx)}{4b}$$

output
```
-3/16*c*d*x/b-3/32*d^2*x^2/b+3/16*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2-3/32
*d^2*sin(b*x+a)^2/b^3+1/8*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)^3/b^2-1/32*d^2*s
in(b*x+a)^4/b^3+1/4*(d*x+c)^2*sin(b*x+a)^4/b
```

### 3.25.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.68

$$\int (c+dx)^2\cos(a+bx)\sin^3(a+bx)\,dx$$
$$=\frac{-16(-d^2+2b^2(c+dx)^2)\cos(2(a+bx))+(-d^2+8b^2(c+dx)^2)\cos(4(a+bx))-4bd(c+dx)(-8\sin(2(a+bx))+\sin(4(a+bx)))}{256b^3}$$

input
```
Integrate[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
(-16*(-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + (-d^2 + 8*b^2*(c + d*x)
^2)*Cos[4*(a + b*x)] - 4*b*d*(c + d*x)*(-8*Sin[2*(a + b*x)] + Sin[4*(a + b
*x)]))/(256*b^3)
```

### 3.25.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4904, 3042, 3791, 3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\sin^3(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \ 4904$$

$$\frac{(c+dx)^2\sin^4(a+bx)}{4b}-\frac{d\int (c+dx)\sin^4(a+bx)dx}{2b}$$

$$\Big\downarrow \ 3042$$

$$\frac{(c+dx)^2\sin^4(a+bx)}{4b}-\frac{d\int (c+dx)\sin(a+bx)^4dx}{2b}$$

$$\Big\downarrow \ 3791$$

$$\frac{(c+dx)^2\sin^4(a+bx)}{4b}-\frac{d\left(\frac{3}{4}\int (c+dx)\sin^2(a+bx)dx+\frac{d\sin^4(a+bx)}{16b^2}-\frac{(c+dx)\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{2b}$$

$$\Big\downarrow \ 3042$$

$$\frac{(c+dx)^2\sin^4(a+bx)}{4b}-\frac{d\left(\frac{3}{4}\int (c+dx)\sin(a+bx)^2dx+\frac{d\sin^4(a+bx)}{16b^2}-\frac{(c+dx)\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{2b}$$

$$\Big\downarrow \text{ 3791}$$

$$\frac{(c+dx)^2\sin^4(a+bx)}{4b}-\frac{d\left(\frac{3}{4}\left(\frac{1}{2}\int (c+dx)dx+\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{d\sin^4(a+bx)}{16b^2}-\frac{(c+dx)\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{2b}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{(c+dx)^2\sin^4(a+bx)}{4b}-\frac{d\left(\frac{3}{4}\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)+\frac{d\sin^4(a+bx)}{16b^2}-\frac{(c+dx)\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{2b}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
((c + d*x)^2*Sin[a + b*x]^4)/(4*b) - (d*(-1/4*((c + d*x)*Cos[a + b*x]*Sin[
a + b*x]^3)/b + (d*Sin[a + b*x]^4)/(16*b^2) + (3*((c + d*x)^2/(4*d) - ((c
+ d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (d*Sin[a + b*x]^2)/(4*b^2)))/4))
/(2*b)
```

### 3.25.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

### 3.25.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

| method | result |
|---|---|
| parallelrisch | $\frac{16\left(-2(dx+c)^2b^2+d^2\right)\cos(2xb+2a)+\left(8(dx+c)^2b^2-d^2\right)\cos(4xb+4a)+32bd(dx+c)\sin(2xb+2a)-4bd(dx+c)\sin(4xb+4}{256b^3}$ |
| risch | $\frac{(8x^2d^2b^2+16b^2cdx+8b^2c^2-d^2)\cos(4xb+4a)}{256b^3}-\frac{d(dx+c)\sin(4xb+4a)}{64b^2}-\frac{(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2)\cos(2xb+2a)}{16b^3}$ |
| derivativedivides | $\frac{\frac{a^2d^2\sin(xb+a)^4}{4b^2}-\frac{acd\sin(xb+a)^4}{2b}-\frac{2a\,d^2\left(\frac{(xb+a)\sin(xb+a)^4}{4}+\frac{\left(\sin(xb+a)^3+\frac{3\sin(xb+a)}{2}\right)\cos(xb+a)}{16}-\frac{3xb}{32}-\frac{3a}{32}\right)}{b^2}+\frac{c^2\sin(xb+a}{4}}{}$ |
| default | $\frac{\frac{a^2d^2\sin(xb+a)^4}{4b^2}-\frac{acd\sin(xb+a)^4}{2b}-\frac{2a\,d^2\left(\frac{(xb+a)\sin(xb+a)^4}{4}+\frac{\left(\sin(xb+a)^3+\frac{3\sin(xb+a)}{2}\right)\cos(xb+a)}{16}-\frac{3xb}{32}-\frac{3a}{32}\right)}{b^2}+\frac{c^2\sin(xb+a}{4}}{}$ |
| norman | $\frac{-\frac{3d^2x^2}{32b}-\frac{3d^2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}{8b^3}-\frac{3d^2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^6}{8b^3}+\frac{\left(16b^2c^2-5d^2\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^4}{4b^3}+\frac{3cd\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{8b^2}+\frac{11cd\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3}{8b^2}-\frac{11cd\tan}{}}{}$ |

input
```
int((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/256*(16*(-2*(d*x+c)^2*b^2+d^2)*cos(2*b*x+2*a)+(8*(d*x+c)^2*b^2-d^2)*cos(
4*b*x+4*a)+32*b*d*(d*x+c)*sin(2*b*x+2*a)-4*b*d*(d*x+c)*sin(4*b*x+4*a)+24*b
^2*c^2-15*d^2)/b^3
```

### 3.25.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.19

$$\int (c+dx)^2 \cos(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{5\,b^2d^2x^2 + 10\,b^2cdx + \left(8\,b^2d^2x^2 + 16\,b^2cdx + 8\,b^2c^2 - d^2\right)\cos\left(bx+a\right)^4 - \left(16\,b^2d^2x^2 + 32\,b^2cdx + 16\,b^2c^2 -}{32\,b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/32*(5*b^2*d^2*x^2 + 10*b^2*c*d*x + (8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2
*c^2 - d^2)*cos(b*x + a)^4 - (16*b^2*d^2*x^2 + 32*b^2*c*d*x + 16*b^2*c^2 -
 5*d^2)*cos(b*x + a)^2 - 2*(2*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - 5*(b*d^2*
x + b*c*d)*cos(b*x + a))*sin(b*x + a))/b^3
```

### 3.25.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(129) = 258$.

Time = 0.43 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.39

$$\int (c+dx)^2 \cos(a+bx)\sin^3(a+bx)\,dx$$
$$= \begin{cases} \frac{c^2\sin^4(a+bx)}{4b} + \frac{5cdx\sin^4(a+bx)}{16b} - \frac{3cdx\sin^2(a+bx)\cos^2(a+bx)}{8b} - \frac{3cdx\cos^4(a+bx)}{16b} + \frac{5d^2x^2\sin^4(a+bx)}{32b} - \frac{3d^2x^2\sin^2(a+bx)\cos^2}{16b} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin^3(a)\cos(a) \end{cases}$$

input
```
integrate((d*x+c)**2*cos(b*x+a)*sin(b*x+a)**3,x)
```

output
```
Piecewise((c**2*sin(a + b*x)**4/(4*b) + 5*c*d*x*sin(a + b*x)**4/(16*b) - 3
*c*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(8*b) - 3*c*d*x*cos(a + b*x)**4/(16
*b) + 5*d**2*x**2*sin(a + b*x)**4/(32*b) - 3*d**2*x**2*sin(a + b*x)**2*cos
(a + b*x)**2/(16*b) - 3*d**2*x**2*cos(a + b*x)**4/(32*b) + 5*c*d*sin(a + b
*x)**3*cos(a + b*x)/(16*b**2) + 3*c*d*sin(a + b*x)*cos(a + b*x)**3/(16*b**
2) + 5*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 3*d**2*x*sin(a + b*
x)*cos(a + b*x)**3/(16*b**2) - 5*d**2*sin(a + b*x)**4/(64*b**3) + 3*d**2*c
os(a + b*x)**4/(64*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*si
n(a)**3*cos(a), True))
```

### 3.25.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(120) = 240$.

Time = 0.22 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.96

$$\int (c+dx)^2 \cos(a+bx) \sin^3(a+bx)\, dx$$

$$= \frac{64\, c^2 \sin(bx+a)^4 - \frac{128\, acd \sin(bx+a)^4}{b} + \frac{64\, a^2 d^2 \sin(bx+a)^4}{b^2} + \frac{4\,(4\,(bx+a)\cos(4\,bx+4\,a) - 16\,(bx+a)\cos(2\,bx+2\,a) - \sin(4\,bx+4\,a) + 8}{b}}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/256*(64*c^2*sin(b*x + a)^4 - 128*a*c*d*sin(b*x + a)^4/b + 64*a^2*d^2*sin
(b*x + a)^4/b^2 + 4*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x
 + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*c*d/b - 4*(4*(b*x + a)*co
s(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(
2*b*x + 2*a))*a*d^2/b^2 + ((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 16*(2*(b
*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x +
 a)*sin(2*b*x + 2*a))*d^2/b^2)/b
```

### 3.25.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.08

$$\int (c+dx)^2 \cos(a+bx) \sin^3(a+bx)\, dx = \frac{(8\,b^2 d^2 x^2 + 16\,b^2 cdx + 8\,b^2 c^2 - d^2)\cos(4\,bx+4\,a)}{256\,b^3} - \frac{(2\,b^2 d^2 x^2 + 4\,b^2 cdx + 2\,b^2 c^2 - d^2)\cos(2\,bx+2\,a)}{16\,b^3} - \frac{(bd^2 x + bcd)\sin(4\,bx+4\,a)}{64\,b^3} + \frac{(bd^2 x + bcd)\sin(2\,bx+2\,a)}{8\,b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/256*(8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(4*b*x + 4*a)/b^
3 - 1/16*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2*a)/
b^3 - 1/64*(b*d^2*x + b*c*d)*sin(4*b*x + 4*a)/b^3 + 1/8*(b*d^2*x + b*c*d)*
sin(2*b*x + 2*a)/b^3
```

### 3.25.9 Mupad [B] (verification not implemented)

Time = 24.53 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.51

$$\int (c+dx)^2 \cos(a+bx) \sin^3(a+bx)\, dx$$
$$= \frac{8\,d^2\,\cos\left(2\,a+2\,b\,x\right)-\frac{d^2\,\cos\left(4\,a+4\,b\,x\right)}{2}-16\,b^2\,c^2\,\cos\left(2\,a+2\,b\,x\right)+4\,b^2\,c^2\,\cos\left(4\,a+4\,b\,x\right)+16\,b\,c\,d\,\sin\left(2\,a\right.}{}$$

input
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^2,x)
```

output
```
(8*d^2*cos(2*a + 2*b*x) - (d^2*cos(4*a + 4*b*x))/2 - 16*b^2*c^2*cos(2*a +
2*b*x) + 4*b^2*c^2*cos(4*a + 4*b*x) + 16*b*c*d*sin(2*a + 2*b*x) - 2*b*c*d*
sin(4*a + 4*b*x) - 16*b^2*d^2*x^2*cos(2*a + 2*b*x) + 4*b^2*d^2*x^2*cos(4*a
 + 4*b*x) + 16*b*d^2*x*sin(2*a + 2*b*x) - 2*b*d^2*x*sin(4*a + 4*b*x) - 32*
b^2*c*d*x*cos(2*a + 2*b*x) + 8*b^2*c*d*x*cos(4*a + 4*b*x))/(128*b^3)
```

# 3.26 $\int (c+dx)\cos(a+bx)\sin^3(a+bx)\,dx$

3.26.1 Optimal result . . . . . 318
3.26.2 Mathematica [A] (verified) . . . . . 318
3.26.3 Rubi [A] (verified) . . . . . 319
3.26.4 Maple [A] (verified) . . . . . 320
3.26.5 Fricas [A] (verification not implemented) . . . . . 321
3.26.6 Sympy [A] (verification not implemented) . . . . . 322
3.26.7 Maxima [A] (verification not implemented) . . . . . 322
3.26.8 Giac [A] (verification not implemented) . . . . . 323
3.26.9 Mupad [B] (verification not implemented) . . . . . 323

## 3.26.1 Optimal result

Integrand size = 20, antiderivative size = 72

$$\int (c+dx)\cos(a+bx)\sin^3(a+bx)\,dx = -\frac{3dx}{32b}+\frac{3d\cos(a+bx)\sin(a+bx)}{32b^2}+\frac{d\cos(a+bx)\sin^3(a+bx)}{16b^2}+\frac{(c+dx)\sin^4(a+bx)}{4b}$$

output
```
-3/32*d*x/b+3/32*d*cos(b*x+a)*sin(b*x+a)/b^2+1/16*d*cos(b*x+a)*sin(b*x+a)^
3/b^2+1/4*(d*x+c)*sin(b*x+a)^4/b
```

## 3.26.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int (c+dx)\cos(a+bx)\sin^3(a+bx)\,dx = \frac{c\sin^4(a+bx)}{4b}+\frac{d(-2bx\cos(2(a+bx))+\sin(2(a+bx)))}{16b^2}-\frac{d(-4bx\cos(4(a+bx))+\sin(4(a+bx)))}{128b^2}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
(c*Sin[a + b*x]^4)/(4*b) + (d*(-2*b*x*Cos[2*(a + b*x)] + Sin[2*(a + b*x)])
)/(16*b^2) - (d*(-4*b*x*Cos[4*(a + b*x)] + Sin[4*(a + b*x)]))/(128*b^2)
```

### 3.26.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4904, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin^3(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{4904}$$

$$\frac{(c+dx)\sin^4(a+bx)}{4b}-\frac{d\int \sin^4(a+bx)dx}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)\sin^4(a+bx)}{4b}-\frac{d\int \sin(a+bx)^4dx}{4b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{(c+dx)\sin^4(a+bx)}{4b}-\frac{d\left(\frac{3}{4}\int \sin^2(a+bx)dx-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)\sin^4(a+bx)}{4b}-\frac{d\left(\frac{3}{4}\int \sin(a+bx)^2dx-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{4b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{(c+dx)\sin^4(a+bx)}{4b}-\frac{d\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{4b}$$

$$\Big\downarrow \text{24}$$

$$\frac{(c+dx)\sin^4(a+bx)}{4b}-\frac{d\left(\frac{3}{4}\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)-\frac{\sin^3(a+bx)\cos(a+bx)}{4b}\right)}{4b}$$

input
```
Int[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
((c + d*x)*Sin[a + b*x]^4)/(4*b) - (d*(-1/4*(Cos[a + b*x]*Sin[a + b*x]^3)/
b + (3*(x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)))/4))/(4*b)
```

#### 3.26.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

### 3.26.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

| method | result |
|---|---|
| parallelrisch | $\frac{-16b(dx+c)\cos(2xb+2a)+4b(dx+c)\cos(4xb+4a)+12cb+8d\sin(2xb+2a)-d\sin(4xb+4a)}{128b^2}$ |
| risch | $\frac{(dx+c)\cos(4xb+4a)}{32b}-\frac{d\sin(4xb+4a)}{128b^2}-\frac{(dx+c)\cos(2xb+2a)}{8b}+\frac{d\sin(2xb+2a)}{16b^2}$ |
| derivativedivides | $\frac{-\frac{da\sin(xb+a)^4}{4b}+\frac{c\sin(xb+a)^4}{4}+\frac{d\left(\frac{(xb+a)\sin(xb+a)^4}{4}+\frac{\left(\sin(xb+a)^3+\frac{3\sin(xb+a)}{2}\right)\cos(xb+a)}{16}-\frac{3xb}{32}-\frac{3a}{32}\right)}{b}}{b}$ |
| default | $\frac{-\frac{da\sin(xb+a)^4}{4b}+\frac{c\sin(xb+a)^4}{4}+\frac{d\left(\frac{(xb+a)\sin(xb+a)^4}{4}+\frac{\left(\sin(xb+a)^3+\frac{3\sin(xb+a)}{2}\right)\cos(xb+a)}{16}-\frac{3xb}{32}-\frac{3a}{32}\right)}{b}}{b}$ |
| norman | $\frac{\frac{3d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{16b^2}+\frac{11d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3}{16b^2}-\frac{11d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^5}{16b^2}-\frac{3d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^7}{16b^2}-\frac{3dx}{32b}+\frac{4c\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^4}{b}-\frac{3dx\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}{8b}+55dx\tan}{\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)^4}$ |

input
```
int((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/128*(-16*b*(d*x+c)*cos(2*b*x+2*a)+4*b*(d*x+c)*cos(4*b*x+4*a)+12*c*b+8*d*
sin(2*b*x+2*a)-d*sin(4*b*x+4*a))/b^2
```

### 3.26.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.06

$$\int (c+dx)\cos(a+bx)\sin^3(a+bx)\,dx$$
$$=\frac{8\,(bdx+bc)\cos\left(bx+a\right)^4+5\,bdx-16\,(bdx+bc)\cos\left(bx+a\right)^2-\left(2\,d\cos\left(bx+a\right)^3-5\,d\cos\left(bx+a\right)\right)\sin}{32\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/32*(8*(b*d*x + b*c)*cos(b*x + a)^4 + 5*b*d*x - 16*(b*d*x + b*c)*cos(b*x
+ a)^2 - (2*d*cos(b*x + a)^3 - 5*d*cos(b*x + a))*sin(b*x + a))/b^2
```

### 3.26.6 Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.92

$$\int (c+dx)\cos(a+bx)\sin^3(a+bx)\,dx$$
$$= \begin{cases} \frac{c\sin^4(a+bx)}{4b} + \frac{5dx\sin^4(a+bx)}{32b} - \frac{3dx\sin^2(a+bx)\cos^2(a+bx)}{16b} - \frac{3dx\cos^4(a+bx)}{32b} + \frac{5d\sin^3(a+bx)\cos(a+bx)}{32b^2} + \frac{3d\sin(a+bx)\cos^3(a+bx)}{32b^2} \\ \left(cx + \frac{dx^2}{2}\right)\sin^3(a)\cos(a) \end{cases}$$

input
```
integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)**3,x)
```

output
```
Piecewise((c*sin(a + b*x)**4/(4*b) + 5*d*x*sin(a + b*x)**4/(32*b) - 3*d*x*
sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 3*d*x*cos(a + b*x)**4/(32*b) + 5*
d*sin(a + b*x)**3*cos(a + b*x)/(32*b**2) + 3*d*sin(a + b*x)*cos(a + b*x)**
3/(32*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*sin(a)**3*cos(a), True))
```

### 3.26.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.28

$$\int (c+dx)\cos(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{32\,c\sin(bx+a)^4 - \frac{32\,ad\sin(bx+a)^4}{b} + \frac{(4\,(bx+a)\cos(4\,bx+4\,a) - 16\,(bx+a)\cos(2\,bx+2\,a) - \sin(4\,bx+4\,a) + 8\,\sin(2\,bx+2\,a))d}{b}}{128\,b}$$

input
```
integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/128*(32*c*sin(b*x + a)^4 - 32*a*d*sin(b*x + a)^4/b + (4*(b*x + a)*cos(4*
b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*
x + 2*a))*d/b)/b
```

### 3.26.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int (c+dx)\cos(a+bx)\sin^3(a+bx)\,dx = \frac{(bdx+bc)\cos(4\,bx+4\,a)}{32\,b^2} - \frac{(bdx+bc)\cos(2\,bx+2\,a)}{8\,b^2} - \frac{d\sin(4\,bx+4\,a)}{128\,b^2} + \frac{d\sin(2\,bx+2\,a)}{16\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/32*(b*d*x + b*c)*cos(4*b*x + 4*a)/b^2 - 1/8*(b*d*x + b*c)*cos(2*b*x + 2*
a)/b^2 - 1/128*d*sin(4*b*x + 4*a)/b^2 + 1/16*d*sin(2*b*x + 2*a)/b^2
```

### 3.26.9 Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.31

$$\int (c+dx)\cos(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{2\,d\,\sin(2\,a+2\,b\,x) - \frac{d\,\sin(4\,a+4\,b\,x)}{4} - 2\,b\,c\sin(2\,a+2\,b\,x)^2 + 8\,b\,c\sin(a+b\,x)^2 + 4\,b\,d\,x\,\left(2\sin(a+b\,x)^2 -\right.}{32\,b^2}$$

input
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x),x)
```

output
```
(2*d*sin(2*a + 2*b*x) - (d*sin(4*a + 4*b*x))/4 - 2*b*c*sin(2*a + 2*b*x)^2
+ 8*b*c*sin(a + b*x)^2 + 4*b*d*x*(2*sin(a + b*x)^2 - 1) - b*d*x*(2*sin(2*a
 + 2*b*x)^2 - 1))/(32*b^2)
```

# 3.27 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx$


3.27.1 Optimal result . . . 324
3.27.2 Mathematica [A] (verified) . . . 324
3.27.3 Rubi [A] (verified) . . . 325
3.27.4 Maple [A] (verified) . . . 326
3.27.5 Fricas [A] (verification not implemented) . . . 327
3.27.6 Sympy [F] . . . 327
3.27.7 Maxima [C] (verification not implemented) . . . 327
3.27.8 Giac [C] (verification not implemented) . . . 328
3.27.9 Mupad [F(-1)] . . . 329


## 3.27.1 Optimal result

Integrand size = 22, antiderivative size = 129

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx = -\frac{\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{8d} + \frac{\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{4d} + \frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{4d} - \frac{\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{8d}$$

output
```
1/4*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d-1/8*cos(4*a-4*b*c/d)*Si(4*b*c/d+4
*b*x)/d-1/8*Ci(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d+1/4*Ci(2*b*c/d+2*b*x)*sin
(2*a-2*b*c/d)/d
```

## 3.27.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.85

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx = -\frac{\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)\sin\left(4a-\frac{4bc}{d}\right) - 2\,\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right) - 2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right) \ldots}{8d}$$

input
```
Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x),x]
```

output
```
-1/8*(CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] - 2*CosIntegral[
(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] - 2*Cos[2*a - (2*b*c)/d]*SinIntegr
al[(2*b*(c + d*x))/d] + Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d
])/d
```

### 3.27.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)\cos(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{\sin(2a+2bx)}{4(c+dx)}-\frac{\sin(4a+4bx)}{8(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{4d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{4d}-\frac{\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{8d}$$

input
```
Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x),x]
```

output
```
-1/8*(CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/d + (CosIntegra
l[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(4*d) + (Cos[2*a - (2*b*c)/d]*S
inIntegral[(2*b*c)/d + 2*b*x])/(4*d) - (Cos[4*a - (4*b*c)/d]*SinIntegral[(
4*b*c)/d + 4*b*x])/(8*d)
```

### 3.27.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.27.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.38

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{b\left(-\frac{2\,\mathrm{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2xb+2a+\frac{-2ad+2cb}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d}\right)}{8}-\frac{b\left(-\frac{4\,\mathrm{Si}\left(-4xb-4a-\frac{4(-ad-}{d}\right.}{d}\right.}{}}{b}$ |
| default | $\frac{\frac{b\left(-\frac{2\,\mathrm{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2xb+2a+\frac{-2ad+2cb}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d}\right)}{8}-\frac{b\left(-\frac{4\,\mathrm{Si}\left(-4xb-4a-\frac{4(-ad-}{d}\right.}{d}\right.}{}}{b}$ |
| risch | $-\frac{i e^{-\frac{2i(ad-cb)}{d}}\mathrm{Ei}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{8d}+\frac{i e^{-\frac{4i(ad-cb)}{d}}\mathrm{Ei}_1\left(4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{16d}+\frac{i e^{\frac{2i(ad-cb)}{d}}\mathrm{Ei}_1\left(-2ibx-2\right.}{8d}$ |

input
```
int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*b*(-2*Si(-2*x*b-2*a-2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*
x*b+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)-1/32*b*(-4*Si(-4*x*b-4*a-4*
(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*x*b+4*a+4*(-a*d+b*c)/d)*sin(4*(
-a*d+b*c)/d)/d))
```

### 3.27.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx$$
$$= \frac{2\,\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right) - \mathrm{Ci}\left(\frac{4\,(bdx+bc)}{d}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right) - \cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{4\,(bdx+bc)}{d}\right) + 2\cos}{8\,d}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
1/8*(2*cos_integral(2*(b*d*x + b*c)/d)*sin(-2*(b*c - a*d)/d) - cos_integra
l(4*(b*d*x + b*c)/d)*sin(-4*(b*c - a*d)/d) - cos(-4*(b*c - a*d)/d)*sin_int
egral(4*(b*d*x + b*c)/d) + 2*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x +
 b*c)/d))/d
```

### 3.27.6 Sympy [F]

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \int \frac{\sin^3(a+bx)\cos(a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x), x)
```

### 3.27.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.18

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx =$$
$$-\frac{2\,b\left(-i\,E_1\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right) + i\,E_1\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right) - b\left(-i\,E_1\left(\frac{4\,(-i\,bc-i\,(bx}{d}\right.\right.}{}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output
```
-1/16*(2*b*(-I*exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I
*exp_integral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a
*d)/d) - b*(-I*exp_integral_e(1, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I
*exp_integral_e(1, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c - a
*d)/d) + 2*b*(exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + ex
p_integral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)
/d) - b*(exp_integral_e(1, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_int
egral_e(1, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d)/d))/
(b*d)
```

### 3.27.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.51 (sec) , antiderivative size = 6046, normalized size of antiderivative = 46.87

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output
```
-1/16*(imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*
b*c/d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a
)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(c
os_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/
d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^
2*tan(b*c/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan
(2*b*c/d)^2*tan(b*c/d)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(
2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 4*real_part(cos_integral(-2*b*
x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 2*real_part(
cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)
^2 + 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2
*b*c/d)*tan(b*c/d)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)
^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*real_part(cos_integral(-2*b*x -
2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_
integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 -
 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d
)^2*tan(b*c/d)^2 + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan
(a)^2*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)
^2*tan(a)^2*tan(2*b*c/d)^2 - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d)...
```

### 3.27.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \int \frac{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^3}{c+d\,x}\,dx$$

input
```
int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x),x)
```

output
```
int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x), x)
```

# 3.28 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$


3.28.1 Optimal result . . . . . 330
3.28.2 Mathematica [A] (verified) . . . . . 331
3.28.3 Rubi [A] (verified) . . . . . 331
3.28.4 Maple [A] (verified) . . . . . 332
3.28.5 Fricas [A] (verification not implemented) . . . . . 333
3.28.6 Sympy [F] . . . . . 333
3.28.7 Maxima [C] (verification not implemented) . . . . . 334
3.28.8 Giac [C] (verification not implemented) . . . . . 334
3.28.9 Mupad [F(-1)] . . . . . 335


## 3.28.1 Optimal result

Integrand size = 22, antiderivative size = 179

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^2} - \frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2} - \frac{\sin(2a+2bx)}{4d(c+dx)} + \frac{\sin(4a+4bx)}{8d(c+dx)} - \frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^2} + \frac{b\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{2d^2}$$

output

```
-1/2*b*Ci(4*b*c/d+4*b*x)*cos(4*a-4*b*c/d)/d^2+1/2*b*Ci(2*b*c/d+2*b*x)*cos(
2*a-2*b*c/d)/d^2+1/2*b*Si(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^2-1/2*b*Si(2*b
*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^2-1/4*sin(2*b*x+2*a)/d/(d*x+c)+1/8*sin(4*b*
x+4*a)/d/(d*x+c)
```

### 3.28.2 Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.84

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{4b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)-4b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)-\frac{2d\sin(2(a+bx))}{c+dx}+\frac{d\sin(4(\cdots}{c+\cdots}}{8d^2}$$

input
```
Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

output
```
(4*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - 4*b*Cos[4*a - (
4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] - (2*d*Sin[2*(a + b*x)])/(c + d*x
) + (d*Sin[4*(a + b*x)])/(c + d*x) - 4*b*Sin[2*a - (2*b*c)/d]*SinIntegral[
(2*b*(c + d*x))/d] + 4*b*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/
d])/(8*d^2)
```

### 3.28.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)\cos(a+bx)}{(c+dx)^2}\,dx$$
$$\Big\downarrow \text{4906}$$
$$\int \left(\frac{\sin(2a+2bx)}{4(c+dx)^2}-\frac{\sin(4a+4bx)}{8(c+dx)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^2}-\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^2}+\frac{b\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{\sin(2a+2bx)}{4d(c+dx)}+\frac{\sin(4a+4bx)}{8d(c+dx)}$$

input
```
Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

output
```
(b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) - (b*Cos[4
*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(2*d^2) - Sin[2*a + 2*b*x]
/(4*d*(c + d*x)) + Sin[4*a + 4*b*x]/(8*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)
/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) + (b*Sin[4*a - (4*b*c)/d]*SinI
ntegral[(4*b*c)/d + 4*b*x])/(2*d^2)
```

### 3.28.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.28.4 Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.43

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{b^2\left(-\frac{2\sin(2xb+2a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{4\,\mathrm{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(2xb+2a+\frac{-2ad+2cb}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}}{d}\right)}{8}-\ldots}{b}$ |
| default | $\frac{\frac{b^2\left(-\frac{2\sin(2xb+2a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{4\,\mathrm{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(2xb+2a+\frac{-2ad+2cb}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}}{d}\right)}{8}-\ldots}{b}$ |
| risch | $-\frac{b\,e^{-\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{4d^2}+\frac{b\,e^{-\frac{4i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{4d^2}-\frac{b\,e^{\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(-2ibx-\ldots\right)}{4d^2}$ |

input
```
int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*b^2*(-2*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d+2*(-2*Si(-2*x*b-2*a
-2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*x*b+2*a+2*(-a*d+b*c)/d)*cos(
2*(-a*d+b*c)/d)/d)/d)-1/32*b^2*(-4*sin(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))/d+4
*(-4*Si(-4*x*b-4*a-4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4*x*b+4*a+4*
(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d)/d))
```

### 3.28.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.08

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{(bdx+bc)\cos\left(-\frac{4(bc-ad)}{d}\right)\operatorname{Ci}\left(\frac{4(bdx+bc)}{d}\right)-(bdx+bc)\cos\left(-\frac{2(bc-ad)}{d}\right)\operatorname{Ci}\left(\frac{2(bdx+bc)}{d}\right)-(bdx+bc)\sin\left(\right.}{}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*((b*d*x + b*c)*cos(-4*(b*c - a*d)/d)*cos_integral(4*(b*d*x + b*c)/d)
- (b*d*x + b*c)*cos(-2*(b*c - a*d)/d)*cos_integral(2*(b*d*x + b*c)/d) - (b
*d*x + b*c)*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + (b*d*x
 + b*c)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) - 2*(d*cos(b
*x + a)^3 - d*cos(b*x + a))*sin(b*x + a))/(d^3*x + c*d^2)
```

### 3.28.6 Sympy [F]

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin^3(a+bx)\cos(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x)**2, x)
```

### 3.28.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.72

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = -\frac{2\,b^2\left(-i\,E_2\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_2\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^2\left(-i\,E_2\left(\frac{4\,(-i\,bc-i\,(\right.\right.}{}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/16*(2*b^2*(-I*exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 I*exp_integral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c -
 a*d)/d) - b^2*(-I*exp_integral_e(2, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d)
 + I*exp_integral_e(2, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c
 - a*d)/d) + 2*b^2*(exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d
) + exp_integral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c
- a*d)/d) - b^2*(exp_integral_e(2, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 exp_integral_e(2, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a
*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

### 3.28.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.66 (sec) , antiderivative size = 63510, normalized size of antiderivative = 354.80

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/4*(b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)
^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - b*d*x*real_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(
2*b*c/d)^2*tan(b*c/d)^2 - b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*
tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 +
b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*ta
n(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*b*d*x*imag_part(cos_inte
gral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b
*c/d)^2*tan(b*c/d) - 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 4*b*d
*x*sin_integral(2*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(
a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 2*b*d*x*imag_part(cos_integral(4*b*x + 4*
b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d
)^2 + 2*b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b
*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 4*b*d*x*sin_integral
(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/
d)*tan(b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b
*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*b*d*x*i
mag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^
2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*b*d*x*sin_integral(2*(b*d*x + ...
```

### 3.28.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(a+b\,x)\,\sin(a+b\,x)^3}{(c+d\,x)^2}\,dx$$

input
```
int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^2,x)
```

output
```
int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^2, x)
```

# 3.29 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$

3.29.1 Optimal result . . . 336
3.29.2 Mathematica [A] (verified) . . . 337
3.29.3 Rubi [A] (verified) . . . 337
3.29.4 Maple [A] (verified) . . . 338
3.29.5 Fricas [A] (verification not implemented) . . . 339
3.29.6 Sympy [F] . . . 339
3.29.7 Maxima [C] (verification not implemented) . . . 340
3.29.8 Giac [C] (verification not implemented) . . . 340
3.29.9 Mupad [F(-1)] . . . 341

## 3.29.1 Optimal result

Integrand size = 22, antiderivative size = 229

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = -\frac{b\cos(2a+2bx)}{4d^2(c+dx)}+\frac{b\cos(4a+4bx)}{4d^2(c+dx)} + \frac{b^2\operatorname{CosIntegral}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{d^3} - \frac{b^2\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d^3}-\frac{\sin(2a+2bx)}{8d(c+dx)^2} + \frac{\sin(4a+4bx)}{16d(c+dx)^2}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^3} + \frac{b^2\cos\left(4a-\frac{4bc}{d}\right)\operatorname{Si}\left(\frac{4bc}{d}+4bx\right)}{d^3}$$

output
```
-1/4*b*cos(2*b*x+2*a)/d^2/(d*x+c)+1/4*b*cos(4*b*x+4*a)/d^2/(d*x+c)-1/2*b^2
*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^3+b^2*cos(4*a-4*b*c/d)*Si(4*b*c/d+4*
b*x)/d^3+b^2*Ci(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^3-1/2*b^2*Ci(2*b*c/d+2*b
*x)*sin(2*a-2*b*c/d)/d^3-1/8*sin(2*b*x+2*a)/d/(d*x+c)^2+1/16*sin(4*b*x+4*a
)/d/(d*x+c)^2
```

### 3.29.2 Mathematica [A] (verified)

Time = 3.12 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.87

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{16b^2 \operatorname{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)\sin\left(4a-\frac{4bc}{d}\right)+\frac{d(4b(c+dx)\cos(4(a+bx))+d\sin(4(a+bx)))}{(c+dx)^2}-2\left(4b^2 \operatorname{CosIntegral}\left(\frac{2b(c+dx)}{d}\right.\right.}{}$$

input
```
Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

output
```
(16*b^2*CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + (d*(4*b*(c +
 d*x)*Cos[4*(a + b*x)] + d*Sin[4*(a + b*x)]))/(c + d*x)^2 - 2*(4*b^2*CosIn
tegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (d*(2*b*(c + d*x)*Cos[2*(
a + b*x)] + d*Sin[2*(a + b*x)]))/(c + d*x)^2 + 4*b^2*Cos[2*a - (2*b*c)/d]*
SinIntegral[(2*b*(c + d*x))/d]) + 16*b^2*Cos[4*a - (4*b*c)/d]*SinIntegral[
(4*b*(c + d*x))/d])/(16*d^3)
```

### 3.29.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)\cos(a+bx)}{(c+dx)^3}\,dx$$
$$\Big\downarrow \; 4906$$
$$\int \left(\frac{\sin(2a+2bx)}{4(c+dx)^3}-\frac{\sin(4a+4bx)}{8(c+dx)^3}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\operatorname{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^3}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^3}+\frac{b^2\cos\left(4a-\frac{4bc}{d}\right)\operatorname{Si}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b\cos(2a+2bx)}{4d^2(c+dx)}+\frac{b\cos(4a+4bx)}{4d^2(c+dx)}-\frac{\sin(2a+2bx)}{8d(c+dx)^2}+\frac{\sin(4a+4bx)}{16d(c+dx)^2}$$

input
```
Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

output
```
-1/4*(b*Cos[2*a + 2*b*x])/(d^2*(c + d*x)) + (b*Cos[4*a + 4*b*x])/(4*d^2*(c
 + d*x)) + (b^2*CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/d^3 -
 (b^2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d^3) - Sin[2
*a + 2*b*x]/(8*d*(c + d*x)^2) + Sin[4*a + 4*b*x]/(16*d*(c + d*x)^2) - (b^2
*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^3) + (b^2*Cos[4
*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/d^3
```

### 3.29.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.29.4 Maple [A] (verified)

Time = 1.64 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.44

| method | result |
|---|---|
| derivativedivides | $\frac{b^3\left(-\frac{\sin(2xb+2a)}{(-ad+cb+d(xb+a))^2 d}+\frac{-\frac{2\cos(2xb+2a)}{(-ad+cb+d(xb+a))d}-\frac{2\left(-\frac{2\,\mathrm{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2xb+2a+\frac{-2ad}{d}\right.}{}\right.}{d}}{d}\right.}{8}$ |
| default | $\frac{b^3\left(-\frac{\sin(2xb+2a)}{(-ad+cb+d(xb+a))^2 d}+\frac{-\frac{2\cos(2xb+2a)}{(-ad+cb+d(xb+a))d}-\frac{2\left(-\frac{2\,\mathrm{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2xb+2a+\frac{-2ad}{d}\right.}{}\right.}{d}}{d}\right.}{8}$ |
| risch | $\frac{ib^2 e^{-\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{4d^3}-\frac{ib^2 e^{-\frac{4i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{2d^3}-\frac{ib^2 e^{\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(-2ib\right.}{4d^3}$ |

input
```
int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*b^3*(-sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^2/d+(-2*cos(2*b*x+2*a)/
(-a*d+c*b+d*(b*x+a))/d-2*(-2*Si(-2*x*b-2*a-2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c
)/d)/d-2*Ci(2*x*b+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d)-1/32*b^
3*(-2*sin(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))^2/d+2*(-4*cos(4*b*x+4*a)/(-a*d+c
*b+d*(b*x+a))/d-4*(-4*Si(-4*x*b-4*a-4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-
4*Ci(4*x*b+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)/d)/d))
```

### 3.29.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.46

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{4\left(bd^2x+bcd\right)\cos\left(bx+a\right)^4+bd^2x+bcd-5\left(bd^2x+bcd\right)\cos\left(bx+a\right)^2-\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\operatorname{Ci}\left(\frac{2}{}\right.}{}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(4*(b*d^2*x + b*c*d)*cos(b*x + a)^4 + b*d^2*x + b*c*d - 5*(b*d^2*x + b
*c*d)*cos(b*x + a)^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(
2*(b*d*x + b*c)/d)*sin(-2*(b*c - a*d)/d) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*cos_integral(4*(b*d*x + b*c)/d)*sin(-4*(b*c - a*d)/d) + 2*(b^2*d^
2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x
 + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-2*(b*c - a*d)/d)*s
in_integral(2*(b*d*x + b*c)/d) + (d^2*cos(b*x + a)^3 - d^2*cos(b*x + a))*s
in(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

### 3.29.6 Sympy [F]

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin^3\left(a+bx\right)\cos\left(a+bx\right)}{\left(c+dx\right)^3}\,dx$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c)**3,x)
```

output
```
Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x)**3, x)
```

### 3.29.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.50

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx =$$
$$-\frac{2\,b^3\left(-i\,E_3\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_3\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^3\left(-i\,E_3\left(\frac{4\,(-i\,bc-i\,\cdots}{}\right.\right.}{}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/16*(2*b^3*(-I*exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 I*exp_integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c -
 a*d)/d) - b^3*(-I*exp_integral_e(3, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d)
 + I*exp_integral_e(3, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c
 - a*d)/d) + 2*b^3*(exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d
) + exp_integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c
- a*d)/d) - b^3*(exp_integral_e(3, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 exp_integral_e(3, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a
*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2
 - a*d^3)*(b*x + a))*b)
```

### 3.29.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.34 (sec) , antiderivative size = 111694, normalized size of antiderivative = 487.75

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/8*(4*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*t
an(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*b^2*d^2*x^2*
imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^
2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*imag_part(cos_integ
ral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b
*c/d)^2*tan(b*c/d)^2 - 4*b^2*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c
/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)
^2 + 8*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2
*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*b^2*d^2*x^2*sin_integ
ral(2*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b
*c/d)^2*tan(b*c/d)^2 - 4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/
d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)
- 4*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan
(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 8*b^2*d^2*x^2*real
_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*ta
n(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 8*b^2*d^2*x^2*real_part(cos_integral(-4
*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*
tan(b*c/d)^2 + 4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(
2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*b^2*
d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)...
```

### 3.29.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^3}{{\left(c+d\,x\right)}^3}\,dx$$

input
```
int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^3,x)
```

output
```
int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^3, x)
```

# 3.30 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$

3.30.1 Optimal result . . . 342
3.30.2 Mathematica [A] (verified) . . . 343
3.30.3 Rubi [A] (verified) . . . 343
3.30.4 Maple [A] (verified) . . . 345
3.30.5 Fricas [A] (verification not implemented) . . . 346
3.30.6 Sympy [F] . . . 346
3.30.7 Maxima [C] (verification not implemented) . . . 347
3.30.8 Giac [C] (verification not implemented) . . . 347
3.30.9 Mupad [F(-1)] . . . 348

## 3.30.1 Optimal result

Integrand size = 22, antiderivative size = 287

$$\begin{aligned}\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = &-\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2}+\frac{b\cos(4a+4bx)}{12d^2(c+dx)^2}\\ &-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}\\ &+\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4}\\ &-\frac{\sin(2a+2bx)}{12d(c+dx)^3}+\frac{b^2\sin(2a+2bx)}{6d^3(c+dx)}+\frac{\sin(4a+4bx)}{24d(c+dx)^3}\\ &-\frac{b^2\sin(4a+4bx)}{3d^3(c+dx)}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}\\ &-\frac{4b^3\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{3d^4}\end{aligned}$$

output
```
4/3*b^3*Ci(4*b*c/d+4*b*x)*cos(4*a-4*b*c/d)/d^4-1/3*b^3*Ci(2*b*c/d+2*b*x)*c
os(2*a-2*b*c/d)/d^4-1/12*b*cos(2*b*x+2*a)/d^2/(d*x+c)^2+1/12*b*cos(4*b*x+4
*a)/d^2/(d*x+c)^2-4/3*b^3*Si(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^4+1/3*b^3*S
i(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^4-1/12*sin(2*b*x+2*a)/d/(d*x+c)^3+1/6*
b^2*sin(2*b*x+2*a)/d^3/(d*x+c)+1/24*sin(4*b*x+4*a)/d/(d*x+c)^3-1/3*b^2*sin
(4*b*x+4*a)/d^3/(d*x+c)
```

### 3.30.2 Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.10

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$$
$$= \frac{-2d\cos(2bx)\left(bd(c+dx)\cos(2a)+\left(d^2-2b^2(c+dx)^2\right)\sin(2a)\right)+d\cos(4bx)\left(2bd(c+dx)\cos(4a)+\left(d^2-\right.\right.}{}$$

input
```
Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

output
```
(-2*d*Cos[2*b*x]*(b*d*(c + d*x)*Cos[2*a] + (d^2 - 2*b^2*(c + d*x)^2)*Sin[2
*a]) + d*Cos[4*b*x]*(2*b*d*(c + d*x)*Cos[4*a] + (d^2 - 8*b^2*(c + d*x)^2)*
Sin[4*a]) + 2*d*((-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*a] + b*d*(c + d*x)*Sin[2
*a])*Sin[2*b*x] - d*((-d^2 + 8*b^2*(c + d*x)^2)*Cos[4*a] + 2*b*d*(c + d*x)
*Sin[4*a])*Sin[4*b*x] - 8*b^3*(c + d*x)^3*(Cos[2*a - (2*b*c)/d]*CosIntegra
l[(2*b*(c + d*x))/d] - Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d]
) + 32*b^3*(c + d*x)^3*(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d
] - Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d]))/(24*d^4*(c + d*x
)^3)
```

### 3.30.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)\cos(a+bx)}{(c+dx)^4}\,dx$$
$$\Big\downarrow \text{4906}$$
$$\int \left(\frac{\sin(2a+2bx)}{4(c+dx)^4}-\frac{\sin(4a+4bx)}{8(c+dx)^4}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{4b^3\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{3d^4}+\frac{b^2\sin(2a+2bx)}{6d^3(c+dx)}-\frac{b^2\sin(4a+4bx)}{3d^3(c+dx)}-\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2}+\frac{b\cos(4a+4bx)}{12d^2(c+dx)^2}-\frac{\sin(2a+2bx)}{12d(c+dx)^3}+\frac{\sin(4a+4bx)}{24d(c+dx)^3}$$

input
```
Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

output
```
-1/12*(b*Cos[2*a + 2*b*x])/(d^2*(c + d*x)^2) + (b*Cos[4*a + 4*b*x])/(12*d^
2*(c + d*x)^2) - (b^3*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])
/(3*d^4) + (4*b^3*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(3*
d^4) - Sin[2*a + 2*b*x]/(12*d*(c + d*x)^3) + (b^2*Sin[2*a + 2*b*x])/(6*d^3
*(c + d*x)) + Sin[4*a + 4*b*x]/(24*d*(c + d*x)^3) - (b^2*Sin[4*a + 4*b*x])
/(3*d^3*(c + d*x)) + (b^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b
*x])/(3*d^4) - (4*b^3*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])
/(3*d^4)
```

#### 3.30.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.30.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.41

| method | result |
|---|---|
| derivativedivides | $\frac{b^4\left(-\frac{2\sin(2xb+2a)}{3(-ad+cb+d(xb+a))^3 d}+\frac{-\frac{2\cos(2xb+2a)}{3(-ad+cb+d(xb+a))^2 d}-\frac{2\left(-\frac{2\sin(2xb+2a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{4\,\mathrm{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\sin\left(\frac{-2ad+}{d}\right.}{d}}{3d}\right)}{d}\right)}{8}$ |
| default | $\frac{b^4\left(-\frac{2\sin(2xb+2a)}{3(-ad+cb+d(xb+a))^3 d}+\frac{-\frac{2\cos(2xb+2a)}{3(-ad+cb+d(xb+a))^2 d}-\frac{2\left(-\frac{2\sin(2xb+2a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{4\,\mathrm{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\sin\left(\frac{-2ad+}{d}\right.}{d}}{3d}\right)}{d}\right)}{8}$ |
| risch | $\frac{b^3 e^{-\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{6d^4}-\frac{2b^3 e^{-\frac{4i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{3d^4}+\frac{b^3 e^{\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(-2ibx\right.}{6d^4}$ |

input
```
int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*b^4*(-2/3*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^3/d+2/3*(-cos(2*b*x
+2*a)/(-a*d+c*b+d*(b*x+a))^2/d-(-2*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d+2
*(-2*Si(-2*x*b-2*a-2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*x*b+2*a+2*
(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)/d)/d)-1/32*b^4*(-4/3*sin(4*b*x+4*a
)/(-a*d+c*b+d*(b*x+a))^3/d+4/3*(-2*cos(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))^2/d
-2*(-4*sin(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))/d+4*(-4*Si(-4*x*b-4*a-4*(-a*d+b
*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4*x*b+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*
c)/d)/d)/d)/d)/d))
```

### 3.30.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.65

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$$
$$= \frac{bd^3x + 4\,(bd^3x + bcd^2)\cos{(bx+a)}^4 + bcd^2 - 5\,(bd^3x + bcd^2)\cos{(bx+a)}^2 + 8\,(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c}{}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/6*(b*d^3*x + 4*(b*d^3*x + b*c*d^2)*cos(b*x + a)^4 + b*c*d^2 - 5*(b*d^3*x
 + b*c*d^2)*cos(b*x + a)^2 + 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*
d*x + b^3*c^3)*cos(-4*(b*c - a*d)/d)*cos_integral(4*(b*d*x + b*c)/d) - 2*(
b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-2*(b*c - a*d
)/d)*cos_integral(2*(b*d*x + b*c)/d) - 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
3*b^3*c^2*d*x + b^3*c^3)*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c
)/d) + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-2*
(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) - 2*((8*b^2*d^3*x^2 + 16*b^
2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^3 - (5*b^2*d^3*x^2 + 10*b^2*c*
d^2*x + 5*b^2*c^2*d - d^3)*cos(b*x + a))*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*
x^2 + 3*c^2*d^5*x + c^3*d^4)
```

### 3.30.6 Sympy [F]

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sin^3{(a+bx)}\cos{(a+bx)}}{(c+dx)^4}\,dx$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c)**4,x)
```

output
```
Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x)**4, x)
```

### 3.30.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.37

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx =$$

$$-\frac{2\,b^4\left(-i\,E_4\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_4\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^4\left(-i\,E_4\left(\frac{4\,(-i\,bc-i\,(}{}\right.\right.}{16}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/16*(2*b^4*(-I*exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 I*exp_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c -
 a*d)/d) - b^4*(-I*exp_integral_e(4, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d)
 + I*exp_integral_e(4, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c
 - a*d)/d) + 2*b^4*(exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d
) + exp_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c
- a*d)/d) - b^4*(exp_integral_e(4, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 exp_integral_e(4, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a
*d)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 -
a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 +
 a^2*d^4)*(b*x + a))*b)
```

### 3.30.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.53 (sec) , antiderivative size = 157526, normalized size of antiderivative = 548.87

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")
```

output
```
1/12*(8*b^3*d^3*x^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*
tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*b^3*d^3*x^3
*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)
^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*b^3*d^3*x^3*real_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*
b*c/d)^2*tan(b*c/d)^2 + 8*b^3*d^3*x^3*real_part(cos_integral(-4*b*x - 4*b*
c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d
)^2 + 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*
tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 4*b^3*d^3*x^3*i
mag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^
2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x
 + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(
b*c/d) - 16*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x
)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 16*b^3*d^3*
x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(
2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 32*b^3*d^3*x^3*sin_integral(4*
(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*
tan(b*c/d)^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(
2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*b^3*
d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)...
```

### 3.30.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^3}{{\left(c+d\,x\right)}^4}\,dx$$

input
```
int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^4,x)
```

output
```
int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^4, x)
```

# 3.31 $\int (c+dx)^m \cot(a+bx)\,dx$

3.31.1 Optimal result . . . 349
3.31.2 Mathematica [N/A] . . . 349
3.31.3 Rubi [N/A] . . . 350
3.31.4 Maple [N/A] (verified) . . . 351
3.31.5 Fricas [N/A] . . . 351
3.31.6 Sympy [N/A] . . . 352
3.31.7 Maxima [N/A] . . . 352
3.31.8 Giac [N/A] . . . 352
3.31.9 Mupad [N/A] . . . 353

## 3.31.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (c+dx)^m \cot(a+bx)\,dx = \mathrm{Int}((c+dx)^m \cot(a+bx),x)$$

output
```
Unintegrable((d*x+c)^m*cot(b*x+a),x)
```

## 3.31.2 Mathematica [N/A]

Not integrable

Time = 3.55 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \cot(a+bx)\,dx = \int (c+dx)^m \cot(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Cot[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Cot[a + b*x], x]
```

### 3.31.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 25, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan\left(a+bx+\frac{\pi}{2}\right)(-(c+dx)^m)\,dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)^m \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\Big\downarrow \text{4222}$$

$$-\int (c+dx)^m \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

input
```
Int[(c + d*x)^m*Cot[a + b*x],x]
```

output
```
$Aborted
```

#### 3.31.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

### 3.31.4 Maple [N/A] (verified)

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int (dx+c)^{m} \cos (xb+a) \csc (xb+a) \, dx$$

input
```
int((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x)
```

output
```
int((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x)
```

### 3.31.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int (c+dx)^m \cot(a+bx) \, dx = \int (dx+c)^{m} \cos (bx+a) \csc (bx+a) \, dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*cos(b*x + a)*csc(b*x + a), x)
```

### 3.31.6 Sympy [N/A]

Not integrable

Time = 18.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int (c+dx)^m \cot(a+bx)\,dx = \int (c+dx)^m \cos(a+bx)\csc(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*cos(b*x+a)*csc(b*x+a),x)
```

output
```
Integral((c + d*x)**m*cos(a + b*x)*csc(a + b*x), x)
```

### 3.31.7 Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int (c+dx)^m \cot(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\csc(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a), x)
```

### 3.31.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int (c+dx)^m \cot(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\csc(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a), x)
```

### 3.31.9 Mupad [N/A]

Not integrable

Time = 23.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int (c+dx)^m \cot(a+bx)\,dx = \int \frac{\cos(a+b\,x)\,\left(c+d\,x\right)^m}{\sin(a+b\,x)}\,dx$$

input
```
int((cos(a + b*x)*(c + d*x)^m)/sin(a + b*x),x)
```

output
```
int((cos(a + b*x)*(c + d*x)^m)/sin(a + b*x), x)
```

# 3.32 $\int (c + dx)^4 \cot(a + bx)\, dx$


3.32.1 Optimal result . . . 354
3.32.2 Mathematica [B] (verified) . . . 355
3.32.3 Rubi [A] (verified) . . . 357
3.32.4 Maple [B] (verified) . . . 362
3.32.5 Fricas [B] (verification not implemented) . . . 363
3.32.6 Sympy [F] . . . 363
3.32.7 Maxima [B] (verification not implemented) . . . 364
3.32.8 Giac [F] . . . 364
3.32.9 Mupad [F(-1)] . . . 365


## 3.32.1 Optimal result

Integrand size = 14, antiderivative size = 151

$$\begin{aligned}\int (c+dx)^4 \cot(a+bx)\,dx = &-\frac{i(c+dx)^5}{5d}+\frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b}\\ &-\frac{2id(c+dx)^3\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}\\ &+\frac{3d^2(c+dx)^2\operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3}\\ &+\frac{3id^3(c+dx)\operatorname{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4}\\ &-\frac{3d^4\operatorname{PolyLog}\left(5,e^{2i(a+bx)}\right)}{2b^5}\end{aligned}$$

output
```
-1/5*I*(d*x+c)^5/d+(d*x+c)^4*ln(1-exp(2*I*(b*x+a)))/b-2*I*d*(d*x+c)^3*poly
log(2,exp(2*I*(b*x+a)))/b^2+3*d^2*(d*x+c)^2*polylog(3,exp(2*I*(b*x+a)))/b^
3+3*I*d^3*(d*x+c)*polylog(4,exp(2*I*(b*x+a)))/b^4-3/2*d^4*polylog(5,exp(2*
I*(b*x+a)))/b^5
```

### 3.32.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 799 vs. $2(151) = 302$.

Time = 6.27 (sec) , antiderivative size = 799, normalized size of antiderivative = 5.29

$$\int (c+dx)^4\cot(a+bx)\,dx = \frac{2ic^3d\pi x}{b}+2ic^2d^2x^3+icd^3x^4+\frac{1}{5}id^4x^5 - \frac{4ic^3dx\arctan(\tan(a))}{b}+2c^3dx^2\cot(a) + \frac{2c^3d\pi\log\left(1+e^{-2ibx}\right)}{b^2}+\frac{6c^2d^2x^2\log\left(1-e^{-i(a+bx)}\right)}{b} + \frac{4cd^3x^3\log\left(1-e^{-i(a+bx)}\right)}{b}+\frac{d^4x^4\log\left(1-e^{-i(a+bx)}\right)}{b} + \frac{6c^2d^2x^2\log\left(1+e^{-i(a+bx)}\right)}{b}+\frac{4cd^3x^3\log\left(1+e^{-i(a+bx)}\right)}{b} + \frac{d^4x^4\log\left(1+e^{-i(a+bx)}\right)}{b}+\frac{4c^3dx\log\left(1-e^{2i(bx+\arctan(\tan(a)))}\right)}{b} + \frac{4c^3d\arctan(\tan(a))\log\left(1-e^{2i(bx+\arctan(\tan(a)))}\right)}{b^2} - \frac{2c^3d\pi\log(\cos(bx))}{b^2}+\frac{c^4\log(\sin(a+bx))}{b} - \frac{4c^3d\arctan(\tan(a))\log(\sin(bx+\arctan(\tan(a))))}{b^2} + \frac{4id^2x(3c^2+3cdx+d^2x^2)\operatorname{PolyLog}\left(2,-e^{-i(a+bx)}\right)}{b^2} + \frac{4id^2x(3c^2+3cdx+d^2x^2)\operatorname{PolyLog}\left(2,e^{-i(a+bx)}\right)}{b^2} - \frac{2ic^3d\operatorname{PolyLog}\left(2,e^{2i(bx+\arctan(\tan(a)))}\right)}{b^2} + \frac{12c^2d^2\operatorname{PolyLog}\left(3,-e^{-i(a+bx)}\right)}{b^3} + \frac{24cd^3x\operatorname{PolyLog}\left(3,-e^{-i(a+bx)}\right)}{b^3} + \frac{12d^4x^2\operatorname{PolyLog}\left(3,-e^{-i(a+bx)}\right)}{b^3} + \frac{12c^2d^2\operatorname{PolyLog}\left(3,e^{-i(a+bx)}\right)}{b^3}+\frac{24cd^3x\operatorname{PolyLog}\left(3,e^{-i(a+bx)}\right)}{b^3} + \frac{12d^4x^2\operatorname{PolyLog}\left(3,e^{-i(a+bx)}\right)}{b^3} - \frac{24icd^3\operatorname{PolyLog}\left(4,-e^{-i(a+bx)}\right)}{b^4} - \frac{24id^4x\operatorname{PolyLog}\left(4,-e^{-i(a+bx)}\right)}{b^4} - \frac{24icd^3\operatorname{PolyLog}\left(4,e^{-i(a+bx)}\right)}{b^4}-\frac{24id^4x\operatorname{PolyLog}\left(4,e^{-i(a+bx)}\right)}{b^4} - \frac{24d^4\operatorname{PolyLog}\left(5,-e^{-i(a+bx)}\right)}{b^5}-\frac{24d^4\operatorname{PolyLog}\left(5,e^{-i(a+bx)}\right)}{b^5} - 2c^3de^{i\arctan(\tan(a))}x^2\cot(a)\sqrt{\sec^2(a)}$$

3.32. $\int (c+dx)^4\cot(a+bx)\,dx$

input
```
Integrate[(c + d*x)^4*Cot[a + b*x],x]
```

output
```
((2*I)*c^3*d*Pi*x)/b + (2*I)*c^2*d^2*x^3 + I*c*d^3*x^4 + (I/5)*d^4*x^5 - (
(4*I)*c^3*d*x*ArcTan[Tan[a]])/b + 2*c^3*d*x^2*Cot[a] + (2*c^3*d*Pi*Log[1 +
 E^((-2*I)*b*x)])/b^2 + (6*c^2*d^2*x^2*Log[1 - E^((-I)*(a + b*x))])/b + (4
*c*d^3*x^3*Log[1 - E^((-I)*(a + b*x))])/b + (d^4*x^4*Log[1 - E^((-I)*(a +
b*x))])/b + (6*c^2*d^2*x^2*Log[1 + E^((-I)*(a + b*x))])/b + (4*c*d^3*x^3*L
og[1 + E^((-I)*(a + b*x))])/b + (d^4*x^4*Log[1 + E^((-I)*(a + b*x))])/b +
(4*c^3*d*x*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))])/b + (4*c^3*d*ArcTan[
Tan[a]]*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))])/b^2 - (2*c^3*d*Pi*Log[C
os[b*x]])/b^2 + (c^4*Log[Sin[a + b*x]])/b - (4*c^3*d*ArcTan[Tan[a]]*Log[Si
n[b*x + ArcTan[Tan[a]]]])/b^2 + ((4*I)*d^2*x*(3*c^2 + 3*c*d*x + d^2*x^2)*P
olyLog[2, -E^((-I)*(a + b*x))])/b^2 + ((4*I)*d^2*x*(3*c^2 + 3*c*d*x + d^2*
x^2)*PolyLog[2, E^((-I)*(a + b*x))])/b^2 - ((2*I)*c^3*d*PolyLog[2, E^((2*I
)*(b*x + ArcTan[Tan[a]]))])/b^2 + (12*c^2*d^2*PolyLog[3, -E^((-I)*(a + b*x
))])/b^3 + (24*c*d^3*x*PolyLog[3, -E^((-I)*(a + b*x))])/b^3 + (12*d^4*x^2*
PolyLog[3, -E^((-I)*(a + b*x))])/b^3 + (12*c^2*d^2*PolyLog[3, E^((-I)*(a +
 b*x))])/b^3 + (24*c*d^3*x*PolyLog[3, E^((-I)*(a + b*x))])/b^3 + (12*d^4*x
^2*PolyLog[3, E^((-I)*(a + b*x))])/b^3 - ((24*I)*c*d^3*PolyLog[4, -E^((-I)
*(a + b*x))])/b^4 - ((24*I)*d^4*x*PolyLog[4, -E^((-I)*(a + b*x))])/b^4 - (
(24*I)*c*d^3*PolyLog[4, E^((-I)*(a + b*x))])/b^4 - ((24*I)*d^4*x*PolyLog[4
, E^((-I)*(a + b*x))])/b^4 - (24*d^4*PolyLog[5, -E^((-I)*(a + b*x))])/b...
```

### 3.32.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.42, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3042, 25, 4202, 2620, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \cot(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -(c+dx)^4 \tan\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \; 25$$

$$-\int (c+dx)^4 \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\Big\downarrow \text{4202}$$

$$2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)^4}{1+e^{i(2a+2bx+\pi)}}dx-\frac{i(c+dx)^5}{5d}$$

$$\Big\downarrow \text{2620}$$

$$2i\left(\frac{2id\int (c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{i(c+dx)^5}{5d}$$

$$\Big\downarrow \text{3011}$$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int (c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$

$$\frac{i(c+dx)^5}{5d}$$

$$\Big\downarrow \text{7163}$$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\left(\frac{id\int (c+dx)\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)dx}{b}-\frac{i(c+dx)^2\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{2b}\right)}{b}-\frac{i(\cdots}{}\right.$$

$$\frac{i(c+dx)^5}{5d}$$

$$\Big\downarrow \text{7163}$$

3.32. $\int (c+dx)^4\cot(a+bx)\,dx$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\left(\frac{id\left(\frac{id\int\operatorname{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}-\frac{i(c+dx)^2\operatorname{Pol}}{}\right.}{2b}\right.}{b}\right.$$

$$\frac{i(c+dx)^5}{5d}$$

$\downarrow$ 2720

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\left(\frac{id\left(\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,-e^{i(2}\right.}{2b}\right.}{b}\right.}{2b}\right.}{b}\right.$$

$$\frac{i(c+dx)^5}{5d}$$

$\downarrow$ 7143

## 3.32. $\int(c+dx)^4\cot(a+bx)\,dx$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\left(\frac{id\left(\frac{d\operatorname{PolyLog}\left(5,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}-\frac{i(c+dx)^2\operatorname{PolyLog}}{}}{2b}\right.}{b}\right.$$

$$\frac{i(c+dx)^5}{5d}$$

input
```
Int[(c + d*x)^4*Cot[a + b*x],x]
```

output
```
((-1/5*I)*(c + d*x)^5)/d + (2*I)*(((-1/2*I)*(c + d*x)^4*Log[1 + E^(I*(2*a
+ Pi + 2*b*x))])/b + ((2*I)*d*(((I/2)*(c + d*x)^3*PolyLog[2, -E^(I*(2*a +
Pi + 2*b*x))])/b - (((3*I)/2)*d*(((-1/2*I)*(c + d*x)^2*PolyLog[3, -E^(I*(2
*a + Pi + 2*b*x))])/b + (I*d*(((-1/2*I)*(c + d*x)*PolyLog[4, -E^(I*(2*a +
Pi + 2*b*x))])/b + (d*PolyLog[5, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2)))/b))
/b))/b)
```

### 3.32.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

## 3.32. $\int (c+dx)^4 \cot(a+bx)\,dx$

### 3.32.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1158 vs. 2(134) = 268.

Time = 1.21 (sec) , antiderivative size = 1159, normalized size of antiderivative = 7.68

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1159 |

input
```
int((d*x+c)^4*cos(b*x+a)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
12*I/b^2*d^2*c^2*a^2*x-12*I/b^2*d^2*c^2*polylog(2,exp(I*(b*x+a)))*x-12*I/b
^2*d^2*c^2*polylog(2,-exp(I*(b*x+a)))*x-8*I/b^3*d^3*c*a^3*x-12*I/b^2*d^3*c
*polylog(2,-exp(I*(b*x+a)))*x^2-12*I/b^2*d^3*c*polylog(2,exp(I*(b*x+a)))*x
^2+I*c^4*x+1/5*I/d*c^5-I*d^3*c*x^4-2*I*d^2*c^2*x^3-2*I*d*c^3*x^2-4/b^2*c^3
*d*a*ln(exp(I*(b*x+a))-1)+4/b^2*d*c^3*ln(1-exp(I*(b*x+a)))*a+4/b^4*d^3*c*l
n(1-exp(I*(b*x+a)))*a^3-12/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a)))+6/b^3*c^2*d^
2*a^2*ln(exp(I*(b*x+a))-1)+8/b^4*c*d^3*a^3*ln(exp(I*(b*x+a)))-4/b^4*c*d^3*
a^3*ln(exp(I*(b*x+a))-1)+4/b*d^3*c*ln(1-exp(I*(b*x+a)))*x^3+8/5*I/b^5*d^4*
a^5+2*I/b^4*d^4*a^4*x+8*I/b^3*d^2*c^2*a^3+24*I/b^4*d^4*polylog(4,exp(I*(b*
x+a)))*x-4*I/b^2*d*c^3*a^2-4*I/b^2*d*c^3*polylog(2,exp(I*(b*x+a)))-6*I/b^4
*d^3*c*a^4+24*I/b^4*d^3*c*polylog(4,exp(I*(b*x+a)))+24*I/b^4*d^3*c*polylog
(4,-exp(I*(b*x+a)))-4*I/b^2*d^4*polylog(2,exp(I*(b*x+a)))*x^3-4*I/b^2*d^4*
polylog(2,-exp(I*(b*x+a)))*x^3-4*I/b^2*d*c^3*polylog(2,-exp(I*(b*x+a)))+24
*I/b^4*d^4*polylog(4,-exp(I*(b*x+a)))*x+1/b*c^4*ln(exp(I*(b*x+a))+1)-2/b*c
^4*ln(exp(I*(b*x+a)))+1/b*c^4*ln(exp(I*(b*x+a))-1)+12/b^3*d^2*c^2*polylog(
3,exp(I*(b*x+a)))+12/b^3*d^2*c^2*polylog(3,-exp(I*(b*x+a)))+1/b*d^4*ln(1-e
xp(I*(b*x+a)))*x^4+1/b*d^4*ln(exp(I*(b*x+a))+1)*x^4+12/b^3*d^4*polylog(3,e
xp(I*(b*x+a)))*x^2+12/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2-2/b^5*d^4*a^4
*ln(exp(I*(b*x+a)))+1/b^5*d^4*a^4*ln(exp(I*(b*x+a))-1)-1/b^5*d^4*ln(1-exp(
I*(b*x+a)))*a^4-1/5*I*d^4*x^5+8/b^2*c^3*d*a*ln(exp(I*(b*x+a)))+6/b*d^2*...
```

### 3.32.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1204 vs. $2(130) = 260$.

Time = 0.31 (sec) , antiderivative size = 1204, normalized size of antiderivative = 7.97

$$\int (c+dx)^4 \cot(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(24*d^4*polylog(5, cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*polylog(5,
 cos(b*x + a) - I*sin(b*x + a)) + 24*d^4*polylog(5, -cos(b*x + a) + I*sin(
b*x + a)) + 24*d^4*polylog(5, -cos(b*x + a) - I*sin(b*x + a)) + 4*(I*b^3*d
^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(cos(b*
x + a) + I*sin(b*x + a)) + 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3
*c^2*d^2*x - I*b^3*c^3*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) + 4*(-I*b^3
*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*d^2*x - I*b^3*c^3*d)*dilog(-cos
(b*x + a) + I*sin(b*x + a)) + 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b
^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^4*d
^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*lo
g(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*
b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(cos(b*x + a) - I*sin(b*x +
a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - (b^4*c^4 - 4*a
*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-1/2*cos(b*x
 + a) - 1/2*I*sin(b*x + a) + 1/2) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4
*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b
*c*d^3 - a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^4*d^4*x^4 +
 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a
^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-cos(b*x + a) - I*sin(b*x...
```

### 3.32.6 Sympy [F]

$$\int (c+dx)^4 \cot(a+bx) \, dx = \int (c+dx)^4 \cos(a+bx) \csc(a+bx) \, dx$$

input
```
integrate((d*x+c)**4*cos(b*x+a)*csc(b*x+a),x)
```

output
```
Integral((c + d*x)**4*cos(a + b*x)*csc(a + b*x), x)
```

### 3.32.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1281 vs. $2(130) = 260$.

Time = 0.41 (sec) , antiderivative size = 1281, normalized size of antiderivative = 8.48

$$\int (c+dx)^4 \cot(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

output
```
1/10*(10*c^4*log(sin(b*x + a)) - 40*a*c^3*d*log(sin(b*x + a))/b + 60*a^2*c
^2*d^2*log(sin(b*x + a))/b^2 - 40*a^3*c*d^3*log(sin(b*x + a))/b^3 + 10*a^4
*d^4*log(sin(b*x + a))/b^4 + (-2*I*(b*x + a)^5*d^4 - 10*(I*b*c*d^3 - I*a*d
^4)*(b*x + a)^4 - 240*d^4*polylog(5, -e^(I*b*x + I*a)) - 240*d^4*polylog(5
, e^(I*b*x + I*a)) - 20*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x +
 a)^3 - 20*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 - I*a^3*d^4)
*(b*x + a)^2 - 10*(-I*(b*x + a)^4*d^4 + 4*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)
^3 + 6*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x + a)^2 + 4*(-I*b^
3*c^3*d + 3*I*a*b^2*c^2*d^2 - 3*I*a^2*b*c*d^3 + I*a^3*d^4)*(b*x + a))*arct
an2(sin(b*x + a), cos(b*x + a) + 1) - 10*(I*(b*x + a)^4*d^4 + 4*(I*b*c*d^3
 - I*a*d^4)*(b*x + a)^3 + 6*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b
*x + a)^2 + 4*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 - I*a^3*d
^4)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 40*(I*b^3*c^3*d
- 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 + I*(b*x + a)^3*d^4 - I*a^3*d^4 + 3*
(I*b*c*d^3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a
^2*d^4)*(b*x + a))*dilog(-e^(I*b*x + I*a)) - 40*(I*b^3*c^3*d - 3*I*a*b^2*c
^2*d^2 + 3*I*a^2*b*c*d^3 + I*(b*x + a)^3*d^4 - I*a^3*d^4 + 3*(I*b*c*d^3 -
I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x
+ a))*dilog(e^(I*b*x + I*a)) + 5*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b
*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^...
```

### 3.32.8 Giac [F]

$$\int (c+dx)^4 \cot(a+bx)\,dx = \int (dx+c)^4 \cos(bx+a)\csc(bx+a)\,dx$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*cos(b*x + a)*csc(b*x + a), x)
```

### 3.32.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \cot(a+bx)\,dx = \int \frac{\cos(a+b\,x)\,\left(c+d\,x\right)^4}{\sin(a+b\,x)}\,dx$$

input
```
int((cos(a + b*x)*(c + d*x)^4)/sin(a + b*x),x)
```

output
```
int((cos(a + b*x)*(c + d*x)^4)/sin(a + b*x), x)
```

# 3.33 $\int (c+dx)^3 \cot(a+bx)\,dx$


3.33.1 Optimal result . . . 366
3.33.2 Mathematica [B] (verified) . . . 366
3.33.3 Rubi [A] (verified) . . . 367
3.33.4 Maple [B] (verified) . . . 370
3.33.5 Fricas [B] (verification not implemented) . . . 371
3.33.6 Sympy [F] . . . 372
3.33.7 Maxima [B] (verification not implemented) . . . 373
3.33.8 Giac [F] . . . 374
3.33.9 Mupad [F(-1)] . . . 374


## 3.33.1 Optimal result

Integrand size = 14, antiderivative size = 127

$$\int (c+dx)^3 \cot(a+bx)\,dx = -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3 \log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2} + \frac{3d^2(c+dx) \operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3} + \frac{3id^3 \operatorname{PolyLog}\left(4,e^{2i(a+bx)}\right)}{4b^4}$$

output
```
-1/4*I*(d*x+c)^4/d+(d*x+c)^3*ln(1-exp(2*I*(b*x+a)))/b-3/2*I*d*(d*x+c)^2*po
lylog(2,exp(2*I*(b*x+a)))/b^2+3/2*d^2*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/
b^3+3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^4
```

## 3.33.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 560 vs. 2(127) = 254.

Time = 2.86 (sec) , antiderivative size = 560, normalized size of antiderivative = 4.41

$$\int (c+dx)^3 \cot(a+bx)\,dx$$
$$= \frac{6ib^3c^2d\pi x + 4ib^4cd^2x^3 + ib^4d^3x^4 - 12ib^3c^2dx\arctan(\tan(a)) + 6b^4c^2dx^2\cot(a) + 6b^2c^2d\pi\log\left(1+e^{-2ibx}\right)}{}$$

input
```
Integrate[(c + d*x)^3*Cot[a + b*x],x]
```

output
```
((6*I)*b^3*c^2*d*Pi*x + (4*I)*b^4*c*d^2*x^3 + I*b^4*d^3*x^4 - (12*I)*b^3*c
^2*d*x*ArcTan[Tan[a]] + 6*b^4*c^2*d*x^2*Cot[a] + 6*b^2*c^2*d*Pi*Log[1 + E^
((-2*I)*b*x)] + 12*b^3*c*d^2*x^2*Log[1 - E^((-I)*(a + b*x))] + 4*b^3*d^3*x
^3*Log[1 - E^((-I)*(a + b*x))] + 12*b^3*c*d^2*x^2*Log[1 + E^((-I)*(a + b*x
))] + 4*b^3*d^3*x^3*Log[1 + E^((-I)*(a + b*x))] + 12*b^3*c^2*d*x*Log[1 - E
^((2*I)*(b*x + ArcTan[Tan[a]]))] + 12*b^2*c^2*d*ArcTan[Tan[a]]*Log[1 - E^(
(2*I)*(b*x + ArcTan[Tan[a]]))] - 6*b^2*c^2*d*Pi*Log[Cos[b*x]] + 4*b^3*c^3*
Log[Sin[a + b*x]] - 12*b^2*c^2*d*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a
]]]] + (12*I)*b^2*d^2*x*(2*c + d*x)*PolyLog[2, -E^((-I)*(a + b*x))] + (12*
I)*b^2*d^2*x*(2*c + d*x)*PolyLog[2, E^((-I)*(a + b*x))] - (6*I)*b^2*c^2*d*
PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))] + 24*b*c*d^2*PolyLog[3, -E^((
-I)*(a + b*x))] + 24*b*d^3*x*PolyLog[3, -E^((-I)*(a + b*x))] + 24*b*c*d^2*
PolyLog[3, E^((-I)*(a + b*x))] + 24*b*d^3*x*PolyLog[3, E^((-I)*(a + b*x))]
 - (24*I)*d^3*PolyLog[4, -E^((-I)*(a + b*x))] - (24*I)*d^3*PolyLog[4, E^((
-I)*(a + b*x))] - 6*b^4*c^2*d*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^
2])/(4*b^4)
```

### 3.33.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.35, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 25, 4202, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \cot(a+bx) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -(c+dx)^3 \tan\left(a+bx+\frac{\pi}{2}\right) dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)^3 \tan\left(\frac{1}{2}(2a+\pi)+bx\right) dx$$

$$\Big\downarrow \text{4202}$$

$$2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)^3}{1+e^{i(2a+2bx+\pi)}}dx-\frac{i(c+dx)^4}{4d}$$

$$\downarrow \text{2620}$$

$$2i\left(\frac{3id\int (c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{i(c+dx)^4}{4d}$$

$$\downarrow \text{3011}$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int (c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-$$

$$\frac{i(c+dx)^4}{4d}$$

$$\downarrow \text{7163}$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\left(\frac{id\int \operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i(c+dx)}{}\right.$$

$$\frac{i(c+dx)^4}{4d}$$

$$\downarrow \text{2720}$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\left(\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(2a+2b}\right.}{2b}\right.}{b}\right.}{2b}\right.$$

$$\frac{i(c+dx)^4}{4d}$$

$$\downarrow \text{7143}$$

3.33. $\int (c+dx)^3\cot(a+bx)\,dx$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b} - \frac{id\left(\frac{d \operatorname{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)}{4b^2} - \frac{i(c+dx) \operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}\right)}{2b} - \frac{i(c+dx)^3 \text{lo}}{}\right.$$

$$\frac{i(c+dx)^4}{4d}$$

input
```
Int[(c + d*x)^3*Cot[a + b*x],x]
```

output
```
((-1/4*I)*(c + d*x)^4)/d + (2*I)*(((-1/2*I)*(c + d*x)^3*Log[1 + E^(I*(2*a
+ Pi + 2*b*x))])/b + (((3*I)/2)*d*(((I/2)*(c + d*x)^2*PolyLog[2, -E^(I*(2*
a + Pi + 2*b*x))])/b - (I*d*(((-1/2*I)*(c + d*x)*PolyLog[3, -E^(I*(2*a + P
i + 2*b*x))])/b + (d*PolyLog[4, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2)))/b))/
b)
```

#### 3.33.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

### 3.33.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(108) = 216$.

Time = 1.16 (sec) , antiderivative size = 792, normalized size of antiderivative = 6.24

| method | result |
|---|---|
| risch | $-\frac{3id^3\operatorname{polylog}(2,e^{i(xb+a)})x^2}{b^2}-\frac{2id^3a^3x}{b^3}+\frac{3c\,d^2a^2\ln(e^{i(xb+a)}-1)}{b^3}+\frac{6c^2da\ln(e^{i(xb+a)})}{b^2}-\frac{3c^2da\ln(e^{i(xb+a)}-1)}{b^2}-\frac{d^3a^3\ln}{}$ |

input
```
int((d*x+c)^3*cos(b*x+a)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*c^3*ln(exp(I*(b*x+a))+1)-2/b*c^3*ln(exp(I*(b*x+a)))+1/b*c^3*ln(exp(I*(
b*x+a))-1)+3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2-3/b^3*c*d^2*ln(1-exp(I*(b*x+
a)))*a^2+3/b^2*d*c^2*ln(1-exp(I*(b*x+a)))*a-6/b^3*c*d^2*a^2*ln(exp(I*(b*x+
a)))-1/4*I*d^3*x^4-I*d^2*c*x^3-3/2*I*d*c^2*x^2-3*I/b^2*d*c^2*a^2-3*I/b^2*d
*c^2*polylog(2,exp(I*(b*x+a)))-3*I/b^2*d*c^2*polylog(2,-exp(I*(b*x+a)))+4*
I/b^3*c*d^2*a^3+3/b*d*c^2*ln(exp(I*(b*x+a))+1)*x+3/b*c*d^2*ln(1-exp(I*(b*x
+a)))*x^2-3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-3*I/b^2*d^3*polylog(2
,exp(I*(b*x+a)))*x^2-2*I/b^3*d^3*a^3*x+I*c^3*x+1/4*I/d*c^4+2/b^4*d^3*a^3*l
n(exp(I*(b*x+a)))+6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+1/b^4*d^3*ln(1-ex
p(I*(b*x+a)))*a^3+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-3/2*I/b^4*d^3*a^4+
6*I/b^4*d^3*polylog(4,-exp(I*(b*x+a)))+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/
b*d^3*ln(exp(I*(b*x+a))+1)*x^3+6/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))+6/b^3
*c*d^2*polylog(3,-exp(I*(b*x+a)))-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+3/b^3
*c*d^2*a^2*ln(exp(I*(b*x+a))-1)+6/b^2*c^2*d*a*ln(exp(I*(b*x+a)))-3/b^2*c^2
*d*a*ln(exp(I*(b*x+a))-1)+3/b*d*c^2*ln(1-exp(I*(b*x+a)))*x-6*I/b^2*c*d^2*p
olylog(2,-exp(I*(b*x+a)))*x-6*I/b*d*c^2*x*a+6*I/b^2*c*d^2*a^2*x-6*I/b^2*c*
d^2*polylog(2,exp(I*(b*x+a)))*x+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4
```

### 3.33.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(104) = 208$.

Time = 0.31 (sec) , antiderivative size = 818, normalized size of antiderivative = 6.44

$$\int (c+dx)^3 \cot(a+bx)\,dx = \frac{6i\,d^3\text{polylog}(4,\cos(bx+a)+i\,\sin(bx+a)) - 6i\,d^3\text{polylog}(4,\cos(bx+a)-i\,\sin(bx+a)) - 6i\,d^3\text{polylo}}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(6*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*polylog(4
, cos(b*x + a) - I*sin(b*x + a)) - 6*I*d^3*polylog(4, -cos(b*x + a) + I*si
n(b*x + a)) + 6*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) - 3*(I*b^
2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(cos(b*x + a) + I*sin(b*x
+ a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(cos(b*x +
 a) - I*sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)
*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*
x + I*b^2*c^2*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) + I*sin(b*x + a)
 + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(
b*x + a) - I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^3*c^3 -
3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin
(b*x + a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^
2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)
 + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*
b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 6*(b*d^3*x +
b*c*d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)
*polylog(3, cos(b*x + a) - I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog
(3, -cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, ...
```

### 3.33.6 Sympy [F]

$$\int (c+dx)^3\cot(a+bx)\,dx = \int (c+dx)^3\cos(a+bx)\csc(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*cos(b*x+a)*csc(b*x+a),x)
```

output
```
Integral((c + d*x)**3*cos(a + b*x)*csc(a + b*x), x)
```

### 3.33.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 759 vs. $2(104) = 208$.

Time = 0.34 (sec) , antiderivative size = 759, normalized size of antiderivative = 5.98

$$\int (c + dx)^3 \cot(a + bx) \, dx$$

$$= \frac{4 c^3 \log(\sin(bx + a)) - \frac{12 ac^2 d \log(\sin(bx+a))}{b} + \frac{12 a^2 cd^2 \log(\sin(bx+a))}{b^2} - \frac{4 a^3 d^3 \log(\sin(bx+a))}{b^3} + \frac{-i (bx+a)^4 d^3 - 4 (i bcd^2 - i ad}{}}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

output
```
1/4*(4*c^3*log(sin(b*x + a)) - 12*a*c^2*d*log(sin(b*x + a))/b + 12*a^2*c*d
^2*log(sin(b*x + a))/b^2 - 4*a^3*d^3*log(sin(b*x + a))/b^3 + (-I*(b*x + a)
^4*d^3 - 4*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^3 + 24*I*d^3*polylog(4, -e^(I*b
*x + I*a)) + 24*I*d^3*polylog(4, e^(I*b*x + I*a)) - 6*(I*b^2*c^2*d - 2*I*a
*b*c*d^2 + I*a^2*d^3)*(b*x + a)^2 - 4*(-I*(b*x + a)^3*d^3 + 3*(-I*b*c*d^2
+ I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*(b*x
 + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 4*(I*(b*x + a)^3*d^3 + 3*
(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2
*d^3)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 12*(I*b^2*c^2*
d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3
)*(b*x + a))*dilog(-e^(I*b*x + I*a)) - 12*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I
*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*dilog(e^
(I*b*x + I*a)) + 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*
(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*
x + a)^2 + 2*cos(b*x + a) + 1) + 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*
(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x
 + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 24*(b*c*d^2 + (b*x + a)*d
^3 - a*d^3)*polylog(3, -e^(I*b*x + I*a)) + 24*(b*c*d^2 + (b*x + a)*d^3 - a
*d^3)*polylog(3, e^(I*b*x + I*a)))/b^3)/b
```

### 3.33.8 Giac [F]

$$\int (c+dx)^3 \cot(a+bx)\,dx = \int (dx+c)^3 \cos(bx+a)\csc(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*cos(b*x + a)*csc(b*x + a), x)
```

### 3.33.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \cot(a+bx)\,dx = \int \frac{\cos(a+b\,x)\,(c+d\,x)^3}{\sin(a+b\,x)}\,dx$$

input
```
int((cos(a + b*x)*(c + d*x)^3)/sin(a + b*x),x)
```

output
```
int((cos(a + b*x)*(c + d*x)^3)/sin(a + b*x), x)
```

# 3.34 $\int (c+dx)^2 \cot(a+bx)\,dx$


3.34.1 Optimal result . . . 375
3.34.2 Mathematica [B] (verified) . . . 375
3.34.3 Rubi [A] (verified) . . . 376
3.34.4 Maple [B] (verified) . . . 378
3.34.5 Fricas [B] (verification not implemented) . . . 379
3.34.6 Sympy [F] . . . 380
3.34.7 Maxima [B] (verification not implemented) . . . 380
3.34.8 Giac [F] . . . 381
3.34.9 Mupad [F(-1)] . . . 381


## 3.34.1 Optimal result

Integrand size = 14, antiderivative size = 93

$$\int (c+dx)^2 \cot(a+bx)\,dx = -\frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2 \log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id(c+dx)\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2} + \frac{d^2\operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}$$

output
```
-1/3*I*(d*x+c)^3/d+(d*x+c)^2*ln(1-exp(2*I*(b*x+a)))/b-I*d*(d*x+c)*polylog(
2,exp(2*I*(b*x+a)))/b^2+1/2*d^2*polylog(3,exp(2*I*(b*x+a)))/b^3
```

## 3.34.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 356 vs. 2(93) = 186.

Time = 1.61 (sec) , antiderivative size = 356, normalized size of antiderivative = 3.83

$$\int (c+dx)^2 \cot(a+bx)\,dx = \frac{3ib^2cd\pi x + ib^3d^2x^3 - 6ib^2cdx\arctan(\tan(a)) + 3b^3cdx^2\cot(a) + 3bcd\pi\log\left(1+e^{-2ibx}\right) + 3b^2d^2x^2\log\left(1-\right.}{}$$

input
```
Integrate[(c + d*x)^2*Cot[a + b*x],x]
```

output
```
((3*I)*b^2*c*d*Pi*x + I*b^3*d^2*x^3 - (6*I)*b^2*c*d*x*ArcTan[Tan[a]] + 3*b
^3*c*d*x^2*Cot[a] + 3*b*c*d*Pi*Log[1 + E^((-2*I)*b*x)] + 3*b^2*d^2*x^2*Log
[1 - E^((-I)*(a + b*x))] + 3*b^2*d^2*x^2*Log[1 + E^((-I)*(a + b*x))] + 6*b
^2*c*d*x*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + 6*b*c*d*ArcTan[Tan[a]
]*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] - 3*b*c*d*Pi*Log[Cos[b*x]] + 3
*b^2*c^2*Log[Sin[a + b*x]] - 6*b*c*d*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[T
an[a]]]] + (6*I)*b*d^2*x*PolyLog[2, -E^((-I)*(a + b*x))] + (6*I)*b*d^2*x*P
olyLog[2, E^((-I)*(a + b*x))] - (3*I)*b*c*d*PolyLog[2, E^((2*I)*(b*x + Arc
Tan[Tan[a]]))] + 6*d^2*PolyLog[3, -E^((-I)*(a + b*x))] + 6*d^2*PolyLog[3,
E^((-I)*(a + b*x))] - 3*b^3*c*d*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a
]^2])/(3*b^3)
```

### 3.34.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.34, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 25, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \cot(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -(c+dx)^2 \tan\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)^2 \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\Big\downarrow \text{4202}$$

$$2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)^2}{1+e^{i(2a+2bx+\pi)}}dx-\frac{i(c+dx)^3}{3d}$$

$$\Big\downarrow \text{2620}$$

$$2i\left(\frac{id\int (c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{i(c+dx)^3}{3d}$$

$$\Big\downarrow \text{3011}$$

$$2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{i(c+dx)^3}{3d}$$

$\big\downarrow$ 2720

$$2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2}\right.}{2b}\right.$$

$$\frac{i(c+dx)^3}{3d}$$

$\big\downarrow$ 7143

$$2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{i(c+dx)^3}{3d}$$

input
```
Int[(c + d*x)^2*Cot[a + b*x],x]
```

output
```
((-1/3*I)*(c + d*x)^3)/d + (2*I)*(((-1/2*I)*(c + d*x)^2*Log[1 + E^(I*(2*a
+ Pi + 2*b*x))])/b + (I*d*(((I/2)*(c + d*x)*PolyLog[2, -E^(I*(2*a + Pi + 2
*b*x))])/b - (d*PolyLog[3, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2)))/b)
```

### 3.34.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.34.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(81) = 162$.

Time = 1.13 (sec) , antiderivative size = 477, normalized size of antiderivative = 5.13

| method | result |
|---|---|
| risch | $\frac{c^2 \ln(\mathrm{e}^{i(xb+a)}+1)}{b} - \frac{2c^2 \ln(\mathrm{e}^{i(xb+a)})}{b} + \frac{c^2 \ln(\mathrm{e}^{i(xb+a)}-1)}{b} + ic^2x + \frac{4id^2a^3}{3b^3} - idc\,x^2 - \frac{d^2 \ln(1-\mathrm{e}^{i(xb+a)})a^2}{b^3} + \frac{d^2 \ln(1\cdots}{}$ |

input
```
int((d*x+c)^2*cos(b*x+a)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))+1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+4/3*I
/b^3*d^2*a^3-I*d*c*x^2-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x-2*I/b^2*d^2
*polylog(2,-exp(I*(b*x+a)))*x+1/b*c^2*ln(exp(I*(b*x+a))+1)-2/b*c^2*ln(exp(
I*(b*x+a)))+1/b*c^2*ln(exp(I*(b*x+a))-1)-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^
2+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2+1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+I*c^2*
x+1/3*I/d*c^3+2/b*d*c*ln(exp(I*(b*x+a))+1)*x+2/b^2*d*c*ln(1-exp(I*(b*x+a))
)*a+4/b^2*c*d*a*ln(exp(I*(b*x+a)))-2/b^2*c*d*a*ln(exp(I*(b*x+a))-1)+2/b*d*
c*ln(1-exp(I*(b*x+a)))*x-2*I/b^2*d*c*a^2-2*I/b^2*d*c*polylog(2,exp(I*(b*x+
a)))-2*I/b^2*d*c*polylog(2,-exp(I*(b*x+a)))+2*I/b^2*d^2*a^2*x-1/3*I*d^2*x^
3-4*I/b*d*c*x*a+2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp(I
*(b*x+a)))/b^3
```

### 3.34.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 502 vs. 2(78) = 156.

Time = 0.27 (sec) , antiderivative size = 502, normalized size of antiderivative = 5.40

$$\int (c+dx)^2 \cot(a+bx)\,dx$$
$$= \frac{2\,d^2\text{polylog}(3,\cos{(bx+a)}+i\,\sin{(bx+a)})+2\,d^2\text{polylog}(3,\cos{(bx+a)}-i\,\sin{(bx+a)})+2\,d^2\text{polylog}(\ldots}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylog(3, co
s(b*x + a) - I*sin(b*x + a)) + 2*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x
+ a)) + 2*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) - 2*(I*b*d^2*x +
I*b*c*d)*dilog(cos(b*x + a) + I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*d
ilog(cos(b*x + a) - I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*dilog(-cos(
b*x + a) + I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*dilog(-cos(b*x + a) -
 I*sin(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a)
+ I*sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x
+ a) - I*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(
b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log
(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*
x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*d^
2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x +
 a) + 1))/b^3
```

### 3.34.6 Sympy [F]

$$\int (c+dx)^2 \cot(a+bx)\,dx = \int (c+dx)^2 \cos(a+bx)\csc(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*cos(b*x+a)*csc(b*x+a),x)
```

output
```
Integral((c + d*x)**2*cos(a + b*x)*csc(a + b*x), x)
```

### 3.34.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(78) = 156$.

Time = 0.30 (sec) , antiderivative size = 411, normalized size of antiderivative = 4.42

$$\int (c+dx)^2 \cot(a+bx)\,dx$$
$$= \frac{6c^2\log(\sin(bx+a)) - \frac{12acd\log(\sin(bx+a))}{b} + \frac{6a^2d^2\log(\sin(bx+a))}{b^2} + \frac{-2i(bx+a)^3d^2 - 6(ibcd - iad^2)(bx+a)^2 + 12d^2\mathrm{Li}_3(-e^{(ibx+}}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

output
```
1/6*(6*c^2*log(sin(b*x + a)) - 12*a*c*d*log(sin(b*x + a))/b + 6*a^2*d^2*lo
g(sin(b*x + a))/b^2 + (-2*I*(b*x + a)^3*d^2 - 6*(I*b*c*d - I*a*d^2)*(b*x +
 a)^2 + 12*d^2*polylog(3, -e^(I*b*x + I*a)) + 12*d^2*polylog(3, e^(I*b*x +
 I*a)) - 6*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*arctan2
(sin(b*x + a), cos(b*x + a) + 1) - 6*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a
*d^2)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 12*(I*b*c*d +
I*(b*x + a)*d^2 - I*a*d^2)*dilog(-e^(I*b*x + I*a)) - 12*(I*b*c*d + I*(b*x
+ a)*d^2 - I*a*d^2)*dilog(e^(I*b*x + I*a)) + 3*((b*x + a)^2*d^2 + 2*(b*c*d
 - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a)
+ 1) + 3*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^
2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1))/b^2)/b
```

### 3.34.8 Giac [F]

$$\int (c+dx)^2 \cot(a+bx)\,dx = \int (dx+c)^2 \cos(bx+a)\csc(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*cos(b*x + a)*csc(b*x + a), x)
```

### 3.34.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \cot(a+bx)\,dx = \int \frac{\cos(a+b\,x)\,(c+d\,x)^2}{\sin(a+b\,x)}\,dx$$

input
```
int((cos(a + b*x)*(c + d*x)^2)/sin(a + b*x),x)
```

output
```
int((cos(a + b*x)*(c + d*x)^2)/sin(a + b*x), x)
```

# 3.35 $\int (c+dx)\cot(a+bx)\,dx$


3.35.1 Optimal result . . . . . 382
3.35.2 Mathematica [B] (verified) . . . . . 382
3.35.3 Rubi [A] (verified) . . . . . 383
3.35.4 Maple [B] (verified) . . . . . 385
3.35.5 Fricas [B] (verification not implemented) . . . . . 385
3.35.6 Sympy [F] . . . . . 386
3.35.7 Maxima [B] (verification not implemented) . . . . . 386
3.35.8 Giac [F] . . . . . 386
3.35.9 Mupad [F(-1)] . . . . . 387


## 3.35.1 Optimal result

Integrand size = 12, antiderivative size = 65

$$\int (c+dx)\cot(a+bx)\,dx = -\frac{i(c+dx)^2}{2d}+\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{id\,\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}$$

output
```
-1/2*I*(d*x+c)^2/d+(d*x+c)*ln(1-exp(2*I*(b*x+a)))/b-1/2*I*d*polylog(2,exp(
2*I*(b*x+a)))/b^2
```

## 3.35.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 192 vs. 2(65) = 130.

Time = 5.47 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.95

$$\int (c+dx)\cot(a+bx)\,dx = \frac{1}{2}dx^2\cot(a)+\frac{c\log(\cos(a+bx))}{b}+\frac{c\log(\tan(a+bx))}{b}-\frac{d\csc(a)\sec(a)\left(b^2e^{i\arctan(\tan(a))}x^2+\frac{(ibx(-\pi+2\arctan(\tan(a)))-\pi\log\left(1+e^{-2ibx}\right)-2(bx+\arctan(\tan(a)))\log\left(1-e^{2i(bx+\arctan(\cdots}\right.}{\cdots}\right.}{2b^2\sqrt{\sec^2(a)\left(\cos^2(a)+\cdots\right.}}$$

input
```
Integrate[(c + d*x)*Cot[a + b*x],x]
```

output
```
(d*x^2*Cot[a])/2 + (c*Log[Cos[a + b*x]])/b + (c*Log[Tan[a + b*x]])/b - (d*
Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[
a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2
*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin
[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*
Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

### 3.35.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 25, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\cot(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\left((c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\Big\downarrow \text{4202}$$

$$2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)}{1+e^{i(2a+2bx+\pi)}}dx-\frac{i(c+dx)^2}{2d}$$

$$\Big\downarrow \text{2620}$$

$$2i\left(\frac{id\int \log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{i(c+dx)^2}{2d}$$

$$\Big\downarrow \text{2715}$$

$$2i\left(\frac{d\int e^{-i(2a+2bx+\pi)}\log\left(1+e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{i(c+dx)^2}{2d}$$

$$\Big\downarrow \text{2838}$$

$$2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{i(c+dx)^2}{2d}$$

input
```
Int[(c + d*x)*Cot[a + b*x],x]
```

output
```
((-1/2*I)*(c + d*x)^2)/d + (2*I)*(((-1/2*I)*(c + d*x)*Log[1 + E^(I*(2*a +
Pi + 2*b*x))])/b - (d*PolyLog[2, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2))
```

### 3.35.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

### 3.35.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 214 vs. 2(55) = 110.

Time = 1.00 (sec) , antiderivative size = 215, normalized size of antiderivative = 3.31

| method | result |
|---|---|
| risch | $-\frac{id x^2}{2}+icx+\frac{c\ln(e^{i(xb+a)}+1)}{b}-\frac{2c\ln(e^{i(xb+a)})}{b}+\frac{c\ln(e^{i(xb+a)}-1)}{b}-\frac{2idxa}{b}-\frac{id a^2}{b^2}+\frac{d\ln(1-e^{i(xb+a)})x}{b}+\frac{d\ln(1$ |

input
```
int((d*x+c)*cos(b*x+a)*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*d*x^2+I*c*x+1/b*c*ln(exp(I*(b*x+a))+1)-2/b*c*ln(exp(I*(b*x+a)))+1/b
*c*ln(exp(I*(b*x+a))-1)-2*I/b*d*x*a-I/b^2*d*a^2+1/b*d*ln(1-exp(I*(b*x+a)))
*x+1/b^2*d*ln(1-exp(I*(b*x+a)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2+1/b*d*
ln(exp(I*(b*x+a))+1)*x-I*d*polylog(2,-exp(I*(b*x+a)))/b^2+2/b^2*d*a*ln(exp
(I*(b*x+a)))-1/b^2*d*a*ln(exp(I*(b*x+a))-1)
```

### 3.35.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(52) = 104.

Time = 0.27 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.85

$$\int (c+dx)\cot(a+bx)\,dx$$
$$=\frac{-i\,d\mathrm{Li}_2(\cos(bx+a)+i\,\sin(bx+a))+i\,d\mathrm{Li}_2(\cos(bx+a)-i\,\sin(bx+a))+i\,d\mathrm{Li}_2(-\cos(bx+a)+i\,\mathrm{si}}{}$$

input
```
integrate((d*x+c)*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(-I*d*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d*dilog(cos(b*x + a) -
I*sin(b*x + a)) + I*d*dilog(-cos(b*x + a) + I*sin(b*x + a)) - I*d*dilog(-c
os(b*x + a) - I*sin(b*x + a)) + (b*d*x + b*c)*log(cos(b*x + a) + I*sin(b*x
 + a) + 1) + (b*d*x + b*c)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b*c -
 a*d)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b*c - a*d)*log(
-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b*d*x + a*d)*log(-cos(b*x
 + a) + I*sin(b*x + a) + 1) + (b*d*x + a*d)*log(-cos(b*x + a) - I*sin(b*x
+ a) + 1))/b^2
```

### 3.35.6 Sympy [F]

$$\int (c+dx)\cot(a+bx)\,dx = \int (c+dx)\cos(a+bx)\csc(a+bx)\,dx$$

input
```
integrate((d*x+c)*cos(b*x+a)*csc(b*x+a),x)
```

output
```
Integral((c + d*x)*cos(a + b*x)*csc(a + b*x), x)
```

### 3.35.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(52) = 104.

Time = 0.31 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.92

$$\int (c+dx)\cot(a+bx)\,dx$$
$$= \frac{-i\,b^2dx^2 - 2i\,b^2cx - 2i\,bdx\arctan(\sin(bx+a), -\cos(bx+a)+1) + 2i\,bc\arctan(\sin(bx+a), \cos(bx+$$

input
```
integrate((d*x+c)*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(-I*b^2*d*x^2 - 2*I*b^2*c*x - 2*I*b*d*x*arctan2(sin(b*x + a), -cos(b*x
 + a) + 1) + 2*I*b*c*arctan2(sin(b*x + a), cos(b*x + a) - 1) - 2*(-I*b*d*x
 - I*b*c)*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*I*d*dilog(-e^(I*b*x
+ I*a)) - 2*I*d*dilog(e^(I*b*x + I*a)) + (b*d*x + b*c)*log(cos(b*x + a)^2
+ sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (b*d*x + b*c)*log(cos(b*x + a)^2
+ sin(b*x + a)^2 - 2*cos(b*x + a) + 1))/b^2
```

### 3.35.8 Giac [F]

$$\int (c+dx)\cot(a+bx)\,dx = \int (dx+c)\cos(bx+a)\csc(bx+a)\,dx$$

input
```
integrate((d*x+c)*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*cos(b*x + a)*csc(b*x + a), x)
```

### 3.35.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\cot(a+bx)\,dx = \int \frac{\cos(a+b\,x)\,(c+d\,x)}{\sin(a+b\,x)}\,dx$$

input
```
int((cos(a + b*x)*(c + d*x))/sin(a + b*x),x)
```

output
```
int((cos(a + b*x)*(c + d*x))/sin(a + b*x), x)
```

# 3.36 $\int \frac{\cot(a+bx)}{c+dx}\,dx$

3.36.1 Optimal result . . . . . 388
3.36.2 Mathematica [N/A] . . . . . 388
3.36.3 Rubi [N/A] . . . . . 389
3.36.4 Maple [N/A] (verified) . . . . . 390
3.36.5 Fricas [N/A] . . . . . 390
3.36.6 Sympy [N/A] . . . . . 391
3.36.7 Maxima [N/A] . . . . . 391
3.36.8 Giac [N/A] . . . . . 391
3.36.9 Mupad [N/A] . . . . . 392

## 3.36.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\cot(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\cot(a+bx)}{c+dx},x\right)$$

output
```
Unintegrable(cot(b*x+a)/(d*x+c),x)
```

## 3.36.2 Mathematica [N/A]

Not integrable

Time = 4.96 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\cot(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+bx)}{c+dx}\,dx$$

input
```
Integrate[Cot[a + b*x]/(c + d*x),x]
```

output
```
Integrate[Cot[a + b*x]/(c + d*x), x]
```

## 3.36.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 25, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{c+dx}dx$$

$$\Big\downarrow \text{4222}$$

$$-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{c+dx}dx$$

input
```
Int[Cot[a + b*x]/(c + d*x),x]
```

output
```
$Aborted
```

### 3.36.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

## 3.36.4 Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{\cos (xb+a) \csc (xb+a)}{dx+c} dx$$

input
```
int(cos(b*x+a)*csc(b*x+a)/(d*x+c),x)
```

output
```
int(cos(b*x+a)*csc(b*x+a)/(d*x+c),x)
```

## 3.36.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{\cot(a+bx)}{c+dx} \, dx = \int \frac{\cos (bx+a) \csc (bx+a)}{dx+c} \, dx$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)*csc(b*x + a)/(d*x + c), x)
```

### 3.36.6 Sympy [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \frac{\cot(a+bx)}{c+dx} \, dx = \int \frac{\cos(a+bx)\csc(a+bx)}{c+dx} \, dx$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c),x)
```

output
```
Integral(cos(a + b*x)*csc(a + b*x)/(c + d*x), x)
```

### 3.36.7 Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{\cot(a+bx)}{c+dx} \, dx = \int \frac{\cos(bx+a)\csc(bx+a)}{dx+c} \, dx$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)*csc(b*x + a)/(d*x + c), x)
```

### 3.36.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{\cot(a+bx)}{c+dx} \, dx = \int \frac{\cos(bx+a)\csc(bx+a)}{dx+c} \, dx$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*csc(b*x + a)/(d*x + c), x)
```

### 3.36.9 Mupad [N/A]

Not integrable

Time = 22.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos(a+b\,x)}{\sin(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(cos(a + b*x)/(sin(a + b*x)*(c + d*x)),x)
```

output
```
int(cos(a + b*x)/(sin(a + b*x)*(c + d*x)), x)
```

# 3.37 $\int \frac{\cot(a+bx)}{(c+dx)^2} dx$

3.37.1 Optimal result . . . 393
3.37.2 Mathematica [N/A] . . . 393
3.37.3 Rubi [N/A] . . . 394
3.37.4 Maple [N/A] (verified) . . . 395
3.37.5 Fricas [N/A] . . . 395
3.37.6 Sympy [N/A] . . . 396
3.37.7 Maxima [N/A] . . . 396
3.37.8 Giac [N/A] . . . 396
3.37.9 Mupad [N/A] . . . 397

## 3.37.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\cot(a+bx)}{(c+dx)^2} dx = \text{Int}\left(\frac{\cot(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Unintegrable(cot(b*x+a)/(d*x+c)^2,x)
```

## 3.37.2 Mathematica [N/A]

Not integrable

Time = 9.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\cot(a+bx)}{(c+dx)^2} dx = \int \frac{\cot(a+bx)}{(c+dx)^2} dx$$

input
```
Integrate[Cot[a + b*x]/(c + d*x)^2,x]
```

output
```
Integrate[Cot[a + b*x]/(c + d*x)^2, x]
```

### 3.37.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 25, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{4222}$$

$$-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx$$

input
```
Int[Cot[a + b*x]/(c + d*x)^2,x]
```

output
```
$Aborted
```

#### 3.37.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

## 3.37.4 Maple [N/A] (verified)

Not integrable

Time = 0.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{\cos (xb+a) \csc (xb+a)}{(dx+c)^2} dx$$

input
```
int(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x)
```

output
```
int(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x)
```

## 3.37.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.36

$$\int \frac{\cot(a+bx)}{(c+dx)^2} dx = \int \frac{\cos (bx+a) \csc (bx+a)}{(dx+c)^2} dx$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)*csc(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.37.6 Sympy [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(cos(a + b*x)*csc(a + b*x)/(c + d*x)**2, x)
```

### 3.37.7 Maxima [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(bx+a)\csc(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(cos(b*x + a)*csc(b*x + a)/(d*x + c)^2, x)
```

### 3.37.8 Giac [N/A]

Not integrable

Time = 2.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(bx+a)\csc(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*csc(b*x + a)/(d*x + c)^2, x)
```

### 3.37.9 Mupad [N/A]

Not integrable

Time = 21.86 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos{(a+b\,x)}}{\sin{(a+b\,x)}\,{(c+d\,x)}^2}\,dx$$

input
```
int(cos(a + b*x)/(sin(a + b*x)*(c + d*x)^2),x)
```

output
```
int(cos(a + b*x)/(sin(a + b*x)*(c + d*x)^2), x)
```

# 3.38 $\int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx$


3.38.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . 398
3.38.2 Mathematica [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . 398
3.38.3 Rubi [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 399
3.38.4 Maple [N/A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 399
3.38.5 Fricas [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 400
3.38.6 Sympy [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 400
3.38.7 Maxima [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . 400
3.38.8 Giac [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 401
3.38.9 Mupad [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 401


## 3.38.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx = \text{Int}((c+dx)^m \cot(a+bx)\csc(a+bx),x)$$

output
```
CannotIntegrate((d*x+c)^m*cot(b*x+a)*csc(b*x+a),x)
```

## 3.38.2 Mathematica [N/A]

Not integrable

Time = 3.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx = \int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x], x]
```

## 3.38.3 Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(a+bx)\csc(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \cot(a+bx)\csc(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x],x]
```

output
```
$Aborted
```

#### 3.38.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.38.4 Maple [N/A] (verified)

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (dx+c)^m \cos(xb+a)\csc(xb+a)^2\,dx$$

input
```
int((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x)
```

### 3.38.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\csc(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^2, x)
```

### 3.38.6 Sympy [N/A]

Not integrable

Time = 66.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx = \int (c+dx)^m \cos(a+bx)\csc^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*cos(b*x+a)*csc(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*cos(a + b*x)*csc(a + b*x)**2, x)
```

### 3.38.7 Maxima [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\csc(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^2, x)
```

### 3.38.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int (c+dx)^m \cot(a+bx) \csc(a+bx)\,dx = \int (dx+c)^m \cos(bx+a) \csc(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^2, x)
```

### 3.38.9 Mupad [N/A]

Not integrable

Time = 21.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int (c+dx)^m \cot(a+bx) \csc(a+bx)\,dx = \int \frac{\cos(a+b\,x)\,(c+d\,x)^m}{\sin(a+b\,x)^2}\,dx$$

input
```
int((cos(a + b*x)*(c + d*x)^m)/sin(a + b*x)^2,x)
```

output
```
int((cos(a + b*x)*(c + d*x)^m)/sin(a + b*x)^2, x)
```

# 3.39 $\int (c+dx)^4 \cot(a+bx)\csc(a+bx)\,dx$


3.39.1 Optimal result . . . 402
3.39.2 Mathematica [A] (verified) . . . 403
3.39.3 Rubi [A] (verified) . . . 403
3.39.4 Maple [B] (verified) . . . 406
3.39.5 Fricas [B] (verification not implemented) . . . 407
3.39.6 Sympy [F] . . . 408
3.39.7 Maxima [B] (verification not implemented) . . . 409
3.39.8 Giac [F] . . . 409
3.39.9 Mupad [F(-1)] . . . 410


## 3.39.1 Optimal result

Integrand size = 20, antiderivative size = 208

$$\begin{aligned}\int (c+dx)^4\cot(a+bx)\csc(a+bx)\,dx =& -\frac{8d(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b^2} \\ &-\frac{(c+dx)^4\csc(a+bx)}{b} \\ &+\frac{12id^2(c+dx)^2\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3} \\ &-\frac{12id^2(c+dx)^2\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3} \\ &-\frac{24d^3(c+dx)\,\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^4} \\ &+\frac{24d^3(c+dx)\,\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^4} \\ &-\frac{24id^4\,\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^5} \\ &+\frac{24id^4\,\text{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^5}\end{aligned}$$

output
```
-8*d*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b^2-(d*x+c)^4*csc(b*x+a)/b+12*I*d^2
*(d*x+c)^2*polylog(2,-exp(I*(b*x+a)))/b^3-12*I*d^2*(d*x+c)^2*polylog(2,exp
(I*(b*x+a)))/b^3-24*d^3*(d*x+c)*polylog(3,-exp(I*(b*x+a)))/b^4+24*d^3*(d*x
+c)*polylog(3,exp(I*(b*x+a)))/b^4-24*I*d^4*polylog(4,-exp(I*(b*x+a)))/b^5+
24*I*d^4*polylog(4,exp(I*(b*x+a)))/b^5
```

### 3.39.2 Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.48

$$\int (c+dx)^4 \cot(a+bx)\csc(a+bx)\,dx$$
$$= \frac{-2b(c+dx)^4\csc(a)+8id\Big(2i(c+dx)^3\text{arctanh}(\cos(a+bx)+i\sin(a+bx))+\frac{3d(b^2(c+dx)^2\,\text{PolyLog}(2,-\cos(a+bx}{}}{}$$

input
```
Integrate[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x],x]
```

output
```
(-2*b*(c + d*x)^4*Csc[a] + (8*I)*d*((2*I)*(c + d*x)^3*ArcTanh[Cos[a + b*x]
 + I*Sin[a + b*x]] + (3*d*(b^2*(c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Si
n[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x
]] - 2*d^2*PolyLog[4, -Cos[a + b*x] - I*Sin[a + b*x]]))/b^3 - (3*d*(b^2*(c
 + d*x)^2*PolyLog[2, Cos[a + b*x] + I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*
PolyLog[3, Cos[a + b*x] + I*Sin[a + b*x]] - 2*d^2*PolyLog[4, Cos[a + b*x]
+ I*Sin[a + b*x]]))/b^3) + b*(c + d*x)^4*Csc[a/2]*Csc[(a + b*x)/2]*Sin[(b*
x)/2] - b*(c + d*x)^4*Sec[a/2]*Sec[(a + b*x)/2]*Sin[(b*x)/2])/(2*b^2)
```

### 3.39.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {4910, 3042, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \cot(a+bx)\csc(a+bx)\,dx$$
$$\Big\downarrow \text{4910}$$
$$\frac{4d\int (c+dx)^3\csc(a+bx)dx}{b}-\frac{(c+dx)^4\csc(a+bx)}{b}$$
$$\Big\downarrow \text{3042}$$
$$\frac{4d\int (c+dx)^3\csc(a+bx)dx}{b}-\frac{(c+dx)^4\csc(a+bx)}{b}$$
$$\Big\downarrow \text{4671}$$

$$-\frac{(c+dx)^4\csc(a+bx)}{b}+\frac{4d\left(-\frac{3d\int (c+dx)^2\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{3d\int (c+dx)^2\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 3011

$$-\frac{(c+dx)^4\csc(a+bx)}{b}+\frac{4d\left(\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\int (c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\int (c+dx)\text{PolyLog}\left(2,e^{i(a+b\cdots}{b}\cdots\right)}{b}\cdots\right)}{b}$$

$\Big\downarrow$ 7163

$$-\frac{(c+dx)^4\csc(a+bx)}{b}+\frac{4d\left(\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{id\int \text{PolyLog}\left(3,-e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}\cdots\right)}{\cdots}\cdots\right)}{b}$$

$\Big\downarrow$ 2720

$$-\frac{(c+dx)^4\csc(a+bx)}{b}+\frac{4d\left(\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\text{PolyLog}\left(3,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\text{Po}\cdots}{\cdots}\cdots\right)}{\cdots}\cdots\right)}{b}$$

$\Big\downarrow$ 7143

3.39. $\int (c+dx)^4\cot(a+bx)\csc(a+bx)\,dx$

$$-\frac{(c+dx)^4\csc(a+bx)}{b}+\frac{4d\left(-\frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\,\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\,\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\right.}{b}$$

input
```
Int[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x],x]
```

output
```
-(((c + d*x)^4*Csc[a + b*x])/b) + (4*d*((-2*(c + d*x)^3*ArcTanh[E^(I*(a +
b*x))])/b + (3*d*((I*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b - ((2*I)*
d*(((-I)*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b + (d*PolyLog[4, -E^(I*(
a + b*x))])/b^2))/b))/b - (3*d*((I*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))]
)/b - ((2*I)*d*(((-I)*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b + (d*PolyLo
g[4, E^(I*(a + b*x))])/b^2))/b))/b))/b
```

### 3.39.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

### 3.39.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 715 vs. 2(190) = 380.

Time = 1.23 (sec) , antiderivative size = 716, normalized size of antiderivative = 3.44

| method | result |
|---|---|
| risch | $-\frac{24 i d^3 c \operatorname{polylog}\left(2, \mathrm{e}^{i(x b+a)}\right) x}{b^3}+\frac{24 i d^3 c \operatorname{polylog}\left(2,-\mathrm{e}^{i(x b+a)}\right) x}{b^3}+\frac{8 d^4 a^3 \operatorname{arctanh}\left(\mathrm{e}^{i(x b+a)}\right)}{b^5}-\frac{8 d c^3 \operatorname{arctanh}\left(\mathrm{e}^{i(x b+a)}\right)}{b^2}-\frac{4 d^4 \mathrm{l}}{}$ |

input
```
int((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
24*I*d^4*polylog(4,exp(I*(b*x+a)))/b^5-24*I*d^4*polylog(4,-exp(I*(b*x+a)))
/b^5-12*d^2/b^3*c^2*ln(exp(I*(b*x+a))+1)*a+24*d^3/b^4*c*polylog(3,exp(I*(b
*x+a)))+8*d^4/b^5*a^3*arctanh(exp(I*(b*x+a)))-8*d/b^2*c^3*arctanh(exp(I*(b
*x+a)))-4*d^4/b^2*ln(exp(I*(b*x+a))+1)*x^3+4*d^4/b^2*ln(1-exp(I*(b*x+a)))*
x^3-24*d^3/b^4*c*polylog(3,-exp(I*(b*x+a)))-4*d^4/b^5*ln(exp(I*(b*x+a))+1)
*a^3+4*d^4/b^5*ln(1-exp(I*(b*x+a)))*a^3-24*d^4/b^4*polylog(3,-exp(I*(b*x+a
)))*x+24*d^4/b^4*polylog(3,exp(I*(b*x+a)))*x-2*I*(d^4*x^4+4*c*d^3*x^3+6*c^
2*d^2*x^2+4*c^3*d*x+c^4)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)-24*d^3/b^4*
c*a^2*arctanh(exp(I*(b*x+a)))+24*d^2/b^3*c^2*a*arctanh(exp(I*(b*x+a)))-12*
d^3/b^2*c*ln(exp(I*(b*x+a))+1)*x^2+12*d^3/b^2*c*ln(1-exp(I*(b*x+a)))*x^2+1
2*d^2/b^2*c^2*ln(1-exp(I*(b*x+a)))*x-12*d^2/b^2*c^2*ln(exp(I*(b*x+a))+1)*x
-12*d^3/b^4*c*ln(1-exp(I*(b*x+a)))*a^2+12*d^2/b^3*c^2*ln(1-exp(I*(b*x+a)))
*a+12*d^3/b^4*c*ln(exp(I*(b*x+a))+1)*a^2+12*I*d^4/b^3*polylog(2,-exp(I*(b*
x+a)))*x^2+12*I*d^2/b^3*c^2*polylog(2,-exp(I*(b*x+a)))-12*I*d^2/b^3*c^2*po
lylog(2,exp(I*(b*x+a)))-12*I*d^4/b^3*polylog(2,exp(I*(b*x+a)))*x^2-24*I*d^
3/b^3*c*polylog(2,exp(I*(b*x+a)))*x+24*I*d^3/b^3*c*polylog(2,-exp(I*(b*x+a
)))*x
```

### 3.39.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1021 vs. 2(184) = 368.

Time = 0.31 (sec) , antiderivative size = 1021, normalized size of antiderivative = 4.91

$$\int (c+dx)^4 \cot(a+bx)\csc(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*
c^4 - 12*I*d^4*polylog(4, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12
*I*d^4*polylog(4, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - 12*I*d^4*p
olylog(4, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*I*d^4*polylog(
4, -cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*(I*b^2*d^4*x^2 + 2*I*b
^2*c*d^3*x + I*b^2*c^2*d^2)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x +
 a) + 6*(-I*b^2*d^4*x^2 - 2*I*b^2*c*d^3*x - I*b^2*c^2*d^2)*dilog(cos(b*x +
 a) - I*sin(b*x + a))*sin(b*x + a) + 6*(I*b^2*d^4*x^2 + 2*I*b^2*c*d^3*x +
I*b^2*c^2*d^2)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*(-I*
b^2*d^4*x^2 - 2*I*b^2*c*d^3*x - I*b^2*c^2*d^2)*dilog(-cos(b*x + a) - I*sin
(b*x + a))*sin(b*x + a) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2
*x + b^3*c^3*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 2*(b
^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*log(cos(b*x +
a) - I*sin(b*x + a) + 1)*sin(b*x + a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3
*a^2*b*c*d^3 - a^3*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*
sin(b*x + a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*l
og(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 2*(b^3*d^4
*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3
 + a^3*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - 2*(b^3*
d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b...
```

### 3.39.6 Sympy [F]

$$\int (c+dx)^4 \cot(a+bx)\csc(a+bx)\,dx = \int (c+dx)^4 \cos(a+bx)\csc^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**4*cos(b*x+a)*csc(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**4*cos(a + b*x)*csc(a + b*x)**2, x)
```

### 3.39.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2948 vs. $2(184) = 368$.

Time = 0.58 (sec) , antiderivative size = 2948, normalized size of antiderivative = 14.17

$$\int (c+dx)^4 \cot(a+bx)\csc(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(2*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x + 2
*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x
+ 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (c
os(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b
*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x + a
))*c^3*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) +
1)*b) - 6*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b
*x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(
2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1
) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log
(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b
*x + a))*a*c^2*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x
 + 2*a) + 1)*b^2) + 6*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x
+ a)*cos(2*b*x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a
)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(
b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2
*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*
x + a)*sin(b*x + a))*a^2*c*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 -
 2*cos(2*b*x + 2*a) + 1)*b^3) - 2*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*
a) - 4*(b*x + a)*cos(2*b*x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + ...
```

### 3.39.8 Giac [F]

$$\int (c+dx)^4 \cot(a+bx)\csc(a+bx)\,dx = \int (dx+c)^4 \cos(bx+a)\csc(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*cos(b*x + a)*csc(b*x + a)^2, x)
```

### 3.39.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \cot(a+bx)\csc(a+bx)\,dx = \int \frac{\cos\left(a+b\,x\right)\,\left(c+d\,x\right)^4}{\sin\left(a+b\,x\right)^2}\,dx$$

input
```
int((cos(a + b*x)*(c + d*x)^4)/sin(a + b*x)^2,x)
```

output
```
int((cos(a + b*x)*(c + d*x)^4)/sin(a + b*x)^2, x)
```

# 3.40 $\int (c+dx)^3 \cot(a+bx)\csc(a+bx)\,dx$

3.40.1 Optimal result . . . 411
3.40.2 Mathematica [B] (verified) . . . 412
3.40.3 Rubi [A] (verified) . . . 412
3.40.4 Maple [B] (verified) . . . 415
3.40.5 Fricas [B] (verification not implemented) . . . 415
3.40.6 Sympy [F] . . . 416
3.40.7 Maxima [B] (verification not implemented) . . . 416
3.40.8 Giac [F] . . . 417
3.40.9 Mupad [F(-1)] . . . 418

## 3.40.1 Optimal result

Integrand size = 20, antiderivative size = 146

$$\int (c+dx)^3 \cot(a+bx)\csc(a+bx)\,dx = -\frac{6d(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^3\csc(a+bx)}{b} + \frac{6id^2(c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3} - \frac{6id^2(c+dx)\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3} - \frac{6d^3\,\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^4} + \frac{6d^3\,\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^4}$$

output
```
-6*d*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b^2-(d*x+c)^3*csc(b*x+a)/b+6*I*d^2*
(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^3-6*I*d^2*(d*x+c)*polylog(2,exp(I*(b*
x+a)))/b^3-6*d^3*polylog(3,-exp(I*(b*x+a)))/b^4+6*d^3*polylog(3,exp(I*(b*x
+a)))/b^4
```

### 3.40.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 311 vs. 2(146) = 292.

Time = 1.34 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.13

$$\int (c+dx)^3 \cot(a+bx)\csc(a+bx)\,dx =$$
$$-\frac{b^3c^3\csc(a+bx)+3b^3c^2dx\csc(a+bx)+3b^3cd^2x^2\csc(a+bx)+b^3d^3x^3\csc(a+bx)-3b^2c^2d\log\left(1-e^{i}\right.}{}$$

input
```
Integrate[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x],x]
```

output
```
-((b^3*c^3*Csc[a + b*x] + 3*b^3*c^2*d*x*Csc[a + b*x] + 3*b^3*c*d^2*x^2*Csc
[a + b*x] + b^3*d^3*x^3*Csc[a + b*x] - 3*b^2*c^2*d*Log[1 - E^(I*(a + b*x))
] - 6*b^2*c*d^2*x*Log[1 - E^(I*(a + b*x))] - 3*b^2*d^3*x^2*Log[1 - E^(I*(a
 + b*x))] + 3*b^2*c^2*d*Log[1 + E^(I*(a + b*x))] + 6*b^2*c*d^2*x*Log[1 + E
^(I*(a + b*x))] + 3*b^2*d^3*x^2*Log[1 + E^(I*(a + b*x))] - (6*I)*b*d^2*(c
+ d*x)*PolyLog[2, -E^(I*(a + b*x))] + (6*I)*b*d^2*(c + d*x)*PolyLog[2, E^(
I*(a + b*x))] + 6*d^3*PolyLog[3, -E^(I*(a + b*x))] - 6*d^3*PolyLog[3, E^(I
*(a + b*x))])/b^4)
```

### 3.40.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4910, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\cot(a+bx)\csc(a+bx)\,dx$$
$$\Big\downarrow \text{4910}$$
$$\frac{3d\int (c+dx)^2\csc(a+bx)dx}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$
$$\Big\downarrow \text{3042}$$
$$\frac{3d\int (c+dx)^2\csc(a+bx)dx}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

$$\Big\downarrow \; 4671$$

$$-\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{3d\left(-\frac{2d\int(c+dx)\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{2d\int(c+dx)\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 3011$$

$$-\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{3d\left(\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{id\int\text{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{id\int\text{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2(c+\cdots}{}\right)}{b}$$

$$\Big\downarrow \; 2720$$

$$-\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{3d\left(\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\cdots}{b\cdots}\right)}{b}\cdots\right)}{b}$$

$$\Big\downarrow \; 7143$$

$$-\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{3d\left(-\frac{2(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\,\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\,\text{Poly}\cdots}{}\right)}{b}\cdots\right)}{b}$$

input
```
Int[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x],x]
```

output
```
-(((c + d*x)^3*Csc[a + b*x])/b) + (3*d*((-2*(c + d*x)^2*ArcTanh[E^(I*(a +
b*x))])/b + (2*d*((I*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b - (d*PolyLo
g[3, -E^(I*(a + b*x))])/b^2))/b - (2*d*((I*(c + d*x)*PolyLog[2, E^(I*(a +
b*x))])/b - (d*PolyLog[3, E^(I*(a + b*x))])/b^2))/b))/b
```

## 3.40. $\int (c+dx)^3\cot(a+bx)\csc(a+bx)\,dx$

### 3.40.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.40.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(134) = 268$.

Time = 1.13 (sec) , antiderivative size = 433, normalized size of antiderivative = 2.97

| method | result |
|---|---|
| risch | $-\frac{2i(d^3x^3+3c\,d^2x^2+3c^2dx+c^3)\mathrm{e}^{i(xb+a)}}{b(\mathrm{e}^{2i(xb+a)}-1)}+\frac{6id^3\,\mathrm{polylog}(2,-\mathrm{e}^{i(xb+a)})x}{b^3}+\frac{6id^2c\,\mathrm{polylog}(2,-\mathrm{e}^{i(xb+a)})}{b^3}+\frac{12d^2ca\,\mathrm{arctanh}(\mathrm{e}^{i(xb-}}{b^3}$ |

input
```
int((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a)
)-1)+6*I*d^3/b^3*polylog(2,-exp(I*(b*x+a)))*x+6*I*d^2/b^3*c*polylog(2,-exp
(I*(b*x+a)))+12*d^2/b^3*c*a*arctanh(exp(I*(b*x+a)))+6*d^2/b^3*c*ln(1-exp(I
*(b*x+a)))*a-6*d^2/b^3*c*ln(exp(I*(b*x+a))+1)*a-6*I*d^2/b^3*c*polylog(2,ex
p(I*(b*x+a)))-6*I*d^3/b^3*polylog(2,exp(I*(b*x+a)))*x-6*d^3/b^4*a^2*arctan
h(exp(I*(b*x+a)))+6*d^2/b^2*c*ln(1-exp(I*(b*x+a)))*x-6*d^2/b^2*c*ln(exp(I*
(b*x+a))+1)*x+3*d^3/b^2*ln(1-exp(I*(b*x+a)))*x^2+6*d^3*polylog(3,exp(I*(b*
x+a)))/b^4-3*d^3/b^2*ln(exp(I*(b*x+a))+1)*x^2-6*d^3*polylog(3,-exp(I*(b*x+
a)))/b^4-3*d^3/b^4*ln(1-exp(I*(b*x+a)))*a^2+3*d^3/b^4*ln(exp(I*(b*x+a))+1)
*a^2-6*d/b^2*c^2*arctanh(exp(I*(b*x+a)))
```

### 3.40.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 669 vs. $2(130) = 260$.

Time = 0.29 (sec) , antiderivative size = 669, normalized size of antiderivative = 4.58

$$\int (c+dx)^3\cot(a+bx)\csc(a+bx)\,dx =$$
$$-\frac{2\,b^3d^3x^3+6\,b^3cd^2x^2+6\,b^3c^2dx+2\,b^3c^3-6\,d^3\mathrm{polylog}(3,\cos(bx+a)+i\,\sin(bx+a))\sin(bx+a)-6\,}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 6*d^3*
polylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 6*d^3*polylog(3,
cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x +
a) + I*sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a) - I*sin
(b*x + a))*sin(b*x + a) + 6*(I*b*d^3*x + I*b*c*d^2)*dilog(cos(b*x + a) + I
*sin(b*x + a))*sin(b*x + a) + 6*(-I*b*d^3*x - I*b*c*d^2)*dilog(cos(b*x + a
) - I*sin(b*x + a))*sin(b*x + a) + 6*(I*b*d^3*x + I*b*c*d^2)*dilog(-cos(b*
x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*(-I*b*d^3*x - I*b*c*d^2)*dilog(-
cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2
*x + b^2*c^2*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(b
^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a) - I*sin(b*x + a)
+ 1)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x
 + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^
2 + a^2*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a
) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(b*x +
 a) + I*sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x +
2*a*b*c*d^2 - a^2*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a
))/(b^4*sin(b*x + a))
```

### 3.40.6 Sympy [F]

$$\int (c+dx)^3 \cot(a+bx)\csc(a+bx)\,dx = \int (c+dx)^3 \cos(a+bx)\csc^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*cos(b*x+a)*csc(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**3*cos(a + b*x)*csc(a + b*x)**2, x)
```

### 3.40.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1770 vs. $2(130) = 260$.

Time = 0.38 (sec) , antiderivative size = 1770, normalized size of antiderivative = 12.12

$$\int (c+dx)^3 \cot(a+bx)\csc(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(3*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x
 + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*
b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
- (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(c
os(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x
 + a))*c^2*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a
) + 1)*b) - 6*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos
(2*b*x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*
cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a)
 + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)
*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*s
in(b*x + a))*a*c*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b
*x + 2*a) + 1)*b^2) + 3*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*
x + a)*cos(2*b*x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2
*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*co
s(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x +
 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(
b*x + a)*sin(b*x + a))*a^2*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 -
 2*cos(2*b*x + 2*a) + 1)*b^3) + 2*c^3/sin(b*x + a) - 6*a*c^2*d/(b*sin(b*x
+ a)) + 6*a^2*c*d^2/(b^2*sin(b*x + a)) - 2*a^3*d^3/(b^3*sin(b*x + a)) -...
```

## 3.40.8 Giac [F]

$$\int (c+dx)^3 \cot(a+bx)\csc(a+bx)\,dx = \int (dx+c)^3 \cos(bx+a)\csc(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*cos(b*x + a)*csc(b*x + a)^2, x)
```

### 3.40.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \cot(a+bx) \csc(a+bx)\, dx = \int \frac{\cos(a+b\,x)\,(c+d\,x)^3}{\sin(a+b\,x)^2}\, dx$$

input
```
int((cos(a + b*x)*(c + d*x)^3)/sin(a + b*x)^2,x)
```

output
```
int((cos(a + b*x)*(c + d*x)^3)/sin(a + b*x)^2, x)
```

# 3.41 $\int (c+dx)^2 \cot(a+bx)\csc(a+bx)\,dx$

3.41.1 Optimal result . . . . . . . . . . 419
3.41.2 Mathematica [B] (verified) . . . . . . . . . . 419
3.41.3 Rubi [A] (verified) . . . . . . . . . . 420
3.41.4 Maple [B] (verified) . . . . . . . . . . 422
3.41.5 Fricas [B] (verification not implemented) . . . . . . . . . . 422
3.41.6 Sympy [F] . . . . . . . . . . 423
3.41.7 Maxima [B] (verification not implemented) . . . . . . . . . . 423
3.41.8 Giac [F] . . . . . . . . . . 424
3.41.9 Mupad [F(-1)] . . . . . . . . . . 424

## 3.41.1 Optimal result

Integrand size = 20, antiderivative size = 90

$$\int (c+dx)^2 \cot(a+bx)\csc(a+bx)\,dx = -\frac{4d(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2\csc(a+bx)}{b} + \frac{2id^2\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3} - \frac{2id^2\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}$$

output
```
-4*d*(d*x+c)*arctanh(exp(I*(b*x+a)))/b^2-(d*x+c)^2*csc(b*x+a)/b+2*I*d^2*po
lylog(2,-exp(I*(b*x+a)))/b^3-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^3
```

## 3.41.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 234 vs. 2(90) = 180.

Time = 2.37 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.60

$$\int (c+dx)^2 \cot(a+bx)\csc(a+bx)\,dx$$

$$= \frac{-8bcd\text{arctanh}\left(\cos(a)-\sin(a)\tan\left(\frac{bx}{2}\right)\right) - 2b^2(c+dx)^2\csc(a) + 4d^2\left(2\arctan(\tan(a))\text{arctanh}(\cos(a)-\text{s}\right.}{}$$

input
```
Integrate[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x],x]
```

output
```
(-8*b*c*d*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] - 2*b^2*(c + d*x)^2*Csc[a]
 + 4*d^2*(2*ArcTan[Tan[a]]*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] + (((b*x
+ ArcTan[Tan[a]])*(Log[1 - E^(I*(b*x + ArcTan[Tan[a]]))] - Log[1 + E^(I*(b
*x + ArcTan[Tan[a]]))]) + I*PolyLog[2, -E^(I*(b*x + ArcTan[Tan[a]]))] - I*
PolyLog[2, E^(I*(b*x + ArcTan[Tan[a]]))])*Sec[a])/Sqrt[Sec[a]^2]) + b^2*(c
 + d*x)^2*Csc[a/2]*Csc[(a + b*x)/2]*Sin[(b*x)/2] - b^2*(c + d*x)^2*Sec[a/2
]*Sec[(a + b*x)/2]*Sin[(b*x)/2])/(2*b^3)
```

### 3.41.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4910, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \cot(a+bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{4910}$$

$$\frac{2d\int (c+dx)\csc(a+bx)dx}{b}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\int (c+dx)\csc(a+bx)dx}{b}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{(c+dx)^2\csc(a+bx)}{b}+\frac{2d\left(-\frac{d\int \log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{d\int \log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{(c+dx)^2\csc(a+bx)}{b}+\frac{2d\left(\frac{id\int e^{-i(a+bx)}\log\left(1-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{id\int e^{-i(a+bx)}\log\left(1+e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{(c+dx)^2\csc(a+bx)}{b}+\frac{2d\left(-\frac{2(c+dx)\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{id\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}\right)}{b}$$

input
```
Int[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x],x]
```

output
```
-(((c + d*x)^2*Csc[a + b*x])/b) + (2*d*((-2*(c + d*x)*ArcTanh[E^(I*(a + b*
x))])/b + (I*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, E^(I*(a
 + b*x))])/b^2))/b
```

### 3.41.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

### 3.41.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(82) = 164$.

Time = 0.78 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.36

| method | result |
|---|---|
| risch | $-\frac{2i(x^2d^2+2cdx+c^2)\mathrm{e}^{i(xb+a)}}{b(\mathrm{e}^{2i(xb+a)}-1)}-\frac{4dc\,\mathrm{arctanh}(\mathrm{e}^{i(xb+a)})}{b^2}+\frac{2d^2\ln(1-\mathrm{e}^{i(xb+a)})x}{b^2}+\frac{2d^2\ln(1-\mathrm{e}^{i(xb+a)})a}{b^3}-\frac{2id^2\,\mathrm{p}}{}$ |
| derivativedivides | $\frac{-\frac{a^2d^2}{b^2\sin(xb+a)}+\frac{2acd}{b\sin(xb+a)}-\frac{2a\,d^2\left(-\frac{xb+a}{\sin(xb+a)}+\ln(\csc(xb+a)-\cot(xb+a))\right)}{b^2}-\frac{c^2}{\sin(xb+a)}+\frac{2cd\left(-\frac{xb+a}{\sin(xb+a)}+\ln(\csc(xb+a)-\cot(\right.}{b}}{b}$ |
| default | $\frac{-\frac{a^2d^2}{b^2\sin(xb+a)}+\frac{2acd}{b\sin(xb+a)}-\frac{2a\,d^2\left(-\frac{xb+a}{\sin(xb+a)}+\ln(\csc(xb+a)-\cot(xb+a))\right)}{b^2}-\frac{c^2}{\sin(xb+a)}+\frac{2cd\left(-\frac{xb+a}{\sin(xb+a)}+\ln(\csc(xb+a)-\cot(\right.}{b}}{b}$ |

input
```
int((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*I*(d^2*x^2+2*c*d*x+c^2)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)-4*d/b^2*c
*arctanh(exp(I*(b*x+a)))+2*d^2/b^2*ln(1-exp(I*(b*x+a)))*x+2*d^2/b^3*ln(1-e
xp(I*(b*x+a)))*a-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^3-2*d^2/b^2*ln(exp(I*
(b*x+a))+1)*x-2*d^2/b^3*ln(exp(I*(b*x+a))+1)*a+2*I*d^2*polylog(2,-exp(I*(b
*x+a)))/b^3+4*d^2/b^3*a*arctanh(exp(I*(b*x+a)))
```

### 3.41.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(78) = 156$.

Time = 0.28 (sec) , antiderivative size = 375, normalized size of antiderivative = 4.17

$$\int (c+dx)^2\cot(a+bx)\csc(a+bx)\,dx =$$
$$-\frac{b^2d^2x^2+2\,b^2cdx+b^2c^2+i\,d^2\mathrm{Li}_2(\cos(bx+a)+i\,\sin(bx+a))\sin(bx+a)-i\,d^2\mathrm{Li}_2(\cos(bx+a)-i\,\mathrm{s}}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*dilog(cos(b*x + a) + I*sin(b
*x + a))*sin(b*x + a) - I*d^2*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x
 + a) + I*d^2*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - I*d^2*d
ilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + (b*d^2*x + b*c*d)*log(
cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + (b*d^2*x + b*c*d)*log(co
s(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - (b*c*d - a*d^2)*log(-1/2*c
os(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - (b*c*d - a*d^2)*log
(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - (b*d^2*x + a
*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - (b*d^2*x + a*
d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a))/(b^3*sin(b*x +
a))
```

## 3.41.6 Sympy [F]

$$\int (c+dx)^2 \cot(a+bx) \csc(a+bx)\, dx = \int (c+dx)^2 \cos(a+bx) \csc^2(a+bx)\, dx$$

input
```
integrate((d*x+c)**2*cos(b*x+a)*csc(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*cos(a + b*x)*csc(a + b*x)**2, x)
```

## 3.41.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 553 vs. $2(78) = 156$.

Time = 0.35 (sec) , antiderivative size = 553, normalized size of antiderivative = 6.14

$$\int (c+dx)^2 \cot(a+bx) \csc(a+bx)\, dx$$
$$= \frac{2\left(bd^2x + bcd - (bd^2x + bcd)\cos(2\,bx + 2\,a) + (-i\,bd^2x - i\,bcd)\sin(2\,bx + 2\,a)\right)\arctan(\sin(bx + a), \cos}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
(2*(b*d^2*x + b*c*d - (b*d^2*x + b*c*d)*cos(2*b*x + 2*a) + (-I*b*d^2*x - I
*b*c*d)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 2*(b*c
*d*cos(2*b*x + 2*a) + I*b*c*d*sin(2*b*x + 2*a) - b*c*d)*arctan2(sin(b*x +
a), cos(b*x + a) - 1) - 2*(b*d^2*x*cos(2*b*x + 2*a) + I*b*d^2*x*sin(2*b*x
+ 2*a) - b*d^2*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a) + 2*(d^2*cos(2*b*x + 2*a) + I*d^2*
sin(2*b*x + 2*a) - d^2)*dilog(-e^(I*b*x + I*a)) - 2*(d^2*cos(2*b*x + 2*a)
+ I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(e^(I*b*x + I*a)) - (I*b*d^2*x + I*b*
c*d + (-I*b*d^2*x - I*b*c*d)*cos(2*b*x + 2*a) + (b*d^2*x + b*c*d)*sin(2*b*
x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-I*
b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(2*b*x + 2*a) - (b*d^2*x + b*
c*d)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a
) + 1) + 2*(-I*b^2*d^2*x^2 - 2*I*b^2*c*d*x - I*b^2*c^2)*sin(b*x + a))/(-I*
b^3*cos(2*b*x + 2*a) + b^3*sin(2*b*x + 2*a) + I*b^3)
```

### 3.41.8 Giac [F]

$$\int (c+dx)^2 \cot(a+bx)\csc(a+bx)\,dx = \int (dx+c)^2 \cos(bx+a)\csc(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*cos(b*x + a)*csc(b*x + a)^2, x)
```

### 3.41.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \cot(a+bx)\csc(a+bx)\,dx = \int \frac{\cos(a+b\,x)\,(c+d\,x)^2}{\sin(a+b\,x)^2}\,dx$$

input
```
int((cos(a + b*x)*(c + d*x)^2)/sin(a + b*x)^2,x)
```

output
```
int((cos(a + b*x)*(c + d*x)^2)/sin(a + b*x)^2, x)
```

# 3.42 $\int (c + dx) \cot(a + bx) \csc(a + bx) \, dx$

3.42.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 425
3.42.2 Mathematica [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 425
3.42.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 426
3.42.4 Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 427
3.42.5 Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . 428
3.42.6 Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 428
3.42.7 Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . 428
3.42.8 Giac [B] (verification not implemented) . . . . . . . . . . . . . . . . . 429
3.42.9 Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . 430

## 3.42.1 Optimal result

Integrand size = 18, antiderivative size = 30

$$\int (c + dx) \cot(a + bx) \csc(a + bx) \, dx = -\frac{d \operatorname{arctanh}(\cos(a + bx))}{b^2} - \frac{(c + dx) \csc(a + bx)}{b}$$

output
```
-d*arctanh(cos(b*x+a))/b^2-(d*x+c)*csc(b*x+a)/b
```

## 3.42.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 131 vs. 2(30) = 60.

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 4.37

$$\begin{aligned}\int (c + dx) \cot(a + bx) \csc(a + bx) \, dx = &-\frac{dx \csc(a)}{b} - \frac{c \csc(a + bx)}{b} \\ &- \frac{d \log\left(\cos\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{b^2} + \frac{d \log\left(\sin\left(\frac{a}{2} + \frac{bx}{2}\right)\right)}{b^2} \\ &+ \frac{dx \csc\left(\frac{a}{2}\right) \csc\left(\frac{a}{2} + \frac{bx}{2}\right) \sin\left(\frac{bx}{2}\right)}{2b} \\ &- \frac{dx \sec\left(\frac{a}{2}\right) \sec\left(\frac{a}{2} + \frac{bx}{2}\right) \sin\left(\frac{bx}{2}\right)}{2b}\end{aligned}$$

input
```
Integrate[(c + d*x)*Cot[a + b*x]*Csc[a + b*x],x]
```

output
```
-((d*x*Csc[a])/b) - (c*Csc[a + b*x])/b - (d*Log[Cos[a/2 + (b*x)/2]])/b^2 +
 (d*Log[Sin[a/2 + (b*x)/2]])/b^2 + (d*x*Csc[a/2]*Csc[a/2 + (b*x)/2]*Sin[(b
*x)/2])/(2*b) - (d*x*Sec[a/2]*Sec[a/2 + (b*x)/2]*Sin[(b*x)/2])/(2*b)
```

### 3.42.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4910, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\cot(a+bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{ 4910}$$

$$\frac{d\int \csc(a+bx)dx}{b}-\frac{(c+dx)\csc(a+bx)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{d\int \csc(a+bx)dx}{b}-\frac{(c+dx)\csc(a+bx)}{b}$$

$$\Big\downarrow \text{ 4257}$$

$$-\frac{d\text{arctanh}(\cos(a+bx))}{b^2}-\frac{(c+dx)\csc(a+bx)}{b}$$

input
```
Int[(c + d*x)*Cot[a + b*x]*Csc[a + b*x],x]
```

output
```
-((d*ArcTanh[Cos[a + b*x]])/b^2) - ((c + d*x)*Csc[a + b*x])/b
```

#### 3.42.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

## 3.42.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.53

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{2\ln\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\right)d-\sec\left(\frac{a}{2}+\frac{xb}{2}\right)\csc\left(\frac{a}{2}+\frac{xb}{2}\right)b(dx+c)}{2b^{2}}$ | 46 |
| derivativedivides | $\frac{\frac{da}{b\sin(xb+a)}-\frac{c}{\sin(xb+a)}+\frac{d\left(-\frac{xb+a}{\sin(xb+a)}+\ln(\csc(xb+a)-\cot(xb+a))\right)}{b}}{b}$ | 68 |
| default | $\frac{\frac{da}{b\sin(xb+a)}-\frac{c}{\sin(xb+a)}+\frac{d\left(-\frac{xb+a}{\sin(xb+a)}+\ln(\csc(xb+a)-\cot(xb+a))\right)}{b}}{b}$ | 68 |
| risch | $-\frac{2i(dx+c)e^{i(xb+a)}}{b\left(e^{2i(xb+a)}-1\right)}+\frac{d\ln\left(e^{i(xb+a)}-1\right)}{b^{2}}-\frac{d\ln\left(e^{i(xb+a)}+1\right)}{b^{2}}$ | 70 |
| norman | $\frac{-\frac{c}{2b}-\frac{c\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}}{2b}-\frac{dx}{2b}-\frac{dx\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}}{2b}}{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}+\frac{d\ln\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\right)}{b^{2}}$ | 78 |

input
```
int((d*x+c)*cos(b*x+a)*csc(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*ln(tan(1/2*a+1/2*x*b))*d-sec(1/2*a+1/2*x*b)*csc(1/2*a+1/2*x*b)*b*(d
*x+c))/b^2
```

### 3.42.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(30) = 60.

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.07

$$\int (c+dx)\cot(a+bx)\csc(a+bx)\,dx = -\frac{2\,bdx + d\log\left(\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)\sin\left(bx+a\right) - d\log\left(-\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)\sin\left(bx+a\right)+2\,bc}{2\,b^2\sin\left(bx+a\right)}$$

input
```
integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*b*d*x + d*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - d*log(-1/2*co
s(b*x + a) + 1/2)*sin(b*x + a) + 2*b*c)/(b^2*sin(b*x + a))
```

### 3.42.6 Sympy [F]

$$\int (c+dx)\cot(a+bx)\csc(a+bx)\,dx = \int (c+dx)\cos\left(a+bx\right)\csc^2\left(a+bx\right)dx$$

input
```
integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*cos(a + b*x)*csc(a + b*x)**2, x)
```

### 3.42.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(30) = 60.

Time = 0.23 (sec) , antiderivative size = 259, normalized size of antiderivative = 8.63

$$\int (c+dx)\cot(a+bx)\csc(a+bx)\,dx = -\frac{\left(4\left(bx+a\right)\cos(bx+a)\sin(2\,bx+2\,a)-4\left(bx+a\right)\cos(2\,bx+2\,a)\sin(bx+a)+\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2-2\,\cos(2\,bx+2\,a)+1\right)\log\left(\cos(bx+a)\right.\right.}{\left(\cos(2\,bx+2\,a)^2+\right.}$$

input
```
integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*((4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x +
 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*
x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) -
(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos
(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x +
 a))*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)
*b) + 2*c/sin(b*x + a) - 2*a*d/(b*sin(b*x + a)))/b
```

### 3.42.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(30) = 60$.

Time = 0.54 (sec) , antiderivative size = 697, normalized size of antiderivative = 23.23

$$\int (c + dx)\cot(a + bx)\csc(a + bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*c*tan(1/2*b*x)^2*tan(1/2*a)^2 +
 b*d*x*tan(1/2*b*x)^2 - d*log(4*(tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b
*x)*tan(1/2*a) + 1)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/
2*a)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a) + d*log(4*(tan(1/2*b*x)^2 + 2*tan(1
/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*
b*x)^2 + tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a) + b*d*x*tan(1/2*a)^2
 - d*log(4*(tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(
tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 + 1))*tan(1/2*
b*x)*tan(1/2*a)^2 + d*log(4*(tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) +
tan(1/2*a)^2)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2
 + 1))*tan(1/2*b*x)*tan(1/2*a)^2 + b*c*tan(1/2*b*x)^2 + b*c*tan(1/2*a)^2 +
 b*d*x + d*log(4*(tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)
+ 1)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 + 1))*ta
n(1/2*b*x) - d*log(4*(tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2
*a)^2)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 + 1))*
tan(1/2*b*x) + d*log(4*(tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1
/2*a) + 1)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 +
1))*tan(1/2*a) - d*log(4*(tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan
(1/2*a)^2)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 +
1))*tan(1/2*a) + b*c)/(b^2*tan(1/2*b*x)^2*tan(1/2*a) + b^2*tan(1/2*b*x)...
```

### 3.42.9 Mupad [B] (verification not implemented)

Time = 24.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.93

$$\int (c+dx)\cot(a+bx)\csc(a+bx)\,dx = -\frac{d\,\ln\left(\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}{b^2} + \frac{d\,\ln\left(d\,2\mathrm{i}-d\,\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,1\mathrm{i}}\,2\mathrm{i}\right)}{b^2} - \frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,(c+d\,x)\,2\mathrm{i}}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}$$

input
```
int((cos(a + b*x)*(c + d*x))/sin(a + b*x)^2,x)
```

output
```
(d*log(d*2i - d*exp(a*1i)*exp(b*x*1i)*2i))/b^2 - (d*log(exp(a*1i + b*x*1i)
*1i + 1i))/b^2 - (exp(a*1i + b*x*1i)*(c + d*x)*2i)/(b*(exp(a*2i + b*x*2i)
- 1))
```

# 3.43 $\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$

3.43.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 431
3.43.2 Mathematica [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 431
3.43.3 Rubi [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 432
3.43.4 Maple [N/A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 432
3.43.5 Fricas [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 433
3.43.6 Sympy [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 433
3.43.7 Maxima [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 433
3.43.8 Giac [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 434
3.43.9 Mupad [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 434

## 3.43.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx=\operatorname{Int}\left(\frac{\cot(a+bx)\csc(a+bx)}{c+dx},x\right)$$

output
```
CannotIntegrate(cot(b*x+a)*csc(b*x+a)/(d*x+c),x)
```

## 3.43.2 Mathematica [N/A]

Not integrable

Time = 19.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx=\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x),x]
```

output
```
Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x), x]
```

### 3.43.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx} \, dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}dx$$

input
```
Int[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x),x]
```

output
```
$Aborted
```

#### 3.43.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

### 3.43.4 Maple [N/A] (verified)

Not integrable

Time = 0.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(xb+a\right)\csc\left(xb+a\right)^{2}}{dx+c}dx$$

input
```
int(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x)
```

output
```
int(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x)
```

### 3.43.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)\csc(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)*csc(b*x + a)^2/(d*x + c), x)
```

### 3.43.6 Sympy [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cos(a+bx)\csc^2(a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(cos(a + b*x)*csc(a + b*x)**2/(c + d*x), x)
```

### 3.43.7 Maxima [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 508, normalized size of antiderivative = 25.40

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)\csc(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
-((b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c
*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a))*integrate(s
in(b*x + a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^
2)*cos(b*x + a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 + 2*(b*
d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)), x) + (b*d^2*x + b*c*d + (b*d^2
*x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 - 2*
(b*d^2*x + b*c*d)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^2*x^2 + 2*
b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)^2 + (b*d^2*
x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2
)*cos(b*x + a)), x) + 2*cos(b*x + a)*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a)
*sin(b*x + a) + 2*sin(b*x + a))/(b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2
+ (b*d*x + b*c)*sin(2*b*x + 2*a)^2 + b*c - 2*(b*d*x + b*c)*cos(2*b*x + 2*a
))
```

## 3.43.8 Giac [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)\csc(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*csc(b*x + a)^2/(d*x + c), x)
```

## 3.43.9 Mupad [N/A]

Not integrable

Time = 23.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cos(a+b\,x)}{\sin(a+b\,x)^2\,(c+d\,x)}\,dx$$

input
```
int(cos(a + b*x)/(sin(a + b*x)^2*(c + d*x)),x)
```

output
```
int(cos(a + b*x)/(sin(a + b*x)^2*(c + d*x)), x)
```

# 3.44 $\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2} dx$

3.44.1 Optimal result . . . . . 435
3.44.2 Mathematica [N/A] . . . . . 435
3.44.3 Rubi [N/A] . . . . . 436
3.44.4 Maple [N/A] (verified) . . . . . 436
3.44.5 Fricas [N/A] . . . . . 437
3.44.6 Sympy [N/A] . . . . . 437
3.44.7 Maxima [N/A] . . . . . 437
3.44.8 Giac [N/A] . . . . . 438
3.44.9 Mupad [N/A] . . . . . 438

## 3.44.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2} dx = \text{Int}\left(\frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}, x\right)$$

output
```
CannotIntegrate(cot(b*x+a)*csc(b*x+a)/(d*x+c)^2,x)
```

## 3.44.2 Mathematica [N/A]

Not integrable

Time = 22.94 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2} dx = \int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2} dx$$

input
```
Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2, x]
```

### 3.44.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2} \, dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}dx$$

input
```
Int[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

#### 3.44.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

### 3.44.4 Maple [N/A] (verified)

Not integrable

Time = 0.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos\left(xb+a\right)\csc\left(xb+a\right)^{2}}{\left(dx+c\right)^{2}}dx$$

input
```
int(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x)
```

### 3.44.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(bx+a\right)\csc\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)*csc(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.44.6 Sympy [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(a+bx\right)\csc^2\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(cos(a + b*x)*csc(a + b*x)**2/(c + d*x)**2, x)
```

### 3.44.7 Maxima [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 745, normalized size of antiderivative = 37.25

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(bx+a\right)\csc\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-2*((b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*
d)*cos(2*b*x + 2*a)^2 + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*sin(2*b*x + 2*
a)^2 - 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a))*integrate(s
in(b*x + a)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3
+ 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)^2 + (b*d^3*x^3 + 3*b*c
*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(b*x + a)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*
x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)), x) + (b*d^3*x^2 + 2*b*c*d^2*x +
b*c^2*d + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a)^2 + (b*d^3*
x^2 + 2*b*c*d^2*x + b*c^2*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^3*x^2 + 2*b*c*d^2
*x + b*c^2*d)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^3*x^3 + 3*b*c*
d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x +
 b*c^3)*cos(b*x + a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)
*sin(b*x + a)^2 - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(
b*x + a)), x) + cos(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)*sin(b*x +
 a) + sin(b*x + a))/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*
x + b*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x
+ 2*a)^2 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a))
```

## 3.44.8 Giac [N/A]

Not integrable

Time = 2.88 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(bx+a\right)\csc\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*csc(b*x + a)^2/(d*x + c)^2, x)
```

## 3.44.9 Mupad [N/A]

Not integrable

Time = 23.87 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(a+b\,x\right)}{\sin\left(a+b\,x\right)^2\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(cos(a + b*x)/(sin(a + b*x)^2*(c + d*x)^2),x)
```

output
```
int(cos(a + b*x)/(sin(a + b*x)^2*(c + d*x)^2), x)
```

## 3.44. $\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$

# 3.45 $\int (c+dx)^m \cot(a+bx)\csc^2(a+bx)\,dx$

3.45.1 Optimal result . . . . . . . . . . 440
3.45.2 Mathematica [N/A] . . . . . . . . . . 440
3.45.3 Rubi [N/A] . . . . . . . . . . 441
3.45.4 Maple [N/A] (verified) . . . . . . . . . . 441
3.45.5 Fricas [N/A] . . . . . . . . . . 442
3.45.6 Sympy [F(-2)] . . . . . . . . . . 442
3.45.7 Maxima [N/A] . . . . . . . . . . 442
3.45.8 Giac [N/A] . . . . . . . . . . 443
3.45.9 Mupad [N/A] . . . . . . . . . . 443

## 3.45.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (c+dx)^m \cot(a+bx)\csc^2(a+bx)\,dx = \text{Int}\left((c+dx)^m \cot(a+bx)\csc^2(a+bx),x\right)$$

output
```
CannotIntegrate((d*x+c)^m*cot(b*x+a)*csc(b*x+a)^2,x)
```

## 3.45.2 Mathematica [N/A]

Not integrable

Time = 7.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cot(a+bx)\csc^2(a+bx)\,dx = \int (c+dx)^m \cot(a+bx)\csc^2(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2, x]
```

### 3.45.3 Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(a+bx)\csc^2(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \cot(a+bx)\csc^2(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

output
```
$Aborted
```

#### 3.45.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

### 3.45.4 Maple [N/A] (verified)

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \cos(xb+a)\csc(xb+a)^3\,dx$$

input
```
int((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x)
```

output
```
int((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x)
```

### 3.45.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cot(a+bx)\csc^2(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\csc(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^3, x)
```

### 3.45.6 Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m \cot(a+bx)\csc^2(a+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d*x+c)**m*cos(b*x+a)*csc(b*x+a)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.45.7 Maxima [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cot(a+bx)\csc^2(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\csc(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^3, x)
```

### 3.45.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cot(a+bx) \csc^2(a+bx)\, dx = \int (dx+c)^m \cos(bx+a) \csc(bx+a)^3\, dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^3, x)
```

### 3.45.9 Mupad [N/A]

Not integrable

Time = 23.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cot(a+bx) \csc^2(a+bx)\, dx = \int \frac{\cos(a+b\,x)\ (c+d\,x)^m}{\sin(a+b\,x)^3}\, dx$$

input
```
int((cos(a + b*x)*(c + d*x)^m)/sin(a + b*x)^3,x)
```

output
```
int((cos(a + b*x)*(c + d*x)^m)/sin(a + b*x)^3, x)
```

# 3.46 $\int (c+dx)^4 \cot(a+bx) \csc^2(a+bx) \, dx$


3.46.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 444
3.46.2 Mathematica [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 445
3.46.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 446
3.46.4 Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 449
3.46.5 Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . 450
3.46.6 Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 451
3.46.7 Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . . 452
3.46.8 Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 452
3.46.9 Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 453


## 3.46.1 Optimal result

Integrand size = 22, antiderivative size = 137

$$\int (c+dx)^4 \cot(a+bx) \csc^2(a+bx) \, dx = -\frac{2id(c+dx)^3}{b^2} - \frac{2d(c+dx)^3 \cot(a+bx)}{b^2} - \frac{(c+dx)^4 \csc^2(a+bx)}{2b} + \frac{6d^2(c+dx)^2 \log\left(1-e^{2i(a+bx)}\right)}{b^3} - \frac{6id^3(c+dx) \operatorname{PolyLog}\left(2, e^{2i(a+bx)}\right)}{b^4} + \frac{3d^4 \operatorname{PolyLog}\left(3, e^{2i(a+bx)}\right)}{b^5}$$

output
```
-2*I*d*(d*x+c)^3/b^2-2*d*(d*x+c)^3*cot(b*x+a)/b^2-1/2*(d*x+c)^4*csc(b*x+a)
^2/b+6*d^2*(d*x+c)^2*ln(1-exp(2*I*(b*x+a)))/b^3-6*I*d^3*(d*x+c)*polylog(2,
exp(2*I*(b*x+a)))/b^4+3*d^4*polylog(3,exp(2*I*(b*x+a)))/b^5
```

### 3.46.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 512 vs. $2(137) = 274$.

Time = 6.64 (sec) , antiderivative size = 512, normalized size of antiderivative = 3.74

$$\int (c+dx)^4 \cot(a+bx)\csc^2(a+bx)\,dx = -\frac{(c+dx)^4\csc^2(a+bx)}{2b}$$
$$-\frac{d^4e^{ia}\csc(a)\left(2b^3e^{-2ia}x^3+3ib^2(1-e^{-2ia})\,x^2\log\left(1-e^{-i(a+bx)}\right)+3ib^2(1-e^{-2ia})\,x^2\log\left(1+e^{-i(a+bx)}\right)-\right.}{}$$
$$+\frac{6c^2d^2\csc(a)(-bx\cos(a)+\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a))}{b^3\left(\cos^2(a)+\sin^2(a)\right)}$$
$$+\frac{2\csc(a)\csc(a+bx)\left(c^3d\sin(bx)+3c^2d^2x\sin(bx)+3cd^3x^2\sin(bx)+d^4x^3\sin(bx)\right)}{b^2}$$
$$-\frac{6cd^3\csc(a)\sec(a)\left(b^2e^{i\arctan(\tan(a))}x^2+\frac{(ibx(-\pi+2\arctan(\tan(a)))-\pi\log(1+e^{-2ibx})-2(bx+\arctan(\tan(a)))\log(1-e^{2i(bx+\text{arc}}}{}\right.}{b^4\sqrt{\sec^2(a)\left(\cos^2(a)+\right.}}$$

input
```
Integrate[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

output
```
-1/2*((c + d*x)^4*Csc[a + b*x]^2)/b - (d^4*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^(
(2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] +
(3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - 6*b*(1 - E^
((-2*I)*a))*x*PolyLog[2, -E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*P
olyLog[2, E^((-I)*(a + b*x))] + (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, -E^((-
I)*(a + b*x))] + (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, E^((-I)*(a + b*x))]))
/b^5 + (6*c^2*d^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin
[b*x]]*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) + (2*Csc[a]*Csc[a + b*x]*(c^3*
d*Sin[b*x] + 3*c^2*d^2*x*Sin[b*x] + 3*c*d^3*x^2*Sin[b*x] + d^4*x^3*Sin[b*x
]))/b^2 - (6*c*d^3*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(
-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan
[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*Arc
Tan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x +
ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^4*Sqrt[Sec[a]^2*(Cos[a]
^2 + Sin[a]^2)])
```

### 3.46.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.31, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {4910, 3042, 4672, 3042, 25, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \cot(a+bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 4910}$$

$$\frac{2d\int (c+dx)^3\csc^2(a+bx)dx}{b}-\frac{(c+dx)^4\csc^2(a+bx)}{2b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2d\int (c+dx)^3\csc(a+bx)^2dx}{b}-\frac{(c+dx)^4\csc^2(a+bx)}{2b}$$

$$\Big\downarrow \text{ 4672}$$

$$\frac{2d\left(\frac{3d\int (c+dx)^2\cot(a+bx)dx}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}\right)}{b}-\frac{(c+dx)^4\csc^2(a+bx)}{2b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2d\left(\frac{3d\int -(c+dx)^2\tan\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}\right)}{b}-\frac{(c+dx)^4\csc^2(a+bx)}{2b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2d\left(-\frac{3d\int (c+dx)^2\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}\right)}{b}-\frac{(c+dx)^4\csc^2(a+bx)}{2b}$$

$$\Big\downarrow \text{ 4202}$$

$$-\frac{(c+dx)^4\csc^2(a+bx)}{2b}+\frac{2d\left(-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\int\frac{e^{i(2a+2bx+\pi)}(c+dx)^2}{1+e^{i(2a+2bx+\pi)}}dx\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{(c+dx)^4\csc^2(a+bx)}{2b}+\frac{2d\left(-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\int(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}\right)}{b}$$

$\downarrow$ 3011

$$-\frac{(c+dx)^4\csc^2(a+bx)}{2b}+ \frac{2d\left(-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a\cdots}\right.}{2b}\cdots\right.\right.}{b}\cdots\right.}{b}$$

$\downarrow$ 2720

$$-\frac{(c+dx)^4\csc^2(a+bx)}{2b}+ \frac{2d\left(-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\cdots}}{4b^2}\cdots\right.}{b}\cdots\right.\right.}{b}\cdots\right.}{b}$$

$\downarrow$ 7143

$$-\frac{(c+dx)^4\csc^2(a+bx)}{2b}+ \frac{2d\left(-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2b\cdots}\right.}{2b}\cdots\right.\right.}{b}\cdots\right.}{b}$$

input
```
Int[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

## 3.46. $\int(c+dx)^4\cot(a+bx)\csc^2(a+bx)\,dx$

output
```
-1/2*((c + d*x)^4*Csc[a + b*x]^2)/b + (2*d*(-(((c + d*x)^3*Cot[a + b*x])/b
) - (3*d*(((I/3)*(c + d*x)^3)/d - (2*I)*(((-1/2*I)*(c + d*x)^2*Log[1 + E^(
I*(2*a + Pi + 2*b*x))])/b + (I*d*(((I/2)*(c + d*x)*PolyLog[2, -E^(I*(2*a +
 Pi + 2*b*x))])/b - (d*PolyLog[3, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2)))/b)
))/b))/b
```

### 3.46.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.46.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 715 vs. $2(127) = 254$.

Time = 1.82 (sec) , antiderivative size = 716, normalized size of antiderivative = 5.23

| method | result |
|---|---|
| risch | $-\frac{12id^4\operatorname{polylog}(2,\mathrm{e}^{i(xb+a)})x}{b^4}+\frac{2b\,d^4x^4\mathrm{e}^{2i(xb+a)}+8bc\,d^3x^3\mathrm{e}^{2i(xb+a)}+12b\,c^2d^2x^2\mathrm{e}^{2i(xb+a)}+8b\,c^3dx\,\mathrm{e}^{2i(xb+a)}-4id^4x^3\mathrm{e}^{2i(xb+a)}+2}{b^2(}$ |

input
```
int((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-12*I*d^4/b^4*polylog(2,exp(I*(b*x+a)))*x+2*(b*d^4*x^4*exp(2*I*(b*x+a))+4*
b*c*d^3*x^3*exp(2*I*(b*x+a))+6*b*c^2*d^2*x^2*exp(2*I*(b*x+a))+4*b*c^3*d*x*
exp(2*I*(b*x+a))-2*I*d^4*x^3*exp(2*I*(b*x+a))+b*c^4*exp(2*I*(b*x+a))-6*I*c
*d^3*x^2*exp(2*I*(b*x+a))-6*I*c^2*d^2*x*exp(2*I*(b*x+a))+2*I*d^4*x^3-2*I*c
^3*d*exp(2*I*(b*x+a))+6*I*c*d^3*x^2+6*I*c^2*d^2*x+2*I*c^3*d)/b^2/(exp(2*I*
(b*x+a))-1)^2+12*d^3/b^4*c*ln(1-exp(I*(b*x+a)))*a+12*d^3/b^3*c*ln(1-exp(I*
(b*x+a)))*x+12*d^3/b^3*c*ln(exp(I*(b*x+a))+1)*x+24*d^3/b^4*c*a*ln(exp(I*(b
*x+a)))-12*d^3/b^4*c*a*ln(exp(I*(b*x+a))-1)-12*I*d^3/b^4*c*polylog(2,-exp(
I*(b*x+a)))+6*d^4/b^3*ln(1-exp(I*(b*x+a)))*x^2+6*d^4/b^3*ln(exp(I*(b*x+a))
+1)*x^2-12*d^4/b^5*a^2*ln(exp(I*(b*x+a)))+6*d^4/b^5*a^2*ln(exp(I*(b*x+a))-
1)-6*d^4/b^5*ln(1-exp(I*(b*x+a)))*a^2+6*d^2/b^3*c^2*ln(exp(I*(b*x+a))+1)-1
2*d^2/b^3*c^2*ln(exp(I*(b*x+a)))+6*d^2/b^3*c^2*ln(exp(I*(b*x+a))-1)-4*I*d^
4/b^2*x^3+8*I*d^4/b^5*a^3-12*I*d^4/b^4*polylog(2,-exp(I*(b*x+a)))*x+12*I*d
^4/b^4*a^2*x-12*I*d^3/b^2*c*x^2-12*I*d^3/b^4*c*a^2-12*I*d^3/b^4*c*polylog(
2,exp(I*(b*x+a)))-24*I*d^3/b^3*c*x*a+12*d^4*polylog(3,-exp(I*(b*x+a)))/b^5
+12*d^4*polylog(3,exp(I*(b*x+a)))/b^5
```

### 3.46.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1075 vs. 2(124) = 248.

Time = 0.30 (sec) , antiderivative size = 1075, normalized size of antiderivative = 7.85

$$\int (c+dx)^4 \cot(a+bx)\csc^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b
^4*c^4 + 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*c
os(b*x + a)*sin(b*x + a) - 12*(-I*b*d^4*x - I*b*c*d^3 + (I*b*d^4*x + I*b*c
*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) + I*sin(b*x + a)) - 12*(I*b*d^4*x
 + I*b*c*d^3 + (-I*b*d^4*x - I*b*c*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a)
 - I*sin(b*x + a)) - 12*(I*b*d^4*x + I*b*c*d^3 + (-I*b*d^4*x - I*b*c*d^3)*
cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 12*(-I*b*d^4*x - I
*b*c*d^3 + (I*b*d^4*x + I*b*c*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x + a) - I
*sin(b*x + a)) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - (b^2*d^4*x
^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin
(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - (b^2*d^4*x
^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) - I*sin
(b*x + a) + 1) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4 - (b^2*c^2*d^2 - 2
*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*
x + a) + 1/2) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4 - (b^2*c^2*d^2 - 2*
a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x
 + a) + 1/2) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4 - (b
^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*cos(b*x + a)^2)*log(-c
os(b*x + a) + I*sin(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b
*c*d^3 - a^2*d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4...
```

### 3.46.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^4 \cot(a+bx) \csc^2(a+bx) \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**4*cos(b*x+a)*csc(b*x+a)**3,x)
```

output
```
Timed out
```

### 3.46.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4551 vs. $2(124) = 248$.

Time = 0.55 (sec) , antiderivative size = 4551, normalized size of antiderivative = 33.22

$$\int (c+dx)^4 \cot(a+bx)\csc^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*(8*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 -
(2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*
x + a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)
 + 1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*c^3*d/((2*(2*cos(2*b*x + 2*a) -
 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b
*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 +
 4*cos(2*b*x + 2*a) - 1)*b) - 24*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x
+ a)*sin(2*b*x + 2*a)^2 - (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)
)*cos(4*b*x + 4*a) - 2*(b*x + a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x
 + 2*a) - cos(2*b*x + 2*a) + 1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*a*c^2
*d^2/((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 -
4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x +
 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^2) + 24*(4*(b*x +
 a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (2*(b*x + a)*cos
(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x + a)*cos(2*b*x
 + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*sin(4*b*x
+ 4*a) + sin(2*b*x + 2*a))*a^2*c*d^3/((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*
x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2
+ 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x
 + 2*a) - 1)*b^3) - 8*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin...
```

### 3.46.8 Giac [F]

$$\int (c+dx)^4 \cot(a+bx)\csc^2(a+bx)\,dx = \int (dx+c)^4 \cos(bx+a)\csc(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*cos(b*x + a)*csc(b*x + a)^3, x)
```

### 3.46.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \cot(a+bx) \csc^2(a+bx)\, dx = \int \frac{\cos(a+b\,x)\ (c+d\,x)^4}{\sin(a+b\,x)^3}\, dx$$

input
```
int((cos(a + b*x)*(c + d*x)^4)/sin(a + b*x)^3,x)
```

output
```
int((cos(a + b*x)*(c + d*x)^4)/sin(a + b*x)^3, x)
```

# 3.47 $\int (c+dx)^3 \cot(a+bx) \csc^2(a+bx) \, dx$


3.47.1 Optimal result . . . . . 454
3.47.2 Mathematica [B] (verified) . . . . . 454
3.47.3 Rubi [A] (verified) . . . . . 455
3.47.4 Maple [B] (verified) . . . . . 458
3.47.5 Fricas [B] (verification not implemented) . . . . . 459
3.47.6 Sympy [F] . . . . . 459
3.47.7 Maxima [B] (verification not implemented) . . . . . 460
3.47.8 Giac [F] . . . . . 460
3.47.9 Mupad [F(-1)] . . . . . 461


## 3.47.1 Optimal result

Integrand size = 22, antiderivative size = 115

$$\int (c+dx)^3 \cot(a+bx) \csc^2(a+bx) \, dx = -\frac{3id(c+dx)^2}{2b^2} - \frac{3d(c+dx)^2 \cot(a+bx)}{2b^2} - \frac{(c+dx)^3 \csc^2(a+bx)}{2b} + \frac{3d^2(c+dx) \log\left(1-e^{2i(a+bx)}\right)}{b^3} - \frac{3id^3 \operatorname{PolyLog}\left(2, e^{2i(a+bx)}\right)}{2b^4}$$

output
```
-3/2*I*d*(d*x+c)^2/b^2-3/2*d*(d*x+c)^2*cot(b*x+a)/b^2-1/2*(d*x+c)^3*csc(b*
x+a)^2/b+3*d^2*(d*x+c)*ln(1-exp(2*I*(b*x+a)))/b^3-3/2*I*d^3*polylog(2,exp(
2*I*(b*x+a)))/b^4
```

## 3.47.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 277 vs. 2(115) = 230.

Time = 6.46 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.41

$$\int (c+dx)^3 \cot(a+bx)\csc^2(a+bx)\,dx = -\frac{(c+dx)^3\csc^2(a+bx)}{2b} + \frac{3cd^2\csc(a)(-bx\cos(a)+\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a))}{b^3(\cos^2(a)+\sin^2(a))} + \frac{3\csc(a)\csc(a+bx)\left(c^2d\sin(bx)+2cd^2x\sin(bx)+d^3x^2\sin(bx)\right)}{2b^2} - \frac{3d^3\csc(a)\sec(a)\left(b^2e^{i\arctan(\tan(a))}x^2+\frac{(ibx(-\pi+2\arctan(\tan(a)))-\pi\log(1+e^{-2ibx})-2(bx+\arctan(\tan(a)))\log(1-e^{2i(bx+\text{arcta}}}{}\right.}{2b^4\sqrt{\sec^2(a)\left(\cos^2(a)+\right.}}$$

input
```
Integrate[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

output
```
-1/2*((c + d*x)^3*Csc[a + b*x]^2)/b + (3*c*d^2*Csc[a]*(-(b*x*Cos[a]) + Log
[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) +
 (3*Csc[a]*Csc[a + b*x]*(c^2*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x] + d^3*x^2*Sin
[b*x]))/(2*b^2) - (3*d^3*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I
*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcT
an[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] +
 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(
b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^4*Sqrt[Sec[a]^2
*(Cos[a]^2 + Sin[a]^2)])
```

### 3.47.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {4910, 3042, 4672, 3042, 25, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\cot(a+bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 4910}$$

$$\frac{3d\int (c+dx)^2\csc^2(a+bx)dx}{2b} - \frac{(c+dx)^3\csc^2(a+bx)}{2b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3d\int (c+dx)^2\csc(a+bx)^2dx}{2b}-\frac{(c+dx)^3\csc^2(a+bx)}{2b}$$

$$\Big\downarrow \text{4672}$$

$$\frac{3d\left(\frac{2d\int (c+dx)\cot(a+bx)dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}\right)}{2b}-\frac{(c+dx)^3\csc^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d\left(\frac{2d\int -\left((c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)\right)dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}\right)}{2b}-\frac{(c+dx)^3\csc^2(a+bx)}{2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{3d\left(-\frac{2d\int (c+dx)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}\right)}{2b}-\frac{(c+dx)^3\csc^2(a+bx)}{2b}$$

$$\Big\downarrow \text{4202}$$

$$-\frac{(c+dx)^3\csc^2(a+bx)}{2b}+\frac{3d\left(-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)}{1+e^{i(2a+2bx+\pi)}}dx\right)}{b}\right)}{2b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{(c+dx)^3\csc^2(a+bx)}{2b}+\frac{3d\left(-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{id\int \log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}\right)}{2b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{(c+dx)^3\csc^2(a+bx)}{2b}+\frac{3d\left(-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{d\int e^{-i(2a+2bx+\pi)}\log\left(1+e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}\right)}{2b}$$

$$\Big\downarrow \text{2838}$$

3.47. $\int (c+dx)^3\cot(a+bx)\csc^2(a+bx)\,dx$

$$-\frac{(c+dx)^3\csc^2(a+bx)}{2b}+\frac{3d\left(-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\,\mathrm{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}\right)}{2b}$$

input
```
Int[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

output
```
-1/2*((c + d*x)^3*Csc[a + b*x]^2)/b + (3*d*(-(((c + d*x)^2*Cot[a + b*x])/b
) - (2*d*(((I/2)*(c + d*x)^2)/d - (2*I)*(((-1/2*I)*(c + d*x)*Log[1 + E^(I*
(2*a + Pi + 2*b*x))])/b - (d*PolyLog[2, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2
))))/b))/(2*b)
```

#### 3.47.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

## 3.47.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(101) = 202$.

Time = 1.67 (sec) , antiderivative size = 409, normalized size of antiderivative = 3.56

| method | result |
|---|---|
| risch | $\frac{2b\,d^3x^3\mathrm{e}^{2i(xb+a)}-3id^3x^2\mathrm{e}^{2i(xb+a)}+6bc\,d^2x^2\mathrm{e}^{2i(xb+a)}-6ic\,d^2x\,\mathrm{e}^{2i(xb+a)}+6b\,c^2dx\,\mathrm{e}^{2i(xb+a)}-3ic^2d\,\mathrm{e}^{2i(xb+a)}+3id^3x^2+2b\,c^3\mathrm{e}^{2i(xb+a}}{b^2\left(\mathrm{e}^{2i(xb+a)}-1\right)^2}$ |

input
```
int((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*e
xp(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-
3*I*c^2*d*exp(2*I*(b*x+a))+3*I*d^3*x^2+2*b*c^3*exp(2*I*(b*x+a))+6*I*c*d^2*
x+3*I*c^2*d)/b^2/(exp(2*I*(b*x+a))-1)^2+3*d^2/b^3*c*ln(exp(I*(b*x+a))+1)-6
*d^2/b^3*c*ln(exp(I*(b*x+a)))+3*d^2/b^3*c*ln(exp(I*(b*x+a))-1)-3*I*d^3/b^2
*x^2-6*I*d^3/b^3*x*a-3*I*d^3/b^4*a^2+3*d^3/b^3*ln(1-exp(I*(b*x+a)))*x+3*d^
3/b^4*ln(1-exp(I*(b*x+a)))*a-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4+3*d^3/b
^3*ln(exp(I*(b*x+a))+1)*x-3*I*d^3/b^4*polylog(2,-exp(I*(b*x+a)))+6*d^3/b^4
*a*ln(exp(I*(b*x+a)))-3*d^3/b^4*a*ln(exp(I*(b*x+a))-1)
```

### 3.47.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 591 vs. 2(98) = 196.

Time = 0.27 (sec) , antiderivative size = 591, normalized size of antiderivative = 5.14

$$\int (c+dx)^3 \cot(a+bx) \csc^2(a+bx)\,dx$$
$$= \frac{b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3 + 3\,(b^2d^3x^2 + 2\,b^2cd^2x + b^2c^2d)\cos(bx+a)\sin(bx+a) - 3\,(i\,d^3\cos}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 + 3*(b^2*d^3*
x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x + a) - 3*(I*d^3*cos(
b*x + a)^2 - I*d^3)*dilog(cos(b*x + a) + I*sin(b*x + a)) - 3*(-I*d^3*cos(b
*x + a)^2 + I*d^3)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*(-I*d^3*cos(b*
x + a)^2 + I*d^3)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 3*(I*d^3*cos(b*x
 + a)^2 - I*d^3)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 3*(b*d^3*x + b*c*
d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a
) + 1) - 3*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*log(co
s(b*x + a) - I*sin(b*x + a) + 1) - 3*(b*c*d^2 - a*d^3 - (b*c*d^2 - a*d^3)*
cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 3*(b*c
*d^2 - a*d^3 - (b*c*d^2 - a*d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1
/2*I*sin(b*x + a) + 1/2) - 3*(b*d^3*x + a*d^3 - (b*d^3*x + a*d^3)*cos(b*x
+ a)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b*d^3*x + a*d^3 - (b*
d^3*x + a*d^3)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1))/(b
^4*cos(b*x + a)^2 - b^4)
```

### 3.47.6 Sympy [F]

$$\int (c+dx)^3 \cot(a+bx) \csc^2(a+bx)\,dx = \int (c+dx)^3 \cos(a+bx) \csc^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*cos(b*x+a)*csc(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**3*cos(a + b*x)*csc(a + b*x)**3, x)
```

### 3.47.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1044 vs. $2(98) = 196$.

Time = 0.49 (sec) , antiderivative size = 1044, normalized size of antiderivative = 9.08

$$\int (c+dx)^3 \cot(a+bx)\csc^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

output
```
(6*b^2*c^2*d + 6*(b*d^3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*cos(4*b*x + 4*a)
 - 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a) - (-I*b*d^3*x - I*b*c*d^2)*sin(4
*b*x + 4*a) - 2*(I*b*d^3*x + I*b*c*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x
+ a), cos(b*x + a) + 1) + 6*(b*c*d^2*cos(4*b*x + 4*a) - 2*b*c*d^2*cos(2*b*
x + 2*a) + I*b*c*d^2*sin(4*b*x + 4*a) - 2*I*b*c*d^2*sin(2*b*x + 2*a) + b*c
*d^2)*arctan2(sin(b*x + a), cos(b*x + a) - 1) - 6*(b*d^3*x*cos(4*b*x + 4*a
) - 2*b*d^3*x*cos(2*b*x + 2*a) + I*b*d^3*x*sin(4*b*x + 4*a) - 2*I*b*d^3*x*
sin(2*b*x + 2*a) + b*d^3*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 6*(
b^2*d^3*x^2 + 2*b^2*c*d^2*x)*cos(4*b*x + 4*a) - 2*(2*I*b^3*d^3*x^3 + 2*I*b
^3*c^3 + 3*b^2*c^2*d + 3*(2*I*b^3*c*d^2 - b^2*d^3)*x^2 + 6*(I*b^3*c^2*d -
b^2*c*d^2)*x)*cos(2*b*x + 2*a) - 6*(d^3*cos(4*b*x + 4*a) - 2*d^3*cos(2*b*x
 + 2*a) + I*d^3*sin(4*b*x + 4*a) - 2*I*d^3*sin(2*b*x + 2*a) + d^3)*dilog(-
e^(I*b*x + I*a)) - 6*(d^3*cos(4*b*x + 4*a) - 2*d^3*cos(2*b*x + 2*a) + I*d^
3*sin(4*b*x + 4*a) - 2*I*d^3*sin(2*b*x + 2*a) + d^3)*dilog(e^(I*b*x + I*a)
) - 3*(I*b*d^3*x + I*b*c*d^2 + (I*b*d^3*x + I*b*c*d^2)*cos(4*b*x + 4*a) +
2*(-I*b*d^3*x - I*b*c*d^2)*cos(2*b*x + 2*a) - (b*d^3*x + b*c*d^2)*sin(4*b*
x + 4*a) + 2*(b*d^3*x + b*c*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + si
n(b*x + a)^2 + 2*cos(b*x + a) + 1) - 3*(I*b*d^3*x + I*b*c*d^2 + (I*b*d^3*x
 + I*b*c*d^2)*cos(4*b*x + 4*a) + 2*(-I*b*d^3*x - I*b*c*d^2)*cos(2*b*x + 2*
a) - (b*d^3*x + b*c*d^2)*sin(4*b*x + 4*a) + 2*(b*d^3*x + b*c*d^2)*sin(2...
```

### 3.47.8 Giac [F]

$$\int (c+dx)^3 \cot(a+bx)\csc^2(a+bx)\,dx = \int (dx+c)^3 \cos(bx+a)\csc(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*cos(b*x + a)*csc(b*x + a)^3, x)
```

### 3.47.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \cot(a+bx) \csc^2(a+bx)\,dx = \int \frac{\cos(a+b\,x)\,\left(c+d\,x\right)^3}{\sin(a+b\,x)^3}\,dx$$

input
```
int((cos(a + b*x)*(c + d*x)^3)/sin(a + b*x)^3,x)
```

output
```
int((cos(a + b*x)*(c + d*x)^3)/sin(a + b*x)^3, x)
```

# 3.48 $\int (c+dx)^2 \cot(a+bx) \csc^2(a+bx) \, dx$

3.48.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 462
3.48.2 Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 462
3.48.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 463
3.48.4 Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 464
3.48.5 Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 465
3.48.6 Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 465
3.48.7 Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . . . 466
3.48.8 Giac [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 466
3.48.9 Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 467

## 3.48.1 Optimal result

Integrand size = 22, antiderivative size = 54

$$\int (c+dx)^2 \cot(a+bx) \csc^2(a+bx) \, dx = -\frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2 \csc^2(a+bx)}{2b} + \frac{d^2 \log(\sin(a+bx))}{b^3}$$

output
```
-d*(d*x+c)*cot(b*x+a)/b^2-1/2*(d*x+c)^2*csc(b*x+a)^2/b+d^2*ln(sin(b*x+a))/
b^3
```

## 3.48.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.74

$$\int (c+dx)^2 \cot(a+bx) \csc^2(a+bx) \, dx = \frac{2ibd^2x - 2id^2 \arctan(\tan(a+bx)) - 2bd^2x\cot(a) - b^2(c+dx)^2\csc^2(a+bx) + d^2\log\left(\sin^2(a+bx)\right) + 2bc}{2b^3}$$

input
```
Integrate[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

output
```
((2*I)*b*d^2*x - (2*I)*d^2*ArcTan[Tan[a + b*x]] - 2*b*d^2*x*Cot[a] - b^2*(
c + d*x)^2*Csc[a + b*x]^2 + d^2*Log[Sin[a + b*x]^2] + 2*b*d*(c + d*x)*Csc[
a]*Csc[a + b*x]*Sin[b*x])/(2*b^3)
```

### 3.48.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4910, 3042, 4672, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \cot(a+bx) \csc^2(a+bx) \, dx$$

$$\Big\downarrow \text{4910}$$

$$\frac{d \int (c+dx) \csc^2(a+bx)dx}{b} - \frac{(c+dx)^2 \csc^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d \int (c+dx) \csc(a+bx)^2dx}{b} - \frac{(c+dx)^2 \csc^2(a+bx)}{2b}$$

$$\Big\downarrow \text{4672}$$

$$\frac{d\left(\frac{d \int \cot(a+bx)dx}{b} - \frac{(c+dx)\cot(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2 \csc^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d\left(\frac{d \int -\tan\left(a+bx+\frac{\pi}{2}\right)dx}{b} - \frac{(c+dx)\cot(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2 \csc^2(a+bx)}{2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{d\left(-\frac{d \int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx}{b} - \frac{(c+dx)\cot(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2 \csc^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3956}$$

$$\frac{d\left(\frac{d\log(-\sin(a+bx))}{b^2} - \frac{(c+dx)\cot(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2 \csc^2(a+bx)}{2b}$$

input
```
Int[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

output
```
-1/2*((c + d*x)^2*Csc[a + b*x]^2)/b + (d*(-(((c + d*x)*Cot[a + b*x])/b) +
(d*Log[-Sin[a + b*x]])/b^2))/b
```

### 3.48.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

## 3.48.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(52) = 104$.

Time = 1.31 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.52

| method | result |
|---|---|
| parallelrisch | $\frac{-8\ln\left(\sec\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)d^{2}+8\ln\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\right)d^{2}-b\left(b(dx+c)^{2}\cot\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}+(4d^{2}x+4cd)\cot\left(\frac{a}{2}+\frac{xb}{2}\right)+b(dx+c)^{2}\tan\left(\cdots\right.\right.}{8b^{3}}$ |
| risch | $-\frac{2id^{2}x}{b^{2}}-\frac{2id^{2}a}{b^{3}}+\frac{2b\,d^{2}x^{2}\mathrm{e}^{2i(xb+a)}+4bcdx\,\mathrm{e}^{2i(xb+a)}+2b\,c^{2}\mathrm{e}^{2i(xb+a)}-2id^{2}x\,\mathrm{e}^{2i(xb+a)}-2icd\,\mathrm{e}^{2i(xb+a)}+2id^{2}x+2id\cdots}{b^{2}\left(\mathrm{e}^{2i(xb+a)}-1\right)^{2}}$ |
| derivativedivides | $\frac{-\frac{a^{2}d^{2}}{2b^{2}\sin(xb+a)^{2}}+\frac{acd}{b\sin(xb+a)^{2}}-\frac{2a\,d^{2}\left(-\frac{xb+a}{2\sin(xb+a)^{2}}-\frac{\cot(xb+a)}{2}\right)}{b^{2}}-\frac{c^{2}}{2\sin(xb+a)^{2}}+\frac{2cd\left(-\frac{xb+a}{2\sin(xb+a)^{2}}-\frac{\cot(xb+a)}{2}\right)}{b}+\frac{d^{2}\left(-\cdots\right.}{\cdots}}{b}$ |
| default | $\frac{-\frac{a^{2}d^{2}}{2b^{2}\sin(xb+a)^{2}}+\frac{acd}{b\sin(xb+a)^{2}}-\frac{2a\,d^{2}\left(-\frac{xb+a}{2\sin(xb+a)^{2}}-\frac{\cot(xb+a)}{2}\right)}{b^{2}}-\frac{c^{2}}{2\sin(xb+a)^{2}}+\frac{2cd\left(-\frac{xb+a}{2\sin(xb+a)^{2}}-\frac{\cot(xb+a)}{2}\right)}{b}+\frac{d^{2}\left(-\cdots\right.}{\cdots}}{b}$ |
| norman | $\frac{-\frac{c^{2}}{8b}-\frac{c^{2}\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{4}}{8b}-\frac{d^{2}x^{2}}{8b}-\frac{cd\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{2b^{2}}+\frac{cd\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{3}}{2b^{2}}-\frac{cdx}{4b}-\frac{d^{2}x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{2b^{2}}+\frac{d^{2}x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{3}}{2b^{2}}-\frac{d^{2}x^{2}\tan\left(\frac{a}{2}+\cdots\right.}{4b}}{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}}$ |

input
```
int((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*(-8*ln(sec(1/2*a+1/2*x*b)^2)*d^2+8*ln(tan(1/2*a+1/2*x*b))*d^2-b*(b*(d*
x+c)^2*cot(1/2*a+1/2*x*b)^2+(4*d^2*x+4*c*d)*cot(1/2*a+1/2*x*b)+b*(d*x+c)^2
*tan(1/2*a+1/2*x*b)^2+(-4*d^2*x-4*c*d)*tan(1/2*a+1/2*x*b)+4*b*(1/2*d*x+c)*
x*d))/b^3
```

### 3.48.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.89

$$\int (c+dx)^2 \cot(a+bx)\csc^2(a+bx)\,dx$$
$$= \frac{b^2d^2x^2 + 2\,b^2cdx + b^2c^2 + 2\,(bd^2x + bcd)\cos(bx+a)\sin(bx+a) + 2\,\left(d^2\cos(bx+a)^2 - d^2\right)\log\left(\frac{1}{2}\,\sin(bx\right.}{2\,\left(b^3\cos(bx+a)^2 - b^3\right)}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a
)*sin(b*x + a) + 2*(d^2*cos(b*x + a)^2 - d^2)*log(1/2*sin(b*x + a)))/(b^3*
cos(b*x + a)^2 - b^3)
```

### 3.48.6 Sympy [F]

$$\int (c+dx)^2 \cot(a+bx)\csc^2(a+bx)\,dx = \int (c+dx)^2\cos(a+bx)\csc^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*cos(b*x+a)*csc(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**2*cos(a + b*x)*csc(a + b*x)**3, x)
```

### 3.48.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1130 vs. $2(52) = 104$.

Time = 0.29 (sec) , antiderivative size = 1130, normalized size of antiderivative = 20.93

$$\int (c + dx)^2 \cot(a + bx) \csc^2(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*(4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 -
(2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*
x + a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)
 + 1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*c*d/((2*(2*cos(2*b*x + 2*a) - 1
)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x
 + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4
*cos(2*b*x + 2*a) - 1)*b) - 4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a
)*sin(2*b*x + 2*a)^2 - (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*c
os(4*b*x + 4*a) - 2*(b*x + a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x +
2*a) - cos(2*b*x + 2*a) + 1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*a*d^2/((
2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2
*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) -
 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^2) + (8*(b*x + a)^2*cos(
2*b*x + 2*a)^2 + 8*(b*x + a)^2*sin(2*b*x + 2*a)^2 - 4*(b*x + a)^2*cos(2*b*
x + 2*a) - 4*((b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2*a))*c
os(4*b*x + 4*a) + (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x
 + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)
*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos
(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (2*(2*cos(2*b*x + 2*a
) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - s...
```

### 3.48.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2978 vs. $2(52) = 104$.

Time = 1.21 (sec) , antiderivative size = 2978, normalized size of antiderivative = 55.15

$$\int (c + dx)^2 \cot(a + bx) \csc^2(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/8*(b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4
*tan(1/2*a)^4 + 2*b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*d^2*x^2*
tan(1/2*b*x)^2*tan(1/2*a)^4 + b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b^2*
c*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 4*b*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^3
+ 4*b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4 - 4*b*d^2*x*tan(1/2*b*x)^3*tan(1
/2*a)^4 + b^2*d^2*x^2*tan(1/2*b*x)^4 + 4*b^2*d^2*x^2*tan(1/2*b*x)^2*tan(1/
2*a)^2 + 2*b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a)^2 - 4*b*c*d*tan(1/2*b*x)^4*ta
n(1/2*a)^3 + b^2*d^2*x^2*tan(1/2*a)^4 + 2*b^2*c^2*tan(1/2*b*x)^2*tan(1/2*a
)^4 - 4*b*c*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4 + 4
*b*d^2*x*tan(1/2*b*x)^4*tan(1/2*a) + 8*b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)
^2 + 24*b*d^2*x*tan(1/2*b*x)^3*tan(1/2*a)^2 - 4*d^2*log(16*(tan(1/2*b*x)^4
*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^
4 - 2*tan(1/2*b*x)^3*tan(1/2*a) - 4*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/
2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2
*a)^2)/(tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*ta
n(1/2*b*x)^2*tan(1/2*a)^4 + tan(1/2*b*x)^4 + 4*tan(1/2*b*x)^2*tan(1/2*a)^2
 + tan(1/2*a)^4 + 2*tan(1/2*b*x)^2 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*t
an(1/2*a)^2 + 24*b*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)^3 - 8*d^2*log(16*(tan(1
/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*ta
n(1/2*a)^4 - 2*tan(1/2*b*x)^3*tan(1/2*a) - 4*tan(1/2*b*x)^2*tan(1/2*a)^...
```

### 3.48.9 Mupad [B] (verification not implemented)

Time = 25.39 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.72

$$\int (c+dx)^2 \cot(a+bx)\csc^2(a+bx)\,dx = \frac{\frac{(c+d\,x)^2}{b}+\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,(c+d\,x)^2}{b}}{1+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}-2\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}}-\frac{d^2\,x\,2\mathrm{i}}{b^2}+\frac{b\,c^2+2\,b\,c\,d\,x-c\,d\,2\mathrm{i}+b\,d^2\,x^2-d^2\,x\,2\mathrm{i}}{b^2\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}+\frac{d^2\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}-1\right)}{b^3}$$

input
```
int((cos(a + b*x)*(c + d*x)^2)/sin(a + b*x)^3,x)
```

output
```
((c + d*x)^2/b + (exp(a*2i + b*x*2i)*(c + d*x)^2)/b)/(exp(a*4i + b*x*4i) -
 2*exp(a*2i + b*x*2i) + 1) - (d^2*x*2i)/b^2 + (b*c^2 - c*d*2i - d^2*x*2i +
 b*d^2*x^2 + 2*b*c*d*x)/(b^2*(exp(a*2i + b*x*2i) - 1)) + (d^2*log(exp(a*2i
)*exp(b*x*2i) - 1))/b^3
```

3.48. $\int (c+dx)^2 \cot(a+bx) \csc^2(a+bx)\, dx$

# 3.49 $\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx$

3.49.1 Optimal result . . . . . 469
3.49.2 Mathematica [A] (verified) . . . . . 469
3.49.3 Rubi [A] (verified) . . . . . 470
3.49.4 Maple [A] (verified) . . . . . 471
3.49.5 Fricas [A] (verification not implemented) . . . . . 472
3.49.6 Sympy [F] . . . . . 472
3.49.7 Maxima [B] (verification not implemented) . . . . . 472
3.49.8 Giac [B] (verification not implemented) . . . . . 473
3.49.9 Mupad [B] (verification not implemented) . . . . . 474

## 3.49.1 Optimal result

Integrand size = 20, antiderivative size = 35

$$\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx=-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\csc^2(a+bx)}{2b}$$

output
```
-1/2*d*cot(b*x+a)/b^2-1/2*(d*x+c)*csc(b*x+a)^2/b
```

## 3.49.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx=-\frac{d\cot(a+bx)}{2b^2}-\frac{c\csc^2(a+bx)}{2b}-\frac{dx\csc^2(a+bx)}{2b}$$

input
```
Integrate[(c + d*x)*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

output
```
-1/2*(d*Cot[a + b*x])/b^2 - (c*Csc[a + b*x]^2)/(2*b) - (d*x*Csc[a + b*x]^2
)/(2*b)
```

### 3.49.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4910, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 4910}$$

$$\frac{d\int \csc^2(a+bx)dx}{2b}-\frac{(c+dx)\csc^2(a+bx)}{2b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{d\int \csc(a+bx)^2dx}{2b}-\frac{(c+dx)\csc^2(a+bx)}{2b}$$

$$\Big\downarrow \text{ 4254}$$

$$-\frac{d\int 1d\cot(a+bx)}{2b^2}-\frac{(c+dx)\csc^2(a+bx)}{2b}$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\csc^2(a+bx)}{2b}$$

input
```
Int[(c + d*x)*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

output
```
-1/2*(d*Cot[a + b*x])/b^2 - ((c + d*x)*Csc[a + b*x]^2)/(2*b)
```

#### 3.49.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

## 3.49.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{da}{2b\sin(xb+a)^2}-\frac{c}{2\sin(xb+a)^2}+\frac{d\left(-\frac{xb+a}{2\sin(xb+a)^2}-\frac{\cot(xb+a)}{2}\right)}{b}}{b}$ | 61 |
| default | $\frac{\frac{da}{2b\sin(xb+a)^2}-\frac{c}{2\sin(xb+a)^2}+\frac{d\left(-\frac{xb+a}{2\sin(xb+a)^2}-\frac{\cot(xb+a)}{2}\right)}{b}}{b}$ | 61 |
| parallelrisch | $-\frac{\sec\left(\frac{a}{2}+\frac{xb}{2}\right)^2\csc\left(\frac{a}{2}+\frac{xb}{2}\right)^2\left(\cos(2xb+2a)bc+2d\sin(2xb+2a)+3\left(\frac{4dx}{3}+c\right)b\right)}{32b^2}$ | 62 |
| risch | $\frac{2bdx\,e^{2i(xb+a)}-id\,e^{2i(xb+a)}+2bc\,e^{2i(xb+a)}+id}{b^2\left(e^{2i(xb+a)}-1\right)^2}$ | 63 |
| norman | $\frac{-\frac{c}{8b}-\frac{c\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^4}{8b}-\frac{d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{4b^2}+\frac{d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3}{4b^2}-\frac{dx}{8b}-\frac{dx\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}{4b}-\frac{dx\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^4}{8b}}{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}$ | 112 |

input
```
int((d*x+c)*cos(b*x+a)*csc(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2/b*d*a/sin(b*x+a)^2-1/2*c/sin(b*x+a)^2+1/b*d*(-1/2*(b*x+a)/sin(b*x
+a)^2-1/2*cot(b*x+a)))
```

### 3.49.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26

$$\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx = \frac{bdx + d\cos{(bx+a)}\sin{(bx+a)} + bc}{2\left(b^2\cos{(bx+a)}^2 - b^2\right)}$$

input
```
integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(b*d*x + d*cos(b*x + a)*sin(b*x + a) + b*c)/(b^2*cos(b*x + a)^2 - b^2)
```

### 3.49.6 Sympy [F]

$$\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx = \int (c+dx)\cos{(a+bx)}\csc^3{(a+bx)}\,dx$$

input
```
integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)**3,x)
```

output
```
Integral((c + d*x)*cos(a + b*x)*csc(a + b*x)**3, x)
```

### 3.49.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(31) = 62.

Time = 0.25 (sec) , antiderivative size = 287, normalized size of antiderivative = 8.20

$$\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx$$
$$= \frac{\frac{2\left(4\,(bx+a)\cos(2\,bx+2\,a)^2+4\,(bx+a)\sin(2\,bx+2\,a)^2-(2\,(bx+a)\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a))\cos(4\,bx+4\,a)-2\,(bx+a)\cos(2\,bx+2\,a)-(2\,(bx+a)}{\left(2\,(2\,\cos(2\,bx+2\,a)-1)\cos(4\,bx+4\,a)-\cos(4\,bx+4\,a)^2-4\,\cos(2\,bx+2\,a)^2-\sin(4\,bx+4\,a)^2+4\,\sin(4\,bx+4\,a)\sin(2\,bx+2\,a)-}}{2\,b}$$

input
```
integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 -
(2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*
x + a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)
 + 1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*d/((2*(2*cos(2*b*x + 2*a) - 1)*
cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x +
 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*c
os(2*b*x + 2*a) - 1)*b) - c/sin(b*x + a)^2 + a*d/(b*sin(b*x + a)^2))/b
```

### 3.49.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(31) = 62$.

Time = 0.35 (sec) , antiderivative size = 526, normalized size of antiderivative = 15.03

$$\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx = -\frac{bdx\tan\left(\frac{1}{2}bx\right)^4\tan\left(\frac{1}{2}a\right)^4 + bc\tan\left(\frac{1}{2}bx\right)^4\tan\left(\frac{1}{2}a\right)^4 + 2\,bdx\tan\left(\frac{1}{2}bx\right)^4\tan\left(\frac{1}{2}a\right)^2 + 2\,bdx\tan\left(\frac{1}{2}bx\right)^2 t}{}$$

input
```
integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/8*(b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4*tan(1/2*a)^4
+ 2*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^
4 + 2*b*c*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*d*tan(1/2*b*x)^4*tan(1/2*a)^3 +
2*b*c*tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + b*d*
x*tan(1/2*b*x)^4 + 4*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*d*x*tan(1/2*a)^
4 + b*c*tan(1/2*b*x)^4 + 2*d*tan(1/2*b*x)^4*tan(1/2*a) + 4*b*c*tan(1/2*b*x
)^2*tan(1/2*a)^2 + 12*d*tan(1/2*b*x)^3*tan(1/2*a)^2 + 12*d*tan(1/2*b*x)^2*
tan(1/2*a)^3 + b*c*tan(1/2*a)^4 + 2*d*tan(1/2*b*x)*tan(1/2*a)^4 + 2*b*d*x*
tan(1/2*b*x)^2 + 2*b*d*x*tan(1/2*a)^2 + 2*b*c*tan(1/2*b*x)^2 - 2*d*tan(1/2
*b*x)^3 - 12*d*tan(1/2*b*x)^2*tan(1/2*a) + 2*b*c*tan(1/2*a)^2 - 12*d*tan(1
/2*b*x)*tan(1/2*a)^2 - 2*d*tan(1/2*a)^3 + b*d*x + b*c + 2*d*tan(1/2*b*x) +
 2*d*tan(1/2*a))/(b^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*tan(1/2*b*x)^3*t
an(1/2*a)^3 + b^2*tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*b^2*tan(1/2*b*x)^3*tan(1
/2*a) - 4*b^2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^2*tan(1/2*b*x)*tan(1/2*a)^
3 + b^2*tan(1/2*b*x)^2 + 2*b^2*tan(1/2*b*x)*tan(1/2*a) + b^2*tan(1/2*a)^2)
```

### 3.49.9 Mupad [B] (verification not implemented)

Time = 24.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx = \frac{d\,1\mathrm{i} - \mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,\left(-b\,(2\,c+2\,d\,x)+d\,1\mathrm{i}\right)}{b^2\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)^2}$$

input
```
int((cos(a + b*x)*(c + d*x))/sin(a + b*x)^3,x)
```

output
```
(d*1i - exp(a*2i + b*x*2i)*(d*1i - b*(2*c + 2*d*x)))/(b^2*(exp(a*2i + b*x*
2i) - 1)^2)
```

# 3.50 $\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx$

3.50.1 Optimal result . . . . . 475
3.50.2 Mathematica [N/A] . . . . . 475
3.50.3 Rubi [N/A] . . . . . 476
3.50.4 Maple [N/A] (verified) . . . . . 476
3.50.5 Fricas [N/A] . . . . . 477
3.50.6 Sympy [N/A] . . . . . 477
3.50.7 Maxima [N/A] . . . . . 477
3.50.8 Giac [N/A] . . . . . 478
3.50.9 Mupad [N/A] . . . . . 479

## 3.50.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\cot(a+bx)\csc^2(a+bx)}{c+dx},x\right)$$

output
```
CannotIntegrate(cot(b*x+a)*csc(b*x+a)^2/(d*x+c),x)
```

## 3.50.2 Mathematica [N/A]

Not integrable

Time = 13.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x),x]
```

output
```
Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x), x]
```

### 3.50.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}dx$$

input
```
Int[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x),x]
```

output
```
$Aborted
```

#### 3.50.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

### 3.50.4 Maple [N/A] (verified)

Not integrable

Time = 0.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cos\left(xb+a\right)\csc\left(xb+a\right)^{3}}{dx+c}dx$$

input
```
int(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x)
```

output
```
int(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x)
```

### 3.50.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)\csc(bx+a)^3}{dx+c}\,dx$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)*csc(b*x + a)^3/(d*x + c), x)
```

### 3.50.6 Sympy [N/A]

Not integrable

Time = 2.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx = \int \frac{\cos(a+bx)\csc^3(a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(cos(a + b*x)*csc(a + b*x)**3/(c + d*x), x)
```

### 3.50.7 Maxima [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 1664, normalized size of antiderivative = 75.64

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)\csc(bx+a)^3}{dx+c}\,dx$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output
```
-(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2
- (2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
 - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c
^2*d^2 + (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(4*b*x + 4*a)^2 +
4*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(2*b*x + 2*a)^2 + (b^2*d^
4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^4*x^2 +
 2*b^2*c*d^3*x + b^2*c^2*d^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d
^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^4*x^2
+ 2*b^2*c*d^3*x + b^2*c^2*d^2 - 2*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d
^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^4*x^2 + 2*b^2*c*d^3*x +
b^2*c^2*d^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b^2*d^3*x^3 + 3*b^2
*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3)*sin(b*x + a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
+ 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)), x) - (b^2*d^4*x^2 + 2*b^2*c*d^3*
x + b^2*c^2*d^2 + (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(4*b*x +
4*a)^2 + 4*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(2*b*x + 2*a)^2
+ (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*
d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) +
 4*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*sin(2*b*x + 2*a)^2 + 2*(...
```

### 3.50.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)\csc(bx+a)^3}{dx+c}\,dx$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*csc(b*x + a)^3/(d*x + c), x)
```

### 3.50.9 Mupad [N/A]

Not integrable

Time = 23.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx = \int \frac{\cos\left(a+b\,x\right)}{{\sin\left(a+b\,x\right)}^3\,\left(c+d\,x\right)}\,dx$$

input
```
int(cos(a + b*x)/(sin(a + b*x)^3*(c + d*x)),x)
```

output
```
int(cos(a + b*x)/(sin(a + b*x)^3*(c + d*x)), x)
```

# 3.51 $\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2} dx$


3.51.1 Optimal result . . . . . . . . . . 480
3.51.2 Mathematica [N/A] . . . . . . . . . . 480
3.51.3 Rubi [N/A] . . . . . . . . . . 481
3.51.4 Maple [N/A] (verified) . . . . . . . . . . 481
3.51.5 Fricas [N/A] . . . . . . . . . . 482
3.51.6 Sympy [N/A] . . . . . . . . . . 482
3.51.7 Maxima [N/A] . . . . . . . . . . 482
3.51.8 Giac [F(-1)] . . . . . . . . . . 483
3.51.9 Mupad [N/A] . . . . . . . . . . 484


## 3.51.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2} dx = \text{Int}\left(\frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2},x\right)$$

output
```
CannotIntegrate(cot(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x)
```

## 3.51.2 Mathematica [N/A]

Not integrable

Time = 12.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2} dx = \int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2} dx$$

input
```
Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2, x]
```

### 3.51.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}dx$$

input
```
Int[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
$Aborted
```

#### 3.51.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

### 3.51.4 Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cos(xb+a)\csc(xb+a)^3}{(dx+c)^2}dx$$

input
```
int(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x)
```

### 3.51.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(bx+a)\csc(bx+a)^3}{(dx+c)^2}\,dx$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)*csc(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.51.6 Sympy [N/A]

Not integrable

Time = 3.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(a+bx)\csc^3(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(cos(a + b*x)*csc(a + b*x)**3/(c + d*x)**2, x)
```

### 3.51.7 Maxima [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 2153, normalized size of antiderivative = 97.86

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(bx+a)\csc(bx+a)^3}{(dx+c)^2}\,dx$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2
- 2*((b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
 - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + 3*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3
*b^2*c^2*d^3*x + b^2*c^3*d^2 + (b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*
d^3*x + b^2*c^3*d^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2
 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*cos(2*b*x + 2*a)^2 + (b^2*d^5*x^3 + 3*b^
2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d
^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*sin(4*b*x + 4*a)
*sin(2*b*x + 2*a) + 4*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b
^2*c^3*d^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*
c^2*d^3*x + b^2*c^3*d^2 - 2*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3
*x + b^2*c^3*d^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^5*x^3 + 3*
b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*cos(2*b*x + 2*a))*integrate
(sin(b*x + a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c
^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*
b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6
*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(b*x + a)^2 + 2*(b^2*d^4*x^
4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x
 + a)), x) - 3*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*
d^2 + (b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*c...
```

### 3.51.8 Giac [F(-1)]

Timed out.

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

### 3.51.9 Mupad [N/A]

Not integrable

Time = 23.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(a+b\,x)}{\sin(a+b\,x)^3\,(c+d\,x)^2}\,dx$$

input
```
int(cos(a + b*x)/(sin(a + b*x)^3*(c + d*x)^2),x)
```

output
```
int(cos(a + b*x)/(sin(a + b*x)^3*(c + d*x)^2), x)
```

# 3.52 $\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx$


3.52.1 Optimal result . . . . . 485
3.52.2 Mathematica [C] (verified) . . . . . 485
3.52.3 Rubi [A] (verified) . . . . . 486
3.52.4 Maple [A] (verified) . . . . . 492
3.52.5 Fricas [A] (verification not implemented) . . . . . 493
3.52.6 Sympy [F] . . . . . 493
3.52.7 Maxima [C] (verification not implemented) . . . . . 494
3.52.8 Giac [C] (verification not implemented) . . . . . 494
3.52.9 Mupad [F(-1)] . . . . . 495


## 3.52.1 Optimal result

Integrand size = 22, antiderivative size = 196

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} - \frac{15d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}} + \frac{15d^{5/2}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{128b^{7/2}} + \frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{16b^2}$$

output
```
-1/4*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b+5/16*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^
2-15/128*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)
/Pi^(1/2))*Pi^(1/2)/b^(7/2)+15/128*d^(5/2)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+15/64*d^2*cos(2*b*x+
2*a)*(d*x+c)^(1/2)/b^3
```

## 3.52.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.73

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \frac{e^{-\frac{2i(bc+ad)}{d}}(c+dx)^{5/2}\left(e^{4ia}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{2ib(c+dx)}{d}\right)+e^{\frac{4ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{2ib(c+dx)}{d}\right)\right)}{32\sqrt{2}b\left(\frac{b^2(c+dx)^2}{d^2}\right)^{3/2}}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
((c + d*x)^(5/2)*(E^((4*I)*a)*Sqrt[(I*b*(c + d*x))/d]*Gamma[7/2, ((-2*I)*b
*(c + d*x))/d] + E^(((4*I)*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[7/2, (
(2*I)*b*(c + d*x))/d]))/(32*Sqrt[2]*b*E^(((2*I)*(b*c + a*d))/d)*((b^2*(c +
 d*x)^2)/d^2)^(3/2))
```

### 3.52.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.11, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {4906, 27, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{ 4906}$$

$$\int \frac{1}{2}(c+dx)^{5/2}\sin(2a+2bx)dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int (c+dx)^{5/2}\sin(2a+2bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int (c+dx)^{5/2}\sin(2a+2bx)dx$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{1}{2}\left(\frac{5d\int (c+dx)^{3/2}\cos(2a+2bx)dx}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\left(\frac{5d\int (c+dx)^{3/2}\sin\left(2a+2bx+\frac{\pi}{2}\right)dx}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{3d\int -\sqrt{c+dx}\sin(2a+2bx)dx}{4b}+\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}\right)}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$\big\downarrow$ 25

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\int \sqrt{c+dx}\sin(2a+2bx)dx}{4b}\right)}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$\big\downarrow$ 3042

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\int \sqrt{c+dx}\sin(2a+2bx)dx}{4b}\right)}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$\big\downarrow$ 3777

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}dx}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)}{4b}\right)}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$\big\downarrow$ 3042

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\int \frac{\sin\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)}{4b}\right)}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$\big\downarrow$ 3787

3.52. $\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\left(\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx-\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)}{4b}\right)}{4b}-\frac{(c+$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\left(\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx-\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)}{4b}\right)}{4b}-\frac{(c$$

$\downarrow$ 3785

3.52. $\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)}{4b}\right)}{4b}-\right.$$

$$\Big\downarrow \text{3786}$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\sin\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right.}{4b}\right.}{4b}\right.$$

$$\Big\downarrow \text{3832}$$

3.52. $\int(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right.}{4b}\right.}{4b}\right.$$

$\downarrow$ 3833

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\left(\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right.}{4b}\right.}{4b}\right.$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

## 3.52. $\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx$

output
```
(-1/2*((c + d*x)^(5/2)*Cos[2*a + 2*b*x])/b + (5*d*((-3*d*(-1/2*(Sqrt[c + d
*x]*Cos[2*a + 2*b*x])/b + (d*((Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*S
qrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(Sqrt[b]*Sqrt[d]) - (Sqrt[Pi]*F
resnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d]
)/(Sqrt[b]*Sqrt[d])))/(4*b)))/(4*b) + ((c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(
2*b)))/(4*b))/2
```

### 3.52.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.52.4 Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.19

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\right.}{}\right)}{4}}{d}$ |
| default | $\frac{-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\right.}{}\right)}{4}}{d}$ |

input
```
int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+5/8/b*d*(1/4/
b*d*(d*x+c)^(3/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+
c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(co
s(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(2*
(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

### 3.52.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.13

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{15\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 15\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right) - 2\,(16}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/128*(15*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(
d*x + c)*sqrt(b/(pi*d))) - 15*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x
 + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*(16*b^3*d^2*x^2 + 32*b^3*c
*d*x + 16*b^3*c^2 - 15*b*d^2 - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c
^2 - 15*b*d^2)*cos(b*x + a)^2 + 40*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)*sin(
b*x + a))*sqrt(d*x + c))/b^4
```

### 3.52.6 Sympy [F]

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \int (c+dx)^{\frac{5}{2}}\sin(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**(5/2)*sin(a + b*x)*cos(a + b*x), x)
```

### 3.52.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.41

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \frac{\sqrt{2}\left(160\sqrt{2}(dx+c)^{\frac{3}{2}}b^2d\sin\left(\frac{2((dx+c)b-bc+ad)}{d}\right)-8\left(16\sqrt{2}(dx+c)^{\frac{5}{2}}b^3-15\sqrt{2}\sqrt{dx+c}bd^2\right)\cos\left(\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/1024*sqrt(2)*(160*sqrt(2)*(d*x + c)^(3/2)*b^2*d*sin(2*((d*x + c)*b - b*c
 + a*d)/d) - 8*(16*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d*x + c)*
b*d^2)*cos(2*((d*x + c)*b - b*c + a*d)/d) - 15*(-(I - 1)*4^(1/4)*sqrt(pi)*
d^3*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(pi)*d^3*(
b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 1
5*((I + 1)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I
 - 1)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt
(d*x + c)*sqrt(-2*I*b/d)))/b^4
```

### 3.52.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 1205, normalized size of antiderivative = 6.15

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
1/256*(64*(sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + s
qrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 12*c*d
^2*((sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x
+ c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d
)/b^2)/d^2 + (sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(I*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*
I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c
+ I*a*d)/d)/b^2)/d^2) - d^3*((sqrt(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b
*c*d^2 + 15*I*d^3)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3)
- 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sq
rt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^
2 - 15*I*sqrt(d*x + c)*d^3)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)
/d^3 + (sqrt(pi)*(64*b^3*c^3 + 48*I*b^2*c^2*d - 36*b*c*d^2 - 15*I*d^3)*d*e
rf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*...
```

### 3.52.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \int \cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)\,\left(c+d\,x\right)^{5/2}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(5/2), x)
```

# 3.53 $\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx$

3.53.1 Optimal result . . . . . . . . . . 496
3.53.2 Mathematica [C] (verified) . . . . . . . . . . 496
3.53.3 Rubi [A] (verified) . . . . . . . . . . 497
3.53.4 Maple [A] (verified) . . . . . . . . . . 501
3.53.5 Fricas [A] (verification not implemented) . . . . . . . . . . 502
3.53.6 Sympy [F] . . . . . . . . . . 502
3.53.7 Maxima [C] (verification not implemented) . . . . . . . . . . 502
3.53.8 Giac [C] (verification not implemented) . . . . . . . . . . 503
3.53.9 Mupad [F(-1)] . . . . . . . . . . 504

## 3.53.1 Optimal result

Integrand size = 22, antiderivative size = 168

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} - \frac{3d^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}} - \frac{3d^{3/2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{32b^{5/2}} + \frac{3d\sqrt{c+dx}\sin(2a+2bx)}{16b^2}$$

output
```
-1/4*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b-3/32*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS
(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(5/2)-3/32*d^(3/2)*F
resnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2
)/b^(5/2)+3/16*d*sin(2*b*x+2*a)*(d*x+c)^(1/2)/b^2
```

## 3.53.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.76

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = \frac{e^{-\frac{2i(bc+ad)}{d}}(c+dx)^{3/2}\left(-\frac{e^{4ia}\Gamma\left(\frac{5}{2},-\frac{2ib(c+dx)}{d}\right)}{\left(-\frac{ib(c+dx)}{d}\right)^{3/2}} - \frac{e^{\frac{4ibc}{d}}\Gamma\left(\frac{5}{2},\frac{2ib(c+dx)}{d}\right)}{\left(\frac{ib(c+dx)}{d}\right)^{3/2}}\right)}{16\sqrt{2}b}$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
((c + d*x)^(3/2)*(-((E^((4*I)*a)*Gamma[5/2, ((-2*I)*b*(c + d*x))/d])/(((-I
)*b*(c + d*x))/d)^(3/2)) - (E^(((4*I)*b*c)/d)*Gamma[5/2, ((2*I)*b*(c + d*x
))/d])/((I*b*(c + d*x))/d)^(3/2)))/(16*Sqrt[2]*b*E^(((2*I)*(b*c + a*d))/d)
)
```

### 3.53.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {4906, 27, 3042, 3777, 3042, 3777, 25, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{ 4906}$$

$$\int \frac{1}{2}(c+dx)^{3/2}\sin(2a+2bx)dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\int (c+dx)^{3/2}\sin(2a+2bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\int (c+dx)^{3/2}\sin(2a+2bx)dx$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{1}{2}\left(\frac{3d\int \sqrt{c+dx}\cos(2a+2bx)dx}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{2}\left(\frac{3d\int \sqrt{c+dx}\sin\left(2a+2bx+\frac{\pi}{2}\right)dx}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{d\int -\frac{\sin(2a+2bx)}{\sqrt{c+dx}}dx}{4b}+\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}dx}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}dx}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{3787}$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\left(\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx+\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{2b}\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\left(\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx+\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2\ldots}{2b}\right.$$

$$\Big\downarrow \text{3785}$$

3.53. $\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\left(\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2a+\right.$$

$\Big\downarrow$ 3786

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\left(\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\sin\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(}{2b}\right.$$

$\Big\downarrow$ 3832

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\left(\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2}{2b}\right.$$

$\Big\downarrow$ 3833

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\left(\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2}{2b}\right.$$

3.53. $\int(c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
(-1/2*((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/b + (3*d*(-1/4*(d*((Sqrt[Pi]*Cos[
2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(
Sqrt[b]*Sqrt[d]) + (Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*S
qrt[Pi])]*Sin[2*a - (2*b*c)/d])/(Sqrt[b]*Sqrt[d])))/b + (Sqrt[c + d*x]*Sin
[2*a + 2*b*x])/(2*b)))/(4*b))/2
```

### 3.53.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.53.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.11

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\text{s}\right.}{8b\sqrt{\frac{b}{d}}}\right.}{4b}}{d}$ |
| default | $\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\text{s}\right.}{8b\sqrt{\frac{b}{d}}}\right.}{4b}}{d}$ |

input
```
int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+3/8/b*d*(1/4/
b*d*(d*x+c)^(1/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^
(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/
d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
)
```

### 3.53.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{3\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-4\,(2\,b^2d}{32\,b^3}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/32*(3*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*
x + c)*sqrt(b/(pi*d))) + 3*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x +
c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(2*b^2*d*x + 3*b*d*cos(b*x +
a)*sin(b*x + a) + 2*b^2*c - 4*(b^2*d*x + b^2*c)*cos(b*x + a)^2)*sqrt(d*x +
 c))/b^3
```

### 3.53.6 Sympy [F]

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)*cos(a + b*x), x)
```

### 3.53.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.52

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{\sqrt{2}\left(32\,\sqrt{2}(dx+c)^{\frac{3}{2}}b^2\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-24\,\sqrt{2}\sqrt{dx+c}bd\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)+3\left((i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\right.\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/256*sqrt(2)*(32*sqrt(2)*(d*x + c)^(3/2)*b^2*cos(2*((d*x + c)*b - b*c +
a*d)/d) - 24*sqrt(2)*sqrt(d*x + c)*b*d*sin(2*((d*x + c)*b - b*c + a*d)/d)
+ 3*((I + 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) -
(I - 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sq
rt(d*x + c)*sqrt(2*I*b/d)) + 3*(-(I - 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1
/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*s
in(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/b^3
```

### 3.53.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 747, normalized size of antiderivative = 4.45

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = \frac{16\left(\frac{\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{i\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2\,(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}+\frac{\sqrt{\pi}d\,\mathrm{erf}\left(\frac{i\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2\,(-i\,bc+i\,ad)}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}\right)c^2+c\cdots}{\cdots}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
1/64*(16*(sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sq
rt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2
*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*((s
qrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)*b^2) + 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b
*c*d - 3*sqrt(d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)
/d^2 + (sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(I*sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)
*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt
(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*
d)/d)/b^2)/d^2) - 8*(sqrt(pi)*(4*b*c - I*d)*d*erf(-I*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d
/sqrt(b^2*d^2) + 1)*b) + sqrt(pi)*(4*b*c + I*d)*d*erf(I*sqrt(b*d)*sqrt(d*x
 + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(
-I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*
b*c + I*a*d)/d)/b + 2*sqrt(d*x + c)*d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*
d)/d)/b)*c)/d
```

### 3.53.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx=\int \cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{3/2}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(3/2), x)
```

# 3.54 $\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx$


3.54.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 505
3.54.2 Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 505
3.54.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 506
3.54.4 Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 509
3.54.5 Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 510
3.54.6 Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 510
3.54.7 Maxima [C] (verification not implemented) . . . . . . . . . . . . . . . . . . 511
3.54.8 Giac [C] (verification not implemented) . . . . . . . . . . . . . . . . . . . 511
3.54.9 Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 512


## 3.54.1 Optimal result

Integrand size = 22, antiderivative size = 142

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = -\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b} + \frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}} - \frac{\sqrt{d}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{8b^{3/2}}$$

output
```
1/8*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*d^
(1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/
2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/4*cos(2*b*x+2*a)*(d*x+c)^(
1/2)/b
```

## 3.54.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.02

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = \frac{1}{2}\left(-\frac{e^{2i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\,\Gamma\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{-2i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\,\Gamma\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{\frac{ib(c+dx)}{d}}}\right)$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
(-1/4*(E^((2*I)*(a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-2*I)*b*(c + d*x
))/d])/(Sqrt[2]*b*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]*Gamma[3/2,
((2*I)*b*(c + d*x))/d])/(4*Sqrt[2]*b*E^((2*I)*(a - (b*c)/d))*Sqrt[(I*b*(c
+ d*x))/d]))/2
```

### 3.54.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4906, 27, 3042, 3777, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \frac{1}{2}\sqrt{c+dx}\sin(2a+2bx)dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \sqrt{c+dx}\sin(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sqrt{c+dx}\sin(2a+2bx)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{2}\left(\frac{d\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}dx}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\left(\frac{d\int \frac{\sin\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \; 3787$$

$$\frac{1}{2}\left(\frac{d\left(\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx-\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\left(\frac{d\left(\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx-\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \; 3785$$

$$\frac{1}{2}\left(\frac{d\left(\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int \cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \; 3786$$

$$\frac{1}{2}\left(\frac{d\left(\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int \cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int \sin\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \; 3832$$

3.54. $\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx$

$$\frac{1}{2}\left(\frac{d\left(\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \; 3833$$

$$\frac{1}{2}\left(\frac{d\left(\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
(-1/2*(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/b + (d*((Sqrt[Pi]*Cos[2*a - (2*b*c)
/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(Sqrt[b]*Sqrt[
d]) - (Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin
[2*a - (2*b*c)/d])/(Sqrt[b]*Sqrt[d])))/(4*b))/2
```

### 3.54.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.54.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00

| method | result | s |
|---|---|---|
| derivativedivides | $\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}}{d}$ | 1 |
| default | $\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}}{d}$ | 1 |

input
```
int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/16/b*d*Pi^(
1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*
x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)
^(1/2)/d)))
```

### 3.54.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.88

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx$$
$$=\frac{\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-2\left(2\,b\cos\left(bx\right.\right.}{8\,b^2}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/8*(pi*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)
*sqrt(b/(pi*d))) - pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/
(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*(2*b*cos(b*x + a)^2 - b)*sqrt(d*x + c))
/b^2
```

### 3.54.6 Sympy [F]

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx=\int \sqrt{c+dx}\sin\left(a+bx\right)\cos\left(a+bx\right)dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)*cos(a + b*x), x)
```

### 3.54.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.47

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = -\frac{\sqrt{2}\left(8\sqrt{2}\sqrt{dx+c}b\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)+\left((i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)+(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d\right.}{}$$

input

```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output

```
-1/64*sqrt(2)*(8*sqrt(2)*sqrt(d*x + c)*b*cos(2*((d*x + c)*b - b*c + a*d)/d
) + ((I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I
 + 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d
*x + c)*sqrt(2*I*b/d)) + (-(I + 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(
-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c
 - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/b^2
```

### 3.54.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.86

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = \frac{4\left(\frac{\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{i\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2\,(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}+\frac{\sqrt{\pi}d\,\mathrm{erf}\left(\frac{i\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2\,(-i\,bc+i\,ad)}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}\right)c-\frac{\sqrt{\pi}(4\,bc-i\,d)d}{}}{}$$

input

```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
1/16*(4*(sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqr
t(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*
(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - sqrt(pi)*(
4*b*c - I*d)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - sqrt(p
i)*(4*b*c + I*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) -
2*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 2*sqrt(d*x
+ c)*d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)/d
```

### 3.54.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = \int \cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)\,\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(1/2), x)
```

# 3.55 $\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx$


3.55.1 Optimal result . . . . . . . . . . 513
3.55.2 Mathematica [C] (verified) . . . . . . . . . . 513
3.55.3 Rubi [A] (verified) . . . . . . . . . . 514
3.55.4 Maple [A] (verified) . . . . . . . . . . 517
3.55.5 Fricas [A] (verification not implemented) . . . . . . . . . . 518
3.55.6 Sympy [F] . . . . . . . . . . 518
3.55.7 Maxima [C] (verification not implemented) . . . . . . . . . . 519
3.55.8 Giac [C] (verification not implemented) . . . . . . . . . . 519
3.55.9 Mupad [F(-1)] . . . . . . . . . . 520


## 3.55.1 Optimal result

Integrand size = 22, antiderivative size = 142

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = -\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b} + \frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}} - \frac{\sqrt{d}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{8b^{3/2}}$$

output
```
1/8*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*d^
(1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/
2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/4*cos(2*b*x+2*a)*(d*x+c)^(
1/2)/b
```

## 3.55.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.02

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = \frac{1}{2}\left(-\frac{e^{2i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\,\Gamma\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{-2i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\,\Gamma\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{\frac{ib(c+dx)}{d}}}\right)$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
(-1/4*(E^((2*I)*(a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-2*I)*b*(c + d*x
))/d])/(Sqrt[2]*b*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]*Gamma[3/2,
((2*I)*b*(c + d*x))/d])/(4*Sqrt[2]*b*E^((2*I)*(a - (b*c)/d))*Sqrt[(I*b*(c
+ d*x))/d]))/2
```

### 3.55.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4906, 27, 3042, 3777, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \frac{1}{2}\sqrt{c+dx}\sin(2a+2bx)dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \sqrt{c+dx}\sin(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int \sqrt{c+dx}\sin(2a+2bx)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{2}\left(\frac{d\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}dx}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{d\int \frac{\sin\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{3787}$$

$$\frac{1}{2}\left(\frac{d\left(\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx-\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{d\left(\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx-\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{3785}$$

$$\frac{1}{2}\left(\frac{d\left(\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int \cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{3786}$$

$$\frac{1}{2}\left(\frac{d\left(\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int \cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int \sin\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{3832}$$

---

3.55. $\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx$

$$\frac{1}{2}\left(\frac{d\left(\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{3833}$$

$$\frac{1}{2}\left(\frac{d\left(\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
(-1/2*(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/b + (d*((Sqrt[Pi]*Cos[2*a - (2*b*c)
/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(Sqrt[b]*Sqrt[
d]) - (Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin
[2*a - (2*b*c)/d])/(Sqrt[b]*Sqrt[d])))/(4*b))/2
```

### 3.55.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.55.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00

| method | result | s |
|---|---|---|
| derivativedivides | $\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}}{d}$ | 1 |
| default | $\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}}{d}$ | 1 |

input
```
int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/16/b*d*Pi^(
1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*
x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)
^(1/2)/d)))
```

### 3.55.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.88

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx$$
$$= \frac{\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)-2\left(2\,b\cos\left(bx\right.\right.}{8\,b^2}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/8*(pi*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)
*sqrt(b/(pi*d))) - pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/
(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*(2*b*cos(b*x + a)^2 - b)*sqrt(d*x + c))
/b^2
```

### 3.55.6 Sympy [F]

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = \int \sqrt{c+dx}\sin\left(a+bx\right)\cos\left(a+bx\right)dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)*cos(a + b*x), x)
```

### 3.55.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.47

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{\sqrt{2}\left(8\sqrt{2}\sqrt{dx+c}b\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)+\left((i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)+(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}d\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/64*sqrt(2)*(8*sqrt(2)*sqrt(d*x + c)*b*cos(2*((d*x + c)*b - b*c + a*d)/d
) + ((I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I
 + 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d
*x + c)*sqrt(2*I*b/d)) + (-(I + 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(
-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c
 - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/b^2
```

### 3.55.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.86

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx$$
$$=\frac{4\left(\frac{\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{i\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}+\frac{\sqrt{\pi}d\,\mathrm{erf}\left(\frac{i\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2(-i\,bc+i\,ad)}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}\right)c-\frac{\sqrt{\pi}(4\,bc-i\,d)d}{}}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
1/16*(4*(sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqr
t(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*
(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - sqrt(pi)*(
4*b*c - I*d)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - sqrt(p
i)*(4*b*c + I*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) -
2*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 2*sqrt(d*x
+ c)*d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)/d
```

### 3.55.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)\,\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(1/2), x)
```

# 3.56 $\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx$

3.56.1 Optimal result . . . 521
3.56.2 Mathematica [C] (verified) . . . 521
3.56.3 Rubi [A] (verified) . . . 522
3.56.4 Maple [A] (verified) . . . 526
3.56.5 Fricas [A] (verification not implemented) . . . 527
3.56.6 Sympy [F] . . . 527
3.56.7 Maxima [C] (verification not implemented) . . . 527
3.56.8 Giac [C] (verification not implemented) . . . 528
3.56.9 Mupad [F(-1)] . . . 529

## 3.56.1 Optimal result

Integrand size = 22, antiderivative size = 168

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} - \frac{3d^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}} - \frac{3d^{3/2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{32b^{5/2}} + \frac{3d\sqrt{c+dx}\sin(2a+2bx)}{16b^2}$$

output
```
-1/4*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b-3/32*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS
(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(5/2)-3/32*d^(3/2)*F
resnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2
)/b^(5/2)+3/16*d*sin(2*b*x+2*a)*(d*x+c)^(1/2)/b^2
```

## 3.56.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.76

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = \frac{e^{-\frac{2i(bc+ad)}{d}}(c+dx)^{3/2}\left(-\frac{e^{4ia}\Gamma\left(\frac{5}{2},-\frac{2ib(c+dx)}{d}\right)}{\left(-\frac{ib(c+dx)}{d}\right)^{3/2}}-\frac{e^{\frac{4ibc}{d}}\Gamma\left(\frac{5}{2},\frac{2ib(c+dx)}{d}\right)}{\left(\frac{ib(c+dx)}{d}\right)^{3/2}}\right)}{16\sqrt{2}b}$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
((c + d*x)^(3/2)*(-((E^((4*I)*a)*Gamma[5/2, ((-2*I)*b*(c + d*x))/d])/(((-I
)*b*(c + d*x))/d)^(3/2)) - (E^(((4*I)*b*c)/d)*Gamma[5/2, ((2*I)*b*(c + d*x
))/d])/((I*b*(c + d*x))/d)^(3/2)))/(16*Sqrt[2]*b*E^(((2*I)*(b*c + a*d))/d)
)
```

### 3.56.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.09, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {4906, 27, 3042, 3777, 3042, 3777, 25, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \frac{1}{2}(c+dx)^{3/2}\sin(2a+2bx)dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int (c+dx)^{3/2}\sin(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int (c+dx)^{3/2}\sin(2a+2bx)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{2}\left(\frac{3d\int \sqrt{c+dx}\cos(2a+2bx)dx}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{3d\int \sqrt{c+dx}\sin\left(2a+2bx+\frac{\pi}{2}\right)dx}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{d\int -\frac{\sin(2a+2bx)}{\sqrt{c+dx}}dx}{4b}+\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}dx}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\int \frac{\sin(2a+2bx)}{\sqrt{c+dx}}dx}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \; 3787$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\left(\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx+\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{2b}\right.$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\left(\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx+\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2a+2}{2b}\right.$$

$$\Big\downarrow \; 3785$$

3.56. $\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\left(\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(2a+}{2b}\right.$$

$$\Big\downarrow \; 3786$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\left(\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\sin\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(}{2b}\right.$$

$$\Big\downarrow \; 3832$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\left(\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(}{2b}\right.$$

$$\Big\downarrow \; 3833$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{\sqrt{c+dx}\sin(2a+2bx)}{2b}-\frac{d\left(\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{4b}\right)}{4b}-\frac{(c+dx)^{3/2}\cos(}{2b}\right.$$

---

3.56. $\int(c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
(-1/2*((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/b + (3*d*(-1/4*(d*((Sqrt[Pi]*Cos[
2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(
Sqrt[b]*Sqrt[d]) + (Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*S
qrt[Pi])]*Sin[2*a - (2*b*c)/d])/(Sqrt[b]*Sqrt[d])))/b + (Sqrt[c + d*x]*Sin
[2*a + 2*b*x])/(2*b)))/(4*b))/2
```

#### 3.56.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.56.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.11

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+s\right.}{8b\sqrt{\frac{b}{d}}}\right.}{4b}}{d}$ |
| default | $\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+s\right.}{8b\sqrt{\frac{b}{d}}}\right.}{4b}}{d}$ |

input
```
int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+3/8/b*d*(1/4/
b*d*(d*x+c)^(1/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^
(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/
d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
)
```

### 3.56.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{3\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-4\,(2\,b^2d \cdots}{32\,b^3}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/32*(3*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*
x + c)*sqrt(b/(pi*d))) + 3*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x +
c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(2*b^2*d*x + 3*b*d*cos(b*x +
a)*sin(b*x + a) + 2*b^2*c - 4*(b^2*d*x + b^2*c)*cos(b*x + a)^2)*sqrt(d*x +
 c))/b^3
```

### 3.56.6 Sympy [F]

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx=\int (c+dx)^{\frac{3}{2}}\sin\left(a+bx\right)\cos\left(a+bx\right)dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)*cos(a + b*x), x)
```

### 3.56.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.52

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{\sqrt{2}\left(32\,\sqrt{2}(dx+c)^{\frac{3}{2}}b^2\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-24\,\sqrt{2}\sqrt{dx+c}bd\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)+3\left((i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi} \cdots\right.\right.}{\cdots}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/256*sqrt(2)*(32*sqrt(2)*(d*x + c)^(3/2)*b^2*cos(2*((d*x + c)*b - b*c +
a*d)/d) - 24*sqrt(2)*sqrt(d*x + c)*b*d*sin(2*((d*x + c)*b - b*c + a*d)/d)
+ 3*((I + 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) -
(I - 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sq
rt(d*x + c)*sqrt(2*I*b/d)) + 3*(-(I - 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1
/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*s
in(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/b^3
```

### 3.56.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 747, normalized size of antiderivative = 4.45

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = \frac{16\left(\frac{\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{i\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2\,(i\,bc-i\,ad)}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}+\frac{\sqrt{\pi}d\,\mathrm{erf}\left(\frac{i\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{2\,(-i\,bc+i\,ad)}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}\right)c^2+c\cdots}{\cdots}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
1/64*(16*(sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sq
rt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2
*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*((s
qrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)*b^2) + 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b
*c*d - 3*sqrt(d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)
/d^2 + (sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(I*sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)
*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt
(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*
d)/d)/b^2)/d^2) - 8*(sqrt(pi)*(4*b*c - I*d)*d*erf(-I*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d
/sqrt(b^2*d^2) + 1)*b) + sqrt(pi)*(4*b*c + I*d)*d*erf(I*sqrt(b*d)*sqrt(d*x
 + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(
-I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*sqrt(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*
b*c + I*a*d)/d)/b + 2*sqrt(d*x + c)*d*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*
d)/d)/b)*c)/d
```

### 3.56.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx=\int \cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{3/2}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(3/2), x)
```

# 3.57 $\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx$


3.57.1 Optimal result . . . . . 530
3.57.2 Mathematica [C] (verified) . . . . . 530
3.57.3 Rubi [A] (verified) . . . . . 531
3.57.4 Maple [A] (verified) . . . . . 537
3.57.5 Fricas [A] (verification not implemented) . . . . . 538
3.57.6 Sympy [F] . . . . . 538
3.57.7 Maxima [C] (verification not implemented) . . . . . 539
3.57.8 Giac [C] (verification not implemented) . . . . . 539
3.57.9 Mupad [F(-1)] . . . . . 540


## 3.57.1 Optimal result

Integrand size = 22, antiderivative size = 196

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} - \frac{15d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{7/2}} + \frac{15d^{5/2}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{128b^{7/2}} + \frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{16b^2}$$

output
```
-1/4*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b+5/16*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^
2-15/128*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)
/Pi^(1/2))*Pi^(1/2)/b^(7/2)+15/128*d^(5/2)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+15/64*d^2*cos(2*b*x+
2*a)*(d*x+c)^(1/2)/b^3
```

## 3.57.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.73

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \frac{e^{-\frac{2i(bc+ad)}{d}}(c+dx)^{5/2}\left(e^{4ia}\sqrt{\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{7}{2},-\frac{2ib(c+dx)}{d}\right)+e^{\frac{4ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\,\Gamma\left(\frac{7}{2},\frac{2ib(c+dx)}{d}\right)\right)}{32\sqrt{2}b\left(\frac{b^2(c+dx)^2}{d^2}\right)^{3/2}}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

output
```
((c + d*x)^(5/2)*(E^((4*I)*a)*Sqrt[(I*b*(c + d*x))/d]*Gamma[7/2, ((-2*I)*b
*(c + d*x))/d] + E^(((4*I)*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[7/2, (
(2*I)*b*(c + d*x))/d]))/(32*Sqrt[2]*b*E^(((2*I)*(b*c + a*d))/d)*((b^2*(c +
 d*x)^2)/d^2)^(3/2))
```

### 3.57.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.11, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {4906, 27, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3787, 3042, 3785, 3786, 3832, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \frac{1}{2}(c+dx)^{5/2}\sin(2a+2bx)dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int (c+dx)^{5/2}\sin(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int (c+dx)^{5/2}\sin(2a+2bx)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{2}\left(\frac{5d\int (c+dx)^{3/2}\cos(2a+2bx)dx}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{5d\int (c+dx)^{3/2}\sin\left(2a+2bx+\frac{\pi}{2}\right)dx}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{3d\int -\sqrt{c+dx}\sin(2a+2bx)dx}{4b}+\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}\right)}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\int \sqrt{c+dx}\sin(2a+2bx)dx}{4b}\right)}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\int \sqrt{c+dx}\sin(2a+2bx)dx}{4b}\right)}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{3777}$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}dx}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)}{4b}\right)}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\int \frac{\sin\left(2a+2bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)}{4b}\right)}{4b}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{3787}$$

3.57. $\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\left(\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx-\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)}{4b}\right)}{4b}-\frac{(c+\right.$$

$\downarrow$ 3042

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\left(\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx-\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)}{4b}\right)}{4b}-\frac{(c\right.$$

$\downarrow$ 3785

3.57. $\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}dx\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)}{4b}\right)}{4b}-\right.$$

$$\downarrow 3786$$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{2\sin\left(2a-\frac{2bc}{d}\right)\int\sin\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right.}{4b}\right.}{4b}\right.$$

$$\downarrow 3832$$

---

3.57. $\int(c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx$

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\left(\frac{2\cos\left(2a-\frac{2bc}{d}\right)\int\cos\left(\frac{2b(c+dx)}{d}\right)d\sqrt{c+dx}}{d}-\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)}{4b}\right)}{4b}\right.$$

$\downarrow$ 3833

$$\frac{1}{2}\left(\frac{5d\left(\frac{(c+dx)^{3/2}\sin(2a+2bx)}{2b}-\frac{3d\left(\frac{d\left(\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}-\frac{\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{\sqrt{b}\sqrt{d}}\right)}{4b}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{2b}\right)}{4b}\right)}{4b}\right.$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

## 3.57. $\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx$

output
```
(-1/2*((c + d*x)^(5/2)*Cos[2*a + 2*b*x])/b + (5*d*((-3*d*(-1/2*(Sqrt[c + d
*x]*Cos[2*a + 2*b*x])/b + (d*((Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*S
qrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(Sqrt[b]*Sqrt[d]) - (Sqrt[Pi]*F
resnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d]
)/(Sqrt[b]*Sqrt[d])))/(4*b)))/(4*b) + ((c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(
2*b)))/(4*b))/2
```

### 3.57.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3787
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Cos
[(d*e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Simp[Sin[(
d*e - c*f)/d]   Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.57.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.19

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\cdots\right.}{\cdots}\right.}{4\cdots}}{d}$ |
| default | $\frac{-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\cdots\right.}{\cdots}\right.}{4\cdots}}{d}$ |

input
```
int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+5/8/b*d*(1/4/
b*d*(d*x+c)^(3/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+
c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(co
s(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(2*
(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

### 3.57.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.13

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{15\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-2\,(16}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/128*(15*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(
d*x + c)*sqrt(b/(pi*d))) - 15*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x
 + c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*(16*b^3*d^2*x^2 + 32*b^3*c
*d*x + 16*b^3*c^2 - 15*b*d^2 - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c
^2 - 15*b*d^2)*cos(b*x + a)^2 + 40*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)*sin(
b*x + a))*sqrt(d*x + c))/b^4
```

### 3.57.6 Sympy [F]

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \int (c+dx)^{\frac{5}{2}}\sin(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**(5/2)*sin(a + b*x)*cos(a + b*x), x)
```

### 3.57.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.41

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \frac{\sqrt{2}\left(160\sqrt{2}(dx+c)^{\frac{3}{2}}b^2d\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-8\left(16\sqrt{2}(dx+c)^{\frac{5}{2}}b^3-15\sqrt{2}\sqrt{dx+c}bd^2\right)\cos\left(\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/1024*sqrt(2)*(160*sqrt(2)*(d*x + c)^(3/2)*b^2*d*sin(2*((d*x + c)*b - b*c
 + a*d)/d) - 8*(16*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d*x + c)*
b*d^2)*cos(2*((d*x + c)*b - b*c + a*d)/d) - 15*(-(I - 1)*4^(1/4)*sqrt(pi)*
d^3*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(pi)*d^3*(
b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 1
5*((I + 1)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I
 - 1)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt
(d*x + c)*sqrt(-2*I*b/d)))/b^4
```

### 3.57.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 1205, normalized size of antiderivative = 6.15

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
1/256*(64*(sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + s
qrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 12*c*d
^2*((sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x
+ c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d
)/b^2)/d^2 + (sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(I*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*
I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-2*(I*(d*x + c)*b - I*b*c
+ I*a*d)/d)/b^2)/d^2) - d^3*((sqrt(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b
*c*d^2 + 15*I*d^3)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3)
- 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sq
rt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^
2 - 15*I*sqrt(d*x + c)*d^3)*e^(-2*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)
/d^3 + (sqrt(pi)*(64*b^3*c^3 + 48*I*b^2*c^2*d - 36*b*c*d^2 - 15*I*d^3)*d*e
rf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*...
```

### 3.57.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx=\int \cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{5/2}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(5/2), x)
```

# 3.58 $\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx$

3.58.1 Optimal result . . . 541
3.58.2 Mathematica [C] (verified) . . . 542
3.58.3 Rubi [A] (verified) . . . 543
3.58.4 Maple [A] (verified) . . . 545
3.58.5 Fricas [A] (verification not implemented) . . . 546
3.58.6 Sympy [F(-1)] . . . 546
3.58.7 Maxima [C] (verification not implemented) . . . 547
3.58.8 Giac [C] (verification not implemented) . . . 547
3.58.9 Mupad [F(-1)] . . . 548

## 3.58.1 Optimal result

Integrand size = 24, antiderivative size = 406

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx=\frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2}+\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}-\frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{144b^{7/2}}+\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{16b^{7/2}}-\frac{15d^2\sqrt{c+dx}\sin(a+bx)}{16b^3}+\frac{(c+dx)^{5/2}\sin(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{144b^3}-\frac{(c+dx)^{5/2}\sin(3a+3bx)}{12b}$$

output
```
5/8*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2-5/72*d*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b^2
+1/4*(d*x+c)^(5/2)*sin(b*x+a)/b-1/12*(d*x+c)^(5/2)*sin(3*b*x+3*a)/b-5/864*
d^(5/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d
^(1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)-5/864*d^(5/2)*FresnelC(b^(1/2)*6^(1/2)/Pi
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(7/2)+15
/32*d^(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d
^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/32*d^(5/2)*FresnelC(b^(1/2)*2^(1/2)/Pi
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)-15/16*
d^2*sin(b*x+a)*(d*x+c)^(1/2)/b^3+5/144*d^2*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b^
3
```

### 3.58.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.50 (sec) , antiderivative size = 1418, normalized size of antiderivative = 3.49

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
-1/288*(c*Sqrt[d]*(12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(1 +
 (2*I)*b*x + E^((6*I)*(a + b*x))*(1 - (2*I)*b*x)) + (1 + I)*(2*b*c + I*d)*
E^(((3*I)*b*(2*c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt
[c + d*x])/Sqrt[d]] - (1 + I)*(2*b*c - I*d)*E^((3*I)*(2*a + b*x))*Sqrt[6*P
i]*Erfi[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(b^(5/2)*E^((
(3*I)*(a*d + b*(c + d*x)))/d)) + (c^2*d*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x
))/d]*Gamma[3/2, ((-I)*b*(c + d*x))/d] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c +
d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d]))/(8*b^2*E^((I*(b*c + a*d))/d)*Sqrt
[c + d*x]) - (c^2*(c + d*x)^(3/2)*(-((E^((6*I)*a)*Gamma[3/2, ((-3*I)*b*(c
+ d*x))/d])/(((-I)*b*(c + d*x))/d)^(3/2)) - (E^(((6*I)*b*c)/d)*Gamma[3/2,
((3*I)*b*(c + d*x))/d])/((I*b*(c + d*x))/d)^(3/2)))/(24*Sqrt[3]*d*E^(((3*I
)*(b*c + a*d))/d)) + (c*Sqrt[d]*(E^(I*(a - (b*c)/d))*(2*Sqrt[b]*Sqrt[d]*E^
((I*b*(c + d*x))/d)*(3 - (2*I)*b*x)*Sqrt[c + d*x] + (-1)^(1/4)*(-2*b*c + (
3*I)*d)*Sqrt[Pi]*Erfi[((-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]) + (2*Sq
rt[b]*Sqrt[d]*(3 + (2*I)*b*x)*Sqrt[c + d*x] + (1 + I)*(2*b*c + (3*I)*d)*Sq
rt[Pi/2]*Erf[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[2]*Sqrt[d])]*(Cos[b*(c/
d + x)] + I*Sin[b*(c/d + x)]))*(Cos[a + b*x] - I*Sin[a + b*x])))/(16*b^(5/
2)) + (Sqrt[d]*((Cos[a - (b*c)/d] + I*Sin[a - (b*c)/d])*((1 + I)*(4*b^2*c^
2 - (12*I)*b*c*d - 15*d^2)*Sqrt[Pi/2]*Erfi[((1 + I)*Sqrt[b]*Sqrt[c + d*x])
/(Sqrt[2]*Sqrt[d])] + 2*Sqrt[b]*Sqrt[d]*Sqrt[c + d*x]*((15*I)*d - (4*I)...
```

### 3.58.3 Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin^2(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{ 4906}$$

$$\int \left(\frac{1}{4}(c+dx)^{5/2}\cos(a+bx)-\frac{1}{4}(c+dx)^{5/2}\cos(3a+3bx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}-\frac{15d^2\sqrt{c+dx}\sin(a+bx)}{16b^3}+\frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{144b^3}+\frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)}{4b}-\frac{(c+dx)^{5/2}\sin(3a+3bx)}{12b}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
(5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(8*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a
+ 3*b*x])/(72*b^2) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqr
t[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi
/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[
d]])/(144*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/2)*Sq
rt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c
)/d])/(16*b^(7/2)) - (15*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(16*b^3) + ((c +
d*x)^(5/2)*Sin[a + b*x])/(4*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(1
44*b^3) - ((c + d*x)^(5/2)*Sin[3*a + 3*b*x])/(12*b)
```

### 3.58.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.58.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.17

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos \cdots\right.}{\cdots}\right.}{\cdots}\right.}{4b}\cdots}{\cdots}$ |
| default | $\frac{\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos \cdots\right.}{\cdots}\right.}{\cdots}\right.}{4b}\cdots}{\cdots}$ |

input
```
int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/8/b*d*(d*x+c)^(5/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-5/8/b*d*(-1/2/b*d*
(d*x+c)^(3/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/2)*
sin(b/d*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*
d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((a*
d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1/24
/b*d*(d*x+c)^(5/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+5/24/b*d*(-1/6/b*d*(d*
x+c)^(3/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2)
*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^
(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*
(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d
)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

### 3.58.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.91

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx =$$
$$-\frac{5\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-405\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(
sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 40
5*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(
pi*d)))*sin(-(b*c - a*d)/d) + 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(
sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(10*(b^2*
d^2*x + b^2*c*d)*cos(b*x + a)^3 - 30*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) -
(12*b^3*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 35*b*d^2 - (12*b^3*d^2*x^2 +
 24*b^3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d
*x + c))/b^4
```

### 3.58.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Timed out
```

### 3.58.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.35

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{\left(240\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)-2160\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)-5\left(-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}bc\right.\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/3456*(240*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d) - 2160
*(d*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d) - 5*(-(I + 1)*9^(1/4
)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9
^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(s
qrt(d*x + c)*sqrt(3*I*b/d)) - 405*((I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2
)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/
4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 405*(-(I - 1)*sqr
t(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(2)*
sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt
(-I*b/d)) - 5*((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(
-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)
*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + 24*(12*(d*x +
c)^(5/2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*sin(3*((d*x + c)*b - b*c + a*d)/d)
 - 216*(4*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*sin(((d*x + c)*b
 - b*c + a*d)/d))*d/b^5
```

### 3.58.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 2478, normalized size of antiderivative = 6.10

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/1728*(72*(3*I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*s
qrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b
*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/
d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(1/2*I*
sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*
c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2*(9*(I
*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(1/2*I*sqrt(2)*sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(s
qrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4
*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c -
 I*a*d)/d)/b^2)/d^2 + (I*sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d
*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 6*(-2*
I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-3
*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 9*(-I*sqrt(2)*sqrt(pi)*(4*
b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/s...
```

### 3.58.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)^2\,(c+d\,x)^{5/2}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

# 3.59 $\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx$

3.59.1 Optimal result . . . 549
3.59.2 Mathematica [C] (verified) . . . 550
3.59.3 Rubi [A] (verified) . . . 551
3.59.4 Maple [A] (verified) . . . 553
3.59.5 Fricas [A] (verification not implemented) . . . 553
3.59.6 Sympy [F] . . . 554
3.59.7 Maxima [C] (verification not implemented) . . . 554
3.59.8 Giac [C] (verification not implemented) . . . 555
3.59.9 Mupad [F(-1)] . . . 556

## 3.59.1 Optimal result

Integrand size = 24, antiderivative size = 353

$$\begin{aligned}\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx &= \frac{3d\sqrt{c+dx}\cos(a+bx)}{8b^2}\\
&-\frac{d\sqrt{c+dx}\cos(3a+3bx)}{24b^2}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}\\
&+\frac{d^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}\\
&-\frac{d^{3/2}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{24b^{5/2}}\\
&+\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{8b^{5/2}}\\
&+\frac{(c+dx)^{3/2}\sin(a+bx)}{4b}-\frac{(c+dx)^{3/2}\sin(3a+3bx)}{12b}\end{aligned}$$

output
```
1/4*(d*x+c)^(3/2)*sin(b*x+a)/b-1/12*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b+1/144*d
^(3/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1/144*d^(3/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^
(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/1
6*d^(3/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d^(3/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/8*d*cos
(b*x+a)*(d*x+c)^(1/2)/b^2-1/24*d*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b^2
```

## 3.59.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 730, normalized size of antiderivative = 2.07

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{\sqrt{d}e^{-\frac{3i(ad+b(c+dx))}{d}}\left(12\sqrt{b}\sqrt{d}e^{\frac{3ibc}{d}}\sqrt{c+dx}\left(1+2ibx+e^{6i(a+bx)}(1-2ibx)\right)+(1+i)(2bc+id)e^{\frac{3ib(2c+dx)}{d}}\sqrt{6\pi}e\cdots\right.}{576b^{5/2}}$$

$$+\frac{cde^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)\right)}{8b^2\sqrt{c+dx}}$$

$$-\frac{ce^{-\frac{3i(bc+ad)}{d}}(c+dx)^{3/2}\left(-\frac{e^{6ia}\Gamma\left(\frac{3}{2},-\frac{3ib(c+dx)}{d}\right)}{\left(-\frac{ib(c+dx)}{d}\right)^{3/2}}-\frac{e^{\frac{6ibc}{d}}\Gamma\left(\frac{3}{2},\frac{3ib(c+dx)}{d}\right)}{\left(\frac{ib(c+dx)}{d}\right)^{3/2}}\right)}{24\sqrt{3}d}$$

$$+\frac{\sqrt{d}\left(e^{i\left(a-\frac{bc}{d}\right)}\left(2\sqrt{b}\sqrt{d}e^{\frac{ib(c+dx)}{d}}(3-2ibx)\sqrt{c+dx}+\sqrt[4]{-1}(-2bc+3id)\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt[4]{-1}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)\right)+\left(2\sqrt{b}\sqrt{c}\cdots\right.\right.}{\cdots}$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
-1/576*(Sqrt[d]*(12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(1 + (
2*I)*b*x + E^((6*I)*(a + b*x))*(1 - (2*I)*b*x)) + (1 + I)*(2*b*c + I*d)*E^
(((3*I)*b*(2*c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c
 + d*x])/Sqrt[d]] - (1 + I)*(2*b*c - I*d)*E^((3*I)*(2*a + b*x))*Sqrt[6*Pi]
*Erfi[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(b^(5/2)*E^(((3
*I)*(a*d + b*(c + d*x)))/d)) + (c*d*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d
]*Gamma[3/2, ((-I)*b*(c + d*x))/d] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x)
)/d]*Gamma[3/2, (I*b*(c + d*x))/d]))/(8*b^2*E^((I*(b*c + a*d))/d)*Sqrt[c +
 d*x]) - (c*(c + d*x)^(3/2)*(-((E^((6*I)*a)*Gamma[3/2, ((-3*I)*b*(c + d*x)
)/d])/(((-I)*b*(c + d*x))/d)^(3/2)) - (E^(((6*I)*b*c)/d)*Gamma[3/2, ((3*I)
*b*(c + d*x))/d])/((I*b*(c + d*x))/d)^(3/2)))/(24*Sqrt[3]*d*E^(((3*I)*(b*c
 + a*d))/d)) + (Sqrt[d]*(E^(I*(a - (b*c)/d))*(2*Sqrt[b]*Sqrt[d]*E^((I*b*(c
 + d*x))/d)*(3 - (2*I)*b*x)*Sqrt[c + d*x] + (-1)^(1/4)*(-2*b*c + (3*I)*d)*
Sqrt[Pi]*Erfi[((-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]) + (2*Sqrt[b]*Sq
rt[d]*(3 + (2*I)*b*x)*Sqrt[c + d*x] + (1 + I)*(2*b*c + (3*I)*d)*Sqrt[Pi/2]
*Erf[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[2]*Sqrt[d])]*(Cos[b*(c/d + x)]
+ I*Sin[b*(c/d + x)]))*(Cos[a + b*x] - I*Sin[a + b*x])))/(32*b^(5/2))
```

### 3.59.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin^2(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{4}(c+dx)^{3/2}\cos(a+bx)-\frac{1}{4}(c+dx)^{3/2}\cos(3a+3bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{3d\sqrt{c+dx}\cos(a+bx)}{8b^2}-\frac{d\sqrt{c+dx}\cos(3a+3bx)}{24b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)}{4b}-\frac{(c+dx)^{3/2}\sin(3a+3bx)}{12b}$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
(3*d*Sqrt[c + d*x]*Cos[a + b*x])/(8*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*
x])/(24*b^2) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sq
rt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*
a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b
^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/
Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(24*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*Fresne
lS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2
)) + ((c + d*x)^(3/2)*Sin[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b
*x])/(12*b)
```

### 3.59.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.59.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.09

| method | result |
|---|---|
| derivativedivides | $\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-cb}{d}\right)\text{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\cdots\right)\right)}{4b\sqrt{\frac{b}{d}}}\cdots\right)}{4b}\cdots$ |
| default | $\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-cb}{d}\right)\text{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\cdots\right)\right)}{4b\sqrt{\frac{b}{d}}}\cdots\right)}{4b}\cdots$ |

input
```
int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/8/b*d*(d*x+c)^(3/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-3/8/b*d*(-1/2/b*d*
(d*x+c)^(1/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^
(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)))-1/24/b*d*(d*x+c)^(3/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/8/b*d*(
-1/6/b*d*(d*x+c)^(1/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*P
i^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*
3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)
/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

### 3.59.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.84

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \frac{\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-27\,\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\cdots\right)\cdots}{\cdots}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqr
t(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(
-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqr
t(2)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)
))*sin(-(b*c - a*d)/d) - sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)
*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*d*cos(b*x + a
)^3 - 3*b*d*cos(b*x + a) - 2*(b^2*d*x + b^2*c - (b^2*d*x + b^2*c)*cos(b*x
+ a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

### 3.59.6 Sympy [F]

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin^2(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x), x)
```

### 3.59.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.41

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{\left(\frac{48\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{144\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+24\sqrt{dx+c}b^2\cos\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)-216\,\sqrt{\cdots}\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/576*(48*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d)/d - 144*
(d*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d + 24*sqrt(d*x + c)*
b^2*cos(3*((d*x + c)*b - b*c + a*d)/d) - 216*sqrt(d*x + c)*b^2*cos(((d*x +
 c)*b - b*c + a*d)/d) + ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1
/4)*cos(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)
^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 27*((I -
1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt
(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(I*b/d)) - 27*(-(I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c -
 a*d)/d) - (I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d
))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*
d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)
*b*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/
d)))*d/b^4
```

### 3.59.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 1547, normalized size of antiderivative = 4.38

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/288*(12*(3*I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d
)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(1/2*I*s
qrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c
 + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*(9*(I*sqrt(
2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqr
t(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d
)/d)/b^2)/d^2 + (I*sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-
1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(
I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 6*(-2*I*(d*x
 + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-3*(-I*(
d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 9*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^
2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^...
```

### 3.59.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx=\int \cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2\,\left(c+d\,x\right)^{3/2}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

# 3.60 $\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx$


3.60.1 Optimal result . . . 557
3.60.2 Mathematica [C] (verified) . . . 558
3.60.3 Rubi [A] (verified) . . . 558
3.60.4 Maple [A] (verified) . . . 560
3.60.5 Fricas [A] (verification not implemented) . . . 560
3.60.6 Sympy [F] . . . 561
3.60.7 Maxima [C] (verification not implemented) . . . 561
3.60.8 Giac [C] (verification not implemented) . . . 562
3.60.9 Mupad [F(-1)] . . . 563


## 3.60.1 Optimal result

Integrand size = 24, antiderivative size = 304

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx = -\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{12b^{3/2}} - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{4b^{3/2}} + \frac{\sqrt{c+dx}\sin(a+bx)}{4b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}$$

output
```
1/72*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)+1/72*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2
)-1/8*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2)
)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/4*si
n(b*x+a)*(d*x+c)^(1/2)/b-1/12*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b
```

### 3.60.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.83

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx = \frac{de^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)\right)}{8b^2\sqrt{c+dx}} - \frac{e^{-\frac{3i(bc+ad)}{d}}(c+dx)^{3/2}\left(-\frac{e^{6ia}\Gamma\left(\frac{3}{2},-\frac{3ib(c+dx)}{d}\right)}{\left(-\frac{ib(c+dx)}{d}\right)^{3/2}}-\frac{e^{\frac{6ibc}{d}}\Gamma\left(\frac{3}{2},\frac{3ib(c+dx)}{d}\right)}{\left(\frac{ib(c+dx)}{d}\right)^{3/2}}\right)}{24\sqrt{3}d}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
(d*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-I)*b*(c + d*x))/d
] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d
]))/(8*b^2*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x]) - ((c + d*x)^(3/2)*(-((E^(
(6*I)*a)*Gamma[3/2, ((-3*I)*b*(c + d*x))/d])/(((-I)*b*(c + d*x))/d)^(3/2))
 - (E^(((6*I)*b*c)/d)*Gamma[3/2, ((3*I)*b*(c + d*x))/d])/((I*b*(c + d*x))/
d)^(3/2)))/(24*Sqrt[3]*d*E^(((3*I)*(b*c + a*d))/d))
```

### 3.60.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^2(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \ 4906$$

$$\int \left(\frac{1}{4}\sqrt{c+dx}\cos(a+bx)-\frac{1}{4}\sqrt{c+dx}\cos(3a+3bx)\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}+\frac{\sqrt{c+dx}\sin(a+bx)}{4b}-\frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
-1/4*(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqr
t[c + d*x])/Sqrt[d]])/b^(3/2) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*F
resnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) + (Sqrt[
d]*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a
 - (3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2
/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2)) + (Sqrt[c + d*x
]*Sin[a + b*x])/(4*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(12*b)
```

### 3.60.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.60.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-cb}{d}\right)\text{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx}}{}}{}$ |
| default | $\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-cb}{d}\right)\text{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx}}{}}{}$ |

input
```
int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/8/b*d*(d*x+c)^(1/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-1/16/b*d*2^(1/2)*P
i^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d))-1/24/b*d*(d*x+c)^(1/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)
/d)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fre
snelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b
*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

### 3.60.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.81

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx$$
$$=\frac{\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\,\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\,\sqrt{}}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6
)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c
- a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi
*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(
b*c - a*d)/d) + sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x +
 c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*cos(b*x + a)^2 - b)*sqrt
(d*x + c)*sin(b*x + a))/b^2
```

### 3.60.6 Sympy [F]

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx = \int \sqrt{c+dx}\sin^2(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x), x)
```

### 3.60.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.39

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{\left(\frac{24\sqrt{dx+c}\,b^2\sin\left(\frac{3\left((dx+c)b-bc+ad\right)}{d}\right)}{d} - \frac{72\sqrt{dx+c}\,b^2\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d} + \left(-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right.\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/288*(24*sqrt(d*x + c)*b^2*sin(3*((d*x + c)*b - b*c + a*d)/d)/d - 72*sqr
t(d*x + c)*b^2*sin(((d*x + c)*b - b*c + a*d)/d)/d + (-(I + 1)*9^(1/4)*sqrt
(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqr
t(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(3*I*b/d)) - 9*(-(I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c -
 a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))
*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 9*((I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^
(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin
(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((I - 1)*9^(1/4)*sqrt(
2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt
(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(-3*I*b/d)))*d/b^3
```

### 3.60.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 848, normalized size of antiderivative = 2.79

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/144*(-9*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(
-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*
(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*s
qrt(pi)*(2*b*c - I*d)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b) + I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(s
qrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 6*(3*I*sqrt(2)*sqrt(pi)*d*erf(1/2
*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d
*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2
)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) -
I*sqrt(6)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d
^2) + 1)))*c + 18*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/
b + 6*I*sqrt(d*x + c)*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 18...
```

### 3.60.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx=\int \cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2\,\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

# 3.61 $\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx$

3.61.1 Optimal result . . . 564
3.61.2 Mathematica [C] (verified) . . . 565
3.61.3 Rubi [A] (verified) . . . 565
3.61.4 Maple [A] (verified) . . . 567
3.61.5 Fricas [A] (verification not implemented) . . . 567
3.61.6 Sympy [F] . . . 568
3.61.7 Maxima [C] (verification not implemented) . . . 568
3.61.8 Giac [C] (verification not implemented) . . . 569
3.61.9 Mupad [F(-1)] . . . 570

## 3.61.1 Optimal result

Integrand size = 24, antiderivative size = 304

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx = -\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}} + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{12b^{3/2}} - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{4b^{3/2}} + \frac{\sqrt{c+dx}\sin(a+bx)}{4b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}$$

output
```
1/72*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)+1/72*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2
)-1/8*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2)
)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/4*si
n(b*x+a)*(d*x+c)^(1/2)/b-1/12*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b
```

### 3.61.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.83

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{de^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)\right)}{8b^2\sqrt{c+dx}}$$
$$- \frac{e^{-\frac{3i(bc+ad)}{d}}(c+dx)^{3/2}\left(-\frac{e^{6ia}\Gamma\left(\frac{3}{2},-\frac{3ib(c+dx)}{d}\right)}{\left(-\frac{ib(c+dx)}{d}\right)^{3/2}}-\frac{e^{\frac{6ibc}{d}}\Gamma\left(\frac{3}{2},\frac{3ib(c+dx)}{d}\right)}{\left(\frac{ib(c+dx)}{d}\right)^{3/2}}\right)}{24\sqrt{3}d}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
(d*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-I)*b*(c + d*x))/d
] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d
]))/(8*b^2*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x]) - ((c + d*x)^(3/2)*(-((E^(
(6*I)*a)*Gamma[3/2, ((-3*I)*b*(c + d*x))/d])/(((-I)*b*(c + d*x))/d)^(3/2))
 - (E^(((6*I)*b*c)/d)*Gamma[3/2, ((3*I)*b*(c + d*x))/d])/((I*b*(c + d*x))/
d)^(3/2)))/(24*Sqrt[3]*d*E^(((3*I)*(b*c + a*d))/d))
```

### 3.61.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^2(a+bx)\cos(a+bx)\,dx$$
$$\Big\downarrow \text{4906}$$
$$\int \left(\frac{1}{4}\sqrt{c+dx}\cos(a+bx)-\frac{1}{4}\sqrt{c+dx}\cos(3a+3bx)\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}+\frac{\sqrt{c+dx}\sin(a+bx)}{4b}-\frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
-1/4*(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqr
t[c + d*x])/Sqrt[d]])/b^(3/2) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*F
resnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) + (Sqrt[
d]*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a
 - (3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2
/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2)) + (Sqrt[c + d*x
]*Sin[a + b*x])/(4*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(12*b)
```

### 3.61.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.61.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-cb}{d}\right)\text{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx}}{}}{}$ |
| default | $\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-cb}{d}\right)\text{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx}}{}}{}$ |

input
```
int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/8/b*d*(d*x+c)^(1/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-1/16/b*d*2^(1/2)*P
i^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*b*(d*x+c)^(1/2)/d))-1/24/b*d*(d*x+c)^(1/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)
/d)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fre
snelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b
*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

### 3.61.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.81

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx$$
$$=\frac{\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\,\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\,\sqrt{}}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6
)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c
- a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi
*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(
b*c - a*d)/d) + sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x +
 c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*cos(b*x + a)^2 - b)*sqrt
(d*x + c)*sin(b*x + a))/b^2
```

### 3.61.6 Sympy [F]

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx=\int \sqrt{c+dx}\sin^2(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x), x)
```

### 3.61.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.39

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx=$$
$$-\frac{\left(\frac{24\sqrt{dx+c}b^2\sin\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{72\sqrt{dx+c}b^2\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+\left(-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right.\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/288*(24*sqrt(d*x + c)*b^2*sin(3*((d*x + c)*b - b*c + a*d)/d)/d - 72*sqr
t(d*x + c)*b^2*sin(((d*x + c)*b - b*c + a*d)/d)/d + (-(I + 1)*9^(1/4)*sqrt
(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqr
t(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(3*I*b/d)) - 9*(-(I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c -
 a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))
*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 9*((I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^
(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin
(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((I - 1)*9^(1/4)*sqrt(
2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt
(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(-3*I*b/d)))*d/b^3
```

### 3.61.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 848, normalized size of antiderivative = 2.79

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/144*(-9*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(
-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*
(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*s
qrt(pi)*(2*b*c - I*d)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b) + I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(s
qrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 6*(3*I*sqrt(2)*sqrt(pi)*d*erf(1/2
*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d
*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2
)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) -
I*sqrt(6)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d
^2) + 1)))*c + 18*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/
b + 6*I*sqrt(d*x + c)*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 18...
```

### 3.61.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx=\int \cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2\,\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

# 3.62 $\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx$

3.62.1 Optimal result . . . 571
3.62.2 Mathematica [C] (verified) . . . 572
3.62.3 Rubi [A] (verified) . . . 573
3.62.4 Maple [A] (verified) . . . 575
3.62.5 Fricas [A] (verification not implemented) . . . 575
3.62.6 Sympy [F] . . . 576
3.62.7 Maxima [C] (verification not implemented) . . . 576
3.62.8 Giac [C] (verification not implemented) . . . 577
3.62.9 Mupad [F(-1)] . . . 578

## 3.62.1 Optimal result

Integrand size = 24, antiderivative size = 353

$$\begin{aligned}\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx &= \frac{3d\sqrt{c+dx}\cos(a+bx)}{8b^2} \\ &- \frac{d\sqrt{c+dx}\cos(3a+3bx)}{24b^2} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} \\ &+ \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}} \\ &- \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{24b^{5/2}} \\ &+ \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{8b^{5/2}} \\ &+ \frac{(c+dx)^{3/2}\sin(a+bx)}{4b} - \frac{(c+dx)^{3/2}\sin(3a+3bx)}{12b}\end{aligned}$$

output
```
1/4*(d*x+c)^(3/2)*sin(b*x+a)/b-1/12*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b+1/144*d
^(3/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1/144*d^(3/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^
(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/1
6*d^(3/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d^(3/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/8*d*cos
(b*x+a)*(d*x+c)^(1/2)/b^2-1/24*d*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b^2
```

### 3.62.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 730, normalized size of antiderivative = 2.07

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{\sqrt{d}e^{-\frac{3i(ad+b(c+dx))}{d}}\left(12\sqrt{b}\sqrt{d}e^{\frac{3ibc}{d}}\sqrt{c+dx}\left(1+2ibx+e^{6i(a+bx)}(1-2ibx)\right)+(1+i)(2bc+id)e^{\frac{3ib(2c+dx)}{d}}\sqrt{6\pi}e\right.}{576b^{5/2}}$$

$$+\frac{cde^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)\right)}{8b^2\sqrt{c+dx}}$$

$$-\frac{ce^{-\frac{3i(bc+ad)}{d}}(c+dx)^{3/2}\left(-\frac{e^{6ia}\Gamma\left(\frac{3}{2},-\frac{3ib(c+dx)}{d}\right)}{\left(-\frac{ib(c+dx)}{d}\right)^{3/2}}-\frac{e^{\frac{6ibc}{d}}\Gamma\left(\frac{3}{2},\frac{3ib(c+dx)}{d}\right)}{\left(\frac{ib(c+dx)}{d}\right)^{3/2}}\right)}{24\sqrt{3}d}$$

$$+\frac{\sqrt{d}\left(e^{i\left(a-\frac{bc}{d}\right)}\left(2\sqrt{b}\sqrt{d}e^{\frac{ib(c+dx)}{d}}(3-2ibx)\sqrt{c+dx}+\sqrt[4]{-1}(-2bc+3id)\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)\right)+\left(2\sqrt{b}\sqrt{c}\right.\right.}{}$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
-1/576*(Sqrt[d]*(12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(1 + (
2*I)*b*x + E^((6*I)*(a + b*x))*(1 - (2*I)*b*x)) + (1 + I)*(2*b*c + I*d)*E^
(((3*I)*b*(2*c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c
 + d*x])/Sqrt[d]] - (1 + I)*(2*b*c - I*d)*E^((3*I)*(2*a + b*x))*Sqrt[6*Pi]
*Erfi[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(b^(5/2)*E^(((3
*I)*(a*d + b*(c + d*x)))/d)) + (c*d*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d
]*Gamma[3/2, ((-I)*b*(c + d*x))/d] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x)
)/d]*Gamma[3/2, (I*b*(c + d*x))/d]))/(8*b^2*E^((I*(b*c + a*d))/d)*Sqrt[c +
 d*x]) - (c*(c + d*x)^(3/2)*(-((E^((6*I)*a)*Gamma[3/2, ((-3*I)*b*(c + d*x)
)/d])/(((-I)*b*(c + d*x))/d)^(3/2)) - (E^(((6*I)*b*c)/d)*Gamma[3/2, ((3*I)
*b*(c + d*x))/d])/((I*b*(c + d*x))/d)^(3/2)))/(24*Sqrt[3]*d*E^(((3*I)*(b*c
 + a*d))/d)) + (Sqrt[d]*(E^(I*(a - (b*c)/d))*(2*Sqrt[b]*Sqrt[d]*E^((I*b*(c
 + d*x))/d)*(3 - (2*I)*b*x)*Sqrt[c + d*x] + (-1)^(1/4)*(-2*b*c + (3*I)*d)*
Sqrt[Pi]*Erfi[((-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]) + (2*Sqrt[b]*Sq
rt[d]*(3 + (2*I)*b*x)*Sqrt[c + d*x] + (1 + I)*(2*b*c + (3*I)*d)*Sqrt[Pi/2]
*Erf[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[2]*Sqrt[d])]*(Cos[b*(c/d + x)]
+ I*Sin[b*(c/d + x)]))*(Cos[a + b*x] - I*Sin[a + b*x])))/(32*b^(5/2))
```

### 3.62.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin^2(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{4}(c+dx)^{3/2}\cos(a+bx)-\frac{1}{4}(c+dx)^{3/2}\cos(3a+3bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{3d\sqrt{c+dx}\cos(a+bx)}{8b^2}-\frac{d\sqrt{c+dx}\cos(3a+3bx)}{24b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)}{4b}-\frac{(c+dx)^{3/2}\sin(3a+3bx)}{12b}$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
(3*d*Sqrt[c + d*x]*Cos[a + b*x])/(8*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*
x])/(24*b^2) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sq
rt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*
a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b
^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/
Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(24*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*Fresne
lS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2
)) + ((c + d*x)^(3/2)*Sin[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b
*x])/(12*b)
```

### 3.62.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.62.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.09

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\right.\right.}{4b\sqrt{\frac{b}{d}}}\right.}{4b}}{}$ |
| default | $\frac{\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\right.\right.}{4b\sqrt{\frac{b}{d}}}\right.}{4b}}{}$ |

input
```
int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/8/b*d*(d*x+c)^(3/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-3/8/b*d*(-1/2/b*d*
(d*x+c)^(1/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^
(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)))-1/24/b*d*(d*x+c)^(3/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/8/b*d*(
-1/6/b*d*(d*x+c)^(1/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*P
i^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*
3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)
/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

### 3.62.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.84

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \frac{\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-27\,\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqr
t(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(
-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqr
t(2)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)
))*sin(-(b*c - a*d)/d) - sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)
*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*d*cos(b*x + a
)^3 - 3*b*d*cos(b*x + a) - 2*(b^2*d*x + b^2*c - (b^2*d*x + b^2*c)*cos(b*x
+ a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

### 3.62.6 Sympy [F]

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin^2(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x), x)
```

### 3.62.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.41

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{\left(\frac{48\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)}{d} - \frac{144\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d} + 24\,\sqrt{dx+c}b^2\cos\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right) - 216\,\sqrt{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/576*(48*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d)/d - 144*
(d*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d + 24*sqrt(d*x + c)*
b^2*cos(3*((d*x + c)*b - b*c + a*d)/d) - 216*sqrt(d*x + c)*b^2*cos(((d*x +
 c)*b - b*c + a*d)/d) + ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1
/4)*cos(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)
^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 27*((I -
1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt
(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(I*b/d)) - 27*(-(I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c -
 a*d)/d) - (I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d
))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*
d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)
*b*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/
d)))*d/b^4
```

### 3.62.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 1547, normalized size of antiderivative = 4.38

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/288*(12*(3*I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d
)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(1/2*I*s
qrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c
 + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*(9*(I*sqrt(
2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqr
t(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d
)/d)/b^2)/d^2 + (I*sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-
1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(
I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 6*(-2*I*(d*x
 + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-3*(-I*(
d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 9*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^
2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^...
```

### 3.62.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)^2\,(c+d\,x)^{3/2}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

# 3.63 $\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx$


3.63.1 Optimal result . . . 579
3.63.2 Mathematica [C] (verified) . . . 580
3.63.3 Rubi [A] (verified) . . . 581
3.63.4 Maple [A] (verified) . . . 583
3.63.5 Fricas [A] (verification not implemented) . . . 584
3.63.6 Sympy [F(-1)] . . . 584
3.63.7 Maxima [C] (verification not implemented) . . . 585
3.63.8 Giac [C] (verification not implemented) . . . 585
3.63.9 Mupad [F(-1)] . . . 586


## 3.63.1 Optimal result

Integrand size = 24, antiderivative size = 406

$$\begin{aligned}\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx &= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} \\ &- \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} \\ &- \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}} \\ &- \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{144b^{7/2}} \\ &+ \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{16b^{7/2}} - \frac{15d^2\sqrt{c+dx}\sin(a+bx)}{16b^3} \\ &+ \frac{(c+dx)^{5/2}\sin(a+bx)}{4b} + \frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{144b^3} - \frac{(c+dx)^{5/2}\sin(3a+3bx)}{12b}\end{aligned}$$

output
```
5/8*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2-5/72*d*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b^2
+1/4*(d*x+c)^(5/2)*sin(b*x+a)/b-1/12*(d*x+c)^(5/2)*sin(3*b*x+3*a)/b-5/864*
d^(5/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d
^(1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)-5/864*d^(5/2)*FresnelC(b^(1/2)*6^(1/2)/Pi
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(7/2)+15
/32*d^(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d
^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/32*d^(5/2)*FresnelC(b^(1/2)*2^(1/2)/Pi
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)-15/16*
d^2*sin(b*x+a)*(d*x+c)^(1/2)/b^3+5/144*d^2*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b^
3
```

### 3.63.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.01 (sec) , antiderivative size = 1418, normalized size of antiderivative = 3.49

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
-1/288*(c*Sqrt[d]*(12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(1 +
 (2*I)*b*x + E^((6*I)*(a + b*x))*(1 - (2*I)*b*x)) + (1 + I)*(2*b*c + I*d)*
E^(((3*I)*b*(2*c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt
[c + d*x])/Sqrt[d]] - (1 + I)*(2*b*c - I*d)*E^((3*I)*(2*a + b*x))*Sqrt[6*P
i]*Erfi[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(b^(5/2)*E^((
(3*I)*(a*d + b*(c + d*x)))/d)) + (c^2*d*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x
))/d]*Gamma[3/2, ((-I)*b*(c + d*x))/d] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c +
d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d]))/(8*b^2*E^((I*(b*c + a*d))/d)*Sqrt
[c + d*x]) - (c^2*(c + d*x)^(3/2)*(-((E^((6*I)*a)*Gamma[3/2, ((-3*I)*b*(c
+ d*x))/d])/(((-I)*b*(c + d*x))/d)^(3/2)) - (E^(((6*I)*b*c)/d)*Gamma[3/2,
((3*I)*b*(c + d*x))/d])/((I*b*(c + d*x))/d)^(3/2)))/(24*Sqrt[3]*d*E^(((3*I
)*(b*c + a*d))/d)) + (c*Sqrt[d]*(E^(I*(a - (b*c)/d))*(2*Sqrt[b]*Sqrt[d]*E^
((I*b*(c + d*x))/d)*(3 - (2*I)*b*x)*Sqrt[c + d*x] + (-1)^(1/4)*(-2*b*c + (
3*I)*d)*Sqrt[Pi]*Erfi[((-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]) + (2*Sq
rt[b]*Sqrt[d]*(3 + (2*I)*b*x)*Sqrt[c + d*x] + (1 + I)*(2*b*c + (3*I)*d)*Sq
rt[Pi/2]*Erf[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[2]*Sqrt[d])]*(Cos[b*(c/
d + x)] + I*Sin[b*(c/d + x)]))*(Cos[a + b*x] - I*Sin[a + b*x])))/(16*b^(5/
2)) + (Sqrt[d]*((Cos[a - (b*c)/d] + I*Sin[a - (b*c)/d])*((1 + I)*(4*b^2*c^
2 - (12*I)*b*c*d - 15*d^2)*Sqrt[Pi/2]*Erfi[((1 + I)*Sqrt[b]*Sqrt[c + d*x])
/(Sqrt[2]*Sqrt[d])] + 2*Sqrt[b]*Sqrt[d]*Sqrt[c + d*x]*((15*I)*d - (4*I)...
```

## 3.63.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin^2(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{4}(c+dx)^{5/2}\cos(a+bx)-\frac{1}{4}(c+dx)^{5/2}\cos(3a+3bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}-\frac{15d^2\sqrt{c+dx}\sin(a+bx)}{16b^3}+\frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{144b^3}+\frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)}{4b}-\frac{(c+dx)^{5/2}\sin(3a+3bx)}{12b}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

output
```
(5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(8*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a
+ 3*b*x])/(72*b^2) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqr
t[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi
/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[
d]])/(144*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/2)*Sq
rt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c
)/d])/(16*b^(7/2)) - (15*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(16*b^3) + ((c +
d*x)^(5/2)*Sin[a + b*x])/(4*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(1
44*b^3) - ((c + d*x)^(5/2)*Sin[3*a + 3*b*x])/(12*b)
```

### 3.63.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.63.4 Maple [A] (verified)

Time = 1.23 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.17

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\mathrm{co}\right.}{}\right.}{}\right)}{4b}}{}$ |
| default | $\frac{\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\mathrm{co}\right.}{}\right.}{}\right)}{4b}}{}$ |

input
```
int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/8/b*d*(d*x+c)^(5/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-5/8/b*d*(-1/2/b*d*
(d*x+c)^(3/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/2)*
sin(b/d*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*
d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((a*
d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1/24
/b*d*(d*x+c)^(5/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+5/24/b*d*(-1/6/b*d*(d*
x+c)^(3/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2)
*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^
(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*
(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d
)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

### 3.63.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.91

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{5\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-405\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(
sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 40
5*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(
pi*d)))*sin(-(b*c - a*d)/d) + 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(
sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(10*(b^2*
d^2*x + b^2*c*d)*cos(b*x + a)^3 - 30*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) -
(12*b^3*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 35*b*d^2 - (12*b^3*d^2*x^2 +
 24*b^3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d
*x + c))/b^4
```

### 3.63.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Timed out
```

### 3.63.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.35

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{\left(240\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)-2160\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)-5\left(-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}bc\right.\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/3456*(240*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d) - 2160
*(d*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d) - 5*(-(I + 1)*9^(1/4
)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9
^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(s
qrt(d*x + c)*sqrt(3*I*b/d)) - 405*((I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2
)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/
4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 405*(-(I - 1)*sqr
t(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(2)*
sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt
(-I*b/d)) - 5*((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(
-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)
*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + 24*(12*(d*x +
c)^(5/2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*sin(3*((d*x + c)*b - b*c + a*d)/d)
 - 216*(4*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*sin(((d*x + c)*b
 - b*c + a*d)/d))*d/b^5
```

### 3.63.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 2478, normalized size of antiderivative = 6.10

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/1728*(72*(3*I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)) + I*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*s
qrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b
*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/
d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(1/2*I*
sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*
c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2*(9*(I
*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(1/2*I*sqrt(2)*sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(s
qrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4
*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c -
 I*a*d)/d)/b^2)/d^2 + (I*sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d
*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 6*(-2*
I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-3
*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 9*(-I*sqrt(2)*sqrt(pi)*(4*
b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/s...
```

## 3.63.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx=\int \cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2\,\left(c+d\,x\right)^{5/2}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

# 3.64 $\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx$

3.64.1 Optimal result . . . 587
3.64.2 Mathematica [C] (verified) . . . 588
3.64.3 Rubi [A] (verified) . . . 589
3.64.4 Maple [A] (verified) . . . 591
3.64.5 Fricas [A] (verification not implemented) . . . 592
3.64.6 Sympy [F(-1)] . . . 592
3.64.7 Maxima [C] (verification not implemented) . . . 593
3.64.8 Giac [C] (verification not implemented) . . . 593
3.64.9 Mupad [F(-1)] . . . 594

## 3.64.1 Optimal result

Integrand size = 24, antiderivative size = 407

$$\begin{aligned}\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx &= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3} \\ &- \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{15d^2\sqrt{c+dx}\cos(4a+4bx)}{2048b^3} \\ &+ \frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} \\ &- \frac{15d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}} \\ &- \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{4096b^{7/2}} \\ &+ \frac{15d^{5/2}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{256b^{7/2}} \\ &+ \frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{32b^2} - \frac{5d(c+dx)^{3/2}\sin(4a+4bx)}{256b^2}\end{aligned}$$

output
```
-1/8*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b+1/32*(d*x+c)^(5/2)*cos(4*b*x+4*a)/b+5/
32*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2-5/256*d*(d*x+c)^(3/2)*sin(4*b*x+4*a)
/b^2+15/8192*d^(5/2)*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*
(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)-15/8192*d^(5/2)*FresnelS(2
*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*
Pi^(1/2)/b^(7/2)-15/256*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c
)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+15/256*d^(5/2)*FresnelS(2*b^(1/
2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+15/12
8*d^2*cos(2*b*x+2*a)*(d*x+c)^(1/2)/b^3-15/2048*d^2*cos(4*b*x+4*a)*(d*x+c)^
(1/2)/b^3
```

### 3.64.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.41 (sec) , antiderivative size = 1332, normalized size of antiderivative = 3.27

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \frac{\left(\frac{1}{128}+\frac{i}{128}\right)c\sqrt{d}e^{-\frac{2i(ad+b(c+dx))}{d}}\left((2+2i)\sqrt{b}\sqrt{d}e^{\frac{2ibc}{d}}\sqrt{c+dx}\left(3+4ibx+e^{4i(a+bx)}(-3+4ibx)\right)+i(4\cdots\right.}{b^{5/2}} - \frac{c\sqrt{d}e^{-\frac{4i(ad+b(c+dx))}{d}}\left(-4\sqrt{b}\sqrt{d}e^{\frac{4ibc}{d}}\sqrt{c+dx}\left(-3i+8bx+e^{8i(a+bx)}(3i+8bx)\right)+(-1)^{3/4}(8bc+3id)e^{\frac{4ib(2c+dx)}{d}}\cdots\right.}{1024b^{5/2}} + \frac{1}{4}c^2\left(-\frac{e^{2i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\,\Gamma\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{-2i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\,\Gamma\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{\frac{ib(c+dx)}{d}}}\right) - \frac{c^2e^{-\frac{4i(bc+ad)}{d}}\sqrt{c+dx}\left(-\cdots\right.}{\cdots}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
((1/128 + I/128)*c*Sqrt[d]*((2 + 2*I)*Sqrt[b]*Sqrt[d]*E^(((2*I)*b*c)/d)*Sq
rt[c + d*x]*(3 + (4*I)*b*x + E^((4*I)*(a + b*x))*(-3 + (4*I)*b*x)) + I*(4*
b*c + (3*I)*d)*E^(((2*I)*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[((1 + I)*Sqrt[b]*S
qrt[c + d*x])/Sqrt[d]] + ((4*I)*b*c + 3*d)*E^((2*I)*(2*a + b*x))*Sqrt[Pi]*
Erfi[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(b^(5/2)*E^(((2*I)*(a*d +
b*(c + d*x)))/d)) - (c*Sqrt[d]*(-4*Sqrt[b]*Sqrt[d]*E^(((4*I)*b*c)/d)*Sqrt[
c + d*x]*(-3*I + 8*b*x + E^((8*I)*(a + b*x))*(3*I + 8*b*x)) + (-1)^(3/4)*(
8*b*c + (3*I)*d)*E^(((4*I)*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[(2*(-1)^(1/4)*Sq
rt[b]*Sqrt[c + d*x])/Sqrt[d]] + (-1)^(1/4)*((8*I)*b*c + 3*d)*E^((4*I)*(2*a
 + b*x))*Sqrt[Pi]*Erfi[(2*(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(10
24*b^(5/2)*E^(((4*I)*(a*d + b*(c + d*x)))/d)) + (c^2*(-1/4*(E^((2*I)*(a -
(b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d])/(Sqrt[2]*b*Sqr
t[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]*Gamma[3/2, ((2*I)*b*(c + d*x))/d
])/(4*Sqrt[2]*b*E^((2*I)*(a - (b*c)/d))*Sqrt[(I*b*(c + d*x))/d])))/4 - (c^
2*Sqrt[c + d*x]*(-((E^((8*I)*a)*Gamma[3/2, ((-4*I)*b*(c + d*x))/d])/Sqrt[(
(-I)*b*(c + d*x))/d]) - (E^(((8*I)*b*c)/d)*Gamma[3/2, ((4*I)*b*(c + d*x))/
d])/Sqrt[(I*b*(c + d*x))/d]))/(128*b*E^(((4*I)*(b*c + a*d))/d)) + (Sqrt[d]
*((1 - I)*E^((2*I)*(a - (b*c)/d))*((2 + 2*I)*Sqrt[b]*Sqrt[d]*E^(((2*I)*b*(
c + d*x))/d)*Sqrt[c + d*x]*(15*d - 16*b^2*d*x^2 + (4*I)*b*(c - 5*d*x)) + (
16*b^2*c^2 - (24*I)*b*c*d - 15*d^2)*Sqrt[Pi]*Erfi[((1 + I)*Sqrt[b]*Sqrt...
```

### 3.64.3 Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin^3(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{4}(c+dx)^{5/2}\sin(2a+2bx)-\frac{1}{8}(c+dx)^{5/2}\sin(4a+4bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15\sqrt{\pi}d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{15d^2\sqrt{c+dx}\cos(4a+4bx)}{2048b^3}+\frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{32b^2}-\frac{5d(c+dx)^{3/2}\sin(4a+4bx)}{256b^2}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
(15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2
*a + 2*b*x])/(8*b) - (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) +
((c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a
 - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(409
6*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]
*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2
]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)
/d])/(4096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*
x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*
x)^(3/2)*Sin[2*a + 2*b*x])/(32*b^2) - (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x
])/(256*b^2)
```

### 3.64.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.64.4 Maple [A] (verified)

Time = 3.85 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.15

| method | result |
|---|---|
| derivativedivides | $-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\cdots\right)}{\cdots}\right)}{8\cdots}$ |
| default | $-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\cdots\right)}{\cdots}\right)}{8\cdots}$ |

input
```
int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+5/16/b*d*(1/
4/b*d*(d*x+c)^(3/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*
x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(
cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(
2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))+1/64/
b*d*(d*x+c)^(5/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-5/64/b*d*(1/8/b*d*(d*x+
c)^(3/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*
cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(co
s(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d
)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/
2)/d)))))
```

### 3.64.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \frac{15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d)
)*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d
) - 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x +
 c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 4*(320*b^3*d^2*x^2 + 640*b^3*c
*d*x + 320*b^3*c^2 + 8*(64*b^3*d^2*x^2 + 128*b^3*c*d*x + 64*b^3*c^2 - 15*b
*d^2)*cos(b*x + a)^4 - 255*b*d^2 - 8*(128*b^3*d^2*x^2 + 256*b^3*c*d*x + 12
8*b^3*c^2 - 75*b*d^2)*cos(b*x + a)^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*cos(b*
x + a)^3 - 5*(b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x +
c))/b^4
```

### 3.64.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

output
```
Timed out
```

### 3.64.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.35

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{\left(640\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)-5120\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-16\left(\frac{64\,(dx+c)^{\frac{5}{2}}b^4}{d}-15\sqrt{\ }\right.\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/32768*(640*(d*x + c)^(3/2)*b^3*sin(4*((d*x + c)*b - b*c + a*d)/d) - 512
0*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 16*(64*(d*x + c
)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(4*((d*x + c)*b - b*c + a*d)/d)
 + 256*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c
)*b - b*c + a*d)/d) - 240*((I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2
)^(1/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^
2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 15*
(-(I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (
I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2
*sqrt(d*x + c)*sqrt(I*b/d)) - 15*((I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)
^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1
/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - 240*(-(I +
1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) -
(I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/
d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*d/b^5
```

### 3.64.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.16 (sec) , antiderivative size = 2435, normalized size of antiderivative = 5.98

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/16384*(512*(sqrt(2)*sqrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)) - 4*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)) - 4*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)))*c^3 + 24*c*d^2*((sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*
d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(-8*I
*(d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(
-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (sqrt(2)*sqrt(pi)*(64*b^
2*c^2 + 16*I*b*c*d - 3*d^2)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
b^2*d^2) + 1)*b^2) - 4*I*(-8*I*(d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*
c*d - 3*sqrt(d*x + c)*d^2)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d
^2 - 16*(sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(b*d)*sqrt
(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)
*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sq...
```

### 3.64.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx=\int \cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^3\,\left(c+d\,x\right)^{5/2}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

# 3.65 $\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx$


3.65.1 Optimal result . . . 595
3.65.2 Mathematica [C] (verified) . . . 596
3.65.3 Rubi [A] (verified) . . . 596
3.65.4 Maple [A] (verified) . . . 598
3.65.5 Fricas [A] (verification not implemented) . . . 598
3.65.6 Sympy [F] . . . 599
3.65.7 Maxima [C] (verification not implemented) . . . 599
3.65.8 Giac [C] (verification not implemented) . . . 600
3.65.9 Mupad [F(-1)] . . . 601


## 3.65.1 Optimal result

Integrand size = 24, antiderivative size = 351

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3d^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{512b^{5/2}} - \frac{3d^{3/2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{64b^{5/2}} + \frac{3d\sqrt{c+dx}\sin(2a+2bx)}{32b^2} - \frac{3d\sqrt{c+dx}\sin(4a+4bx)}{256b^2}$$

output
```
-1/8*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b+1/32*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b+3/
1024*d^(3/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)+3/1024*d^(3/2)*FresnelC(2*b^(1/2)*
2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/
b^(5/2)-3/64*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(
1/2)/Pi^(1/2))*Pi^(1/2)/b^(5/2)-3/64*d^(3/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1
/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)+3/32*d*sin(2*b*x+2
*a)*(d*x+c)^(1/2)/b^2-3/256*d*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b^2
```

### 3.65.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.94 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.97

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \frac{e^{-4ia}\left(8e^{2i\left(a-\frac{b(c+dx)}{d}\right)}\left(-4\sqrt{b}de^{\frac{2ibc}{d}}\sqrt{c+dx}\left(-3i+4bx+e^{4i(a+bx)}(3i+4bx)\right)-(1-i)(4bc+3id)\right.\right.}{}$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
(8*E^((2*I)*(a - (b*(c + d*x))/d))*(-4*Sqrt[b]*d*E^(((2*I)*b*c)/d)*Sqrt[c
+ d*x]*(-3*I + 4*b*x + E^((4*I)*(a + b*x))*(3*I + 4*b*x)) - (1 - I)*(4*b*c
 + (3*I)*d)*Sqrt[d]*E^(((2*I)*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[((1 + I)*Sqrt
[b]*Sqrt[c + d*x])/Sqrt[d]] + (1 + I)*Sqrt[d]*((4*I)*b*c + 3*d)*E^((2*I)*(
2*a + b*x))*Sqrt[Pi]*Erfi[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]) - (Sqr
t[d]*(-4*Sqrt[b]*Sqrt[d]*E^(((4*I)*b*c)/d)*Sqrt[c + d*x]*(-3*I + 8*b*x + E
^((8*I)*(a + b*x))*(3*I + 8*b*x)) + (-1)^(3/4)*(8*b*c + (3*I)*d)*E^(((4*I)
*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[(2*(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[
d]] + (-1)^(1/4)*((8*I)*b*c + 3*d)*E^((4*I)*(2*a + b*x))*Sqrt[Pi]*Erfi[(2*
(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/E^(((4*I)*b*(c + d*x))/d) - (
64*Sqrt[2]*b^(3/2)*c*E^((2*I)*(a - (b*c)/d))*Sqrt[c + d*x]*(E^((4*I)*a)*Sq
rt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d] + E^(((4*I)*b*c)/
d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((2*I)*b*(c + d*x))/d]))/Sqrt[(b^
2*(c + d*x)^2)/d^2] - (16*b^(3/2)*c*Sqrt[c + d*x]*(-((E^((8*I)*a)*Gamma[3/
2, ((-4*I)*b*(c + d*x))/d])/Sqrt[((-I)*b*(c + d*x))/d]) - (E^(((8*I)*b*c)/
d)*Gamma[3/2, ((4*I)*b*(c + d*x))/d])/Sqrt[(I*b*(c + d*x))/d]))/E^(((4*I)*
b*c)/d))/(2048*b^(5/2)*E^((4*I)*a))
```

### 3.65.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin^3(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \; 4906$$

$$\int \left(\frac{1}{4}(c+dx)^{3/2}\sin(2a+2bx)-\frac{1}{8}(c+dx)^{3/2}\sin(4a+4bx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(2a+2bx)}{32b^2}-\frac{3d\sqrt{c+dx}\sin(4a+4bx)}{256b^2}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}+\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
-1/8*((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/b + ((c + d*x)^(3/2)*Cos[4*a + 4*b
*x])/(32*b) + (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[
b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]
*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi]
)])/(64*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqr
t[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*
b*c)/d])/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) - (3
*d*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

### 3.65.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.65.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.07

| method | result |
|---|---|
| derivativedivides | $-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\text{s}\right.}{8b\sqrt{\frac{b}{d}}}\right.}{8b}$ |
| default | $-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\text{s}\right.}{8b\sqrt{\frac{b}{d}}}\right.}{8b}$ |

input
```
int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+3/16/b*d*(1/
4/b*d*(d*x+c)^(1/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d
)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2
)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)
))+1/64/b*d*(d*x+c)^(3/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-3/64/b*d*(1/8/b
*d*(d*x+c)^(1/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2
)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*
b*(d*x+c)^(1/2)/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d)))
```

## 3.65.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.90

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*
fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d)
- 48*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x +
c)*sqrt(b/(pi*d))) - 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*
sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x +
 a)^4 + 10*b^2*d*x + 10*b^2*c - 32*(b^2*d*x + b^2*c)*cos(b*x + a)^2 - 3*(2
*b*d*cos(b*x + a)^3 - 5*b*d*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^3
```

### 3.65.6 Sympy [F]

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx=\int (c+dx)^{\frac{3}{2}}\sin^3{(a+bx)}\cos{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)**3*cos(a + b*x), x)
```

### 3.65.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.43

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx=\frac{\left(\frac{128\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{512\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)}{d}-48\sqrt{dx+c}b^2\sin\left(\frac{4\,((dx+c)b-bc+c}{d}\right.\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4096*(128*(d*x + c)^(3/2)*b^3*cos(4*((d*x + c)*b - b*c + a*d)/d)/d - 512
*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 48*sqrt(d*x +
c)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) + 384*sqrt(d*x + c)*b^2*sin(2*((
d*x + c)*b - b*c + a*d)/d) + 24*(-(I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^
2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*
(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) +
3*((I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I
 - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sq
rt(d*x + c)*sqrt(I*b/d)) + 3*(-(I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4
)*cos(-4*(b*c - a*d)/d) + (I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin
(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + 24*((I - 1)*4^(1/4
)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(
1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(
d*x + c)*sqrt(-2*I*b/d)))*d/b^4
```

### 3.65.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 1515, normalized size of antiderivative = 4.32

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/2048*(64*(sqrt(2)*sqrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)) - 4*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)) - 4*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)))*c^2 + d^2*((sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-
I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*
c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(-8*I*(d*x +
 c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-4*(-I*
(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (sqrt(2)*sqrt(pi)*(64*b^2*c^2 +
 16*I*b*c*d - 3*d^2)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(
b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2
) + 1)*b^2) - 4*I*(-8*I*(d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d - 3
*sqrt(d*x + c)*d^2)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 - 16
*(sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d
/sqrt(b^2*d^2) + 1)*b^2) + 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x ...
```

### 3.65.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx=\int \cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^3\,\left(c+d\,x\right)^{3/2}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

# 3.66 $\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx$

3.66.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 602
3.66.2 Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 603
3.66.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 603
3.66.4 Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 605
3.66.5 Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . 605
3.66.6 Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 606
3.66.7 Maxima [C] (verification not implemented) . . . . . . . . . . . . . . . . 606
3.66.8 Giac [C] (verification not implemented) . . . . . . . . . . . . . . . . . 607
3.66.9 Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 608

## 3.66.1 Optimal result

Integrand size = 24, antiderivative size = 299

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx = -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}+\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b} - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} + \frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}} + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{64b^{3/2}} - \frac{\sqrt{d}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{16b^{3/2}}$$

output
```
-1/128*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/
d^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/128*FresnelS(2*b^(1/2)*2^(1/2)
/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)
/b^(3/2)+1/16*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi
^(1/2))*d^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(
1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/8*cos(2*b*x+2*a
)*(d*x+c)^(1/2)/b+1/32*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b
```

### 3.66.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.84

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{e^{-\frac{4i(bc+ad)}{d}}\sqrt{c+dx}\left(-4\sqrt{2}e^{2i\left(3a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)-4\sqrt{2}e^{2i\left(a+\frac{3bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right)\right.}{128b\sqrt{\frac{b^2(c+dx)^2}{d^2}}}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
(Sqrt[c + d*x]*(-4*Sqrt[2]*E^((2*I)*(3*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/
d]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d] - 4*Sqrt[2]*E^((2*I)*(a + (3*b*c)/d)
)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((2*I)*b*(c + d*x))/d] + E^((8*I)*
a)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-4*I)*b*(c + d*x))/d] + E^(((8*I)*
b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((4*I)*b*(c + d*x))/d]))/(12
8*b*E^(((4*I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

### 3.66.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^3(a+bx)\cos(a+bx)\,dx$$
$$\Big\downarrow \text{4906}$$
$$\int \left(\frac{1}{4}\sqrt{c+dx}\sin(2a+2bx)-\frac{1}{8}\sqrt{c+dx}\sin(4a+4bx)\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}+\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
-1/8*(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/b + (Sqrt[c + d*x]*Cos[4*a + 4*b*x])
/(32*b) - (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqr
t[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a
- (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b
^(3/2)) + (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x]
)/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS
[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b
^(3/2))
```

### 3.66.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.66.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.96

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}+\frac{d\sqrt{\cdots}}{\cdots}}{d}$ |
| default | $\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}+\frac{d\sqrt{\cdots}}{\cdots}}{d}$ |

input
```
int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/32/b*d*Pi^
(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c
)^(1/2)/d))+1/64/b*d*(d*x+c)^(1/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-1/256/
b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi
^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2
)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

### 3.66.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.82

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)\cdots}{\cdots}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sq
rt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_
sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 8*pi*d
*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(
pi*d))) + 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d))
)*sin(-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 16*b*cos(b*x + a)^2 + 5*
b)*sqrt(d*x + c))/b^2
```

### 3.66.6 Sympy [F]

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx = \int \sqrt{c+dx}\sin^3(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x), x)
```

### 3.66.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.43

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx = \frac{\left(\frac{16\sqrt{dx+c}b^2\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right)}{d} - \frac{64\sqrt{dx+c}b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d} - 4\left(\left(i-1\right)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right.\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/512*(16*sqrt(d*x + c)*b^2*cos(4*((d*x + c)*b - b*c + a*d)/d)/d - 64*sqrt
(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 4*((I - 1)*4^(1/4)*sq
rt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*s
qrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)
*sqrt(2*I*b/d)) - (-(I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-4*(b*c
 - a*d)/d) - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)
/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - ((I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d
^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/
4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - 4*(-(I + 1)*
4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)
*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(-2*I*b/d)))*d/b^3
```

### 3.66.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 826, normalized size of antiderivative = 2.76

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/256*(sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(I*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*
d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*(sqrt(2)*sqrt(pi)*d*erf
(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*
b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d
*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4
*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 4*sqrt(pi)*d
*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c
- I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 4*sqrt(pi)*d*erf(I*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 8*sqrt(pi)*(4*b*c - I*d)*d*
erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c -
 I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*sqrt(pi)*(4*b*c + I
*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-
I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 16*sqrt(d*x +
 c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-4
*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 16*sqrt(d*x + c)*d*e^(-2*(-I*(d*x
+ c)*b + I*b*c - I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-4*(-I*(d*x + c)*b ...
```

### 3.66.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx=\int \cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^3\,\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

# 3.67 $\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx$

3.67.1 Optimal result . . . 609
3.67.2 Mathematica [C] (verified) . . . 610
3.67.3 Rubi [A] (verified) . . . 610
3.67.4 Maple [A] (verified) . . . 612
3.67.5 Fricas [A] (verification not implemented) . . . 612
3.67.6 Sympy [F] . . . 613
3.67.7 Maxima [C] (verification not implemented) . . . 613
3.67.8 Giac [C] (verification not implemented) . . . 614
3.67.9 Mupad [F(-1)] . . . 615

## 3.67.1 Optimal result

Integrand size = 24, antiderivative size = 299

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx = -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}+\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}-\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{64b^{3/2}}-\frac{\sqrt{d}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{16b^{3/2}}$$

output
```
-1/128*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/
d^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/128*FresnelS(2*b^(1/2)*2^(1/2)
/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)
/b^(3/2)+1/16*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi
^(1/2))*d^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(
1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/8*cos(2*b*x+2*a
)*(d*x+c)^(1/2)/b+1/32*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b
```

### 3.67.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.84

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{e^{-\frac{4i(bc+ad)}{d}}\sqrt{c+dx}\left(-4\sqrt{2}e^{2i\left(3a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)-4\sqrt{2}e^{2i\left(a+\frac{3bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right)\right.}{128b\sqrt{\frac{b^2(c+dx)^2}{d^2}}}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
(Sqrt[c + d*x]*(-4*Sqrt[2]*E^((2*I)*(3*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/
d]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d] - 4*Sqrt[2]*E^((2*I)*(a + (3*b*c)/d)
)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((2*I)*b*(c + d*x))/d] + E^((8*I)*
a)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-4*I)*b*(c + d*x))/d] + E^(((8*I)*
b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((4*I)*b*(c + d*x))/d]))/(12
8*b*E^(((4*I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

### 3.67.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^3(a+bx)\cos(a+bx)\,dx$$
$$\Big\downarrow \text{4906}$$
$$\int \left(\frac{1}{4}\sqrt{c+dx}\sin(2a+2bx)-\frac{1}{8}\sqrt{c+dx}\sin(4a+4bx)\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}+\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
-1/8*(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/b + (Sqrt[c + d*x]*Cos[4*a + 4*b*x])
/(32*b) - (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqr
t[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a
- (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b
^(3/2)) + (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x]
)/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS
[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b
^(3/2))
```

### 3.67.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.67.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.96

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}+\frac{d\sqrt{}}{}}{d}$ |
| default | $\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}+\frac{d\sqrt{}}{}}{d}$ |

input
```
int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/32/b*d*Pi^
(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c
)^(1/2)/d))+1/64/b*d*(d*x+c)^(1/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-1/256/
b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi
^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2
)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

## 3.67.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.82

$$\int\sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sq
rt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_
sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 8*pi*d
*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(
pi*d))) + 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d))
)*sin(-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 16*b*cos(b*x + a)^2 + 5*
b)*sqrt(d*x + c))/b^2
```

### 3.67.6 Sympy [F]

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx = \int \sqrt{c+dx}\sin^3{(a+bx)}\cos{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x), x)
```

### 3.67.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.43

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{\left(\frac{16\sqrt{dx+c}b^2\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right)}{d} - \frac{64\sqrt{dx+c}b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d} - 4\left(\left(i-1\right)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right.\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/512*(16*sqrt(d*x + c)*b^2*cos(4*((d*x + c)*b - b*c + a*d)/d)/d - 64*sqrt
(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 4*((I - 1)*4^(1/4)*sq
rt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*s
qrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)
*sqrt(2*I*b/d)) - (-(I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-4*(b*c
 - a*d)/d) - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)
/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - ((I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d
^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/
4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - 4*(-(I + 1)*
4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)
*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(-2*I*b/d)))*d/b^3
```

### 3.67.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 826, normalized size of antiderivative = 2.76

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/256*(sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(I*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*
d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*(sqrt(2)*sqrt(pi)*d*erf
(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*
b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d
*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4
*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 4*sqrt(pi)*d
*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c
- I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 4*sqrt(pi)*d*erf(I*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 8*sqrt(pi)*(4*b*c - I*d)*d*
erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c -
 I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*sqrt(pi)*(4*b*c + I
*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-
I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 16*sqrt(d*x +
 c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-4
*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 16*sqrt(d*x + c)*d*e^(-2*(-I*(d*x
+ c)*b + I*b*c - I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-4*(-I*(d*x + c)*b ...
```

### 3.67.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)^3\,\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

# 3.68 $\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx$


3.68.1 Optimal result . . . 616
3.68.2 Mathematica [C] (verified) . . . 617
3.68.3 Rubi [A] (verified) . . . 617
3.68.4 Maple [A] (verified) . . . 619
3.68.5 Fricas [A] (verification not implemented) . . . 619
3.68.6 Sympy [F] . . . 620
3.68.7 Maxima [C] (verification not implemented) . . . 620
3.68.8 Giac [C] (verification not implemented) . . . 621
3.68.9 Mupad [F(-1)] . . . 622


## 3.68.1 Optimal result

Integrand size = 24, antiderivative size = 351

$$\begin{aligned}\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx &= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} \\ &+\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} \\ &-\frac{3d^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}} \\ &+\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{512b^{5/2}} \\ &-\frac{3d^{3/2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{64b^{5/2}} \\ &+\frac{3d\sqrt{c+dx}\sin(2a+2bx)}{32b^2}-\frac{3d\sqrt{c+dx}\sin(4a+4bx)}{256b^2}\end{aligned}$$

output
```
-1/8*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b+1/32*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b+3/
1024*d^(3/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)+3/1024*d^(3/2)*FresnelC(2*b^(1/2)*
2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/
b^(5/2)-3/64*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(
1/2)/Pi^(1/2))*Pi^(1/2)/b^(5/2)-3/64*d^(3/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1
/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)+3/32*d*sin(2*b*x+2
*a)*(d*x+c)^(1/2)/b^2-3/256*d*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b^2
```

### 3.68.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.54 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.97

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \frac{e^{-4ia}\left(8e^{2i\left(a-\frac{b(c+dx)}{d}\right)}\left(-4\sqrt{b}de^{\frac{2ibc}{d}}\sqrt{c+dx}\left(-3i+4bx+e^{4i(a+bx)}(3i+4bx)\right)-(1-i)(4bc+3id)\right.\right.}{}$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
(8*E^((2*I)*(a - (b*(c + d*x))/d))*(-4*Sqrt[b]*d*E^(((2*I)*b*c)/d)*Sqrt[c
+ d*x]*(-3*I + 4*b*x + E^((4*I)*(a + b*x))*(3*I + 4*b*x)) - (1 - I)*(4*b*c
 + (3*I)*d)*Sqrt[d]*E^(((2*I)*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[((1 + I)*Sqrt
[b]*Sqrt[c + d*x])/Sqrt[d]] + (1 + I)*Sqrt[d]*((4*I)*b*c + 3*d)*E^((2*I)*(
2*a + b*x))*Sqrt[Pi]*Erfi[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]) - (Sqr
t[d]*(-4*Sqrt[b]*Sqrt[d]*E^(((4*I)*b*c)/d)*Sqrt[c + d*x]*(-3*I + 8*b*x + E
^((8*I)*(a + b*x))*(3*I + 8*b*x)) + (-1)^(3/4)*(8*b*c + (3*I)*d)*E^(((4*I)
*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[(2*(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[
d]] + (-1)^(1/4)*((8*I)*b*c + 3*d)*E^((4*I)*(2*a + b*x))*Sqrt[Pi]*Erfi[(2*
(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/E^(((4*I)*b*(c + d*x))/d) - (
64*Sqrt[2]*b^(3/2)*c*E^((2*I)*(a - (b*c)/d))*Sqrt[c + d*x]*(E^((4*I)*a)*Sq
rt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d] + E^(((4*I)*b*c)/
d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((2*I)*b*(c + d*x))/d]))/Sqrt[(b^
2*(c + d*x)^2)/d^2] - (16*b^(3/2)*c*Sqrt[c + d*x]*(-((E^((8*I)*a)*Gamma[3/
2, ((-4*I)*b*(c + d*x))/d])/Sqrt[((-I)*b*(c + d*x))/d]) - (E^(((8*I)*b*c)/
d)*Gamma[3/2, ((4*I)*b*(c + d*x))/d])/Sqrt[(I*b*(c + d*x))/d]))/E^(((4*I)*
b*c)/d))/(2048*b^(5/2)*E^((4*I)*a))
```

### 3.68.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin^3(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{4}(c+dx)^{3/2}\sin(2a+2bx)-\frac{1}{8}(c+dx)^{3/2}\sin(4a+4bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(2a+2bx)}{32b^2}-\frac{3d\sqrt{c+dx}\sin(4a+4bx)}{256b^2}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}+\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
-1/8*((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/b + ((c + d*x)^(3/2)*Cos[4*a + 4*b
*x])/(32*b) + (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[
b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]
*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi]
)])/(64*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqr
t[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*
b*c)/d])/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) - (3
*d*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

### 3.68.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.68.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.07

| method | result |
|---|---|
| derivativedivides | $-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\mathrm{s}\right.}{8b\sqrt{\frac{b}{d}}}\right.}{8b}$ |
| default | $-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\mathrm{s}\right.}{8b\sqrt{\frac{b}{d}}}\right.}{8b}$ |

input
```
int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+3/16/b*d*(1/
4/b*d*(d*x+c)^(1/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d
)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2
)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)
))+1/64/b*d*(d*x+c)^(3/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-3/64/b*d*(1/8/b
*d*(d*x+c)^(1/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2
)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*
b*(d*x+c)^(1/2)/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d)))
```

### 3.68.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 316, normalized size of antiderivative = 0.90

$$\int(c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx=\frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*
fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d)
- 48*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x +
c)*sqrt(b/(pi*d))) - 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*
sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x +
 a)^4 + 10*b^2*d*x + 10*b^2*c - 32*(b^2*d*x + b^2*c)*cos(b*x + a)^2 - 3*(2
*b*d*cos(b*x + a)^3 - 5*b*d*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^3
```

### 3.68.6 Sympy [F]

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin^3(a+bx)\cos(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)**3*cos(a + b*x), x)
```

### 3.68.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.43

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \frac{\left(\frac{128\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)}{d} - \frac{512\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)}{d} - 48\sqrt{dx+c}\,b^2\sin\left(\frac{4\,((dx+c)b-bc+c}{d}\right.\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4096*(128*(d*x + c)^(3/2)*b^3*cos(4*((d*x + c)*b - b*c + a*d)/d)/d - 512
*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 48*sqrt(d*x +
c)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) + 384*sqrt(d*x + c)*b^2*sin(2*((
d*x + c)*b - b*c + a*d)/d) + 24*(-(I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^
2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*
(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) +
3*((I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I
 - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sq
rt(d*x + c)*sqrt(I*b/d)) + 3*(-(I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4
)*cos(-4*(b*c - a*d)/d) + (I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin
(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + 24*((I - 1)*4^(1/4
)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(
1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(
d*x + c)*sqrt(-2*I*b/d)))*d/b^4
```

### 3.68.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 1515, normalized size of antiderivative = 4.32

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/2048*(64*(sqrt(2)*sqrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)) - 4*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)) - 4*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)))*c^2 + d^2*((sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-
I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*
c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(-8*I*(d*x +
 c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-4*(-I*
(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (sqrt(2)*sqrt(pi)*(64*b^2*c^2 +
 16*I*b*c*d - 3*d^2)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(
b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2
) + 1)*b^2) - 4*I*(-8*I*(d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d - 3
*sqrt(d*x + c)*d^2)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 - 16
*(sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d
/sqrt(b^2*d^2) + 1)*b^2) + 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x ...
```

### 3.68.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx=\int \cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^3\,\left(c+d\,x\right)^{3/2}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

# 3.69 $\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx$

3.69.1 Optimal result . . . 623
3.69.2 Mathematica [C] (verified) . . . 624
3.69.3 Rubi [A] (verified) . . . 625
3.69.4 Maple [A] (verified) . . . 627
3.69.5 Fricas [A] (verification not implemented) . . . 628
3.69.6 Sympy [F(-1)] . . . 628
3.69.7 Maxima [C] (verification not implemented) . . . 629
3.69.8 Giac [C] (verification not implemented) . . . 629
3.69.9 Mupad [F(-1)] . . . 630

## 3.69.1 Optimal result

Integrand size = 24, antiderivative size = 407

$$\begin{aligned}\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx &= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3} \\ &- \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{15d^2\sqrt{c+dx}\cos(4a+4bx)}{2048b^3} \\ &+ \frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} \\ &- \frac{15d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}} \\ &- \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{4096b^{7/2}} \\ &+ \frac{15d^{5/2}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{256b^{7/2}} \\ &+ \frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{32b^2} - \frac{5d(c+dx)^{3/2}\sin(4a+4bx)}{256b^2}\end{aligned}$$

output
```
-1/8*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b+1/32*(d*x+c)^(5/2)*cos(4*b*x+4*a)/b+5/
32*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2-5/256*d*(d*x+c)^(3/2)*sin(4*b*x+4*a)
/b^2+15/8192*d^(5/2)*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*
(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)-15/8192*d^(5/2)*FresnelS(2
*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*
Pi^(1/2)/b^(7/2)-15/256*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c
)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+15/256*d^(5/2)*FresnelS(2*b^(1/
2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+15/12
8*d^2*cos(2*b*x+2*a)*(d*x+c)^(1/2)/b^3-15/2048*d^2*cos(4*b*x+4*a)*(d*x+c)^
(1/2)/b^3
```

### 3.69.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.03 (sec) , antiderivative size = 1332, normalized size of antiderivative = 3.27

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \frac{\left(\frac{1}{128}+\frac{i}{128}\right)c\sqrt{d}e^{-\frac{2i(ad+b(c+dx))}{d}}\left((2+2i)\sqrt{b}\sqrt{d}e^{\frac{2ibc}{d}}\sqrt{c+dx}\left(3+4ibx+e^{4i(a+bx)}(-3+4ibx)\right)+i(4\right.}{b^{5/2}}$$
$$-\frac{c\sqrt{d}e^{-\frac{4i(ad+b(c+dx))}{d}}\left(-4\sqrt{b}\sqrt{d}e^{\frac{4ibc}{d}}\sqrt{c+dx}\left(-3i+8bx+e^{8i(a+bx)}(3i+8bx)\right)+(-1)^{3/4}(8bc+3id)e^{\frac{4ib(2c+dx)}{d}}\right.}{1024b^{5/2}}$$
$$+\frac{1}{4}c^2\left(-\frac{e^{2i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\,\Gamma\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{-2i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\,\Gamma\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{\frac{ib(c+dx)}{d}}}\right)-\frac{c^2e^{-\frac{4i(bc+ad)}{d}}\sqrt{c+dx}\left(-\right.}{}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
((1/128 + I/128)*c*Sqrt[d]*((2 + 2*I)*Sqrt[b]*Sqrt[d]*E^(((2*I)*b*c)/d)*Sq
rt[c + d*x]*(3 + (4*I)*b*x + E^((4*I)*(a + b*x))*(-3 + (4*I)*b*x)) + I*(4*
b*c + (3*I)*d)*E^(((2*I)*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[((1 + I)*Sqrt[b]*S
qrt[c + d*x])/Sqrt[d]] + ((4*I)*b*c + 3*d)*E^((2*I)*(2*a + b*x))*Sqrt[Pi]*
Erfi[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(b^(5/2)*E^(((2*I)*(a*d +
b*(c + d*x)))/d)) - (c*Sqrt[d]*(-4*Sqrt[b]*Sqrt[d]*E^(((4*I)*b*c)/d)*Sqrt[
c + d*x]*(-3*I + 8*b*x + E^((8*I)*(a + b*x))*(3*I + 8*b*x)) + (-1)^(3/4)*(
8*b*c + (3*I)*d)*E^(((4*I)*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[(2*(-1)^(1/4)*Sq
rt[b]*Sqrt[c + d*x])/Sqrt[d]] + (-1)^(1/4)*((8*I)*b*c + 3*d)*E^((4*I)*(2*a
 + b*x))*Sqrt[Pi]*Erfi[(2*(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(10
24*b^(5/2)*E^(((4*I)*(a*d + b*(c + d*x)))/d)) + (c^2*(-1/4*(E^((2*I)*(a -
(b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d])/(Sqrt[2]*b*Sqr
t[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]*Gamma[3/2, ((2*I)*b*(c + d*x))/d
])/(4*Sqrt[2]*b*E^((2*I)*(a - (b*c)/d))*Sqrt[(I*b*(c + d*x))/d])))/4 - (c^
2*Sqrt[c + d*x]*(-((E^((8*I)*a)*Gamma[3/2, ((-4*I)*b*(c + d*x))/d])/Sqrt[(
(-I)*b*(c + d*x))/d]) - (E^(((8*I)*b*c)/d)*Gamma[3/2, ((4*I)*b*(c + d*x))/
d])/Sqrt[(I*b*(c + d*x))/d]))/(128*b*E^(((4*I)*(b*c + a*d))/d)) + (Sqrt[d]
*((1 - I)*E^((2*I)*(a - (b*c)/d))*((2 + 2*I)*Sqrt[b]*Sqrt[d]*E^(((2*I)*b*(
c + d*x))/d)*Sqrt[c + d*x]*(15*d - 16*b^2*d*x^2 + (4*I)*b*(c - 5*d*x)) + (
16*b^2*c^2 - (24*I)*b*c*d - 15*d^2)*Sqrt[Pi]*Erfi[((1 + I)*Sqrt[b]*Sqrt...
```

## 3.69.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin^3(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{4}(c+dx)^{5/2}\sin(2a+2bx)-\frac{1}{8}(c+dx)^{5/2}\sin(4a+4bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15\sqrt{\pi}d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}-\frac{15d^2\sqrt{c+dx}\cos(4a+4bx)}{2048b^3}+\frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{32b^2}-\frac{5d(c+dx)^{3/2}\sin(4a+4bx)}{256b^2}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

output
```
(15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2
*a + 2*b*x])/(8*b) - (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) +
((c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a
 - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(409
6*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]
*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2
]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)
/d])/(4096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*
x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*
x)^(3/2)*Sin[2*a + 2*b*x])/(32*b^2) - (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x
])/(256*b^2)
```

### 3.69.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.69.4 Maple [A] (verified)

Time = 3.56 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.15

| method | result |
|---|---|
| derivativedivides | $-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\right.}{}\right.}{8}$ |
| default | $-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\right.}{}\right.}{8}$ |

input
```
int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+5/16/b*d*(1/
4/b*d*(d*x+c)^(3/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*
x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(
cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(
2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))+1/64/
b*d*(d*x+c)^(5/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-5/64/b*d*(1/8/b*d*(d*x+
c)^(3/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*
cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(co
s(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d
)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/
2)/d)))))
```

### 3.69.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = 15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d)
)*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d
) - 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x +
 c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 4*(320*b^3*d^2*x^2 + 640*b^3*c
*d*x + 320*b^3*c^2 + 8*(64*b^3*d^2*x^2 + 128*b^3*c*d*x + 64*b^3*c^2 - 15*b
*d^2)*cos(b*x + a)^4 - 255*b*d^2 - 8*(128*b^3*d^2*x^2 + 256*b^3*c*d*x + 12
8*b^3*c^2 - 75*b*d^2)*cos(b*x + a)^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*cos(b*
x + a)^3 - 5*(b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x +
c))/b^4
```

### 3.69.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

output
```
Timed out
```

### 3.69.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.35

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{\left(640\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)-5120\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-16\left(\frac{64\,(dx+c)^{\frac{5}{2}}b^4}{d}-15\sqrt{\right.\right.}{}}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/32768*(640*(d*x + c)^(3/2)*b^3*sin(4*((d*x + c)*b - b*c + a*d)/d) - 512
0*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 16*(64*(d*x + c
)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(4*((d*x + c)*b - b*c + a*d)/d)
 + 256*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c
)*b - b*c + a*d)/d) - 240*((I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2
)^(1/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^
2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 15*
(-(I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (
I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2
*sqrt(d*x + c)*sqrt(I*b/d)) - 15*((I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)
^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1
/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - 240*(-(I +
1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) -
(I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/
d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*d/b^5
```

### 3.69.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 2435, normalized size of antiderivative = 5.98

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/16384*(512*(sqrt(2)*sqrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)) - 4*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)) - 4*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)))*c^3 + 24*c*d^2*((sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*
d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(-8*I
*(d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(
-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (sqrt(2)*sqrt(pi)*(64*b^
2*c^2 + 16*I*b*c*d - 3*d^2)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
b^2*d^2) + 1)*b^2) - 4*I*(-8*I*(d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*
c*d - 3*sqrt(d*x + c)*d^2)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d
^2 - 16*(sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(b*d)*sqrt
(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)
*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sq...
```

### 3.69.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \int \cos(a+b\,x)\,\sin(a+b\,x)^3\,(c+d\,x)^{5/2}\,dx$$

input
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

# 3.70 $\int (c+dx)^m \cos^2(a+bx) \sin(a+bx) \, dx$


3.70.1 Optimal result . . . 631
3.70.2 Mathematica [A] (verified) . . . 632
3.70.3 Rubi [A] (verified) . . . 632
3.70.4 Maple [F] . . . 633
3.70.5 Fricas [A] (verification not implemented) . . . 634
3.70.6 Sympy [F] . . . 634
3.70.7 Maxima [F] . . . 634
3.70.8 Giac [F] . . . 635
3.70.9 Mupad [F(-1)] . . . 635


## 3.70.1 Optimal result

Integrand size = 22, antiderivative size = 267

$$\begin{aligned}\int (c+dx)^m \cos^2(a+bx) \sin(a+bx) \, dx &= -\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{8b} \\ &\quad - \frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)}{8b} \\ &\quad - \frac{3^{-1-m}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m,-\frac{3ib(c+dx)}{d}\right)}{8b} \\ &\quad - \frac{3^{-1-m}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m,\frac{3ib(c+dx)}{d}\right)}{8b}\end{aligned}$$

output
```
-1/8*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)
/d)^m)-1/8*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*
x+c)/d)^m)-1/8*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3*I*b*(d*x
+c)/d)/b/((-I*b*(d*x+c)/d)^m)-1/8*3^(-1-m)*(d*x+c)^m*GAMMA(1+m,3*I*b*(d*x+
c)/d)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

### 3.70.2 Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.94

$$\int (c+dx)^m \cos^2(a+bx)\sin(a+bx)\,dx$$

$$= \frac{e^{-\frac{3i(bc+ad)}{d}}(c+dx)^m\left(3e^{\frac{2i(bc+ad)}{d}}\left(-e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)-e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\cdots\right.\right.\right.}{\cdots}$$

input
```
Integrate[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
((c + d*x)^m*(3*E^(((2*I)*(b*c + a*d))/d)*(-((E^((2*I)*a)*Gamma[1 + m, ((-
I)*b*(c + d*x))/d])/(((-I)*b*(c + d*x))/d)^m) - (E^(((2*I)*b*c)/d)*Gamma[1
 + m, (I*b*(c + d*x))/d])/((I*b*(c + d*x))/d)^m) - (E^((6*I)*a)*((I*b*(c +
 d*x))/d)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d] + E^(((6*I)*b*c)/d)*(((-I
)*b*(c + d*x))/d)^m*Gamma[1 + m, ((3*I)*b*(c + d*x))/d])/(3^m*((b^2*(c + d
*x)^2)/d^2)^m)))/(24*b*E^(((3*I)*(b*c + a*d))/d))
```

### 3.70.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cos^2(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \ 4906$$

$$\int \left(\frac{1}{4}\sin(a+bx)(c+dx)^m+\frac{1}{4}\sin(3a+3bx)(c+dx)^m\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}-\frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{8b}-\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{8b}-\frac{3^{-m-1}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{3ib(c+dx)}{d}\right)}{8b}$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/8*(E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/
(b*(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d
])/(8*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) - (3^(-1 - m)*E^((3*I)*
(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(8*b*(((-
I)*b*(c + d*x))/d)^m) - (3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((3*I)*b*(c +
 d*x))/d])/(8*b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

### 3.70.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.70.4 Maple [F]

$$\int (dx+c)^m \cos(xb+a)^2 \sin(xb+a)\,dx$$

input
```
int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x)
```

output
```
int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x)
```

### 3.70.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.70

$$\int (c+dx)^m \cos^2(a+bx)\sin(a+bx)\,dx = -\frac{3\,e^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-i\,bc+i\,ad}{d}\right)}\Gamma\left(m+1,\frac{i\,bdx+i\,bc}{d}\right)+e^{\left(-\frac{dm\log\left(-\frac{3i\,b}{d}\right)+3i\,bc-3i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{3\,(i\,bdx+i\,bc)}{d}\right)+3\,e^{\left(-\frac{dm\log}{}\right.}}{24\,b}$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/24*(3*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x + I
*b*c)/d) + e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d)*gamma(m + 1, -3*
(I*b*d*x + I*b*c)/d) + 3*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gamma(m
+ 1, (-I*b*d*x - I*b*c)/d) + e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*a*d)/d)
*gamma(m + 1, -3*(-I*b*d*x - I*b*c)/d))/b
```

### 3.70.6 Sympy [F]

$$\int (c+dx)^m \cos^2(a+bx)\sin(a+bx)\,dx = \int (c+dx)^m \sin(a+bx)\cos^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*cos(b*x+a)**2*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**m*sin(a + b*x)*cos(a + b*x)**2, x)
```

### 3.70.7 Maxima [F]

$$\int (c+dx)^m \cos^2(a+bx)\sin(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)^2 \sin(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a), x)
```

### 3.70.8 Giac [F]

$$\int (c+dx)^m \cos^2(a+bx)\sin(a+bx)\,dx = \int (dx+c)^m \cos{(bx+a)}^2 \sin{(bx+a)}\ dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a), x)
```

### 3.70.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cos^2(a+bx)\sin(a+bx)\,dx = \int \cos{(a+b\,x)}^2\ \sin{(a+b\,x)}\ (c+d\,x)^m\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^m, x)
```

# 3.71 $\int (c+dx)^4 \cos^2(a+bx) \sin(a+bx) \, dx$


3.71.1 Optimal result . . . . . 636
3.71.2 Mathematica [A] (verified) . . . . . 637
3.71.3 Rubi [A] (verified) . . . . . 637
3.71.4 Maple [A] (verified) . . . . . 643
3.71.5 Fricas [A] (verification not implemented) . . . . . 643
3.71.6 Sympy [B] (verification not implemented) . . . . . 644
3.71.7 Maxima [B] (verification not implemented) . . . . . 644
3.71.8 Giac [A] (verification not implemented) . . . . . 645
3.71.9 Mupad [B] (verification not implemented) . . . . . 647


## 3.71.1 Optimal result

Integrand size = 22, antiderivative size = 205

$$\begin{aligned}\int (c+dx)^4 \cos^2(a+bx) \sin(a+bx) \, dx = & -\frac{160 d^4 \cos(a+bx)}{27 b^5} + \frac{8 d^2 (c+dx)^2 \cos(a+bx)}{3 b^3} \\ & - \frac{8 d^4 \cos^3(a+bx)}{81 b^5} + \frac{4 d^2 (c+dx)^2 \cos^3(a+bx)}{9 b^3} \\ & - \frac{(c+dx)^4 \cos^3(a+bx)}{3 b} \\ & - \frac{160 d^3 (c+dx) \sin(a+bx)}{27 b^4} \\ & + \frac{8 d (c+dx)^3 \sin(a+bx)}{9 b^2} \\ & - \frac{8 d^3 (c+dx) \cos^2(a+bx) \sin(a+bx)}{27 b^4} \\ & + \frac{4 d (c+dx)^3 \cos^2(a+bx) \sin(a+bx)}{9 b^2}\end{aligned}$$

output
```
-160/27*d^4*cos(b*x+a)/b^5+8/3*d^2*(d*x+c)^2*cos(b*x+a)/b^3-8/81*d^4*cos(b
*x+a)^3/b^5+4/9*d^2*(d*x+c)^2*cos(b*x+a)^3/b^3-1/3*(d*x+c)^4*cos(b*x+a)^3/
b-160/27*d^3*(d*x+c)*sin(b*x+a)/b^4+8/9*d*(d*x+c)^3*sin(b*x+a)/b^2-8/27*d^
3*(d*x+c)*cos(b*x+a)^2*sin(b*x+a)/b^4+4/9*d*(d*x+c)^3*cos(b*x+a)^2*sin(b*x
+a)/b^2
```

### 3.71.2 Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.73

$$\int (c+dx)^4 \cos^2(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{81(24d^4-12b^2d^2(c+dx)^2+b^4(c+dx)^4)\cos(a+bx)+(8d^4-36b^2d^2(c+dx)^2+27b^4(c+dx)^4)\cos(3}{324}$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/324*(81*(24*d^4 - 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cos[a + b*x
] + (8*d^4 - 36*b^2*d^2*(c + d*x)^2 + 27*b^4*(c + d*x)^4)*Cos[3*(a + b*x)]
 - 24*b*d*(c + d*x)*(-82*d^2 + 15*b^2*(c + d*x)^2 + (-2*d^2 + 3*b^2*(c + d
*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/b^5
```

### 3.71.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.20, number of steps used = 19, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.864$, Rules used = {4905, 3042, 3792, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118, 3791, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sin(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \; 4905$$

$$\frac{4d\int (c+dx)^3\cos^3(a+bx)dx}{3b}-\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$$\Big\downarrow \; 3042$$

$$\frac{4d\int (c+dx)^3\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b}-\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$$\Big\downarrow \; 3792$$

$$\frac{4d\left(-\frac{2d^2\int(c+dx)\cos^3(a+bx)dx}{3b^2}+\frac{2}{3}\int(c+dx)^3\cos(a+bx)dx+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b}-\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$\Big\downarrow$ 3042

$$\frac{4d\left(-\frac{2d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\int(c+dx)^3\sin\left(a+bx+\frac{\pi}{2}\right)dx+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sin(a+bx)\cos^2(a+bx)}{3b}\right.}{3b}$$
$$\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$\Big\downarrow$ 3777

$$\frac{4d\left(-\frac{2d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{3d\int-(c+dx)^2\sin(a+bx)dx}{b}+\frac{(c+dx)^3\sin(a+bx)}{b}\right)+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sin(a+}{3}\right.}{3b}$$
$$\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$\Big\downarrow$ 25

$$\frac{4d\left(-\frac{2d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\int(c+dx)^2\sin(a+bx)dx}{b}\right)+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sin(a+bx}{3b}\right.}{3b}$$
$$\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$\Big\downarrow$ 3042

$$\frac{4d\left(-\frac{2d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\int(c+dx)^2\sin(a+bx)dx}{b}\right)+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sin(a+bx}{3b}\right.}{3b}$$
$$\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$\Big\downarrow$ 3777

$$\frac{4d\left(-\frac{2d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\int(c+dx)\cos(a+bx)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)+\frac{d(c+dx)^2\cos^3(a+b}{3b^2}\right.}{3b}$$
$$\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$\Big\downarrow$ 3042

---

3.71. $\int(c+dx)^4\cos^2(a+bx)\sin(a+bx)\,dx$

$$\frac{4d\left(-\frac{2d^2\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)+\frac{d(c+dx)^2\cos^3(c}{3b^2}}{3b}$$

$$\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$\downarrow$ 3777

$$\frac{4d\left(-\frac{2d^2\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\int -\sin(a+bx)dx}{b}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)+\frac{d}{}}{3b}$$

$$\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$\downarrow$ 25

$$\frac{4d\left(-\frac{2d^2\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int \sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)+\frac{d(c}{}}{3b}$$

$$\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$\downarrow$ 3042

$$\frac{4d\left(-\frac{2d^2\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int \sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)+\frac{d(c}{}}{3b}$$

$$\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$\downarrow$ 3118

3.71. $\int (c+dx)^4\cos^2(a+bx)\sin(a+bx)\,dx$

$$\frac{4d\left(-\frac{2d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2}{}\right.}{b}\right.\right.}{3b}$$

$$\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$\downarrow$ 3791

$$\frac{4d\left(-\frac{2d^2\left(\frac{2}{3}\int(c+dx)\cos(a+bx)dx+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2}{}\right.}{}\right.\right.}{3b}$$

$$\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$\downarrow$ 3042

$$\frac{4d\left(-\frac{2d^2\left(\frac{2}{3}\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d}{}\right.\right.}{3b}$$

$$\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$\downarrow$ 3777

$$\frac{4d\left(-\frac{2d^2\left(\frac{2}{3}\left(\frac{d\int-\sin(a+bx)dx}{b}+\frac{(c+dx)\sin(a+bx)}{b}\right)+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a}{b}\right.\right.}{3b}$$

$$\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$\downarrow$ 25

3.71. $\int(c+dx)^4\cos^2(a+bx)\sin(a+bx)\,dx$

$$\frac{4d\left(-\frac{2d^2\left(\frac{2}{3}\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+\cdots}{b}\right.\right.}{3b}$$

$$\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$\big\downarrow$ 3042

$$\frac{4d\left(-\frac{2d^2\left(\frac{2}{3}\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+\cdots}{b}\right.\right.}{3b}$$

$$\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

$\big\downarrow$ 3118

$$\frac{4d\left(-\frac{2d^2\left(\frac{2}{3}\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b^2}+\frac{d(c+dx)^2\cos^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sin(a+bx)}{b}\right.\right.}{3b}$$

$$\frac{(c+dx)^4\cos^3(a+bx)}{3b}$$

input
```
Int[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/3*((c + d*x)^4*Cos[a + b*x]^3)/b + (4*d*((d*(c + d*x)^2*Cos[a + b*x]^3)
/(3*b^2) + ((c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x])/(3*b) - (2*d^2*((d*Co
s[a + b*x]^3)/(9*b^2) + ((c + d*x)*Cos[a + b*x]^2*Sin[a + b*x])/(3*b) + (2
*((d*Cos[a + b*x])/b^2 + ((c + d*x)*Sin[a + b*x])/b))/3))/(3*b^2) + (2*(((
c + d*x)^3*Sin[a + b*x])/b - (3*d*(-(((c + d*x)^2*Cos[a + b*x])/b) + (2*d*
((d*Cos[a + b*x])/b^2 + ((c + d*x)*Sin[a + b*x])/b))/b))/b))/3))/(3*b)
```

## 3.71. $\int(c+dx)^4\cos^2(a+bx)\sin(a+bx)\,dx$

### 3.71.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

### 3.71.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.88

| method | result |
|---|---|
| parallelrisch | $\frac{\left(-27b^4(dx+c)^4+36d^2(dx+c)^2b^2-8d^4\right)\cos(3xb+3a)+36bd\left((dx+c)^2b^2-\frac{2d^2}{3}\right)(dx+c)\sin(3xb+3a)+\left(-81b^4(dx+c)^4+9\right.}{324b^5}$ |
| risch | $-\frac{(d^4x^4b^4+4b^4c\,d^3x^3+6b^4c^2d^2x^2+4b^4c^3dx+b^4c^4-12b^2d^4x^2-24b^2c\,d^3x-12b^2c^2d^2+24d^4)\cos(xb+a)}{4b^5}+\frac{d(b^2d^3x^3+3}{}$ |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |

input
```
int((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/324*((-27*b^4*(d*x+c)^4+36*d^2*(d*x+c)^2*b^2-8*d^4)*cos(3*b*x+3*a)+36*b*
d*((d*x+c)^2*b^2-2/3*d^2)*(d*x+c)*sin(3*b*x+3*a)+(-81*b^4*(d*x+c)^4+972*d^
2*(d*x+c)^2*b^2-1944*d^4)*cos(b*x+a)+324*b*d*((d*x+c)^2*b^2-6*d^2)*(d*x+c)
*sin(b*x+a)-108*b^4*c^4+1008*b^2*c^2*d^2-1952*d^4)/b^5
```

### 3.71.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.43

$$\int (c+dx)^4\cos^2(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{(27\,b^4d^4x^4+108\,b^4cd^3x^3+27\,b^4c^4-36\,b^2c^2d^2+8\,d^4+18\,(9\,b^4c^2d^2-2\,b^2d^4)x^2+36\,(3\,b^4c^3d-2\,b^2cd^3)}{}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/81*((27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 - 36*b^2*c^2*d^2 +
 8*d^4 + 18*(9*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 36*(3*b^4*c^3*d - 2*b^2*c*d^
3)*x)*cos(b*x + a)^3 - 24*(9*b^2*d^4*x^2 + 18*b^2*c*d^3*x + 9*b^2*c^2*d^2
- 20*d^4)*cos(b*x + a) - 12*(6*b^3*d^4*x^3 + 18*b^3*c*d^3*x^2 + 6*b^3*c^3*
d - 40*b*c*d^3 + (3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d - 2*b*c*d^
3 + (9*b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 + 2*(9*b^3*c^2*d^2 - 20*b*
d^4)*x)*sin(b*x + a))/b^5
```

### 3.71.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 646 vs. $2(207) = 414$.

Time = 0.64 (sec) , antiderivative size = 646, normalized size of antiderivative = 3.15

$$\int (c+dx)^4 \cos^2(a+bx)\sin(a+bx)\,dx$$
$$= \begin{cases} -\frac{c^4\cos^3(a+bx)}{3b} - \frac{4c^3dx\cos^3(a+bx)}{3b} - \frac{2c^2d^2x^2\cos^3(a+bx)}{b} - \frac{4cd^3x^3\cos^3(a+bx)}{3b} - \frac{d^4x^4\cos^3(a+bx)}{3b} + \frac{8c^3d\sin^3(a+bx)}{9b^2} + \frac{4c^3d}{} \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\sin(a)\cos^2(a) \end{cases}$$

input
```
integrate((d*x+c)**4*cos(b*x+a)**2*sin(b*x+a),x)
```

output
```
Piecewise((-c**4*cos(a + b*x)**3/(3*b) - 4*c**3*d*x*cos(a + b*x)**3/(3*b)
- 2*c**2*d**2*x**2*cos(a + b*x)**3/b - 4*c*d**3*x**3*cos(a + b*x)**3/(3*b)
 - d**4*x**4*cos(a + b*x)**3/(3*b) + 8*c**3*d*sin(a + b*x)**3/(9*b**2) + 4
*c**3*d*sin(a + b*x)*cos(a + b*x)**2/(3*b**2) + 8*c**2*d**2*x*sin(a + b*x)
**3/(3*b**2) + 4*c**2*d**2*x*sin(a + b*x)*cos(a + b*x)**2/b**2 + 8*c*d**3*
x**2*sin(a + b*x)**3/(3*b**2) + 4*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**2
/b**2 + 8*d**4*x**3*sin(a + b*x)**3/(9*b**2) + 4*d**4*x**3*sin(a + b*x)*co
s(a + b*x)**2/(3*b**2) + 8*c**2*d**2*sin(a + b*x)**2*cos(a + b*x)/(3*b**3)
 + 28*c**2*d**2*cos(a + b*x)**3/(9*b**3) + 16*c*d**3*x*sin(a + b*x)**2*cos
(a + b*x)/(3*b**3) + 56*c*d**3*x*cos(a + b*x)**3/(9*b**3) + 8*d**4*x**2*si
n(a + b*x)**2*cos(a + b*x)/(3*b**3) + 28*d**4*x**2*cos(a + b*x)**3/(9*b**3
) - 160*c*d**3*sin(a + b*x)**3/(27*b**4) - 56*c*d**3*sin(a + b*x)*cos(a +
b*x)**2/(9*b**4) - 160*d**4*x*sin(a + b*x)**3/(27*b**4) - 56*d**4*x*sin(a
+ b*x)*cos(a + b*x)**2/(9*b**4) - 160*d**4*sin(a + b*x)**2*cos(a + b*x)/(2
7*b**5) - 488*d**4*cos(a + b*x)**3/(81*b**5), Ne(b, 0)), ((c**4*x + 2*c**3
*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)*cos(a)**2,
True))
```

### 3.71.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 889 vs. $2(187) = 374$.

Time = 0.30 (sec) , antiderivative size = 889, normalized size of antiderivative = 4.34

$$\int (c+dx)^4 \cos^2(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/324*(108*c^4*cos(b*x + a)^3 - 432*a*c^3*d*cos(b*x + a)^3/b + 648*a^2*c^
2*d^2*cos(b*x + a)^3/b^2 - 432*a^3*c*d^3*cos(b*x + a)^3/b^3 + 108*a^4*d^4*
cos(b*x + a)^3/b^4 + 36*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*
x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*c^3*d/b - 108*(3*(b*x + a)*cos
(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x +
a))*a*c^2*d^2/b^2 + 108*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*
x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*a^2*c*d^3/b^3 - 36*(3*(b*x + a
)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b
*x + a))*a^3*d^4/b^4 + 18*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x
 + a)^2 - 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*si
n(b*x + a))*c^2*d^2/b^2 - 36*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((
b*x + a)^2 - 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)
*sin(b*x + a))*a*c*d^3/b^3 + 18*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27
*((b*x + a)^2 - 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x +
 a)*sin(b*x + a))*a^2*d^4/b^4 + 12*(3*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*
b*x + 3*a) + 27*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) - (9*(b*x + a)^2
- 2)*sin(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*sin(b*x + a))*c*d^3/b^3 - 12*
(3*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) + 27*((b*x + a)^3 - 6*b*
x - 6*a)*cos(b*x + a) - (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 81*((b*x +
a)^2 - 2)*sin(b*x + a))*a*d^4/b^4 + ((27*(b*x + a)^4 - 36*(b*x + a)^2 +...
```

### 3.71.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.71

$$\int (c+dx)^4\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{(27\,b^4d^4x^4+108\,b^4cd^3x^3+162\,b^4c^2d^2x^2+108\,b^4c^3dx+27\,b^4c^4-36\,b^2d^4x^2-72\,b^2cd^3x-36\,b^2c^2d^2+8}{324\,b^5} - \frac{(b^4d^4x^4+4\,b^4cd^3x^3+6\,b^4c^2d^2x^2+4\,b^4c^3dx+b^4c^4-12\,b^2d^4x^2-24\,b^2cd^3x-12\,b^2c^2d^2+24\,d^4)\cos(bx}{4\,b^5} + \frac{(3\,b^3d^4x^3+9\,b^3cd^3x^2+9\,b^3c^2d^2x+3\,b^3c^3d-2\,bd^4x-2\,bcd^3)\sin(3\,bx+3\,a)}{27\,b^5} + \frac{(b^3d^4x^3+3\,b^3cd^3x^2+3\,b^3c^2d^2x+b^3c^3d-6\,bd^4x-6\,bcd^3)\sin(bx+a)}{b^5}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/324*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4
*c^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 +
 8*d^4)*cos(3*b*x + 3*a)/b^5 - 1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*
c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x -
12*b^2*c^2*d^2 + 24*d^4)*cos(b*x + a)/b^5 + 1/27*(3*b^3*d^4*x^3 + 9*b^3*c*
d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*b*c*d^3)*sin(3*b*x
 + 3*a)/b^5 + (b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d
 - 6*b*d^4*x - 6*b*c*d^3)*sin(b*x + a)/b^5
```

3.71. $\int (c+dx)^4 \cos^2(a+bx) \sin(a+bx) \, dx$

### 3.71.9 Mupad [B] (verification not implemented)

Time = 25.72 (sec) , antiderivative size = 448, normalized size of antiderivative = 2.19

$$\int (c+dx)^4\cos^2(a+bx)\sin(a+bx)\,dx = \frac{4\,x\cos\left(a+b\,x\right)^3\,\left(14\,c\,d^3-3\,b^2\,c^3\,d\right)}{9\,b^3} - \frac{\cos\left(a+b\,x\right)^3\,\left(27\,b^4\,c^4-252\,b^2\,c^2\,d^2+488\,d^4\right)}{81\,b^5} - \frac{8\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2\,\left(20\,d^4-9\,b^2\,c^2\,d^2\right)}{27\,b^5} - \frac{4\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\left(14\,c\,d^3-3\,b^2\,c^3\,d\right)}{9\,b^4} - \frac{d^4\,x^4\cos\left(a+b\,x\right)^3}{3\,b} - \frac{8\sin\left(a+b\,x\right)^3\,\left(20\,c\,d^3-3\,b^2\,c^3\,d\right)}{27\,b^4} + \frac{8\,d^4\,x^3\sin\left(a+b\,x\right)^3}{9\,b^2} - \frac{8\,x\sin\left(a+b\,x\right)^3\,\left(20\,d^4-9\,b^2\,c^2\,d^2\right)}{27\,b^4} + \frac{2\,x^2\cos\left(a+b\,x\right)^3\,\left(14\,d^4-9\,b^2\,c^2\,d^2\right)}{9\,b^3} - \frac{4\,c\,d^3\,x^3\cos\left(a+b\,x\right)^3}{3\,b} + \frac{4\,d^4\,x^3\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{3\,b^2} + \frac{8\,d^4\,x^2\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2}{3\,b^3} + \frac{8\,c\,d^3\,x^2\sin\left(a+b\,x\right)^3}{3\,b^2} - \frac{4\,x\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\left(14\,d^4-9\,b^2\,c^2\,d^2\right)}{9\,b^4} + \frac{4\,c\,d^3\,x^2\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{b^2} + \frac{16\,c\,d^3\,x\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2}{3\,b^3}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^4,x)
```

output

```
(4*x*cos(a + b*x)^3*(14*c*d^3 - 3*b^2*c^3*d))/(9*b^3) - (cos(a + b*x)^3*(4
88*d^4 + 27*b^4*c^4 - 252*b^2*c^2*d^2))/(81*b^5) - (8*cos(a + b*x)*sin(a +
 b*x)^2*(20*d^4 - 9*b^2*c^2*d^2))/(27*b^5) - (4*cos(a + b*x)^2*sin(a + b*x
)*(14*c*d^3 - 3*b^2*c^3*d))/(9*b^4) - (d^4*x^4*cos(a + b*x)^3)/(3*b) - (8*
sin(a + b*x)^3*(20*c*d^3 - 3*b^2*c^3*d))/(27*b^4) + (8*d^4*x^3*sin(a + b*x
)^3)/(9*b^2) - (8*x*sin(a + b*x)^3*(20*d^4 - 9*b^2*c^2*d^2))/(27*b^4) + (2
*x^2*cos(a + b*x)^3*(14*d^4 - 9*b^2*c^2*d^2))/(9*b^3) - (4*c*d^3*x^3*cos(a
 + b*x)^3)/(3*b) + (4*d^4*x^3*cos(a + b*x)^2*sin(a + b*x))/(3*b^2) + (8*d^
4*x^2*cos(a + b*x)*sin(a + b*x)^2)/(3*b^3) + (8*c*d^3*x^2*sin(a + b*x)^3)/
(3*b^2) - (4*x*cos(a + b*x)^2*sin(a + b*x)*(14*d^4 - 9*b^2*c^2*d^2))/(9*b^
4) + (4*c*d^3*x^2*cos(a + b*x)^2*sin(a + b*x))/b^2 + (16*c*d^3*x*cos(a + b
*x)*sin(a + b*x)^2)/(3*b^3)
```

3.71. $\int (c+dx)^4 \cos^2(a+bx)\sin(a+bx)\,dx$

# 3.72 $\int (c+dx)^3 \cos^2(a+bx) \sin(a+bx)\,dx$


3.72.1 Optimal result . . . . . 649
3.72.2 Mathematica [A] (verified) . . . . . 650
3.72.3 Rubi [A] (verified) . . . . . 650
3.72.4 Maple [A] (verified) . . . . . 653
3.72.5 Fricas [A] (verification not implemented) . . . . . 654
3.72.6 Sympy [B] (verification not implemented) . . . . . 655
3.72.7 Maxima [B] (verification not implemented) . . . . . 655
3.72.8 Giac [A] (verification not implemented) . . . . . 656
3.72.9 Mupad [B] (verification not implemented) . . . . . 657


## 3.72.1 Optimal result

Integrand size = 22, antiderivative size = 151

$$\int (c+dx)^3 \cos^2(a+bx)\sin(a+bx)\,dx = \frac{4d^2(c+dx)\cos(a+bx)}{3b^3} + \frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3} - \frac{(c+dx)^3\cos^3(a+bx)}{3b} - \frac{14d^3\sin(a+bx)}{9b^4} + \frac{2d(c+dx)^2\sin(a+bx)}{3b^2} + \frac{d(c+dx)^2\cos^2(a+bx)\sin(a+bx)}{3b^2} + \frac{2d^3\sin^3(a+bx)}{27b^4}$$

output
```
4/3*d^2*(d*x+c)*cos(b*x+a)/b^3+2/9*d^2*(d*x+c)*cos(b*x+a)^3/b^3-1/3*(d*x+c
)^3*cos(b*x+a)^3/b-14/9*d^3*sin(b*x+a)/b^4+2/3*d*(d*x+c)^2*sin(b*x+a)/b^2+
1/3*d*(d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)/b^2+2/27*d^3*sin(b*x+a)^3/b^4
```

### 3.72.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.84

$$\int (c+dx)^3\cos^2(a+bx)\sin(a+bx)\,dx$$
$$=\frac{-27b(c+dx)\left(-6d^2+b^2(c+dx)^2\right)\cos(a+bx)-3b(c+dx)\left(-2d^2+3b^2(c+dx)^2\right)\cos(3(a+bx))+2d(\cdots}{108b^4}$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
(-27*b*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 3*b*(c + d*x)*(
-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 2*d*(-82*d^2 + 45*b^2*(c +
d*x)^2 + (-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/(108
*b^4)
```

### 3.72.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {4905, 3042, 3792, 3042, 3113, 2009, 3777, 25, 3042, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\sin(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{4905}$$

$$\frac{d\int (c+dx)^2\cos^3(a+bx)dx}{b}-\frac{(c+dx)^3\cos^3(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d\int (c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{b}-\frac{(c+dx)^3\cos^3(a+bx)}{3b}$$

$$\Big\downarrow \text{3792}$$

$$\frac{d\left(-\frac{2d^2\int\cos^3(a+bx)dx}{9b^2}+\frac{2}{3}\int (c+dx)^2\cos(a+bx)dx+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{b}-\frac{(c+dx)^3\cos^3(a+bx)}{3b}$$

$$\Big\downarrow \; 3042$$

$$\frac{d\left(-\frac{2d^2\int \sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{9b^2}+\frac{2}{3}\int (c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)dx+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{b}-\frac{(c+dx)^3\cos^3(a+bx)}{3b}$$

$$\Big\downarrow \; 3113$$

$$\frac{d\left(\frac{2d^2\int \left(1-\sin^2(a+bx)\right)d(-\sin(a+bx))}{9b^3}+\frac{2}{3}\int (c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)dx+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{b}-\frac{(c+dx)^3\cos^3(a+bx)}{3b}$$

$$\Big\downarrow \; 2009$$

$$\frac{d\left(\frac{2}{3}\int (c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)dx+\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{b}-\frac{(c+dx)^3\cos^3(a+bx)}{3b}$$

$$\Big\downarrow \; 3777$$

$$\frac{d\left(\frac{2}{3}\left(\frac{2d\int -((c+dx)\sin(a+bx))dx}{b}+\frac{(c+dx)^2\sin(a+bx)}{b}\right)+\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{b}-\frac{(c+dx)^3\cos^3(a+bx)}{3b}$$

$$\Big\downarrow \; 25$$

$$\frac{d\left(\frac{2}{3}\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int (c+dx)\sin(a+bx)dx}{b}\right)+\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{b}-\frac{(c+dx)^3\cos^3(a+bx)}{3b}$$

$$\Big\downarrow \; 3042$$

$$\frac{d\left(\frac{2}{3}\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int (c+dx)\sin(a+bx)dx}{b}\right)+\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{b}-\frac{(c+dx)^3\cos^3(a+bx)}{3b}$$

$$\Big\downarrow \; 3777$$

## 3.72. $\int (c+dx)^3\cos^2(a+bx)\sin(a+bx)\,dx$

$$\frac{d\left(\frac{2}{3}\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\cos(a+bx)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)+\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c+dx)^2}{}\right.}{b}$$
$$\frac{(c+dx)^3\cos^3(a+bx)}{3b}$$

$\big\downarrow$ 3042

$$\frac{d\left(\frac{2}{3}\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)+\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{(c}{}\right.}{b}$$
$$\frac{(c+dx)^3\cos^3(a+bx)}{3b}$$

$\big\downarrow$ 3117

$$\frac{d\left(\frac{2d^2\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{9b^3}+\frac{2d(c+dx)\cos^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)+\frac{(c+dx)^2\sin}{}\right.}{b}$$
$$\frac{(c+dx)^3\cos^3(a+bx)}{3b}$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/3*((c + d*x)^3*Cos[a + b*x]^3)/b + (d*((2*d*(c + d*x)*Cos[a + b*x]^3)/(
9*b^2) + ((c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x])/(3*b) + (2*d^2*(-Sin[a
+ b*x] + Sin[a + b*x]^3/3))/(9*b^3) + (2*(((c + d*x)^2*Sin[a + b*x])/b - (
2*d*(-(((c + d*x)*Cos[a + b*x])/b) + (d*Sin[a + b*x])/b^2))/b))/3))/b
```

### 3.72.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

### 3.72.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.48

| method | result |
|---|---|
| risch | $-\frac{(b^2d^3x^3+3b^2c\,d^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6c\,d^2)\cos(xb+a)}{4b^3}+\frac{3d(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2)\sin(xb+a)}{4b^4}-\frac{(3b^2}{}$ |
| parallelrisch | $\frac{3bx\left(\left(\frac{1}{3}x^2d^2+cdx+c^2\right)b^2-\frac{14d^2}{9}\right)d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^6+6d\left((dx+c)^2b^2-\frac{14d^2}{9}\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^5-6b\left((x^2d^2+cdx+c^2)\left(\frac{dx}{2}+c\right)b^2\right.}{}$ |
| derivativedivides | $\frac{\frac{a^3d^3\cos(xb+a)^3}{3b^3}-\frac{a^2c\,d^2\cos(xb+a)^3}{b^2}+\frac{3a^2d^3\left(-\frac{\cos(xb+a)^3(xb+a)}{3}+\frac{(2+\cos(xb+a)^2)\sin(xb+a)}{9}\right)}{b^3}+\frac{a\,c^2d\cos(xb+a)^3}{b}-\frac{6ac\,d^2\left(\right.}{}}{}$ |
| default | $\frac{\frac{a^3d^3\cos(xb+a)^3}{3b^3}-\frac{a^2c\,d^2\cos(xb+a)^3}{b^2}+\frac{3a^2d^3\left(-\frac{\cos(xb+a)^3(xb+a)}{3}+\frac{(2+\cos(xb+a)^2)\sin(xb+a)}{9}\right)}{b^3}+\frac{a\,c^2d\cos(xb+a)^3}{b}-\frac{6ac\,d^2\left(\right.}{}}{}$ |
| norman | $\frac{\frac{(-2b^2c^3+4c\,d^2)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^4}{b^3}+\frac{d^3x^3\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}{b}+\frac{c\,d^2x^2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^6}{b}+\frac{-6b^2c^3+28c\,d^2}{9b^3}-\frac{d^3x^3}{3b}+\frac{16c\,d^2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}{3b^3}+\frac{2d(}{}}{}$ |

input
```
int((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^3*(b^2*d^3*x^3+3*b^2*c*d^2*x^2+3*b^2*c^2*d*x+b^2*c^3-6*d^3*x-6*c*d^
2)*cos(b*x+a)+3/4*d*(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2-2*d^2)/b^4*sin(b*x+a)
-1/36/b^3*(3*b^2*d^3*x^3+9*b^2*c*d^2*x^2+9*b^2*c^2*d*x+3*b^2*c^3-2*d^3*x-2
*c*d^2)*cos(3*b*x+3*a)+1/108*d*(9*b^2*d^2*x^2+18*b^2*c*d*x+9*b^2*c^2-2*d^2
)/b^4*sin(3*b*x+3*a)
```

### 3.72.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.21

$$\int (c+dx)^3\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{3\left(3\,b^3d^3x^3+9\,b^3cd^2x^2+3\,b^3c^3-2\,bcd^2+(9\,b^3c^2d-2\,bd^3)x\right)\cos(bx+a)^3-36\left(bd^3x+bcd^2\right)\cos(bx+}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/27*(3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 2*b*c*d^2 + (9*b^3
*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^3 - 36*(b*d^3*x + b*c*d^2)*cos(b*x + a)
- (18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - 40*d^3 + (9*b^2*d^3*x^
2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a)^2)*sin(b*x + a))/b^
4
```

### 3.72.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(150) = 300$.

Time = 0.45 (sec) , antiderivative size = 391, normalized size of antiderivative = 2.59

$$\int (c+dx)^3\cos^2(a+bx)\sin(a+bx)\,dx$$
$$= \begin{cases} -\frac{c^3\cos^3(a+bx)}{3b} - \frac{c^2dx\cos^3(a+bx)}{b} - \frac{cd^2x^2\cos^3(a+bx)}{b} - \frac{d^3x^3\cos^3(a+bx)}{3b} + \frac{2c^2d\sin^3(a+bx)}{3b^2} + \frac{c^2d\sin(a+bx)\cos^2(a+bx)}{b^2} + \frac{4}{} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin(a)\cos^2(a) \end{cases}$$

input
```
integrate((d*x+c)**3*cos(b*x+a)**2*sin(b*x+a),x)
```

output
```
Piecewise((-c**3*cos(a + b*x)**3/(3*b) - c**2*d*x*cos(a + b*x)**3/b - c*d*
*2*x**2*cos(a + b*x)**3/b - d**3*x**3*cos(a + b*x)**3/(3*b) + 2*c**2*d*sin
(a + b*x)**3/(3*b**2) + c**2*d*sin(a + b*x)*cos(a + b*x)**2/b**2 + 4*c*d**
2*x*sin(a + b*x)**3/(3*b**2) + 2*c*d**2*x*sin(a + b*x)*cos(a + b*x)**2/b**
2 + 2*d**3*x**2*sin(a + b*x)**3/(3*b**2) + d**3*x**2*sin(a + b*x)*cos(a +
b*x)**2/b**2 + 4*c*d**2*sin(a + b*x)**2*cos(a + b*x)/(3*b**3) + 14*c*d**2*
cos(a + b*x)**3/(9*b**3) + 4*d**3*x*sin(a + b*x)**2*cos(a + b*x)/(3*b**3)
+ 14*d**3*x*cos(a + b*x)**3/(9*b**3) - 40*d**3*sin(a + b*x)**3/(27*b**4) -
 14*d**3*sin(a + b*x)*cos(a + b*x)**2/(9*b**4), Ne(b, 0)), ((c**3*x + 3*c*
*2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)*cos(a)**2, True))
```

### 3.72.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(137) = 274$.

Time = 0.27 (sec) , antiderivative size = 505, normalized size of antiderivative = 3.34

$$\int (c+dx)^3\cos^2(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{36\,c^3\cos(bx+a)^3 - \frac{108\,ac^2d\cos(bx+a)^3}{b} + \frac{108\,a^2cd^2\cos(bx+a)^3}{b^2} - \frac{36\,a^3d^3\cos(bx+a)^3}{b^3} + \frac{9\,(3\,(bx+a)\cos(3\,bx+3\,a)+9\,(bx+a)}{}}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/108*(36*c^3*cos(b*x + a)^3 - 108*a*c^2*d*cos(b*x + a)^3/b + 108*a^2*c*d
^2*cos(b*x + a)^3/b^2 - 36*a^3*d^3*cos(b*x + a)^3/b^3 + 9*(3*(b*x + a)*cos
(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x +
a))*c^2*d/b - 18*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a)
- sin(3*b*x + 3*a) - 9*sin(b*x + a))*a*c*d^2/b^2 + 9*(3*(b*x + a)*cos(3*b*
x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*a
^2*d^3/b^3 + 3*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2 - 2
)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a))
*c*d^2/b^2 - 3*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2 - 2
)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a))
*a*d^3/b^3 + (3*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) + 27*((b*x
+ a)^3 - 6*b*x - 6*a)*cos(b*x + a) - (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a)
- 81*((b*x + a)^2 - 2)*sin(b*x + a))*d^3/b^3)/b
```

### 3.72.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.53

$$\begin{aligned}\int (c+dx)^3\cos^2(a+bx)\sin(a+bx)\,dx &= -\frac{(3\,b^3d^3x^3+9\,b^3cd^2x^2+9\,b^3c^2dx+3\,b^3c^3-2\,bd^3x-2\,bcd^2)\cos{(3\,bx+3\,a)}}{36\,b^4}\\ &\quad-\frac{(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3-6\,bd^3x-6\,bcd^2)\cos{(bx+a)}}{4\,b^4}\\ &\quad+\frac{(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3)\sin{(3\,bx+3\,a)}}{108\,b^4}\\ &\quad+\frac{3\,(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d-2\,d^3)\sin{(bx+a)}}{4\,b^4}\end{aligned}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/36*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d
^3*x - 2*b*c*d^2)*cos(3*b*x + 3*a)/b^4 - 1/4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^
2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*cos(b*x + a)/b^4 + 1/
108*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*sin(3*b*x + 3*a
)/b^4 + 3/4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sin(b*x + a)
/b^4
```

### 3.72.9 Mupad [B] (verification not implemented)

Time = 24.41 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.92

$$\int (c+dx)^3\cos^2(a+bx)\sin(a+bx)\,dx = \frac{\cos(a+b\,x)^3\,(14\,c\,d^2-3\,b^2\,c^3)}{9\,b^3} - \frac{2\sin(a+b\,x)^3\,(20\,d^3-9\,b^2\,c^2\,d)}{27\,b^4} - \frac{\cos(a+b\,x)^2\,\sin(a+b\,x)\,\left(14\,d^3-9\,b^2\,c^2\,d\right)}{9\,b^4} + \frac{x\cos(a+b\,x)^3\,(14\,d^3-9\,b^2\,c^2\,d)}{9\,b^3} - \frac{d^3\,x^3\cos(a+b\,x)^3}{3\,b} + \frac{2\,d^3\,x^2\sin(a+b\,x)^3}{3\,b^2} + \frac{4\,c\,d^2\,\cos(a+b\,x)\,\sin(a+b\,x)^2}{3\,b^3} + \frac{4\,d^3\,x\,\cos(a+b\,x)\,\sin(a+b\,x)^2}{3\,b^3} + \frac{4\,c\,d^2\,x\sin(a+b\,x)^3}{3\,b^2} - \frac{c\,d^2\,x^2\cos(a+b\,x)^3}{b} + \frac{d^3\,x^2\cos(a+b\,x)^2\,\sin(a+b\,x)}{b^2} + \frac{2\,c\,d^2\,x\cos(a+b\,x)^2\,\sin(a+b\,x)}{b^2}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^3,x)
```

output
```
(cos(a + b*x)^3*(14*c*d^2 - 3*b^2*c^3))/(9*b^3) - (2*sin(a + b*x)^3*(20*d^
3 - 9*b^2*c^2*d))/(27*b^4) - (cos(a + b*x)^2*sin(a + b*x)*(14*d^3 - 9*b^2*
c^2*d))/(9*b^4) + (x*cos(a + b*x)^3*(14*d^3 - 9*b^2*c^2*d))/(9*b^3) - (d^3
*x^3*cos(a + b*x)^3)/(3*b) + (2*d^3*x^2*sin(a + b*x)^3)/(3*b^2) + (4*c*d^2
*cos(a + b*x)*sin(a + b*x)^2)/(3*b^3) + (4*d^3*x*cos(a + b*x)*sin(a + b*x)
^2)/(3*b^3) + (4*c*d^2*x*sin(a + b*x)^3)/(3*b^2) - (c*d^2*x^2*cos(a + b*x)
^3)/b + (d^3*x^2*cos(a + b*x)^2*sin(a + b*x))/b^2 + (2*c*d^2*x*cos(a + b*x
)^2*sin(a + b*x))/b^2
```

# 3.73 $\int (c+dx)^2 \cos^2(a+bx) \sin(a+bx) \, dx$


3.73.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 658
3.73.2 Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 658
3.73.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 659
3.73.4 Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 661
3.73.5 Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 661
3.73.6 Sympy [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 662
3.73.7 Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 662
3.73.8 Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 663
3.73.9 Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 663


## 3.73.1 Optimal result

Integrand size = 22, antiderivative size = 103

$$\int (c+dx)^2 \cos^2(a+bx) \sin(a+bx) \, dx = \frac{4d^2 \cos(a+bx)}{9b^3} + \frac{2d^2 \cos^3(a+bx)}{27b^3} - \frac{(c+dx)^2 \cos^3(a+bx)}{3b} + \frac{4d(c+dx) \sin(a+bx)}{9b^2} + \frac{2d(c+dx) \cos^2(a+bx) \sin(a+bx)}{9b^2}$$

output
```
4/9*d^2*cos(b*x+a)/b^3+2/27*d^2*cos(b*x+a)^3/b^3-1/3*(d*x+c)^2*cos(b*x+a)^
3/b+4/9*d*(d*x+c)*sin(b*x+a)/b^2+2/9*d*(d*x+c)*cos(b*x+a)^2*sin(b*x+a)/b^2
```

## 3.73.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int (c+dx)^2 \cos^2(a+bx) \sin(a+bx) \, dx = -\frac{27(-2d^2+b^2(c+dx)^2) \cos(a+bx) + (-2d^2+9b^2(c+dx)^2) \cos(3(a+bx)) - 6bd(c+dx)(9 \sin(a+bx)}{108b^3}$$

input
```
Integrate[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/108*(27*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + (-2*d^2 + 9*b^2*(c +
d*x)^2)*Cos[3*(a + b*x)] - 6*b*d*(c + d*x)*(9*Sin[a + b*x] + Sin[3*(a + b*
x)]))/b^3
```

### 3.73.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4905, 3042, 3791, 3042, 3777, 25, 3042, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{4905}$$

$$\frac{2d\int (c+dx)\cos^3(a+bx)dx}{3b}-\frac{(c+dx)^2\cos^3(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b}-\frac{(c+dx)^2\cos^3(a+bx)}{3b}$$

$$\Big\downarrow \text{3791}$$

$$\frac{2d\left(\frac{2}{3}\int (c+dx)\cos(a+bx)dx+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b}-\frac{(c+dx)^2\cos^3(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\left(\frac{2}{3}\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b}-\frac{(c+dx)^2\cos^3(a+bx)}{3b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{2d\left(\frac{2}{3}\left(\frac{d\int -\sin(a+bx)dx}{b}+\frac{(c+dx)\sin(a+bx)}{b}\right)+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b}-\frac{(c+dx)^2\cos^3(a+bx)}{3b}$$

$$\Big\downarrow \text{25}$$

$$\frac{2d\left(\frac{2}{3}\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int \sin(a+bx)dx}{b}\right)+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b}-\frac{(c+dx)^2\cos^3(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\left(\frac{2}{3}\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b}-\frac{(c+dx)^2\cos^3(a+bx)}{3b}$$

$$\Big\downarrow \; 3118$$

$$\frac{2d\left(\frac{2}{3}\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)+\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin(a+bx)\cos^2(a+bx)}{3b}\right)}{3b}-\frac{(c+dx)^2\cos^3(a+bx)}{3b}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/3*((c + d*x)^2*Cos[a + b*x]^3)/b + (2*d*((d*Cos[a + b*x]^3)/(9*b^2) + (
(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x])/(3*b) + (2*((d*Cos[a + b*x])/b^2 +
((c + d*x)*Sin[a + b*x])/b))/3))/(3*b)
```

### 3.73.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

## 3.73.4 Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.24

| method | result |
|---|---|
| risch | $-\frac{(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2)\cos(xb+a)}{4b^3}+\frac{d(dx+c)\sin(xb+a)}{2b^2}-\frac{(9x^2d^2b^2+18b^2cdx+9b^2c^2-2d^2)\cos(3xb+3a)}{108b^3}$ |
| derivativedivides | $\frac{-\frac{a^2d^2\cos(xb+a)^3}{3b^2}+\frac{2acd\cos(xb+a)^3}{3b}-\frac{2a\,d^2\left(-\frac{\cos(xb+a)^3(xb+a)}{3}+\frac{(2+\cos(xb+a)^2)\sin(xb+a)}{9}\right)}{b^2}-\frac{c^2\cos(xb+a)^3}{3}+\frac{2cd\left(-\frac{\cos(}{}\right.}{}}{}$ |
| default | $\frac{-\frac{a^2d^2\cos(xb+a)^3}{3b^2}+\frac{2acd\cos(xb+a)^3}{3b}-\frac{2a\,d^2\left(-\frac{\cos(xb+a)^3(xb+a)}{3}+\frac{(2+\cos(xb+a)^2)\sin(xb+a)}{9}\right)}{b^2}-\frac{c^2\cos(xb+a)^3}{3}+\frac{2cd\left(-\frac{\cos(}{}\right.}{}}{}$ |
| parallelrisch | $\frac{18b^2xd\left(\frac{dx}{2}+c\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^6+36bd(dx+c)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^5+((-27x^2d^2-54cdx-54c^2)b^2+36d^2)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^4+24bd(d}{27b^3\left(1+\mathrm{ta}\right.}$ |
| norman | $\frac{\frac{d^2x^2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}{b}+\frac{-18b^2c^2+28d^2}{27b^3}-\frac{d^2x^2}{3b}+\frac{16d^2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}{9b^3}+\frac{(-6b^2c^2+4d^2)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^4}{3b^3}+\frac{4cd\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{3b^2}+\frac{8cd\tan\left(\frac{a}{2}\right.}{9b^2}}{}$ |

input
```
int((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2-2*d^2)/b^3*cos(b*x+a)+1/2*d*(d*x+c)*
sin(b*x+a)/b^2-1/108*(9*b^2*d^2*x^2+18*b^2*c*d*x+9*b^2*c^2-2*d^2)/b^3*cos(
3*b*x+3*a)+1/18*d*(d*x+c)*sin(3*b*x+3*a)/b^2
```

## 3.73.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.97

$$\int (c+dx)^2\cos^2(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2)\cos{(bx+a)}^3-12\,d^2\cos{(bx+a)}-6\,(2\,bd^2x+2\,bcd+(bd^2x+bc}{27\,b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/27*((9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(b*x + a)^3 -
 12*d^2*cos(b*x + a) - 6*(2*b*d^2*x + 2*b*c*d + (b*d^2*x + b*c*d)*cos(b*x
+ a)^2)*sin(b*x + a))/b^3
```

## 3.73.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(102) = 204$.

Time = 0.34 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.10

$$\int (c+dx)^2\cos^2(a+bx)\sin(a+bx)\,dx$$
$$= \begin{cases} -\frac{c^2\cos^3(a+bx)}{3b} - \frac{2cdx\cos^3(a+bx)}{3b} - \frac{d^2x^2\cos^3(a+bx)}{3b} + \frac{4cd\sin^3(a+bx)}{9b^2} + \frac{2cd\sin(a+bx)\cos^2(a+bx)}{3b^2} + \frac{4d^2x\sin^3(a+bx)}{9b^2} + \frac{2d^2}{} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin(a)\cos^2(a) \end{cases}$$

input
```
integrate((d*x+c)**2*cos(b*x+a)**2*sin(b*x+a),x)
```

output
```
Piecewise((-c**2*cos(a + b*x)**3/(3*b) - 2*c*d*x*cos(a + b*x)**3/(3*b) - d
**2*x**2*cos(a + b*x)**3/(3*b) + 4*c*d*sin(a + b*x)**3/(9*b**2) + 2*c*d*si
n(a + b*x)*cos(a + b*x)**2/(3*b**2) + 4*d**2*x*sin(a + b*x)**3/(9*b**2) +
2*d**2*x*sin(a + b*x)*cos(a + b*x)**2/(3*b**2) + 4*d**2*sin(a + b*x)**2*co
s(a + b*x)/(9*b**3) + 14*d**2*cos(a + b*x)**3/(27*b**3), Ne(b, 0)), ((c**2
*x + c*d*x**2 + d**2*x**3/3)*sin(a)*cos(a)**2, True))
```

## 3.73.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(93) = 186$.

Time = 0.26 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.36

$$\int (c+dx)^2\cos^2(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{36\,c^2\cos(bx+a)^3 - \frac{72\,acd\cos(bx+a)^3}{b} + \frac{36\,a^2d^2\cos(bx+a)^3}{b^2} + \frac{6\,(3\,(bx+a)\cos(3\,bx+3\,a)+9\,(bx+a)\cos(bx+a)-\sin(3\,bx+3\,a)-9}{b}}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/108*(36*c^2*cos(b*x + a)^3 - 72*a*c*d*cos(b*x + a)^3/b + 36*a^2*d^2*cos
(b*x + a)^3/b^2 + 6*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x +
a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*c*d/b - 6*(3*(b*x + a)*cos(3*b*x +
 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*a*d^
2/b^2 + ((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2 - 2)*cos(b
*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a))*d^2/b^
2)/b
```

### 3.73.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.33

$$\int (c+dx)^2 \cos^2(a+bx)\sin(a+bx)\,dx = -\frac{(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2)\cos(3\,bx+3\,a)}{108\,b^3} - \frac{(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,d^2)\cos(bx+a)}{4\,b^3} + \frac{(bd^2x+bcd)\sin(3\,bx+3\,a)}{18\,b^3} + \frac{(bd^2x+bcd)\sin(bx+a)}{2\,b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/108*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(3*b*x + 3*a)
/b^3 - 1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)/b^3
+ 1/18*(b*d^2*x + b*c*d)*sin(3*b*x + 3*a)/b^3 + 1/2*(b*d^2*x + b*c*d)*sin(
b*x + a)/b^3
```

### 3.73.9 Mupad [B] (verification not implemented)

Time = 24.43 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.41

$$\int (c+dx)^2 \cos^2(a+bx)\sin(a+bx)\,dx = \frac{12\,d^2\,\cos(a+b\,x)+2\,d^2\cos(a+b\,x)^3-9\,b^2\,c^2\cos(a+b\,x)^3+12\,b\,d^2\,x\,\sin(a+b\,x)-9\,b^2\,d^2\,x^2\cos(a+$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^2,x)
```

output
```
(12*d^2*cos(a + b*x) + 2*d^2*cos(a + b*x)^3 - 9*b^2*c^2*cos(a + b*x)^3 + 1
2*b*d^2*x*sin(a + b*x) - 9*b^2*d^2*x^2*cos(a + b*x)^3 + 12*b*c*d*sin(a + b
*x) - 18*b^2*c*d*x*cos(a + b*x)^3 + 6*b*d^2*x*cos(a + b*x)^2*sin(a + b*x)
+ 6*b*c*d*cos(a + b*x)^2*sin(a + b*x))/(27*b^3)
```

## 3.73. $\int (c+dx)^2 \cos^2(a+bx) \sin(a+bx) \, dx$

# 3.74 $\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx$

3.74.1 Optimal result . . . 665
3.74.2 Mathematica [A] (verified) . . . 665
3.74.3 Rubi [A] (verified) . . . 666
3.74.4 Maple [A] (verified) . . . 667
3.74.5 Fricas [A] (verification not implemented) . . . 668
3.74.6 Sympy [A] (verification not implemented) . . . 668
3.74.7 Maxima [A] (verification not implemented) . . . 668
3.74.8 Giac [A] (verification not implemented) . . . 669
3.74.9 Mupad [B] (verification not implemented) . . . 669

## 3.74.1 Optimal result

Integrand size = 20, antiderivative size = 51

$$\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx=-\frac{(c+dx)\cos^3(a+bx)}{3b}+\frac{d\sin(a+bx)}{3b^2}-\frac{d\sin^3(a+bx)}{9b^2}$$

output
```
-1/3*(d*x+c)*cos(b*x+a)^3/b+1/3*d*sin(b*x+a)/b^2-1/9*d*sin(b*x+a)^3/b^2
```

## 3.74.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.39

$$\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx=-\frac{c\cos^3(a+bx)}{3b}+\frac{d(-bx\cos(a+bx)+\sin(a+bx))}{4b^2}+\frac{d(-3bx\cos(3(a+bx))+\sin(3(a+bx)))}{36b^2}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/3*(c*Cos[a + b*x]^3)/b + (d*(-(b*x*Cos[a + b*x]) + Sin[a + b*x]))/(4*b^
2) + (d*(-3*b*x*Cos[3*(a + b*x)] + Sin[3*(a + b*x)]))/(36*b^2)
```

### 3.74.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4905, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{4905}$$

$$\frac{d\int \cos^3(a+bx)dx}{3b}-\frac{(c+dx)\cos^3(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d\int \sin\left(a+bx+\frac{\pi}{2}\right)^3dx}{3b}-\frac{(c+dx)\cos^3(a+bx)}{3b}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{d\int \left(1-\sin^2(a+bx)\right)d(-\sin(a+bx))}{3b^2}-\frac{(c+dx)\cos^3(a+bx)}{3b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\left(\frac{1}{3}\sin^3(a+bx)-\sin(a+bx)\right)}{3b^2}-\frac{(c+dx)\cos^3(a+bx)}{3b}$$

input
```
Int[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/3*((c + d*x)*Cos[a + b*x]^3)/b - (d*(-Sin[a + b*x] + Sin[a + b*x]^3/3))
/(3*b^2)
```

#### 3.74.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

## 3.74.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.25

| method | result |
|---|---|
| risch | $-\frac{(dx+c)\cos(xb+a)}{4b}+\frac{d\sin(xb+a)}{4b^2}-\frac{(dx+c)\cos(3xb+3a)}{12b}+\frac{d\sin(3xb+3a)}{36b^2}$ |
| derivativedivides | $\frac{\frac{da\cos(xb+a)^3}{3b}-\frac{c\cos(xb+a)^3}{3}+\frac{d\left(-\frac{\cos(xb+a)^3(xb+a)}{3}+\frac{\left(2+\cos(xb+a)^2\right)\sin(xb+a)}{9}\right)}{b}}{b}$ |
| default | $\frac{\frac{da\cos(xb+a)^3}{3b}-\frac{c\cos(xb+a)^3}{3}+\frac{d\left(-\frac{\cos(xb+a)^3(xb+a)}{3}+\frac{\left(2+\cos(xb+a)^2\right)\sin(xb+a)}{9}\right)}{b}}{b}$ |
| parallelrisch | $\frac{3d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^6xb+6d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^5-18b\left(\frac{dx}{2}+c\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^4+4d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3+9d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2xb+6d\tan\left(\frac{a}{2}+\cdots\right.}{9b^2\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)^3}$ |
| norman | $\frac{-\frac{2c\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^4}{b}+\frac{dx\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}{b}-\frac{2c}{3b}+\frac{2d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{3b^2}+\frac{4d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3}{9b^2}+\frac{2d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^5}{3b^2}-\frac{dx}{3b}-\frac{dx\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^4}{b}+\cdots}{\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)^3}$ |

input
```
int((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(d*x+c)*cos(b*x+a)/b+1/4*d*sin(b*x+a)/b^2-1/12*(d*x+c)*cos(3*b*x+3*a)
/b+1/36*d*sin(3*b*x+3*a)/b^2
```

### 3.74.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{3\,(bdx+bc)\cos\left(bx+a\right)^3-\left(d\cos\left(bx+a\right)^2+2\,d\right)\sin\left(bx+a\right)}{9\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/9*(3*(b*d*x + b*c)*cos(b*x + a)^3 - (d*cos(b*x + a)^2 + 2*d)*sin(b*x +
a))/b^2
```

### 3.74.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.67

$$\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx = \begin{cases} -\frac{c\cos^3(a+bx)}{3b}-\frac{dx\cos^3(a+bx)}{3b}+\frac{2d\sin^3(a+bx)}{9b^2}+\frac{d\sin(a+bx)\cos^2(a+bx)}{3b^2} & \text{for } b\neq 0 \\ \left(cx+\frac{dx^2}{2}\right)\sin(a)\cos^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*cos(b*x+a)**2*sin(b*x+a),x)
```

output
```
Piecewise((-c*cos(a + b*x)**3/(3*b) - d*x*cos(a + b*x)**3/(3*b) + 2*d*sin(
a + b*x)**3/(9*b**2) + d*sin(a + b*x)*cos(a + b*x)**2/(3*b**2), Ne(b, 0)),
 ((c*x + d*x**2/2)*sin(a)*cos(a)**2, True))
```

### 3.74.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.69

$$\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{12\,c\cos\left(bx+a\right)^3-\frac{12\,ad\cos(bx+a)^3}{b}+\frac{(3\,(bx+a)\cos(3\,bx+3\,a)+9\,(bx+a)\cos(bx+a)-\sin(3\,bx+3\,a)-9\,\sin(bx+a))d}{b}}{36\,b}$$

input
```
integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/36*(12*c*cos(b*x + a)^3 - 12*a*d*cos(b*x + a)^3/b + (3*(b*x + a)*cos(3*
b*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))
*d/b)/b
```

### 3.74.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.35

$$\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{(bdx+bc)\cos(3\,bx+3\,a)}{12\,b^2} - \frac{(bdx+bc)\cos(bx+a)}{4\,b^2} + \frac{d\sin(3\,bx+3\,a)}{36\,b^2} + \frac{d\sin(bx+a)}{4\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/12*(b*d*x + b*c)*cos(3*b*x + 3*a)/b^2 - 1/4*(b*d*x + b*c)*cos(b*x + a)/
b^2 + 1/36*d*sin(3*b*x + 3*a)/b^2 + 1/4*d*sin(b*x + a)/b^2
```

### 3.74.9 Mupad [B] (verification not implemented)

Time = 24.00 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.14

$$\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx = \frac{\frac{2\,d\,\sin(a+b\,x)}{9} - b\left(\frac{c\cos(a+b\,x)^3}{3} + \frac{d\,x\cos(a+b\,x)^3}{3}\right) + \frac{d\cos(a+b\,x)^2\,\sin(a+b\,x)}{9}}{b^2}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x),x)
```

output
```
((2*d*sin(a + b*x))/9 - b*((c*cos(a + b*x)^3)/3 + (d*x*cos(a + b*x)^3)/3)
+ (d*cos(a + b*x)^2*sin(a + b*x))/9)/b^2
```

# 3.75 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx$

3.75.1 Optimal result . . . 670
3.75.2 Mathematica [A] (verified) . . . 670
3.75.3 Rubi [A] (verified) . . . 671
3.75.4 Maple [A] (verified) . . . 672
3.75.5 Fricas [A] (verification not implemented) . . . 672
3.75.6 Sympy [F] . . . 673
3.75.7 Maxima [C] (verification not implemented) . . . 673
3.75.8 Giac [C] (verification not implemented) . . . 674
3.75.9 Mupad [F(-1)] . . . 674

## 3.75.1 Optimal result

Integrand size = 22, antiderivative size = 121

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx = \frac{\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{4d} + \frac{\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{4d} + \frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d} + \frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

output
```
1/4*cos(a-b*c/d)*Si(b*c/d+b*x)/d+1/4*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d+
1/4*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d+1/4*Ci(b*c/d+b*x)*sin(a-b*c/d)/d
```

## 3.75.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx = \frac{\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right)+\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\frac{bc}{d}\right)+\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)+}{4d}$$

input
```
Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x),x]
```

output
```
(CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + CosIntegral[b*(c/d
+ x)]*Sin[a - (b*c)/d] + Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Cos[3
*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d)
```

### 3.75.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\cos^2(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{\sin(a+bx)}{4(c+dx)}+\frac{\sin(3a+3bx)}{4(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

input
```
Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x),x]
```

output
```
(CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(4*d) + (CosIntegral
[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(4*d) + (Cos[a - (b*c)/d]*SinIntegral[(b
*c)/d + b*x])/(4*d) + (Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x]
)/(4*d)
```

### 3.75.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.75.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.42

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{b\left(-\frac{3\,\mathrm{Si}\left(-3xb-3a-\frac{3(-ad+cb)}{d}\right)\cos\left(\frac{-3ad+3cb}{d}\right)}{d}-\frac{3\,\mathrm{Ci}\left(3xb+3a+\frac{-3ad+3cb}{d}\right)\sin\left(\frac{-3ad+3cb}{d}\right)}{d}\right)}{12}+\frac{b\left(-\frac{\mathrm{Si}\left(-xb-a-\frac{-ad+cb}{d}\right)\cos}{d}\right.}{}}{b}$ |
| default | $\frac{\frac{b\left(-\frac{3\,\mathrm{Si}\left(-3xb-3a-\frac{3(-ad+cb)}{d}\right)\cos\left(\frac{-3ad+3cb}{d}\right)}{d}-\frac{3\,\mathrm{Ci}\left(3xb+3a+\frac{-3ad+3cb}{d}\right)\sin\left(\frac{-3ad+3cb}{d}\right)}{d}\right)}{12}+\frac{b\left(-\frac{\mathrm{Si}\left(-xb-a-\frac{-ad+cb}{d}\right)\cos}{d}\right.}{}}{b}$ |
| risch | $-\frac{i e^{-\frac{3i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{8d}-\frac{i e^{-\frac{i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(ibx+ia-\frac{i(ad-cb)}{d}\right)}{8d}+\frac{i e^{\frac{i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(-ibx-ia-\frac{-i}{}\right.}{8d}$ |

input
```
int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/12*b*(-3*Si(-3*x*b-3*a-3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3
*x*b+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)+1/4*b*(-Si(-x*b-a-(-a*d+b*
c)/d)*cos((-a*d+b*c)/d)/d-Ci(x*b+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d))
```

## 3.75.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx$$
$$=\frac{\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)\sin\left(-\frac{bc-ad}{d}\right)+\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)+\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3\,(bdx+bc)}{d}\right)+\cos\left(-\frac{bc-ad}{d}\right)}{4\,d}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
1/4*(cos_integral((b*d*x + b*c)/d)*sin(-(b*c - a*d)/d) + cos_integral(3*(b
*d*x + b*c)/d)*sin(-3*(b*c - a*d)/d) + cos(-3*(b*c - a*d)/d)*sin_integral(
3*(b*d*x + b*c)/d) + cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

### 3.75.6 Sympy [F]

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx = \int \frac{\sin{(a+bx)}\cos^2{(a+bx)}}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x), x)
```

### 3.75.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.27

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx = -\frac{b\left(iE_1\left(\frac{ibc+i(bx+a)d-iad}{d}\right)-iE_1\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b\left(-iE_1\left(\frac{3(-ibc-i(bx+a)d+iad)}{d}\right)+i\right.}{}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
-1/8*(b*(I*exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_in
tegral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b*(
-I*exp_integral_e(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integra
l_e(1, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) + b*(
exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(1, -
(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b*(exp_integral_
e(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(1, -3*(-I*b*c
- I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/(b*d)
```

### 3.75.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.51 (sec) , antiderivative size = 6279, normalized size of antiderivative = 51.89

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
1/8*(imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral(b*x + b*c/d))*tan
(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_part(cos_i
ntegral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d)^2 - imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*
a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)
*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integ
ral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*
c/d)^2 + 2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*real_part(cos_integral(-b*x - b*c/d))*
tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*real_part(co
s_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(
1/2*b*c/d)^2 + 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(b*x
+ b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*re
al_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)
^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a
)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_integra
l(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 + imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)...
```

### 3.75.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx = \int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{c+d\,x}\,dx$$

input
```
int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x),x)
```

output
```
int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x), x)
```

# 3.76 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx$


3.76.1 Optimal result . . . 675
3.76.2 Mathematica [A] (verified) . . . 675
3.76.3 Rubi [A] (verified) . . . 676
3.76.4 Maple [A] (verified) . . . 677
3.76.5 Fricas [A] (verification not implemented) . . . 678
3.76.6 Sympy [F] . . . 678
3.76.7 Maxima [C] (verification not implemented) . . . 678
3.76.8 Giac [C] (verification not implemented) . . . 679
3.76.9 Mupad [F(-1)] . . . 680


## 3.76.1 Optimal result

Integrand size = 22, antiderivative size = 168

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \frac{b\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2} + \frac{3b\cos\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2} - \frac{\sin(a+bx)}{4d(c+dx)} - \frac{\sin(3a+3bx)}{4d(c+dx)} - \frac{b\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{4d^2} - \frac{3b\sin\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3bc}{d}+3bx\right)}{4d^2}$$

output
```
3/4*b*Ci(3*b*c/d+3*b*x)*cos(3*a-3*b*c/d)/d^2+1/4*b*Ci(b*c/d+b*x)*cos(a-b*c
/d)/d^2-3/4*b*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^2-1/4*b*Si(b*c/d+b*x)*s
in(a-b*c/d)/d^2-1/4*sin(b*x+a)/d/(d*x+c)-1/4*sin(3*b*x+3*a)/d/(d*x+c)
```

## 3.76.2 Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.83

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = -\frac{-b\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right) - 3b\cos\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3b(c+dx)}{d}\right) + \frac{d\sin(a+bx)}{c+dx} + \frac{d\sin(3(a}{c+d}}{4d^2}$$

input
```
Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^2,x]
```

output
```
-1/4*(-(b*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)]) - 3*b*Cos[3*a - (3*b*
c)/d]*CosIntegral[(3*b*(c + d*x))/d] + (d*Sin[a + b*x])/(c + d*x) + (d*Sin
[3*(a + b*x)])/(c + d*x) + b*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + 3
*b*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/d^2
```

### 3.76.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\cos^2(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{\sin(a+bx)}{4(c+dx)^2}+\frac{\sin(3a+3bx)}{4(c+dx)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{\sin(a+bx)}{4d(c+dx)}-\frac{\sin(3a+3bx)}{4d(c+dx)}$$

input
```
Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^2,x]
```

output
```
(b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d^2) + (3*b*Cos[3*a - (
3*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d^2) - Sin[a + b*x]/(4*d*(c +
 d*x)) - Sin[3*a + 3*b*x]/(4*d*(c + d*x)) - (b*Sin[a - (b*c)/d]*SinIntegra
l[(b*c)/d + b*x])/(4*d^2) - (3*b*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/
d + 3*b*x])/(4*d^2)
```

### 3.76.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.76.4 Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.46

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{b^2\left(-\frac{3\sin(3xb+3a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{9\,\mathrm{Si}\left(-3xb-3a-\frac{3(-ad+cb)}{d}\right)\sin\left(\frac{-3ad+3cb}{d}\right)}{d}+\frac{9\,\mathrm{Ci}\left(3xb+3a+\frac{-3ad+3cb}{d}\right)\cos\left(\frac{-3ad+3cb}{d}\right)}{d}}{d}\right)}{12}+\frac{b^2 \cdots}{\cdots}}{b}$ |
| default | $\frac{\frac{b^2\left(-\frac{3\sin(3xb+3a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{9\,\mathrm{Si}\left(-3xb-3a-\frac{3(-ad+cb)}{d}\right)\sin\left(\frac{-3ad+3cb}{d}\right)}{d}+\frac{9\,\mathrm{Ci}\left(3xb+3a+\frac{-3ad+3cb}{d}\right)\cos\left(\frac{-3ad+3cb}{d}\right)}{d}}{d}\right)}{12}+\frac{b^2 \cdots}{\cdots}}{b}$ |
| risch | $-\frac{3b\,e^{-\frac{3i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{8d^2}-\frac{b\,e^{-\frac{i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(ibx+ia-\frac{i(ad-cb)}{d}\right)}{8d^2}-\frac{b\,e^{\frac{i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(-ibx-ia-\cdots\right.}{8d^2}$ |

input
```
int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/12*b^2*(-3*sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d+3*(-3*Si(-3*x*b-3*
a-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*x*b+3*a+3*(-a*d+b*c)/d)*cos
(3*(-a*d+b*c)/d)/d)/d)+1/4*b^2*(-sin(b*x+a)/(-a*d+c*b+d*(b*x+a))/d+(-Si(-x
*b-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(x*b+a+(-a*d+b*c)/d)*cos((-a*d+b*
c)/d)/d)/d))
```

### 3.76.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.08

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = -\frac{4\,d\cos\left(bx+a\right)^2\sin\left(bx+a\right) - 3\,(bdx+bc)\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right) - (bdx+bc)\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Ci}}{4\,(d^3x+cd^2)}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*d*cos(b*x + a)^2*sin(b*x + a) - 3*(b*d*x + b*c)*cos(-3*(b*c - a*d)
/d)*cos_integral(3*(b*d*x + b*c)/d) - (b*d*x + b*c)*cos(-(b*c - a*d)/d)*co
s_integral((b*d*x + b*c)/d) + 3*(b*d*x + b*c)*sin(-3*(b*c - a*d)/d)*sin_in
tegral(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*sin(-(b*c - a*d)/d)*sin_integral
((b*d*x + b*c)/d))/(d^3*x + c*d^2)
```

### 3.76.6 Sympy [F]

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin\left(a+bx\right)\cos^2\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x)**2, x)
```

### 3.76.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.80

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = -\frac{b^2\left(i\,E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right) - i\,E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right) + b^2\left(-i\,E_2\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right) +\right.}{}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/8*(b^2*(I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_
integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b
^2*(-I*exp_integral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_int
egral_e(2, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) +
 b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_
e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^2*(exp_i
ntegral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(2, -3*
(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b*c*d + (b*x
 + a)*d^2 - a*d^2)*b)
```

### 3.76.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.45 (sec) , antiderivative size = 66726, normalized size of antiderivative = 397.18

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/8*(3*b*d*x*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + b*
d*x*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan
(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + b*d*x*real_part
(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b*d*x*real_part(cos_integ
ral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*
a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(b*
x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d) + 2*b*d*x*imag_part(cos_integral(-b*x - b*c/d))*
tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d) - 4*b*d*x*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 6*b*
d*x*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2
*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 6*b*d*x*imag_
part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2
*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 12*b*d*x*sin_integral
(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^
2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*b*d*x*imag_part(cos_integral(b*x + b
*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*...
```

### 3.76.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{\left(c+d\,x\right)^2}\,dx$$

input
```
int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^2,x)
```

output
```
int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^2, x)
```

# 3.77 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$

3.77.1 Optimal result . . . 681
3.77.2 Mathematica [A] (verified) . . . 682
3.77.3 Rubi [A] (verified) . . . 682
3.77.4 Maple [A] (verified) . . . 683
3.77.5 Fricas [A] (verification not implemented) . . . 684
3.77.6 Sympy [F] . . . 684
3.77.7 Maxima [C] (verification not implemented) . . . 685
3.77.8 Giac [C] (verification not implemented) . . . 685
3.77.9 Mupad [F(-1)] . . . 686

## 3.77.1 Optimal result

Integrand size = 22, antiderivative size = 221

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = -\frac{b\cos(a+bx)}{8d^2(c+dx)} - \frac{3b\cos(3a+3bx)}{8d^2(c+dx)} - \frac{9b^2\,\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{8d^3} - \frac{b^2\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{8d^3} - \frac{\sin(a+bx)}{8d(c+dx)^2} - \frac{\sin(3a+3bx)}{8d(c+dx)^2} - \frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^3} - \frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^3}$$

output
```
-1/8*b*cos(b*x+a)/d^2/(d*x+c)-3/8*b*cos(3*b*x+3*a)/d^2/(d*x+c)-1/8*b^2*cos
(a-b*c/d)*Si(b*c/d+b*x)/d^3-9/8*b^2*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^3
-9/8*b^2*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^3-1/8*b^2*Ci(b*c/d+b*x)*sin(
a-b*c/d)/d^3-1/8*sin(b*x+a)/d/(d*x+c)^2-1/8*sin(3*b*x+3*a)/d/(d*x+c)^2
```

### 3.77.2 Mathematica [A] (verified)

Time = 2.69 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.82

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx =$$

$$-\frac{9b^2\,\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right)+b^2\,\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)\sin\left(a-\frac{bc}{d}\right)+\frac{d(b(c+dx)\cos(a+bx)+d\sin}{(c+dx)^2}}{8}$$

input
```
Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^3,x]
```

output
```
-1/8*(9*b^2*CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + b^2*CosI
ntegral[b*(c/d + x)]*Sin[a - (b*c)/d] + (d*(b*(c + d*x)*Cos[a + b*x] + d*S
in[a + b*x]))/(c + d*x)^2 + (d*(3*b*(c + d*x)*Cos[3*(a + b*x)] + d*Sin[3*(
a + b*x)]))/(c + d*x)^2 + b^2*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] +
9*b^2*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/d^3
```

### 3.77.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\cos^2(a+bx)}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{ 4906}$$

$$\int \left(\frac{\sin(a+bx)}{4(c+dx)^3}+\frac{\sin(3a+3bx)}{4(c+dx)^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3}-\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^3}-\frac{b\cos(a+bx)}{8d^2(c+dx)}-\frac{3b\cos(3a+3bx)}{8d^2(c+dx)}-\frac{\sin(a+bx)}{8d(c+dx)^2}-\frac{\sin(3a+3bx)}{8d(c+dx)^2}$$

input
```
Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^3,x]
```

output
```
-1/8*(b*Cos[a + b*x])/(d^2*(c + d*x)) - (3*b*Cos[3*a + 3*b*x])/(8*d^2*(c +
 d*x)) - (9*b^2*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(8*d^
3) - (b^2*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(8*d^3) - Sin[a + b
*x]/(8*d*(c + d*x)^2) - Sin[3*a + 3*b*x]/(8*d*(c + d*x)^2) - (b^2*Cos[a -
(b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^3) - (9*b^2*Cos[3*a - (3*b*c)/d]
*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^3)
```

### 3.77.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.77.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.44

| method | result |
|---|---|
| derivativedivides | $\frac{b^3\left(-\frac{3\sin(3xb+3a)}{2(-ad+cb+d(xb+a))^2 d}+\frac{-\frac{9\cos(3xb+3a)}{2(-ad+cb+d(xb+a))d}-\frac{9\left(-\frac{3\,\mathrm{Si}\left(-3xb-3a-\frac{3(-ad+cb)}{d}\right)\cos\left(\frac{-3ad+3cb}{d}\right)}{d}-\frac{3\,\mathrm{Ci}\left(3xb+3a+\frac{-3c\cdots}{\cdots}\right)\cdots}{\cdots}\right)}{2d}}{d}\cdots\right)}{12}$ |
| default | $\frac{b^3\left(-\frac{3\sin(3xb+3a)}{2(-ad+cb+d(xb+a))^2 d}+\frac{-\frac{9\cos(3xb+3a)}{2(-ad+cb+d(xb+a))d}-\frac{9\left(-\frac{3\,\mathrm{Si}\left(-3xb-3a-\frac{3(-ad+cb)}{d}\right)\cos\left(\frac{-3ad+3cb}{d}\right)}{d}-\frac{3\,\mathrm{Ci}\left(3xb+3a+\frac{-3c\cdots}{\cdots}\right)\cdots}{\cdots}\right)}{2d}}{d}\cdots\right)}{12}$ |
| risch | $\frac{9ib^2\mathrm{e}^{-\frac{3i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{16d^3}+\frac{ib^2\mathrm{e}^{-\frac{i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(ibx+ia-\frac{i(ad-cb)}{d}\right)}{16d^3}-\frac{ib^2\mathrm{e}^{\frac{i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(-ibx-ia\cdots\right)}{16d^3}\cdots$ |

input
```
int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/12*b^3*(-3/2*sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))^2/d+3/2*(-3*cos(3*
b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d-3*(-3*Si(-3*x*b-3*a-3*(-a*d+b*c)/d)*cos(3*
(-a*d+b*c)/d)/d-3*Ci(3*x*b+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)/d)/d
)+1/4*b^3*(-1/2*sin(b*x+a)/(-a*d+c*b+d*(b*x+a))^2/d+1/2*(-cos(b*x+a)/(-a*d
+c*b+d*(b*x+a))/d-(-Si(-x*b-a-(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(x*b+a+(
-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)/d))
```

### 3.77.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.38

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx =$$

$$-\frac{4\,d^2\cos\left(bx+a\right)^2\sin\left(bx+a\right)+12\left(bd^2x+bcd\right)\cos\left(bx+a\right)^3+\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\operatorname{Ci}\left(\frac{bdx+bc}{d}\right)\sin\cdots}{\cdots}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/8*(4*d^2*cos(b*x + a)^2*sin(b*x + a) + 12*(b*d^2*x + b*c*d)*cos(b*x + a
)^3 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral((b*d*x + b*c)/d)*
sin(-(b*c - a*d)/d) + 9*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral
(3*(b*d*x + b*c)/d)*sin(-3*(b*c - a*d)/d) + 9*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*cos(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) + (b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*
c)/d) - 8*(b*d^2*x + b*c*d)*cos(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

### 3.77.6 Sympy [F]

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin\left(a+bx\right)\cos^2\left(a+bx\right)}{\left(c+dx\right)^3}\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c)**3,x)
```

output
```
Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x)**3, x)
```

### 3.77.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.52

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx =$$

$$-\frac{b^3\left(i\,E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^3\left(-i\,E_3\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+\cdots\right.}{\cdots}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/8*(b^3*(I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_
integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b
^3*(-I*exp_integral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_int
egral_e(3, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) +
 b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_
e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^3*(exp_i
ntegral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(3, -3*
(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b^2*c^2*d -
2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b
)
```

### 3.77.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.48 (sec) , antiderivative size = 118262, normalized size of antiderivative = 535.12

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/16*(9*b^2*d^2*x^2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x)
^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
)^2 + b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(
1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - b
^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*
x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^2*d
^2*x^2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*
x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 18*b^2*
d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(
3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^2*x^2*si
n_integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan
(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^2*x^2*real_part(cos_
integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*
a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*b^2*d^2*x^2*real_part(cos_integra
l(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d) + 18*b^2*d^2*x^2*real_part(cos_integral(3*b
*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan
(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 18*b^2*d^2*x^2*real_part(cos_integral(-3*b*
x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(
3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*real_part(cos_integral(b*x ...
```

### 3.77.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{{\left(c+d\,x\right)}^3}\,dx$$

input
```
int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^3,x)
```

output
```
int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^3, x)
```

# 3.78 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$

3.78.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 687
3.78.2 Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 688
3.78.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 688
3.78.4 Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 690
3.78.5 Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 691
3.78.6 Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 691
3.78.7 Maxima [C] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 692
3.78.8 Giac [C] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . . 692
3.78.9 Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 693

## 3.78.1 Optimal result

Integrand size = 22, antiderivative size = 270

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = -\frac{b\cos(a+bx)}{24d^2(c+dx)^2}-\frac{b\cos(3a+3bx)}{8d^2(c+dx)^2} - \frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{24d^4} - \frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^4} - \frac{\sin(a+bx)}{12d(c+dx)^3}+\frac{b^2\sin(a+bx)}{24d^3(c+dx)}-\frac{\sin(3a+3bx)}{12d(c+dx)^3} + \frac{3b^2\sin(3a+3bx)}{8d^3(c+dx)}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{24d^4} + \frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^4}$$

output
```
-9/8*b^3*Ci(3*b*c/d+3*b*x)*cos(3*a-3*b*c/d)/d^4-1/24*b^3*Ci(b*c/d+b*x)*cos
(a-b*c/d)/d^4-1/24*b*cos(b*x+a)/d^2/(d*x+c)^2-1/8*b*cos(3*b*x+3*a)/d^2/(d*
x+c)^2+9/8*b^3*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^4+1/24*b^3*Si(b*c/d+b*
x)*sin(a-b*c/d)/d^4-1/12*sin(b*x+a)/d/(d*x+c)^3+1/24*b^2*sin(b*x+a)/d^3/(d
*x+c)-1/12*sin(3*b*x+3*a)/d/(d*x+c)^3+3/8*b^2*sin(3*b*x+3*a)/d^3/(d*x+c)
```

### 3.78.2 Mathematica [A] (verified)

Time = 1.86 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.11

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx =$$

$$-\frac{d\cos(bx)\left(bd(c+dx)\cos(a)-\left(-2d^2+b^2(c+dx)^2\right)\sin(a)\right)+d\cos(3bx)\left(3bd(c+dx)\cos(3a)-\left(-2d^2\right.\right.}{}$$

input
```
Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^4,x]
```

output
```
-1/24*(d*Cos[b*x]*(b*d*(c + d*x)*Cos[a] - (-2*d^2 + b^2*(c + d*x)^2)*Sin[a
]) + d*Cos[3*b*x]*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*x)^2)
*Sin[3*a]) - d*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*d*(c + d*x)*Sin[a])*
Sin[b*x] - d*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*x)*Sin[
3*a])*Sin[3*b*x] + b^3*(c + d*x)^3*(Cos[a - (b*c)/d]*CosIntegral[b*(c/d +
x)] - Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)]) + 27*b^3*(c + d*x)^3*(Cos
[3*a - (3*b*c)/d]*CosIntegral[(3*b*(c + d*x))/d] - Sin[3*a - (3*b*c)/d]*Si
nIntegral[(3*b*(c + d*x))/d]))/(d^4*(c + d*x)^3)
```

### 3.78.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\cos^2(a+bx)}{(c+dx)^4}\,dx$$

$$\Big\downarrow \text{ 4906}$$

$$\int \left(\frac{\sin(a+bx)}{4(c+dx)^4}+\frac{\sin(3a+3bx)}{4(c+dx)^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{24d^4}-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{24d^4}+\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{8d^4}+\frac{b^2\sin(a+bx)}{24d^3(c+dx)}+\frac{3b^2\sin(3a+3bx)}{8d^3(c+dx)}-\frac{b\cos(a+bx)}{24d^2(c+dx)^2}-\frac{b\cos(3a+3bx)}{8d^2(c+dx)^2}-\frac{\sin(a+bx)}{12d(c+dx)^3}-\frac{\sin(3a+3bx)}{12d(c+dx)^3}$$

input
```
Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^4,x]
```

output
```
-1/24*(b*Cos[a + b*x])/(d^2*(c + d*x)^2) - (b*Cos[3*a + 3*b*x])/(8*d^2*(c
+ d*x)^2) - (b^3*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(24*d^4) - (
9*b^3*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(8*d^4) - Sin[a
 + b*x]/(12*d*(c + d*x)^3) + (b^2*Sin[a + b*x])/(24*d^3*(c + d*x)) - Sin[3
*a + 3*b*x]/(12*d*(c + d*x)^3) + (3*b^2*Sin[3*a + 3*b*x])/(8*d^3*(c + d*x)
) + (b^3*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(24*d^4) + (9*b^3*Si
n[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^4)
```

### 3.78.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.78.4 Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.43

| method | result |
|---|---|
| derivativedivides | $\frac{b^4\left(-\frac{\sin(3xb+3a)}{(-ad+cb+d(xb+a))^3 d}+\frac{-\frac{3\cos(3xb+3a)}{2(-ad+cb+d(xb+a))^2 d}-\frac{3\left(-\frac{3\sin(3xb+3a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{9\,\mathrm{Si}\left(-3xb-3a-\frac{3(-ad+cb)}{d}\right)\sin\left(\frac{-3ad+3\cdots}{d}\right)}{d}\cdots}{2d}\right)}{d}\cdots\right)}{12}\cdots$ |
| default | $\frac{b^4\left(-\frac{\sin(3xb+3a)}{(-ad+cb+d(xb+a))^3 d}+\frac{-\frac{3\cos(3xb+3a)}{2(-ad+cb+d(xb+a))^2 d}-\frac{3\left(-\frac{3\sin(3xb+3a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{9\,\mathrm{Si}\left(-3xb-3a-\frac{3(-ad+cb)}{d}\right)\sin\left(\frac{-3ad+3\cdots}{d}\right)}{d}\cdots}{2d}\right)}{d}\cdots\right)}{12}\cdots$ |
| risch | $\frac{9b^3 \mathrm{e}^{-\frac{3i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{16d^4}+\frac{b^3 \mathrm{e}^{-\frac{i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(ibx+ia-\frac{i(ad-cb)}{d}\right)}{48d^4}+\frac{b^3 \mathrm{e}^{\frac{i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(-ibx-ia-\cdots\right)}{48d^4}\cdots$ |

input
```
int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/12*b^4*(-sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))^3/d+(-3/2*cos(3*b*x+3*
a)/(-a*d+c*b+d*(b*x+a))^2/d-3/2*(-3*sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d+
3*(-3*Si(-3*x*b-3*a-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*x*b+3*a+3
*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d)/d)/d)/d)+1/4*b^4*(-1/3*sin(b*x+a)/(-
a*d+c*b+d*(b*x+a))^3/d+1/3*(-1/2*cos(b*x+a)/(-a*d+c*b+d*(b*x+a))^2/d-1/2*(
-sin(b*x+a)/(-a*d+c*b+d*(b*x+a))/d+(-Si(-x*b-a-(-a*d+b*c)/d)*sin((-a*d+b*c
)/d)/d+Ci(x*b+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d)/d))
```

### 3.78.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.64

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx =$$

$$-\frac{12\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^3+27\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)\operatorname{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)}{}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/24*(12*(b*d^3*x + b*c*d^2)*cos(b*x + a)^3 + 27*(b^3*d^3*x^3 + 3*b^3*c*d
^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-3*(b*c - a*d)/d)*cos_integral(3*(b*
d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*
cos(-(b*c - a*d)/d)*cos_integral((b*d*x + b*c)/d) - 27*(b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-3*(b*c - a*d)/d)*sin_integral(
3*(b*d*x + b*c)/d) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*
c^3)*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) - 8*(b*d^3*x + b*c*
d^2)*cos(b*x + a) + 4*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - (9*b^
2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a)^2)*sin(b*x
+ a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

### 3.78.6 Sympy [F]

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sin\left(a+bx\right)\cos^2\left(a+bx\right)}{\left(c+dx\right)^4}\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c)**4,x)
```

output
```
Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x)**4, x)
```

### 3.78.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.43

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx =$$
$$-\frac{b^4\left(i\,E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^4\left(-i\,E_4\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+\right.}{8\,(b^3c^3d-3\,ab^2}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/8*(b^4*(I*exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_
integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b
^4*(-I*exp_integral_e(4, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_int
egral_e(4, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) +
 b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_
e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^4*(exp_i
ntegral_e(4, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(4, -3*
(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b^3*c^3*d -
3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 -
 a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b
)
```

### 3.78.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.75 (sec) , antiderivative size = 166374, normalized size of antiderivative = 616.20

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

output
```
-1/48*(27*b^3*d^3*x^3*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*x
)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 + b^3*d^3*x^3*real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan
(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 +
b^3*d^3*x^3*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b
*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 27*b^3
*d^3*x^3*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b^3
*d^3*x^3*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^
2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*b^3*d^3*x^
3*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(
3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*b^3*d^3*x^3*sin_
integral((b*d*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1
/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 54*b^3*d^3*x^3*imag_part(cos_int
egral(3*b*x + 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2
*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 54*b^3*d^3*x^3*imag_part(cos_integ
ral(-3*b*x - 3*b*c/d))*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*
a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 108*b^3*d^3*x^3*sin_integral(3*(b*d
*x + b*c)/d)*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(3
/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*b^3*d^3*x^3*imag_part(cos_integral(b*x +...
```

### 3.78.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{\left(c+d\,x\right)^4}\,dx$$

input
```
int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^4,x)
```

output
```
int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^4, x)
```

# 3.79 $\int (c+dx)^m \cos^2(a+bx)\sin^2(a+bx)\,dx$

3.79.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 694
3.79.2 Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 694
3.79.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 695
3.79.4 Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 696
3.79.5 Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . 696
3.79.6 Sympy [F(-2)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 697
3.79.7 Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 697
3.79.8 Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 697
3.79.9 Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 698

## 3.79.1 Optimal result

Integrand size = 24, antiderivative size = 162

$$\int (c+dx)^m \cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{(c+dx)^{1+m}}{8d(1+m)} + \frac{i2^{-2(3+m)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{4ib(c+dx)}{d}\right)}{b} - \frac{i2^{-2(3+m)}e^{-4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{4ib(c+dx)}{d}\right)}{b}$$

output

```
1/8*(d*x+c)^(1+m)/d/(1+m)+I*exp(4*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-4*I*b*
(d*x+c)/d)/(2^(6+2*m))/b/((-I*b*(d*x+c)/d)^m)-I*(d*x+c)^m*GAMMA(1+m,4*I*b*
(d*x+c)/d)/(2^(6+2*m))/b/exp(4*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

## 3.79.2 Mathematica [A] (verified)

Time = 3.78 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.96

$$\int (c+dx)^m \cos^2(a+bx)\sin^2(a+bx)\,dx = -\frac{(c+dx)^m\left(-8b(c+dx) - i4^{-m}de^{4i\left(a-\frac{bc}{d}\right)}(1+m)\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{4ib(c+dx)}{d}\right) + i4^{-m}de^{-4i(a-}\right.}{64bd(1+m)}$$

input
```
Integrate[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
-1/64*((c + d*x)^m*(-8*b*(c + d*x) - (I*d*E^((4*I)*(a - (b*c)/d))*(1 + m)*
Gamma[1 + m, ((-4*I)*b*(c + d*x))/d])/(4^m*(((-I)*b*(c + d*x))/d)^m) + (I*
d*(1 + m)*Gamma[1 + m, ((4*I)*b*(c + d*x))/d])/(4^m*E^((4*I)*(a - (b*c)/d)
)*((I*b*(c + d*x))/d)^m)))/(b*d*(1 + m))
```

### 3.79.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cos^2(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}(c+dx)^m-\frac{1}{8}\cos(4a+4bx)(c+dx)^m\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{i2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{4ib(c+dx)}{d}\right)}{b}-\frac{i2^{-2(m+3)}e^{-4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{4ib(c+dx)}{d}\right)}{b}+\frac{(c+dx)^{m+1}}{8d(m+1)}$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^(1 + m)/(8*d*(1 + m)) + (I*E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*G
amma[1 + m, ((-4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*(((-I)*b*(c + d*x))/
d)^m) - (I*(c + d*x)^m*Gamma[1 + m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))
*b*E^((4*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

#### 3.79.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.79.4 Maple [F]

$$\int (dx+c)^m \cos(xb+a)^2 \sin(xb+a)^2\, dx$$

input
```
int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

### 3.79.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84

$$\int (c+dx)^m \cos^2(a+bx)\sin^2(a+bx)\, dx$$

$$= \frac{(i\,dm+i\,d)e^{\left(-\frac{dm\log\left(-\frac{4i\,b}{d}\right)+4i\,bc-4i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{4\,(i\,bdx+i\,bc)}{d}\right)+(-i\,dm-i\,d)e^{\left(-\frac{dm\log\left(\frac{4i\,b}{d}\right)-4i\,bc+4i\,ad}{d}\right)}\Gamma\Big(m+}{64\,(bdm+bd)}$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/64*((I*d*m + I*d)*e^(-(d*m*log(-4*I*b/d) + 4*I*b*c - 4*I*a*d)/d)*gamma(m
 + 1, -4*(I*b*d*x + I*b*c)/d) + (-I*d*m - I*d)*e^(-(d*m*log(4*I*b/d) - 4*I
*b*c + 4*I*a*d)/d)*gamma(m + 1, -4*(-I*b*d*x - I*b*c)/d) + 8*(b*d*x + b*c)
*(d*x + c)^m)/(b*d*m + b*d)
```

### 3.79.6 Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m \cos^2(a+bx)\sin^2(a+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d*x+c)**m*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.79.7 Maxima [F]

$$\int (c+dx)^m \cos^2(a+bx)\sin^2(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)^2 \sin(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/8*((d*m + d)*integrate((d*x + c)^m*cos(4*b*x + 4*a), x) - e^(m*log(d*x
+ c) + log(d*x + c)))/(d*m + d)
```

### 3.79.8 Giac [F]

$$\int (c+dx)^m \cos^2(a+bx)\sin^2(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)^2 \sin(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a)^2, x)
```

### 3.79.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cos^2(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)^2 \sin(a+b\,x)^2 (c+d\,x)^m\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^m, x)
```

# 3.80 $\int (c+dx)^4 \cos^2(a+bx) \sin^2(a+bx) \, dx$

3.80.1 Optimal result . . . . . 699
3.80.2 Mathematica [A] (verified) . . . . . 699
3.80.3 Rubi [A] (verified) . . . . . 700
3.80.4 Maple [A] (verified) . . . . . 701
3.80.5 Fricas [B] (verification not implemented) . . . . . 701
3.80.6 Sympy [B] (verification not implemented) . . . . . 702
3.80.7 Maxima [B] (verification not implemented) . . . . . 703
3.80.8 Giac [A] (verification not implemented) . . . . . 704
3.80.9 Mupad [B] (verification not implemented) . . . . . 705

## 3.80.1 Optimal result

Integrand size = 24, antiderivative size = 131

$$\int (c+dx)^4 \cos^2(a+bx) \sin^2(a+bx) \, dx = \frac{(c+dx)^5}{40d} + \frac{3d^3(c+dx)\cos(4a+4bx)}{256b^4} - \frac{d(c+dx)^3\cos(4a+4bx)}{32b^2} - \frac{3d^4\sin(4a+4bx)}{1024b^5} + \frac{3d^2(c+dx)^2\sin(4a+4bx)}{128b^3} - \frac{(c+dx)^4\sin(4a+4bx)}{32b}$$

output
```
1/40*(d*x+c)^5/d+3/256*d^3*(d*x+c)*cos(4*b*x+4*a)/b^4-1/32*d*(d*x+c)^3*cos
(4*b*x+4*a)/b^2-3/1024*d^4*sin(4*b*x+4*a)/b^5+3/128*d^2*(d*x+c)^2*sin(4*b*
x+4*a)/b^3-1/32*(d*x+c)^4*sin(4*b*x+4*a)/b
```

## 3.80.2 Mathematica [A] (verified)

Time = 1.25 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.01

$$\int (c+dx)^4 \cos^2(a+bx) \sin^2(a+bx) \, dx = \frac{128b^5x(5c^4+10c^3dx+10c^2d^2x^2+5cd^3x^3+d^4x^4)+20bd(c+dx)\left(3d^2-8b^2(c+dx)^2\right)\cos(4(a+bx)) -}{5120b^5}$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(128*b^5*x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4) +
 20*b*d*(c + d*x)*(3*d^2 - 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] - 5*(3*d^4
- 24*b^2*d^2*(c + d*x)^2 + 32*b^4*(c + d*x)^4)*Sin[4*(a + b*x)])/(5120*b^5
)
```

### 3.80.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4\sin^2(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}(c+dx)^4-\frac{1}{8}(c+dx)^4\cos(4a+4bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3d^4\sin(4a+4bx)}{1024b^5}+\frac{3d^3(c+dx)\cos(4a+4bx)}{256b^4}+\frac{3d^2(c+dx)^2\sin(4a+4bx)}{128b^3}-\frac{d(c+dx)^3\cos(4a+4bx)}{32b^2}-\frac{(c+dx)^4\sin(4a+4bx)}{32b}+\frac{(c+dx)^5}{40d}$$

input
```
Int[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^5/(40*d) + (3*d^3*(c + d*x)*Cos[4*a + 4*b*x])/(256*b^4) - (d*(c
+ d*x)^3*Cos[4*a + 4*b*x])/(32*b^2) - (3*d^4*Sin[4*a + 4*b*x])/(1024*b^5)
+ (3*d^2*(c + d*x)^2*Sin[4*a + 4*b*x])/(128*b^3) - ((c + d*x)^4*Sin[4*a +
4*b*x])/(32*b)
```

#### 3.80.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.80.4 Maple [A] (verified)

Time = 2.63 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

| method | result |
|---|---|
| parallelrisch | $\frac{\left(-32b^4(dx+c)^4+24d^2(dx+c)^2b^2-3d^4\right)\sin(4xb+4a)+128b\left(-\frac{\left((dx+c)^2b^2-\frac{3d^2}{8}\right)d(dx+c)\cos(4xb+4a)}{4}+x\left(\frac{1}{5}d^4x^4+c\,d^3x^3\right.\right.}{1024b^5}$ |
| risch | $\frac{d^4x^5}{40}+\frac{d^3c\,x^4}{8}+\frac{d^2c^2x^3}{4}+\frac{d\,c^3x^2}{4}+\frac{c^4x}{8}+\frac{c^5}{40d}-\frac{d(8b^2d^3x^3+24b^2c\,d^2x^2+24b^2c^2dx+8b^2c^3-3d^3x-3c\,d^2)\cos}{256b^4}$ |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |

input
```
int((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/1024*((-32*b^4*(d*x+c)^4+24*d^2*(d*x+c)^2*b^2-3*d^4)*sin(4*b*x+4*a)+128*
b*(-1/4*((d*x+c)^2*b^2-3/8*d^2)*d*(d*x+c)*cos(4*b*x+4*a)+x*(1/5*d^4*x^4+c*
d^3*x^3+2*c^2*d^2*x^2+2*c^3*d*x+c^4)*b^4+1/4*b^2*c^3*d-3/32*d^3*c))/b^5
```

### 3.80.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. 2(119) = 238.

Time = 0.26 (sec) , antiderivative size = 466, normalized size of antiderivative = 3.56

$$\int (c+dx)^4\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$=\frac{32\,b^5d^4x^5+160\,b^5cd^3x^4-40\,(8\,b^3d^4x^3+24\,b^3cd^3x^2+8\,b^3c^3d-3\,bcd^3+3\,(8\,b^3c^2d^2-bd^4)x)\cos\,(bx+a)^4}{}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/1280*(32*b^5*d^4*x^5 + 160*b^5*c*d^3*x^4 - 40*(8*b^3*d^4*x^3 + 24*b^3*c*
d^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(8*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x +
 a)^4 + 40*(8*b^5*c^2*d^2 - b^3*d^4)*x^3 + 40*(8*b^5*c^3*d - 3*b^3*c*d^3)*
x^2 + 40*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(
8*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^2 + 5*(32*b^5*c^4 - 24*b^3*c^2*d^2
+ 3*b*d^4)*x - 5*(2*(32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 32*b^4*c^4 - 24*
b^2*c^2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*x^2 + 16*(8*b^4*c^3*d -
 3*b^2*c*d^3)*x)*cos(b*x + a)^3 - (32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 32
*b^4*c^4 - 24*b^2*c^2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*x^2 + 16*
(8*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a))*sin(b*x + a))/b^5
```

### 3.80.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1231 vs. $2(126) = 252$.

Time = 0.91 (sec) , antiderivative size = 1231, normalized size of antiderivative = 9.40

$$\int (c+dx)^4 \cos^2(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**4*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

output
```
Piecewise((c**4*x*sin(a + b*x)**4/8 + c**4*x*sin(a + b*x)**2*cos(a + b*x)*
*2/4 + c**4*x*cos(a + b*x)**4/8 + c**3*d*x**2*sin(a + b*x)**4/4 + c**3*d*x
**2*sin(a + b*x)**2*cos(a + b*x)**2/2 + c**3*d*x**2*cos(a + b*x)**4/4 + c*
*2*d**2*x**3*sin(a + b*x)**4/4 + c**2*d**2*x**3*sin(a + b*x)**2*cos(a + b*
x)**2/2 + c**2*d**2*x**3*cos(a + b*x)**4/4 + c*d**3*x**4*sin(a + b*x)**4/8
 + c*d**3*x**4*sin(a + b*x)**2*cos(a + b*x)**2/4 + c*d**3*x**4*cos(a + b*x
)**4/8 + d**4*x**5*sin(a + b*x)**4/40 + d**4*x**5*sin(a + b*x)**2*cos(a +
b*x)**2/20 + d**4*x**5*cos(a + b*x)**4/40 + c**4*sin(a + b*x)**3*cos(a + b
*x)/(8*b) - c**4*sin(a + b*x)*cos(a + b*x)**3/(8*b) + c**3*d*x*sin(a + b*x
)**3*cos(a + b*x)/(2*b) - c**3*d*x*sin(a + b*x)*cos(a + b*x)**3/(2*b) + 3*
c**2*d**2*x**2*sin(a + b*x)**3*cos(a + b*x)/(4*b) - 3*c**2*d**2*x**2*sin(a
 + b*x)*cos(a + b*x)**3/(4*b) + c*d**3*x**3*sin(a + b*x)**3*cos(a + b*x)/(
2*b) - c*d**3*x**3*sin(a + b*x)*cos(a + b*x)**3/(2*b) + d**4*x**4*sin(a +
b*x)**3*cos(a + b*x)/(8*b) - d**4*x**4*sin(a + b*x)*cos(a + b*x)**3/(8*b)
- c**3*d*sin(a + b*x)**4/(8*b**2) - c**3*d*cos(a + b*x)**4/(8*b**2) - 3*c*
*2*d**2*x*sin(a + b*x)**4/(32*b**2) + 9*c**2*d**2*x*sin(a + b*x)**2*cos(a
+ b*x)**2/(16*b**2) - 3*c**2*d**2*x*cos(a + b*x)**4/(32*b**2) - 3*c*d**3*x
**2*sin(a + b*x)**4/(32*b**2) + 9*c*d**3*x**2*sin(a + b*x)**2*cos(a + b*x)
**2/(16*b**2) - 3*c*d**3*x**2*cos(a + b*x)**4/(32*b**2) - d**4*x**3*sin(a
+ b*x)**4/(32*b**2) + 3*d**4*x**3*sin(a + b*x)**2*cos(a + b*x)**2/(16*b...
```

### 3.80.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 735 vs. $2(119) = 238$.

Time = 0.27 (sec) , antiderivative size = 735, normalized size of antiderivative = 5.61

$$\int (c+dx)^4\cos^2(a+bx)\sin^2(a+bx)\,dx$$

$$= \frac{160\,(4\,bx+4\,a-\sin(4\,bx+4\,a))c^4 - \frac{640\,(4\,bx+4\,a-\sin(4\,bx+4\,a))ac^3d}{b} + \frac{960\,(4\,bx+4\,a-\sin(4\,bx+4\,a))a^2c^2d^2}{b^2} - \frac{640\,(4\,bx+4}{}}{}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/5120*(160*(4*b*x + 4*a - sin(4*b*x + 4*a))*c^4 - 640*(4*b*x + 4*a - sin(
4*b*x + 4*a))*a*c^3*d/b + 960*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^2*c^2*d^2
/b^2 - 640*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^3*c*d^3/b^3 + 160*(4*b*x + 4
*a - sin(4*b*x + 4*a))*a^4*d^4/b^4 + 160*(8*(b*x + a)^2 - 4*(b*x + a)*sin(
4*b*x + 4*a) - cos(4*b*x + 4*a))*c^3*d/b - 480*(8*(b*x + a)^2 - 4*(b*x + a
)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a*c^2*d^2/b^2 + 480*(8*(b*x + a)^2
- 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a^2*c*d^3/b^3 - 160*(8*
(b*x + a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a^3*d^4/b^4
 + 40*(32*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 -
 1)*sin(4*b*x + 4*a))*c^2*d^2/b^2 - 80*(32*(b*x + a)^3 - 12*(b*x + a)*cos(
4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a))*a*c*d^3/b^3 + 40*(3
2*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*sin(
4*b*x + 4*a))*a^2*d^4/b^4 + 20*(32*(b*x + a)^4 - 3*(8*(b*x + a)^2 - 1)*cos
(4*b*x + 4*a) - 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*sin(4*b*x + 4*a))*c*d^3/b^
3 - 20*(32*(b*x + a)^4 - 3*(8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 4*(8*(b*
x + a)^3 - 3*b*x - 3*a)*sin(4*b*x + 4*a))*a*d^4/b^4 + (128*(b*x + a)^5 - 2
0*(8*(b*x + a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) - 5*(32*(b*x + a)^4 - 24*
(b*x + a)^2 + 3)*sin(4*b*x + 4*a))*d^4/b^4)/b
```

### 3.80.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.71

$$\int (c+dx)^4\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{1}{40}\,d^4x^5+\frac{1}{8}\,cd^3x^4+\frac{1}{4}\,c^2d^2x^3+\frac{1}{4}\,c^3dx^2+\frac{1}{8}\,c^4x - \frac{(8\,b^3d^4x^3+24\,b^3cd^3x^2+24\,b^3c^2d^2x+8\,b^3c^3d-3\,bd^4x-3\,bcd^3)\cos(4\,bx+4\,a)}{256\,b^5} - \frac{(32\,b^4d^4x^4+128\,b^4cd^3x^3+192\,b^4c^2d^2x^2+128\,b^4c^3dx+32\,b^4c^4-24\,b^2d^4x^2-48\,b^2cd^3x-24\,b^2c^2d^2+3}{1024\,b^5}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/40*d^4*x^5 + 1/8*c*d^3*x^4 + 1/4*c^2*d^2*x^3 + 1/4*c^3*d*x^2 + 1/8*c^4*x
 - 1/256*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 24*b^3*c^2*d^2*x + 8*b^3*c^3*
d - 3*b*d^4*x - 3*b*c*d^3)*cos(4*b*x + 4*a)/b^5 - 1/1024*(32*b^4*d^4*x^4 +
 128*b^4*c*d^3*x^3 + 192*b^4*c^2*d^2*x^2 + 128*b^4*c^3*d*x + 32*b^4*c^4 -
24*b^2*d^4*x^2 - 48*b^2*c*d^3*x - 24*b^2*c^2*d^2 + 3*d^4)*sin(4*b*x + 4*a)
/b^5
```

### 3.80.9 Mupad [B] (verification not implemented)

Time = 23.66 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.66

$$\int (c+dx)^4\cos^2(a+bx)\sin^2(a+bx)\,dx =$$
$$-\frac{15\,d^4\,\sin(4\,a+4\,b\,x)-640\,b^5\,c^4\,x+160\,b^4\,c^4\,\sin(4\,a+4\,b\,x)-128\,b^5\,d^4\,x^5+160\,b^3\,c^3\,d\,\cos(4\,a+4\,b\,x)}{}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^4,x)
```

output
```
-(15*d^4*sin(4*a + 4*b*x) - 640*b^5*c^4*x + 160*b^4*c^4*sin(4*a + 4*b*x) -
 128*b^5*d^4*x^5 + 160*b^3*c^3*d*cos(4*a + 4*b*x) - 1280*b^5*c^3*d*x^2 - 6
40*b^5*c*d^3*x^4 - 120*b^2*c^2*d^2*sin(4*a + 4*b*x) + 160*b^3*d^4*x^3*cos(
4*a + 4*b*x) - 1280*b^5*c^2*d^2*x^3 - 120*b^2*d^4*x^2*sin(4*a + 4*b*x) + 1
60*b^4*d^4*x^4*sin(4*a + 4*b*x) - 60*b*c*d^3*cos(4*a + 4*b*x) - 60*b*d^4*x
*cos(4*a + 4*b*x) + 960*b^4*c^2*d^2*x^2*sin(4*a + 4*b*x) - 240*b^2*c*d^3*x
*sin(4*a + 4*b*x) + 640*b^4*c^3*d*x*sin(4*a + 4*b*x) + 480*b^3*c^2*d^2*x*c
os(4*a + 4*b*x) + 480*b^3*c*d^3*x^2*cos(4*a + 4*b*x) + 640*b^4*c*d^3*x^3*s
in(4*a + 4*b*x))/(5120*b^5)
```

# 3.81 $\int (c+dx)^3 \cos^2(a+bx) \sin^2(a+bx) \, dx$


3.81.1 Optimal result . . . 706
3.81.2 Mathematica [A] (verified) . . . 706
3.81.3 Rubi [A] (verified) . . . 707
3.81.4 Maple [A] (verified) . . . 708
3.81.5 Fricas [B] (verification not implemented) . . . 708
3.81.6 Sympy [B] (verification not implemented) . . . 709
3.81.7 Maxima [B] (verification not implemented) . . . 710
3.81.8 Giac [A] (verification not implemented) . . . 711
3.81.9 Mupad [B] (verification not implemented) . . . 712


## 3.81.1 Optimal result

Integrand size = 24, antiderivative size = 105

$$\int (c+dx)^3 \cos^2(a+bx) \sin^2(a+bx) \, dx = \frac{(c+dx)^4}{32d} + \frac{3d^3 \cos(4a+4bx)}{1024b^4} - \frac{3d(c+dx)^2 \cos(4a+4bx)}{128b^2} + \frac{3d^2(c+dx) \sin(4a+4bx)}{256b^3} - \frac{(c+dx)^3 \sin(4a+4bx)}{32b}$$

output
```
1/32*(d*x+c)^4/d+3/1024*d^3*cos(4*b*x+4*a)/b^4-3/128*d*(d*x+c)^2*cos(4*b*x
+4*a)/b^2+3/256*d^2*(d*x+c)*sin(4*b*x+4*a)/b^3-1/32*(d*x+c)^3*sin(4*b*x+4*
a)/b
```

## 3.81.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.01

$$\int (c+dx)^3 \cos^2(a+bx) \sin^2(a+bx) \, dx = \frac{32b^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3) - 3d(-d^2+8b^2(c+dx)^2) \cos(4(a+bx)) - 4b(c+dx)\left(-3d^2+8b^2(c\right.}{1024b^4}$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(32*b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 3*d*(-d^2 + 8*b^2*
(c + d*x)^2)*Cos[4*(a + b*x)] - 4*b*(c + d*x)*(-3*d^2 + 8*b^2*(c + d*x)^2)
*Sin[4*(a + b*x)])/(1024*b^4)
```

### 3.81.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin^2(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}(c+dx)^3-\frac{1}{8}(c+dx)^3\cos(4a+4bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3d^3\cos(4a+4bx)}{1024b^4}+\frac{3d^2(c+dx)\sin(4a+4bx)}{256b^3}-\frac{3d(c+dx)^2\cos(4a+4bx)}{128b^2}-\frac{(c+dx)^3\sin(4a+4bx)}{32b}+\frac{(c+dx)^4}{32d}$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^4/(32*d) + (3*d^3*Cos[4*a + 4*b*x])/(1024*b^4) - (3*d*(c + d*x)^
2*Cos[4*a + 4*b*x])/(128*b^2) + (3*d^2*(c + d*x)*Sin[4*a + 4*b*x])/(256*b^
3) - ((c + d*x)^3*Sin[4*a + 4*b*x])/(32*b)
```

### 3.81.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.81.4 Maple [A] (verified)

Time = 1.81 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.15

| method | result |
|---|---|
| parallelrisch | $\frac{-32\left(\left((dx+c)^2b^2-\frac{3d^2}{8}\right)b(dx+c)\sin(4xb+4a)-24\left((dx+c)^2b^2-\frac{d^2}{8}\right)d\cos(4xb+4a)+32(d^3x^4+4d^2cx^3+6dc^2x^2+4c^3x)b^4}{1024b^4}$ |
| risch | $\frac{d^3x^4}{32}+\frac{d^2cx^3}{8}+\frac{3dc^2x^2}{16}+\frac{c^3x}{8}+\frac{c^4}{32d}-\frac{3d(8x^2d^2b^2+16b^2cdx+8b^2c^2-d^2)\cos(4xb+4a)}{1024b^4}-\frac{(8b^2d^3x^3+24b^2c}{}$ |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |

input
```
int((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/1024*(-32*((d*x+c)^2*b^2-3/8*d^2)*b*(d*x+c)*sin(4*b*x+4*a)-24*((d*x+c)^2
*b^2-1/8*d^2)*d*cos(4*b*x+4*a)+32*(d^3*x^4+4*c*d^2*x^3+6*c^2*d*x^2+4*c^3*x
)*b^4+24*b^2*c^2*d-3*d^3)/b^4
```

## 3.81.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(95) = 190.

Time = 0.25 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.93

$$\int (c+dx)^3\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$=\frac{4\,b^4d^3x^4+16\,b^4cd^2x^3-3\,(8\,b^2d^3x^2+16\,b^2cd^2x+8\,b^2c^2d-d^3)\cos{(bx+a)}^4+3\,(8\,b^4c^2d-b^2d^3)x^2+3\,(8}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/128*(4*b^4*d^3*x^4 + 16*b^4*c*d^2*x^3 - 3*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*
x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^4 + 3*(8*b^4*c^2*d - b^2*d^3)*x^2 + 3*
(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^2 + 2*(8
*b^4*c^3 - 3*b^2*c*d^2)*x - 2*(2*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 8*b^3
*c^3 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^3 - (8*b^3*d^3*
x^3 + 24*b^3*c*d^2*x^2 + 8*b^3*c^3 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x
)*cos(b*x + a))*sin(b*x + a))/b^4
```

## 3.81.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 835 vs. $2(100) = 200$.

Time = 0.63 (sec) , antiderivative size = 835, normalized size of antiderivative = 7.95

$$\int (c+dx)^3 \cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= \begin{cases} \frac{c^3 x \sin^4(a+bx)}{8} + \frac{c^3 x \sin^2(a+bx)\cos^2(a+bx)}{4} + \frac{c^3 x \cos^4(a+bx)}{8} + \frac{3c^2 dx^2 \sin^4(a+bx)}{16} + \frac{3c^2 dx^2 \sin^2(a+bx)\cos^2(a+bx)}{8} + \frac{3c^2 dx^2 c \\ \left(c^3 x + \frac{3c^2 dx^2}{2} + cd^2 x^3 + \frac{d^3 x^4}{4}\right)\sin^2(a)\cos^2(a) \end{cases}$$

input
```
integrate((d*x+c)**3*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

output
```
Piecewise((c**3*x*sin(a + b*x)**4/8 + c**3*x*sin(a + b*x)**2*cos(a + b*x)*
*2/4 + c**3*x*cos(a + b*x)**4/8 + 3*c**2*d*x**2*sin(a + b*x)**4/16 + 3*c**
2*d*x**2*sin(a + b*x)**2*cos(a + b*x)**2/8 + 3*c**2*d*x**2*cos(a + b*x)**4
/16 + c*d**2*x**3*sin(a + b*x)**4/8 + c*d**2*x**3*sin(a + b*x)**2*cos(a +
b*x)**2/4 + c*d**2*x**3*cos(a + b*x)**4/8 + d**3*x**4*sin(a + b*x)**4/32 +
 d**3*x**4*sin(a + b*x)**2*cos(a + b*x)**2/16 + d**3*x**4*cos(a + b*x)**4/
32 + c**3*sin(a + b*x)**3*cos(a + b*x)/(8*b) - c**3*sin(a + b*x)*cos(a + b
*x)**3/(8*b) + 3*c**2*d*x*sin(a + b*x)**3*cos(a + b*x)/(8*b) - 3*c**2*d*x*
sin(a + b*x)*cos(a + b*x)**3/(8*b) + 3*c*d**2*x**2*sin(a + b*x)**3*cos(a +
 b*x)/(8*b) - 3*c*d**2*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b) + d**3*x**3
*sin(a + b*x)**3*cos(a + b*x)/(8*b) - d**3*x**3*sin(a + b*x)*cos(a + b*x)*
*3/(8*b) - 3*c**2*d*sin(a + b*x)**4/(32*b**2) - 3*c**2*d*cos(a + b*x)**4/(
32*b**2) - 3*c*d**2*x*sin(a + b*x)**4/(64*b**2) + 9*c*d**2*x*sin(a + b*x)*
*2*cos(a + b*x)**2/(32*b**2) - 3*c*d**2*x*cos(a + b*x)**4/(64*b**2) - 3*d*
*3*x**2*sin(a + b*x)**4/(128*b**2) + 9*d**3*x**2*sin(a + b*x)**2*cos(a + b
*x)**2/(64*b**2) - 3*d**3*x**2*cos(a + b*x)**4/(128*b**2) - 3*c*d**2*sin(a
 + b*x)**3*cos(a + b*x)/(64*b**3) + 3*c*d**2*sin(a + b*x)*cos(a + b*x)**3/
(64*b**3) - 3*d**3*x*sin(a + b*x)**3*cos(a + b*x)/(64*b**3) + 3*d**3*x*sin
(a + b*x)*cos(a + b*x)**3/(64*b**3) + 3*d**3*sin(a + b*x)**4/(256*b**4) +
3*d**3*cos(a + b*x)**4/(256*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/...
```

### 3.81.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(95) = 190$.

Time = 0.25 (sec) , antiderivative size = 442, normalized size of antiderivative = 4.21

$$\int (c+dx)^3 \cos^2(a+bx) \sin^2(a+bx)\,dx$$

$$= \frac{32\,(4\,bx+4\,a-\sin\,(4\,bx+4\,a))c^3 - \frac{96\,(4\,bx+4\,a-\sin(4\,bx+4\,a))ac^2d}{b} + \frac{96\,(4\,bx+4\,a-\sin(4\,bx+4\,a))a^2cd^2}{b^2} - \frac{32\,(4\,bx+4\,a-\sin}{b^3}}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/1024*(32*(4*b*x + 4*a - sin(4*b*x + 4*a))*c^3 - 96*(4*b*x + 4*a - sin(4*
b*x + 4*a))*a*c^2*d/b + 96*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^2*c*d^2/b^2
- 32*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^3*d^3/b^3 + 24*(8*(b*x + a)^2 - 4*
(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*c^2*d/b - 48*(8*(b*x + a)^2
 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a*c*d^2/b^2 + 24*(8*(b
*x + a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a^2*d^3/b^3 +
 4*(32*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)
*sin(4*b*x + 4*a))*c*d^2/b^2 - 4*(32*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x
+ 4*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a))*a*d^3/b^3 + (32*(b*x + a)
^4 - 3*(8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 4*(8*(b*x + a)^3 - 3*b*x - 3
*a)*sin(4*b*x + 4*a))*d^3/b^3)/b
```

### 3.81.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.46

$$\begin{aligned}\int(c+dx)^3\cos^2(a+bx)\sin^2(a+bx)\,dx &= \frac{1}{32}\,d^3x^4+\frac{1}{8}\,cd^2x^3+\frac{3}{16}\,c^2dx^2+\frac{1}{8}\,c^3x \\ &\quad-\frac{3\,(8\,b^2d^3x^2+16\,b^2cd^2x+8\,b^2c^2d-d^3)\cos(4\,bx+4\,a)}{1024\,b^4} \\ &\quad-\frac{(8\,b^3d^3x^3+24\,b^3cd^2x^2+24\,b^3c^2dx+8\,b^3c^3-3\,bd^3x-3\,bcd^2)\sin(4\,bx+4\,a)}{256\,b^4}\end{aligned}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/32*d^3*x^4 + 1/8*c*d^2*x^3 + 3/16*c^2*d*x^2 + 1/8*c^3*x - 3/1024*(8*b^2*
d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(4*b*x + 4*a)/b^4 - 1/256
*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 - 3*b*d^3*
x - 3*b*c*d^2)*sin(4*b*x + 4*a)/b^4
```

### 3.81.9 Mupad [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 329, normalized size of antiderivative = 3.13

$$\int (c+dx)^3\cos^2(a+bx)\sin^2(a+bx)\,dx = x^2\left(\frac{3c^2d}{64}+\frac{9d^3}{512b^2}\right)+x^2\left(\frac{9c^2d}{64}-\frac{9d^3}{512b^2}\right)+x\left(\frac{c^3}{32}+\frac{9cd^2}{256b^2}\right)+x\left(\frac{3c^3}{32}-\frac{9cd^2}{256b^2}\right)+\frac{d^3x^4}{32}-\frac{x\cos(4a+4bx)\left(\frac{c^3}{4}+\frac{9cd^2}{32b^2}\right)}{8}+\frac{x\cos(4a+4bx)\left(\frac{c^3}{8}-\frac{3cd^2}{64b^2}\right)}{4}+\frac{cd^2x^3}{8}+\frac{\cos(4a+4bx)\left(\frac{3d^3}{128}-\frac{3b^2c^2d}{16}\right)}{8b^4}+\frac{\sin(4a+4bx)\,(3cd^2-8b^2c^3)}{256b^3}-\frac{x^2\cos(4a+4bx)\left(\frac{3c^2d}{8}+\frac{9d^3}{64b^2}\right)}{8}+\frac{x^2\cos(4a+4bx)\left(\frac{3c^2d}{16}-\frac{3d^3}{128b^2}\right)}{4}-\frac{d^3x^3\sin(4a+4bx)}{32b}+\frac{3x\sin(4a+4bx)\,(d^3-8b^2c^2d)}{256b^3}-\frac{3cd^2x^2\sin(4a+4bx)}{32b}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^3,x)
```

output
```
x^2*((3*c^2*d)/64 + (9*d^3)/(512*b^2)) + x^2*((9*c^2*d)/64 - (9*d^3)/(512*
b^2)) + x*(c^3/32 + (9*c*d^2)/(256*b^2)) + x*((3*c^3)/32 - (9*c*d^2)/(256*
b^2)) + (d^3*x^4)/32 - (x*cos(4*a + 4*b*x)*(c^3/4 + (9*c*d^2)/(32*b^2)))/8
 + (x*cos(4*a + 4*b*x)*(c^3/8 - (3*c*d^2)/(64*b^2)))/4 + (c*d^2*x^3)/8 + (
cos(4*a + 4*b*x)*((3*d^3)/128 - (3*b^2*c^2*d)/16))/(8*b^4) + (sin(4*a + 4*
b*x)*(3*c*d^2 - 8*b^2*c^3))/(256*b^3) - (x^2*cos(4*a + 4*b*x)*((3*c^2*d)/8
 + (9*d^3)/(64*b^2)))/8 + (x^2*cos(4*a + 4*b*x)*((3*c^2*d)/16 - (3*d^3)/(1
28*b^2)))/4 - (d^3*x^3*sin(4*a + 4*b*x))/(32*b) + (3*x*sin(4*a + 4*b*x)*(d
^3 - 8*b^2*c^2*d))/(256*b^3) - (3*c*d^2*x^2*sin(4*a + 4*b*x))/(32*b)
```

# 3.82 $\int (c+dx)^2 \cos^2(a+bx)\sin^2(a+bx)\,dx$

3.82.1 Optimal result . . . . . 713
3.82.2 Mathematica [A] (verified) . . . . . 713
3.82.3 Rubi [A] (verified) . . . . . 714
3.82.4 Maple [A] (verified) . . . . . 715
3.82.5 Fricas [B] (verification not implemented) . . . . . 715
3.82.6 Sympy [B] (verification not implemented) . . . . . 716
3.82.7 Maxima [B] (verification not implemented) . . . . . 717
3.82.8 Giac [A] (verification not implemented) . . . . . 717
3.82.9 Mupad [B] (verification not implemented) . . . . . 718

## 3.82.1 Optimal result

Integrand size = 24, antiderivative size = 79

$$\int (c+dx)^2\cos^2(a+bx)\sin^2(a+bx)\,dx=\frac{(c+dx)^3}{24d}-\frac{d(c+dx)\cos(4a+4bx)}{64b^2}+\frac{d^2\sin(4a+4bx)}{256b^3}-\frac{(c+dx)^2\sin(4a+4bx)}{32b}$$

output
```
1/24*(d*x+c)^3/d-1/64*d*(d*x+c)*cos(4*b*x+4*a)/b^2+1/256*d^2*sin(4*b*x+4*a
)/b^3-1/32*(d*x+c)^2*sin(4*b*x+4*a)/b
```

## 3.82.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$\int (c+dx)^2\cos^2(a+bx)\sin^2(a+bx)\,dx=\frac{32b^3x(3c^2+3cdx+d^2x^2)-12bd(c+dx)\cos(4(a+bx))-3(-d^2+8b^2(c+dx)^2)\sin(4(a+bx))}{768b^3}$$

input
```
Integrate[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(32*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) - 12*b*d*(c + d*x)*Cos[4*(a + b*x)]
- 3*(-d^2 + 8*b^2*(c + d*x)^2)*Sin[4*(a + b*x)])/(768*b^3)
```

### 3.82.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin^2(a+bx)\cos^2(a+bx) \, dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}(c+dx)^2-\frac{1}{8}(c+dx)^2\cos(4a+4bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2\sin(4a+4bx)}{256b^3}-\frac{d(c+dx)\cos(4a+4bx)}{64b^2}-\frac{(c+dx)^2\sin(4a+4bx)}{32b}+\frac{(c+dx)^3}{24d}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^3/(24*d) - (d*(c + d*x)*Cos[4*a + 4*b*x])/(64*b^2) + (d^2*Sin[4*
a + 4*b*x])/(256*b^3) - ((c + d*x)^2*Sin[4*a + 4*b*x])/(32*b)
```

#### 3.82.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.82.4 Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

| method | result |
|---|---|
| parallelrisch | $\frac{\left(-8(dx+c)^{2}b^{2}+d^{2}\right)\sin(4xb+4a)+32b\left(-\frac{d(dx+c)\cos(4xb+4a)}{8}+x\left(\frac{1}{3}x^{2}d^{2}+cdx+c^{2}\right)b^{2}+\frac{cd}{8}\right)}{256b^{3}}$ |
| risch | $\frac{d^{2}x^{3}}{24}+\frac{cd\,x^{2}}{8}+\frac{c^{2}x}{8}+\frac{c^{3}}{24d}-\frac{d(dx+c)\cos(4xb+4a)}{64b^{2}}-\frac{(8x^{2}d^{2}b^{2}+16b^{2}cdx+8b^{2}c^{2}-d^{2})\sin(4xb+4a)}{256b^{3}}$ |
| derivativedivides | $\frac{\frac{a^{2}d^{2}\left(-\frac{\cos(xb+a)^{3}\sin(xb+a)}{4}+\frac{\cos(xb+a)\sin(xb+a)}{8}+\frac{xb}{8}+\frac{a}{8}\right)}{b^{2}}-\frac{2acd\left(-\frac{\cos(xb+a)^{3}\sin(xb+a)}{4}+\frac{\cos(xb+a)\sin(xb+a)}{8}+\frac{xb}{8}+\frac{a}{8}\right)}{b}-}{}$ |
| default | $\frac{\frac{a^{2}d^{2}\left(-\frac{\cos(xb+a)^{3}\sin(xb+a)}{4}+\frac{\cos(xb+a)\sin(xb+a)}{8}+\frac{xb}{8}+\frac{a}{8}\right)}{b^{2}}-\frac{2acd\left(-\frac{\cos(xb+a)^{3}\sin(xb+a)}{4}+\frac{\cos(xb+a)\sin(xb+a)}{8}+\frac{xb}{8}+\frac{a}{8}\right)}{b}-}{}$ |
| norman | $\frac{\frac{cdx\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{7}}{2b}+\frac{\left(8b^{2}c^{2}+7d^{2}\right)x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}}{16b^{2}}+\frac{\left(24b^{2}c^{2}-35d^{2}\right)x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{4}}{32b^{2}}+\frac{\left(8b^{2}c^{2}+7d^{2}\right)x\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{6}}{16b^{2}}+\frac{\left(8b^{2}c^{2}-d^{2}\right)}{6}}{}$ |

input
```
int((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/256*((-8*(d*x+c)^2*b^2+d^2)*sin(4*b*x+4*a)+32*b*(-1/8*d*(d*x+c)*cos(4*b*
x+4*a)+x*(1/3*x^2*d^2+c*d*x+c^2)*b^2+1/8*c*d))/b^3
```

### 3.82.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(71) = 142.

Time = 0.26 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.28

$$\int (c+dx)^2\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$=\frac{8\,b^3d^2x^3+24\,b^3cdx^2-24\,(bd^2x+bcd)\cos(bx+a)^4+24\,(bd^2x+bcd)\cos(bx+a)^2+3\,(8\,b^3c^2-bd^2)x-}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/192*(8*b^3*d^2*x^3 + 24*b^3*c*d*x^2 - 24*(b*d^2*x + b*c*d)*cos(b*x + a)^
4 + 24*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + 3*(8*b^3*c^2 - b*d^2)*x - 3*(2*(
8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(b*x + a)^3 - (8*b^2*d^
2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(b*x + a))*sin(b*x + a))/b^3
```

### 3.82.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(70) = 140$.

Time = 0.47 (sec) , antiderivative size = 493, normalized size of antiderivative = 6.24

$$\int (c+dx)^2\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= \begin{cases} \frac{c^2x\sin^4(a+bx)}{8} + \frac{c^2x\sin^2(a+bx)\cos^2(a+bx)}{4} + \frac{c^2x\cos^4(a+bx)}{8} + \frac{cdx^2\sin^4(a+bx)}{8} + \frac{cdx^2\sin^2(a+bx)\cos^2(a+bx)}{4} + \frac{cdx^2\cos^4(a+ \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin^2(a)\cos^2(a) \end{cases}$$

input
```
integrate((d*x+c)**2*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

output
```
Piecewise((c**2*x*sin(a + b*x)**4/8 + c**2*x*sin(a + b*x)**2*cos(a + b*x)*
*2/4 + c**2*x*cos(a + b*x)**4/8 + c*d*x**2*sin(a + b*x)**4/8 + c*d*x**2*si
n(a + b*x)**2*cos(a + b*x)**2/4 + c*d*x**2*cos(a + b*x)**4/8 + d**2*x**3*s
in(a + b*x)**4/24 + d**2*x**3*sin(a + b*x)**2*cos(a + b*x)**2/12 + d**2*x*
*3*cos(a + b*x)**4/24 + c**2*sin(a + b*x)**3*cos(a + b*x)/(8*b) - c**2*sin
(a + b*x)*cos(a + b*x)**3/(8*b) + c*d*x*sin(a + b*x)**3*cos(a + b*x)/(4*b)
 - c*d*x*sin(a + b*x)*cos(a + b*x)**3/(4*b) + d**2*x**2*sin(a + b*x)**3*co
s(a + b*x)/(8*b) - d**2*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b) - c*d*sin(
a + b*x)**4/(16*b**2) - c*d*cos(a + b*x)**4/(16*b**2) - d**2*x*sin(a + b*x
)**4/(64*b**2) + 3*d**2*x*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**2) - d**2
*x*cos(a + b*x)**4/(64*b**2) - d**2*sin(a + b*x)**3*cos(a + b*x)/(64*b**3)
 + d**2*sin(a + b*x)*cos(a + b*x)**3/(64*b**3), Ne(b, 0)), ((c**2*x + c*d*
x**2 + d**2*x**3/3)*sin(a)**2*cos(a)**2, True))
```

### 3.82.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. 2(71) = 142.

Time = 0.24 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.94

$$\int (c+dx)^2\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{24\,(4\,bx+4\,a-\sin(4\,bx+4\,a))c^2 - \frac{48\,(4\,bx+4\,a-\sin(4\,bx+4\,a))acd}{b} + \frac{24\,(4\,bx+4\,a-\sin(4\,bx+4\,a))a^2d^2}{b^2} + \frac{12\left(8\,(bx+a)^2-4\,(b\right.}{}}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/768*(24*(4*b*x + 4*a - sin(4*b*x + 4*a))*c^2 - 48*(4*b*x + 4*a - sin(4*b
*x + 4*a))*a*c*d/b + 24*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^2*d^2/b^2 + 12*
(8*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*c*d/b -
12*(8*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a*d^2
/b^2 + (32*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2
- 1)*sin(4*b*x + 4*a))*d^2/b^2)/b
```

### 3.82.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.19

$$\int (c+dx)^2\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{1}{24}\,d^2x^3 + \frac{1}{8}\,cdx^2 + \frac{1}{8}\,c^2x - \frac{(bd^2x+bcd)\cos(4\,bx+4\,a)}{64\,b^3} - \frac{(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-d^2)\sin(4\,bx+4\,a)}{256\,b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/24*d^2*x^3 + 1/8*c*d*x^2 + 1/8*c^2*x - 1/64*(b*d^2*x + b*c*d)*cos(4*b*x
+ 4*a)/b^3 - 1/256*(8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*sin(4*
b*x + 4*a)/b^3
```

### 3.82.9 Mupad [B] (verification not implemented)

Time = 23.13 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.27

$$\int (c+dx)^2\cos^2(a+bx)\sin^2(a+bx)\,dx = x\left(\frac{c^2}{32}+\frac{3\,d^2}{256\,b^2}\right)+x\left(\frac{3\,c^2}{32}-\frac{3\,d^2}{256\,b^2}\right)+\frac{d^2\,x^3}{24}+\frac{\sin(4\,a+4\,b\,x)\,(d^2-8\,b^2\,c^2)}{256\,b^3}-\frac{x\,\cos(4\,a+4\,b\,x)\,\left(\frac{c^2}{4}+\frac{3\,d^2}{32\,b^2}\right)}{8}+\frac{x\,\cos(4\,a+4\,b\,x)\,\left(\frac{c^2}{8}-\frac{d^2}{64\,b^2}\right)}{4}+\frac{c\,d\,x^2}{8}-\frac{d^2\,x^2\,\sin(4\,a+4\,b\,x)}{32\,b}-\frac{c\,d\,\cos(4\,a+4\,b\,x)}{64\,b^2}-\frac{c\,d\,x\,\sin(4\,a+4\,b\,x)}{16\,b}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^2,x)
```

output
```
x*(c^2/32 + (3*d^2)/(256*b^2)) + x*((3*c^2)/32 - (3*d^2)/(256*b^2)) + (d^2
*x^3)/24 + (sin(4*a + 4*b*x)*(d^2 - 8*b^2*c^2))/(256*b^3) - (x*cos(4*a + 4
*b*x)*(c^2/4 + (3*d^2)/(32*b^2)))/8 + (x*cos(4*a + 4*b*x)*(c^2/8 - d^2/(64
*b^2)))/4 + (c*d*x^2)/8 - (d^2*x^2*sin(4*a + 4*b*x))/(32*b) - (c*d*cos(4*a
 + 4*b*x))/(64*b^2) - (c*d*x*sin(4*a + 4*b*x))/(16*b)
```

# 3.83 $\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx$

3.83.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 719
3.83.2 Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 719
3.83.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 720
3.83.4 Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 721
3.83.5 Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 721
3.83.6 Sympy [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 722
3.83.7 Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . . . 722
3.83.8 Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 723
3.83.9 Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 723

## 3.83.1 Optimal result

Integrand size = 22, antiderivative size = 53

$$\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx=\frac{(c+dx)^2}{16d}-\frac{d\cos(4a+4bx)}{128b^2}-\frac{(c+dx)\sin(4a+4bx)}{32b}$$

output
```
1/16*(d*x+c)^2/d-1/128*d*cos(4*b*x+4*a)/b^2-1/32*(d*x+c)*sin(4*b*x+4*a)/b
```

## 3.83.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.02

$$\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$=-\frac{8(a+bx)(-2bc+ad-bdx)+d\cos(4(a+bx))+4b(c+dx)\sin(4(a+bx))}{128b^2}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
-1/128*(8*(a + b*x)*(-2*b*c + a*d - b*d*x) + d*Cos[4*(a + b*x)] + 4*b*(c +
 d*x)*Sin[4*(a + b*x)])/b^2
```

### 3.83.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin^2(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}(c+dx)-\frac{1}{8}(c+dx)\cos(4a+4bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\cos(4a+4bx)}{128b^2}-\frac{(c+dx)\sin(4a+4bx)}{32b}+\frac{(c+dx)^2}{16d}$$

input
```
Int[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^2/(16*d) - (d*Cos[4*a + 4*b*x])/(128*b^2) - ((c + d*x)*Sin[4*a +
 4*b*x])/(32*b)
```

#### 3.83.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.83.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

| method | result |
|---|---|
| risch | $\frac{d\,x^2}{16}+\frac{cx}{8}-\frac{d\cos(4xb+4a)}{128b^2}-\frac{(dx+c)\sin(4xb+4a)}{32b}$ |
| derivativedivides | $\frac{-\frac{da\left(-\frac{\cos(xb+a)^3\sin(xb+a)}{4}+\frac{\cos(xb+a)\sin(xb+a)}{8}+\frac{xb}{8}+\frac{a}{8}\right)}{b}+c\left(-\frac{\cos(xb+a)^3\sin(xb+a)}{4}+\frac{\cos(xb+a)\sin(xb+a)}{8}+\frac{xb}{8}+\frac{a}{8}\right)+\frac{d}{}}{}$ |
| default | $\frac{-\frac{da\left(-\frac{\cos(xb+a)^3\sin(xb+a)}{4}+\frac{\cos(xb+a)\sin(xb+a)}{8}+\frac{xb}{8}+\frac{a}{8}\right)}{b}+c\left(-\frac{\cos(xb+a)^3\sin(xb+a)}{4}+\frac{\cos(xb+a)\sin(xb+a)}{8}+\frac{xb}{8}+\frac{a}{8}\right)+\frac{d}{}}{}$ |
| norman | $\frac{\frac{cx}{8}+\frac{d\,x^2}{16}-\frac{c\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{4b}+\frac{7c\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3}{4b}-\frac{7c\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^5}{4b}+\frac{c\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^7}{4b}+\frac{cx\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}{2}+\frac{3cx\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^4}{4}+\frac{cx\tan(}{}}{}$ |

input
```
int((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*d*x^2+1/8*c*x-1/128*d*cos(4*b*x+4*a)/b^2-1/32*(d*x+c)*sin(4*b*x+4*a)/
b
```

### 3.83.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.60

$$\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$=\frac{b^2dx^2-d\cos(bx+a)^4+2\,b^2cx+d\cos(bx+a)^2-2\left(2\left(bdx+bc\right)\cos(bx+a)^3-(bdx+bc)\cos(bx+a)\right.}{16\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/16*(b^2*d*x^2 - d*cos(b*x + a)^4 + 2*b^2*c*x + d*cos(b*x + a)^2 - 2*(2*(
b*d*x + b*c)*cos(b*x + a)^3 - (b*d*x + b*c)*cos(b*x + a))*sin(b*x + a))/b^
2
```

### 3.83.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(46) = 92$.

Time = 0.33 (sec) , antiderivative size = 238, normalized size of antiderivative = 4.49

$$\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= \begin{cases} \frac{cx\sin^4(a+bx)}{8} + \frac{cx\sin^2(a+bx)\cos^2(a+bx)}{4} + \frac{cx\cos^4(a+bx)}{8} + \frac{dx^2\sin^4(a+bx)}{16} + \frac{dx^2\sin^2(a+bx)\cos^2(a+bx)}{8} + \frac{dx^2\cos^4(a+bx)}{16} + \\ \left(cx + \frac{dx^2}{2}\right)\sin^2(a)\cos^2(a) \end{cases}$$

input
```
integrate((d*x+c)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

output
```
Piecewise((c*x*sin(a + b*x)**4/8 + c*x*sin(a + b*x)**2*cos(a + b*x)**2/4 +
 c*x*cos(a + b*x)**4/8 + d*x**2*sin(a + b*x)**4/16 + d*x**2*sin(a + b*x)**
2*cos(a + b*x)**2/8 + d*x**2*cos(a + b*x)**4/16 + c*sin(a + b*x)**3*cos(a
+ b*x)/(8*b) - c*sin(a + b*x)*cos(a + b*x)**3/(8*b) + d*x*sin(a + b*x)**3*
cos(a + b*x)/(8*b) - d*x*sin(a + b*x)*cos(a + b*x)**3/(8*b) - d*sin(a + b*
x)**4/(32*b**2) - d*cos(a + b*x)**4/(32*b**2), Ne(b, 0)), ((c*x + d*x**2/2
)*sin(a)**2*cos(a)**2, True))
```

### 3.83.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(47) = 94$.

Time = 0.23 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.81

$$\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{4(4bx+4a-\sin(4bx+4a))c - \frac{4(4bx+4a-\sin(4bx+4a))ad}{b} + \frac{\left(8(bx+a)^2-4(bx+a)\sin(4bx+4a)-\cos(4bx+4a)\right)d}{b}}{128b}$$

input
```
integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/128*(4*(4*b*x + 4*a - sin(4*b*x + 4*a))*c - 4*(4*b*x + 4*a - sin(4*b*x +
 4*a))*a*d/b + (8*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x +
 4*a))*d/b)/b
```

### 3.83.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{1}{16}\,dx^2 + \frac{1}{8}\,cx - \frac{d\cos(4\,bx+4\,a)}{128\,b^2} - \frac{(bdx+bc)\sin(4\,bx+4\,a)}{32\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
1/16*d*x^2 + 1/8*c*x - 1/128*d*cos(4*b*x + 4*a)/b^2 - 1/32*(b*d*x + b*c)*s
in(4*b*x + 4*a)/b^2
```

### 3.83.9 Mupad [B] (verification not implemented)

Time = 23.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.08

$$\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{c\,x}{8} + \frac{d\,x^2}{16} - \frac{d\,\cos(4\,a+4\,b\,x)}{128\,b^2} - \frac{c\,\sin(4\,a+4\,b\,x)}{32\,b} - \frac{d\,x\,\sin(4\,a+4\,b\,x)}{32\,b}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x),x)
```

output
```
(c*x)/8 + (d*x^2)/16 - (d*cos(4*a + 4*b*x))/(128*b^2) - (c*sin(4*a + 4*b*x
))/(32*b) - (d*x*sin(4*a + 4*b*x))/(32*b)
```

# 3.84 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx$


3.84.1 Optimal result . . . . . . . . . . . . 724
3.84.2 Mathematica [A] (verified) . . . . . . . . . . . . 724
3.84.3 Rubi [A] (verified) . . . . . . . . . . . . 725
3.84.4 Maple [C] (verified) . . . . . . . . . . . . 726
3.84.5 Fricas [A] (verification not implemented) . . . . . . . . . . . . 726
3.84.6 Sympy [F] . . . . . . . . . . . . 727
3.84.7 Maxima [C] (verification not implemented) . . . . . . . . . . . . 727
3.84.8 Giac [C] (verification not implemented) . . . . . . . . . . . . 727
3.84.9 Mupad [F(-1)] . . . . . . . . . . . . 728


## 3.84.1 Optimal result

Integrand size = 24, antiderivative size = 78

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx = -\frac{\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{8d} + \frac{\log(c+dx)}{8d} + \frac{\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{8d}$$

output
```
-1/8*Ci(4*b*c/d+4*b*x)*cos(4*a-4*b*c/d)/d+1/8*ln(d*x+c)/d+1/8*Si(4*b*c/d+4
*b*x)*sin(4*a-4*b*c/d)/d
```

## 3.84.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.83

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \frac{-\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)+\log(c+dx)+\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4b(c+dx)}{d}\right)}{8d}$$

input
```
Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x),x]
```

output
```
(-(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d]) + Log[c + d*x] + S
in[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(8*d)
```

### 3.84.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\cos^2(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8(c+dx)}-\frac{\cos(4a+4bx)}{8(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\log(c+dx)}{8d}$$

input
```
Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x),x]
```

output
```
-1/8*(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/d + Log[c + d*x
]/(8*d) + (Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(8*d)
```

#### 3.84.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.84.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.37

| method | result |
|---|---|
| risch | $\frac{\ln(dx+c)}{8d}+\frac{e^{-\frac{4i(ad-cb)}{d}}\mathrm{Ei}_1\left(4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{16d}+\frac{e^{\frac{4i(ad-cb)}{d}}\mathrm{Ei}_1\left(-4ibx-4ia-\frac{4(-iad+icb)}{d}\right)}{16d}$ |
| derivativedivides | $\frac{-\frac{b\left(-\frac{4\,\mathrm{Si}\left(-4xb-4a-\frac{4(-ad+cb)}{d}\right)\sin\left(\frac{-4ad+4cb}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(4xb+4a+\frac{-4ad+4cb}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right)}{d}\right)}{32}+\frac{b\ln(-ad+cb+d(xb+a))}{8d}}{b}$ |
| default | $\frac{-\frac{b\left(-\frac{4\,\mathrm{Si}\left(-4xb-4a-\frac{4(-ad+cb)}{d}\right)\sin\left(\frac{-4ad+4cb}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(4xb+4a+\frac{-4ad+4cb}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right)}{d}\right)}{32}+\frac{b\ln(-ad+cb+d(xb+a))}{8d}}{b}$ |

input
```
int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(d*x+c)/d+1/16/d*exp(-4*I*(a*d-b*c)/d)*Ei(1,4*I*b*x+4*I*a-4*I*(a*d-b
*c)/d)+1/16/d*exp(4*I*(a*d-b*c)/d)*Ei(1,-4*I*x*b-4*I*a-4*(-I*a*d+I*c*b)/d)
```

### 3.84.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx = -\frac{\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{4(bdx+bc)}{d}\right)-\sin\left(-\frac{4(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{4(bdx+bc)}{d}\right)-\log(dx+c)}{8\,d}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
-1/8*(cos(-4*(b*c - a*d)/d)*cos_integral(4*(b*d*x + b*c)/d) - sin(-4*(b*c
- a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) - log(d*x + c))/d
```

### 3.84.6 Sympy [F]

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \int \frac{\sin^2(a+bx)\cos^2(a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x), x)
```

### 3.84.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.08

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \frac{b\left(E_1\left(\frac{4(-ibc-i(bx+a)d+iad)}{d}\right)+E_1\left(-\frac{4(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{4(bc-ad)}{d}\right)+b\left(iE_1\left(\frac{4(-ibc-i(bx+a)d+iad)}{d}\right)\right.}{16\,bd}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
1/16*(b*(exp_integral_e(1, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_int
egral_e(1, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c - a*d)/d) +
 b*(I*exp_integral_e(1, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_inte
gral_e(1, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d)/d) +
2*b*log(b*c + (b*x + a)*d - a*d))/(b*d)
```

### 3.84.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.33 (sec) , antiderivative size = 669, normalized size of antiderivative = 8.58

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
1/16*(2*log(abs(d*x + c))*tan(2*a)^2*tan(2*b*c/d)^2 - real_part(cos_integr
al(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 - real_part(cos_integral(-4
*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(4*b*
x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) - 2*imag_part(cos_integral(-4*b*x -
4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 4*sin_integral(4*(b*d*x + b*c)/d)*tan(
2*a)^2*tan(2*b*c/d) - 2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*
tan(2*b*c/d)^2 + 2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(
2*b*c/d)^2 - 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d)^2 + 2
*log(abs(d*x + c))*tan(2*a)^2 + real_part(cos_integral(4*b*x + 4*b*c/d))*t
an(2*a)^2 + real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2 - 4*real_
part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d) - 4*real_part(co
s_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d) + 2*log(abs(d*x + c))*
tan(2*b*c/d)^2 + real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)^2 +
 real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d)^2 + 2*imag_part(co
s_integral(4*b*x + 4*b*c/d))*tan(2*a) - 2*imag_part(cos_integral(-4*b*x -
4*b*c/d))*tan(2*a) + 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a) - 2*imag_p
art(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d) + 2*imag_part(cos_integral
(-4*b*x - 4*b*c/d))*tan(2*b*c/d) - 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2
*b*c/d) + 2*log(abs(d*x + c)) - real_part(cos_integral(4*b*x + 4*b*c/d)) -
 real_part(cos_integral(-4*b*x - 4*b*c/d)))/(d*tan(2*a)^2*tan(2*b*c/d)^...
```

### 3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \int \frac{\cos(a+b\,x)^2\sin(a+b\,x)^2}{c+d\,x}\,dx$$

input
```
int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x),x)
```

output
```
int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x), x)
```

# 3.85 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$


3.85.1 Optimal result . . . 729
3.85.2 Mathematica [A] (verified) . . . 729
3.85.3 Rubi [A] (verified) . . . 730
3.85.4 Maple [C] (verified) . . . 731
3.85.5 Fricas [A] (verification not implemented) . . . 732
3.85.6 Sympy [F] . . . 732
3.85.7 Maxima [C] (verification not implemented) . . . 732
3.85.8 Giac [C] (verification not implemented) . . . 733
3.85.9 Mupad [F(-1)] . . . 734


## 3.85.1 Optimal result

Integrand size = 24, antiderivative size = 104

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = -\frac{1}{8d(c+dx)}+\frac{\cos(4a+4bx)}{8d(c+dx)}+\frac{b\,\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{2d^2}$$

output
```
-1/8/d/(d*x+c)+1/8*cos(4*b*x+4*a)/d/(d*x+c)+1/2*b*cos(4*a-4*b*c/d)*Si(4*b*
c/d+4*b*x)/d^2+1/2*b*Ci(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^2
```

## 3.85.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.78

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \frac{\frac{d(-1+\cos(4(a+bx)))}{c+dx}+4b\,\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)\sin\left(4a-\frac{4bc}{d}\right)+4b\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4b(c+dx)}{d}\right)}{8d^2}$$

input
```
Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
((d*(-1 + Cos[4*(a + b*x)]))/(c + d*x) + 4*b*CosIntegral[(4*b*(c + d*x))/d
]*Sin[4*a - (4*b*c)/d] + 4*b*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*
x))/d])/(8*d^2)
```

### 3.85.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\cos^2(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8(c+dx)^2}-\frac{\cos(4a+4bx)}{8(c+dx)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{2d^2}+\frac{\cos(4a+4bx)}{8d(c+dx)}-\frac{1}{8d(c+dx)}$$

input
```
Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
-1/8*1/(d*(c + d*x)) + Cos[4*a + 4*b*x]/(8*d*(c + d*x)) + (b*CosIntegral[(
4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/(2*d^2) + (b*Cos[4*a - (4*b*c)/d]*
SinIntegral[(4*b*c)/d + 4*b*x])/(2*d^2)
```

### 3.85.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.85.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.49

| method | result |
|---|---|
| risch | $-\frac{1}{8d(dx+c)}-\frac{ib\,e^{-\frac{4i(ad-cb)}{d}}\operatorname{Ei}_1\left(4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{4d^2}+\frac{ib\,e^{\frac{4i(ad-cb)}{d}}\operatorname{Ei}_1\left(-4ibx-4ia-\frac{4(-iad+icb)}{d}\right)}{4d^2}+\frac{(-2dx}{16d}$ |
| derivativedivides | $\frac{-\frac{b^2\left(-\frac{4\cos(4xb+4a)}{(-ad+cb+d(xb+a))d}-\frac{4\left(-\frac{4\operatorname{Si}\left(-4xb-4a-\frac{4(-ad+cb)}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right)}{d}-\frac{4\operatorname{Ci}\left(4xb+4a+\frac{-4ad+4cb}{d}\right)\sin\left(\frac{-4ad+4cb}{d}\right)}{d}\right)}{d}\right)}{32}}{b}$ |
| default | $\frac{-\frac{b^2\left(-\frac{4\cos(4xb+4a)}{(-ad+cb+d(xb+a))d}-\frac{4\left(-\frac{4\operatorname{Si}\left(-4xb-4a-\frac{4(-ad+cb)}{d}\right)\cos\left(\frac{-4ad+4cb}{d}\right)}{d}-\frac{4\operatorname{Ci}\left(4xb+4a+\frac{-4ad+4cb}{d}\right)\sin\left(\frac{-4ad+4cb}{d}\right)}{d}\right)}{d}\right)}{32}}{b}$ |

input
```
int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8/d/(d*x+c)-1/4*I*b/d^2*exp(-4*I*(a*d-b*c)/d)*Ei(1,4*I*b*x+4*I*a-4*I*(a
*d-b*c)/d)+1/4*I*b/d^2*exp(4*I*(a*d-b*c)/d)*Ei(1,-4*I*x*b-4*I*a-4*(-I*a*d+
I*c*b)/d)+1/16/d*(-2*b*d*x-2*b*c)/(-b*d*x-b*c)/(d*x+c)*cos(4*b*x+4*a)
```

### 3.85.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.08

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{2\,d\cos\left(bx+a\right)^4 - 2\,d\cos\left(bx+a\right)^2 + \left(bdx+bc\right)\operatorname{Ci}\left(\frac{4\,(bdx+bc)}{d}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right) + \left(bdx+bc\right)\cos\left(-\frac{4\,(bc-a}{d}\right.}{2\,(d^3x+cd^2)}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(2*d*cos(b*x + a)^4 - 2*d*cos(b*x + a)^2 + (b*d*x + b*c)*cos_integral(
4*(b*d*x + b*c)/d)*sin(-4*(b*c - a*d)/d) + (b*d*x + b*c)*cos(-4*(b*c - a*d
)/d)*sin_integral(4*(b*d*x + b*c)/d))/(d^3*x + c*d^2)
```

### 3.85.6 Sympy [F]

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin^2\left(a+bx\right)\cos^2\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x)**2, x)
```

### 3.85.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.64

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{b^2\left(E_2\left(\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right) + E_2\left(-\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right) + b^2\left(i\,E_2\left(\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right.\right.}{16\,(bcd+(bx+a)d^2-ad^2)b}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/16*(b^2*(exp_integral_e(2, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_i
ntegral_e(2, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c - a*d)/d)
 + b^2*(I*exp_integral_e(2, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_
integral_e(2, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d)/d
) - 2*b^2)/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

### 3.85.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.76 (sec) , antiderivative size = 3218, normalized size of antiderivative = 30.94

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/4*(b*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^
2*tan(2*b*c/d)^2 - b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b
*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 2*b*d*x*sin_integral(4*(b*d*x + b*c)/d)*
tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 2*b*d*x*real_part(cos_integral(4*
b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 2*b*d*x*real_part(c
os_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 2*b*
d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b
*c/d)^2 - 2*b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*t
an(2*a)*tan(2*b*c/d)^2 + b*c*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(
2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - b*c*imag_part(cos_integral(-4*b*x - 4
*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 2*b*c*sin_integral(4*(b*
d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - b*d*x*imag_part(cos
_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + b*d*x*imag_part(cos_
integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 2*b*d*x*sin_integral
(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2 + 4*b*d*x*imag_part(cos_integr
al(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) - 4*b*d*x*imag_par
t(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 8*b
*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) +
2*b*c*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan
(2*b*c/d) + 2*b*c*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^...
```

### 3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\,dx$$

input
```
int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^2,x)
```

output
```
int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^2, x)
```

# 3.86 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$

3.86.1 Optimal result . . . 735
3.86.2 Mathematica [A] (verified) . . . 735
3.86.3 Rubi [A] (verified) . . . 736
3.86.4 Maple [A] (verified) . . . 737
3.86.5 Fricas [A] (verification not implemented) . . . 738
3.86.6 Sympy [F] . . . 738
3.86.7 Maxima [C] (verification not implemented) . . . 738
3.86.8 Giac [C] (verification not implemented) . . . 739
3.86.9 Mupad [F(-1)] . . . 740

## 3.86.1 Optimal result

Integrand size = 24, antiderivative size = 127

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = -\frac{1}{16d(c+dx)^2}+\frac{\cos(4a+4bx)}{16d(c+dx)^2} + \frac{b^2\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{d^3} - \frac{b\sin(4a+4bx)}{4d^2(c+dx)} - \frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{d^3}$$

output
```
-1/16/d/(d*x+c)^2+b^2*Ci(4*b*c/d+4*b*x)*cos(4*a-4*b*c/d)/d^3+1/16*cos(4*b*
x+4*a)/d/(d*x+c)^2-b^2*Si(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^3-1/4*b*sin(4*
b*x+4*a)/d^2/(d*x+c)
```

## 3.86.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.83

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \frac{16b^2\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)+\frac{d(-d+d\cos(4(a+bx))-4b(c+dx)\sin(4(a+bx)))}{(c+dx)^2}-16b^2\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\right.}{16d^3}$$

input
```
Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

output
```
(16*b^2*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] + (d*(-d + d*C
os[4*(a + b*x)] - 4*b*(c + d*x)*Sin[4*(a + b*x)]))/(c + d*x)^2 - 16*b^2*Si
n[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(16*d^3)
```

### 3.86.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\cos^2(a+bx)}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8(c+dx)^3}-\frac{\cos(4a+4bx)}{8(c+dx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b\sin(4a+4bx)}{4d^2(c+dx)}+\frac{\cos(4a+4bx)}{16d(c+dx)^2}-\frac{1}{16d(c+dx)^2}$$

input
```
Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

output
```
-1/16*1/(d*(c + d*x)^2) + Cos[4*a + 4*b*x]/(16*d*(c + d*x)^2) + (b^2*Cos[4
*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/d^3 - (b*Sin[4*a + 4*b*x])
/(4*d^2*(c + d*x)) - (b^2*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b
*x])/d^3
```

### 3.86.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.86.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.52

| method | result |
|---|---|
| derivativedivides | $-\frac{b^3\left(-\frac{2\cos(4xb+4a)}{(-ad+cb+d(xb+a))^2d}-\frac{2\left(-\frac{4\sin(4xb+4a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{16\,\mathrm{Si}\left(-4xb-4a-\frac{4(-ad+cb)}{d}\right)\sin\left(\frac{-4ad+4cb}{d}\right)}{d}+\frac{16\,\mathrm{Ci}\left(4xb+4a+\cdots\right.}{}}{d}\right.}{d}\right.}{32}}{b}$ |
| default | $-\frac{b^3\left(-\frac{2\cos(4xb+4a)}{(-ad+cb+d(xb+a))^2d}-\frac{2\left(-\frac{4\sin(4xb+4a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{16\,\mathrm{Si}\left(-4xb-4a-\frac{4(-ad+cb)}{d}\right)\sin\left(\frac{-4ad+4cb}{d}\right)}{d}+\frac{16\,\mathrm{Ci}\left(4xb+4a+\cdots\right.}{}}{d}\right.}{d}\right.}{32}}{b}$ |
| risch | $-\frac{1}{16d(dx+c)^2}-\frac{b^2e^{-\frac{4i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{2d^3}-\frac{b^2e^{\frac{4i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(-4ibx-4ia-\frac{4(-iad+icb)}{d}\right)}{2d^3}-\frac{(-2\cdots}{}$ |

input
```
int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/32*b^3*(-2*cos(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))^2/d-2*(-4*sin(4*b*x
+4*a)/(-a*d+c*b+d*(b*x+a))/d+4*(-4*Si(-4*x*b-4*a-4*(-a*d+b*c)/d)*sin(4*(-a
*d+b*c)/d)/d+4*Ci(4*x*b+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d)/d)/d)-1
/16*b^3/(-a*d+c*b+d*(b*x+a))^2/d)
```

### 3.86.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.68

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{d^2\cos(bx+a)^4 - d^2\cos(bx+a)^2 + 2\left(b^2d^2x^2 + 2b^2cdx + b^2c^2\right)\cos\left(-\frac{4(bc-ad)}{d}\right)\operatorname{Ci}\left(\frac{4(bdx+bc)}{d}\right) - 2\left(b^2d^2x\right.}{2\,(c}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/2*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*cos(-4*(b*c - a*d)/d)*cos_integral(4*(b*d*x + b*c)/d) - 2*(b^
2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b
*d*x + b*c)/d) - 2*(2*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - (b*d^2*x + b*c*d)
*cos(b*x + a))*sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

### 3.86.6 Sympy [F]

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin^2(a+bx)\cos^2(a+bx)}{(c+dx)^3}\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c)**3,x)
```

output
```
Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x)**3, x)
```

### 3.86.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.62

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{b^3\left(E_3\left(\frac{4(-ibc-i(bx+a)d+iad)}{d}\right) + E_3\left(-\frac{4(-ibc-i(bx+a)d+iad)}{d}\right)\right)\cos\left(-\frac{4(bc-ad)}{d}\right) + b^3\left(i\,E_3\left(\frac{4(-ibc-i(bx+a)d+iad)}{d}\right.\right.}{16\left(b^2c^2d - 2abcd^2 + (bx+a)^2d^3 + a^2d^3 + 2\left(bcd^2 - ad^3\right)(b\right.}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/16*(b^3*(exp_integral_e(3, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_i
ntegral_e(3, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c - a*d)/d)
 + b^3*(I*exp_integral_e(3, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_
integral_e(3, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d)/d
) - b^3)/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^
2 - a*d^3)*(b*x + a))*b)
```

### 3.86.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.49 (sec) , antiderivative size = 5600, normalized size of antiderivative = 44.09

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/8*(4*b^2*d^2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*t
an(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*d^2*x^2*real_part(cos_integral(-4*b*x - 4
*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 8*b^2*d^2*x^2*imag_part(
cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 8*b^
2*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^
2*tan(2*b*c/d) - 16*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)
^2*tan(2*a)^2*tan(2*b*c/d) + 8*b^2*d^2*x^2*imag_part(cos_integral(4*b*x +
4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 - 8*b^2*d^2*x^2*imag_part(c
os_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 16*b
^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c
/d)^2 + 8*b^2*c*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*
tan(2*a)^2*tan(2*b*c/d)^2 + 8*b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*
b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 4*b^2*d^2*x^2*real_part(c
os_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 4*b^2*d^2*x^2*real
_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + 16*b^2*d^2
*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*
b*c/d) + 16*b^2*d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*
x)^2*tan(2*a)*tan(2*b*c/d) - 16*b^2*c*d*x*imag_part(cos_integral(4*b*x + 4
*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 16*b^2*c*d*x*imag_part(cos
_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 32*...
```

### 3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos(a+b\,x)^2\sin(a+b\,x)^2}{(c+d\,x)^3}\,dx$$

input
```
int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^3,x)
```

output
```
int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^3, x)
```

# 3.87 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx$

3.87.1 Optimal result . . . 741
3.87.2 Mathematica [A] (verified) . . . 741
3.87.3 Rubi [A] (verified) . . . 742
3.87.4 Maple [A] (verified) . . . 743
3.87.5 Fricas [B] (verification not implemented) . . . 744
3.87.6 Sympy [F] . . . 744
3.87.7 Maxima [C] (verification not implemented) . . . 745
3.87.8 Giac [C] (verification not implemented) . . . 745
3.87.9 Mupad [F(-1)] . . . 746

## 3.87.1 Optimal result

Integrand size = 24, antiderivative size = 158

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = -\frac{1}{24d(c+dx)^3}+\frac{\cos(4a+4bx)}{24d(c+dx)^3}-\frac{b^2\cos(4a+4bx)}{3d^3(c+dx)} - \frac{4b^3\,\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{3d^4} - \frac{b\sin(4a+4bx)}{12d^2(c+dx)^2}-\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{3d^4}$$

output
```
-1/24/d/(d*x+c)^3+1/24*cos(4*b*x+4*a)/d/(d*x+c)^3-1/3*b^2*cos(4*b*x+4*a)/d
^3/(d*x+c)-4/3*b^3*cos(4*a-4*b*c/d)*Si(4*b*c/d+4*b*x)/d^4-4/3*b^3*Ci(4*b*c
/d+4*b*x)*sin(4*a-4*b*c/d)/d^4-1/12*b*sin(4*b*x+4*a)/d^2/(d*x+c)^2
```

## 3.87.2 Mathematica [A] (verified)

Time = 1.80 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.78

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = -\frac{32b^3\,\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)\sin\left(4a-\frac{4bc}{d}\right)+\frac{d\left(\left(-d^2+8b^2(c+dx)^2\right)\cos(4(a+bx))+d(d+2b(c+dx)\sin(4(a+bx)))\right)}{(c+dx)^3}+32b^3\,\text{c}}{24d^4}$$

input
```
Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

output
```
-1/24*(32*b^3*CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + (d*((-
d^2 + 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] + d*(d + 2*b*(c + d*x)*Sin[4*(a
+ b*x)])))/(c + d*x)^3 + 32*b^3*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c +
 d*x))/d])/d^4
```

### 3.87.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\cos^2(a+bx)}{(c+dx)^4}dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8(c+dx)^4}-\frac{\cos(4a+4bx)}{8(c+dx)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4b^3\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4}-\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{3d^4}-\frac{b^2\cos(4a+4bx)}{3d^3(c+dx)}-\frac{b\sin(4a+4bx)}{12d^2(c+dx)^2}+\frac{\cos(4a+4bx)}{24d(c+dx)^3}-\frac{1}{24d(c+dx)^3}$$

input
```
Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

output
```
-1/24*1/(d*(c + d*x)^3) + Cos[4*a + 4*b*x]/(24*d*(c + d*x)^3) - (b^2*Cos[4
*a + 4*b*x])/(3*d^3*(c + d*x)) - (4*b^3*CosIntegral[(4*b*c)/d + 4*b*x]*Sin
[4*a - (4*b*c)/d])/(3*d^4) - (b*Sin[4*a + 4*b*x])/(12*d^2*(c + d*x)^2) - (
4*b^3*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(3*d^4)
```

### 3.87.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.87.4 Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.46

| method | result |
|---|---|
| derivativedivides | $-\frac{b^4\left(-\frac{4\cos(4xb+4a)}{3(-ad+cb+d(xb+a))^3 d}-\frac{4\left(-\frac{2\sin(4xb+4a)}{(-ad+cb+d(xb+a))^2 d}+\frac{-\frac{8\cos(4xb+4a)}{(-ad+cb+d(xb+a))d}-\frac{8\left(-\frac{4\,\mathrm{Si}\left(-4xb-4a-\frac{4(-ad+cb)}{d}\right)\cos\left(\frac{-4\cdots}{\cdots}\right)}{d}\cdots\right)}{\cdots}}{d}\cdots\right)}{3d}\cdots\right)}{32}\cdots}{b}$ |
| default | $-\frac{b^4\left(-\frac{4\cos(4xb+4a)}{3(-ad+cb+d(xb+a))^3 d}-\frac{4\left(-\frac{2\sin(4xb+4a)}{(-ad+cb+d(xb+a))^2 d}+\frac{-\frac{8\cos(4xb+4a)}{(-ad+cb+d(xb+a))d}-\frac{8\left(-\frac{4\,\mathrm{Si}\left(-4xb-4a-\frac{4(-ad+cb)}{d}\right)\cos\left(\frac{-4\cdots}{\cdots}\right)}{d}\cdots\right)}{\cdots}}{d}\cdots\right)}{3d}\cdots\right)}{32}\cdots}{b}$ |
| risch | $-\frac{1}{24d(dx+c)^3}+\frac{2ib^3 e^{-\frac{4i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{3d^4}-\frac{2ib^3 e^{\frac{4i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(-4ibx-4ia-\frac{4(-iad+icb)}{d}\right)}{3d^4}+$ |

input
```
int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/32*b^4*(-4/3*cos(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))^3/d-4/3*(-2*sin(4
*b*x+4*a)/(-a*d+c*b+d*(b*x+a))^2/d+2*(-4*cos(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a
))/d-4*(-4*Si(-4*x*b-4*a-4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*x*b+
4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)/d)/d)/d)-1/24*b^4/(-a*d+c*b+d*(
b*x+a))^3/d)
```

### 3.87.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(146) = 292.

Time = 0.27 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.21

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = -\frac{b^2d^3x^2 + 2\,b^2cd^2x + b^2c^2d + (8\,b^2d^3x^2 + 16\,b^2cd^2x + 8\,b^2c^2d - d^3)\cos{(bx+a)}^4 - (8\,b^2d^3x^2 + 16\,b^2cd^2x}{}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d + (8*b^2*d^3*x^2 + 16*b^2*c*
d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^4 - (8*b^2*d^3*x^2 + 16*b^2*c*d^2*
x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^2 + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(4*(b*d*x + b*c)/d)*sin(-4*(b*c - a*
d)/d) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-4
*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + (2*(b*d^3*x + b*c*d^2)*c
os(b*x + a)^3 - (b*d^3*x + b*c*d^2)*cos(b*x + a))*sin(b*x + a))/(d^7*x^3 +
 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

### 3.87.6 Sympy [F]

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sin^2{(a+bx)}\cos^2{(a+bx)}}{(c+dx)^4}\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c)**4,x)
```

output
```
Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x)**4, x)
```

### 3.87.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.63

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \frac{3\,b^4\left(E_4\left(\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_4\left(-\frac{4\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)+3\,b^4\left(i\,E_4\left(\frac{4\,(-i\,bc-i\,(bx+a)d+}{d}\right.\right.}{48\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3+(bx+a)^3d^4-a^3d^4+3\,(bcd^3-ad^4)(bx+a)^2+\right.}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/48*(3*b^4*(exp_integral_e(4, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp
_integral_e(4, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c - a*d)/
d) + 3*b^4*(I*exp_integral_e(4, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*
exp_integral_e(4, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*
d)/d) - 2*b^4)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3
*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*
c*d^3 + a^2*d^4)*(b*x + a))*b)
```

### 3.87.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.57 (sec) , antiderivative size = 8508, normalized size of antiderivative = 53.85

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

output
```
-1/12*(8*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2
*tan(2*a)^2*tan(2*b*c/d)^2 - 8*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x -
 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 16*b^3*d^3*x^3*sin_int
egral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 16*b^3*d
^3*x^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*ta
n(2*b*c/d) + 16*b^3*d^3*x^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(
2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 16*b^3*d^3*x^3*real_part(cos_integral(4
*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*rea
l_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^
2 + 24*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2
*tan(2*a)^2*tan(2*b*c/d)^2 - 24*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*
x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 48*b^3*c*d^2*x^2*si
n_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 8*b
^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^
2 + 8*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*t
an(2*a)^2 - 16*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*ta
n(2*a)^2 + 32*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b
*x)^2*tan(2*a)*tan(2*b*c/d) - 32*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x
 - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 64*b^3*d^3*x^3*sin_integ
ral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 48*b^3*c*d^...
```

### 3.87.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^4}\,dx$$

input
```
int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^4,x)
```

output
```
int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^4, x)
```

# 3.88 $\int (c+dx)^m \cos^2(a+bx) \sin^3(a+bx)\,dx$


3.88.1 Optimal result . . . 747
3.88.2 Mathematica [A] (verified) . . . 748
3.88.3 Rubi [A] (verified) . . . 749
3.88.4 Maple [F] . . . 750
3.88.5 Fricas [A] (verification not implemented) . . . 751
3.88.6 Sympy [F(-2)] . . . 751
3.88.7 Maxima [F] . . . 752
3.88.8 Giac [F] . . . 752
3.88.9 Mupad [F(-1)] . . . 752


## 3.88.1 Optimal result

Integrand size = 24, antiderivative size = 407

$$\begin{aligned}
&\int (c+dx)^m \cos^2(a+bx)\sin^3(a+bx)\,dx \\
&= -\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{16b} \\
&\quad-\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)}{16b} \\
&\quad-\frac{3^{-1-m}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3ib(c+dx)}{d}\right)}{32b} \\
&\quad-\frac{3^{-1-m}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3ib(c+dx)}{d}\right)}{32b} \\
&\quad+\frac{5^{-1-m}e^{5i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{5ib(c+dx)}{d}\right)}{32b} \\
&\quad+\frac{5^{-1-m}e^{-5i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{5ib(c+dx)}{d}\right)}{32b}
\end{aligned}$$

output
```
-1/16*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c
)/d)^m)-1/16*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(
d*x+c)/d)^m)-1/32*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3*I*b*(
d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)-1/32*3^(-1-m)*(d*x+c)^m*GAMMA(1+m,3*I*b*(
d*x+c)/d)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)+1/32*5^(-1-m)*exp(5*I*(
a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-5*I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)+1/3
2*5^(-1-m)*(d*x+c)^m*GAMMA(1+m,5*I*b*(d*x+c)/d)/b/exp(5*I*(a-b*c/d))/((I*b
*(d*x+c)/d)^m)
```

### 3.88.2 Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.92

$$\int (c+dx)^m \cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{e^{-\frac{5i(bc+ad)}{d}}(c+dx)^m\left(30e^{\frac{4i(bc+ad)}{d}}\left(-e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)-e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\right.\right.\right.}{}$$

input
```
Integrate[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
((c + d*x)^m*(30*E^(((4*I)*(b*c + a*d))/d)*(-((E^((2*I)*a)*Gamma[1 + m, ((
-I)*b*(c + d*x))/d])/(((-I)*b*(c + d*x))/d)^m) - (E^(((2*I)*b*c)/d)*Gamma[
1 + m, (I*b*(c + d*x))/d])/((I*b*(c + d*x))/d)^m) - (5*E^(((2*I)*(b*c + a*
d))/d)*(E^((6*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x)
)/d] + E^(((6*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((3*I)*b*(c
 + d*x))/d]))/(3^m*((b^2*(c + d*x)^2)/d^2)^m) + (3*(E^((10*I)*a)*((I*b*(c
+ d*x))/d)^m*Gamma[1 + m, ((-5*I)*b*(c + d*x))/d] + E^(((10*I)*b*c)/d)*(((
-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((5*I)*b*(c + d*x))/d]))/(5^m*((b^2*(c
+ d*x)^2)/d^2)^m)))/(480*b*E^(((5*I)*(b*c + a*d))/d))
```

### 3.88.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\cos^2(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \ 4906$$

$$\int \left(\frac{1}{8}\sin(a+bx)(c+dx)^m+\frac{1}{16}\sin(3a+3bx)(c+dx)^m-\frac{1}{16}\sin(5a+5bx)(c+dx)^m\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{16b}-\frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{32b}+\frac{5^{-m-1}e^{5i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{5ib(c+dx)}{d}\right)}{32b}-\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{16b}-\frac{3^{-m-1}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{3ib(c+dx)}{d}\right)}{32b}+\frac{5^{-m-1}e^{-5i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{5ib(c+dx)}{d}\right)}{32b}$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
-1/16*(E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])
/(b*(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/
d])/(16*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) - (3^(-1 - m)*E^((3*I
)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(32*b*(
((-I)*b*(c + d*x))/d)^m) - (3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((3*I)*b*(
c + d*x))/d])/(32*b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + (5^(-
1 - m)*E^((5*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-5*I)*b*(c + d*x
))/d])/(32*b*(((-I)*b*(c + d*x))/d)^m) + (5^(-1 - m)*(c + d*x)^m*Gamma[1 +
 m, ((5*I)*b*(c + d*x))/d])/(32*b*E^((5*I)*(a - (b*c)/d))*((I*b*(c + d*x))
/d)^m)
```

### 3.88.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.88.4 Maple [F]

$$\int (dx + c)^m \cos(xb + a)^2 \sin(xb + a)^3 \, dx$$

input
```
int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

output
```
int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

### 3.88.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.69

$$\int (c+dx)^m \cos^2(a+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{30\,e^{\left(-\frac{dm\log\left(\frac{i\,b}{d}\right)-i\,bc+i\,ad}{d}\right)}\Gamma\left(m+1,\frac{i\,bdx+i\,bc}{d}\right)+5\,e^{\left(-\frac{dm\log\left(-\frac{3i\,b}{d}\right)+3i\,bc-3i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{3\,(i\,bdx+i\,bc)}{d}\right)-3\,e^{\left(-\cdots\right)}\cdots}{\cdots}$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/480*(30*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x +
 I*b*c)/d) + 5*e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d)*gamma(m + 1,
 -3*(I*b*d*x + I*b*c)/d) - 3*e^(-(d*m*log(-5*I*b/d) + 5*I*b*c - 5*I*a*d)/d
)*gamma(m + 1, -5*(I*b*d*x + I*b*c)/d) + 30*e^(-(d*m*log(-I*b/d) + I*b*c -
 I*a*d)/d)*gamma(m + 1, (-I*b*d*x - I*b*c)/d) + 5*e^(-(d*m*log(3*I*b/d) -
3*I*b*c + 3*I*a*d)/d)*gamma(m + 1, -3*(-I*b*d*x - I*b*c)/d) - 3*e^(-(d*m*l
og(5*I*b/d) - 5*I*b*c + 5*I*a*d)/d)*gamma(m + 1, -5*(-I*b*d*x - I*b*c)/d))
/b
```

### 3.88.6 Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m \cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d*x+c)**m*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.88.7 Maxima [F]

$$\int (c+dx)^m \cos^2(a+bx)\sin^3(a+bx)\,dx = \int (dx+c)^m \cos{(bx+a)}^2 \sin{(bx+a)}^3\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a)^3, x)
```

### 3.88.8 Giac [F]

$$\int (c+dx)^m \cos^2(a+bx)\sin^3(a+bx)\,dx = \int (dx+c)^m \cos{(bx+a)}^2 \sin{(bx+a)}^3\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a)^3, x)
```

### 3.88.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cos^2(a+bx)\sin^3(a+bx)\,dx = \int \cos{(a+b\,x)}^2 \sin{(a+b\,x)}^3\,(c+d\,x)^m\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^m, x)
```

# 3.89 $\int (c+dx)^4 \cos^2(a+bx) \sin^3(a+bx) \, dx$


3.89.1 Optimal result . . . 753
3.89.2 Mathematica [A] (verified) . . . 754
3.89.3 Rubi [A] (verified) . . . 754
3.89.4 Maple [A] (verified) . . . 756
3.89.5 Fricas [A] (verification not implemented) . . . 756
3.89.6 Sympy [B] (verification not implemented) . . . 757
3.89.7 Maxima [B] (verification not implemented) . . . 758
3.89.8 Giac [A] (verification not implemented) . . . 759
3.89.9 Mupad [B] (verification not implemented) . . . 760


## 3.89.1 Optimal result

Integrand size = 24, antiderivative size = 330

$$\begin{aligned}
&\int (c+dx)^4 \cos^2(a+bx) \sin^3(a+bx) \, dx \\
&= -\frac{3d^4 \cos(a+bx)}{b^5} + \frac{3d^2(c+dx)^2 \cos(a+bx)}{2b^3} - \frac{(c+dx)^4 \cos(a+bx)}{8b} \\
&\quad - \frac{d^4 \cos(3a+3bx)}{162b^5} + \frac{d^2(c+dx)^2 \cos(3a+3bx)}{36b^3} - \frac{(c+dx)^4 \cos(3a+3bx)}{48b} \\
&\quad + \frac{3d^4 \cos(5a+5bx)}{6250b^5} - \frac{3d^2(c+dx)^2 \cos(5a+5bx)}{500b^3} + \frac{(c+dx)^4 \cos(5a+5bx)}{80b} \\
&\quad - \frac{3d^3(c+dx) \sin(a+bx)}{b^4} + \frac{d(c+dx)^3 \sin(a+bx)}{2b^2} - \frac{d^3(c+dx) \sin(3a+3bx)}{54b^4} \\
&\quad + \frac{d(c+dx)^3 \sin(3a+3bx)}{36b^2} + \frac{3d^3(c+dx) \sin(5a+5bx)}{1250b^4} - \frac{d(c+dx)^3 \sin(5a+5bx)}{100b^2}
\end{aligned}$$

output
```
-3*d^4*cos(b*x+a)/b^5+3/2*d^2*(d*x+c)^2*cos(b*x+a)/b^3-1/8*(d*x+c)^4*cos(b
*x+a)/b-1/162*d^4*cos(3*b*x+3*a)/b^5+1/36*d^2*(d*x+c)^2*cos(3*b*x+3*a)/b^3
-1/48*(d*x+c)^4*cos(3*b*x+3*a)/b+3/6250*d^4*cos(5*b*x+5*a)/b^5-3/500*d^2*(
d*x+c)^2*cos(5*b*x+5*a)/b^3+1/80*(d*x+c)^4*cos(5*b*x+5*a)/b-3*d^3*(d*x+c)*
sin(b*x+a)/b^4+1/2*d*(d*x+c)^3*sin(b*x+a)/b^2-1/54*d^3*(d*x+c)*sin(3*b*x+3
*a)/b^4+1/36*d*(d*x+c)^3*sin(3*b*x+3*a)/b^2+3/1250*d^3*(d*x+c)*sin(5*b*x+5
*a)/b^4-1/100*d*(d*x+c)^3*sin(5*b*x+5*a)/b^2
```

### 3.89.2 Mathematica [A] (verified)

Time = 3.58 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.72

$$\int (c+dx)^4 \cos^2(a+bx) \sin^3(a+bx) \, dx$$
$$= \frac{-506250(24d^4-12b^2d^2(c+dx)^2+b^4(c+dx)^4)\cos(a+bx)-3125(8d^4-36b^2d^2(c+dx)^2+27b^4(c+dx}{}$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
(-506250*(24*d^4 - 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cos[a + b*x]
- 3125*(8*d^4 - 36*b^2*d^2*(c + d*x)^2 + 27*b^4*(c + d*x)^4)*Cos[3*(a + b*
x)] + 81*(24*d^4 - 300*b^2*d^2*(c + d*x)^2 + 625*b^4*(c + d*x)^4)*Cos[5*(a
 + b*x)] + 120*b*d*(c + d*x)*(17475*b^2*c^2 - 101794*d^2 + 34950*b^2*c*d*x
 + 17475*b^2*d^2*x^2 + 16*(-68*d^2 + 75*b^2*(c + d*x)^2)*Cos[2*(a + b*x)]
- 27*(-6*d^2 + 25*b^2*(c + d*x)^2)*Cos[4*(a + b*x)])*Sin[a + b*x])/(405000
0*b^5)
```

### 3.89.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sin^3(a+bx) \cos^2(a+bx) \, dx$$
$$\Big\downarrow \text{4906}$$
$$\int \left(\frac{1}{8}(c+dx)^4 \sin(a+bx) + \frac{1}{16}(c+dx)^4 \sin(3a+3bx) - \frac{1}{16}(c+dx)^4 \sin(5a+5bx)\right) dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{3d^4\cos(a+bx)}{b^5}-\frac{d^4\cos(3a+3bx)}{162b^5}+\frac{3d^4\cos(5a+5bx)}{6250b^5}-\frac{3d^3(c+dx)\sin(a+bx)}{b^4}-\frac{d^3(c+dx)\sin(3a+3bx)}{54b^4}+\frac{3d^3(c+dx)\sin(5a+5bx)}{1250b^4}+\frac{3d^2(c+dx)^2\cos(a+bx)}{2b^3}+\frac{d^2(c+dx)^2\cos(3a+3bx)}{36b^3}-\frac{3d^2(c+dx)^2\cos(5a+5bx)}{500b^3}+\frac{d(c+dx)^3\sin(a+bx)}{2b^2}+\frac{d(c+dx)^3\sin(3a+3bx)}{36b^2}-\frac{d(c+dx)^3\sin(5a+5bx)}{100b^2}-\frac{(c+dx)^4\cos(a+bx)}{8b}-\frac{(c+dx)^4\cos(3a+3bx)}{48b}+\frac{(c+dx)^4\cos(5a+5bx)}{80b}$$

input
```
Int[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
(-3*d^4*Cos[a + b*x])/b^5 + (3*d^2*(c + d*x)^2*Cos[a + b*x])/(2*b^3) - ((c
 + d*x)^4*Cos[a + b*x])/(8*b) - (d^4*Cos[3*a + 3*b*x])/(162*b^5) + (d^2*(c
 + d*x)^2*Cos[3*a + 3*b*x])/(36*b^3) - ((c + d*x)^4*Cos[3*a + 3*b*x])/(48*
b) + (3*d^4*Cos[5*a + 5*b*x])/(6250*b^5) - (3*d^2*(c + d*x)^2*Cos[5*a + 5*
b*x])/(500*b^3) + ((c + d*x)^4*Cos[5*a + 5*b*x])/(80*b) - (3*d^3*(c + d*x)
*Sin[a + b*x])/b^4 + (d*(c + d*x)^3*Sin[a + b*x])/(2*b^2) - (d^3*(c + d*x)
*Sin[3*a + 3*b*x])/(54*b^4) + (d*(c + d*x)^3*Sin[3*a + 3*b*x])/(36*b^2) +
(3*d^3*(c + d*x)*Sin[5*a + 5*b*x])/(1250*b^4) - (d*(c + d*x)^3*Sin[5*a + 5
*b*x])/(100*b^2)
```

### 3.89.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.89.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.78

| method | result |
|---|---|
| parallelrisch | $\frac{\left(-84375b^4(dx+c)^4+112500d^2(dx+c)^2b^2-25000d^4\right)\cos(3xb+3a)+\left(50625b^4(dx+c)^4-24300d^2(dx+c)^2b^2+1944d^4\right)\mathrm{co}}{}$ |
| risch | $-\frac{\left(d^4x^4b^4+4b^4c\,d^3x^3+6b^4c^2d^2x^2+4b^4c^3dx+b^4c^4-12b^2d^4x^2-24b^2c\,d^3x-12b^2c^2d^2+24d^4\right)\cos(xb+a)}{8b^5}+\frac{d\left(b^2d^3x^3+3\right.}{}$ |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |

input
```
int((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4050000*((-84375*b^4*(d*x+c)^4+112500*d^2*(d*x+c)^2*b^2-25000*d^4)*cos(3
*b*x+3*a)+(50625*b^4*(d*x+c)^4-24300*d^2*(d*x+c)^2*b^2+1944*d^4)*cos(5*b*x
+5*a)+112500*b*d*((d*x+c)^2*b^2-2/3*d^2)*(d*x+c)*sin(3*b*x+3*a)-40500*b*((
d*x+c)^2*b^2-6/25*d^2)*d*(d*x+c)*sin(5*b*x+5*a)+(-506250*b^4*(d*x+c)^4+607
5000*d^2*(d*x+c)^2*b^2-12150000*d^4)*cos(b*x+a)+2025000*b*d*((d*x+c)^2*b^2
-6*d^2)*(d*x+c)*sin(b*x+a)-540000*b^4*c^4+6163200*b^2*c^2*d^2-12173056*d^4
)/b^5
```

### 3.89.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.43

$$\int (c+dx)^4\cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$=\frac{81\,(625\,b^4d^4x^4+2500\,b^4cd^3x^3+625\,b^4c^4-300\,b^2c^2d^2+24\,d^4+150\,(25\,b^4c^2d^2-2\,b^2d^4)x^2+100\,(25\,b^4c^3d}{}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/253125*(81*(625*b^4*d^4*x^4 + 2500*b^4*c*d^3*x^3 + 625*b^4*c^4 - 300*b^2
*c^2*d^2 + 24*d^4 + 150*(25*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 100*(25*b^4*c^3
*d - 6*b^2*c*d^3)*x)*cos(b*x + a)^5 - 5*(16875*b^4*d^4*x^4 + 67500*b^4*c*d
^3*x^3 + 16875*b^4*c^4 - 11700*b^2*c^2*d^2 + 1736*d^4 + 450*(225*b^4*c^2*d
^2 - 26*b^2*d^4)*x^2 + 900*(75*b^4*c^3*d - 26*b^2*c*d^3)*x)*cos(b*x + a)^3
 + 120*(2925*b^2*d^4*x^2 + 5850*b^2*c*d^3*x + 2925*b^2*c^2*d^2 - 6284*d^4)
*cos(b*x + a) + 60*(1950*b^3*d^4*x^3 + 5850*b^3*c*d^3*x^2 + 1950*b^3*c^3*d
 - 12568*b*c*d^3 - 27*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 25*b^3*c^3*d -
6*b*c*d^3 + 3*(25*b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^4 + (975*b^3*d^4*
x^3 + 2925*b^3*c*d^3*x^2 + 975*b^3*c^3*d - 434*b*c*d^3 + (2925*b^3*c^2*d^2
 - 434*b*d^4)*x)*cos(b*x + a)^2 + 2*(2925*b^3*c^2*d^2 - 6284*b*d^4)*x)*sin
(b*x + a))/b^5
```

### 3.89.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1098 vs. 2(325) = 650.

Time = 1.13 (sec) , antiderivative size = 1098, normalized size of antiderivative = 3.33

$$\int (c+dx)^4 \cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**4*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

output
```
Piecewise((-c**4*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c**4*cos(a + b*
x)**5/(15*b) - 4*c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 8*c**3*d
*x*cos(a + b*x)**5/(15*b) - 2*c**2*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)*
*3/b - 4*c**2*d**2*x**2*cos(a + b*x)**5/(5*b) - 4*c*d**3*x**3*sin(a + b*x)
**2*cos(a + b*x)**3/(3*b) - 8*c*d**3*x**3*cos(a + b*x)**5/(15*b) - d**4*x*
*4*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*d**4*x**4*cos(a + b*x)**5/(15
*b) + 104*c**3*d*sin(a + b*x)**5/(225*b**2) + 52*c**3*d*sin(a + b*x)**3*co
s(a + b*x)**2/(45*b**2) + 8*c**3*d*sin(a + b*x)*cos(a + b*x)**4/(15*b**2)
+ 104*c**2*d**2*x*sin(a + b*x)**5/(75*b**2) + 52*c**2*d**2*x*sin(a + b*x)*
*3*cos(a + b*x)**2/(15*b**2) + 8*c**2*d**2*x*sin(a + b*x)*cos(a + b*x)**4/
(5*b**2) + 104*c*d**3*x**2*sin(a + b*x)**5/(75*b**2) + 52*c*d**3*x**2*sin(
a + b*x)**3*cos(a + b*x)**2/(15*b**2) + 8*c*d**3*x**2*sin(a + b*x)*cos(a +
 b*x)**4/(5*b**2) + 104*d**4*x**3*sin(a + b*x)**5/(225*b**2) + 52*d**4*x**
3*sin(a + b*x)**3*cos(a + b*x)**2/(45*b**2) + 8*d**4*x**3*sin(a + b*x)*cos
(a + b*x)**4/(15*b**2) + 104*c**2*d**2*sin(a + b*x)**4*cos(a + b*x)/(75*b*
*3) + 676*c**2*d**2*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 1712*c**2
*d**2*cos(a + b*x)**5/(1125*b**3) + 208*c*d**3*x*sin(a + b*x)**4*cos(a + b
*x)/(75*b**3) + 1352*c*d**3*x*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) +
 3424*c*d**3*x*cos(a + b*x)**5/(1125*b**3) + 104*d**4*x**2*sin(a + b*x)**4
*cos(a + b*x)/(75*b**3) + 676*d**4*x**2*sin(a + b*x)**2*cos(a + b*x)**3...
```

### 3.89.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1339 vs. $2(304) = 608$.

Time = 0.31 (sec) , antiderivative size = 1339, normalized size of antiderivative = 4.06

$$\int (c+dx)^4\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4050000*(270000*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c^4 - 1080000*(3*c
os(b*x + a)^5 - 5*cos(b*x + a)^3)*a*c^3*d/b + 1620000*(3*cos(b*x + a)^5 -
5*cos(b*x + a)^3)*a^2*c^2*d^2/b^2 - 1080000*(3*cos(b*x + a)^5 - 5*cos(b*x
+ a)^3)*a^3*c*d^3/b^3 + 270000*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*a^4*d
^4/b^4 + 4500*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*
a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a)
 + 450*sin(b*x + a))*c^3*d/b - 13500*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(
b*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a
) + 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*a*c^2*d^2/b^2 + 13500*(45*(b*x
 + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos
(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*a
^2*c*d^3/b^3 - 4500*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*
x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x
+ 3*a) + 450*sin(b*x + a))*a^3*d^4/b^4 + 450*(27*(25*(b*x + a)^2 - 2)*cos(
5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 6750*((b*x + a)^
2 - 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*(b*x + a)*sin(3
*b*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*c^2*d^2/b^2 - 900*(27*(25*(b*x
 + a)^2 - 2)*cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) -
 6750*((b*x + a)^2 - 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 75
0*(b*x + a)*sin(3*b*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*a*c*d^3/b^...
```

### 3.89.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.61

$$\int (c+dx)^4\cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{(625\,b^4d^4x^4+2500\,b^4cd^3x^3+3750\,b^4c^2d^2x^2+2500\,b^4c^3dx+625\,b^4c^4-300\,b^2d^4x^2-600\,b^2cd^3x-300\,b^2c}{50000\,b^5}$$
$$-\frac{(27\,b^4d^4x^4+108\,b^4cd^3x^3+162\,b^4c^2d^2x^2+108\,b^4c^3dx+27\,b^4c^4-36\,b^2d^4x^2-72\,b^2cd^3x-36\,b^2c^2d^2+8}{1296\,b^5}$$
$$-\frac{(b^4d^4x^4+4\,b^4cd^3x^3+6\,b^4c^2d^2x^2+4\,b^4c^3dx+b^4c^4-12\,b^2d^4x^2-24\,b^2cd^3x-12\,b^2c^2d^2+24\,d^4)\cos(bx}{8\,b^5}$$
$$-\frac{(25\,b^3d^4x^3+75\,b^3cd^3x^2+75\,b^3c^2d^2x+25\,b^3c^3d-6\,bd^4x-6\,bcd^3)\sin(5\,bx+5\,a)}{2500\,b^5}$$
$$+\frac{(3\,b^3d^4x^3+9\,b^3cd^3x^2+9\,b^3c^2d^2x+3\,b^3c^3d-2\,bd^4x-2\,bcd^3)\sin(3\,bx+3\,a)}{108\,b^5}$$
$$+\frac{(b^3d^4x^3+3\,b^3cd^3x^2+3\,b^3c^2d^2x+b^3c^3d-6\,bd^4x-6\,bcd^3)\sin(bx+a)}{2\,b^5}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/50000*(625*b^4*d^4*x^4 + 2500*b^4*c*d^3*x^3 + 3750*b^4*c^2*d^2*x^2 + 250
0*b^4*c^3*d*x + 625*b^4*c^4 - 300*b^2*d^4*x^2 - 600*b^2*c*d^3*x - 300*b^2*
c^2*d^2 + 24*d^4)*cos(5*b*x + 5*a)/b^5 - 1/1296*(27*b^4*d^4*x^4 + 108*b^4*
c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4*c^3*d*x + 27*b^4*c^4 - 36*b^2*d^
4*x^2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 + 8*d^4)*cos(3*b*x + 3*a)/b^5 - 1/
8*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4
*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b^2*c^2*d^2 + 24*d^4)*cos(b*x
+ a)/b^5 - 1/2500*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 75*b^3*c^2*d^2*x +
25*b^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*sin(5*b*x + 5*a)/b^5 + 1/108*(3*b^3*
d^4*x^3 + 9*b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*
b*c*d^3)*sin(3*b*x + 3*a)/b^5 + 1/2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3
*c^2*d^2*x + b^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*sin(b*x + a)/b^5
```

### 3.89.9 Mupad [B] (verification not implemented)

Time = 27.72 (sec) , antiderivative size = 816, normalized size of antiderivative = 2.47

$$\int (c+dx)^4 \cos^2(a+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{3\,d^4\,\cos\left(a+b\,x\right)+\frac{d^4\,\cos(3\,a+3\,b\,x)}{162}-\frac{3\,d^4\,\cos(5\,a+5\,b\,x)}{6250}+\frac{b^4\,c^4\,\cos(a+b\,x)}{8}+\frac{b^4\,c^4\,\cos(3\,a+3\,b\,x)}{48}-\frac{b^4\,c^4\,\cos(5\,a+5\,b\,x)}{80}-$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^4,x)
```

output

```
-(3*d^4*cos(a + b*x) + (d^4*cos(3*a + 3*b*x))/162 - (3*d^4*cos(5*a + 5*b*x
))/6250 + (b^4*c^4*cos(a + b*x))/8 + (b^4*c^4*cos(3*a + 3*b*x))/48 - (b^4*
c^4*cos(5*a + 5*b*x))/80 - (3*b^2*c^2*d^2*cos(a + b*x))/2 - (b^3*c^3*d*sin
(3*a + 3*b*x))/36 + (b^3*c^3*d*sin(5*a + 5*b*x))/100 - (3*b^2*d^4*x^2*cos(
a + b*x))/2 + (b^4*d^4*x^4*cos(a + b*x))/8 - (b^3*d^4*x^3*sin(a + b*x))/2
+ 3*b*c*d^3*sin(a + b*x) - (b^2*c^2*d^2*cos(3*a + 3*b*x))/36 + (3*b^2*c^2*
d^2*cos(5*a + 5*b*x))/500 + 3*b*d^4*x*sin(a + b*x) - (b^2*d^4*x^2*cos(3*a
+ 3*b*x))/36 + (3*b^2*d^4*x^2*cos(5*a + 5*b*x))/500 + (b^4*d^4*x^4*cos(3*a
 + 3*b*x))/48 - (b^4*d^4*x^4*cos(5*a + 5*b*x))/80 - (b^3*d^4*x^3*sin(3*a +
 3*b*x))/36 + (b^3*d^4*x^3*sin(5*a + 5*b*x))/100 + (b*c*d^3*sin(3*a + 3*b*
x))/54 - (3*b*c*d^3*sin(5*a + 5*b*x))/1250 - (b^3*c^3*d*sin(a + b*x))/2 +
(b*d^4*x*sin(3*a + 3*b*x))/54 - (3*b*d^4*x*sin(5*a + 5*b*x))/1250 - 3*b^2*
c*d^3*x*cos(a + b*x) + (b^4*c^3*d*x*cos(a + b*x))/2 + (b^4*c^2*d^2*x^2*cos
(3*a + 3*b*x))/8 - (3*b^4*c^2*d^2*x^2*cos(5*a + 5*b*x))/40 - (b^2*c*d^3*x*
cos(3*a + 3*b*x))/18 + (b^4*c^3*d*x*cos(3*a + 3*b*x))/12 + (3*b^2*c*d^3*x*
cos(5*a + 5*b*x))/250 - (b^4*c^3*d*x*cos(5*a + 5*b*x))/20 + (b^4*c*d^3*x^3
*cos(a + b*x))/2 - (3*b^3*c^2*d^2*x*sin(a + b*x))/2 - (3*b^3*c*d^3*x^2*sin
(a + b*x))/2 + (b^4*c*d^3*x^3*cos(3*a + 3*b*x))/12 - (b^4*c*d^3*x^3*cos(5*
a + 5*b*x))/20 + (3*b^4*c^2*d^2*x^2*cos(a + b*x))/4 - (b^3*c^2*d^2*x*sin(3
*a + 3*b*x))/12 - (b^3*c*d^3*x^2*sin(3*a + 3*b*x))/12 + (3*b^3*c^2*d^2*...
```

## 3.89. $\int (c + dx)^4 \cos^2(a + bx) \sin^3(a + bx)\, dx$

# 3.90 $\int (c+dx)^3 \cos^2(a+bx) \sin^3(a+bx) \, dx$


3.90.1 Optimal result . . . . . . . . . . 762
3.90.2 Mathematica [A] (verified) . . . . . . . . . . 763
3.90.3 Rubi [A] (verified) . . . . . . . . . . 763
3.90.4 Maple [A] (verified) . . . . . . . . . . 764
3.90.5 Fricas [A] (verification not implemented) . . . . . . . . . . 765
3.90.6 Sympy [B] (verification not implemented) . . . . . . . . . . 765
3.90.7 Maxima [B] (verification not implemented) . . . . . . . . . . 766
3.90.8 Giac [A] (verification not implemented) . . . . . . . . . . 767
3.90.9 Mupad [B] (verification not implemented) . . . . . . . . . . 768


## 3.90.1 Optimal result

Integrand size = 24, antiderivative size = 259

$$\int (c+dx)^3 \cos^2(a+bx) \sin^3(a+bx) \, dx = \frac{3d^2(c+dx)\cos(a+bx)}{4b^3} - \frac{(c+dx)^3\cos(a+bx)}{8b} + \frac{d^2(c+dx)\cos(3a+3bx)}{72b^3} - \frac{(c+dx)^3\cos(3a+3bx)}{48b} - \frac{3d^2(c+dx)\cos(5a+5bx)}{1000b^3} + \frac{(c+dx)^3\cos(5a+5bx)}{80b} - \frac{3d^3\sin(a+bx)}{4b^4} + \frac{3d(c+dx)^2\sin(a+bx)}{8b^2} - \frac{d^3\sin(3a+3bx)}{216b^4} + \frac{d(c+dx)^2\sin(3a+3bx)}{48b^2} + \frac{3d^3\sin(5a+5bx)}{5000b^4} - \frac{3d(c+dx)^2\sin(5a+5bx)}{400b^2}$$

output
```
3/4*d^2*(d*x+c)*cos(b*x+a)/b^3-1/8*(d*x+c)^3*cos(b*x+a)/b+1/72*d^2*(d*x+c)
*cos(3*b*x+3*a)/b^3-1/48*(d*x+c)^3*cos(3*b*x+3*a)/b-3/1000*d^2*(d*x+c)*cos
(5*b*x+5*a)/b^3+1/80*(d*x+c)^3*cos(5*b*x+5*a)/b-3/4*d^3*sin(b*x+a)/b^4+3/8
*d*(d*x+c)^2*sin(b*x+a)/b^2-1/216*d^3*sin(3*b*x+3*a)/b^4+1/48*d*(d*x+c)^2*
sin(3*b*x+3*a)/b^2+3/5000*d^3*sin(5*b*x+5*a)/b^4-3/400*d*(d*x+c)^2*sin(5*b
*x+5*a)/b^2
```

### 3.90.2 Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.42

$$\int (c+dx)^3 \cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{-33750b(c+dx)\left(-6d^2+b^2(c+dx)^2\right)\cos(a+bx)-1875b(c+dx)\left(-2d^2+3b^2(c+dx)^2\right)\cos(3(a+bx))}{}$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
(-33750*b*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 1875*b*(c +
d*x)*(-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 3375*b^3*c^3*Cos[5*(a
 + b*x)] - 810*b*c*d^2*Cos[5*(a + b*x)] + 10125*b^3*c^2*d*x*Cos[5*(a + b*x
)] - 810*b*d^3*x*Cos[5*(a + b*x)] + 10125*b^3*c*d^2*x^2*Cos[5*(a + b*x)] +
 3375*b^3*d^3*x^3*Cos[5*(a + b*x)] + 101250*b^2*c^2*d*Sin[a + b*x] - 20250
0*d^3*Sin[a + b*x] + 202500*b^2*c*d^2*x*Sin[a + b*x] + 101250*b^2*d^3*x^2*
Sin[a + b*x] + 5625*b^2*c^2*d*Sin[3*(a + b*x)] - 1250*d^3*Sin[3*(a + b*x)]
 + 11250*b^2*c*d^2*x*Sin[3*(a + b*x)] + 5625*b^2*d^3*x^2*Sin[3*(a + b*x)]
- 2025*b^2*c^2*d*Sin[5*(a + b*x)] + 162*d^3*Sin[5*(a + b*x)] - 4050*b^2*c*
d^2*x*Sin[5*(a + b*x)] - 2025*b^2*d^3*x^2*Sin[5*(a + b*x)])/(270000*b^4)
```

### 3.90.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin^3(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 4906}$$

$$\int \left(\frac{1}{8}(c+dx)^3\sin(a+bx)+\frac{1}{16}(c+dx)^3\sin(3a+3bx)-\frac{1}{16}(c+dx)^3\sin(5a+5bx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$
-\frac{3d^3\sin(a+bx)}{4b^4}-\frac{d^3\sin(3a+3bx)}{216b^4}+\frac{3d^3\sin(5a+5bx)}{5000b^4}+\frac{3d^2(c+dx)\cos(a+bx)}{4b^3}+\frac{d^2(c+dx)\cos(3a+3bx)}{72b^3}-\frac{3d^2(c+dx)\cos(5a+5bx)}{1000b^3}+\frac{3d(c+dx)^2\sin(a+bx)}{8b^2}+\frac{d(c+dx)^2\sin(3a+3bx)}{48b^2}-\frac{3d(c+dx)^2\sin(5a+5bx)}{400b^2}-\frac{(c+dx)^3\cos(a+bx)}{8b}-\frac{(c+dx)^3\cos(3a+3bx)}{48b}+\frac{(c+dx)^3\cos(5a+5bx)}{80b}
$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
(3*d^2*(c + d*x)*Cos[a + b*x])/(4*b^3) - ((c + d*x)^3*Cos[a + b*x])/(8*b)
+ (d^2*(c + d*x)*Cos[3*a + 3*b*x])/(72*b^3) - ((c + d*x)^3*Cos[3*a + 3*b*x
])/(48*b) - (3*d^2*(c + d*x)*Cos[5*a + 5*b*x])/(1000*b^3) + ((c + d*x)^3*C
os[5*a + 5*b*x])/(80*b) - (3*d^3*Sin[a + b*x])/(4*b^4) + (3*d*(c + d*x)^2*
Sin[a + b*x])/(8*b^2) - (d^3*Sin[3*a + 3*b*x])/(216*b^4) + (d*(c + d*x)^2*
Sin[3*a + 3*b*x])/(48*b^2) + (3*d^3*Sin[5*a + 5*b*x])/(5000*b^4) - (3*d*(c
 + d*x)^2*Sin[5*a + 5*b*x])/(400*b^2)
```

### 3.90.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.90.4 Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| parallelrisch | $\frac{-375b\left((dx+c)^2b^2-\frac{2d^2}{3}\right)(dx+c)\cos(3xb+3a)+225b\left((dx+c)^2b^2-\frac{6d^2}{25}\right)(dx+c)\cos(5xb+5a)+375d\left((dx+c)^2b^2-\frac{2d^2}{9}\right)\mathrm{si}}{}$ |
| risch | $-\frac{\left(b^2d^3x^3+3b^2c\,d^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6c\,d^2\right)\cos(xb+a)}{8b^3}+\frac{3d\left(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2\right)\sin(xb+a)}{8b^4}+\frac{(25}{}$ |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |

input
```
int((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/18000*(-375*b*((d*x+c)^2*b^2-2/3*d^2)*(d*x+c)*cos(3*b*x+3*a)+225*b*((d*x
+c)^2*b^2-6/25*d^2)*(d*x+c)*cos(5*b*x+5*a)+375*d*((d*x+c)^2*b^2-2/9*d^2)*s
in(3*b*x+3*a)-135*d*((d*x+c)^2*b^2-2/25*d^2)*sin(5*b*x+5*a)-2250*b*((d*x+c
)^2*b^2-6*d^2)*(d*x+c)*cos(b*x+a)+6750*d*((d*x+c)^2*b^2-2*d^2)*sin(b*x+a)-
2400*b^3*c^3+13696*c*d^2*b)/b^4
```

### 3.90.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.14

$$\int (c+dx)^3\cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{135\,(25\,b^3d^3x^3+75\,b^3cd^2x^2+25\,b^3c^3-6\,bcd^2+3\,(25\,b^3c^2d-2\,bd^3)x)\cos{(bx+a)}^5-75\,(75\,b^3d^3x^3+225}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/16875*(135*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 25*b^3*c^3 - 6*b*c*d^2 +
 3*(25*b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^5 - 75*(75*b^3*d^3*x^3 + 225*b
^3*c*d^2*x^2 + 75*b^3*c^3 - 26*b*c*d^2 + (225*b^3*c^2*d - 26*b*d^3)*x)*cos
(b*x + a)^3 + 11700*(b*d^3*x + b*c*d^2)*cos(b*x + a) + (5850*b^2*d^3*x^2 +
 11700*b^2*c*d^2*x + 5850*b^2*c^2*d - 81*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x
+ 25*b^2*c^2*d - 2*d^3)*cos(b*x + a)^4 - 12568*d^3 + (2925*b^2*d^3*x^2 + 5
850*b^2*c*d^2*x + 2925*b^2*c^2*d - 434*d^3)*cos(b*x + a)^2)*sin(b*x + a))/
b^4
```

### 3.90.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(255) = 510$.

Time = 0.84 (sec) , antiderivative size = 690, normalized size of antiderivative = 2.66

$$\int (c+dx)^3\cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$= \begin{cases} -\frac{c^3\sin^2{(a+bx)}\cos^3{(a+bx)}}{3b} - \frac{2c^3\cos^5{(a+bx)}}{15b} - \frac{c^2dx\sin^2{(a+bx)}\cos^3{(a+bx)}}{b} - \frac{2c^2dx\cos^5{(a+bx)}}{5b} - \frac{cd^2x^2\sin^2{(a+bx)}\cos^3{(a+bx)}}{b} \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\sin^3{(a)}\cos^2{(a)} \end{cases}$$

input
```
integrate((d*x+c)**3*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

output
```
Piecewise((-c**3*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c**3*cos(a + b*
x)**5/(15*b) - c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**3/b - 2*c**2*d*x*cos
(a + b*x)**5/(5*b) - c*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**3/b - 2*c*d
**2*x**2*cos(a + b*x)**5/(5*b) - d**3*x**3*sin(a + b*x)**2*cos(a + b*x)**3
/(3*b) - 2*d**3*x**3*cos(a + b*x)**5/(15*b) + 26*c**2*d*sin(a + b*x)**5/(7
5*b**2) + 13*c**2*d*sin(a + b*x)**3*cos(a + b*x)**2/(15*b**2) + 2*c**2*d*s
in(a + b*x)*cos(a + b*x)**4/(5*b**2) + 52*c*d**2*x*sin(a + b*x)**5/(75*b**
2) + 26*c*d**2*x*sin(a + b*x)**3*cos(a + b*x)**2/(15*b**2) + 4*c*d**2*x*si
n(a + b*x)*cos(a + b*x)**4/(5*b**2) + 26*d**3*x**2*sin(a + b*x)**5/(75*b**
2) + 13*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)**2/(15*b**2) + 2*d**3*x**2*
sin(a + b*x)*cos(a + b*x)**4/(5*b**2) + 52*c*d**2*sin(a + b*x)**4*cos(a +
b*x)/(75*b**3) + 338*c*d**2*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 8
56*c*d**2*cos(a + b*x)**5/(1125*b**3) + 52*d**3*x*sin(a + b*x)**4*cos(a +
b*x)/(75*b**3) + 338*d**3*x*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 8
56*d**3*x*cos(a + b*x)**5/(1125*b**3) - 12568*d**3*sin(a + b*x)**5/(16875*
b**4) - 5114*d**3*sin(a + b*x)**3*cos(a + b*x)**2/(3375*b**4) - 856*d**3*s
in(a + b*x)*cos(a + b*x)**4/(1125*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x*
*2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**3*cos(a)**2, True))
```

### 3.90.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 766 vs. $2(235) = 470$.

Time = 0.27 (sec) , antiderivative size = 766, normalized size of antiderivative = 2.96

$$\int (c+dx)^3 \cos^2(a+bx) \sin^3(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/270000*(18000*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c^3 - 54000*(3*cos(b
*x + a)^5 - 5*cos(b*x + a)^3)*a*c^2*d/b + 54000*(3*cos(b*x + a)^5 - 5*cos(
b*x + a)^3)*a^2*c*d^2/b^2 - 18000*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*a^
3*d^3/b^3 + 225*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x +
3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*
a) + 450*sin(b*x + a))*c^2*d/b - 450*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(
b*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a
) + 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*a*c*d^2/b^2 + 225*(45*(b*x + a
)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x
 + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*a^2*d
^3/b^3 + 15*(27*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2
 - 2)*cos(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*cos(b*x + a) - 270*(b*x +
a)*sin(5*b*x + 5*a) + 750*(b*x + a)*sin(3*b*x + 3*a) + 13500*(b*x + a)*sin
(b*x + a))*c*d^2/b^2 - 15*(27*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) - 125*
(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*cos(b*x + a)
 - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*(b*x + a)*sin(3*b*x + 3*a) + 13500
*(b*x + a)*sin(b*x + a))*a*d^3/b^3 + (135*(25*(b*x + a)^3 - 6*b*x - 6*a)*c
os(5*b*x + 5*a) - 1875*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) - 33
750*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) - 81*(25*(b*x + a)^2 - 2)*sin
(5*b*x + 5*a) + 625*(9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 101250*((b*x...
```

### 3.90.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.36

$$\int (c+dx)^3\cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{(25\,b^3d^3x^3+75\,b^3cd^2x^2+75\,b^3c^2dx+25\,b^3c^3-6\,bd^3x-6\,bcd^2)\cos{(5\,bx+5\,a)}}{2000\,b^4} - \frac{(3\,b^3d^3x^3+9\,b^3cd^2x^2+9\,b^3c^2dx+3\,b^3c^3-2\,bd^3x-2\,bcd^2)\cos{(3\,bx+3\,a)}}{144\,b^4} - \frac{(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3-6\,bd^3x-6\,bcd^2)\cos{(bx+a)}}{8\,b^4} - \frac{3\,(25\,b^2d^3x^2+50\,b^2cd^2x+25\,b^2c^2d-2\,d^3)\sin{(5\,bx+5\,a)}}{10000\,b^4} + \frac{(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3)\sin{(3\,bx+3\,a)}}{432\,b^4} + \frac{3\,(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d-2\,d^3)\sin{(bx+a)}}{8\,b^4}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/2000*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 75*b^3*c^2*d*x + 25*b^3*c^3 -
6*b*d^3*x - 6*b*c*d^2)*cos(5*b*x + 5*a)/b^4 - 1/144*(3*b^3*d^3*x^3 + 9*b^3
*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d^3*x - 2*b*c*d^2)*cos(3*b*x
+ 3*a)/b^4 - 1/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3
- 6*b*d^3*x - 6*b*c*d^2)*cos(b*x + a)/b^4 - 3/10000*(25*b^2*d^3*x^2 + 50*b
^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*sin(5*b*x + 5*a)/b^4 + 1/432*(9*b^2*d^3
*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*sin(3*b*x + 3*a)/b^4 + 3/8*(b
^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sin(b*x + a)/b^4
```

### 3.90.9 Mupad [B] (verification not implemented)

Time = 3.00 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.99

$$\int (c+dx)^3 \cos^2(a+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{3\,d^3\,\sin(a+b\,x)}{4}+\frac{d^3\,\sin(3\,a+3\,b\,x)}{216}-\frac{3\,d^3\,\sin(5\,a+5\,b\,x)}{5000}+\frac{b^3\,c^3\,\cos(a+b\,x)}{8}+\frac{b^3\,c^3\,\cos(3\,a+3\,b\,x)}{48}-\frac{b^3\,c^3\,\cos(5\,a+5\,b\,x)}{80}-\frac{b^2\,c^2\,d}{}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^3,x)
```

output
```
-((3*d^3*sin(a + b*x))/4 + (d^3*sin(3*a + 3*b*x))/216 - (3*d^3*sin(5*a + 5
*b*x))/5000 + (b^3*c^3*cos(a + b*x))/8 + (b^3*c^3*cos(3*a + 3*b*x))/48 - (
b^3*c^3*cos(5*a + 5*b*x))/80 - (b^2*c^2*d*sin(3*a + 3*b*x))/48 + (3*b^2*c^
2*d*sin(5*a + 5*b*x))/400 + (b^3*d^3*x^3*cos(a + b*x))/8 - (3*b^2*d^3*x^2*
sin(a + b*x))/8 - (3*b*c*d^2*cos(a + b*x))/4 - (3*b*d^3*x*cos(a + b*x))/4
+ (b^3*d^3*x^3*cos(3*a + 3*b*x))/48 - (b^3*d^3*x^3*cos(5*a + 5*b*x))/80 -
(b^2*d^3*x^2*sin(3*a + 3*b*x))/48 + (3*b^2*d^3*x^2*sin(5*a + 5*b*x))/400 -
 (b*c*d^2*cos(3*a + 3*b*x))/72 + (3*b*c*d^2*cos(5*a + 5*b*x))/1000 - (3*b^
2*c^2*d*sin(a + b*x))/8 - (b*d^3*x*cos(3*a + 3*b*x))/72 + (3*b*d^3*x*cos(5
*a + 5*b*x))/1000 + (3*b^3*c^2*d*x*cos(a + b*x))/8 - (3*b^2*c*d^2*x*sin(a
+ b*x))/4 + (b^3*c^2*d*x*cos(3*a + 3*b*x))/16 - (3*b^3*c^2*d*x*cos(5*a + 5
*b*x))/80 + (3*b^3*c*d^2*x^2*cos(a + b*x))/8 - (b^2*c*d^2*x*sin(3*a + 3*b*
x))/24 + (3*b^2*c*d^2*x*sin(5*a + 5*b*x))/200 + (b^3*c*d^2*x^2*cos(3*a + 3
*b*x))/16 - (3*b^3*c*d^2*x^2*cos(5*a + 5*b*x))/80)/b^4
```

# 3.91 $\int (c+dx)^2 \cos^2(a+bx) \sin^3(a+bx)\,dx$


3.91.1 Optimal result . . . . . 769
3.91.2 Mathematica [A] (verified) . . . . . 770
3.91.3 Rubi [A] (verified) . . . . . 770
3.91.4 Maple [A] (verified) . . . . . 771
3.91.5 Fricas [A] (verification not implemented) . . . . . 772
3.91.6 Sympy [B] (verification not implemented) . . . . . 772
3.91.7 Maxima [B] (verification not implemented) . . . . . 773
3.91.8 Giac [A] (verification not implemented) . . . . . 774
3.91.9 Mupad [B] (verification not implemented) . . . . . 774


## 3.91.1 Optimal result

Integrand size = 24, antiderivative size = 184

$$\begin{aligned}\int (c+dx)^2 \cos^2(a+bx)\sin^3(a+bx)\,dx &= \frac{d^2\cos(a+bx)}{4b^3} - \frac{(c+dx)^2\cos(a+bx)}{8b} \\ &+ \frac{d^2\cos(3a+3bx)}{216b^3} - \frac{(c+dx)^2\cos(3a+3bx)}{48b} \\ &- \frac{d^2\cos(5a+5bx)}{1000b^3} + \frac{(c+dx)^2\cos(5a+5bx)}{80b} \\ &+ \frac{d(c+dx)\sin(a+bx)}{4b^2} \\ &+ \frac{d(c+dx)\sin(3a+3bx)}{72b^2} \\ &- \frac{d(c+dx)\sin(5a+5bx)}{200b^2}\end{aligned}$$

output
```
1/4*d^2*cos(b*x+a)/b^3-1/8*(d*x+c)^2*cos(b*x+a)/b+1/216*d^2*cos(3*b*x+3*a)
/b^3-1/48*(d*x+c)^2*cos(3*b*x+3*a)/b-1/1000*d^2*cos(5*b*x+5*a)/b^3+1/80*(d
*x+c)^2*cos(5*b*x+5*a)/b+1/4*d*(d*x+c)*sin(b*x+a)/b^2+1/72*d*(d*x+c)*sin(3
*b*x+3*a)/b^2-1/200*d*(d*x+c)*sin(5*b*x+5*a)/b^2
```

### 3.91.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

$$\int (c+dx)^2\cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$=\frac{-6750(-2d^2+b^2(c+dx)^2)\cos(a+bx)-125(-2d^2+9b^2(c+dx)^2)\cos(3(a+bx))+27(-2d^2+25b^2(c+}{54000b^3}$$

input
```
Integrate[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
(-6750*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 125*(-2*d^2 + 9*b^2*(c +
d*x)^2)*Cos[3*(a + b*x)] + 27*(-2*d^2 + 25*b^2*(c + d*x)^2)*Cos[5*(a + b*x
)] + 30*b*d*(c + d*x)*(450*Sin[a + b*x] + 25*Sin[3*(a + b*x)] - 9*Sin[5*(a
 + b*x)]))/(54000*b^3)
```

### 3.91.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\sin^3(a+bx)\cos^2(a+bx)\,dx$$
$$\Big\downarrow \text{4906}$$
$$\int \left(\frac{1}{8}(c+dx)^2\sin(a+bx)+\frac{1}{16}(c+dx)^2\sin(3a+3bx)-\frac{1}{16}(c+dx)^2\sin(5a+5bx)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{d^2\cos(a+bx)}{4b^3}+\frac{d^2\cos(3a+3bx)}{216b^3}-\frac{d^2\cos(5a+5bx)}{1000b^3}+\frac{d(c+dx)\sin(a+bx)}{4b^2}+\frac{d(c+dx)\sin(3a+3bx)}{72b^2}-\frac{d(c+dx)\sin(5a+5bx)}{200b^2}-\frac{(c+dx)^2\cos(a+bx)}{8b}-\frac{(c+dx)^2\cos(3a+3bx)}{48b}+\frac{(c+dx)^2\cos(5a+5bx)}{80b}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
(d^2*Cos[a + b*x])/(4*b^3) - ((c + d*x)^2*Cos[a + b*x])/(8*b) + (d^2*Cos[3
*a + 3*b*x])/(216*b^3) - ((c + d*x)^2*Cos[3*a + 3*b*x])/(48*b) - (d^2*Cos[
5*a + 5*b*x])/(1000*b^3) + ((c + d*x)^2*Cos[5*a + 5*b*x])/(80*b) + (d*(c +
 d*x)*Sin[a + b*x])/(4*b^2) + (d*(c + d*x)*Sin[3*a + 3*b*x])/(72*b^2) - (d
*(c + d*x)*Sin[5*a + 5*b*x])/(200*b^2)
```

### 3.91.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.91.4 Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.83

| method | result |
|---|---|
| parallelrisch | $\frac{\left(-1125(dx+c)^2b^2+250d^2\right)\cos(3xb+3a)+\left(675(dx+c)^2b^2-54d^2\right)\cos(5xb+5a)+750bd(dx+c)\sin(3xb+3a)-270bd(dx+}{54000b^3}$ |
| risch | $-\frac{\left(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2\right)\cos(xb+a)}{8b^3}+\frac{d(dx+c)\sin(xb+a)}{4b^2}+\frac{\left(25x^2d^2b^2+50b^2cdx+25b^2c^2-2d^2\right)\cos(5xb+5a}{2000b^3}$ |
| derivativedivides | $\frac{\frac{a^2d^2\left(-\frac{\sin(xb+a)^2\cos(xb+a)^3}{5}-\frac{2\cos(xb+a)^3}{15}\right)}{b^2}-\frac{2acd\left(-\frac{\sin(xb+a)^2\cos(xb+a)^3}{5}-\frac{2\cos(xb+a)^3}{15}\right)}{b}-\frac{2a\,d^2\left(-\frac{(xb+a)\left(2+\sin(xb+}{3}\right.}{}}{}$ |
| default | $\frac{\frac{a^2d^2\left(-\frac{\sin(xb+a)^2\cos(xb+a)^3}{5}-\frac{2\cos(xb+a)^3}{15}\right)}{b^2}-\frac{2acd\left(-\frac{\sin(xb+a)^2\cos(xb+a)^3}{5}-\frac{2\cos(xb+a)^3}{15}\right)}{b}-\frac{2a\,d^2\left(-\frac{(xb+a)\left(2+\sin(xb+}{3}\right.}{}}{}$ |

input
```
int((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/54000*((-1125*(d*x+c)^2*b^2+250*d^2)*cos(3*b*x+3*a)+(675*(d*x+c)^2*b^2-5
4*d^2)*cos(5*b*x+5*a)+750*b*d*(d*x+c)*sin(3*b*x+3*a)-270*b*d*(d*x+c)*sin(5
*b*x+5*a)+(-6750*(d*x+c)^2*b^2+13500*d^2)*cos(b*x+a)+13500*b*d*(d*x+c)*sin
(b*x+a)-7200*b^2*c^2+13696*d^2)/b^3
```

### 3.91.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.90

$$\int (c+dx)^2\cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$=\frac{27\,(25\,b^2d^2x^2+50\,b^2cdx+25\,b^2c^2-2\,d^2)\cos(bx+a)^5-5\,(225\,b^2d^2x^2+450\,b^2cdx+225\,b^2c^2-26\,d^2)\,\mathrm{co}}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/3375*(27*(25*b^2*d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*cos(b*x +
a)^5 - 5*(225*b^2*d^2*x^2 + 450*b^2*c*d*x + 225*b^2*c^2 - 26*d^2)*cos(b*x
+ a)^3 + 780*d^2*cos(b*x + a) - 30*(9*(b*d^2*x + b*c*d)*cos(b*x + a)^4 - 2
6*b*d^2*x - 26*b*c*d - 13*(b*d^2*x + b*c*d)*cos(b*x + a)^2)*sin(b*x + a))/
b^3
```

### 3.91.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(172) = 344$.

Time = 0.58 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.08

$$\int (c+dx)^2\cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$=\begin{cases}-\frac{c^2\sin^2(a+bx)\cos^3(a+bx)}{3b}-\frac{2c^2\cos^5(a+bx)}{15b}-\frac{2cdx\sin^2(a+bx)\cos^3(a+bx)}{3b}-\frac{4cdx\cos^5(a+bx)}{15b}-\frac{d^2x^2\sin^2(a+bx)\cos^3(a+bx)}{3b}- \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sin^3(a)\cos^2(a)\end{cases}$$

input
```
integrate((d*x+c)**2*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

output
```
Piecewise((-c**2*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c**2*cos(a + b*
x)**5/(15*b) - 2*c*d*x*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 4*c*d*x*cos
(a + b*x)**5/(15*b) - d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*
d**2*x**2*cos(a + b*x)**5/(15*b) + 52*c*d*sin(a + b*x)**5/(225*b**2) + 26*
c*d*sin(a + b*x)**3*cos(a + b*x)**2/(45*b**2) + 4*c*d*sin(a + b*x)*cos(a +
 b*x)**4/(15*b**2) + 52*d**2*x*sin(a + b*x)**5/(225*b**2) + 26*d**2*x*sin(
a + b*x)**3*cos(a + b*x)**2/(45*b**2) + 4*d**2*x*sin(a + b*x)*cos(a + b*x)
**4/(15*b**2) + 52*d**2*sin(a + b*x)**4*cos(a + b*x)/(225*b**3) + 338*d**2
*sin(a + b*x)**2*cos(a + b*x)**3/(675*b**3) + 856*d**2*cos(a + b*x)**5/(33
75*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**3*cos(a)**
2, True))
```

### 3.91.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(166) = 332$.

Time = 0.26 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.04

$$\int (c+dx)^2 \cos^2(a+bx) \sin^3(a+bx)\, dx$$

$$= \frac{3600\left(3\cos(bx+a)^5 - 5\cos(bx+a)^3\right)c^2 - \frac{7200\left(3\cos(bx+a)^5-5\cos(bx+a)^3\right)acd}{b} + \frac{3600\left(3\cos(bx+a)^5-5\cos(bx+a)^3\right)a}{b^2}}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/54000*(3600*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c^2 - 7200*(3*cos(b*x
+ a)^5 - 5*cos(b*x + a)^3)*a*c*d/b + 3600*(3*cos(b*x + a)^5 - 5*cos(b*x +
a)^3)*a^2*d^2/b^2 + 30*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3
*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b
*x + 3*a) + 450*sin(b*x + a))*c*d/b - 30*(45*(b*x + a)*cos(5*b*x + 5*a) -
75*(b*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x +
 5*a) + 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*a*d^2/b^2 + (27*(25*(b*x +
 a)^2 - 2)*cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 6
750*((b*x + a)^2 - 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*
(b*x + a)*sin(3*b*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*d^2/b^2)/b
```

## 3.91.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.14

$$\int (c+dx)^2\cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{(25\,b^2d^2x^2+50\,b^2cdx+25\,b^2c^2-2\,d^2)\cos(5\,bx+5\,a)}{2000\,b^3} - \frac{(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2)\cos(3\,bx+3\,a)}{432\,b^3} - \frac{(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,d^2)\cos(bx+a)}{8\,b^3} - \frac{(bd^2x+bcd)\sin(5\,bx+5\,a)}{200\,b^3} + \frac{(bd^2x+bcd)\sin(3\,bx+3\,a)}{72\,b^3} + \frac{(bd^2x+bcd)\sin(bx+a)}{4\,b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/2000*(25*b^2*d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*cos(5*b*x + 5*
a)/b^3 - 1/432*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(3*b*
x + 3*a)/b^3 - 1/8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x +
 a)/b^3 - 1/200*(b*d^2*x + b*c*d)*sin(5*b*x + 5*a)/b^3 + 1/72*(b*d^2*x + b
*c*d)*sin(3*b*x + 3*a)/b^3 + 1/4*(b*d^2*x + b*c*d)*sin(b*x + a)/b^3
```

## 3.91.9 Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.35

$$\int (c+dx)^2\cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{780\,d^2\,\cos(a+b\,x)+130\,d^2\cos(a+b\,x)^3-54\,d^2\cos(a+b\,x)^5-1125\,b^2\,c^2\cos(a+b\,x)^3+675\,b^2\,c^2\cos}{}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^2,x)
```

output
```
(780*d^2*cos(a + b*x) + 130*d^2*cos(a + b*x)^3 - 54*d^2*cos(a + b*x)^5 - 1
125*b^2*c^2*cos(a + b*x)^3 + 675*b^2*c^2*cos(a + b*x)^5 + 780*b*d^2*x*sin(
a + b*x) - 1125*b^2*d^2*x^2*cos(a + b*x)^3 + 675*b^2*d^2*x^2*cos(a + b*x)^
5 + 780*b*c*d*sin(a + b*x) - 2250*b^2*c*d*x*cos(a + b*x)^3 + 1350*b^2*c*d*
x*cos(a + b*x)^5 + 390*b*d^2*x*cos(a + b*x)^2*sin(a + b*x) - 270*b*d^2*x*c
os(a + b*x)^4*sin(a + b*x) + 390*b*c*d*cos(a + b*x)^2*sin(a + b*x) - 270*b
*c*d*cos(a + b*x)^4*sin(a + b*x))/(3375*b^3)
```

# 3.92 $\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx$


3.92.1 Optimal result . . . . . . . . . . 775
3.92.2 Mathematica [A] (verified) . . . . . . . . . . 775
3.92.3 Rubi [A] (verified) . . . . . . . . . . 776
3.92.4 Maple [A] (verified) . . . . . . . . . . 777
3.92.5 Fricas [A] (verification not implemented) . . . . . . . . . . 778
3.92.6 Sympy [A] (verification not implemented) . . . . . . . . . . 778
3.92.7 Maxima [A] (verification not implemented) . . . . . . . . . . 779
3.92.8 Giac [A] (verification not implemented) . . . . . . . . . . 779
3.92.9 Mupad [B] (verification not implemented) . . . . . . . . . . 780


## 3.92.1 Optimal result

Integrand size = 22, antiderivative size = 109

$$\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx = -\frac{(c+dx)\cos(a+bx)}{8b} - \frac{(c+dx)\cos(3a+3bx)}{48b} + \frac{(c+dx)\cos(5a+5bx)}{80b} + \frac{d\sin(a+bx)}{8b^2} + \frac{d\sin(3a+3bx)}{144b^2} - \frac{d\sin(5a+5bx)}{400b^2}$$

output
```
-1/8*(d*x+c)*cos(b*x+a)/b-1/48*(d*x+c)*cos(3*b*x+3*a)/b+1/80*(d*x+c)*cos(5
*b*x+5*a)/b+1/8*d*sin(b*x+a)/b^2+1/144*d*sin(3*b*x+3*a)/b^2-1/400*d*sin(5*
b*x+5*a)/b^2
```

## 3.92.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.86

$$\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{-450b(c+dx)\cos(a+bx) - 75b(c+dx)\cos(3(a+bx)) + 45bc\cos(5(a+bx)) + 45bdx\cos(5(a+bx)) +}{3600b^2}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
(-450*b*(c + d*x)*Cos[a + b*x] - 75*b*(c + d*x)*Cos[3*(a + b*x)] + 45*b*c*
Cos[5*(a + b*x)] + 45*b*d*x*Cos[5*(a + b*x)] + 450*d*Sin[a + b*x] + 25*d*S
in[3*(a + b*x)] - 9*d*Sin[5*(a + b*x)])/(3600*b^2)
```

### 3.92.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin^3(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}(c+dx)\sin(a+bx)+\frac{1}{16}(c+dx)\sin(3a+3bx)-\frac{1}{16}(c+dx)\sin(5a+5bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\sin(a+bx)}{8b^2}+\frac{d\sin(3a+3bx)}{144b^2}-\frac{d\sin(5a+5bx)}{400b^2}-\frac{(c+dx)\cos(a+bx)}{8b}-\frac{(c+dx)\cos(3a+3bx)}{48b}+\frac{(c+dx)\cos(5a+5bx)}{80b}$$

input
```
Int[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
-1/8*((c + d*x)*Cos[a + b*x])/b - ((c + d*x)*Cos[3*a + 3*b*x])/(48*b) + ((
c + d*x)*Cos[5*a + 5*b*x])/(80*b) + (d*Sin[a + b*x])/(8*b^2) + (d*Sin[3*a
+ 3*b*x])/(144*b^2) - (d*Sin[5*a + 5*b*x])/(400*b^2)
```

### 3.92.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.92.4 Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

| method | result |
|---|---|
| parallelrisch | $\frac{-75b(dx+c)\cos(3xb+3a)+45b(dx+c)\cos(5xb+5a)+25\sin(3xb+3a)d-9d\sin(5xb+5a)-450b(dx+c)\cos(xb+a)-480cb-}{3600b^2}$ |
| risch | $-\frac{(dx+c)\cos(xb+a)}{8b}-\frac{(dx+c)\cos(3xb+3a)}{48b}+\frac{(dx+c)\cos(5xb+5a)}{80b}+\frac{d\sin(xb+a)}{8b^2}+\frac{d\sin(3xb+3a)}{144b^2}-\frac{d\sin(5xb}{400b}$ |
| derivativedivides | $\frac{-\frac{da\left(-\frac{\sin(xb+a)^2\cos(xb+a)^3}{5}-\frac{2\cos(xb+a)^3}{15}\right)}{b}+c\left(-\frac{\sin(xb+a)^2\cos(xb+a)^3}{5}-\frac{2\cos(xb+a)^3}{15}\right)+\frac{d\left(-\frac{(xb+a)\left(2+\sin(xb+a)^2\right)\cos}{3}\right.}{}}{b}$ |
| default | $\frac{-\frac{da\left(-\frac{\sin(xb+a)^2\cos(xb+a)^3}{5}-\frac{2\cos(xb+a)^3}{15}\right)}{b}+c\left(-\frac{\sin(xb+a)^2\cos(xb+a)^3}{5}-\frac{2\cos(xb+a)^3}{15}\right)+\frac{d\left(-\frac{(xb+a)\left(2+\sin(xb+a)^2\right)\cos}{3}\right.}{}}{b}$ |
| norman | $\frac{-\frac{4c}{15b}+\frac{4d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{15b^2}+\frac{56d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3}{45b^2}+\frac{152d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^5}{225b^2}+\frac{56d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^7}{45b^2}+\frac{4d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^9}{15b^2}-\frac{2dx}{15b}-\frac{4c\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^6}{b}}{\left(1+\mathrm{ta}\right.}$ |

input
```
int((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3600*(-75*b*(d*x+c)*cos(3*b*x+3*a)+45*b*(d*x+c)*cos(5*b*x+5*a)+25*sin(3*
b*x+3*a)*d-9*d*sin(5*b*x+5*a)-450*b*(d*x+c)*cos(b*x+a)-480*c*b+450*d*sin(b
*x+a))/b^2
```

### 3.92.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.70

$$\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$=\frac{45\,(bdx+bc)\cos{(bx+a)}^5-75\,(bdx+bc)\cos{(bx+a)}^3-\left(9\,d\cos{(bx+a)}^4-13\,d\cos{(bx+a)}^2-26\,d\right)\sin{(bx+a)}}{225\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/225*(45*(b*d*x + b*c)*cos(b*x + a)^5 - 75*(b*d*x + b*c)*cos(b*x + a)^3 -
 (9*d*cos(b*x + a)^4 - 13*d*cos(b*x + a)^2 - 26*d)*sin(b*x + a))/b^2
```

### 3.92.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.50

$$\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$=\begin{cases}-\frac{c\sin^2(a+bx)\cos^3(a+bx)}{3b}-\frac{2c\cos^5(a+bx)}{15b}-\frac{dx\sin^2(a+bx)\cos^3(a+bx)}{3b}-\frac{2dx\cos^5(a+bx)}{15b}+\frac{26d\sin^5(a+bx)}{225b^2}+\frac{13d\sin^3(a+bx)\cos^2(a+bx)}{45b^2}+\frac{2d\sin(a+bx)\cos^4(a+bx)}{15b^2} & \text{for}\ b\neq 0\\ \left(cx+\frac{dx^2}{2}\right)\sin^3(a)\cos^2(a) & \text{otherwise}\end{cases}$$

input
```
integrate((d*x+c)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

output
```
Piecewise((-c*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c*cos(a + b*x)**5/
(15*b) - d*x*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*d*x*cos(a + b*x)**5
/(15*b) + 26*d*sin(a + b*x)**5/(225*b**2) + 13*d*sin(a + b*x)**3*cos(a + b
*x)**2/(45*b**2) + 2*d*sin(a + b*x)*cos(a + b*x)**4/(15*b**2), Ne(b, 0)),
((c*x + d*x**2/2)*sin(a)**3*cos(a)**2, True))
```

### 3.92.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.28

$$\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{240\left(3\cos(bx+a)^5-5\cos(bx+a)^3\right)c-\frac{240\left(3\cos(bx+a)^5-5\cos(bx+a)^3\right)ad}{b}+\frac{(45\,(bx+a)\cos(5\,bx+5\,a)-75\,(bx+a)\cos(3}{}}{3600\,b}$$

input
```
integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/3600*(240*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c - 240*(3*cos(b*x + a)^
5 - 5*cos(b*x + a)^3)*a*d/b + (45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a
)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*
sin(3*b*x + 3*a) + 450*sin(b*x + a))*d/b)/b
```

### 3.92.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.97

$$\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{(bdx+bc)\cos(5\,bx+5\,a)}{80\,b^2} - \frac{(bdx+bc)\cos(3\,bx+3\,a)}{48\,b^2} - \frac{(bdx+bc)\cos(bx+a)}{8\,b^2} - \frac{d\sin(5\,bx+5\,a)}{400\,b^2} + \frac{d\sin(3\,bx+3\,a)}{144\,b^2} + \frac{d\sin(bx+a)}{8\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/80*(b*d*x + b*c)*cos(5*b*x + 5*a)/b^2 - 1/48*(b*d*x + b*c)*cos(3*b*x + 3
*a)/b^2 - 1/8*(b*d*x + b*c)*cos(b*x + a)/b^2 - 1/400*d*sin(5*b*x + 5*a)/b^
2 + 1/144*d*sin(3*b*x + 3*a)/b^2 + 1/8*d*sin(b*x + a)/b^2
```

### 3.92.9 Mupad [B] (verification not implemented)

Time = 23.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.91

$$\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$=\frac{26\,d\,\sin\left(a+b\,x\right)-75\,b\,c\cos\left(a+b\,x\right)^3+45\,b\,c\cos\left(a+b\,x\right)^5+13\,d\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)-9\,d\cos\left(a\right.}{225\,b^2}$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x),x)
```

output
```
(26*d*sin(a + b*x) - 75*b*c*cos(a + b*x)^3 + 45*b*c*cos(a + b*x)^5 + 13*d*
cos(a + b*x)^2*sin(a + b*x) - 9*d*cos(a + b*x)^4*sin(a + b*x) - 75*b*d*x*c
os(a + b*x)^3 + 45*b*d*x*cos(a + b*x)^5)/(225*b^2)
```

# 3.93 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx$

3.93.1 Optimal result . . . . . . 781
3.93.2 Mathematica [A] (verified) . . . . . . 782
3.93.3 Rubi [A] (verified) . . . . . . 782
3.93.4 Maple [A] (verified) . . . . . . 783
3.93.5 Fricas [A] (verification not implemented) . . . . . . 784
3.93.6 Sympy [F] . . . . . . 784
3.93.7 Maxima [C] (verification not implemented) . . . . . . 785
3.93.8 Giac [C] (verification not implemented) . . . . . . 785
3.93.9 Mupad [F(-1)] . . . . . . 786

## 3.93.1 Optimal result

Integrand size = 24, antiderivative size = 185

$$\begin{aligned}\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx = &-\frac{\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)\sin\left(5a-\frac{5bc}{d}\right)}{16d}\\ &+\frac{\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{16d}\\ &+\frac{\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{8d}\\ &+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d}+\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{16d}\\ &-\frac{\cos\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{16d}\end{aligned}$$

output
```
1/8*cos(a-b*c/d)*Si(b*c/d+b*x)/d+1/16*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d
-1/16*cos(5*a-5*b*c/d)*Si(5*b*c/d+5*b*x)/d-1/16*Ci(5*b*c/d+5*b*x)*sin(5*a-
5*b*c/d)/d+1/16*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d+1/8*Ci(b*c/d+b*x)*sin
(a-b*c/d)/d
```

### 3.93.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.83

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx$$
$$= \frac{-\operatorname{CosIntegral}\left(\frac{5b(c+dx)}{d}\right)\sin\left(5a-\frac{5bc}{d}\right)+\operatorname{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right)+2\operatorname{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)}{16d}$$

input
```
Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x),x]
```

output
```
(-(CosIntegral[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d]) + CosIntegral[(3*b
*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + 2*CosIntegral[b*(c/d + x)]*Sin[a - (
b*c)/d] + 2*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Cos[3*a - (3*b*c)/
d]*SinIntegral[(3*b*(c + d*x))/d] - Cos[5*a - (5*b*c)/d]*SinIntegral[(5*b*
(c + d*x))/d])/(16*d)
```

### 3.93.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)\cos^2(a+bx)}{c+dx}\,dx$$
$$\Big\downarrow \ 4906$$
$$\int \left(\frac{\sin(a+bx)}{8(c+dx)}+\frac{\sin(3a+3bx)}{16(c+dx)}-\frac{\sin(5a+5bx)}{16(c+dx)}\right)dx$$
$$\Big\downarrow \ 2009$$
$$-\frac{\sin\left(5a-\frac{5bc}{d}\right)\operatorname{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{16d}+\frac{\sin\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d}+\frac{\sin\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d}+\frac{\cos\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{8d}+\frac{\cos\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3bc}{d}+3bx\right)}{16d}-\frac{\cos\left(5a-\frac{5bc}{d}\right)\operatorname{Si}\left(\frac{5bc}{d}+5bx\right)}{16d}$$

input
```
Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x),x]
```

output
```
-1/16*(CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*a - (5*b*c)/d])/d + (CosIntegr
al[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(16*d) + (CosIntegral[(b*c)/d
+ b*x]*Sin[a - (b*c)/d])/(8*d) + (Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b
*x])/(8*d) + (Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d)
- (Cos[5*a - (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d)
```

### 3.93.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.93.4 Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.39

| method | result |
|---|---|
| derivativedivides | $\frac{b\left(-\frac{3\ \mathrm{Si}\left(-3xb-3a-\frac{3(-ad+cb)}{d}\right)\cos\left(\frac{-3ad+3cb}{d}\right)}{d}-\frac{3\ \mathrm{Ci}\left(3xb+3a+\frac{-3ad+3cb}{d}\right)\sin\left(\frac{-3ad+3cb}{d}\right)}{d}\right)}{48}+\frac{b\left(-\frac{\mathrm{Si}\left(-xb-a-\frac{-ad+cb}{d}\right)\mathrm{co}}{d}\right.}{}$ |
| default | $\frac{b\left(-\frac{3\ \mathrm{Si}\left(-3xb-3a-\frac{3(-ad+cb)}{d}\right)\cos\left(\frac{-3ad+3cb}{d}\right)}{d}-\frac{3\ \mathrm{Ci}\left(3xb+3a+\frac{-3ad+3cb}{d}\right)\sin\left(\frac{-3ad+3cb}{d}\right)}{d}\right)}{48}+\frac{b\left(-\frac{\mathrm{Si}\left(-xb-a-\frac{-ad+cb}{d}\right)\mathrm{co}}{d}\right.}{}$ |
| risch | $\frac{i\mathrm{e}^{-\frac{5i(ad-cb)}{d}}\ \mathrm{Ei}_1\left(5ibx+5ia-\frac{5i(ad-cb)}{d}\right)}{32d}-\frac{i\mathrm{e}^{-\frac{3i(ad-cb)}{d}}\ \mathrm{Ei}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{32d}-\frac{i\mathrm{e}^{-\frac{i(ad-cb)}{d}}\ \mathrm{Ei}_1\left(ibx+ia-\frac{i(}{}\right.}{16d}$ |

input
```
int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/48*b*(-3*Si(-3*x*b-3*a-3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3
*x*b+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)+1/8*b*(-Si(-x*b-a-(-a*d+b*
c)/d)*cos((-a*d+b*c)/d)/d-Ci(x*b+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)-1/80
*b*(-5*Si(-5*x*b-5*a-5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d-5*Ci(5*x*b+5*a+
5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d))
```

### 3.93.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.98

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx$$
$$= \frac{2\,\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)\sin\left(-\frac{bc-ad}{d}\right)+\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)-\mathrm{Ci}\left(\frac{5\,(bdx+bc)}{d}\right)\sin\left(-\frac{5\,(bc-ad)}{d}\right)-\cos\left(-\frac{5\,(bc\cdots}{d}\right.}{16\,d}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
1/16*(2*cos_integral((b*d*x + b*c)/d)*sin(-(b*c - a*d)/d) + cos_integral(3
*(b*d*x + b*c)/d)*sin(-3*(b*c - a*d)/d) - cos_integral(5*(b*d*x + b*c)/d)*
sin(-5*(b*c - a*d)/d) - cos(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)
/d) + cos(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) + 2*cos(-(b*c
- a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

### 3.93.6 Sympy [F]

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \int \frac{\sin^3(a+bx)\cos^2(a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)**3*cos(a + b*x)**2/(c + d*x), x)
```

### 3.93.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.24

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx = -\frac{2\,b\left(i\,E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b\left(i\,E_1\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i}{}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output
```
-1/32*(2*b*(I*exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp
_integral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) -
b*(I*exp_integral_e(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integ
ral_e(1, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b
*(-I*exp_integral_e(1, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integ
ral_e(1, -5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-5*(b*c - a*d)/d) + 2
*b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(
1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b*(exp_integ
ral_e(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(1, -3*(-I*
b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d) - b*(exp_integral_e
(1, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(1, -5*(-I*b*c -
 I*(b*x + a)*d + I*a*d)/d))*sin(-5*(b*c - a*d)/d))/(b*d)
```

### 3.93.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.76 (sec) , antiderivative size = 46675, normalized size of antiderivative = 252.30

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output
```
-1/32*(imag_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*
tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_par
t(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integra
l(b*x + b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(-b*x - b*c/d))*
tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d)^2 + imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*a)^2*t
an(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^
2 - imag_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_inte
gral(5*(b*d*x + b*c)/d)*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c
/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*sin_integral(3*(b*d*x + b*c)/d
)*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d)^2 - 4*sin_integral((b*d*x + b*c)/d)*tan(5/2*a)^2*tan(3/2*a
)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 4*re
al_part(cos_integral(b*x + b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*
tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*real_part(cos_integra
l(-b*x - b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*real_part(cos_integral(3*b*x + 3*b*c...
```

### 3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx=\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3}{c+d\,x}\,dx$$

input
```
int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x),x)
```

output
```
int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x), x)
```

# 3.94 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$

3.94.1 Optimal result . . . 787
3.94.2 Mathematica [A] (verified) . . . 788
3.94.3 Rubi [A] (verified) . . . 788
3.94.4 Maple [A] (verified) . . . 790
3.94.5 Fricas [A] (verification not implemented) . . . 790
3.94.6 Sympy [F] . . . 791
3.94.7 Maxima [C] (verification not implemented) . . . 791
3.94.8 Giac [C] (verification not implemented) . . . 792
3.94.9 Mupad [F(-1)] . . . 793

## 3.94.1 Optimal result

Integrand size = 24, antiderivative size = 257

$$\begin{aligned}\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx &= \frac{b\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^2} \\ &+ \frac{3b\cos\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d^2} \\ &- \frac{5b\cos\left(5a-\frac{5bc}{d}\right)\operatorname{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{16d^2} \\ &- \frac{\sin(a+bx)}{8d(c+dx)} - \frac{\sin(3a+3bx)}{16d(c+dx)} \\ &+ \frac{\sin(5a+5bx)}{16d(c+dx)} - \frac{b\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{8d^2} \\ &- \frac{3b\sin\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3bc}{d}+3bx\right)}{16d^2} \\ &+ \frac{5b\sin\left(5a-\frac{5bc}{d}\right)\operatorname{Si}\left(\frac{5bc}{d}+5bx\right)}{16d^2}\end{aligned}$$

output
```
-5/16*b*Ci(5*b*c/d+5*b*x)*cos(5*a-5*b*c/d)/d^2+3/16*b*Ci(3*b*c/d+3*b*x)*co
s(3*a-3*b*c/d)/d^2+1/8*b*Ci(b*c/d+b*x)*cos(a-b*c/d)/d^2+5/16*b*Si(5*b*c/d+
5*b*x)*sin(5*a-5*b*c/d)/d^2-3/16*b*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^2-
1/8*b*Si(b*c/d+b*x)*sin(a-b*c/d)/d^2-1/8*sin(b*x+a)/d/(d*x+c)-1/16*sin(3*b
*x+3*a)/d/(d*x+c)+1/16*sin(5*b*x+5*a)/d/(d*x+c)
```

### 3.94.2 Mathematica [A] (verified)

Time = 1.59 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.83

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{2b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)-5b\cos\left(5a-\frac{5bc}{d}\right)\text{CosI}}{}$$

input
```
Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

output
```
(2*b*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] + 3*b*Cos[3*a - (3*b*c)/d]*
CosIntegral[(3*b*(c + d*x))/d] - 5*b*Cos[5*a - (5*b*c)/d]*CosIntegral[(5*b
*(c + d*x))/d] - (2*d*Sin[a + b*x])/(c + d*x) - (d*Sin[3*(a + b*x)])/(c +
d*x) + (d*Sin[5*(a + b*x)])/(c + d*x) - 2*b*Sin[a - (b*c)/d]*SinIntegral[b
*(c/d + x)] - 3*b*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d] + 5*
b*Sin[5*a - (5*b*c)/d]*SinIntegral[(5*b*(c + d*x))/d])/(16*d^2)
```

### 3.94.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)\cos^2(a+bx)}{(c+dx)^2}\,dx$$
$$\Big\downarrow \text{4906}$$
$$\int \left(\frac{\sin(a+bx)}{8(c+dx)^2}+\frac{\sin(3a+3bx)}{16(c+dx)^2}-\frac{\sin(5a+5bx)}{16(c+dx)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d^2}-\frac{5b\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{16d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^2}-\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{16d^2}+\frac{5b\sin\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{16d^2}-\frac{\sin(a+bx)}{8d(c+dx)}-\frac{\sin(3a+3bx)}{16d(c+dx)}+\frac{\sin(5a+5bx)}{16d(c+dx)}$$

input
```
Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

output
```
(b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d^2) + (3*b*Cos[3*a - (
3*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) - (5*b*Cos[5*a - (5*b*c
)/d]*CosIntegral[(5*b*c)/d + 5*b*x])/(16*d^2) - Sin[a + b*x]/(8*d*(c + d*x
)) - Sin[3*a + 3*b*x]/(16*d*(c + d*x)) + Sin[5*a + 5*b*x]/(16*d*(c + d*x))
 - (b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^2) - (3*b*Sin[3*a
- (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) + (5*b*Sin[5*a - (5*
b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d^2)
```

### 3.94.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.94.4 Maple [A] (verified)

Time = 1.54 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.44

| method | result |
|---|---|
| derivativedivides | $-\frac{b^2\left(-\frac{5\sin(5xb+5a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{25\,\mathrm{Si}\left(-5xb-5a-\frac{5(-ad+cb)}{d}\right)\sin\left(\frac{-5ad+5cb}{d}\right)}{d}+\frac{25\,\mathrm{Ci}\left(5xb+5a+\frac{-5ad+5cb}{d}\right)\cos\left(\frac{-5ad+5cb}{d}\right)}{d}}{d}\right)}{80}$ |
| default | $-\frac{b^2\left(-\frac{5\sin(5xb+5a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{25\,\mathrm{Si}\left(-5xb-5a-\frac{5(-ad+cb)}{d}\right)\sin\left(\frac{-5ad+5cb}{d}\right)}{d}+\frac{25\,\mathrm{Ci}\left(5xb+5a+\frac{-5ad+5cb}{d}\right)\cos\left(\frac{-5ad+5cb}{d}\right)}{d}}{d}\right)}{80}$ |
| risch | $\frac{5b\,\mathrm{e}^{-\frac{5i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(5ibx+5ia-\frac{5i(ad-cb)}{d}\right)}{32d^2}-\frac{3b\,\mathrm{e}^{-\frac{3i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{32d^2}-\frac{b\,\mathrm{e}^{-\frac{i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(ibx+ia\right.}{16d^2}$ |

input
```
int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/80*b^2*(-5*sin(5*b*x+5*a)/(-a*d+c*b+d*(b*x+a))/d+5*(-5*Si(-5*x*b-5
*a-5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d+5*Ci(5*x*b+5*a+5*(-a*d+b*c)/d)*co
s(5*(-a*d+b*c)/d)/d)/d)+1/8*b^2*(-sin(b*x+a)/(-a*d+c*b+d*(b*x+a))/d+(-Si(-
x*b-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(x*b+a+(-a*d+b*c)/d)*cos((-a*d+b
*c)/d)/d)/d)+1/48*b^2*(-3*sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d+3*(-3*Si(-
3*x*b-3*a-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*x*b+3*a+3*(-a*d+b*c
)/d)*cos(3*(-a*d+b*c)/d)/d)/d))
```

### 3.94.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.06

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{5\,(bdx+bc)\cos\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{5\,(bdx+bc)}{d}\right)-3\,(bdx+bc)\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)-2\,(bdx+bc)}{}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/16*(5*(b*d*x + b*c)*cos(-5*(b*c - a*d)/d)*cos_integral(5*(b*d*x + b*c)/
d) - 3*(b*d*x + b*c)*cos(-3*(b*c - a*d)/d)*cos_integral(3*(b*d*x + b*c)/d)
 - 2*(b*d*x + b*c)*cos(-(b*c - a*d)/d)*cos_integral((b*d*x + b*c)/d) - 5*(
b*d*x + b*c)*sin(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d) + 3*(b*
d*x + b*c)*sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) + 2*(b*d*
x + b*c)*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) - 16*(d*cos(b*x
 + a)^4 - d*cos(b*x + a)^2)*sin(b*x + a))/(d^3*x + c*d^2)
```

### 3.94.6 Sympy [F]

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin^3(a+bx)\cos^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)**3*cos(a + b*x)**2/(c + d*x)**2, x)
```

### 3.94.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.73

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = -\frac{2\,b^2\left(i\,E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^2\left(i\,E_2\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-\cdots}{\cdots}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/32*(2*b^2*(I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*e
xp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
- b^2*(I*exp_integral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_i
ntegral_e(2, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d)
 - b^2*(-I*exp_integral_e(2, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp
_integral_e(2, -5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-5*(b*c - a*d)/
d) + 2*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_int
egral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^2*
(exp_integral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(
2, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d) - b^2*(ex
p_integral_e(2, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(2,
-5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-5*(b*c - a*d)/d))/((b*c*d + (
b*x + a)*d^2 - a*d^2)*b)
```

### 3.94.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 47.39 (sec) , antiderivative size = 1014406, normalized size of antiderivative = 3947.11

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/32*(5*b*d*x*real_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*x)^2*tan
(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*
b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 3*b*d*x*real_part(cos_integra
l(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a
)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*
c/d)^2 - 2*b*d*x*real_part(cos_integral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(3
/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*
c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b*d*x*real_part(cos_integral(
-b*x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*t
an(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^
2 - 3*b*d*x*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*b*x)^2*tan(3
/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*
c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 5*b*d*x*real_part(cos_integral(
-5*b*x - 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)
^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d)^2 + 4*b*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(3/
2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c
/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*b*d*x*imag_part(cos_integral(-b*
x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(
3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) ...
```

### 3.94.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3}{\left(c+d\,x\right)^2}\,dx$$

input
```
int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^2,x)
```

output
```
int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^2, x)
```

# 3.95 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$

3.95.1 Optimal result . . . . . 794
3.95.2 Mathematica [A] (verified) . . . . . 795
3.95.3 Rubi [A] (verified) . . . . . 795
3.95.4 Maple [A] (verified) . . . . . 797
3.95.5 Fricas [A] (verification not implemented) . . . . . 797
3.95.6 Sympy [F] . . . . . 798
3.95.7 Maxima [C] (verification not implemented) . . . . . 798
3.95.8 Giac [C] (verification not implemented) . . . . . 799
3.95.9 Mupad [F(-1)] . . . . . 800

## 3.95.1 Optimal result

Integrand size = 24, antiderivative size = 338

$$\begin{aligned}\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx =& -\frac{b\cos(a+bx)}{16d^2(c+dx)}-\frac{3b\cos(3a+3bx)}{32d^2(c+dx)}+\frac{5b\cos(5a+5bx)}{32d^2(c+dx)}\\ &+\frac{25b^2\operatorname{CosIntegral}\left(\frac{5bc}{d}+5bx\right)\sin\left(5a-\frac{5bc}{d}\right)}{32d^3}\\ &-\frac{9b^2\operatorname{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{32d^3}\\ &-\frac{b^2\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{16d^3}\\ &-\frac{\sin(a+bx)}{16d(c+dx)^2}-\frac{\sin(3a+3bx)}{32d(c+dx)^2}\\ &+\frac{\sin(5a+5bx)}{32d(c+dx)^2}-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{16d^3}\\ &-\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3bc}{d}+3bx\right)}{32d^3}\\ &+\frac{25b^2\cos\left(5a-\frac{5bc}{d}\right)\operatorname{Si}\left(\frac{5bc}{d}+5bx\right)}{32d^3}\end{aligned}$$

output
```
-1/16*b*cos(b*x+a)/d^2/(d*x+c)-3/32*b*cos(3*b*x+3*a)/d^2/(d*x+c)+5/32*b*co
s(5*b*x+5*a)/d^2/(d*x+c)-1/16*b^2*cos(a-b*c/d)*Si(b*c/d+b*x)/d^3-9/32*b^2*
cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^3+25/32*b^2*cos(5*a-5*b*c/d)*Si(5*b*c
/d+5*b*x)/d^3+25/32*b^2*Ci(5*b*c/d+5*b*x)*sin(5*a-5*b*c/d)/d^3-9/32*b^2*Ci
(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^3-1/16*b^2*Ci(b*c/d+b*x)*sin(a-b*c/d)/d
^3-1/16*sin(b*x+a)/d/(d*x+c)^2-1/32*sin(3*b*x+3*a)/d/(d*x+c)^2+1/32*sin(5*
b*x+5*a)/d/(d*x+c)^2
```

### 3.95.2 Mathematica [A] (verified)

Time = 3.97 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.83

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \frac{25b^2\,\text{CosIntegral}\left(\frac{5b(c+dx)}{d}\right)\sin\left(5a-\frac{5bc}{d}\right)-9b^2\,\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\frac{3bc}{d}\right)-\frac{d(3b(c+dx)\cos(3(a+bx)}{(c+dx)}}{}$$

input
```
Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

output
```
(25*b^2*CosIntegral[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d] - 9*b^2*CosInt
egral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] - (d*(3*b*(c + d*x)*Cos[3*(a
 + b*x)] + d*Sin[3*(a + b*x)]))/(c + d*x)^2 + (d*(5*b*(c + d*x)*Cos[5*(a +
 b*x)] + d*Sin[5*(a + b*x)]))/(c + d*x)^2 - 2*(b^2*CosIntegral[b*(c/d + x)
]*Sin[a - (b*c)/d] + (d*(b*(c + d*x)*Cos[a + b*x] + d*Sin[a + b*x]))/(c +
d*x)^2 + b^2*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)]) - 9*b^2*Cos[3*a -
(3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d] + 25*b^2*Cos[5*a - (5*b*c)/d]*Si
nIntegral[(5*b*(c + d*x))/d])/(32*d^3)
```

### 3.95.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)\cos^2(a+bx)}{(c+dx)^3}\,dx$$
$$\Big\downarrow \text{4906}$$
$$\int \left(\frac{\sin(a+bx)}{8(c+dx)^3}+\frac{\sin(3a+3bx)}{16(c+dx)^3}-\frac{\sin(5a+5bx)}{16(c+dx)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{25b^2\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{32d^3}-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^3}-\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{16d^3}-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{16d^3}-\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{32d^3}+\frac{25b^2\cos\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{32d^3}-\frac{b\cos(a+bx)}{16d^2(c+dx)}-\frac{3b\cos(3a+3bx)}{32d^2(c+dx)}+\frac{5b\cos(5a+5bx)}{32d^2(c+dx)}-\frac{\sin(a+bx)}{16d(c+dx)^2}-\frac{\sin(3a+3bx)}{32d(c+dx)^2}+\frac{\sin(5a+5bx)}{32d(c+dx)^2}$$

input
```
Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

output
```
-1/16*(b*Cos[a + b*x])/(d^2*(c + d*x)) - (3*b*Cos[3*a + 3*b*x])/(32*d^2*(c
 + d*x)) + (5*b*Cos[5*a + 5*b*x])/(32*d^2*(c + d*x)) + (25*b^2*CosIntegral
[(5*b*c)/d + 5*b*x]*Sin[5*a - (5*b*c)/d])/(32*d^3) - (9*b^2*CosIntegral[(3
*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(32*d^3) - (b^2*CosIntegral[(b*c)/d
 + b*x]*Sin[a - (b*c)/d])/(16*d^3) - Sin[a + b*x]/(16*d*(c + d*x)^2) - Sin
[3*a + 3*b*x]/(32*d*(c + d*x)^2) + Sin[5*a + 5*b*x]/(32*d*(c + d*x)^2) - (
b^2*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(16*d^3) - (9*b^2*Cos[3*a
 - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(32*d^3) + (25*b^2*Cos[5*a -
 (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(32*d^3)
```

### 3.95.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.95.4 Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.42

| method | result |
|---|---|
| derivativedivides | $-\frac{b^3\left(-\frac{5\sin(5xb+5a)}{2(-ad+cb+d(xb+a))^2 d}+\frac{-\frac{25\cos(5xb+5a)}{2(-ad+cb+d(xb+a))d}-\frac{25\left(-\frac{5\,\mathrm{Si}\left(-5xb-5a-\frac{5(-ad+cb)}{d}\right)\cos\left(\frac{-5ad+5cb}{d}\right)}{d}-\frac{5\,\mathrm{Ci}\left(5xb+5a+\right.}{}\right.}{2d}}{d}\right)}{80}$ |
| default | $-\frac{b^3\left(-\frac{5\sin(5xb+5a)}{2(-ad+cb+d(xb+a))^2 d}+\frac{-\frac{25\cos(5xb+5a)}{2(-ad+cb+d(xb+a))d}-\frac{25\left(-\frac{5\,\mathrm{Si}\left(-5xb-5a-\frac{5(-ad+cb)}{d}\right)\cos\left(\frac{-5ad+5cb}{d}\right)}{d}-\frac{5\,\mathrm{Ci}\left(5xb+5a+\right.}{}\right.}{2d}}{d}\right)}{80}$ |
| risch | $-\frac{25ib^2\mathrm{e}^{-\frac{5i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(5ibx+5ia-\frac{5i(ad-cb)}{d}\right)}{64d^3}+\frac{9ib^2\mathrm{e}^{-\frac{3i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{64d^3}+\frac{ib^2\mathrm{e}^{-\frac{i(ad-cb)}{d}}\,\mathrm{Ei}_1}{3}$ |

input
```
int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/80*b^3*(-5/2*sin(5*b*x+5*a)/(-a*d+c*b+d*(b*x+a))^2/d+5/2*(-5*cos(5
*b*x+5*a)/(-a*d+c*b+d*(b*x+a))/d-5*(-5*Si(-5*x*b-5*a-5*(-a*d+b*c)/d)*cos(5
*(-a*d+b*c)/d)/d-5*Ci(5*x*b+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d)/d)/
d)+1/8*b^3*(-1/2*sin(b*x+a)/(-a*d+c*b+d*(b*x+a))^2/d+1/2*(-cos(b*x+a)/(-a*
d+c*b+d*(b*x+a))/d-(-Si(-x*b-a-(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(x*b+a+
(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)/d)+1/48*b^3*(-3/2*sin(3*b*x+3*a)/(-a
*d+c*b+d*(b*x+a))^2/d+3/2*(-3*cos(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d-3*(-3*
Si(-3*x*b-3*a-3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*x*b+3*a+3*(-a*d
+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)/d)/d))
```

### 3.95.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 456, normalized size of antiderivative = 1.35

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$$

$$= \frac{80\,(bd^2x+bcd)\cos{(bx+a)}^5-112\,(bd^2x+bcd)\cos{(bx+a)}^3-2\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\,\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)\sin}{}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/32*(80*(b*d^2*x + b*c*d)*cos(b*x + a)^5 - 112*(b*d^2*x + b*c*d)*cos(b*x
+ a)^3 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral((b*d*x + b*c
)/d)*sin(-(b*c - a*d)/d) - 9*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_int
egral(3*(b*d*x + b*c)/d)*sin(-3*(b*c - a*d)/d) + 25*(b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*cos_integral(5*(b*d*x + b*c)/d)*sin(-5*(b*c - a*d)/d) + 25
*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-5*(b*c - a*d)/d)*sin_integral(
5*(b*d*x + b*c)/d) - 9*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-3*(b*c -
 a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 32*(b*d^2*x +
 b*c*d)*cos(b*x + a) + 16*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2)*sin(b*
x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

### 3.95.6 Sympy [F]

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin^3(a+bx)\cos^2(a+bx)}{(c+dx)^3}\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c)**3,x)
```

output
```
Integral(sin(a + b*x)**3*cos(a + b*x)**2/(c + d*x)**3, x)
```

### 3.95.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.42

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = -\frac{2\,b^3\left(i\,E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^3\left(i\,E_3\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-\right.}{}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/32*(2*b^3*(I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*e
xp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
- b^3*(I*exp_integral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_i
ntegral_e(3, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d)
 - b^3*(-I*exp_integral_e(3, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp
_integral_e(3, -5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-5*(b*c - a*d)/
d) + 2*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_int
egral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^3*
(exp_integral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(
3, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d) - b^3*(ex
p_integral_e(3, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(3,
-5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-5*(b*c - a*d)/d))/((b^2*c^2*d
 - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a)
)*b)
```

### 3.95.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 99.43 (sec) , antiderivative size = 1737414, normalized size of antiderivative = 5140.28

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/64*(25*b^2*d^2*x^2*imag_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*x)
^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 9*b^2*d^2*x^2*imag_part
(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)
^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^
2*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_integral(b*x + b*c/d))*ta
n(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1
/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*b^2*d^2*x^2
*imag_part(cos_integral(-b*x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*
b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*imag_part(cos_integral(-3*b*x -
3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/
2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2
5*b^2*d^2*x^2*imag_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*b*x)^2*tan
(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*
b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 50*b^2*d^2*x^2*sin_integral(5
*(b*d*x + b*c)/d)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^
2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 - 18*b^2*d^2*x^2*sin_integral(3*(b*d*x + b*c)/d)*tan(5/2*b*x)^2*tan(3
/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2...
```

### 3.95.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^3}{\left(c+d\,x\right)^3}\,dx$$

input
```
int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^3,x)
```

output
```
int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^3, x)
```

# 3.96 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$

3.96.1 Optimal result . . . 801
3.96.2 Mathematica [A] (verified) . . . 802
3.96.3 Rubi [A] (verified) . . . 803
3.96.4 Maple [A] (verified) . . . 804
3.96.5 Fricas [A] (verification not implemented) . . . 805
3.96.6 Sympy [F] . . . 806
3.96.7 Maxima [C] (verification not implemented) . . . 806
3.96.8 Giac [C] (verification not implemented) . . . 807
3.96.9 Mupad [F(-1)] . . . 807

## 3.96.1 Optimal result

Integrand size = 24, antiderivative size = 413

$$\begin{aligned}\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx =& -\frac{b\cos(a+bx)}{48d^2(c+dx)^2} - \frac{b\cos(3a+3bx)}{32d^2(c+dx)^2} + \frac{5b\cos(5a+5bx)}{96d^2(c+dx)^2} \\ &- \frac{b^3\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{48d^4} \\ &- \frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\operatorname{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^4} \\ &+ \frac{125b^3\cos\left(5a-\frac{5bc}{d}\right)\operatorname{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{96d^4} \\ &- \frac{\sin(a+bx)}{24d(c+dx)^3} + \frac{b^2\sin(a+bx)}{48d^3(c+dx)} - \frac{\sin(3a+3bx)}{48d(c+dx)^3} \\ &+ \frac{3b^2\sin(3a+3bx)}{32d^3(c+dx)} + \frac{\sin(5a+5bx)}{48d(c+dx)^3} \\ &- \frac{25b^2\sin(5a+5bx)}{96d^3(c+dx)} + \frac{b^3\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{48d^4} \\ &+ \frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3bc}{d}+3bx\right)}{32d^4} \\ &- \frac{125b^3\sin\left(5a-\frac{5bc}{d}\right)\operatorname{Si}\left(\frac{5bc}{d}+5bx\right)}{96d^4}\end{aligned}$$

output
```
125/96*b^3*Ci(5*b*c/d+5*b*x)*cos(5*a-5*b*c/d)/d^4-9/32*b^3*Ci(3*b*c/d+3*b*
x)*cos(3*a-3*b*c/d)/d^4-1/48*b^3*Ci(b*c/d+b*x)*cos(a-b*c/d)/d^4-1/48*b*cos
(b*x+a)/d^2/(d*x+c)^2-1/32*b*cos(3*b*x+3*a)/d^2/(d*x+c)^2+5/96*b*cos(5*b*x
+5*a)/d^2/(d*x+c)^2-125/96*b^3*Si(5*b*c/d+5*b*x)*sin(5*a-5*b*c/d)/d^4+9/32
*b^3*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^4+1/48*b^3*Si(b*c/d+b*x)*sin(a-b
*c/d)/d^4-1/24*sin(b*x+a)/d/(d*x+c)^3+1/48*b^2*sin(b*x+a)/d^3/(d*x+c)-1/48
*sin(3*b*x+3*a)/d/(d*x+c)^3+3/32*b^2*sin(3*b*x+3*a)/d^3/(d*x+c)+1/48*sin(5
*b*x+5*a)/d/(d*x+c)^3-25/96*b^2*sin(5*b*x+5*a)/d^3/(d*x+c)
```

### 3.96.2 Mathematica [A] (verified)

Time = 2.54 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.11

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$$
$$= \frac{-d\cos(3bx)\left(3bd(c+dx)\cos(3a)-\left(-2d^2+9b^2(c+dx)^2\right)\sin(3a)\right)+d\cos(5bx)\left(5bd(c+dx)\cos(5a)-(\cdots\right.}{}$$

input
```
Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

output
```
(-(d*Cos[3*b*x]*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*x)^2)*S
in[3*a])) + d*Cos[5*b*x]*(5*b*d*(c + d*x)*Cos[5*a] - (-2*d^2 + 25*b^2*(c +
 d*x)^2)*Sin[5*a]) + d*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c +
 d*x)*Sin[3*a])*Sin[3*b*x] - d*((-2*d^2 + 25*b^2*(c + d*x)^2)*Cos[5*a] + 5
*b*d*(c + d*x)*Sin[5*a])*Sin[5*b*x] - 2*(d*Cos[b*x]*(b*d*(c + d*x)*Cos[a]
- (-2*d^2 + b^2*(c + d*x)^2)*Sin[a]) - d*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a
] + b*d*(c + d*x)*Sin[a])*Sin[b*x] + b^3*(c + d*x)^3*(Cos[a - (b*c)/d]*Cos
Integral[b*(c/d + x)] - Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)])) - 27*b
^3*(c + d*x)^3*(Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*(c + d*x))/d] - Sin[
3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d]) + 125*b^3*(c + d*x)^3*(Co
s[5*a - (5*b*c)/d]*CosIntegral[(5*b*(c + d*x))/d] - Sin[5*a - (5*b*c)/d]*S
inIntegral[(5*b*(c + d*x))/d]))/(96*d^4*(c + d*x)^3)
```

### 3.96.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)\cos^2(a+bx)}{(c+dx)^4}dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{\sin(a+bx)}{8(c+dx)^4}+\frac{\sin(3a+3bx)}{16(c+dx)^4}-\frac{\sin(5a+5bx)}{16(c+dx)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{48d^4}-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^4}+\frac{125b^3\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{96d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{48d^4}+\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{32d^4}-\frac{125b^3\sin\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{96d^4}+\frac{b^2\sin(a+bx)}{48d^3(c+dx)}+\frac{3b^2\sin(3a+3bx)}{32d^3(c+dx)}-\frac{25b^2\sin(5a+5bx)}{96d^3(c+dx)}-\frac{b\cos(a+bx)}{48d^2(c+dx)^2}-\frac{b\cos(3a+3bx)}{32d^2(c+dx)^2}+\frac{5b\cos(5a+5bx)}{96d^2(c+dx)^2}-\frac{\sin(a+bx)}{24d(c+dx)^3}-\frac{\sin(3a+3bx)}{48d(c+dx)^3}+\frac{\sin(5a+5bx)}{48d(c+dx)^3}$$

input
```
Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

output
```
-1/48*(b*Cos[a + b*x])/(d^2*(c + d*x)^2) - (b*Cos[3*a + 3*b*x])/(32*d^2*(c
 + d*x)^2) + (5*b*Cos[5*a + 5*b*x])/(96*d^2*(c + d*x)^2) - (b^3*Cos[a - (b
*c)/d]*CosIntegral[(b*c)/d + b*x])/(48*d^4) - (9*b^3*Cos[3*a - (3*b*c)/d]*
CosIntegral[(3*b*c)/d + 3*b*x])/(32*d^4) + (125*b^3*Cos[5*a - (5*b*c)/d]*C
osIntegral[(5*b*c)/d + 5*b*x])/(96*d^4) - Sin[a + b*x]/(24*d*(c + d*x)^3)
+ (b^2*Sin[a + b*x])/(48*d^3*(c + d*x)) - Sin[3*a + 3*b*x]/(48*d*(c + d*x)
^3) + (3*b^2*Sin[3*a + 3*b*x])/(32*d^3*(c + d*x)) + Sin[5*a + 5*b*x]/(48*d
*(c + d*x)^3) - (25*b^2*Sin[5*a + 5*b*x])/(96*d^3*(c + d*x)) + (b^3*Sin[a
- (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(48*d^4) + (9*b^3*Sin[3*a - (3*b*c)
/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(32*d^4) - (125*b^3*Sin[5*a - (5*b*c)/
d]*SinIntegral[(5*b*c)/d + 5*b*x])/(96*d^4)
```

### 3.96.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.96.4 Maple [A] (verified)

Time = 2.89 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.42

| method | result |
|---|---|
| derivativedivides | $-\frac{b^4\left(-\frac{5\sin(5xb+5a)}{3(-ad+cb+d(xb+a))^3d}+\frac{-\frac{25\cos(5xb+5a)}{6(-ad+cb+d(xb+a))^2d}-\frac{25\left(-\frac{5\sin(5xb+5a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{25\,\mathrm{Si}\left(-5xb-5a-\frac{5(-ad+cb)}{d}\right)\sin\left(\frac{-5}{}\right.}{d}}{}\right.}{6d}}{d}\right.}{80}$ |
| default | $-\frac{b^4\left(-\frac{5\sin(5xb+5a)}{3(-ad+cb+d(xb+a))^3d}+\frac{-\frac{25\cos(5xb+5a)}{6(-ad+cb+d(xb+a))^2d}-\frac{25\left(-\frac{5\sin(5xb+5a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{25\,\mathrm{Si}\left(-5xb-5a-\frac{5(-ad+cb)}{d}\right)\sin\left(\frac{-5}{}\right.}{d}}{}\right.}{6d}}{d}\right.}{80}$ |
| risch | Expression too large to display |

input
```
int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/80*b^4*(-5/3*sin(5*b*x+5*a)/(-a*d+c*b+d*(b*x+a))^3/d+5/3*(-5/2*cos
(5*b*x+5*a)/(-a*d+c*b+d*(b*x+a))^2/d-5/2*(-5*sin(5*b*x+5*a)/(-a*d+c*b+d*(b
*x+a))/d+5*(-5*Si(-5*x*b-5*a-5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d+5*Ci(5*
x*b+5*a+5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d)/d)/d)/d)+1/8*b^4*(-1/3*sin(
b*x+a)/(-a*d+c*b+d*(b*x+a))^3/d+1/3*(-1/2*cos(b*x+a)/(-a*d+c*b+d*(b*x+a))^
2/d-1/2*(-sin(b*x+a)/(-a*d+c*b+d*(b*x+a))/d+(-Si(-x*b-a-(-a*d+b*c)/d)*sin(
(-a*d+b*c)/d)/d+Ci(x*b+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d)/d)+1/48*
b^4*(-sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))^3/d+(-3/2*cos(3*b*x+3*a)/(-a*d+c
*b+d*(b*x+a))^2/d-3/2*(-3*sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d+3*(-3*Si(-
3*x*b-3*a-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*x*b+3*a+3*(-a*d+b*c
)/d)*cos(3*(-a*d+b*c)/d)/d)/d)/d)/d))
```

### 3.96.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.58

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$$

$$= \frac{80\,(bd^3x+bcd^2)\cos{(bx+a)}^5 - 112\,(bd^3x+bcd^2)\cos{(bx+a)}^3 + 125\,(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3}{}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/96*(80*(b*d^3*x + b*c*d^2)*cos(b*x + a)^5 - 112*(b*d^3*x + b*c*d^2)*cos(
b*x + a)^3 + 125*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)
*cos(-5*(b*c - a*d)/d)*cos_integral(5*(b*d*x + b*c)/d) - 27*(b^3*d^3*x^3 +
 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-3*(b*c - a*d)/d)*cos_inte
gral(3*(b*d*x + b*c)/d) - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x
 + b^3*c^3)*cos(-(b*c - a*d)/d)*cos_integral((b*d*x + b*c)/d) - 125*(b^3*d
^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-5*(b*c - a*d)/d)*
sin_integral(5*(b*d*x + b*c)/d) + 27*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + b^3*c^3)*sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d)
 + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-(b*c -
 a*d)/d)*sin_integral((b*d*x + b*c)/d) + 32*(b*d^3*x + b*c*d^2)*cos(b*x +
a) - 16*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d + (25*b^2*d^3*x^2 + 5
0*b^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*cos(b*x + a)^4 - (21*b^2*d^3*x^2 + 4
2*b^2*c*d^2*x + 21*b^2*c^2*d - 2*d^3)*cos(b*x + a)^2)*sin(b*x + a))/(d^7*x
^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

### 3.96.6 Sympy [F]

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sin^3{(a+bx)}\cos^2{(a+bx)}}{(c+dx)^4}\,dx$$

input
```
integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c)**4,x)
```

output
```
Integral(sin(a + b*x)**3*cos(a + b*x)**2/(c + d*x)**4, x)
```

### 3.96.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.28

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx =$$
$$-\frac{2\,b^4\left(i\,E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^4\left(i\,E_4\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-\right.}{}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/32*(2*b^4*(I*exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*e
xp_integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
- b^4*(I*exp_integral_e(4, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_i
ntegral_e(4, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d)
 - b^4*(-I*exp_integral_e(4, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp
_integral_e(4, -5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-5*(b*c - a*d)/
d) + 2*b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_int
egral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^4*
(exp_integral_e(4, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(
4, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d) - b^4*(ex
p_integral_e(4, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(4,
-5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-5*(b*c - a*d)/d))/((b^3*c^3*d
 - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^
3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)
)*b)
```

### 3.96.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 207.31 (sec) , antiderivative size = 2449286, normalized size of antiderivative = 5930.47

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")
```

output
```
1/192*(125*b^3*d^3*x^3*real_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*
x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*
tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 27*b^3*d^3*x^3*real_p
art(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b
*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/
d)^2*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3*real_part(cos_integral(b*x + b*c/d))
*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b^3*d^3*
x^3*real_part(cos_integral(-b*x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*ta
n(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 27*b^3*d^3*x^3*real_part(cos_integral(-3*b*
x - 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*ta
n(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 + 125*b^3*d^3*x^3*real_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*b*x)^
2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan
(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*b^3*d^3*x^3*imag_part(
cos_integral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*ta
n(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d) - 4*b^3*d^3*x^3*imag_part(cos_integral(-b*x - b*c/d))*tan(5/2*
b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*...
```

### 3.96.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos(a+b\,x)^2\sin(a+b\,x)^3}{(c+d\,x)^4}\,dx$$

input
```
int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^4,x)
```

output
```
int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^4, x)
```

## 3.96. $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$

# 3.97 $\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx$


3.97.1 Optimal result . . . . . . . . . . 809
3.97.2 Mathematica [N/A] . . . . . . . . . . 809
3.97.3 Rubi [N/A] . . . . . . . . . . 810
3.97.4 Maple [N/A] (verified) . . . . . . . . . . 811
3.97.5 Fricas [N/A] . . . . . . . . . . 812
3.97.6 Sympy [N/A] . . . . . . . . . . 812
3.97.7 Maxima [N/A] . . . . . . . . . . 812
3.97.8 Giac [N/A] . . . . . . . . . . 813
3.97.9 Mupad [N/A] . . . . . . . . . . 813


## 3.97.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx = \frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b} + \frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)}{2b} + \mathrm{Int}\left((c+dx)^m\csc(a+bx),x\right)$$

output
```
1/2*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/
d)^m)+1/2*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*x
+c)/d)^m)+Unintegrable((d*x+c)^m*csc(b*x+a),x)
```

## 3.97.2 Mathematica [N/A]

Not integrable

Time = 11.87 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx = \int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x], x]
```

### 3.97.3 Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4908, 3042, 3789, 2612, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cot(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{4908}$$

$$\int (c+dx)^m\csc(a+bx)dx-\int (c+dx)^m\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m\csc(a+bx)dx-\int (c+dx)^m\sin(a+bx)dx$$

$$\Big\downarrow \text{3789}$$

$$-\frac{1}{2}i\int e^{-i(a+bx)}(c+dx)^m dx+\frac{1}{2}i\int e^{i(a+bx)}(c+dx)^m dx+\int (c+dx)^m\csc(a+bx)dx$$

$$\Big\downarrow \text{2612}$$

$$\int (c+dx)^m\csc(a+bx)dx+\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}+\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}$$

$$\Big\downarrow \text{4680}$$

$$\int (c+dx)^m\csc(a+bx)dx+\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}+\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x],x]
```

output
```
$Aborted
```

### 3.97.3.1 Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

## 3.97.4 Maple [N/A] (verified)

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \cos(xb+a)\cot(xb+a)\,dx$$

input
```
int((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x)
```

output
```
int((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x)
```

### 3.97.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\cot(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*cos(b*x + a)*cot(b*x + a), x)
```

### 3.97.6 Sympy [N/A]

Not integrable

Time = 7.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx = \int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*cos(b*x+a)*cot(b*x+a),x)
```

output
```
Integral((c + d*x)**m*cos(a + b*x)*cot(a + b*x), x)
```

### 3.97.7 Maxima [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\cot(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a), x)
```

### 3.97.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\cot(bx+a)\;dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a), x)
```

### 3.97.9 Mupad [N/A]

Not integrable

Time = 23.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx = \int \cos(a+b\,x)\;\cot(a+b\,x)\;(c+d\,x)^m\,dx$$

input
```
int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^m, x)
```

# 3.98 $\int (c+dx)^4 \cos(a+bx) \cot(a+bx) \, dx$

3.98.1 Optimal result . . . . . 815
3.98.2 Mathematica [B] (verified) . . . . . 816
3.98.3 Rubi [A] (verified) . . . . . 817
3.98.4 Maple [B] (verified) . . . . . 823
3.98.5 Fricas [B] (verification not implemented) . . . . . 824
3.98.6 Sympy [F] . . . . . 825
3.98.7 Maxima [B] (verification not implemented) . . . . . 826
3.98.8 Giac [F] . . . . . 826
3.98.9 Mupad [F(-1)] . . . . . 827

### 3.98.1 Optimal result

Integrand size = 20, antiderivative size = 333

$$\int (c+dx)^4\cos(a+bx)\cot(a+bx)\,dx = -\frac{2(c+dx)^4\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{24d^4\cos(a+bx)}{b^5} - \frac{12d^2(c+dx)^2\cos(a+bx)}{b^3} + \frac{(c+dx)^4\cos(a+bx)}{b} + \frac{4id(c+dx)^3\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} - \frac{4id(c+dx)^3\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} - \frac{12d^2(c+dx)^2\,\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} + \frac{12d^2(c+dx)^2\,\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3} - \frac{24id^3(c+dx)\,\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4} + \frac{24id^3(c+dx)\,\text{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^4} + \frac{24d^4\,\text{PolyLog}\left(5,-e^{i(a+bx)}\right)}{b^5} - \frac{24d^4\,\text{PolyLog}\left(5,e^{i(a+bx)}\right)}{b^5} + \frac{24d^3(c+dx)\sin(a+bx)}{b^4} - \frac{4d(c+dx)^3\sin(a+bx)}{b^2}$$

output
```
-2*(d*x+c)^4*arctanh(exp(I*(b*x+a)))/b+24*d^4*cos(b*x+a)/b^5-12*d^2*(d*x+c
)^2*cos(b*x+a)/b^3+(d*x+c)^4*cos(b*x+a)/b+4*I*d*(d*x+c)^3*polylog(2,-exp(I
*(b*x+a)))/b^2-4*I*d*(d*x+c)^3*polylog(2,exp(I*(b*x+a)))/b^2-12*d^2*(d*x+c
)^2*polylog(3,-exp(I*(b*x+a)))/b^3+12*d^2*(d*x+c)^2*polylog(3,exp(I*(b*x+a
)))/b^3-24*I*d^3*(d*x+c)*polylog(4,-exp(I*(b*x+a)))/b^4+24*I*d^3*(d*x+c)*p
olylog(4,exp(I*(b*x+a)))/b^4+24*d^4*polylog(5,-exp(I*(b*x+a)))/b^5-24*d^4*
polylog(5,exp(I*(b*x+a)))/b^5+24*d^3*(d*x+c)*sin(b*x+a)/b^4-4*d*(d*x+c)^3*
sin(b*x+a)/b^2
```

### 3.98.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 837 vs. $2(333) = 666$.

Time = 1.33 (sec) , antiderivative size = 837, normalized size of antiderivative = 2.51

$$\int (c+dx)^4 \cos(a+bx)\cot(a+bx)\,dx$$
$$= \frac{b^4c^4\cos(a+bx) - 12b^2c^2d^2\cos(a+bx) + 24d^4\cos(a+bx) + 4b^4c^3dx\cos(a+bx) - 24b^2cd^3x\cos(a+bx)}{}$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x],x]
```

output
```
(b^4*c^4*Cos[a + b*x] - 12*b^2*c^2*d^2*Cos[a + b*x] + 24*d^4*Cos[a + b*x]
+ 4*b^4*c^3*d*x*Cos[a + b*x] - 24*b^2*c*d^3*x*Cos[a + b*x] + 6*b^4*c^2*d^2
*x^2*Cos[a + b*x] - 12*b^2*d^4*x^2*Cos[a + b*x] + 4*b^4*c*d^3*x^3*Cos[a +
b*x] + b^4*d^4*x^4*Cos[a + b*x] + b^4*c^4*Log[1 - E^(I*(a + b*x))] + 4*b^4
*c^3*d*x*Log[1 - E^(I*(a + b*x))] + 6*b^4*c^2*d^2*x^2*Log[1 - E^(I*(a + b*
x))] + 4*b^4*c*d^3*x^3*Log[1 - E^(I*(a + b*x))] + b^4*d^4*x^4*Log[1 - E^(I
*(a + b*x))] - b^4*c^4*Log[1 + E^(I*(a + b*x))] - 4*b^4*c^3*d*x*Log[1 + E^
(I*(a + b*x))] - 6*b^4*c^2*d^2*x^2*Log[1 + E^(I*(a + b*x))] - 4*b^4*c*d^3*
x^3*Log[1 + E^(I*(a + b*x))] - b^4*d^4*x^4*Log[1 + E^(I*(a + b*x))] + (4*I
)*b^3*d*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))] - (4*I)*b^3*d*(c + d*x)^3
*PolyLog[2, E^(I*(a + b*x))] - 12*b^2*c^2*d^2*PolyLog[3, -E^(I*(a + b*x))]
 - 24*b^2*c*d^3*x*PolyLog[3, -E^(I*(a + b*x))] - 12*b^2*d^4*x^2*PolyLog[3,
 -E^(I*(a + b*x))] + 12*b^2*c^2*d^2*PolyLog[3, E^(I*(a + b*x))] + 24*b^2*c
*d^3*x*PolyLog[3, E^(I*(a + b*x))] + 12*b^2*d^4*x^2*PolyLog[3, E^(I*(a + b
*x))] - (24*I)*b*c*d^3*PolyLog[4, -E^(I*(a + b*x))] - (24*I)*b*d^4*x*PolyL
og[4, -E^(I*(a + b*x))] + (24*I)*b*c*d^3*PolyLog[4, E^(I*(a + b*x))] + (24
*I)*b*d^4*x*PolyLog[4, E^(I*(a + b*x))] + 24*d^4*PolyLog[5, -E^(I*(a + b*x
))] - 24*d^4*PolyLog[5, E^(I*(a + b*x))] - 4*b^3*c^3*d*Sin[a + b*x] + 24*b
*c*d^3*Sin[a + b*x] - 12*b^3*c^2*d^2*x*Sin[a + b*x] + 24*b*d^4*x*Sin[a + b
*x] - 12*b^3*c*d^3*x^2*Sin[a + b*x] - 4*b^3*d^4*x^3*Sin[a + b*x])/b^5
```

### 3.98.3 Rubi [A] (verified)

Time = 1.67 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.13, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.950$, Rules used = {4908, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118, 4671, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \cos(a+bx)\cot(a+bx)\,dx$$

$$\Big\downarrow \text{ 4908}$$

$$\int (c+dx)^4 \csc(a+bx)dx-\int (c+dx)^4 \sin(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^4 \csc(a+bx)dx-\int (c+dx)^4 \sin(a+bx)dx$$

$$\Big\downarrow \text{ 3777}$$

$$-\frac{4d\int (c+dx)^3\cos(a+bx)dx}{b}+\int (c+dx)^4\csc(a+bx)dx+\frac{(c+dx)^4\cos(a+bx)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{4d\int (c+dx)^3\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}+\int (c+dx)^4\csc(a+bx)dx+\frac{(c+dx)^4\cos(a+bx)}{b}$$

$$\Big\downarrow \text{ 3777}$$

$$-\frac{4d\left(\frac{3d\int -(c+dx)^2\sin(a+bx)dx}{b}+\frac{(c+dx)^3\sin(a+bx)}{b}\right)}{b}+\int (c+dx)^4\csc(a+bx)dx+\frac{(c+dx)^4\cos(a+bx)}{b}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\int (c+dx)^2\sin(a+bx)dx}{b}\right)}{b}+\int (c+dx)^4\csc(a+bx)dx+\frac{(c+dx)^4\cos(a+bx)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\int (c+dx)^2\sin(a+bx)dx}{b}\right)}{b}+\int (c+dx)^4\csc(a+bx)dx+\frac{(c+dx)^4\cos(a+bx)}{b}$$

$$\Big\downarrow \text{ 3777}$$

$$\int (c+dx)^4 \csc(a+bx)dx - \frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b} - \frac{3d\left(\frac{2d\int (c+dx)\cos(a+bx)dx}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b} + \frac{(c+dx)^4\cos(a+bx)}{b}$$

$\downarrow$ 3042

$$\int (c+dx)^4 \csc(a+bx)dx - \frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b} - \frac{3d\left(\frac{2d\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b} + \frac{(c+dx)^4\cos(a+bx)}{b}$$

$\downarrow$ 3777

$$\int (c+dx)^4 \csc(a+bx)dx - \frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b} - \frac{3d\left(\frac{2d\left(\frac{d\int -\sin(a+bx)dx}{b} + \frac{(c+dx)\sin(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b} + \frac{(c+dx)^4\cos(a+bx)}{b}$$

$\downarrow$ 25

$$\int (c+dx)^4 \csc(a+bx)dx - \frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b} - \frac{3d\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b} - \frac{d\int \sin(a+bx)dx}{b}\right)}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b} + \frac{(c+dx)^4\cos(a+bx)}{b}$$

$\downarrow$ 3042

$$\int (c+dx)^4 \csc(a+bx)dx - \frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b} - \frac{3d\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b} - \frac{d\int \sin(a+bx)dx}{b}\right)}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b} + \frac{(c+dx)^4\cos(a+bx)}{b}$$

$\downarrow$ 3118

3.98. $\int (c+dx)^4 \cos(a+bx)\cot(a+bx)\,dx$

$$\int (c+dx)^4 \csc(a+bx)dx - \frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b} - \frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b} + \frac{(c+dx)^4\cos(a+bx)}{b}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{4d\int (c+dx)^3\log\left(1-e^{i(a+bx)}\right)dx}{b} + \frac{4d\int (c+dx)^3\log\left(1+e^{i(a+bx)}\right)dx}{b} - \frac{2(c+dx)^4\text{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b} - \frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b} + \frac{(c+dx)^4\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3011}$$

$$\frac{4d\left(\frac{i(c+dx)^3\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b} - \frac{3id\int (c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b} - \frac{4d\left(\frac{i(c+dx)^3\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b} - \frac{3id\int (c+dx)^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b} - \frac{2(c+dx)^4\text{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b} - \frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b} + \frac{(c+dx)^4\cos(a+bx)}{b}$$

$$\Big\downarrow \text{7163}$$

---

3.98. $\int (c+dx)^4\cos(a+bx)\cot(a+bx)\,dx$

$$\frac{4d\left(\frac{i(c+dx)^3 \text{PolyLog}(2,-e^{i(a+bx)})}{b}-\frac{3id\left(\frac{2id\int(c+dx)\text{PolyLog}(3,-e^{i(a+bx)})dx}{b}-\frac{i(c+dx)^2\text{PolyLog}(3,-e^{i(a+bx)})}{b}\right)}{b}\right)}{b}-\frac{4d\left(\frac{i(c+dx)^3 \text{PolyLog}(2,e^{i(a+bx)})}{b}-\frac{3id\left(\frac{2id\int(c+dx)\text{PolyLog}(3,e^{i(a+bx)})dx}{b}-\frac{i(c+dx)^2\text{PolyLog}(3,e^{i(a+bx)})}{b}\right)}{b}\right)}{b}-\frac{2(c+dx)^4\text{arctanh}(e^{i(a+bx)})}{b}-\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^4\cos(a+bx)}{b}$$

$$\Big\downarrow \text{ 7163}$$

$$\frac{4d\left(\frac{i(c+dx)^3 \text{PolyLog}(2,-e^{i(a+bx)})}{b}-\frac{3id\left(\frac{2id\left(\frac{id\int\text{PolyLog}(4,-e^{i(a+bx)})dx}{b}-\frac{i(c+dx)\text{PolyLog}(4,-e^{i(a+bx)})}{b}\right)}{b}-\frac{i(c+dx)^2\text{PolyLog}(3,-e^{i(a+bx)})}{b}\right)}{b}\right)}{b}-\frac{4d\left(\frac{i(c+dx)^3 \text{PolyLog}(2,e^{i(a+bx)})}{b}-\frac{3id\left(\frac{2id\left(\frac{id\int\text{PolyLog}(4,e^{i(a+bx)})dx}{b}-\frac{i(c+dx)\text{PolyLog}(4,e^{i(a+bx)})}{b}\right)}{b}-\frac{i(c+dx)^2\text{PolyLog}(3,e^{i(a+bx)})}{b}\right)}{b}\right)}{b}-\frac{2(c+dx)^4\text{arctanh}(e^{i(a+bx)})}{b}-\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^4\cos(a+bx)}{b}$$

$$\Big\downarrow \text{ 2720}$$

3.98. $\int(c+dx)^4\cos(a+bx)\cot(a+bx)\,dx$

$$\frac{4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(4,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{i(c+dx)^2\operatorname{PolyLo}}{}\right.}{b}\right.}{b}$$

$$\frac{4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(4,e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{i(c+dx)^2\operatorname{PolyLog}\left(3,e\right.}{b}\right.}{b}\right.}{b}$$

$$\frac{2(c+dx)^4\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^4\cos(a+bx)}{b}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{2(c+dx)^4\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{d\operatorname{PolyLog}\left(5,-e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{i(c+dx)^2\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}$$

$$\frac{4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{d\operatorname{PolyLog}\left(5,e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{i(c+dx)^2\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-$$

$$\frac{4d\left(\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^4\cos(a+bx)}{b}$$

3.98. $\int(c+dx)^4\cos(a+bx)\cot(a+bx)\,dx$

input
```
Int[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x],x]
```

output
```
(-2*(c + d*x)^4*ArcTanh[E^(I*(a + b*x))])/b + ((c + d*x)^4*Cos[a + b*x])/b
 + (4*d*((I*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b - ((3*I)*d*(((-I)*
(c + d*x)^2*PolyLog[3, -E^(I*(a + b*x))])/b + ((2*I)*d*(((-I)*(c + d*x)*Po
lyLog[4, -E^(I*(a + b*x))])/b + (d*PolyLog[5, -E^(I*(a + b*x))])/b^2))/b))
/b))/b - (4*d*((I*(c + d*x)^3*PolyLog[2, E^(I*(a + b*x))])/b - ((3*I)*d*((
(-I)*(c + d*x)^2*PolyLog[3, E^(I*(a + b*x))])/b + ((2*I)*d*(((-I)*(c + d*x
)*PolyLog[4, E^(I*(a + b*x))])/b + (d*PolyLog[5, E^(I*(a + b*x))])/b^2))/b
))/b))/b - (4*d*(((c + d*x)^3*Sin[a + b*x])/b - (3*d*(-(((c + d*x)^2*Cos[a
 + b*x])/b) + (2*d*((d*Cos[a + b*x])/b^2 + ((c + d*x)*Sin[a + b*x])/b))/b)
)/b))/b
```

### 3.98.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

### 3.98.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1294 vs. $2(311) = 622$.

Time = 2.60 (sec) , antiderivative size = 1295, normalized size of antiderivative = 3.89

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1295 |

input
```
int((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(d^4*x^4*b^4+4*b^4*c*d^3*x^3+6*b^4*c^2*d^2*x^2+4*b^4*c^3*d*x+4*I*b^3*d
^4*x^3+b^4*c^4-12*b^2*d^4*x^2+12*I*b^3*c*d^3*x^2-24*b^2*c*d^3*x+12*I*b^3*c
^2*d^2*x-12*b^2*c^2*d^2+4*I*b^3*c^3*d-24*I*b*d^4*x+24*d^4-24*I*b*c*d^3)/b^
5*exp(I*(b*x+a))+1/2*(d^4*x^4*b^4+4*b^4*c*d^3*x^3+6*b^4*c^2*d^2*x^2+4*b^4*
c^3*d*x-4*I*b^3*d^4*x^3+b^4*c^4-12*b^2*d^4*x^2-12*I*b^3*c*d^3*x^2-24*b^2*c
*d^3*x-12*I*b^3*c^2*d^2*x-12*b^2*c^2*d^2-4*I*b^3*c^3*d+24*I*b*d^4*x+24*d^4
+24*I*b*c*d^3)/b^5*exp(-I*(b*x+a))-2/b*c^4*arctanh(exp(I*(b*x+a)))-12/b^3*
c^2*d^2*a^2*arctanh(exp(I*(b*x+a)))+8/b^2*c^3*d*a*arctanh(exp(I*(b*x+a)))-
24/b^3*c*d^3*polylog(3,-exp(I*(b*x+a)))*x-4/b^4*c*d^3*ln(exp(I*(b*x+a))+1)
*a^3+4/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a^3-6/b^3*d^2*c^2*ln(1-exp(I*(b*x+a)
))*a^2+6/b^3*d^2*c^2*ln(exp(I*(b*x+a))+1)*a^2+4/b^2*c^3*d*ln(1-exp(I*(b*x+
a)))*a-4/b^2*c^3*d*ln(exp(I*(b*x+a))+1)*a+4/b*c*d^3*ln(1-exp(I*(b*x+a)))*x
^3-4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3+6/b*d^2*c^2*ln(1-exp(I*(b*x+a)))*x^2
-6/b*d^2*c^2*ln(exp(I*(b*x+a))+1)*x^2+4/b*c^3*d*ln(1-exp(I*(b*x+a)))*x-4/b
*c^3*d*ln(exp(I*(b*x+a))+1)*x+24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x+24*
I/b^4*c*d^3*polylog(4,exp(I*(b*x+a)))-24*I/b^4*c*d^3*polylog(4,-exp(I*(b*x
+a)))-4*I/b^2*c^3*d*polylog(2,exp(I*(b*x+a)))-24*I/b^4*d^4*polylog(4,-exp(
I*(b*x+a)))*x+4*I/b^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3-4*I/b^2*d^4*polyl
og(2,exp(I*(b*x+a)))*x^3+4*I/b^2*c^3*d*polylog(2,-exp(I*(b*x+a)))+24*d^4*p
olylog(5,-exp(I*(b*x+a)))/b^5-24*d^4*polylog(5,exp(I*(b*x+a)))/b^5+12*I...
```

### 3.98.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1367 vs. $2(305) = 610$.

Time = 0.33 (sec) , antiderivative size = 1367, normalized size of antiderivative = 4.11

$$\int (c+dx)^4 \cos(a+bx)\cot(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(24*d^4*polylog(5, cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*polylog(5,
 cos(b*x + a) - I*sin(b*x + a)) - 24*d^4*polylog(5, -cos(b*x + a) + I*sin(
b*x + a)) - 24*d^4*polylog(5, -cos(b*x + a) - I*sin(b*x + a)) - 2*(b^4*d^4
*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 24*d^4 + 6*(b^4*c^2*d^
2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a) + 4*(I*b^
3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(cos
(b*x + a) + I*sin(b*x + a)) + 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*
b^3*c^2*d^2*x - I*b^3*c^3*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) + 4*(I*b
^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(-c
os(b*x + a) + I*sin(b*x + a)) + 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*
I*b^3*c^2*d^2*x - I*b^3*c^3*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (b^
4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)
*log(cos(b*x + a) + I*sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 +
 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(cos(b*x + a) - I*sin(b*x
 + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - (b^4*c^4 -
4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-1/2*cos(
b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*
b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^
3*b*c*d^3 - a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^4*d^4...
```

### 3.98.6 Sympy [F]

$$\int (c+dx)^4 \cos(a+bx)\cot(a+bx)\,dx = \int (c+dx)^4 \cos{(a+bx)}\cot{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**4*cos(b*x+a)*cot(b*x+a),x)
```

output
```
Integral((c + d*x)**4*cos(a + b*x)*cot(a + b*x), x)
```

### 3.98.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1548 vs. 2(305) = 610.

Time = 0.53 (sec) , antiderivative size = 1548, normalized size of antiderivative = 4.65

$$\int (c+dx)^4 \cos(a+bx)\cot(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(c^4*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
- 4*a*c^3*d*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1
))/b + 6*a^2*c^2*d^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x
 + a) - 1))/b^2 - 4*a^3*c*d^3*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + lo
g(cos(b*x + a) - 1))/b^3 + a^4*d^4*(2*cos(b*x + a) - log(cos(b*x + a) + 1)
 + log(cos(b*x + a) - 1))/b^4 + (48*d^4*polylog(5, -e^(I*b*x + I*a)) - 48*
d^4*polylog(5, e^(I*b*x + I*a)) + 2*(-I*(b*x + a)^4*d^4 + 4*(-I*b*c*d^3 +
I*a*d^4)*(b*x + a)^3 + 6*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x
 + a)^2 + 4*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 - 3*I*a^2*b*c*d^3 + I*a^3*d^
4)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 2*(-I*(b*x + a)^4*
d^4 + 4*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 6*(-I*b^2*c^2*d^2 + 2*I*a*b*c
*d^3 - I*a^2*d^4)*(b*x + a)^2 + 4*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 - 3*I*
a^2*b*c*d^3 + I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) +
1) + 2*((b*x + a)^4*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*(a^2 - 2)*d^4
 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 -
 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^
3 - (a^3 - 6*a)*d^4)*(b*x + a))*cos(b*x + a) + 8*(I*b^3*c^3*d - 3*I*a*b^2*
c^2*d^2 + 3*I*a^2*b*c*d^3 + I*(b*x + a)^3*d^4 - I*a^3*d^4 + 3*(I*b*c*d^3 -
 I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x
 + a))*dilog(-e^(I*b*x + I*a)) + 8*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 - ...
```

### 3.98.8 Giac [F]

$$\int (c+dx)^4 \cos(a+bx)\cot(a+bx)\,dx = \int (dx+c)^4 \cos(bx+a)\cot(bx+a)\,dx$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*cos(b*x + a)*cot(b*x + a), x)
```

### 3.98.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \cos(a+bx)\cot(a+bx)\,dx = \int \cos(a+b\,x)\,\cot(a+b\,x)\,(c+d\,x)^4\,dx$$

input
```
int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^4,x)
```

output
```
int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^4, x)
```

# 3.99 $\int (c+dx)^3 \cos(a+bx) \cot(a+bx)\,dx$


3.99.1 Optimal result . . . . . 828
3.99.2 Mathematica [A] (verified) . . . . . 829
3.99.3 Rubi [A] (verified) . . . . . 829
3.99.4 Maple [B] (verified) . . . . . 835
3.99.5 Fricas [B] (verification not implemented) . . . . . 836
3.99.6 Sympy [F] . . . . . 836
3.99.7 Maxima [B] (verification not implemented) . . . . . 837
3.99.8 Giac [F] . . . . . 837
3.99.9 Mupad [F(-1)] . . . . . 838


## 3.99.1 Optimal result

Integrand size = 20, antiderivative size = 254

$$\begin{aligned}\int (c+dx)^3 \cos(a+bx)\cot(a+bx)\,dx = & -\frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} \\ & -\frac{6d^2(c+dx)\cos(a+bx)}{b^3}+\frac{(c+dx)^3\cos(a+bx)}{b} \\ & +\frac{3id(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} \\ & -\frac{3id(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} \\ & -\frac{6d^2(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} \\ & +\frac{6d^2(c+dx)\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3} \\ & -\frac{6id^3\operatorname{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4} \\ & +\frac{6id^3\operatorname{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^4} \\ & +\frac{6d^3\sin(a+bx)}{b^4}-\frac{3d(c+dx)^2\sin(a+bx)}{b^2}\end{aligned}$$

output
```
-2*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b-6*d^2*(d*x+c)*cos(b*x+a)/b^3+(d*x+c
)^3*cos(b*x+a)/b+3*I*d*(d*x+c)^2*polylog(2,-exp(I*(b*x+a)))/b^2-3*I*d*(d*x
+c)^2*polylog(2,exp(I*(b*x+a)))/b^2-6*d^2*(d*x+c)*polylog(3,-exp(I*(b*x+a)
))/b^3+6*d^2*(d*x+c)*polylog(3,exp(I*(b*x+a)))/b^3-6*I*d^3*polylog(4,-exp(
I*(b*x+a)))/b^4+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+6*d^3*sin(b*x+a)/b^4
-3*d*(d*x+c)^2*sin(b*x+a)/b^2
```

### 3.99.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.30

$$\int (c+dx)^3\cos(a+bx)\cot(a+bx)\,dx$$
$$= \frac{-2b^3(c+dx)^3\text{arctanh}(\cos(a+bx)+i\sin(a+bx))+3id(b^2(c+dx)^2\,\text{PolyLog}(2,-\cos(a+bx)-i\sin(a+$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x],x]
```

output
```
(-2*b^3*(c + d*x)^3*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]] + (3*I)*d*(b^2*
(c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]] + (2*I)*b*d*(c + d*
x)*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] - 2*d^2*PolyLog[4, -Cos[a +
b*x] - I*Sin[a + b*x]]) - (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, Cos[a + b*x]
 + I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, Cos[a + b*x] + I*Sin[a
 + b*x]] - 2*d^2*PolyLog[4, Cos[a + b*x] + I*Sin[a + b*x]]) + Cos[b*x]*(b*
(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a] - 3*d*(-2*d^2 + b^2*(c + d*x)^
2)*Sin[a]) - (3*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*(c + d*x)*(-6*d^2
+ b^2*(c + d*x)^2)*Sin[a])*Sin[b*x])/b^4
```

### 3.99.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {4908, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \cos(a+bx)\cot(a+bx)\,dx$$

$$\Big\downarrow \text{4908}$$

$$\int (c+dx)^3 \csc(a+bx)dx - \int (c+dx)^3 \sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3 \csc(a+bx)dx - \int (c+dx)^3 \sin(a+bx)dx$$

$$\Big\downarrow \text{3777}$$

$$-\frac{3d\int (c+dx)^2 \cos(a+bx)dx}{b} + \int (c+dx)^3 \csc(a+bx)dx + \frac{(c+dx)^3 \cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3d\int (c+dx)^2 \sin\left(a+bx+\frac{\pi}{2}\right)dx}{b} + \int (c+dx)^3 \csc(a+bx)dx + \frac{(c+dx)^3 \cos(a+bx)}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{3d\left(\frac{2d\int -((c+dx)\sin(a+bx))dx}{b} + \frac{(c+dx)^2 \sin(a+bx)}{b}\right)}{b} + \int (c+dx)^3 \csc(a+bx)dx + \frac{(c+dx)^3 \cos(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{3d\left(\frac{(c+dx)^2 \sin(a+bx)}{b} - \frac{2d\int (c+dx)\sin(a+bx)dx}{b}\right)}{b} + \int (c+dx)^3 \csc(a+bx)dx + \frac{(c+dx)^3 \cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3d\left(\frac{(c+dx)^2 \sin(a+bx)}{b} - \frac{2d\int (c+dx)\sin(a+bx)dx}{b}\right)}{b} + \int (c+dx)^3 \csc(a+bx)dx + \frac{(c+dx)^3 \cos(a+bx)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\int (c+dx)^3 \csc(a+bx)dx - \frac{3d\left(\frac{(c+dx)^2 \sin(a+bx)}{b} - \frac{2d\left(\frac{d\int \cos(a+bx)dx}{b} - \frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b} + \frac{(c+dx)^3 \cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

3.99. $\int (c+dx)^3 \cos(a+bx)\cot(a+bx)\,dx$

$$\int (c+dx)^3 \csc(a+bx)dx - \frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b} - \frac{2d\left(\frac{d\int \sin\left(a+bx+\frac{\pi}{2}\right)dx}{b} - \frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b} + \frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\Big\downarrow \; 3117$$

$$\int (c+dx)^3 \csc(a+bx)dx - \frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b} - \frac{2d\left(\frac{d\sin(a+bx)}{b^2} - \frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b} + \frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\Big\downarrow \; 4671$$

$$-\frac{3d\int (c+dx)^2\log\left(1-e^{i(a+bx)}\right)dx}{b} + \frac{3d\int (c+dx)^2\log\left(1+e^{i(a+bx)}\right)dx}{b} - \frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b} - \frac{2d\left(\frac{d\sin(a+bx)}{b^2} - \frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b} + \frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\Big\downarrow \; 3011$$

$$\frac{3d\left(\frac{i(c+dx)^2\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b} - \frac{2id\int (c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b} - \frac{3d\left(\frac{i(c+dx)^2\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b} - \frac{2id\int (c+dx)\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b} - \frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b} - \frac{2d\left(\frac{d\sin(a+bx)}{b^2} - \frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b} + \frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\Big\downarrow \; 7163$$

---

3.99. $\int (c+dx)^3\cos(a+bx)\cot(a+bx)\,dx$

$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{id\int\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{id\int\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^3\cos(a+bx)}{b}$$

$\Big\downarrow$ 2720

$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^3\cos(a+bx)}{b}$$

$\Big\downarrow$ 7143

3.99. $\int(c+dx)^3\cos(a+bx)\cot(a+bx)\,dx$

$$-\frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\operatorname{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\operatorname{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^3\cos(a+bx)}{b}$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x],x]
```

output
```
(-2*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b + ((c + d*x)^3*Cos[a + b*x])/b
 + (3*d*((I*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*
(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b + (d*PolyLog[4, -E^(I*(a + b*x))
])/b^2))/b))/b - (3*d*((I*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b - ((2
*I)*d*(((-I)*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b + (d*PolyLog[4, E^(I
*(a + b*x))])/b^2))/b))/b - (3*d*(((c + d*x)^2*Sin[a + b*x])/b - (2*d*(-((
(c + d*x)*Cos[a + b*x])/b) + (d*Sin[a + b*x])/b^2))/b))/b
```

### 3.99.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.99. $\int (c+dx)^3 \cos(a+bx) \cot(a+bx)\,dx$

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

### 3.99.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 846 vs. 2(236) = 472.

Time = 1.79 (sec) , antiderivative size = 847, normalized size of antiderivative = 3.33

| method | result |
|---|---|
| risch | $\frac{\left(d^3x^3b^3+3b^3c\,d^2x^2-3ib^2d^3x^2+3b^3c^2dx-6ib^2c\,d^2x+b^3c^3-3ib^2c^2d-6b\,d^3x-6c\,d^2b+6id^3\right)\mathrm{e}^{-i(xb+a)}}{2b^4}+\frac{\left(d^3x^3b^3+3b^3c\,d^2x^2+3ib^2d^3\right.}{}$ |

input
```
int((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b
*d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a)
)+2/b^4*d^3*a^3*arctanh(exp(I*(b*x+a)))+6/b^3*d^3*polylog(3,exp(I*(b*x+a))
)*x-6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+1/b^4*d^3*ln(1-exp(I*(b*x+a)))*
a^3-1/b^4*d^3*ln(exp(I*(b*x+a))+1)*a^3+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3-1/
b*d^3*ln(exp(I*(b*x+a))+1)*x^3+6/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))-6/b^3
*c*d^2*polylog(3,-exp(I*(b*x+a)))+1/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c
^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2
*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))-6/b^3*c*d^2*a^2*arctanh(exp(I*(b*x+a)))
+6/b^2*c^2*d*a*arctanh(exp(I*(b*x+a)))+3/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a-
3/b^2*c^2*d*ln(exp(I*(b*x+a))+1)*a+3/b^3*c*d^2*ln(exp(I*(b*x+a))+1)*a^2-3*
I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))+3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+
a)))-3*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2+3*I/b^2*d^3*polylog(2,-exp(
I*(b*x+a)))*x^2-6*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4-2/b*c^3*arctanh(exp
(I*(b*x+a)))+3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-3/b*c*d^2*ln(exp(I*(b*x+a)
)+1)*x^2+3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x-3/b*c^2*d*ln(exp(I*(b*x+a))+1)*x
-3/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^
4-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x+6*I/b^2*c*d^2*polylog(2,-exp(I
*(b*x+a)))*x
```

### 3.99.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(230) = 460$.

Time = 0.32 (sec) , antiderivative size = 925, normalized size of antiderivative = 3.64

$$\int (c+dx)^3\cos(a+bx)\cot(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(6*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*polylog(4
, cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*polylog(4, -cos(b*x + a) + I*si
n(b*x + a)) - 6*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + 2*(b^3*
d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*
x)*cos(b*x + a) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(
cos(b*x + a) + I*sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b
^2*c^2*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^
2*c*d^2*x + I*b^2*c^2*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 3*(-I*b^2
*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-cos(b*x + a) - I*sin(b*x
+ a)) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(
b*x + a) + I*sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^
2*d*x + b^3*c^3)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b
^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x
+ a) + 1/2) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2
*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x +
 a) + I*sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x
 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x
+ a) + 1) + 6*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)
) + 6*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 6...
```

### 3.99.6 Sympy [F]

$$\int (c+dx)^3\cos(a+bx)\cot(a+bx)\,dx = \int (c+dx)^3\cos(a+bx)\cot(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*cos(b*x+a)*cot(b*x+a),x)
```

output
```
Integral((c + d*x)**3*cos(a + b*x)*cot(a + b*x), x)
```

### 3.99.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 929 vs. 2(230) = 460.

Time = 0.38 (sec) , antiderivative size = 929, normalized size of antiderivative = 3.66

$$\int (c+dx)^3 \cos(a+bx)\cot(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(c^3*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
- 3*a*c^2*d*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1
))/b + 3*a^2*c*d^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x +
 a) - 1))/b^2 - a^3*d^3*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(
b*x + a) - 1))/b^3 - (12*I*d^3*polylog(4, -e^(I*b*x + I*a)) - 12*I*d^3*pol
ylog(4, e^(I*b*x + I*a)) - 2*(-I*(b*x + a)^3*d^3 + 3*(-I*b*c*d^2 + I*a*d^3
)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*(b*x + a))*ar
ctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*(-I*(b*x + a)^3*d^3 + 3*(-I*b*c*
d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*
(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*((b*x + a)^3*d^3 -
 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*
a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos(b*x + a) - 6*(I*b^2*c^2*d - 2*I*
a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x +
 a))*dilog(-e^(I*b*x + I*a)) - 6*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x +
a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*dilog(e^(I*b*x
+ I*a)) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2
+ 2*cos(b*x + a) + 1) - ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2
 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + s
in(b*x + a)^2 - 2*cos(b*x + a) + 1) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d...
```

### 3.99.8 Giac [F]

$$\int (c+dx)^3 \cos(a+bx)\cot(a+bx)\,dx = \int (dx+c)^3 \cos(bx+a)\cot(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*cos(b*x + a)*cot(b*x + a), x)
```

### 3.99.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \cos(a+bx)\cot(a+bx)\,dx = \int \cos\left(a+b\,x\right)\,\cot\left(a+b\,x\right)\,\left(c+d\,x\right)^3\,dx$$

input
```
int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^3,x)
```

output
```
int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^3, x)
```

# 3.100 $\int (c+dx)^2 \cos(a+bx) \cot(a+bx) \, dx$

3.100.1 Optimal result . . . 839
3.100.2 Mathematica [A] (verified) . . . 840
3.100.3 Rubi [A] (verified) . . . 840
3.100.4 Maple [B] (verified) . . . 844
3.100.5 Fricas [B] (verification not implemented) . . . 844
3.100.6 Sympy [F] . . . 845
3.100.7 Maxima [B] (verification not implemented) . . . 845
3.100.8 Giac [F] . . . 846
3.100.9 Mupad [F(-1)] . . . 846

## 3.100.1 Optimal result

Integrand size = 20, antiderivative size = 171

$$\int (c+dx)^2 \cos(a+bx) \cot(a+bx) \, dx = -\frac{2(c+dx)^2 \text{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{2d^2 \cos(a+bx)}{b^3} + \frac{(c+dx)^2 \cos(a+bx)}{b} + \frac{2id(c+dx) \text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} - \frac{2id(c+dx) \text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} - \frac{2d^2 \text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} + \frac{2d^2 \text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3} - \frac{2d(c+dx) \sin(a+bx)}{b^2}$$

output
```
-2*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b-2*d^2*cos(b*x+a)/b^3+(d*x+c)^2*cos(
b*x+a)/b+2*I*d*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^2-2*I*d*(d*x+c)*polylo
g(2,exp(I*(b*x+a)))/b^2-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog
(3,exp(I*(b*x+a)))/b^3-2*d*(d*x+c)*sin(b*x+a)/b^2
```

### 3.100.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.29

$$\int (c+dx)^2\cos(a+bx)\cot(a+bx)\,dx$$
$$= \frac{b^2(c+dx)^2\log\left(1-e^{i(a+bx)}\right)-b^2(c+dx)^2\log\left(1+e^{i(a+bx)}\right)+2ibd(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)-2ibd}{}$$

input
```
Integrate[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x],x]
```

output
```
(b^2*(c + d*x)^2*Log[1 - E^(I*(a + b*x))] - b^2*(c + d*x)^2*Log[1 + E^(I*(
a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] - (2*I)*b*d*
(c + d*x)*PolyLog[2, E^(I*(a + b*x))] - 2*d^2*PolyLog[3, -E^(I*(a + b*x))]
 + 2*d^2*PolyLog[3, E^(I*(a + b*x))] + Cos[b*x]*((-2*d^2 + b^2*(c + d*x)^2
)*Cos[a] - 2*b*d*(c + d*x)*Sin[a]) - (2*b*d*(c + d*x)*Cos[a] + (-2*d^2 + b
^2*(c + d*x)^2)*Sin[a])*Sin[b*x])/b^3
```

### 3.100.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4908, 3042, 3777, 3042, 3777, 25, 3042, 3118, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\cos(a+bx)\cot(a+bx)\,dx$$

$$\Big\downarrow \text{4908}$$

$$\int (c+dx)^2\csc(a+bx)dx-\int (c+dx)^2\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2\csc(a+bx)dx-\int (c+dx)^2\sin(a+bx)dx$$

$$\Big\downarrow \text{3777}$$

$$-\frac{2d\int (c+dx)\cos(a+bx)dx}{b}+\int (c+dx)^2\csc(a+bx)dx+\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2d\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}+\int(c+dx)^2\csc(a+bx)dx+\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{2d\left(\frac{d\int-\sin(a+bx)dx}{b}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}+\int(c+dx)^2\csc(a+bx)dx+\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)}{b}+\int(c+dx)^2\csc(a+bx)dx+\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)}{b}+\int(c+dx)^2\csc(a+bx)dx+\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3118}$$

$$\int(c+dx)^2\csc(a+bx)dx-\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}+\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{2d\int(c+dx)\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{2d\int(c+dx)\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}+\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3011}$$

$$\frac{2d\left(\frac{i(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{id\int\text{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{id\int\text{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}+\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\Big\downarrow \text{2720}$$

---

3.100. $\int(c+dx)^2\cos(a+bx)\cot(a+bx)\,dx$

$$\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}+\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}+\frac{(c+dx)^2\cos(a+bx)}{b}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x],x]
```

output
```
(-2*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b + ((c + d*x)^2*Cos[a + b*x])/b
 + (2*d*((I*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b - (d*PolyLog[3, -E^(
I*(a + b*x))])/b^2))/b - (2*d*((I*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b
 - (d*PolyLog[3, E^(I*(a + b*x))])/b^2))/b - (2*d*((d*Cos[a + b*x])/b^2 +
((c + d*x)*Sin[a + b*x])/b))/b
```

### 3.100.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.100. $\int (c+dx)^2 \cos(a+bx) \cot(a+bx)\, dx$

### 3.100.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(159) = 318.

Time = 1.43 (sec) , antiderivative size = 479, normalized size of antiderivative = 2.80

| method | result |
|---|---|
| risch | $\frac{(x^2d^2b^2+2b^2cdx+2ib\,d^2x+b^2c^2+2ibcd-2d^2)\mathrm{e}^{i(xb+a)}}{2b^3}+\frac{(x^2d^2b^2+2b^2cdx-2ib\,d^2x+b^2c^2-2ibcd-2d^2)\mathrm{e}^{-i(xb+a)}}{2b^3}-\frac{2d^2a^2\,\text{arctanh}}{b^3}$ |

input
```
int((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp(
I*(b*x+a))+1/2*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*
d)/b^3*exp(-I*(b*x+a))-2/b^3*d^2*a^2*arctanh(exp(I*(b*x+a)))+2*I/b^2*c*d*p
olylog(2,-exp(I*(b*x+a)))+4/b^2*c*d*a*arctanh(exp(I*(b*x+a)))+2/b^2*d*c*ln
(1-exp(I*(b*x+a)))*a-2/b^2*c*d*ln(exp(I*(b*x+a))+1)*a-2*I/b^2*d^2*polylog(
2,exp(I*(b*x+a)))*x+2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-1/b^3*d^2*ln(
1-exp(I*(b*x+a)))*a^2+1/b^3*d^2*ln(exp(I*(b*x+a))+1)*a^2+1/b*d^2*ln(1-exp(
I*(b*x+a)))*x^2+2*d^2*polylog(3,exp(I*(b*x+a)))/b^3-1/b*d^2*ln(exp(I*(b*x+
a))+1)*x^2-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3-2*I/b^2*d*c*polylog(2,exp(
I*(b*x+a)))+2/b*d*c*ln(1-exp(I*(b*x+a)))*x-2/b*d*c*ln(exp(I*(b*x+a))+1)*x-
2/b*c^2*arctanh(exp(I*(b*x+a)))
```

### 3.100.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 562 vs. 2(155) = 310.

Time = 0.30 (sec) , antiderivative size = 562, normalized size of antiderivative = 3.29

$$\int (c+dx)^2\cos(a+bx)\cot(a+bx)\,dx$$
$$=\frac{2\,d^2\text{polylog}(3,\cos{(bx+a)}+i\,\sin{(bx+a)})+2\,d^2\text{polylog}(3,\cos{(bx+a)}-i\,\sin{(bx+a)})-2\,d^2\text{polylog}(}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylog(3, co
s(b*x + a) - I*sin(b*x + a)) - 2*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x
+ a)) - 2*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + 2*(b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a) - 2*(I*b*d^2*x + I*b*c*d)*di
log(cos(b*x + a) + I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*dilog(cos(b*
x + a) - I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*dilog(-cos(b*x + a) + I
*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*dilog(-cos(b*x + a) - I*sin(b*x
+ a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) + I*sin(b*x
 + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) - I*si
n(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x + a) +
1/2*I*sin(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b
*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c
*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b
^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) -
4*(b*d^2*x + b*c*d)*sin(b*x + a))/b^3
```

## 3.100.6 Sympy [F]

$$\int (c+dx)^2\cos(a+bx)\cot(a+bx)\,dx=\int (c+dx)^2\cos{(a+bx)}\cot{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**2*cos(b*x+a)*cot(b*x+a),x)
```

output
```
Integral((c + d*x)**2*cos(a + b*x)*cot(a + b*x), x)
```

## 3.100.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(155) = 310$.

Time = 0.30 (sec) , antiderivative size = 513, normalized size of antiderivative = 3.00

$$\int (c+dx)^2\cos(a+bx)\cot(a+bx)\,dx$$

$$= \frac{c^2(2\cos(bx+a)-\log(\cos(bx+a)+1)+\log(\cos(bx+a)-1))-\frac{2acd(2\cos(bx+a)-\log(\cos(bx+a)+1)+\log(\cos(b}{b}}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(c^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
- 2*a*c*d*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b + a^2*d^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) -
1))/b^2 - (4*d^2*polylog(3, -e^(I*b*x + I*a)) - 4*d^2*polylog(3, e^(I*b*x
+ I*a)) - 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*arctan
2(sin(b*x + a), cos(b*x + a) + 1) - 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d +
I*a*d^2)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*((b*x + a
)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(b*x + a) - 4*(I*b*c*d +
 I*(b*x + a)*d^2 - I*a*d^2)*dilog(-e^(I*b*x + I*a)) - 4*(-I*b*c*d - I*(b*x
 + a)*d^2 + I*a*d^2)*dilog(e^(I*b*x + I*a)) + ((b*x + a)^2*d^2 + 2*(b*c*d
- a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) +
 1) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 +
 sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*
sin(b*x + a))/b^2)/b
```

## 3.100.8 Giac [F]

$$\int (c+dx)^2 \cos(a+bx) \cot(a+bx) \, dx = \int (dx+c)^2 \cos(bx+a) \cot(bx+a) \, dx$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*cos(b*x + a)*cot(b*x + a), x)
```

## 3.100.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \cos(a+bx) \cot(a+bx) \, dx = \int \cos(a+b\,x) \cot(a+b\,x) \left(c+d\,x\right)^2 \, dx$$

input
```
int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^2,x)
```

output
```
int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^2, x)
```

# 3.101 $\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx$


3.101.1 Optimal result . . . . . 847
3.101.2 Mathematica [A] (verified) . . . . . 847
3.101.3 Rubi [A] (verified) . . . . . 848
3.101.4 Maple [B] (verified) . . . . . 850
3.101.5 Fricas [B] (verification not implemented) . . . . . 851
3.101.6 Sympy [F] . . . . . 851
3.101.7 Maxima [B] (verification not implemented) . . . . . 852
3.101.8 Giac [F] . . . . . 852
3.101.9 Mupad [F(-1)] . . . . . 852


## 3.101.1 Optimal result

Integrand size = 18, antiderivative size = 94

$$\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx = -\frac{2(c+dx)\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)\cos(a+bx)}{b} + \frac{id\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} - \frac{id\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} - \frac{d\sin(a+bx)}{b^2}$$

output
```
-2*(d*x+c)*arctanh(exp(I*(b*x+a)))/b+(d*x+c)*cos(b*x+a)/b+I*d*polylog(2,-e
xp(I*(b*x+a)))/b^2-I*d*polylog(2,exp(I*(b*x+a)))/b^2-d*sin(b*x+a)/b^2
```

## 3.101.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.87

$$\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx$$
$$= \frac{c\cos(a+bx)}{b} - \frac{c\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b} + \frac{c\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b} + \frac{d\left((a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)-a\log\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)+i\left(\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)\right.\right.}{b^2} + \frac{d\cos(bx)(bx\cos(a)-\sin(a))}{b^2} - \frac{d(\cos(a)+bx\sin(a))\sin(bx)}{b^2}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x]*Cot[a + b*x],x]
```

output
```
(c*Cos[a + b*x])/b - (c*Log[Cos[(a + b*x)/2]])/b + (c*Log[Sin[(a + b*x)/2]
])/b + (d*((a + b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))])
 - a*Log[Tan[(a + b*x)/2]] + I*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2,
E^(I*(a + b*x))])))/b^2 + (d*Cos[b*x]*(b*x*Cos[a] - Sin[a]))/b^2 - (d*(Cos
[a] + b*x*Sin[a])*Sin[b*x])/b^2
```

## 3.101.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {4908, 3042, 3777, 3042, 3117, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx$$

$$\Big\downarrow \text{ 4908}$$

$$\int (c+dx)\csc(a+bx)dx-\int (c+dx)\sin(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)\csc(a+bx)dx-\int (c+dx)\sin(a+bx)dx$$

$$\Big\downarrow \text{ 3777}$$

$$\int (c+dx)\csc(a+bx)dx-\frac{d\int \cos(a+bx)dx}{b}+\frac{(c+dx)\cos(a+bx)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)\csc(a+bx)dx-\frac{d\int \sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}+\frac{(c+dx)\cos(a+bx)}{b}$$

$$\Big\downarrow \text{ 3117}$$

$$\int (c+dx)\csc(a+bx)dx-\frac{d\sin(a+bx)}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}$$

$$\Big\downarrow \text{ 4671}$$

$$-\frac{d\int \log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{d\int \log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{d\sin(a+bx)}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}$$

$\Big\downarrow$ 2715

$$\frac{id\int e^{-i(a+bx)}\log\left(1-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{id\int e^{-i(a+bx)}\log\left(1+e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{d\sin(a+bx)}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}$$

$\Big\downarrow$ 2838

$$-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{d\sin(a+bx)}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}$$

input
```
Int[(c + d*x)*Cos[a + b*x]*Cot[a + b*x],x]
```

output
```
(-2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b + ((c + d*x)*Cos[a + b*x])/b + (
I*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, E^(I*(a + b*x))])/
b^2 - (d*Sin[a + b*x])/b^2
```

### 3.101.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

## 3.101.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(86) = 172$.

Time = 1.17 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.88

| method | result |
|---|---|
| default | $\frac{-\frac{da\cos(xb+a)}{b}+c\cos(xb+a)-\frac{d(\sin(xb+a)-(xb+a)\cos(xb+a))}{b}}{b}+\frac{-\frac{da\ln(\csc(xb+a)-\cot(xb+a))}{b}+c\ln(\csc(xb+a)-\cot(xb+a))+\frac{d\left((xb+a)\right.}{}}{}$ |
| risch | $\frac{(dxb+cb+id)\mathrm{e}^{i(xb+a)}}{2b^2}+\frac{(dxb+cb-id)\mathrm{e}^{-i(xb+a)}}{2b^2}-\frac{2c\,\operatorname{arctanh}\left(\mathrm{e}^{i(xb+a)}\right)}{b}+\frac{d\ln\left(1-\mathrm{e}^{i(xb+a)}\right)x}{b}+\frac{d\ln\left(1-\mathrm{e}^{i(xb+a)}\right)a}{b^2}-\frac{id\,\mathrm{poly}}{}$ |

input
```
int((d*x+c)*cos(b*x+a)*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/b*d*a*cos(b*x+a)+c*cos(b*x+a)-1/b*d*(sin(b*x+a)-(b*x+a)*cos(b*x+a)
))+1/b*(-1/b*d*a*ln(csc(b*x+a)-cot(b*x+a))+c*ln(csc(b*x+a)-cot(b*x+a))+1/b
*d*((b*x+a)*ln(1-exp(I*(b*x+a)))-(b*x+a)*ln(exp(I*(b*x+a))+1)+I*dilog(exp(
I*(b*x+a))+1)-I*dilog(1-exp(I*(b*x+a)))))
```

### 3.101.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(82) = 164$.

Time = 0.27 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.95

$$\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx$$
$$= \frac{2\,(bdx+bc)\cos{(bx+a)} - i\,d\mathrm{Li}_2(\cos{(bx+a)} + i\,\sin{(bx+a)}) + i\,d\mathrm{Li}_2(\cos{(bx+a)} - i\,\sin{(bx+a)}) - i}{}$$

input
```
integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*(b*d*x + b*c)*cos(b*x + a) - I*d*dilog(cos(b*x + a) + I*sin(b*x + a
)) + I*d*dilog(cos(b*x + a) - I*sin(b*x + a)) - I*d*dilog(-cos(b*x + a) +
I*sin(b*x + a)) + I*d*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b*d*x + b*c
)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b*d*x + b*c)*log(cos(b*x + a)
- I*sin(b*x + a) + 1) + (b*c - a*d)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x
+ a) + 1/2) + (b*c - a*d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2
) + (b*d*x + a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b*d*x + a*d)*
log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 2*d*sin(b*x + a))/b^2
```

### 3.101.6 Sympy [F]

$$\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx = \int (c+dx)\cos{(a+bx)}\cot{(a+bx)}\,dx$$

input
```
integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x)
```

output
```
Integral((c + d*x)*cos(a + b*x)*cot(a + b*x), x)
```

### 3.101.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(82) = 164.

Time = 0.33 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.13

$$\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx =$$

$$-\frac{2i\,bdx\arctan\left(\sin\left(bx+a\right),-\cos\left(bx+a\right)+1\right)-2i\,bc\arctan\left(\sin\left(bx+a\right),\cos\left(bx+a\right)-1\right)-2\left(-i\,bc\right.}{}$$

input
```
integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(2*I*b*d*x*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*I*b*c*arctan2
(sin(b*x + a), cos(b*x + a) - 1) - 2*(-I*b*d*x - I*b*c)*arctan2(sin(b*x +
a), cos(b*x + a) + 1) - 2*(b*d*x + b*c)*cos(b*x + a) - 2*I*d*dilog(-e^(I*b
*x + I*a)) + 2*I*d*dilog(e^(I*b*x + I*a)) + (b*d*x + b*c)*log(cos(b*x + a)
^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (b*d*x + b*c)*log(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 2*d*sin(b*x + a))/b^2
```

### 3.101.8 Giac [F]

$$\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx = \int (dx+c)\cos\left(bx+a\right)\cot\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*cos(b*x + a)*cot(b*x + a), x)
```

### 3.101.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx = \int \cos\left(a+b\,x\right)\,\cot(a+b\,x)\,\left(c+d\,x\right)\,dx$$

input
```
int(cos(a + b*x)*cot(a + b*x)*(c + d*x),x)
```

output
```
int(cos(a + b*x)*cot(a + b*x)*(c + d*x), x)
```

# 3.102 $\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx$


3.102.1 Optimal result . . . . . . . . . . 853
3.102.2 Mathematica [N/A] . . . . . . . . . . 853
3.102.3 Rubi [N/A] . . . . . . . . . . 854
3.102.4 Maple [N/A] (verified) . . . . . . . . . . 856
3.102.5 Fricas [N/A] . . . . . . . . . . 856
3.102.6 Sympy [N/A] . . . . . . . . . . 856
3.102.7 Maxima [N/A] . . . . . . . . . . 857
3.102.8 Giac [N/A] . . . . . . . . . . 857
3.102.9 Mupad [N/A] . . . . . . . . . . 858


## 3.102.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx = -\frac{\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d} - \frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d} + \text{Int}\left(\frac{\csc(a+bx)}{c+dx},x\right)$$

output
```
-cos(a-b*c/d)*Si(b*c/d+b*x)/d-Ci(b*c/d+b*x)*sin(a-b*c/d)/d+Unintegrable(cs
c(b*x+a)/(d*x+c),x)
```

## 3.102.2 Mathematica [N/A]

Not integrable

Time = 9.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x),x]
```

output
```
Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x), x]
```

### 3.102.3 Rubi [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4908, 3042, 3784, 3042, 3780, 3783, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{4908}$$

$$\int \frac{\csc(a+bx)}{c+dx}dx-\int \frac{\sin(a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(a+bx)}{c+dx}dx-\int \frac{\sin(a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{3784}$$

$$\int \frac{\csc(a+bx)}{c+dx}dx-\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx-\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx$$

$$\Big\downarrow \text{3042}$$

$$-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx+\int \frac{\csc(a+bx)}{c+dx}dx-\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx$$

$$\Big\downarrow \text{3780}$$

$$-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx+\int \frac{\csc(a+bx)}{c+dx}dx-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

$$\Big\downarrow \text{3783}$$

$$\int \frac{\csc(a+bx)}{c+dx}dx-\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

$$\Big\downarrow \text{4680}$$

$$\int \frac{\csc(a+bx)}{c+dx}dx-\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

input
```
Int[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x),x]
```

output
```
$Aborted
```

### 3.102.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

## 3.102. $\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx$

### 3.102.4 Maple [N/A] (verified)

Not integrable

Time = 0.78 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos (xb+a)\cot (xb+a)}{dx+c}dx$$

input
```
int(cos(b*x+a)*cot(b*x+a)/(d*x+c),x)
```

output
```
int(cos(b*x+a)*cot(b*x+a)/(d*x+c),x)
```

### 3.102.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos (bx+a)\cot (bx+a)}{dx+c}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)*cot(b*x + a)/(d*x + c), x)
```

### 3.102.6 Sympy [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos (a+bx)\cot (a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x)
```

output
```
Integral(cos(a + b*x)*cot(a + b*x)/(c + d*x), x)
```

### 3.102.7 Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 228, normalized size of antiderivative = 11.40

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)\cot(bx+a)}{dx+c}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
1/2*((I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(1, -(I*b
*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + 2*d*integrate(sin(b*x + a)/((d*x +
 c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)*cos(b*x
+ a) + c), x) + 2*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d*
x + c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) + (exp_int
egral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))
*sin(-(b*c - a*d)/d))/d
```

### 3.102.8 Giac [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)\cot(bx+a)}{dx+c}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*cot(b*x + a)/(d*x + c), x)
```

### 3.102.9 Mupad [N/A]

Not integrable

Time = 23.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx=\int \frac{\cos\left(a+b\,x\right)\,\cot(a+b\,x)}{c+d\,x}\,dx$$

input
```
int((cos(a + b*x)*cot(a + b*x))/(c + d*x),x)
```

output
```
int((cos(a + b*x)*cot(a + b*x))/(c + d*x), x)
```

# 3.103 $\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$

3.103.1 Optimal result . . . 859
3.103.2 Mathematica [N/A] . . . 859
3.103.3 Rubi [N/A] . . . 860
3.103.4 Maple [N/A] (verified) . . . 862
3.103.5 Fricas [N/A] . . . 862
3.103.6 Sympy [N/A] . . . 863
3.103.7 Maxima [N/A] . . . 863
3.103.8 Giac [N/A] . . . 864
3.103.9 Mupad [N/A] . . . 864

## 3.103.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = -\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2} + \frac{\sin(a+bx)}{d(c+dx)} + \frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2} + \text{Int}\left(\frac{\csc(a+bx)}{(c+dx)^2},x\right)$$

output
```
-b*Ci(b*c/d+b*x)*cos(a-b*c/d)/d^2+b*Si(b*c/d+b*x)*sin(a-b*c/d)/d^2+sin(b*x
+a)/d/(d*x+c)+Unintegrable(csc(b*x+a)/(d*x+c)^2,x)
```

## 3.103.2 Mathematica [N/A]

Not integrable

Time = 4.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x)^2, x]
```

### 3.103.3 Rubi [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4908, 3042, 3778, 3042, 3784, 3042, 3780, 3783, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{4908}$$

$$\int \frac{\csc(a+bx)}{(c+dx)^2}dx-\int \frac{\sin(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(a+bx)}{(c+dx)^2}dx-\int \frac{\sin(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3778}$$

$$-\frac{b\int \frac{\cos(a+bx)}{c+dx}dx}{d}+\int \frac{\csc(a+bx)}{(c+dx)^2}dx+\frac{\sin(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx}{d}+\int \frac{\csc(a+bx)}{(c+dx)^2}dx+\frac{\sin(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3784}$$

$$\int \frac{\csc(a+bx)}{(c+dx)^2}dx-\frac{b\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}+\frac{\sin(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(a+bx)}{(c+dx)^2}dx-\frac{b\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}+\frac{\sin(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3780}$$

$$-\frac{b\left(\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx-\frac{\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}+\int\frac{\csc(a+bx)}{(c+dx)^2}dx+\frac{\sin(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3783}$$

$$\int\frac{\csc(a+bx)}{(c+dx)^2}dx-\frac{b\left(\frac{\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}+\frac{\sin(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{4680}$$

$$\int\frac{\csc(a+bx)}{(c+dx)^2}dx-\frac{b\left(\frac{\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}+\frac{\sin(a+bx)}{d(c+dx)}$$

input
```
Int[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

### 3.103.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

3.103. $\int\frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

### 3.103.4 Maple [N/A] (verified)

Not integrable

Time = 0.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos (xb+a) \cot (xb+a)}{(dx+c)^2} dx$$

input
```
int(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x)
```

output
```
int(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x)
```

### 3.103.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2} dx = \int \frac{\cos (bx+a) \cot (bx+a)}{(dx+c)^2} dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)*cot(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.103.6 Sympy [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(cos(a + b*x)*cot(a + b*x)/(c + d*x)**2, x)
```

### 3.103.7 Maxima [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 340, normalized size of antiderivative = 17.00

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(bx+a)\cot(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*((I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(2, -(I*b
*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + 2*(d^2*x + c*d)*integrate(sin(b*x
+ a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*
x^2 + 2*c*d*x + c^2)*sin(b*x + a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*co
s(b*x + a)), x) + 2*(d^2*x + c*d)*integrate(sin(b*x + a)/(d^2*x^2 + 2*c*d*
x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2 + 2*c*d*x + c^2)*s
in(b*x + a)^2 + c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)), x) + (exp
_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)
/d))*sin(-(b*c - a*d)/d))/(d^2*x + c*d)
```

### 3.103.8 Giac [N/A]

Not integrable

Time = 2.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(bx+a)\cot(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*cot(b*x + a)/(d*x + c)^2, x)
```

### 3.103.9 Mupad [N/A]

Not integrable

Time = 22.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(a+b\,x)\,\cot(a+b\,x)}{(c+d\,x)^2}\,dx$$

input
```
int((cos(a + b*x)*cot(a + b*x))/(c + d*x)^2,x)
```

output
```
int((cos(a + b*x)*cot(a + b*x))/(c + d*x)^2, x)
```

# 3.104 $\int (c+dx)^m \cot^2(a+bx)\,dx$

3.104.1 Optimal result . . . . . . . . . . 865
3.104.2 Mathematica [N/A] . . . . . . . . . . 865
3.104.3 Rubi [N/A] . . . . . . . . . . 866
3.104.4 Maple [N/A] (verified) . . . . . . . . . . 867
3.104.5 Fricas [N/A] . . . . . . . . . . 867
3.104.6 Sympy [N/A] . . . . . . . . . . 867
3.104.7 Maxima [N/A] . . . . . . . . . . 868
3.104.8 Giac [N/A] . . . . . . . . . . 868
3.104.9 Mupad [N/A] . . . . . . . . . . 868

## 3.104.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (c+dx)^m \cot^2(a+bx)\,dx = \operatorname{Int}\left((c+dx)^m \cot^2(a+bx),x\right)$$

output
```
Unintegrable((d*x+c)^m*cot(b*x+a)^2,x)
```

## 3.104.2 Mathematica [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \cot^2(a+bx)\,dx = \int (c+dx)^m \cot^2(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Cot[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^m*Cot[a + b*x]^2, x]
```

### 3.104.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan\left(a+bx+\frac{\pi}{2}\right)^2 (c+dx)^m dx$$

$$\Big\downarrow \ 4222$$

$$\int \cot^2(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Cot[a + b*x]^2,x]
```

output
```
$Aborted
```

#### 3.104.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

### 3.104.4 Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \cot(xb+a)^2\,dx$$

input
```
int((d*x+c)^m*cot(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*cot(b*x+a)^2,x)
```

### 3.104.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \cot^2(a+bx)\,dx = \int (dx+c)^m \cot(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*cot(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*cot(b*x + a)^2, x)
```

### 3.104.6 Sympy [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (c+dx)^m \cot^2(a+bx)\,dx = \int (c+dx)^m \cot^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*cot(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*cot(a + b*x)**2, x)
```

### 3.104.7 Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \cot^2(a+bx)\,dx = \int (dx+c)^m \cot(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*cot(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cot(b*x + a)^2, x)
```

### 3.104.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \cot^2(a+bx)\,dx = \int (dx+c)^m \cot(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*cot(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cot(b*x + a)^2, x)
```

### 3.104.9 Mupad [N/A]

Not integrable

Time = 22.99 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \cot^2(a+bx)\,dx = \int \cot(a+b\,x)^2\,(c+d\,x)^m\,dx$$

input
```
int(cot(a + b*x)^2*(c + d*x)^m,x)
```

output
```
int(cot(a + b*x)^2*(c + d*x)^m, x)
```

# 3.105 $\int (c+dx)^4 \cot^2(a+bx)\,dx$

3.105.1 Optimal result . . . 869
3.105.2 Mathematica [B] (verified) . . . 870
3.105.3 Rubi [A] (verified) . . . 871
3.105.4 Maple [B] (verified) . . . 874
3.105.5 Fricas [B] (verification not implemented) . . . 875
3.105.6 Sympy [F] . . . 876
3.105.7 Maxima [B] (verification not implemented) . . . 877
3.105.8 Giac [F] . . . 877
3.105.9 Mupad [F(-1)] . . . 878

## 3.105.1 Optimal result

Integrand size = 16, antiderivative size = 155

$$\begin{aligned}\int (c+dx)^4 \cot^2(a+bx)\,dx = & -\frac{i(c+dx)^4}{b}-\frac{(c+dx)^5}{5d}-\frac{(c+dx)^4\cot(a+bx)}{b} \\ & +\frac{4d(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b^2} \\ & -\frac{6id^2(c+dx)^2\,\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3} \\ & +\frac{6d^3(c+dx)\,\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^4} \\ & +\frac{3id^4\,\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^5}\end{aligned}$$

output
```
-I*(d*x+c)^4/b-1/5*(d*x+c)^5/d-(d*x+c)^4*cot(b*x+a)/b+4*d*(d*x+c)^3*ln(1-e
xp(2*I*(b*x+a)))/b^2-6*I*d^2*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^3+6*d
^3*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/b^4+3*I*d^4*polylog(4,exp(2*I*(b*x+
a)))/b^5
```

### 3.105.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 833 vs. $2(155) = 310$.

Time = 6.65 (sec) , antiderivative size = 833, normalized size of antiderivative = 5.37

$$\int (c+dx)^4 \cot^2(a+bx)\,dx = -\frac{1}{5}x\left(5c^4+10c^3dx+10c^2d^2x^2+5cd^3x^3+d^4x^4\right)$$
$$-\frac{2cd^3e^{ia}\csc(a)\left(2b^3e^{-2ia}x^3+3ib^2(1-e^{-2ia})\,x^2\log\left(1-e^{-i(a+bx)}\right)+3ib^2(1-e^{-2ia})\,x^2\log\left(1+e^{-i(a+bx)}\right)\right.}{}$$
$$-\frac{d^4e^{ia}\csc(a)\left(b^4e^{-2ia}x^4+2ib^3(1-e^{-2ia})\,x^3\log\left(1-e^{-i(a+bx)}\right)+2ib^3(1-e^{-2ia})\,x^3\log\left(1+e^{-i(a+bx)}\right)-\right.}{}$$
$$+\frac{4c^3d\csc(a)(-bx\cos(a)+\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a))}{b^2\left(\cos^2(a)+\sin^2(a)\right)}$$
$$+\frac{\csc(a)\csc(a+bx)\left(c^4\sin(bx)+4c^3dx\sin(bx)+6c^2d^2x^2\sin(bx)+4cd^3x^3\sin(bx)+d^4x^4\sin(bx)\right)}{b}$$
$$-\frac{6c^2d^2\csc(a)\sec(a)\left(b^2e^{i\arctan(\tan(a))}x^2+\frac{(ibx(-\pi+2\arctan(\tan(a)))-\pi\log(1+e^{-2ibx})-2(bx+\arctan(\tan(a)))\log(1-e^{2i(bx+a}}{}\right.}{b^3\sqrt{\sec^2(a)\left(\cos^2(a)+\right.}}$$

input
```
Integrate[(c + d*x)^4*Cot[a + b*x]^2,x]
```

output
```
-1/5*(x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4)) - (
2*c*d^3*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)
*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Lo
g[1 + E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, -E^((-I)*(
a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, E^((-I)*(a + b*x))] + (6*
I)*(1 - E^((-2*I)*a))*PolyLog[3, -E^((-I)*(a + b*x))] + (6*I)*(1 - E^((-2*
I)*a))*PolyLog[3, E^((-I)*(a + b*x))]))/b^4 - (d^4*E^(I*a)*Csc[a]*((b^4*x^
4)/E^((2*I)*a) + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a + b*x
))] + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - 6*b^2
*(1 - E^((-2*I)*a))*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - 6*b^2*(1 - E^((-
2*I)*a))*x^2*PolyLog[2, E^((-I)*(a + b*x))] + (12*I)*b*(1 - E^((-2*I)*a))*
x*PolyLog[3, -E^((-I)*(a + b*x))] + (12*I)*b*(1 - E^((-2*I)*a))*x*PolyLog[
3, E^((-I)*(a + b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, -E^((-I)*(a + b*
x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, E^((-I)*(a + b*x))]))/b^5 + (4*c^3
*d*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))
/(b^2*(Cos[a]^2 + Sin[a]^2)) + (Csc[a]*Csc[a + b*x]*(c^4*Sin[b*x] + 4*c^3*
d*x*Sin[b*x] + 6*c^2*d^2*x^2*Sin[b*x] + 4*c*d^3*x^3*Sin[b*x] + d^4*x^4*Sin
[b*x]))/b - (6*c^2*d^2*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b
*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan
[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] ...
```

### 3.105.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.35, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 4203, 17, 25, 3042, 25, 4202, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4\cot^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^4\tan\left(a+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{4203}$$

$$-\frac{4d\int -(c+dx)^3\cot(a+bx)dx}{b}-\int (c+dx)^4dx-\frac{(c+dx)^4\cot(a+bx)}{b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{4d\int -(c+dx)^3\cot(a+bx)dx}{b}-\frac{(c+dx)^4\cot(a+bx)}{b}-\frac{(c+dx)^5}{5d}$$

$$\Big\downarrow \text{25}$$

$$\frac{4d\int (c+dx)^3\cot(a+bx)dx}{b}-\frac{(c+dx)^4\cot(a+bx)}{b}-\frac{(c+dx)^5}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4d\int -(c+dx)^3\tan\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^4\cot(a+bx)}{b}-\frac{(c+dx)^5}{5d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{4d\int (c+dx)^3\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx}{b}-\frac{(c+dx)^4\cot(a+bx)}{b}-\frac{(c+dx)^5}{5d}$$

$$\Big\downarrow \text{4202}$$

$$-\frac{4d\left(\frac{i(c+dx)^4}{4d}-2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)^3}{1+e^{i(2a+2bx+\pi)}}dx\right)}{b}-\frac{(c+dx)^4\cot(a+bx)}{b}-\frac{(c+dx)^5}{5d}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{4d\left(\frac{i(c+dx)^4}{4d}-2i\left(\frac{3id\int (c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^4\cot(a+bx)}{b}-\frac{(c+dx)^5}{5d}$$

$$\Big\downarrow \; 3011$$

$$-\frac{4d\left(\frac{i(c+dx)^4}{4d}-2i\left(\frac{3id\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int (c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.\right.}{b}$$

$$\frac{(c+dx)^4\cot(a+bx)}{b}-\frac{(c+dx)^5}{5d}$$

$$\Big\downarrow \; 7163$$

$$-\frac{4d\left(\frac{i(c+dx)^4}{4d}-2i\left(\frac{3id\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\left(\frac{id\int \operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}\right)}{2b}\right.\right.}{b}$$

$$\frac{(c+dx)^4\cot(a+bx)}{b}-\frac{(c+dx)^5}{5d}$$

$$\Big\downarrow \; 2720$$

$$-\frac{4d\left(\frac{i(c+dx)^4}{4d}-2i\left(\frac{3id\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\left(\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\operatorname{PolyL}}{}\right.}{b}\right.}{2b}\right.\right.}{b}$$

$$\frac{(c+dx)^4\cot(a+bx)}{b}-\frac{(c+dx)^5}{5d}$$

$$\Big\downarrow \; 7143$$

$$-\frac{4d\left(\frac{i(c+dx)^4}{4d}-2i\left(\frac{3id\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\left(\frac{d\operatorname{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i}{}\right.\right.}{b}$$

$$\frac{(c+dx)^4\cot(a+bx)}{b}-\frac{(c+dx)^5}{5d}$$

---

3.105. $\int (c+dx)^4\cot^2(a+bx)\,dx$

input
```
Int[(c + d*x)^4*Cot[a + b*x]^2,x]
```

output
```
-1/5*(c + d*x)^5/d - ((c + d*x)^4*Cot[a + b*x])/b - (4*d*(((I/4)*(c + d*x)
^4)/d - (2*I)*(((-1/2*I)*(c + d*x)^3*Log[1 + E^(I*(2*a + Pi + 2*b*x))])/b
+ (((3*I)/2)*d*(((I/2)*(c + d*x)^2*PolyLog[2, -E^(I*(2*a + Pi + 2*b*x))])/
b - (I*d*(((-1/2*I)*(c + d*x)*PolyLog[3, -E^(I*(2*a + Pi + 2*b*x))])/b + (
d*PolyLog[4, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2)))/b))/b)))/b
```

### 3.105.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

### 3.105.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 920 vs. $2(142) = 284$.

Time = 1.65 (sec) , antiderivative size = 921, normalized size of antiderivative = 5.94

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 921 |

input
```
int((d*x+c)^4*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
24*I*d^4*polylog(4,exp(I*(b*x+a)))/b^5+4*d^4/b^5*ln(1-exp(I*(b*x+a)))*a^3+
4*d^4/b^2*ln(exp(I*(b*x+a))+1)*x^3+4*d^4/b^2*ln(1-exp(I*(b*x+a)))*x^3-4*d^
4/b^5*a^3*ln(exp(I*(b*x+a))-1)+8*d^4/b^5*a^3*ln(exp(I*(b*x+a)))+4*d/b^2*c^
3*ln(exp(I*(b*x+a))+1)-8*d/b^2*c^3*ln(exp(I*(b*x+a)))+4*d/b^2*c^3*ln(exp(I
*(b*x+a))-1)-2*I*(d^4*x^4+4*c*d^3*x^3+6*c^2*d^2*x^2+4*c^3*d*x+c^4)/b/(exp(
2*I*(b*x+a))-1)+24*I*d^4/b^5*polylog(4,-exp(I*(b*x+a)))-6*I*d^4/b^5*a^4-2*
I*d^4/b*x^4+24*d^3/b^4*c*polylog(3,exp(I*(b*x+a)))+24*d^3/b^4*c*polylog(3,
-exp(I*(b*x+a)))+24*d^4/b^4*polylog(3,exp(I*(b*x+a)))*x+24*d^4/b^4*polylog
(3,-exp(I*(b*x+a)))*x-d^3*c*x^4-2*d^2*c^2*x^3-2*d*c^3*x^2-c^4*x+12*d^2/b^2
*c^2*ln(exp(I*(b*x+a))+1)*x+24*d^2/b^3*c^2*a*ln(exp(I*(b*x+a)))-12*d^2/b^3
*c^2*a*ln(exp(I*(b*x+a))-1)+12*d^3/b^4*c*a^2*ln(exp(I*(b*x+a))-1)-24*d^3/b
^4*c*a^2*ln(exp(I*(b*x+a)))-8*I*d^3/b*c*x^3+16*I*d^3/b^4*c*a^3-12*I*d^2/b*
c^2*x^2-12*I*d^2/b^3*c^2*a^2-12*I*d^2/b^3*c^2*polylog(2,exp(I*(b*x+a)))-12
*I*d^2/b^3*c^2*polylog(2,-exp(I*(b*x+a)))-8*I*d^4/b^4*a^3*x-12*I*d^4/b^3*p
olylog(2,-exp(I*(b*x+a)))*x^2-12*I*d^4/b^3*polylog(2,exp(I*(b*x+a)))*x^2+2
4*I*d^3/b^3*c*a^2*x-24*I*d^3/b^3*c*polylog(2,exp(I*(b*x+a)))*x-24*I*d^3/b^
3*c*polylog(2,-exp(I*(b*x+a)))*x-24*I*d^2/b^2*c^2*x*a+12*d^3/b^2*c*ln(1-ex
p(I*(b*x+a)))*x^2+12*d^3/b^2*c*ln(exp(I*(b*x+a))+1)*x^2+12*d^2/b^2*c^2*ln(
1-exp(I*(b*x+a)))*x-12*d^3/b^4*c*ln(1-exp(I*(b*x+a)))*a^2+12*d^2/b^3*c^2*l
n(1-exp(I*(b*x+a)))*a-1/5*d^4*x^5-1/5/d*c^5
```

### 3.105.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 856 vs. $2(138) = 276$.

Time = 0.28 (sec) , antiderivative size = 856, normalized size of antiderivative = 5.52

$$\int (c+dx)^4 \cot^2(a+bx)\,dx =$$
$$-\frac{10\,b^4d^4x^4+40\,b^4cd^3x^3+60\,b^4c^2d^2x^2+40\,b^4c^3dx+10\,b^4c^4-15i\,d^4\mathrm{polylog}(4,\cos(2\,bx+2\,a)+i\,\sin(2}{}$$

input
```
integrate((d*x+c)^4*cot(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/10*(10*b^4*d^4*x^4 + 40*b^4*c*d^3*x^3 + 60*b^4*c^2*d^2*x^2 + 40*b^4*c^3
*d*x + 10*b^4*c^4 - 15*I*d^4*polylog(4, cos(2*b*x + 2*a) + I*sin(2*b*x + 2
*a))*sin(2*b*x + 2*a) + 15*I*d^4*polylog(4, cos(2*b*x + 2*a) - I*sin(2*b*x
 + 2*a))*sin(2*b*x + 2*a) + 30*(I*b^2*d^4*x^2 + 2*I*b^2*c*d^3*x + I*b^2*c^
2*d^2)*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) + 30*
(-I*b^2*d^4*x^2 - 2*I*b^2*c*d^3*x - I*b^2*c^2*d^2)*dilog(cos(2*b*x + 2*a)
- I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 20*(b^3*c^3*d - 3*a*b^2*c^2*d^2 +
 3*a^2*b*c*d^3 - a^3*d^4)*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*
a) + 1/2)*sin(2*b*x + 2*a) - 20*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d
^3 - a^3*d^4)*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2)*si
n(2*b*x + 2*a) - 20*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a
*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*log(-cos(2*b*x + 2*a) + I*sin(2*b*
x + 2*a) + 1)*sin(2*b*x + 2*a) - 20*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3
*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*log(-cos(2*b*x + 2
*a) - I*sin(2*b*x + 2*a) + 1)*sin(2*b*x + 2*a) - 30*(b*d^4*x + b*c*d^3)*po
lylog(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 30*(b*d
^4*x + b*c*d^3)*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a))*sin(2*b*
x + 2*a) + 10*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c
^3*d*x + b^4*c^4)*cos(2*b*x + 2*a) + 2*(b^5*d^4*x^5 + 5*b^5*c*d^3*x^4 + 10
*b^5*c^2*d^2*x^3 + 10*b^5*c^3*d*x^2 + 5*b^5*c^4*x)*sin(2*b*x + 2*a))/(b...
```

### 3.105.6 Sympy [F]

$$\int (c+dx)^4 \cot^2(a+bx)\,dx = \int (c+dx)^4 \cot^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**4*cot(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**4*cot(a + b*x)**2, x)
```

### 3.105.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3242 vs. 2(138) = 276.

Time = 0.68 (sec) , antiderivative size = 3242, normalized size of antiderivative = 20.92

$$\int (c+dx)^4 \cot^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cot(b*x+a)^2,x, algorithm="maxima")
```

output
```
-((b*x + a + 1/tan(b*x + a))*c^4 - 4*(b*x + a + 1/tan(b*x + a))*a*c^3*d/b
+ 6*(b*x + a + 1/tan(b*x + a))*a^2*c^2*d^2/b^2 - 4*(b*x + a + 1/tan(b*x +
a))*a^3*c*d^3/b^3 + (b*x + a + 1/tan(b*x + a))*a^4*d^4/b^4 + 2*((b*x + a)^
2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(
2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*
cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a)
 + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)
*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*s
in(2*b*x + 2*a))*c^3*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2
*b*x + 2*a) + 1)*b) - 6*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(
2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x
 + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^
2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x
 + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 -
2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*a*c^2*d^2/((cos(2*b*x
+ 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b^2) + 6*((b*x + a
)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*co
s(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 -
2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x +
a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)...
```

### 3.105.8 Giac [F]

$$\int (c+dx)^4 \cot^2(a+bx)\,dx = \int (dx+c)^4 \cot(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^4*cot(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*cot(b*x + a)^2, x)
```

### 3.105.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \cot^2(a+bx)\,dx = \int \cot(a+b\,x)^2\,(c+d\,x)^4\,dx$$

input
```
int(cot(a + b*x)^2*(c + d*x)^4,x)
```

output
```
int(cot(a + b*x)^2*(c + d*x)^4, x)
```

# 3.106 $\int (c+dx)^3 \cot^2(a+bx)\,dx$


3.106.1 Optimal result . . . 879
3.106.2 Mathematica [B] (verified) . . . 879
3.106.3 Rubi [A] (verified) . . . 880
3.106.4 Maple [B] (verified) . . . 884
3.106.5 Fricas [B] (verification not implemented) . . . 884
3.106.6 Sympy [F] . . . 885
3.106.7 Maxima [B] (verification not implemented) . . . 885
3.106.8 Giac [F] . . . 886
3.106.9 Mupad [F(-1)] . . . 887


## 3.106.1 Optimal result

Integrand size = 16, antiderivative size = 127

$$\int (c+dx)^3 \cot^2(a+bx)\,dx = -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} - \frac{(c+dx)^3\cot(a+bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\,\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3} + \frac{3d^3\,\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^4}$$

output
```
-I*(d*x+c)^3/b-1/4*(d*x+c)^4/d-(d*x+c)^3*cot(b*x+a)/b+3*d*(d*x+c)^2*ln(1-e
xp(2*I*(b*x+a)))/b^2-3*I*d^2*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/b^3+3/2*d
^3*polylog(3,exp(2*I*(b*x+a)))/b^4
```

## 3.106.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 520 vs. 2(127) = 254.

Time = 6.55 (sec) , antiderivative size = 520, normalized size of antiderivative = 4.09

$$\int (c+dx)^3 \cot^2(a+bx)\,dx = -\frac{1}{4}x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)$$
$$-\frac{d^3e^{ia}\csc(a)\left(2b^3e^{-2ia}x^3+3ib^2(1-e^{-2ia})\,x^2\log\left(1-e^{-i(a+bx)}\right)+3ib^2(1-e^{-2ia})\,x^2\log\left(1+e^{-i(a+bx)}\right)-\right.}{}$$
$$+\frac{3c^2d\csc(a)(-bx\cos(a)+\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a))}{b^2\left(\cos^2(a)+\sin^2(a)\right)}$$
$$+\frac{\csc(a)\csc(a+bx)\left(c^3\sin(bx)+3c^2dx\sin(bx)+3cd^2x^2\sin(bx)+d^3x^3\sin(bx)\right)}{b}$$
$$-\frac{3cd^2\csc(a)\sec(a)\left(b^2e^{i\arctan(\tan(a))}x^2+\frac{(ibx(-\pi+2\arctan(\tan(a)))-\pi\log(1+e^{-2ibx})-2(bx+\arctan(\tan(a)))\log(1-e^{2i(bx+\mathrm{arc}}}{}\right.}{b^3\sqrt{\sec^2(a)\left(\cos^2(a)+\right.}}$$

input
```
Integrate[(c + d*x)^3*Cot[a + b*x]^2,x]
```

output
```
-1/4*(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)) - (d^3*E^(I*a)*Csc[a]
*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-
I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x
))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, -E^((-I)*(a + b*x))] - 6*b*(1 -
E^((-2*I)*a))*x*PolyLog[2, E^((-I)*(a + b*x))] + (6*I)*(1 - E^((-2*I)*a))*
PolyLog[3, -E^((-I)*(a + b*x))] + (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, E^((
-I)*(a + b*x))]))/(2*b^4) + (3*c^2*d*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*
Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^2*(Cos[a]^2 + Sin[a]^2)) + (Csc[a]*C
sc[a + b*x]*(c^3*Sin[b*x] + 3*c^2*d*x*Sin[b*x] + 3*c*d^2*x^2*Sin[b*x] + d^
3*x^3*Sin[b*x]))/b - (3*c*d^2*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2
+ ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x +
 ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*
x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2
*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^3*Sqrt[Sec[a
]^2*(Cos[a]^2 + Sin[a]^2)])
```

### 3.106.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.29, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {3042, 4203, 17, 25, 3042, 25, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \cot^2(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (c+dx)^3 \tan\left(a+bx+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \; 4203$$

$$-\frac{3d\int -(c+dx)^2\cot(a+bx)dx}{b}-\int (c+dx)^3dx-\frac{(c+dx)^3\cot(a+bx)}{b}$$

$$\Big\downarrow \; 17$$

$$-\frac{3d\int -(c+dx)^2\cot(a+bx)dx}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}$$

$$\Big\downarrow \; 25$$

$$\frac{3d\int (c+dx)^2\cot(a+bx)dx}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}$$

$$\Big\downarrow \; 3042$$

$$\frac{3d\int -(c+dx)^2\tan\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}$$

$$\Big\downarrow \; 25$$

$$-\frac{3d\int (c+dx)^2\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}$$

$$\Big\downarrow \; 4202$$

$$-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)^2}{1+e^{i(2a+2bx+\pi)}}dx\right)}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}$$

$$\Big\downarrow \; 2620$$

$$-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\int (c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}$$

$$\Big\downarrow \; 3011$$

3.106. $\int (c+dx)^3 \cot^2(a+bx)\,dx$

$$-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}$$

$\Big\downarrow$ 2720

$$-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}$$

$\Big\downarrow$ 7143

$$-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}$$

input
```
Int[(c + d*x)^3*Cot[a + b*x]^2,x]
```

output
```
-1/4*(c + d*x)^4/d - ((c + d*x)^3*Cot[a + b*x])/b - (3*d*(((I/3)*(c + d*x)
^3)/d - (2*I)*(((-1/2*I)*(c + d*x)^2*Log[1 + E^(I*(2*a + Pi + 2*b*x))])/b
+ (I*d*(((I/2)*(c + d*x)*PolyLog[2, -E^(I*(2*a + Pi + 2*b*x))])/b - (d*Pol
yLog[3, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2)))/b)))/b
```

### 3.106.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.106. $\int (c+dx)^3 \cot^2(a+bx)\, dx$

### 3.106.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(115) = 230.

Time = 1.63 (sec) , antiderivative size = 581, normalized size of antiderivative = 4.57

| method | result |
|---|---|
| risch | $\frac{4id^3a^3}{b^4} + \frac{6d^2c\ln(1-e^{i(xb+a)})x}{b^2} - \frac{6id^3 \operatorname{polylog}(2,e^{i(xb+a)})x}{b^3} + \frac{12d^2ca\ln(e^{i(xb+a)})}{b^3} - \frac{6d^2ca\ln(e^{i(xb+a)}-1)}{b^3} - \frac{2i(d^3x^3+3c}{b(e^{2i(}}$ |

input
```
int((d*x+c)^3*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-6*d^2/b^3*c*a*ln(exp(I*(b*x+a))-1)+6*d^2/b^2*c*ln(1-exp(I*(b*x+a)))*x-6*I
*d^3/b^3*polylog(2,exp(I*(b*x+a)))*x-6*I*d^2/b*c*x^2+6*I*d^3/b^3*a^2*x-6*I
*d^3/b^3*polylog(2,-exp(I*(b*x+a)))*x-6*I*d^2/b^3*c*a^2-6*I*d^2/b^3*c*poly
log(2,exp(I*(b*x+a)))-6*I*d^2/b^3*c*polylog(2,-exp(I*(b*x+a)))+6*d^2/b^2*c
*ln(exp(I*(b*x+a))+1)*x-2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(exp(2*I
*(b*x+a))-1)-d^2*c*x^3-3/2*d*c^2*x^2-c^3*x+6*d^2/b^3*c*ln(1-exp(I*(b*x+a))
)*a+12*d^2/b^3*c*a*ln(exp(I*(b*x+a)))+3*d^3/b^2*ln(1-exp(I*(b*x+a)))*x^2+3
*d^3/b^2*ln(exp(I*(b*x+a))+1)*x^2-3*d^3/b^4*ln(1-exp(I*(b*x+a)))*a^2-6*d^3
/b^4*a^2*ln(exp(I*(b*x+a)))+3*d^3/b^4*a^2*ln(exp(I*(b*x+a))-1)+3*d/b^2*c^2
*ln(exp(I*(b*x+a))+1)-6*d/b^2*c^2*ln(exp(I*(b*x+a)))+3*d/b^2*c^2*ln(exp(I*
(b*x+a))-1)-2*I*d^3/b*x^3+4*I*d^3/b^4*a^3+6*d^3*polylog(3,-exp(I*(b*x+a)))
/b^4+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4-12*I*d^2/b^2*c*x*a-1/4*d^3*x^4-1/
4/d*c^4
```

### 3.106.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 599 vs. 2(112) = 224.

Time = 0.27 (sec) , antiderivative size = 599, normalized size of antiderivative = 4.72

$$\int (c+dx)^3 \cot^2(a+bx)\,dx =$$
$$-\frac{4\,b^3d^3x^3 + 12\,b^3cd^2x^2 + 12\,b^3c^2dx + 4\,b^3c^3 - 3\,d^3\mathrm{polylog}(3,\cos(2\,bx+2\,a) + i\,\sin(2\,bx+2\,a))\sin(2\,ba}{}$$

input
```
integrate((d*x+c)^3*cot(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x + 4*b^3*c^3 - 3*d^
3*polylog(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 3*d
^3*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) + 6*
(I*b*d^3*x + I*b*c*d^2)*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2
*b*x + 2*a) + 6*(-I*b*d^3*x - I*b*c*d^2)*dilog(cos(2*b*x + 2*a) - I*sin(2*
b*x + 2*a))*sin(2*b*x + 2*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-
1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) - 6*
(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(
2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*
a*b*c*d^2 - a^2*d^3)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1)*sin(2
*b*x + 2*a) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log(
-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1)*sin(2*b*x + 2*a) + 4*(b^3*d^3*
x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(2*b*x + 2*a) + (b^4*d
^3*x^4 + 4*b^4*c*d^2*x^3 + 6*b^4*c^2*d*x^2 + 4*b^4*c^3*x)*sin(2*b*x + 2*a)
)/(b^4*sin(2*b*x + 2*a))
```

### 3.106.6 Sympy [F]

$$\int (c+dx)^3 \cot^2(a+bx)\,dx = \int (c+dx)^3 \cot^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*cot(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**3*cot(a + b*x)**2, x)
```

### 3.106.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1953 vs. $2(112) = 224$.

Time = 0.46 (sec) , antiderivative size = 1953, normalized size of antiderivative = 15.38

$$\int (c+dx)^3 \cot^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*cot(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(b*x + a + 1/tan(b*x + a))*c^3 - 6*(b*x + a + 1/tan(b*x + a))*a*c^
2*d/b + 6*(b*x + a + 1/tan(b*x + a))*a^2*c*d^2/b^2 - 2*(b*x + a + 1/tan(b*
x + a))*a^3*d^3/b^3 + 3*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(
2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x
 + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^
2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x
 + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 -
2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*c^2*d/((cos(2*b*x + 2*
a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b) - 6*((b*x + a)^2*co
s(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*
x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(
2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1
) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log
(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2
*b*x + 2*a))*a*c*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b
*x + 2*a) + 1)*b^2) + 3*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(
2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x
 + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^
2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x
 + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2...
```

### 3.106.8 Giac [F]

$$\int (c+dx)^3 \cot^2(a+bx)\,dx = \int (dx+c)^3 \cot(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^3*cot(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*cot(b*x + a)^2, x)
```

### 3.106.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \cot^2(a+bx)\,dx = \int \cot(a+b\,x)^2\,(c+d\,x)^3\,dx$$

input
```
int(cot(a + b*x)^2*(c + d*x)^3,x)
```

output
```
int(cot(a + b*x)^2*(c + d*x)^3, x)
```

# 3.107 $\int (c+dx)^2 \cot^2(a+bx)\,dx$


3.107.1 Optimal result . . . . . . . . . . 888
3.107.2 Mathematica [B] (verified) . . . . . . . . . . 888
3.107.3 Rubi [A] (verified) . . . . . . . . . . 889
3.107.4 Maple [B] (verified) . . . . . . . . . . 892
3.107.5 Fricas [B] (verification not implemented) . . . . . . . . . . 892
3.107.6 Sympy [F] . . . . . . . . . . 893
3.107.7 Maxima [B] (verification not implemented) . . . . . . . . . . 893
3.107.8 Giac [F] . . . . . . . . . . 894
3.107.9 Mupad [F(-1)] . . . . . . . . . . 894


## 3.107.1 Optimal result

Integrand size = 16, antiderivative size = 97

$$\int (c+dx)^2 \cot^2(a+bx)\,dx = -\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}-\frac{(c+dx)^2\cot(a+bx)}{b} + \frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{id^2\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}$$

output
```
-I*(d*x+c)^2/b-1/3*(d*x+c)^3/d-(d*x+c)^2*cot(b*x+a)/b+2*d*(d*x+c)*ln(1-exp
(2*I*(b*x+a)))/b^2-I*d^2*polylog(2,exp(2*I*(b*x+a)))/b^3
```

## 3.107.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 268 vs. 2(97) = 194.

Time = 6.53 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.76

$$\int (c+dx)^2 \cot^2(a+bx)\,dx = -\frac{1}{3}x\left(3c^2+3cdx+d^2x^2\right) + \frac{2cd\csc(a)(-bx\cos(a)+\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a))}{b^2\left(\cos^2(a)+\sin^2(a)\right)} + \frac{\csc(a)\csc(a+bx)\left(c^2\sin(bx)+2cdx\sin(bx)+d^2x^2\sin(bx)\right)}{b} - \frac{d^2\csc(a)\sec(a)\left(b^2e^{i\arctan(\tan(a))}x^2+\frac{(ibx(-\pi+2\arctan(\tan(a)))-\pi\log(1+e^{-2ibx})-2(bx+\arctan(\tan(a)))\log(1-e^{2i(bx+\arctan}}{}\right.}{b^3\sqrt{\sec^2(a)\left(\cos^2(a)+\text{s}\right.}}$$

input
```
Integrate[(c + d*x)^2*Cot[a + b*x]^2,x]
```

output
```
-1/3*(x*(3*c^2 + 3*c*d*x + d^2*x^2)) + (2*c*d*Csc[a]*(-(b*x*Cos[a]) + Log[
Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^2*(Cos[a]^2 + Sin[a]^2)) +
(Csc[a]*Csc[a + b*x]*(c^2*Sin[b*x] + 2*c*d*x*Sin[b*x] + d^2*x^2*Sin[b*x]))
/b - (d^2*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*A
rcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log
[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a
]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Ta
n[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^3*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[
a]^2)])
```

## 3.107.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.24, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 4203, 17, 25, 3042, 25, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \cot^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2 \tan\left(a+bx+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{4203}$$

$$-\frac{2d\int -((c+dx)\cot(a+bx))dx}{b}-\int (c+dx)^2dx-\frac{(c+dx)^2\cot(a+bx)}{b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{2d\int -((c+dx)\cot(a+bx))dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

$$\Big\downarrow \text{25}$$

$$\frac{2d\int (c+dx)\cot(a+bx)dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\int -\left((c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)\right)dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

$$\Big\downarrow \; 25$$

$$-\frac{2d\int (c+dx)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

$$\Big\downarrow \; 4202$$

$$-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)}{1+e^{i(2a+2bx+\pi)}}dx\right)}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

$$\Big\downarrow \; 2620$$

$$-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{id\int \log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

$$\Big\downarrow \; 2715$$

$$-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{d\int e^{-i(2a+2bx+\pi)}\log\left(1+e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

$$\Big\downarrow \; 2838$$

$$-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\,\text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

input
```
Int[(c + d*x)^2*Cot[a + b*x]^2,x]
```

output
```
-1/3*(c + d*x)^3/d - ((c + d*x)^2*Cot[a + b*x])/b - (2*d*(((I/2)*(c + d*x)
^2)/d - (2*I)*(((-1/2*I)*(c + d*x)*Log[1 + E^(I*(2*a + Pi + 2*b*x))])/b -
(d*PolyLog[2, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2))))/b
```

## 3.107. $\int (c+dx)^2\cot^2(a+bx)\,dx$

### 3.107.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

### 3.107.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(89) = 178.

Time = 1.58 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.14

| method | result |
|---|---|
| risch | $-\frac{d^2x^3}{3}-dc\,x^2-c^2x-\frac{c^3}{3d}-\frac{2id^2\,\mathrm{polylog}\left(2,\mathrm{e}^{i(xb+a)}\right)}{b^3}+\frac{2dc\ln\left(\mathrm{e}^{i(xb+a)}+1\right)}{b^2}-\frac{4dc\ln\left(\mathrm{e}^{i(xb+a)}\right)}{b^2}+\frac{2dc\ln\left(\mathrm{e}^{i(xb+a)}-1\right)}{b^2}$ |

input
```
int((d*x+c)^2*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*d^2*x^3-d*c*x^2-c^2*x-1/3/d*c^3-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^3
+2*d/b^2*c*ln(exp(I*(b*x+a))+1)-4*d/b^2*c*ln(exp(I*(b*x+a)))+2*d/b^2*c*ln(
exp(I*(b*x+a))-1)-2*I*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*x+a))-1)-2*I*d^2
/b^3*a^2-4*I*d^2/b^2*x*a+2*d^2/b^2*ln(1-exp(I*(b*x+a)))*x+2*d^2/b^3*ln(1-e
xp(I*(b*x+a)))*a-2*I*d^2/b^3*polylog(2,-exp(I*(b*x+a)))+2*d^2/b^2*ln(exp(I
*(b*x+a))+1)*x-2*I*d^2/b*x^2+4*d^2/b^3*a*ln(exp(I*(b*x+a)))-2*d^2/b^3*a*ln
(exp(I*(b*x+a))-1)
```

### 3.107.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(86) = 172.

Time = 0.26 (sec) , antiderivative size = 384, normalized size of antiderivative = 3.96

$$\int (c+dx)^2\cot^2(a+bx)\,dx =$$
$$-\frac{6\,b^2d^2x^2+12\,b^2cdx+6\,b^2c^2+3i\,d^2\mathrm{Li}_2(\cos(2\,bx+2\,a)+i\,\sin(2\,bx+2\,a))\sin(2\,bx+2\,a)-3i\,d^2\mathrm{Li}_2(\mathrm{c}}{}$$

input
```
integrate((d*x+c)^2*cot(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/6*(6*b^2*d^2*x^2 + 12*b^2*c*d*x + 6*b^2*c^2 + 3*I*d^2*dilog(cos(2*b*x +
 2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 3*I*d^2*dilog(cos(2*b*x + 2
*a) - I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 6*(b*c*d - a*d^2)*log(-1/2*co
s(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) - 6*(b*c*d
 - a*d^2)*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*
b*x + 2*a) - 6*(b*d^2*x + a*d^2)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a
) + 1)*sin(2*b*x + 2*a) - 6*(b*d^2*x + a*d^2)*log(-cos(2*b*x + 2*a) - I*si
n(2*b*x + 2*a) + 1)*sin(2*b*x + 2*a) + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*
c^2)*cos(2*b*x + 2*a) + 2*(b^3*d^2*x^3 + 3*b^3*c*d*x^2 + 3*b^3*c^2*x)*sin(
2*b*x + 2*a))/(b^3*sin(2*b*x + 2*a))
```

### 3.107.6 Sympy [F]

$$\int (c+dx)^2\cot^2(a+bx)\,dx = \int (c+dx)^2\cot^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*cot(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*cot(a + b*x)**2, x)
```

### 3.107.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 645 vs. 2(86) = 172.

Time = 0.45 (sec) , antiderivative size = 645, normalized size of antiderivative = 6.65

$$\int (c+dx)^2\cot^2(a+bx)\,dx$$
$$= \frac{-i\,b^3d^2x^3 - 3i\,b^3cdx^2 - 3i\,b^3c^2x - 6\,b^2c^2 - 6\,(bd^2x + bcd - (bd^2x + bcd)\cos(2\,bx + 2\,a) + (-i\,bd^2x - i\,bc}{}$$

input
```
integrate((d*x+c)^2*cot(b*x+a)^2,x, algorithm="maxima")
```

output
```
(-I*b^3*d^2*x^3 - 3*I*b^3*c*d*x^2 - 3*I*b^3*c^2*x - 6*b^2*c^2 - 6*(b*d^2*x
 + b*c*d - (b*d^2*x + b*c*d)*cos(2*b*x + 2*a) + (-I*b*d^2*x - I*b*c*d)*sin
(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 6*(b*c*d*cos(2*b*
x + 2*a) + I*b*c*d*sin(2*b*x + 2*a) - b*c*d)*arctan2(sin(b*x + a), cos(b*x
 + a) - 1) - 6*(b*d^2*x*cos(2*b*x + 2*a) + I*b*d^2*x*sin(2*b*x + 2*a) - b*
d^2*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (I*b^3*d^2*x^3 - 3*(-I*b
^3*c*d + 2*b^2*d^2)*x^2 - 3*(-I*b^3*c^2 + 4*b^2*c*d)*x)*cos(2*b*x + 2*a) -
 6*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(-e^(I*b*x +
 I*a)) - 6*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(e^(
I*b*x + I*a)) - 3*(-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(2*b*x
+ 2*a) - (b*d^2*x + b*c*d)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x
+ a)^2 + 2*cos(b*x + a) + 1) - 3*(-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*
c*d)*cos(2*b*x + 2*a) - (b*d^2*x + b*c*d)*sin(2*b*x + 2*a))*log(cos(b*x +
a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (b^3*d^2*x^3 + 3*(b^3*c*d +
2*I*b^2*d^2)*x^2 + 3*(b^3*c^2 + 4*I*b^2*c*d)*x)*sin(2*b*x + 2*a))/(-3*I*b^
3*cos(2*b*x + 2*a) + 3*b^3*sin(2*b*x + 2*a) + 3*I*b^3)
```

### 3.107.8 Giac [F]

$$\int (c+dx)^2 \cot^2(a+bx)\,dx = \int (dx+c)^2 \cot(bx+a)^2 \,dx$$

input
```
integrate((d*x+c)^2*cot(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*cot(b*x + a)^2, x)
```

### 3.107.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \cot^2(a+bx)\,dx = \int \cot(a+b\,x)^2\,(c+d\,x)^2\,dx$$

input
```
int(cot(a + b*x)^2*(c + d*x)^2,x)
```

output
```
int(cot(a + b*x)^2*(c + d*x)^2, x)
```

# 3.108 $\int (c+dx)\cot^2(a+bx)\,dx$

3.108.1 Optimal result . . . . . . . . . . 895
3.108.2 Mathematica [C] (verified) . . . . . . . . . . 895
3.108.3 Rubi [A] (verified) . . . . . . . . . . 896
3.108.4 Maple [A] (verified) . . . . . . . . . . 897
3.108.5 Fricas [B] (verification not implemented) . . . . . . . . . . 898
3.108.6 Sympy [B] (verification not implemented) . . . . . . . . . . 898
3.108.7 Maxima [B] (verification not implemented) . . . . . . . . . . 899
3.108.8 Giac [B] (verification not implemented) . . . . . . . . . . 899
3.108.9 Mupad [B] (verification not implemented) . . . . . . . . . . 900

## 3.108.1 Optimal result

Integrand size = 14, antiderivative size = 41

$$\int (c+dx)\cot^2(a+bx)\,dx = -cx - \frac{dx^2}{2} - \frac{(c+dx)\cot(a+bx)}{b} + \frac{d\log(\sin(a+bx))}{b^2}$$

output
```
-c*x-1/2*d*x^2-(d*x+c)*cot(b*x+a)/b+d*ln(sin(b*x+a))/b^2
```

## 3.108.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.59 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.00

$$\int (c+dx)\cot^2(a+bx)\,dx = -\frac{c\cot(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(a+bx)\right)}{b} + \frac{d\log(\sin(a+bx))}{b^2} - \frac{dx\csc(a)(2\cos(a)+bx\sin(a))}{2b} + \frac{dx\csc(a)\csc(a+bx)\sin(bx)}{b}$$

input
```
Integrate[(c + d*x)*Cot[a + b*x]^2,x]
```

output
```
-((c*Cot[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[a + b*x]^2])/b) + (
d*Log[Sin[a + b*x]])/b^2 - (d*x*Csc[a]*(2*Cos[a] + b*x*Sin[a]))/(2*b) + (d
*x*Csc[a]*Csc[a + b*x]*Sin[b*x])/b
```

### 3.108.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4203, 17, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\cot^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{4203}$$

$$-\frac{d\int -\cot(a+bx)dx}{b}-\int (c+dx)dx-\frac{(c+dx)\cot(a+bx)}{b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{d\int -\cot(a+bx)dx}{b}-\frac{(c+dx)\cot(a+bx)}{b}-\frac{(c+dx)^2}{2d}$$

$$\Big\downarrow \text{25}$$

$$\frac{d\int \cot(a+bx)dx}{b}-\frac{(c+dx)\cot(a+bx)}{b}-\frac{(c+dx)^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d\int -\tan\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cot(a+bx)}{b}-\frac{(c+dx)^2}{2d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{d\int \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx}{b}-\frac{(c+dx)\cot(a+bx)}{b}-\frac{(c+dx)^2}{2d}$$

$$\Big\downarrow \text{3956}$$

$$\frac{d\log(-\sin(a+bx))}{b^2}-\frac{(c+dx)\cot(a+bx)}{b}-\frac{(c+dx)^2}{2d}$$

input
```
Int[(c + d*x)*Cot[a + b*x]^2,x]
```

output
```
-1/2*(c + d*x)^2/d - ((c + d*x)*Cot[a + b*x])/b + (d*Log[-Sin[a + b*x]])/b
^2
```

#### 3.108.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

### 3.108.4 Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.56

| method | result | size |
|---|---|---|
| default | $-\frac{d\,x^{2}}{2}-cx+\frac{\frac{da\cot\left(xb+a\right)}{b}-c\cot\left(xb+a\right)+\frac{d\left(-\left(xb+a\right)\cot\left(xb+a\right)+\ln\left(\sin\left(xb+a\right)\right)\right)}{b}}{b}$ | 64 |
| parallelrisch | $\frac{-d\,x^{2}b^{2}-2\cot\left(xb+a\right)dxb-2cx\,b^{2}-2\cot\left(xb+a\right)cb+2d\ln\left(\tan\left(xb+a\right)\right)-d\ln\left(\sec\left(xb+a\right)^{2}\right)}{2b^{2}}$ | 66 |
| risch | $-\frac{d\,x^{2}}{2}-cx-\frac{2idx}{b}-\frac{2ida}{b^{2}}-\frac{2i\left(dx+c\right)}{b\left(\mathrm{e}^{2i\left(xb+a\right)}-1\right)}+\frac{d\ln\left(\mathrm{e}^{2i\left(xb+a\right)}-1\right)}{b^{2}}$ | 69 |
| norman | $\frac{-\frac{c}{b}-cx\tan\left(xb+a\right)-\frac{dx}{b}-\frac{d\,x^{2}\tan\left(xb+a\right)}{2}}{\tan\left(xb+a\right)}+\frac{d\ln\left(\tan\left(xb+a\right)\right)}{b^{2}}-\frac{d\ln\left(1+\tan\left(xb+a\right)^{2}\right)}{2b^{2}}$ | 76 |

input
```
int((d*x+c)*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*d*x^2-c*x+1/b*(1/b*d*a*cot(b*x+a)-c*cot(b*x+a)+1/b*d*(-(b*x+a)*cot(b*
x+a)+ln(sin(b*x+a))))
```

### 3.108.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(39) = 78.

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.37

$$\int (c+dx)\cot^2(a+bx)\,dx = -\frac{2\,bdx - d\log\left(-\frac{1}{2}\,\cos\left(2\,bx+2\,a\right)+\frac{1}{2}\right)\sin\left(2\,bx+2\,a\right)+2\,bc+2\,(bdx+bc)\cos\left(2\,bx+2\,a\right)+(b^2dx^2+}{2\,b^2\sin\left(2\,bx+2\,a\right)}$$

input
```
integrate((d*x+c)*cot(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*b*d*x - d*log(-1/2*cos(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) + 2*b*
c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d*x^2 + 2*b^2*c*x)*sin(2*b*x +
 2*a))/(b^2*sin(2*b*x + 2*a))
```

### 3.108.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(36) = 72.

Time = 0.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.54

$$\int (c+dx)\cot^2(a+bx)\,dx = \begin{cases} \tilde{\infty}\left(cx+\frac{dx^2}{2}\right) & \text{for } a=0\wedge b=0 \\ \left(cx+\frac{dx^2}{2}\right)\cot^2\left(a\right) & \text{for } b=0 \\ \tilde{\infty}\left(cx+\frac{dx^2}{2}\right) & \text{for } a=-bx \\ -cx-\frac{dx^2}{2}-\frac{c}{b\tan\left(a+bx\right)}-\frac{dx}{b\tan\left(a+bx\right)}-\frac{d\log\left(\tan^2\left(a+bx\right)+1\right)}{2b^2}+\frac{d\log\left(\tan\left(a+bx\right)\right)}{b^2} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*cot(b*x+a)**2,x)
```

output
```
Piecewise((zoo*(c*x + d*x**2/2), Eq(a, 0) & Eq(b, 0)), ((c*x + d*x**2/2)*c
ot(a)**2, Eq(b, 0)), (zoo*(c*x + d*x**2/2), Eq(a, -b*x)), (-c*x - d*x**2/2
 - c/(b*tan(a + b*x)) - d*x/(b*tan(a + b*x)) - d*log(tan(a + b*x)**2 + 1)/
(2*b**2) + d*log(tan(a + b*x))/b**2, True))
```

### 3.108.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(39) = 78$.

Time = 0.36 (sec) , antiderivative size = 292, normalized size of antiderivative = 7.12

$$\int (c+dx)\cot^2(a+bx)\,dx = -\frac{2\left(bx+a+\frac{1}{\tan(bx+a)}\right)c - \frac{2\left(bx+a+\frac{1}{\tan(bx+a)}\right)ad}{b} + \frac{\left((bx+a)^2\cos(2\,bx+2\,a)^2+(bx+a)^2\sin(2\,bx+2\,a)^2-2\,(bx+a)^2\cos(2\,bx+2\,a)-\right.}{}}{}$$

input
```
integrate((d*x+c)*cot(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(b*x + a + 1/tan(b*x + a))*c - 2*(b*x + a + 1/tan(b*x + a))*a*d/b
+ ((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*
x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x
+ 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2
*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*
x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) +
4*(b*x + a)*sin(2*b*x + 2*a))*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
- 2*cos(2*b*x + 2*a) + 1)*b))/b
```

### 3.108.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1151 vs. $2(39) = 78$.

Time = 0.55 (sec) , antiderivative size = 1151, normalized size of antiderivative = 28.07

$$\int (c+dx)\cot^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*cot(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*(b^2*d*x^2*tan(1/2*b*x)^2*tan(1/2*a) + b^2*d*x^2*tan(1/2*b*x)*tan(1/2
*a)^2 + 2*b^2*c*x*tan(1/2*b*x)^2*tan(1/2*a) + 2*b^2*c*x*tan(1/2*b*x)*tan(1
/2*a)^2 - b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - b^2*d*x^2*tan(1/2*b*x) - b^2
*d*x^2*tan(1/2*a) - b*c*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^2*c*x*tan(1/2*b*
x) + b*d*x*tan(1/2*b*x)^2 - 2*b^2*c*x*tan(1/2*a) + 4*b*d*x*tan(1/2*b*x)*ta
n(1/2*a) - d*log(16*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3*tan(1/
2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*tan(1/2*b*x)^3*tan(1/2*a) - 4*tan
(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 +
2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*b*x)^4*tan(1/2*a)^4 + 2
*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2*tan(1/2*a)^4 + tan(1/2*b*x
)^4 + 4*tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*a)^4 + 2*tan(1/2*b*x)^2 + 2*
tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a) + b*d*x*tan(1/2*a)^2 - d*log(
16*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*
b*x)^2*tan(1/2*a)^4 - 2*tan(1/2*b*x)^3*tan(1/2*a) - 4*tan(1/2*b*x)^2*tan(1
/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*ta
n(1/2*a) + tan(1/2*a)^2)/(tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4*t
an(1/2*a)^2 + 2*tan(1/2*b*x)^2*tan(1/2*a)^4 + tan(1/2*b*x)^4 + 4*tan(1/2*b
*x)^2*tan(1/2*a)^2 + tan(1/2*a)^4 + 2*tan(1/2*b*x)^2 + 2*tan(1/2*a)^2 + 1)
)*tan(1/2*b*x)*tan(1/2*a)^2 + b*c*tan(1/2*b*x)^2 + 4*b*c*tan(1/2*b*x)*tan(
1/2*a) + b*c*tan(1/2*a)^2 - b*d*x + d*log(16*(tan(1/2*b*x)^4*tan(1/2*a)...
```

### 3.108.9 Mupad [B] (verification not implemented)

Time = 22.93 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.63

$$\int (c+dx)\cot^2(a+bx)\,dx = -\frac{d\,x^2}{2}+\frac{d\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}-1\right)}{b^2}-\frac{(c+d\,x)\,2\mathrm{i}}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}-\frac{x\,(b\,c+d\,2\mathrm{i})}{b}$$

input
```
int(cot(a + b*x)^2*(c + d*x),x)
```

output
```
(d*log(exp(a*2i)*exp(b*x*2i) - 1))/b^2 - (d*x^2)/2 - ((c + d*x)*2i)/(b*(ex
p(a*2i + b*x*2i) - 1)) - (x*(d*2i + b*c))/b
```

# 3.109 $\int \frac{\cot^2(a+bx)}{c+dx} \, dx$

3.109.1 Optimal result . . . 901
3.109.2 Mathematica [N/A] . . . 901
3.109.3 Rubi [N/A] . . . 902
3.109.4 Maple [N/A] (verified) . . . 903
3.109.5 Fricas [N/A] . . . 903
3.109.6 Sympy [N/A] . . . 903
3.109.7 Maxima [N/A] . . . 904
3.109.8 Giac [N/A] . . . 904
3.109.9 Mupad [N/A] . . . 905

## 3.109.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\cot^2(a+bx)}{c+dx} \, dx = \text{Int}\left(\frac{\cot^2(a+bx)}{c+dx},x\right)$$

output
```
Unintegrable(cot(b*x+a)^2/(d*x+c),x)
```

## 3.109.2 Mathematica [N/A]

Not integrable

Time = 6.46 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^2(a+bx)}{c+dx} \, dx = \int \frac{\cot^2(a+bx)}{c+dx} \, dx$$

input
```
Integrate[Cot[a + b*x]^2/(c + d*x),x]
```

output
```
Integrate[Cot[a + b*x]^2/(c + d*x), x]
```

## 3.109.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(a+bx)}{c+dx} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan\left(a+bx+\frac{\pi}{2}\right)^2}{c+dx}dx$$

$$\Big\downarrow \text{ 4222}$$

$$\int \frac{\cot^2(a+bx)}{c+dx}dx$$

input
```
Int[Cot[a + b*x]^2/(c + d*x),x]
```

output
```
$Aborted
```

### 3.109.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

### 3.109.4 Maple [N/A] (verified)

Not integrable

Time = 0.66 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cot (xb+a)^2}{dx+c}dx$$

input
```
int(cot(b*x+a)^2/(d*x+c),x)
```

output
```
int(cot(b*x+a)^2/(d*x+c),x)
```

### 3.109.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^2(a+bx)}{c+dx}\,dx=\int \frac{\cot (bx+a)^2}{dx+c}\,dx$$

input
```
integrate(cot(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(cot(b*x + a)^2/(d*x + c), x)
```

### 3.109.6 Sympy [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\cot^2(a+bx)}{c+dx}\,dx=\int \frac{\cot^2(a+bx)}{c+dx}\,dx$$

input
```
integrate(cot(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(cot(a + b*x)**2/(c + d*x), x)
```

### 3.109.7 Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 581, normalized size of antiderivative = 36.31

$$\int \frac{\cot^2(a+bx)}{c+dx}\,dx = \int \frac{\cot(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(cot(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
((b*d^3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a)^2 + (b*d^3*x +
b*c*d^2)*sin(2*b*x + 2*a)^2 - 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*inte
grate(sin(b*x + a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x
 + b*c^2)*cos(b*x + a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2
+ 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)), x) - (b*d^3*x + b*c*d^2
 + (b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a)^2 + (b*d^3*x + b*c*d^2)*sin(2*b*x
+ 2*a)^2 - 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/
(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x +
 a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 - 2*(b*d^2*x^2 + 2*
b*c*d*x + b*c^2)*cos(b*x + a)), x) - (b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*
a)^2 + (b*d*x + b*c)*sin(2*b*x + 2*a)^2 + b*c - 2*(b*d*x + b*c)*cos(2*b*x
+ 2*a))*log(d*x + c) - 2*d*sin(2*b*x + 2*a))/(b*d^2*x + b*c*d + (b*d^2*x +
 b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 - 2*(b*d
^2*x + b*c*d)*cos(2*b*x + 2*a))
```

### 3.109.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^2(a+bx)}{c+dx}\,dx = \int \frac{\cot(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(cot(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(cot(b*x + a)^2/(d*x + c), x)
```

### 3.109.9 Mupad [N/A]

Not integrable

Time = 22.93 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^2(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+b\,x)^2}{c+d\,x}\,dx$$

input
```
int(cot(a + b*x)^2/(c + d*x),x)
```

output
```
int(cot(a + b*x)^2/(c + d*x), x)
```

# 3.110 $\int \frac{\cot^2(a+bx)}{(c+dx)^2} dx$

3.110.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 906
3.110.2 Mathematica [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 906
3.110.3 Rubi [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 907
3.110.4 Maple [N/A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 908
3.110.5 Fricas [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 908
3.110.6 Sympy [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 908
3.110.7 Maxima [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 909
3.110.8 Giac [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 909
3.110.9 Mupad [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 910

## 3.110.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2} \, dx = \text{Int}\left(\frac{\cot^2(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Unintegrable(cot(b*x+a)^2/(d*x+c)^2,x)
```

## 3.110.2 Mathematica [N/A]

Not integrable

Time = 2.79 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2} \, dx = \int \frac{\cot^2(a+bx)}{(c+dx)^2} \, dx$$

input
```
Integrate[Cot[a + b*x]^2/(c + d*x)^2,x]
```

output
```
Integrate[Cot[a + b*x]^2/(c + d*x)^2, x]
```

## 3.110.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan\left(a+bx+\frac{\pi}{2}\right)^2}{(c+dx)^2}dx$$
$$\Big\downarrow \text{4222}$$
$$\int \frac{\cot^2(a+bx)}{(c+dx)^2}dx$$

input
```
Int[Cot[a + b*x]^2/(c + d*x)^2,x]
```

output
```
$Aborted
```

### 3.110.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

### 3.110.4 Maple [N/A] (verified)

Not integrable

Time = 0.65 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cot(xb+a)^2}{(dx+c)^2}dx$$

input
```
int(cot(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(cot(b*x+a)^2/(d*x+c)^2,x)
```

### 3.110.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(cot(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(cot(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.110.6 Sympy [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(cot(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(cot(a + b*x)**2/(c + d*x)**2, x)
```

### 3.110.7 Maxima [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 813, normalized size of antiderivative = 50.81

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2} dx = \int \frac{\cot(bx+a)^2}{(dx+c)^2} dx$$

input
```
integrate(cot(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
(b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*b*x + 2*a)
^2 + b*c - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + 2*(b*d^4*x^2 + 2*b*c*d^3*x +
 b*c^2*d^2 + (b*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2)*cos(2*b*x + 2*a)^2 + (b
*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2)*sin(2*b*x + 2*a)^2 - 2*(b*d^4*x^2 + 2*
b*c*d^3*x + b*c^2*d^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^3*x^3
 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*
c^2*d*x + b*c^3)*cos(b*x + a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x
 + b*c^3)*sin(b*x + a)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*
c^3)*cos(b*x + a)), x) - 2*(b*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2 + (b*d^4*x
^2 + 2*b*c*d^3*x + b*c^2*d^2)*cos(2*b*x + 2*a)^2 + (b*d^4*x^2 + 2*b*c*d^3*
x + b*c^2*d^2)*sin(2*b*x + 2*a)^2 - 2*(b*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2
)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*
b*c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(
b*x + a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(b*x + a
)^2 - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)), x
) - 2*d*sin(2*b*x + 2*a))/(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d + (b*d^3*x^2
+ 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a)^2 + (b*d^3*x^2 + 2*b*c*d^2*x + b
*c^2*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b
*x + 2*a))
```

### 3.110.8 Giac [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2} dx = \int \frac{\cot(bx+a)^2}{(dx+c)^2} dx$$

input
```
integrate(cot(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(cot(b*x + a)^2/(d*x + c)^2, x)
```

### 3.110.9 Mupad [N/A]

Not integrable

Time = 22.77 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2} \, dx = \int \frac{\cot(a+b\,x)^2}{\left(c+d\,x\right)^2} \, dx$$

input
```
int(cot(a + b*x)^2/(c + d*x)^2,x)
```

output
```
int(cot(a + b*x)^2/(c + d*x)^2, x)
```

# 3.111 $\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx$

3.111.1 Optimal result . . . . . . . . . . 911
3.111.2 Mathematica [N/A] . . . . . . . . . . 911
3.111.3 Rubi [N/A] . . . . . . . . . . 912
3.111.4 Maple [N/A] (verified) . . . . . . . . . . 913
3.111.5 Fricas [N/A] . . . . . . . . . . 913
3.111.6 Sympy [N/A] . . . . . . . . . . 914
3.111.7 Maxima [N/A] . . . . . . . . . . 914
3.111.8 Giac [N/A] . . . . . . . . . . 914
3.111.9 Mupad [N/A] . . . . . . . . . . 915

## 3.111.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx = -\text{Int}((c+dx)^m\csc(a+bx),x) + \text{Int}\left((c+dx)^m\csc^3(a+bx),x\right)$$

output
```
-Unintegrable((d*x+c)^m*csc(b*x+a),x)+Unintegrable((d*x+c)^m*csc(b*x+a)^3,
x)
```

## 3.111.2 Mathematica [N/A]

Not integrable

Time = 41.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx = \int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Cot[a + b*x]^2*Csc[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Cot[a + b*x]^2*Csc[a + b*x], x]
```

## 3.111.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4915, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(a+bx)\csc(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{4915}$$

$$\int (c+dx)^m\csc^3(a+bx)dx-\int (c+dx)^m\csc(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m\csc(a+bx)^3dx-\int (c+dx)^m\csc(a+bx)dx$$

$$\Big\downarrow \text{4680}$$

$$\int (c+dx)^m\csc^3(a+bx)dx-\int (c+dx)^m\csc(a+bx)dx$$

input
```
Int[(c + d*x)^m*Cot[a + b*x]^2*Csc[a + b*x],x]
```

output
```
$Aborted
```

### 3.111.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4915
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

## 3.111.4 Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \cot(xb+a)^2 \csc(xb+a)\,dx$$

input
```
int((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x)
```

output
```
int((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x)
```

## 3.111.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx = \int (dx+c)^m \cot(bx+a)^2 \csc(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*cot(b*x + a)^2*csc(b*x + a), x)
```

### 3.111.6 Sympy [N/A]

Not integrable

Time = 7.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx = \int (c+dx)^m \cot^2{(a+bx)}\csc{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**m*cot(b*x+a)**2*csc(b*x+a),x)
```

output
```
Integral((c + d*x)**m*cot(a + b*x)**2*csc(a + b*x), x)
```

### 3.111.7 Maxima [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx = \int (dx+c)^m \cot\left(bx+a\right)^2 \csc\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cot(b*x + a)^2*csc(b*x + a), x)
```

### 3.111.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx = \int (dx+c)^m \cot\left(bx+a\right)^2 \csc\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cot(b*x + a)^2*csc(b*x + a), x)
```

### 3.111.9 Mupad [N/A]

Not integrable

Time = 23.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx = \int \frac{\cot(a+b\,x)^2\,(c+d\,x)^m}{\sin(a+b\,x)}\,dx$$

input

```
int((cot(a + b*x)^2*(c + d*x)^m)/sin(a + b*x),x)
```

output

```
int((cot(a + b*x)^2*(c + d*x)^m)/sin(a + b*x), x)
```

# 3.112 $\int (c+dx)^4 \cot^2(a+bx) \csc(a+bx) \, dx$

3.112.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 917
3.112.2 Mathematica [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 918
3.112.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 919
3.112.4 Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 927
3.112.5 Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . . 928
3.112.6 Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 929
3.112.7 Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . . 930
3.112.8 Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 930
3.112.9 Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 931

### 3.112.1 Optimal result

Integrand size = 22, antiderivative size = 416

$$\int (c+dx)^4\cot^2(a+bx)\csc(a+bx)\,dx = -\frac{12d^2(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+dx)^3\csc(a+bx)}{b^2} - \frac{(c+dx)^4\cot(a+bx)\csc(a+bx)}{2b} + \frac{12id^3(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^4} - \frac{2id(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} - \frac{12id^3(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^4} + \frac{2id(c+dx)^3\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} - \frac{12d^4\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^5} + \frac{6d^2(c+dx)^2\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} + \frac{12d^4\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^5} - \frac{6d^2(c+dx)^2\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3} + \frac{12id^3(c+dx)\operatorname{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4} - \frac{12id^3(c+dx)\operatorname{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^4} - \frac{12d^4\operatorname{PolyLog}\left(5,-e^{i(a+bx)}\right)}{b^5} + \frac{12d^4\operatorname{PolyLog}\left(5,e^{i(a+bx)}\right)}{b^5}$$

output
```
-12*d^2*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b^3+(d*x+c)^4*arctanh(exp(I*(b*x
+a)))/b-2*d*(d*x+c)^3*csc(b*x+a)/b^2-1/2*(d*x+c)^4*cot(b*x+a)*csc(b*x+a)/b
+12*I*d^3*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^4-12*I*d^3*(d*x+c)*polylog(
2,exp(I*(b*x+a)))/b^4-2*I*d*(d*x+c)^3*polylog(2,-exp(I*(b*x+a)))/b^2+12*I*
d^3*(d*x+c)*polylog(4,-exp(I*(b*x+a)))/b^4-12*d^4*polylog(3,-exp(I*(b*x+a)
))/b^5+6*d^2*(d*x+c)^2*polylog(3,-exp(I*(b*x+a)))/b^3+12*d^4*polylog(3,exp
(I*(b*x+a)))/b^5-6*d^2*(d*x+c)^2*polylog(3,exp(I*(b*x+a)))/b^3+2*I*d*(d*x+
c)^3*polylog(2,exp(I*(b*x+a)))/b^2-12*I*d^3*(d*x+c)*polylog(4,exp(I*(b*x+a
)))/b^4-12*d^4*polylog(5,-exp(I*(b*x+a)))/b^5+12*d^4*polylog(5,exp(I*(b*x+
a)))/b^5
```

## 3.112.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 966 vs. 2(416) = 832.

Time = 8.09 (sec) , antiderivative size = 966, normalized size of antiderivative = 2.32

$$\int (c+dx)^4 \cot^2(a+bx) \csc(a+bx)\, dx$$
$$= \frac{-b^4c^4 \log\left(1-e^{i(a+bx)}\right) + 12b^2c^2d^2 \log\left(1-e^{i(a+bx)}\right) - 4b^4c^3dx \log\left(1-e^{i(a+bx)}\right) + 24b^2cd^3x \log\left(1-e^{i(a+\cdots}\right.}{\cdots}$$
$$- \frac{\csc^2(a+bx)\left(bc^4\cos(a+bx) + 4bc^3dx\cos(a+bx) + 6bc^2d^2x^2\cos(a+bx) + 4bcd^3x^3\cos(a+bx) + bd^4x\cdots\right.}{2b^2\cdots}$$

input
```
Integrate[(c + d*x)^4*Cot[a + b*x]^2*Csc[a + b*x],x]
```

output
```
(-(b^4*c^4*Log[1 - E^(I*(a + b*x))]) + 12*b^2*c^2*d^2*Log[1 - E^(I*(a + b*
x))] - 4*b^4*c^3*d*x*Log[1 - E^(I*(a + b*x))] + 24*b^2*c*d^3*x*Log[1 - E^(
I*(a + b*x))] - 6*b^4*c^2*d^2*x^2*Log[1 - E^(I*(a + b*x))] + 12*b^2*d^4*x^
2*Log[1 - E^(I*(a + b*x))] - 4*b^4*c*d^3*x^3*Log[1 - E^(I*(a + b*x))] - b^
4*d^4*x^4*Log[1 - E^(I*(a + b*x))] + b^4*c^4*Log[1 + E^(I*(a + b*x))] - 12
*b^2*c^2*d^2*Log[1 + E^(I*(a + b*x))] + 4*b^4*c^3*d*x*Log[1 + E^(I*(a + b*
x))] - 24*b^2*c*d^3*x*Log[1 + E^(I*(a + b*x))] + 6*b^4*c^2*d^2*x^2*Log[1 +
 E^(I*(a + b*x))] - 12*b^2*d^4*x^2*Log[1 + E^(I*(a + b*x))] + 4*b^4*c*d^3*
x^3*Log[1 + E^(I*(a + b*x))] + b^4*d^4*x^4*Log[1 + E^(I*(a + b*x))] - (4*I
)*b*d*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*PolyLog[2, -E^(I*(a + b*x))] +
(4*I)*b*d*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*PolyLog[2, E^(I*(a + b*x))]
 + 12*b^2*c^2*d^2*PolyLog[3, -E^(I*(a + b*x))] - 24*d^4*PolyLog[3, -E^(I*(
a + b*x))] + 24*b^2*c*d^3*x*PolyLog[3, -E^(I*(a + b*x))] + 12*b^2*d^4*x^2*
PolyLog[3, -E^(I*(a + b*x))] - 12*b^2*c^2*d^2*PolyLog[3, E^(I*(a + b*x))]
+ 24*d^4*PolyLog[3, E^(I*(a + b*x))] - 24*b^2*c*d^3*x*PolyLog[3, E^(I*(a +
 b*x))] - 12*b^2*d^4*x^2*PolyLog[3, E^(I*(a + b*x))] + (24*I)*b*c*d^3*Poly
Log[4, -E^(I*(a + b*x))] + (24*I)*b*d^4*x*PolyLog[4, -E^(I*(a + b*x))] - (
24*I)*b*c*d^3*PolyLog[4, E^(I*(a + b*x))] - (24*I)*b*d^4*x*PolyLog[4, E^(I
*(a + b*x))] - 24*d^4*PolyLog[5, -E^(I*(a + b*x))] + 24*d^4*PolyLog[5, E^(
I*(a + b*x))])/(2*b^5) - (Csc[a + b*x]^2*(b*c^4*Cos[a + b*x] + 4*b*c^3*...
```

### 3.112.3 Rubi [A] (verified)

Time = 2.95 (sec) , antiderivative size = 742, normalized size of antiderivative = 1.78, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {4915, 3042, 4671, 3011, 4674, 3042, 4671, 3011, 2720, 7143, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \cot^2(a+bx) \csc(a+bx)\,dx$$

$$\Big\downarrow \text{4915}$$

$$\int (c+dx)^4 \csc^3(a+bx)dx - \int (c+dx)^4 \csc(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^4 \csc(a+bx)^3dx - \int (c+dx)^4 \csc(a+bx)dx$$

$$\Big\downarrow \text{4671}$$

$$\frac{4d\int(c+dx)^3\log\left(1-e^{i(a+bx)}\right)dx}{b}-\frac{4d\int(c+dx)^3\log\left(1+e^{i(a+bx)}\right)dx}{b}+\int(c+dx)^4\csc(a+bx)^3dx+\frac{2(c+dx)^4\text{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{4d\left(\frac{i(c+dx)^3\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\int(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{4d\left(\frac{i(c+dx)^3\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{3id\int(c+dx)^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}+\int(c+dx)^4\csc(a+bx)^3dx+\frac{2(c+dx)^4\text{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{4674}$$

$$\frac{6d^2\int(c+dx)^2\csc(a+bx)dx}{b^2}-\frac{4d\left(\frac{i(c+dx)^3\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\int(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{4d\left(\frac{i(c+dx)^3\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{3id\int(c+dx)^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{1}{2}\int(c+dx)^4\csc(a+bx)dx+\frac{2(c+dx)^4\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{2d(c+dx)^3\csc(a+bx)}{b^2}-\frac{(c+dx)^4\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6d^2\int(c+dx)^2\csc(a+bx)dx}{b^2}-\frac{4d\left(\frac{i(c+dx)^3\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\int(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{4d\left(\frac{i(c+dx)^3\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{3id\int(c+dx)^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{1}{2}\int(c+dx)^4\csc(a+bx)dx+\frac{2(c+dx)^4\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{2d(c+dx)^3\csc(a+bx)}{b^2}-\frac{(c+dx)^4\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{4671}$$

$$\frac{6d^2\left(-\frac{2d\int(c+dx)\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{2d\int(c+dx)\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)}{b^2}+\frac{1}{2}\left(-\frac{4d\int(c+dx)^3\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{4d\int(c+dx)^3\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^4\text{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)-\frac{4d\left(\frac{i(c+dx)^3\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\int(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{4d\left(\frac{i(c+dx)^3\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{3id\int(c+dx)^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{2(c+dx)^4\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{2d(c+dx)^3\csc(a+bx)}{b^2}-\frac{(c+dx)^4\cot(a+bx)\csc(a+bx)}{2b}$$

---

## 3.112. $\int(c+dx)^4\cot^2(a+bx)\csc(a+bx)\,dx$

$\downarrow$ 3011

$$\frac{6d^2\left(\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{id\int \text{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{id\int \text{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2(c\text{-}}{}\right.}{b^2}$$

$$\frac{1}{2}\left(\frac{4d\left(\frac{i(c+dx)^3\,\text{PolyLog}(2,-e^{i(a+bx)})}{b}-\frac{3id\int(c+dx)^2\,\text{PolyLog}(2,-e^{i(a+bx)})dx}{b}\right)}{b}-\frac{4d\left(\frac{i(c+dx)^3\,\text{PolyLog}(2,e^{i(a+bx)})}{b}-\frac{3id\int(c+dx)^2}{}\right.}{b}\right.$$

$$\frac{4d\left(\frac{i(c+dx)^3\,\text{PolyLog}(2,-e^{i(a+bx)})}{b}-\frac{3id\int(c+dx)^2\,\text{PolyLog}(2,-e^{i(a+bx)})dx}{b}\right)}{b}+$$

$$\frac{4d\left(\frac{i(c+dx)^3\,\text{PolyLog}(2,e^{i(a+bx)})}{b}-\frac{3id\int(c+dx)^2\,\text{PolyLog}(2,e^{i(a+bx)})dx}{b}\right)}{b}+\frac{2(c+dx)^4\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-$$

$$\frac{2d(c+dx)^3\csc(a+bx)}{b^2}-\frac{(c+dx)^4\cot(a+bx)\csc(a+bx)}{2b}$$

$\downarrow$ 2720

$$\frac{6d^2\left(\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\text{PolyLo}}{}\right.}{b}\right.}{b^2}$$

$$\frac{1}{2}\left(\frac{4d\left(\frac{i(c+dx)^3\,\text{PolyLog}(2,-e^{i(a+bx)})}{b}-\frac{3id\int(c+dx)^2\,\text{PolyLog}(2,-e^{i(a+bx)})dx}{b}\right)}{b}-\frac{4d\left(\frac{i(c+dx)^3\,\text{PolyLog}(2,e^{i(a+bx)})}{b}-\frac{3id\int(c+dx)^2}{}\right.}{b}\right.$$

$$\frac{4d\left(\frac{i(c+dx)^3\,\text{PolyLog}(2,-e^{i(a+bx)})}{b}-\frac{3id\int(c+dx)^2\,\text{PolyLog}(2,-e^{i(a+bx)})dx}{b}\right)}{b}+$$

$$\frac{4d\left(\frac{i(c+dx)^3\,\text{PolyLog}(2,e^{i(a+bx)})}{b}-\frac{3id\int(c+dx)^2\,\text{PolyLog}(2,e^{i(a+bx)})dx}{b}\right)}{b}+\frac{2(c+dx)^4\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-$$

$$\frac{2d(c+dx)^3\csc(a+bx)}{b^2}-\frac{(c+dx)^4\cot(a+bx)\csc(a+bx)}{2b}$$

$\downarrow$ 7143

3.112. $\int (c+dx)^4\cot^2(a+bx)\csc(a+bx)\,dx$

$$\frac{1}{2}\left(\frac{4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}(2,-e^{i(a+bx)})}{b}-\frac{3id\int(c+dx)^2\operatorname{PolyLog}(2,-e^{i(a+bx)})dx}{b}\right)}{b}-\frac{4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}(2,e^{i(a+bx)})}{b}-\frac{3id\int(c+dx)^2\ldots}{}\right.}{b}\right.$$

$$\frac{4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}(2,-e^{i(a+bx)})}{b}-\frac{3id\int(c+dx)^2\operatorname{PolyLog}(2,-e^{i(a+bx)})dx}{b}\right)}{b}+$$

$$\frac{4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}(2,e^{i(a+bx)})}{b}-\frac{3id\int(c+dx)^2\operatorname{PolyLog}(2,e^{i(a+bx)})dx}{b}\right)}{b}+$$

$$\frac{6d^2\left(-\frac{2(c+dx)^2\operatorname{arctanh}(e^{i(a+bx)})}{b}+\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{i(a+bx)})}{b}-\frac{d\operatorname{PolyLog}(3,-e^{i(a+bx)})}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,e^{i(a+bx)})}{b}-\frac{d\operatorname{Po}\ldots}{}\right)}{b}\right.}{b^2}$$

$$\frac{2(c+dx)^4\operatorname{arctanh}(e^{i(a+bx)})}{b}-\frac{2d(c+dx)^3\csc(a+bx)}{b^2}-\frac{(c+dx)^4\cot(a+bx)\csc(a+bx)}{2b}$$

$\downarrow$ 7163

$$\frac{1}{2}\left(\frac{4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}(2,-e^{i(a+bx)})}{b}-\frac{3id\left(\frac{2id\int(c+dx)\operatorname{PolyLog}(3,-e^{i(a+bx)})dx}{b}-\frac{i(c+dx)^2\operatorname{PolyLog}(3,-e^{i(a+bx)})}{b}\right)}{b}\right)}{b}-\frac{4d\left(\frac{i(c+dx)^3\operatorname{P}\ldots}{}\right.}{}\right.$$

$$\frac{4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}(2,-e^{i(a+bx)})}{b}-\frac{3id\left(\frac{2id\int(c+dx)\operatorname{PolyLog}(3,-e^{i(a+bx)})dx}{b}-\frac{i(c+dx)^2\operatorname{PolyLog}(3,-e^{i(a+bx)})}{b}\right)}{b}\right)}{b}+$$

$$\frac{4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}(2,e^{i(a+bx)})}{b}-\frac{3id\left(\frac{2id\int(c+dx)\operatorname{PolyLog}(3,e^{i(a+bx)})dx}{b}-\frac{i(c+dx)^2\operatorname{PolyLog}(3,e^{i(a+bx)})}{b}\right)}{b}\right)}{b}+$$

$$\frac{6d^2\left(-\frac{2(c+dx)^2\operatorname{arctanh}(e^{i(a+bx)})}{b}+\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{i(a+bx)})}{b}-\frac{d\operatorname{PolyLog}(3,-e^{i(a+bx)})}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,e^{i(a+bx)})}{b}-\frac{d\operatorname{Po}\ldots}{}\right)}{b}\right.}{b^2}$$

$$\frac{2(c+dx)^4\operatorname{arctanh}(e^{i(a+bx)})}{b}-\frac{2d(c+dx)^3\csc(a+bx)}{b^2}-\frac{(c+dx)^4\cot(a+bx)\csc(a+bx)}{2b}$$

$\downarrow$ 7163

3.112. $\int(c+dx)^4\cot^2(a+bx)\csc(a+bx)\,dx$

$$\frac{1}{2}\left(\frac{4d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{id\int \mathrm{PolyLog}\left(4,-e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(3,-e^{i(a+b}\right.}{b}\right.}{b}\right.}{b}\right.$$

$$\frac{4d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{id\int \mathrm{PolyLog}\left(4,-e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right.}{b}$$

$$\left.\frac{4d\left(\frac{i(c+dx)^3\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{id\int \mathrm{PolyLog}\left(4,e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\,\mathrm{PolyLog}\left(4,e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{i(c+dx)^2\,\mathrm{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}+\right.$$

$$\frac{6d^2\left(-\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\,\mathrm{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\,\mathrm{Po}}{}\right.}{b}\right.}{b^2}$$

$$\frac{2(c+dx)^4\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{2d(c+dx)^3\csc(a+bx)}{b^2}-\frac{(c+dx)^4\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \ 2720$$

3.112. $\int (c+dx)^4\cot^2(a+bx)\csc(a+bx)\,dx$

$$\frac{2\text{arctanh}\left(e^{i(a+bx)}\right)(c+dx)^4}{b}-\frac{\cot(a+bx)\csc(a+bx)(c+dx)^4}{2b}-\frac{2d\csc(a+bx)(c+dx)^3}{b^2}+$$

$$\frac{6d^2\left(-\frac{2\text{arctanh}\left(e^{i(a+bx)}\right)(c+dx)^2}{b}+\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\,\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\,\text{Po}}{}\right.}{b}\right.}{b^2}$$

$$\frac{4d\left(\frac{i(c+dx)^3\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\left(4,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\,\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{i(c+dx)^2\,\text{PolyLo}}{}\right.}{b}\right.}{b}$$

$$\frac{4d\left(\frac{i(c+dx)^3\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\left(4,e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\,\text{PolyLog}\left(4,e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{i(c+dx)^2\,\text{PolyLog}\left(3,e\right.}{b}\right.}{b}\right.}{b}$$

$$\frac{1}{2}\left(-\frac{2\text{arctanh}\left(e^{i(a+bx)}\right)(c+dx)^4}{b}+\frac{4d\left(\frac{i(c+dx)^3\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\left(4,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\right.}{b}\right.}{}\right.}{b}\right.$$

$\downarrow$ 7143

---

3.112. $\int (c+dx)^4 \cot^2(a+bx)\csc(a+bx)\,dx$

$$\frac{6d^2\left(-\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\operatorname{Po}\cdots}{}\right.}{b}\cdots\right.}{b^2}$$

$$\frac{1}{2}\left(-\frac{2(c+dx)^4\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{d\operatorname{PolyLog}\left(5,-e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,-e^{i\cdots}\right.}{b}\cdots\right.}{b}\cdots\right.}{b}\cdots\right.}{b}\cdots\right.$$

$$\frac{2(c+dx)^4\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-$$

$$\frac{4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{d\operatorname{PolyLog}\left(5,-e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{i(c+dx)^2\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}$$

$$\frac{4d\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{3id\left(\frac{2id\left(\frac{d\operatorname{PolyLog}\left(5,e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{i(c+dx)^2\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-$$

$$\frac{2d(c+dx)^3\csc(a+bx)}{b^2}-\frac{(c+dx)^4\cot(a+bx)\csc(a+bx)}{2b}$$

input

```
Int[(c + d*x)^4*Cot[a + b*x]^2*Csc[a + b*x],x]
```

## 3.112. $\int (c+dx)^4\cot^2(a+bx)\csc(a+bx)\,dx$

output
```
(2*(c + d*x)^4*ArcTanh[E^(I*(a + b*x))])/b - (2*d*(c + d*x)^3*Csc[a + b*x]
)/b^2 - ((c + d*x)^4*Cot[a + b*x]*Csc[a + b*x])/(2*b) + (6*d^2*((-2*(c + d
*x)^2*ArcTanh[E^(I*(a + b*x))])/b + (2*d*((I*(c + d*x)*PolyLog[2, -E^(I*(a
 + b*x))])/b - (d*PolyLog[3, -E^(I*(a + b*x))])/b^2))/b - (2*d*((I*(c + d*
x)*PolyLog[2, E^(I*(a + b*x))])/b - (d*PolyLog[3, E^(I*(a + b*x))])/b^2))/
b))/b^2 - (4*d*((I*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b - ((3*I)*d*
(((-I)*(c + d*x)^2*PolyLog[3, -E^(I*(a + b*x))])/b + ((2*I)*d*(((-I)*(c +
d*x)*PolyLog[4, -E^(I*(a + b*x))])/b + (d*PolyLog[5, -E^(I*(a + b*x))])/b^
2))/b))/b))/b + (4*d*((I*(c + d*x)^3*PolyLog[2, E^(I*(a + b*x))])/b - ((3*
I)*d*(((-I)*(c + d*x)^2*PolyLog[3, E^(I*(a + b*x))])/b + ((2*I)*d*(((-I)*(
c + d*x)*PolyLog[4, E^(I*(a + b*x))])/b + (d*PolyLog[5, E^(I*(a + b*x))])/
b^2))/b))/b))/b + ((-2*(c + d*x)^4*ArcTanh[E^(I*(a + b*x))])/b + (4*d*((I*
(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b - ((3*I)*d*(((-I)*(c + d*x)^2*
PolyLog[3, -E^(I*(a + b*x))])/b + ((2*I)*d*(((-I)*(c + d*x)*PolyLog[4, -E^
(I*(a + b*x))])/b + (d*PolyLog[5, -E^(I*(a + b*x))])/b^2))/b))/b))/b - (4*
d*((I*(c + d*x)^3*PolyLog[2, E^(I*(a + b*x))])/b - ((3*I)*d*(((-I)*(c + d*
x)^2*PolyLog[3, E^(I*(a + b*x))])/b + ((2*I)*d*(((-I)*(c + d*x)*PolyLog[4,
 E^(I*(a + b*x))])/b + (d*PolyLog[5, E^(I*(a + b*x))])/b^2))/b))/b))/b)/2
```

### 3.112.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4915
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

### 3.112.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1672 vs. 2(380) = 760.

Time = 1.49 (sec) , antiderivative size = 1673, normalized size of antiderivative = 4.02

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1673 |

input
```
int((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2/(exp(2*I*(b*x+a))-1)^2*(d^4*x^4*b*exp(3*I*(b*x+a))+4*c*d^3*x^3*b*exp
(3*I*(b*x+a))+6*c^2*d^2*x^2*b*exp(3*I*(b*x+a))+d^4*x^4*b*exp(I*(b*x+a))+4*
c^3*d*x*b*exp(3*I*(b*x+a))+4*c*d^3*x^3*b*exp(I*(b*x+a))-4*I*c^3*d*exp(3*I*
(b*x+a))+c^4*b*exp(3*I*(b*x+a))+6*c^2*d^2*x^2*b*exp(I*(b*x+a))-12*I*c^2*d^
2*x*exp(3*I*(b*x+a))+4*c^3*d*x*b*exp(I*(b*x+a))-4*I*d^4*x^3*exp(3*I*(b*x+a
))+4*I*c^3*d*exp(I*(b*x+a))+c^4*b*exp(I*(b*x+a))+12*I*c*d^3*x^2*exp(I*(b*x
+a))+12*I*c^2*d^2*x*exp(I*(b*x+a))+4*I*d^4*x^3*exp(I*(b*x+a))-12*I*c*d^3*x
^2*exp(3*I*(b*x+a)))-1/2/b^5*d^4*ln(exp(I*(b*x+a))+1)*a^4-1/2/b*d^4*ln(1-e
xp(I*(b*x+a)))*x^4+1/2/b*d^4*ln(exp(I*(b*x+a))+1)*x^4+1/b^5*d^4*a^4*arctan
h(exp(I*(b*x+a)))-12/b^5*d^4*a^2*arctanh(exp(I*(b*x+a)))-6/b^3*c^2*d^2*pol
ylog(3,exp(I*(b*x+a)))+6/b^3*c^2*d^2*polylog(3,-exp(I*(b*x+a)))-6/b^3*d^4*
ln(exp(I*(b*x+a))+1)*x^2-6/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x^2+6/b^3*d^4
*polylog(3,-exp(I*(b*x+a)))*x^2-12/b^3*c^2*d^2*arctanh(exp(I*(b*x+a)))+6/b
^3*d^4*ln(1-exp(I*(b*x+a)))*x^2+1/2/b^5*d^4*ln(1-exp(I*(b*x+a)))*a^4-6/b^5
*d^4*ln(1-exp(I*(b*x+a)))*a^2+6/b^5*d^4*ln(exp(I*(b*x+a))+1)*a^2-12*I/b^4*
c*d^3*polylog(2,exp(I*(b*x+a)))+12*I/b^4*c*d^3*polylog(2,-exp(I*(b*x+a)))+
2*I/b^2*d^4*polylog(2,exp(I*(b*x+a)))*x^3-2*I/b^2*d^4*polylog(2,-exp(I*(b*
x+a)))*x^3-12*I/b^4*d^4*polylog(2,exp(I*(b*x+a)))*x+12*I/b^4*d^4*polylog(2
,-exp(I*(b*x+a)))*x-12*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x+12*I/b^4*d^4*
polylog(4,-exp(I*(b*x+a)))*x-12*I/b^4*c*d^3*polylog(4,exp(I*(b*x+a)))+1...
```

### 3.112.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2770 vs. $2(370) = 740$.

Time = 0.35 (sec) , antiderivative size = 2770, normalized size of antiderivative = 6.66

$$\int (c+dx)^4 \cot^2(a+bx) \csc(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x
+ b^4*c^4)*cos(b*x + a) - 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + I*b^3*c^3
*d - 6*I*b*c*d^3 + (-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - I*b^3*c^3*d + 6*I
*b*c*d^3 - 3*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 + 3*I*(b^3*c^2*d^
2 - 2*b*d^4)*x)*dilog(cos(b*x + a) + I*sin(b*x + a)) - 4*(-I*b^3*d^4*x^3 -
 3*I*b^3*c*d^3*x^2 - I*b^3*c^3*d + 6*I*b*c*d^3 + (I*b^3*d^4*x^3 + 3*I*b^3*
c*d^3*x^2 + I*b^3*c^3*d - 6*I*b*c*d^3 + 3*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*cos
(b*x + a)^2 - 3*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*dilog(cos(b*x + a) - I*sin(b*
x + a)) - 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + I*b^3*c^3*d - 6*I*b*c*d^3
 + (-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - I*b^3*c^3*d + 6*I*b*c*d^3 - 3*I*(
b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 + 3*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*
dilog(-cos(b*x + a) + I*sin(b*x + a)) - 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*
x^2 - I*b^3*c^3*d + 6*I*b*c*d^3 + (I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + I*b
^3*c^3*d - 6*I*b*c*d^3 + 3*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 - 3
*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^4
*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 6*(b^4*c^2*d^2 - 2
*b^2*d^4)*x^2 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2
+ 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x
 + a)^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*log(cos(b*x + a) + I*sin(b*x + a)
 + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 6*(...
```

### 3.112.6 Sympy [F]

$$\int (c+dx)^4 \cot^2(a+bx)\csc(a+bx)\,dx = \int (c+dx)^4 \cot^2{(a+bx)}\csc{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**4*cot(b*x+a)**2*csc(b*x+a),x)
```

output
```
Integral((c + d*x)**4*cot(a + b*x)**2*csc(a + b*x), x)
```

### 3.112.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 7004 vs. $2(370) = 740$.

Time = 3.26 (sec) , antiderivative size = 7004, normalized size of antiderivative = 16.84

$$\int (c+dx)^4 \cot^2(a+bx)\csc(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

output
```
1/4*(c^4*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - lo
g(cos(b*x + a) - 1)) - 4*a*c^3*d*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + lo
g(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b + 6*a^2*c^2*d^2*(2*cos(b*x
+ a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))
/b^2 - 4*a^3*c*d^3*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a)
 + 1) - log(cos(b*x + a) - 1))/b^3 + a^4*d^4*(2*cos(b*x + a)/(cos(b*x + a)
^2 - 1) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b^4 + 4*(2*((b*x
+ a)^4*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*a^2*d^4 + 4*(b*c*d^3 - a*d
^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^
2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4
)*(b*x + a) + ((b*x + a)^4*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*a^2*d^
4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2
- 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d
^3 - (a^3 - 6*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) - 2*((b*x + a)^4*d^4 - 1
2*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^
3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3
*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a))*c
os(2*b*x + 2*a) - (-I*(b*x + a)^4*d^4 + 12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3
+ 12*I*a^2*d^4 + 4*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 6*(-I*b^2*c^2*d^2
+ 2*I*a*b*c*d^3 + (-I*a^2 + 2*I)*d^4)*(b*x + a)^2 + 4*(-I*b^3*c^3*d + 3...
```

### 3.112.8 Giac [F]

$$\int (c+dx)^4 \cot^2(a+bx)\csc(a+bx)\,dx = \int (dx+c)^4 \cot(bx+a)^2 \csc(bx+a)\,dx$$

input
```
integrate((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*cot(b*x + a)^2*csc(b*x + a), x)
```

### 3.112.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \cot^2(a+bx) \csc(a+bx) \, dx = \text{Hanged}$$

input
```
int((cot(a + b*x)^2*(c + d*x)^4)/sin(a + b*x),x)
```

output
```
\text{Hanged}
```

# 3.113 $\int (c+dx)^3 \cot^2(a+bx) \csc(a+bx) \, dx$


3.113.1 Optimal result . . . . . 932
3.113.2 Mathematica [A] (verified) . . . . . 933
3.113.3 Rubi [A] (verified) . . . . . 933
3.113.4 Maple [B] (verified) . . . . . 940
3.113.5 Fricas [B] (verification not implemented) . . . . . 941
3.113.6 Sympy [F] . . . . . 941
3.113.7 Maxima [B] (verification not implemented) . . . . . 942
3.113.8 Giac [F] . . . . . 942
3.113.9 Mupad [F(-1)] . . . . . 943


## 3.113.1 Optimal result

Integrand size = 22, antiderivative size = 308

$$\begin{aligned}
&\int (c+dx)^3 \cot^2(a+bx) \csc(a+bx) \, dx \\
&= -\frac{6d^2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b} \\
&\quad - \frac{3d(c+dx)^2 \csc(a+bx)}{2b^2} - \frac{(c+dx)^3 \cot(a+bx) \csc(a+bx)}{2b} \\
&\quad + \frac{3id^3 \operatorname{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^4} - \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2, -e^{i(a+bx)}\right)}{2b^2} \\
&\quad - \frac{3id^3 \operatorname{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^4} + \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2, e^{i(a+bx)}\right)}{2b^2} \\
&\quad + \frac{3d^2(c+dx) \operatorname{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^3} - \frac{3d^2(c+dx) \operatorname{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^3} \\
&\quad + \frac{3id^3 \operatorname{PolyLog}\left(4, -e^{i(a+bx)}\right)}{b^4} - \frac{3id^3 \operatorname{PolyLog}\left(4, e^{i(a+bx)}\right)}{b^4}
\end{aligned}$$

output
```
-6*d^2*(d*x+c)*arctanh(exp(I*(b*x+a)))/b^3+(d*x+c)^3*arctanh(exp(I*(b*x+a)
))/b-3/2*d*(d*x+c)^2*csc(b*x+a)/b^2-1/2*(d*x+c)^3*cot(b*x+a)*csc(b*x+a)/b+
3*I*d^3*polylog(2,-exp(I*(b*x+a)))/b^4-3/2*I*d*(d*x+c)^2*polylog(2,-exp(I*
(b*x+a)))/b^2-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4+3/2*I*d*(d*x+c)^2*poly
log(2,exp(I*(b*x+a)))/b^2+3*d^2*(d*x+c)*polylog(3,-exp(I*(b*x+a)))/b^3-3*d
^2*(d*x+c)*polylog(3,exp(I*(b*x+a)))/b^3+3*I*d^3*polylog(4,-exp(I*(b*x+a))
)/b^4-3*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4
```

### 3.113.2 Mathematica [A] (verified)

Time = 5.42 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.71

$$\int (c+dx)^3 \cot^2(a+bx)\csc(a+bx)\,dx =$$
$$-\frac{b^2(c+dx)^2(3d+b(c+dx)\cot(a+bx))\csc(a+bx)+b^3c^3\log\left(1-e^{i(a+bx)}\right)-6bcd^2\log\left(1-e^{i(a+bx)}\right)+$$

input
```
Integrate[(c + d*x)^3*Cot[a + b*x]^2*Csc[a + b*x],x]
```

output
```
-1/2*(b^2*(c + d*x)^2*(3*d + b*(c + d*x)*Cot[a + b*x])*Csc[a + b*x] + b^3*
c^3*Log[1 - E^(I*(a + b*x))] - 6*b*c*d^2*Log[1 - E^(I*(a + b*x))] + 3*b^3*
c^2*d*x*Log[1 - E^(I*(a + b*x))] - 6*b*d^3*x*Log[1 - E^(I*(a + b*x))] + 3*
b^3*c*d^2*x^2*Log[1 - E^(I*(a + b*x))] + b^3*d^3*x^3*Log[1 - E^(I*(a + b*x
))] - b^3*c^3*Log[1 + E^(I*(a + b*x))] + 6*b*c*d^2*Log[1 + E^(I*(a + b*x))
] - 3*b^3*c^2*d*x*Log[1 + E^(I*(a + b*x))] + 6*b*d^3*x*Log[1 + E^(I*(a + b
*x))] - 3*b^3*c*d^2*x^2*Log[1 + E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + E^(
I*(a + b*x))] + (3*I)*d*(-2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, -E^(I*(a + b
*x))] - (3*I)*d*(-2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, E^(I*(a + b*x))] - 6
*b*c*d^2*PolyLog[3, -E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, -E^(I*(a + b*
x))] + 6*b*c*d^2*PolyLog[3, E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, E^(I*(
a + b*x))] - (6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4,
 E^(I*(a + b*x))])/b^4
```

### 3.113.3 Rubi [A] (verified)

Time = 1.95 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.73, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {4915, 3042, 4671, 3011, 4674, 3042, 4671, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \cot^2(a+bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \; 4915$$

$$\int (c+dx)^3\csc^3(a+bx)dx - \int (c+dx)^3\csc(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$\int (c+dx)^3\csc(a+bx)^3dx-\int (c+dx)^3\csc(a+bx)dx$$

$$\Big\downarrow \text{4671}$$

$$\frac{3d\int (c+dx)^2\log\left(1-e^{i(a+bx)}\right)dx}{b}-\frac{3d\int (c+dx)^2\log\left(1+e^{i(a+bx)}\right)dx}{b}+\int (c+dx)^3\csc(a+bx)^3dx+\frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\int (c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\int (c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}+\int (c+dx)^3\csc(a+bx)^3dx+\frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{4674}$$

$$\frac{3d^2\int (c+dx)\csc(a+bx)dx}{b^2}-\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\int (c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\int (c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{1}{2}\int (c+dx)^3\csc(a+bx)dx+\frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d^2\int (c+dx)\csc(a+bx)dx}{b^2}-\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\int (c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\int (c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{1}{2}\int (c+dx)^3\csc(a+bx)dx+\frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{4671}$$

3.113. $\int (c+dx)^3\cot^2(a+bx)\csc(a+bx)\,dx$

$$\frac{3d^2\left(-\frac{d\int\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{d\int\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)}{b^2}+\frac{1}{2}\left(-\frac{3d\int(c+dx)^2\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{3d\int(c+dx)^2\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)-\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{2715}$$

$$\frac{3d^2\left(\frac{id\int e^{-i(a+bx)}\log\left(1-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{id\int e^{-i(a+bx)}\log\left(1+e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)}{b^2}+\frac{1}{2}\left(-\frac{3d\int(c+dx)^2\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{3d\int(c+dx)^2\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)-\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{2838}$$

$$\frac{1}{2}\left(-\frac{3d\int(c+dx)^2\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{3d\int(c+dx)^2\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)-\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{3d^2\left(-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}\right)}{b^2}+\frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{3011}$$

3.113. $\int(c+dx)^3\cot^2(a+bx)\csc(a+bx)\,dx$

$$\frac{1}{2}\left(\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\operatorname{Po}}{}\right.}{b}\right.$$

$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}+$$

$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}+$$

$$\frac{3d^2\left(-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{id\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}\right)}{b^2}+$$

$$\frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{7163}$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{id\int\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}(2}{b}\right.}{}\right.$$

$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{id\int\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}+$$

$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{id\int\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}+$$

$$\frac{3d^2\left(-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{id\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}\right)}{b^2}+$$

$$\frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{2720}$$

---

3.113. $\int(c+dx)^3\cot^2(a+bx)\csc(a+bx)\,dx$

$$\frac{1}{2}\left(\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c\cdots}{}\right.}{}\right.$$

$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}+$$

$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}+$$

$$\frac{3d^2\left(-\frac{2(c+dx)\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{id\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}\right)}{b^2}+$$

$$\frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \; 7143$$

$$\frac{3d^2\left(-\frac{2(c+dx)\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{id\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}\right)}{b^2}+$$

$$\frac{1}{2}\left(-\frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\operatorname{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+b\cdots}\right.}{b}\right.}{b}\right.}{b}\right.$$

$$\frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-$$

$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\operatorname{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}+$$

$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\operatorname{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-$$

$$\frac{3d(c+dx)^2\csc(a+bx)}{2b^2}-\frac{(c+dx)^3\cot(a+bx)\csc(a+bx)}{2b}$$

3.113. $\int(c+dx)^3\cot^2(a+bx)\csc(a+bx)\,dx$

input
```
Int[(c + d*x)^3*Cot[a + b*x]^2*Csc[a + b*x],x]
```

output
```
(2*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b - (3*d*(c + d*x)^2*Csc[a + b*x]
)/(2*b^2) - ((c + d*x)^3*Cot[a + b*x]*Csc[a + b*x])/(2*b) + (3*d^2*((-2*(c
 + d*x)*ArcTanh[E^(I*(a + b*x))])/b + (I*d*PolyLog[2, -E^(I*(a + b*x))])/b
^2 - (I*d*PolyLog[2, E^(I*(a + b*x))])/b^2))/b^2 - (3*d*((I*(c + d*x)^2*Po
lyLog[2, -E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(c + d*x)*PolyLog[3, -E^(I
*(a + b*x))])/b + (d*PolyLog[4, -E^(I*(a + b*x))])/b^2))/b))/b + (3*d*((I*
(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(c + d*x)*Pol
yLog[3, E^(I*(a + b*x))])/b + (d*PolyLog[4, E^(I*(a + b*x))])/b^2))/b))/b
+ ((-2*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b + (3*d*((I*(c + d*x)^2*Poly
Log[2, -E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(c + d*x)*PolyLog[3, -E^(I*(
a + b*x))])/b + (d*PolyLog[4, -E^(I*(a + b*x))])/b^2))/b))/b - (3*d*((I*(c
 + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(c + d*x)*PolyL
og[3, E^(I*(a + b*x))])/b + (d*PolyLog[4, E^(I*(a + b*x))])/b^2))/b))/b)/2
```

### 3.113.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4915
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

## 3.113. $\int (c+dx)^3 \cot^2(a+bx) \csc(a+bx) \, dx$

### 3.113.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1055 vs. 2(274) = 548.

Time = 1.32 (sec) , antiderivative size = 1056, normalized size of antiderivative = 3.43

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1056 |

input
```
int((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*d^3*ln(exp(I*(b*x+a))+1)*x^3-6/b^3*c*d^2*arctanh(exp(I*(b*x+a)))-1/b
^4*d^3*a^3*arctanh(exp(I*(b*x+a)))+6/b^4*d^3*a*arctanh(exp(I*(b*x+a)))-3/b
^3*d^3*polylog(3,exp(I*(b*x+a)))*x+3/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+
3/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))-3/b^3*c*d^2*polylog(3,exp(I*(b*x+a)
))+3*d^3/b^3*ln(1-exp(I*(b*x+a)))*x+3*d^3/b^4*ln(1-exp(I*(b*x+a)))*a-3*d^3
/b^3*ln(exp(I*(b*x+a))+1)*x-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4-3*I*d^3*
polylog(4,exp(I*(b*x+a)))/b^4-3/b^4*d^3*ln(exp(I*(b*x+a))+1)*a+1/2/b^4*d^3
*ln(exp(I*(b*x+a))+1)*a^3-1/2/b*d^3*ln(1-exp(I*(b*x+a)))*x^3-1/2/b^4*d^3*l
n(1-exp(I*(b*x+a)))*a^3+1/b^2/(exp(2*I*(b*x+a))-1)^2*(d^3*x^3*b*exp(3*I*(b
*x+a))+3*c*d^2*x^2*b*exp(3*I*(b*x+a))+3*c^2*d*x*b*exp(3*I*(b*x+a))+d^3*x^3
*b*exp(I*(b*x+a))+c^3*b*exp(3*I*(b*x+a))+3*c*d^2*x^2*b*exp(I*(b*x+a))-3*I*
d^3*x^2*exp(3*I*(b*x+a))+3*c^2*d*x*b*exp(I*(b*x+a))-6*I*c*d^2*x*exp(3*I*(b
*x+a))+c^3*b*exp(I*(b*x+a))-3*I*c^2*d*exp(3*I*(b*x+a))+3*I*d^3*x^2*exp(I*(
b*x+a))+6*I*c*d^2*x*exp(I*(b*x+a))+3*I*c^2*d*exp(I*(b*x+a)))+1/b*c^3*arcta
nh(exp(I*(b*x+a)))-3*I/b^2*c*d^2*polylog(2,-exp(I*(b*x+a)))*x+3*I/b^2*c*d^
2*polylog(2,exp(I*(b*x+a)))*x-3/2*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))-3
/2/b^3*c*d^2*ln(exp(I*(b*x+a))+1)*a^2-3/b^2*c^2*d*a*arctanh(exp(I*(b*x+a))
)+3/b^3*c*d^2*a^2*arctanh(exp(I*(b*x+a)))-3/2/b*c*d^2*ln(1-exp(I*(b*x+a)))
*x^2+3/2/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2-3/2/b*c^2*d*ln(1-exp(I*(b*x+a)))
*x-3/2/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a+3/2/b*c^2*d*ln(exp(I*(b*x+a))+1...
```

### 3.113.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1742 vs. $2(264) = 528$.

Time = 0.32 (sec) , antiderivative size = 1742, normalized size of antiderivative = 5.66

$$\int (c+dx)^3 \cot^2(a+bx)\csc(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x +
 a) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d - 2*I*d^3 + (-I*b^2
*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a)^2)*dilog(
cos(b*x + a) + I*sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b
^2*c^2*d + 2*I*d^3 + (I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d - 2*I*
d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*(I*b^2*d^3*x
^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d - 2*I*d^3 + (-I*b^2*d^3*x^2 - 2*I*b^2*c
*d^2*x - I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*si
n(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d + 2*I*d^3
+ (I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d - 2*I*d^3)*cos(b*x + a)^2
)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
 b^3*c^3 - 6*b*c*d^2 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^
2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)
*log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
 b^3*c^3 - 6*b*c*d^2 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^
2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)
*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^
2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*
b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*s
in(b*x + a) + 1/2) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (...
```

### 3.113.6 Sympy [F]

$$\int (c+dx)^3 \cot^2(a+bx)\csc(a+bx)\,dx = \int (c+dx)^3 \cot^2(a+bx)\csc(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*cot(b*x+a)**2*csc(b*x+a),x)
```

output
```
Integral((c + d*x)**3*cot(a + b*x)**2*csc(a + b*x), x)
```

### 3.113.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3887 vs. $2(264) = 528$.

Time = 1.37 (sec) , antiderivative size = 3887, normalized size of antiderivative = 12.62

$$\int (c+dx)^3 \cot^2(a+bx) \csc(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

output
```
1/4*(c^3*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - lo
g(cos(b*x + a) - 1)) - 3*a*c^2*d*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + lo
g(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*cos(b*x +
a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b
^2 - a^3*d^3*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1)
- log(cos(b*x + a) - 1))/b^3 + 4*(2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3
 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2
)*d^3)*(b*x + a) + ((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a
*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))
*cos(4*b*x + 4*a) - 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2
- a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x +
a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + 3*(
-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*
a^2 + 2*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*(I*(b*x + a)^3*d^3 - 6*I*b
*c*d^2 + 6*I*a*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d
- 2*I*a*b*c*d^2 + (I*a^2 - 2*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(
sin(b*x + a), cos(b*x + a) + 1) + 12*(b*c*d^2 - a*d^3 + (b*c*d^2 - a*d^3)*
cos(4*b*x + 4*a) - 2*(b*c*d^2 - a*d^3)*cos(2*b*x + 2*a) - (-I*b*c*d^2 + I*
a*d^3)*sin(4*b*x + 4*a) - 2*(I*b*c*d^2 - I*a*d^3)*sin(2*b*x + 2*a))*arctan
2(sin(b*x + a), cos(b*x + a) - 1) + 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a...
```

### 3.113.8 Giac [F]

$$\int (c+dx)^3 \cot^2(a+bx) \csc(a+bx) \, dx = \int (dx+c)^3 \cot (bx+a)^2 \csc (bx+a) \, dx$$

input
```
integrate((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*cot(b*x + a)^2*csc(b*x + a), x)
```

### 3.113.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \cot^2(a+bx) \csc(a+bx) \, dx = \text{Hanged}$$

input
```
int((cot(a + b*x)^2*(c + d*x)^3)/sin(a + b*x),x)
```

output
```
\text{Hanged}
```

# 3.114 $\int (c+dx)^2 \cot^2(a+bx) \csc(a+bx)\,dx$


3.114.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 944
3.114.2 Mathematica [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 945
3.114.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 945
3.114.4 Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 950
3.114.5 Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . 951
3.114.6 Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 952
3.114.7 Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . 953
3.114.8 Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 953
3.114.9 Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 954


## 3.114.1 Optimal result

Integrand size = 22, antiderivative size = 179

$$\begin{aligned}\int (c+dx)^2 \cot^2(a+bx) \csc(a+bx)\,dx &= \frac{(c+dx)^2 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} \\ &- \frac{d^2 \operatorname{arctanh}(\cos(a+bx))}{b^3} - \frac{d(c+dx)\csc(a+bx)}{b^2} \\ &- \frac{(c+dx)^2 \cot(a+bx)\csc(a+bx)}{2b} \\ &- \frac{id(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} \\ &+ \frac{id(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} \\ &+ \frac{d^2 \operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} \\ &- \frac{d^2 \operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}\end{aligned}$$

output
```
(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b-d^2*arctanh(cos(b*x+a))/b^3-d*(d*x+c)*
csc(b*x+a)/b^2-1/2*(d*x+c)^2*cot(b*x+a)*csc(b*x+a)/b-I*d*(d*x+c)*polylog(2
,-exp(I*(b*x+a)))/b^2+I*d*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^2+d^2*polylo
g(3,-exp(I*(b*x+a)))/b^3-d^2*polylog(3,exp(I*(b*x+a)))/b^3
```

### 3.114.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 471 vs. $2(179) = 358$.

Time = 7.51 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.63

$$\int (c+dx)^2\cot^2(a+bx)\csc(a+bx)\,dx$$
$$= -\frac{d(c+dx)\csc(a)}{b^2} + \frac{(-c^2-2cdx-d^2x^2)\csc^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}$$
$$+ \frac{-b^2c^2\log\left(1-e^{i(a+bx)}\right)+2d^2\log\left(1-e^{i(a+bx)}\right)-2b^2cdx\log\left(1-e^{i(a+bx)}\right)-b^2d^2x^2\log\left(1-e^{i(a+bx)}\right)+}{}$$
$$+ \frac{(c^2+2cdx+d^2x^2)\sec^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b} + \frac{\sec\left(\frac{a}{2}\right)\sec\left(\frac{a}{2}+\frac{bx}{2}\right)\left(-cd\sin\left(\frac{bx}{2}\right)-d^2x\sin\left(\frac{bx}{2}\right)\right)}{2b^2}$$
$$+ \frac{\csc\left(\frac{a}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)\left(cd\sin\left(\frac{bx}{2}\right)+d^2x\sin\left(\frac{bx}{2}\right)\right)}{2b^2}$$

input
```
Integrate[(c + d*x)^2*Cot[a + b*x]^2*Csc[a + b*x],x]
```

output
```
-((d*(c + d*x)*Csc[a])/b^2) + ((-c^2 - 2*c*d*x - d^2*x^2)*Csc[a/2 + (b*x)/
2]^2)/(8*b) + (-(b^2*c^2*Log[1 - E^(I*(a + b*x))]) + 2*d^2*Log[1 - E^(I*(a
 + b*x))] - 2*b^2*c*d*x*Log[1 - E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 - E^(
I*(a + b*x))] + b^2*c^2*Log[1 + E^(I*(a + b*x))] - 2*d^2*Log[1 + E^(I*(a +
 b*x))] + 2*b^2*c*d*x*Log[1 + E^(I*(a + b*x))] + b^2*d^2*x^2*Log[1 + E^(I*
(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] + (2*I)*b*d
*(c + d*x)*PolyLog[2, E^(I*(a + b*x))] + 2*d^2*PolyLog[3, -E^(I*(a + b*x))
] - 2*d^2*PolyLog[3, E^(I*(a + b*x))])/(2*b^3) + ((c^2 + 2*c*d*x + d^2*x^2
)*Sec[a/2 + (b*x)/2]^2)/(8*b) + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c*d*Sin[(b
*x)/2]) - d^2*x*Sin[(b*x)/2]))/(2*b^2) + (Csc[a/2]*Csc[a/2 + (b*x)/2]*(c*d
*Sin[(b*x)/2] + d^2*x*Sin[(b*x)/2]))/(2*b^2)
```

### 3.114.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.81, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {4915, 3042, 4671, 3011, 2720, 4674, 3042, 4257, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \cot^2(a+bx) \csc(a+bx) \, dx$$

$$\Big\downarrow \text{ 4915}$$

$$\int (c+dx)^2 \csc^3(a+bx)dx - \int (c+dx)^2 \csc(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^2 \csc(a+bx)^3dx - \int (c+dx)^2 \csc(a+bx)dx$$

$$\Big\downarrow \text{ 4671}$$

$$\frac{2d \int (c+dx) \log \left(1-e^{i(a+bx)}\right) dx}{b} - \frac{2d \int (c+dx) \log \left(1+e^{i(a+bx)}\right) dx}{b} + \int (c+dx)^2 \csc(a+bx)^3dx + \frac{2(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{ 3011}$$

$$-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b} - \frac{id \int \text{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b} + \frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b} - \frac{id \int \text{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b} + \int (c+dx)^2 \csc(a+bx)^3dx + \frac{2(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{ 2720}$$

$$-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b} - \frac{d \int e^{-i(a+bx)}\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b} + \frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b} - \frac{d \int e^{-i(a+bx)}\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b} + \int (c+dx)^2 \csc(a+bx)^3dx + \frac{2(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{ 4674}$$

$$-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b} - \frac{d \int e^{-i(a+bx)}\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b} + \frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b} - \frac{d \int e^{-i(a+bx)}\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b} + \frac{d^2 \int \csc(a+bx)dx}{b^2} + \frac{1}{2}\int (c+dx)^2 \csc(a+bx)dx + \frac{2(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{d(c+dx)\csc(a+bx)}{b^2} - \frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}$$

---

3.114. $\int (c+dx)^2 \cot^2(a+bx) \csc(a+bx) \, dx$

$$\Big\downarrow \text{3042}$$

$$-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+\frac{d^2\int\csc(a+bx)dx}{b^2}+\frac{1}{2}\int(c+dx)^2\csc(a+bx)dx+\frac{2(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{d(c+dx)\csc(a+bx)}{b^2}-\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+\frac{1}{2}\int(c+dx)^2\csc(a+bx)dx-\frac{d^2\text{arctanh}(\cos(a+bx))}{b^3}+\frac{2(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{d(c+dx)\csc(a+bx)}{b^2}-\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{4671}$$

$$\frac{1}{2}\left(-\frac{2d\int(c+dx)\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{2d\int(c+dx)\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-\frac{d^2\text{arctanh}(\cos(a+bx))}{b^3}+\frac{2(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{d(c+dx)\csc(a+bx)}{b^2}-\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{3011}$$

## 3.114. $\int (c+dx)^2\cot^2(a+bx)\csc(a+bx)\,dx$

$$\frac{1}{2}\left(\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{i(a+bx)})}{b}-\frac{id\int \operatorname{PolyLog}(2,-e^{i(a+bx)})dx}{b}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,e^{i(a+bx)})}{b}-\frac{id\int \operatorname{PolyLog}(2,e^{i(a+bx)})\right.}{b}}{b}\right.$$

$$\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{i(a+bx)})}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}(2,-e^{i(a+bx)})de^{i(a+bx)}}{b^2}\right)}{b}+$$

$$\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,e^{i(a+bx)})}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}(2,e^{i(a+bx)})de^{i(a+bx)}}{b^2}\right)}{b}-\frac{d^2\operatorname{arctanh}(\cos(a+bx))}{b^3}+$$

$$\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{d(c+dx)\csc(a+bx)}{b^2}-\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}$$

$\downarrow$ 2720

$$\frac{1}{2}\left(\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{i(a+bx)})}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}(2,-e^{i(a+bx)})de^{i(a+bx)}}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,e^{i(a+bx)})}{b}-\frac{d\int e^{-i(a-}}{}\right.}{b}\right.$$

$$\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{i(a+bx)})}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}(2,-e^{i(a+bx)})de^{i(a+bx)}}{b^2}\right)}{b}+$$

$$\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,e^{i(a+bx)})}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}(2,e^{i(a+bx)})de^{i(a+bx)}}{b^2}\right)}{b}-\frac{d^2\operatorname{arctanh}(\cos(a+bx))}{b^3}+$$

$$\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{d(c+dx)\csc(a+bx)}{b^2}-\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}$$

$\downarrow$ 7143

$$-\frac{d^2\operatorname{arctanh}(\cos(a+bx))}{b^3}+$$

$$\frac{1}{2}\left(-\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{i(a+bx)})}{b}-\frac{d\operatorname{PolyLog}(3,-e^{i(a+bx)})}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,e^{i}}{b}\right.}{}\right.$$

$$\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{i(a+bx)})}{b}-\frac{d\operatorname{PolyLog}(3,-e^{i(a+bx)})}{b^2}\right)}{b}+$$

$$\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,e^{i(a+bx)})}{b}-\frac{d\operatorname{PolyLog}(3,e^{i(a+bx)})}{b^2}\right)}{b}-\frac{d(c+dx)\csc(a+bx)}{b^2}-$$

$$\frac{(c+dx)^2\cot(a+bx)\csc(a+bx)}{2b}$$

input
```
Int[(c + d*x)^2*Cot[a + b*x]^2*Csc[a + b*x],x]
```

3.114. $\int (c+dx)^2\cot^2(a+bx)\csc(a+bx)\,dx$

output
```
(2*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (d^2*ArcTanh[Cos[a + b*x]])/b
^3 - (d*(c + d*x)*Csc[a + b*x])/b^2 - ((c + d*x)^2*Cot[a + b*x]*Csc[a + b*
x])/(2*b) - (2*d*((I*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b - (d*PolyLo
g[3, -E^(I*(a + b*x))])/b^2))/b + (2*d*((I*(c + d*x)*PolyLog[2, E^(I*(a +
b*x))])/b - (d*PolyLog[3, E^(I*(a + b*x))])/b^2))/b + ((-2*(c + d*x)^2*Arc
Tanh[E^(I*(a + b*x))])/b + (2*d*((I*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))]
)/b - (d*PolyLog[3, -E^(I*(a + b*x))])/b^2))/b - (2*d*((I*(c + d*x)*PolyLo
g[2, E^(I*(a + b*x))])/b - (d*PolyLog[3, E^(I*(a + b*x))])/b^2))/b)/2
```

### 3.114.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4915
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.114.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 545 vs. 2(165) = 330.

Time = 1.20 (sec) , antiderivative size = 546, normalized size of antiderivative = 3.05

| method | result |
|---|---|
| risch | $\frac{x^2 d^2 b\, \mathrm{e}^{3i(xb+a)}+2cdxb\, \mathrm{e}^{3i(xb+a)}+c^2 b\, \mathrm{e}^{3i(xb+a)}+x^2 d^2 b\, \mathrm{e}^{i(xb+a)}+2cdxb\, \mathrm{e}^{i(xb+a)}-2id^2 x\, \mathrm{e}^{3i(xb+a)}+c^2 b\, \mathrm{e}^{i(xb+a)}-2idc\, \mathrm{e}^{3i(xb+a)}+2id}{b^2\left(\mathrm{e}^{2i(xb+a)}-1\right)^2}$ |

input
```
int((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2/(exp(2*I*(b*x+a))-1)^2*(x^2*d^2*b*exp(3*I*(b*x+a))+2*c*d*x*b*exp(3*I
*(b*x+a))+c^2*b*exp(3*I*(b*x+a))+x^2*d^2*b*exp(I*(b*x+a))+2*c*d*x*b*exp(I*
(b*x+a))-2*I*d^2*x*exp(3*I*(b*x+a))+c^2*b*exp(I*(b*x+a))-2*I*d*c*exp(3*I*(
b*x+a))+2*I*d^2*x*exp(I*(b*x+a))+2*I*d*c*exp(I*(b*x+a)))+I/b^2*d^2*polylog
(2,exp(I*(b*x+a)))*x+1/2/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+1/2/b^3*d^2*ln(1-e
xp(I*(b*x+a)))*a^2-1/2/b^3*d^2*ln(exp(I*(b*x+a))+1)*a^2-I/b^2*c*d*polylog(
2,-exp(I*(b*x+a)))+I/b^2*c*d*polylog(2,exp(I*(b*x+a)))-I/b^2*d^2*polylog(2
,-exp(I*(b*x+a)))*x-2/b^2*c*d*a*arctanh(exp(I*(b*x+a)))+1/b*c^2*arctanh(ex
p(I*(b*x+a)))+1/b^3*d^2*a^2*arctanh(exp(I*(b*x+a)))-1/2/b*d^2*ln(1-exp(I*(
b*x+a)))*x^2-1/b*d*c*ln(1-exp(I*(b*x+a)))*x-1/b^2*d*c*ln(1-exp(I*(b*x+a)))
*a+1/b*d*c*ln(exp(I*(b*x+a))+1)*x+1/b^2*c*d*ln(exp(I*(b*x+a))+1)*a-d^2*pol
ylog(3,exp(I*(b*x+a)))/b^3+d^2*polylog(3,-exp(I*(b*x+a)))/b^3-2/b^3*d^2*ar
ctanh(exp(I*(b*x+a)))
```

### 3.114.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 970 vs. $2(161) = 322$.

Time = 0.31 (sec) , antiderivative size = 970, normalized size of antiderivative = 5.42

$$\int (c+dx)^2\cot^2(a+bx)\csc(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a) - 2*(I*b*d^2*x +
 I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(b*x + a)^2)*dilog(cos(b*x + a) + I*s
in(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(b*x + a
)^2)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d + (-I*b
*d^2*x - I*b*c*d)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a)) -
2*(-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2)*dilog(-cos
(b*x + a) - I*sin(b*x + a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - (b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - 2*d^2)*log(cos(b
*x + a) + I*sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - (b^
2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - 2*d^2)*log(cos
(b*x + a) - I*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2 - (
b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a)
 + 1/2*I*sin(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2 - (b^2
*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) -
1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d
^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log
(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*
c*d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x
+ a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 2*(d^2*cos(b*x + a)^2 -
d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 2*(d^2*cos(b*x + a)^2 ...
```

## 3.114.6 Sympy [F]

$$\int (c+dx)^2 \cot^2(a+bx)\csc(a+bx)\,dx = \int (c+dx)^2 \cot^2(a+bx)\csc(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*cot(b*x+a)**2*csc(b*x+a),x)
```

output
```
Integral((c + d*x)**2*cot(a + b*x)**2*csc(a + b*x), x)
```

### 3.114.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1938 vs. 2(161) = 322.

Time = 0.53 (sec) , antiderivative size = 1938, normalized size of antiderivative = 10.83

$$\int (c+dx)^2 \cot^2(a+bx)\csc(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

output
```
1/4*(c^2*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - lo
g(cos(b*x + a) - 1)) - 2*a*c*d*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(
cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b + a^2*d^2*(2*cos(b*x + a)/(co
s(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b^2 + 4
*(2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2 + ((b*x + a)^2*
d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(4*b*x + 4*a) - 2*((b*x + a)
^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(2*b*x + 2*a) - (-I*(b*x
+ a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(4*b*x + 4*a)
- 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) - 2*I*d^2)*sin(2*
b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 4*(d^2*cos(4*b*x + 4
*a) - 2*d^2*cos(2*b*x + 2*a) + I*d^2*sin(4*b*x + 4*a) - 2*I*d^2*sin(2*b*x
+ 2*a) + d^2)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 2*((b*x + a)^2*d^2
 + 2*(b*c*d - a*d^2)*(b*x + a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
 + a))*cos(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a)
)*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a
))*sin(4*b*x + 4*a) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x +
a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 4*(I*(b*x
 + a)^2*d^2 + 2*b*c*d - 2*a*d^2 + 2*(I*b*c*d + (-I*a + 1)*d^2)*(b*x + a))*
cos(3*b*x + 3*a) - 4*(I*(b*x + a)^2*d^2 - 2*b*c*d + 2*a*d^2 + 2*(I*b*c*d +
 (-I*a - 1)*d^2)*(b*x + a))*cos(b*x + a) - 4*(b*c*d + (b*x + a)*d^2 - a...
```

### 3.114.8 Giac [F]

$$\int (c+dx)^2 \cot^2(a+bx)\csc(a+bx)\,dx = \int (dx+c)^2 \cot(bx+a)^2 \csc(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*cot(b*x + a)^2*csc(b*x + a), x)
```

### 3.114.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \cot^2(a+bx)\csc(a+bx)\,dx = \text{Hanged}$$

input
```
int((cot(a + b*x)^2*(c + d*x)^2)/sin(a + b*x),x)
```

output
```
\text{Hanged}
```

# 3.115 $\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx$

3.115.1 Optimal result . . . . . 955
3.115.2 Mathematica [B] (verified) . . . . . 955
3.115.3 Rubi [A] (verified) . . . . . 956
3.115.4 Maple [B] (verified) . . . . . 959
3.115.5 Fricas [B] (verification not implemented) . . . . . 960
3.115.6 Sympy [F] . . . . . 960
3.115.7 Maxima [B] (verification not implemented) . . . . . 961
3.115.8 Giac [F] . . . . . 962
3.115.9 Mupad [F(-1)] . . . . . 962

## 3.115.1 Optimal result

Integrand size = 20, antiderivative size = 108

$$\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx = \frac{(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b} - \frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2} + \frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}$$

output
```
(d*x+c)*arctanh(exp(I*(b*x+a)))/b-1/2*d*csc(b*x+a)/b^2-1/2*(d*x+c)*cot(b*x
+a)*csc(b*x+a)/b-1/2*I*d*polylog(2,-exp(I*(b*x+a)))/b^2+1/2*I*d*polylog(2,
exp(I*(b*x+a)))/b^2
```

## 3.115.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 260 vs. 2(108) = 216.

Time = 1.94 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.41

$$\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx$$
$$= -\frac{d\cot\left(\frac{1}{2}(a+bx)\right)}{4b^2} - \frac{c\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{dx\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b}$$
$$+ \frac{c\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{2b} - \frac{c\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b} + \frac{ad\log\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)}{2b^2}$$
$$- \frac{d\left((a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)+i\left(\text{PolyLog}\left(2,-e^{i(a+bx)}\right)-\text{PolyLog}\left(2,e^{i(a+bx)}\right)\right)\right)}{2b^2}$$
$$+ \frac{c\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b} + \frac{dx\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b} - \frac{d\tan\left(\frac{1}{2}(a+bx)\right)}{4b^2}$$

input
```
Integrate[(c + d*x)*Cot[a + b*x]^2*Csc[a + b*x],x]
```

output
```
-1/4*(d*Cot[(a + b*x)/2])/b^2 - (c*Csc[(a + b*x)/2]^2)/(8*b) - (d*x*Csc[(a
 + b*x)/2]^2)/(8*b) + (c*Log[Cos[(a + b*x)/2]])/(2*b) - (c*Log[Sin[(a + b*
x)/2]])/(2*b) + (a*d*Log[Tan[(a + b*x)/2]])/(2*b^2) - (d*((a + b*x)*(Log[1
 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + I*(PolyLog[2, -E^(I*(a +
 b*x))] - PolyLog[2, E^(I*(a + b*x))])))/(2*b^2) + (c*Sec[(a + b*x)/2]^2)/
(8*b) + (d*x*Sec[(a + b*x)/2]^2)/(8*b) - (d*Tan[(a + b*x)/2])/(4*b^2)
```

## 3.115.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.63, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4915, 3042, 4671, 2715, 2838, 4673, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx$$
$$\Big\downarrow \text{4915}$$
$$\int (c+dx)\csc^3(a+bx)dx - \int (c+dx)\csc(a+bx)dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)\csc(a+bx)^3dx - \int (c+dx)\csc(a+bx)dx$$

$$\Big\downarrow \text{4671}$$

$$\int (c+dx)\csc(a+bx)^3dx+\frac{d\int \log\left(1-e^{i(a+bx)}\right)dx}{b}-\frac{d\int \log\left(1+e^{i(a+bx)}\right)dx}{b}+\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{id\int e^{-i(a+bx)}\log\left(1-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}+\frac{id\int e^{-i(a+bx)}\log\left(1+e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}+\int (c+dx)\csc(a+bx)^3dx+\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{2838}$$

$$\int (c+dx)\csc(a+bx)^3dx+\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}+\frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}$$

$$\Big\downarrow \text{4673}$$

$$\frac{1}{2}\int (c+dx)\csc(a+bx)dx+\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}+\frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{d\csc(a+bx)}{2b^2}-\frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\int (c+dx)\csc(a+bx)dx+\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}+\frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{d\csc(a+bx)}{2b^2}-\frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{4671}$$

$$\frac{1}{2}\left(-\frac{d\int \log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{d\int \log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)+\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}+\frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{d\csc(a+bx)}{2b^2}-\frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b}$$

$$\Big\downarrow \text{2715}$$

3.115. $\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx$

$$\frac{1}{2}\left(\frac{id\int e^{-i(a+bx)}\log\left(1-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{id\int e^{-i(a+bx)}\log\left(1+e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}\right.$$
$$\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}+\frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{d\csc(a+bx)}{2b^2}-\frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b}$$

$\downarrow$ 2838

$$\frac{1}{2}\left(-\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}\right)+\frac{2(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}+\frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{d\csc(a+bx)}{2b^2}-\frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b}$$

input
```
Int[(c + d*x)*Cot[a + b*x]^2*Csc[a + b*x],x]
```

output
```
(2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b - (d*Csc[a + b*x])/(2*b^2) - ((c
+ d*x)*Cot[a + b*x]*Csc[a + b*x])/(2*b) - (I*d*PolyLog[2, -E^(I*(a + b*x))
])/b^2 + (I*d*PolyLog[2, E^(I*(a + b*x))])/b^2 + ((-2*(c + d*x)*ArcTanh[E^
(I*(a + b*x))])/b + (I*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[
2, E^(I*(a + b*x))])/b^2)/2
```

### 3.115.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 4915
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

### 3.115.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(92) = 184$.

Time = 0.88 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.28

| method | result |
|---|---|
| risch | $\frac{dxb\,e^{3i(xb+a)}+cb\,e^{3i(xb+a)}+dxb\,e^{i(xb+a)}+cb\,e^{i(xb+a)}-id\,e^{3i(xb+a)}+id\,e^{i(xb+a)}}{b^2\left(e^{2i(xb+a)}-1\right)^2}+\frac{c\,\mathrm{arctanh}\left(e^{i(xb+a)}\right)}{b}-\frac{d\ln\left(1-e^{i(xb+a)}\right)x}{2b}-$ |

input
```
int((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2/(exp(2*I*(b*x+a))-1)^2*(d*x*b*exp(3*I*(b*x+a))+c*b*exp(3*I*(b*x+a))+
d*x*b*exp(I*(b*x+a))+c*b*exp(I*(b*x+a))-I*d*exp(3*I*(b*x+a))+I*d*exp(I*(b*
x+a)))+1/b*c*arctanh(exp(I*(b*x+a)))-1/2/b*d*ln(1-exp(I*(b*x+a)))*x-1/2/b^
2*d*ln(1-exp(I*(b*x+a)))*a+1/2*I*d*polylog(2,exp(I*(b*x+a)))/b^2+1/2/b*d*l
n(exp(I*(b*x+a))+1)*x+1/2/b^2*d*ln(exp(I*(b*x+a))+1)*a-1/2*I*d*polylog(2,-
exp(I*(b*x+a)))/b^2-1/b^2*d*a*arctanh(exp(I*(b*x+a)))
```

### 3.115.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 454 vs. 2(88) = 176.

Time = 0.27 (sec) , antiderivative size = 454, normalized size of antiderivative = 4.20

$$\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx$$
$$= \frac{2\,(bdx+bc)\cos{(bx+a)} + \left(i\,d\cos{(bx+a)}^2 - i\,d\right)\mathrm{Li}_2(\cos{(bx+a)} + i\,\sin{(bx+a)}) + \left(-i\,d\cos{(bx+a)}^2\right.}{}$$

input
```
integrate((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(2*(b*d*x + b*c)*cos(b*x + a) + (I*d*cos(b*x + a)^2 - I*d)*dilog(cos(b
*x + a) + I*sin(b*x + a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(cos(b*x + a)
 - I*sin(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(-cos(b*x + a) + I*si
n(b*x + a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(-cos(b*x + a) - I*sin(b*x
+ a)) - (b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a) + I*
sin(b*x + a) + 1) - (b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b
*x + a) - I*sin(b*x + a) + 1) - ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*l
og(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - ((b*c - a*d)*cos(b*x +
a)^2 - b*c + a*d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b*d
*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d)*log(-cos(b*x + a) + I*sin(b*x + a
) + 1) + (b*d*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d)*log(-cos(b*x + a) -
I*sin(b*x + a) + 1) + 2*d*sin(b*x + a))/(b^2*cos(b*x + a)^2 - b^2)
```

### 3.115.6 Sympy [F]

$$\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx = \int (c+dx)\cot^2{(a+bx)}\csc{(a+bx)}\,dx$$

input
```
integrate((d*x+c)*cot(b*x+a)**2*csc(b*x+a),x)
```

output
```
Integral((c + d*x)*cot(a + b*x)**2*csc(a + b*x), x)
```

### 3.115.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(88) = 176$.

Time = 0.35 (sec) , antiderivative size = 762, normalized size of antiderivative = 7.06

$$\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx$$
$$= \frac{2\,(bdx + bc + (bdx + bc)\cos{(4\,bx + 4\,a)} - 2\,(bdx + bc)\cos{(2\,bx + 2\,a)} - (-i\,bdx - i\,bc)\sin{(4\,bx + 4\,a)} -}{}$$

input
```
integrate((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

output
```
(2*(b*d*x + b*c + (b*d*x + b*c)*cos(4*b*x + 4*a) - 2*(b*d*x + b*c)*cos(2*b
*x + 2*a) - (-I*b*d*x - I*b*c)*sin(4*b*x + 4*a) - 2*(I*b*d*x + I*b*c)*sin(
2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*(b*c*cos(4*b*x +
 4*a) - 2*b*c*cos(2*b*x + 2*a) + I*b*c*sin(4*b*x + 4*a) - 2*I*b*c*sin(2*b*
x + 2*a) + b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 2*(b*d*x*cos(4*b
*x + 4*a) - 2*b*d*x*cos(2*b*x + 2*a) + I*b*d*x*sin(4*b*x + 4*a) - 2*I*b*d*
x*sin(2*b*x + 2*a) + b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 4*(
I*b*d*x + I*b*c + d)*cos(3*b*x + 3*a) - 4*(I*b*d*x + I*b*c - d)*cos(b*x +
a) - 2*(d*cos(4*b*x + 4*a) - 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) -
 2*I*d*sin(2*b*x + 2*a) + d)*dilog(-e^(I*b*x + I*a)) + 2*(d*cos(4*b*x + 4*
a) - 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) - 2*I*d*sin(2*b*x + 2*a)
+ d)*dilog(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(4
*b*x + 4*a) - 2*(-I*b*d*x - I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(4*
b*x + 4*a) - 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*
x + a)^2 + 2*cos(b*x + a) + 1) + (I*b*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(
4*b*x + 4*a) - 2*(I*b*d*x + I*b*c)*cos(2*b*x + 2*a) - (b*d*x + b*c)*sin(4*
b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*
x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*d*x + b*c - I*d)*sin(3*b*x + 3*a) +
4*(b*d*x + b*c + I*d)*sin(b*x + a))/(-4*I*b^2*cos(4*b*x + 4*a) + 8*I*b^2*c
os(2*b*x + 2*a) + 4*b^2*sin(4*b*x + 4*a) - 8*b^2*sin(2*b*x + 2*a) - 4*I...
```

### 3.115.8 Giac [F]

$$\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx=\int (dx+c)\cot\left(bx+a\right)^2\csc\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*cot(b*x + a)^2*csc(b*x + a), x)
```

### 3.115.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx=\text{Hanged}$$

input
```
int((cot(a + b*x)^2*(c + d*x))/sin(a + b*x),x)
```

output
```
\text{Hanged}
```

# 3.116 $\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$

3.116.1 Optimal result . . . . . 963
3.116.2 Mathematica [N/A] . . . . . 963
3.116.3 Rubi [N/A] . . . . . 964
3.116.4 Maple [N/A] (verified) . . . . . 965
3.116.5 Fricas [N/A] . . . . . 965
3.116.6 Sympy [N/A] . . . . . 966
3.116.7 Maxima [N/A] . . . . . 966
3.116.8 Giac [N/A] . . . . . 967
3.116.9 Mupad [N/A] . . . . . 968

## 3.116.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx = -\text{Int}\left(\frac{\csc(a+bx)}{c+dx},x\right) + \text{Int}\left(\frac{\csc^3(a+bx)}{c+dx},x\right)$$

output
```
-Unintegrable(csc(b*x+a)/(d*x+c),x)+Unintegrable(csc(b*x+a)^3/(d*x+c),x)
```

## 3.116.2 Mathematica [N/A]

Not integrable

Time = 40.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x),x]
```

output
```
Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x), x]
```

### 3.116.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4915, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{4915}$$

$$\int \frac{\csc^3(a+bx)}{c+dx}dx-\int \frac{\csc(a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc(a+bx)^3}{c+dx}dx-\int \frac{\csc(a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{4680}$$

$$\int \frac{\csc^3(a+bx)}{c+dx}dx-\int \frac{\csc(a+bx)}{c+dx}dx$$

input
```
Int[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x),x]
```

output
```
$Aborted
```

#### 3.116.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4915
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

## 3.116.4 Maple [N/A] (verified)

Not integrable

Time = 0.52 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cot (xb+a)^2 \csc (xb+a)}{dx+c} dx$$

input
```
int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x)
```

output
```
int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x)
```

## 3.116.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot^2(a+bx) \csc(a+bx)}{c+dx} dx = \int \frac{\cot (bx+a)^2 \csc (bx+a)}{dx+c} dx$$

input
```
integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(cot(b*x + a)^2*csc(b*x + a)/(d*x + c), x)
```

### 3.116.6 Sympy [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$$

input
```
integrate(cot(b*x+a)**2*csc(b*x+a)/(d*x+c),x)
```

output
```
Integral(cot(a + b*x)**2*csc(a + b*x)/(c + d*x), x)
```

### 3.116.7 Maxima [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 1793, normalized size of antiderivative = 81.50

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cot(bx+a)^2\csc(bx+a)}{dx+c}\,dx$$

input
```
integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
(((b*d*x + b*c)*cos(3*b*x + 3*a) + (b*d*x + b*c)*cos(b*x + a) - d*sin(3*b*
x + 3*a) + d*sin(b*x + a))*cos(4*b*x + 4*a) + (b*d*x + b*c - 2*(b*d*x + b*
c)*cos(2*b*x + 2*a) - 2*d*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) - 2*((b*d*x +
 b*c)*cos(b*x + a) + d*sin(b*x + a))*cos(2*b*x + 2*a) + (b*d*x + b*c)*cos(
b*x + a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*
a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x
+ 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2)*cos(2*b*x + 2*a))*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
 - 2*d^2)*sin(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^
2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x
+ a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(b*x
 + a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(
b*x + a)), x) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^
2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x
 + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*si...
```

### 3.116.8 Giac [N/A]

Not integrable

Time = 4.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cot(bx+a)^2\csc(bx+a)}{dx+c}\,dx$$

input
```
integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(cot(b*x + a)^2*csc(b*x + a)/(d*x + c), x)
```

### 3.116.9 Mupad [N/A]

Not integrable

Time = 24.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+b\,x)^2}{\sin(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(cot(a + b*x)^2/(sin(a + b*x)*(c + d*x)),x)
```

output
```
int(cot(a + b*x)^2/(sin(a + b*x)*(c + d*x)), x)
```

# 3.117 $\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$

3.117.1 Optimal result . . . 969
3.117.2 Mathematica [N/A] . . . 969
3.117.3 Rubi [N/A] . . . 970
3.117.4 Maple [N/A] (verified) . . . 971
3.117.5 Fricas [N/A] . . . 971
3.117.6 Sympy [N/A] . . . 972
3.117.7 Maxima [N/A] . . . 972
3.117.8 Giac [F(-2)] . . . 973
3.117.9 Mupad [N/A] . . . 974

## 3.117.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = -\text{Int}\left(\frac{\csc(a+bx)}{(c+dx)^2},x\right)+\text{Int}\left(\frac{\csc^3(a+bx)}{(c+dx)^2},x\right)$$

output
```
-Unintegrable(csc(b*x+a)/(d*x+c)^2,x)+Unintegrable(csc(b*x+a)^3/(d*x+c)^2,
x)
```

## 3.117.2 Mathematica [N/A]

Not integrable

Time = 45.77 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x)^2, x]
```

## 3.117.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4915, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \; 4915$$

$$\int \frac{\csc^3(a+bx)}{(c+dx)^2}dx-\int \frac{\csc(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\csc(a+bx)^3}{(c+dx)^2}dx-\int \frac{\csc(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \; 4680$$

$$\int \frac{\csc^3(a+bx)}{(c+dx)^2}dx-\int \frac{\csc(a+bx)}{(c+dx)^2}dx$$

input
```
Int[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

### 3.117.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4915
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

### 3.117.4 Maple [N/A] (verified)

Not integrable

Time = 0.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cot (xb+a)^2 \csc (xb+a)}{(dx+c)^2} dx$$

input
```
int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x)
```

output
```
int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x)
```

### 3.117.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot (bx+a)^2 \csc (bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(cot(b*x + a)^2*csc(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.117.6 Sympy [N/A]

Not integrable

Time = 3.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot^2{(a+bx)}\csc{(a+bx)}}{(c+dx)^2}\,dx$$

input
```
integrate(cot(b*x+a)**2*csc(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(cot(a + b*x)**2*csc(a + b*x)/(c + d*x)**2, x)
```

### 3.117.7 Maxima [N/A]

Not integrable

Time = 9.88 (sec) , antiderivative size = 2289, normalized size of antiderivative = 104.05

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot{(bx+a)}^2\csc{(bx+a)}}{(dx+c)^2}\,dx$$

input
```
integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
(((b*d*x + b*c)*cos(3*b*x + 3*a) + (b*d*x + b*c)*cos(b*x + a) - 2*d*sin(3*
b*x + 3*a) + 2*d*sin(b*x + a))*cos(4*b*x + 4*a) + (b*d*x + b*c - 2*(b*d*x
+ b*c)*cos(2*b*x + 2*a) - 4*d*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) - 2*((b*d
*x + b*c)*cos(b*x + a) + 2*d*sin(b*x + a))*cos(2*b*x + 2*a) + (b*d*x + b*c
)*cos(b*x + a) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3
+ (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*
a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b
*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*si
n(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*
c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
+ 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d
^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^
2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^3*x^3 +
 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*integrate(1/
2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 6*d^2)*sin(b*x + a)/(b^2*d^4*x^4
+ 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4
*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(
b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^
3*d*x + b^2*c^4)*sin(b*x + a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2
*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x) - (b^2*d^3*x^...
```

### 3.117.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

### 3.117.9 Mupad [N/A]

Not integrable

Time = 24.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(a+b\,x)^2}{\sin\left(a+b\,x\right)\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(cot(a + b*x)^2/(sin(a + b*x)*(c + d*x)^2),x)
```

output
```
int(cot(a + b*x)^2/(sin(a + b*x)*(c + d*x)^2), x)
```

# 3.118 $\int (c+dx)^{5/2} \cos^2(a+bx) \sin(a+bx) \, dx$


3.118.1 Optimal result . . . . . 975
3.118.2 Mathematica [C] (verified) . . . . . 976
3.118.3 Rubi [A] (verified) . . . . . 977
3.118.4 Maple [A] (verified) . . . . . 979
3.118.5 Fricas [A] (verification not implemented) . . . . . 980
3.118.6 Sympy [F(-1)] . . . . . 980
3.118.7 Maxima [C] (verification not implemented) . . . . . 981
3.118.8 Giac [C] (verification not implemented) . . . . . 981
3.118.9 Mupad [F(-1)] . . . . . 982


## 3.118.1 Optimal result

Integrand size = 24, antiderivative size = 406

$$
\begin{aligned}
\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx &= \frac{15d^2\sqrt{c+dx}\cos(a+bx)}{16b^3} \\
&- \frac{(c+dx)^{5/2}\cos(a+bx)}{4b} + \frac{5d^2\sqrt{c+dx}\cos(3a+3bx)}{144b^3} \\
&- \frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b} - \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} \\
&- \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}} \\
&+ \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{144b^{7/2}} \\
&+ \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{16b^{7/2}} \\
&+ \frac{5d(c+dx)^{3/2}\sin(a+bx)}{8b^2} + \frac{5d(c+dx)^{3/2}\sin(3a+3bx)}{72b^2}
\end{aligned}
$$

output
```
-1/4*(d*x+c)^(5/2)*cos(b*x+a)/b-1/12*(d*x+c)^(5/2)*cos(3*b*x+3*a)/b+5/8*d*
(d*x+c)^(3/2)*sin(b*x+a)/b^2+5/72*d*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b^2-5/864
*d^(5/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/
d^(1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)+5/864*d^(5/2)*FresnelS(b^(1/2)*6^(1/2)/P
i^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(7/2)-1
5/32*d^(5/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/
d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/32*d^(5/2)*FresnelS(b^(1/2)*2^(1/2)/P
i^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/16
*d^2*cos(b*x+a)*(d*x+c)^(1/2)/b^3+5/144*d^2*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b
^3
```

### 3.118.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.76 (sec) , antiderivative size = 1445, normalized size of antiderivative = 3.56

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
(c*Sqrt[d]*(-12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(-I + 2*b*
x + E^((6*I)*(a + b*x))*(I + 2*b*x)) - (1 - I)*(2*b*c + I*d)*E^(((3*I)*b*(
2*c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sq
rt[d]] + (1 + I)*((2*I)*b*c + d)*E^((3*I)*(2*a + b*x))*Sqrt[6*Pi]*Erfi[((1
 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(288*b^(5/2)*E^(((3*I)*(
a*d + b*(c + d*x)))/d)) + ((I/8)*c^2*d*(-(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*
x))/d]*Gamma[3/2, ((-I)*b*(c + d*x))/d]) + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c
+ d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d]))/(b^2*E^((I*(b*c + a*d))/d)*Sqrt
[c + d*x]) + (c^2*(-1/6*(E^((3*I)*(a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2,
((-3*I)*b*(c + d*x))/d])/(Sqrt[3]*b*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c
+ d*x]*Gamma[3/2, ((3*I)*b*(c + d*x))/d])/(6*Sqrt[3]*b*E^((3*I)*(a - (b*c)
/d))*Sqrt[(I*b*(c + d*x))/d])))/4 + (c*Sqrt[d]*(E^(I*(a - (b*c)/d))*(-2*Sq
rt[b]*Sqrt[d]*E^((I*b*(c + d*x))/d)*(3*I + 2*b*x)*Sqrt[c + d*x] + (-1)^(1/
4)*((2*I)*b*c + 3*d)*Sqrt[Pi]*Erfi[((-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]]) + I*(2*Sqrt[b]*Sqrt[d]*(3 + (2*I)*b*x)*Sqrt[c + d*x] + (1 + I)*(2*b*
c + (3*I)*d)*Sqrt[Pi/2]*Erf[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[2]*Sqrt[
d])]*(Cos[b*(c/d + x)] + I*Sin[b*(c/d + x)]))*(Cos[a + b*x] - I*Sin[a + b*
x])))/(16*b^(5/2)) - ((I/64)*Sqrt[d]*((Cos[a - (b*c)/d] + I*Sin[a - (b*c)/
d])*((1 + I)*(4*b^2*c^2 - (12*I)*b*c*d - 15*d^2)*Sqrt[Pi/2]*Erfi[((1 + I)*
Sqrt[b]*Sqrt[c + d*x])/(Sqrt[2]*Sqrt[d])] + 2*Sqrt[b]*Sqrt[d]*Sqrt[c + ...
```

### 3.118.3 Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \; 4906$$

$$\int \left(\frac{1}{4}(c+dx)^{5/2}\sin(a+bx)+\frac{1}{4}(c+dx)^{5/2}\sin(3a+3bx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(a+bx)}{16b^3}+\frac{5d^2\sqrt{c+dx}\cos(3a+3bx)}{144b^3}+\frac{5d(c+dx)^{3/2}\sin(a+bx)}{8b^2}+\frac{5d(c+dx)^{3/2}\sin(3a+3bx)}{72b^2}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
(15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^3) - ((c + d*x)^(5/2)*Cos[a + b*
x])/(4*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(144*b^3) - ((c + d*x)^
(5/2)*Cos[3*a + 3*b*x])/(12*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*F
resnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(
5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(144*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*S
qrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (1
5*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*
Sin[a - (b*c)/d])/(16*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[a + b*x])/(8*b^2
) + (5*d*(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(72*b^2)
```

### 3.118.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.118.4 Maple [A] (verified)

Time = 1.44 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.17

| method | result |
|---|---|
| derivativedivides | $-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,(\cdots}{\cdots}\right.}{\cdots}\right.}{4b}$ |
| default | $-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,(\cdots}{\cdots}\right.}{\cdots}\right.}{4b}$ |

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+5/8/b*d*(1/2/b*d*
(d*x+c)^(3/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(1/2)
*cos(b/d*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a
*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin((a
*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))-1/2
4/b*d*(d*x+c)^(5/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+5/24/b*d*(1/6/b*d*(d*
x+c)^(3/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1/2
)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)
^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b
*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/
d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

### 3.118.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.84

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{5\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+405\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(
sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 405*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 40
5*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(
pi*d)))*sin(-(b*c - a*d)/d) - 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(
sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(30*b*d^2
*cos(b*x + a) - (12*b^3*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos
(b*x + a)^3 + 10*(2*b^2*d^2*x + 2*b^2*c*d + (b^2*d^2*x + b^2*c*d)*cos(b*x
+ a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^4
```

### 3.118.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

output
```
Timed out
```

### 3.118.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.35

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \frac{\left(240\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)+2160\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)-24\left(\frac{12\,(dx+c)^{\frac{5}{2}}b^4}{d}-\right.\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/3456*(240*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d) + 2160*
(d*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d) - 24*(12*(d*x + c)^(5
/2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*cos(3*((d*x + c)*b - b*c + a*d)/d) - 21
6*(4*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(((d*x + c)*b - b*
c + a*d)/d) - 5*(-(I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*c
os(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1
/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 405*(-(I - 1
)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqr
t(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)
*sqrt(I*b/d)) - 405*((I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(
b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c -
 a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - 5*((I + 1)*9^(1/4)*sqrt(2)*sqr
t(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2
)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*
sqrt(-3*I*b/d)))*d/b^5
```

### 3.118.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 2468, normalized size of antiderivative = 6.08

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
1/1728*(72*(3*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d
*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(
I*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*s
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d
)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2*(9*(sqrt(2)*sq
rt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*
x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)
/b^2)/d^2 + (sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*I*
sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c
- I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 6*I*(2*I*(d*x + c)
^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^(-3*(-I*(d*x +
 c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 9*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*
b*c*d - 3*d^2)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +...
```

### 3.118.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx=\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\left(c+d\,x\right)^{5/2}\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(5/2), x)
```

# 3.119 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx$

3.119.1 Optimal result . . . 983
3.119.2 Mathematica [C] (verified) . . . 984
3.119.3 Rubi [A] (verified) . . . 985
3.119.4 Maple [A] (verified) . . . 987
3.119.5 Fricas [A] (verification not implemented) . . . 987
3.119.6 Sympy [F] . . . 988
3.119.7 Maxima [C] (verification not implemented) . . . 988
3.119.8 Giac [C] (verification not implemented) . . . 989
3.119.9 Mupad [F(-1)] . . . 990

## 3.119.1 Optimal result

Integrand size = 24, antiderivative size = 353

$$\begin{aligned}\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx &= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b} \\
&- \frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} \\
&- \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}} \\
&- \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{24b^{5/2}} \\
&- \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{8b^{5/2}} \\
&+ \frac{3d\sqrt{c+dx}\sin(a+bx)}{8b^2} + \frac{d\sqrt{c+dx}\sin(3a+3bx)}{24b^2}\end{aligned}$$

output
```
-1/4*(d*x+c)^(3/2)*cos(b*x+a)/b-1/12*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b-1/144*
d^(3/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d
^(1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1/144*d^(3/2)*FresnelC(b^(1/2)*6^(1/2)/Pi
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/
16*d^(3/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)-3/16*d^(3/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(
1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/8*d*si
n(b*x+a)*(d*x+c)^(1/2)/b^2+1/24*d*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b^2
```

## 3.119.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.75 (sec) , antiderivative size = 751, normalized size of antiderivative = 2.13

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = \frac{\sqrt{d}e^{-\frac{3i(ad+b(c+dx))}{d}}\left(-12\sqrt{b}\sqrt{d}e^{\frac{3ibc}{d}}\sqrt{c+dx}\left(-i+2bx+e^{6i(a+bx)}(i+2bx)\right)-(1-i)(2bc+id)e^{\frac{3ib(2}{}}\right.}{576b^{5/2}}$$
$$+\frac{icde^{-\frac{i(bc+ad)}{d}}\left(-e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)\right)}{8b^2\sqrt{c+dx}}$$
$$+\frac{1}{4}c\left(-\frac{e^{3i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\Gamma\left(\frac{3}{2},-\frac{3ib(c+dx)}{d}\right)}{6\sqrt{3}b\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{-3i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\Gamma\left(\frac{3}{2},\frac{3ib(c+dx)}{d}\right)}{6\sqrt{3}b\sqrt{\frac{ib(c+dx)}{d}}}\right)$$
$$+\frac{\sqrt{d}\left(e^{i\left(a-\frac{bc}{d}\right)}\left(-2\sqrt{b}\sqrt{d}e^{\frac{ib(c+dx)}{d}}(3i+2bx)\sqrt{c+dx}+\sqrt[4]{-1}(2ibc+3d)\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)\right)+i\left(2\sqrt{b}\sqrt{}\right.\right.}{}$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
(Sqrt[d]*(-12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(-I + 2*b*x
+ E^((6*I)*(a + b*x))*(I + 2*b*x)) - (1 - I)*(2*b*c + I*d)*E^(((3*I)*b*(2*
c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]] + (1 + I)*((2*I)*b*c + d)*E^((3*I)*(2*a + b*x))*Sqrt[6*Pi]*Erfi[((1 +
 I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(576*b^(5/2)*E^(((3*I)*(a*
d + b*(c + d*x)))/d)) + ((I/8)*c*d*(-(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/
d]*Gamma[3/2, ((-I)*b*(c + d*x))/d]) + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*
x))/d]*Gamma[3/2, (I*b*(c + d*x))/d]))/(b^2*E^((I*(b*c + a*d))/d)*Sqrt[c +
 d*x]) + (c*(-1/6*(E^((3*I)*(a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-3*I
)*b*(c + d*x))/d])/(Sqrt[3]*b*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]
*Gamma[3/2, ((3*I)*b*(c + d*x))/d])/(6*Sqrt[3]*b*E^((3*I)*(a - (b*c)/d))*S
qrt[(I*b*(c + d*x))/d])))/4 + (Sqrt[d]*(E^(I*(a - (b*c)/d))*(-2*Sqrt[b]*Sq
rt[d]*E^((I*b*(c + d*x))/d)*(3*I + 2*b*x)*Sqrt[c + d*x] + (-1)^(1/4)*((2*I
)*b*c + 3*d)*Sqrt[Pi]*Erfi[((-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]) +
I*(2*Sqrt[b]*Sqrt[d]*(3 + (2*I)*b*x)*Sqrt[c + d*x] + (1 + I)*(2*b*c + (3*I
)*d)*Sqrt[Pi/2]*Erf[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[2]*Sqrt[d])]*(Co
s[b*(c/d + x)] + I*Sin[b*(c/d + x)]))*(Cos[a + b*x] - I*Sin[a + b*x])))/(3
2*b^(5/2))
```

### 3.119.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{4}(c+dx)^{3/2}\sin(a+bx)+\frac{1}{4}(c+dx)^{3/2}\sin(3a+3bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(a+bx)}{8b^2}+\frac{d\sqrt{c+dx}\sin(3a+3bx)}{24b^2}-\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/4*((c + d*x)^(3/2)*Cos[a + b*x])/b - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])
/(12*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (
3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/2
)) - (d^(3/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[
d]]*Sin[3*a - (3*b*c)/d])/(24*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(S
qrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) +
(3*d*Sqrt[c + d*x]*Sin[a + b*x])/(8*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*
x])/(24*b^2)
```

### 3.119.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.119.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.09

| method | result |
|---|---|
| derivativedivides | $-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\cdots\right)\right)}{4b\sqrt{\frac{b}{d}}}\cdots\right)}{4b}\cdots$ |
| default | $-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\cdots\right)\right)}{4b\sqrt{\frac{b}{d}}}\cdots\right)}{4b}\cdots$ |

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+3/8/b*d*(1/2/b*d*
(d*x+c)^(1/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^
(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)))-1/24/b*d*(d*x+c)^(3/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/8/b*d*(
1/6/b*d*(d*x+c)^(1/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi
^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3
^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

### 3.119.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.79

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+27\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+2\cdots}{\cdots}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sq
rt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos
(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sq
rt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-(b*c - a*d)/d) + sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6
)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(2*(b^2*d*x + b
^2*c)*cos(b*x + a)^3 - (b*d*cos(b*x + a)^2 + 2*b*d)*sin(b*x + a))*sqrt(d*x
 + c))/b^3
```

### 3.119.6 Sympy [F]

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx=\int(c+dx)^{\frac{3}{2}}\sin\left(a+bx\right)\cos^2\left(a+bx\right)dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)*cos(a + b*x)**2, x)
```

### 3.119.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.41

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx=$$
$$-\frac{\left(\frac{48\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{144\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}-24\sqrt{dx+c}b^2\sin\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)-216\,\sqrt{\cdots}\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/576*(48*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d)/d + 144*
(d*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/d - 24*sqrt(d*x + c)*
b^2*sin(3*((d*x + c)*b - b*c + a*d)/d) - 216*sqrt(d*x + c)*b^2*sin(((d*x +
 c)*b - b*c + a*d)/d) - (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(
1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2
)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + 27*((I +
 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqr
t(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(I*b/d)) + 27*(-(I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c
- a*d)/d) + (I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/
d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*
d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)
*b*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/
d)))*d/b^4
```

### 3.119.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 1540, normalized size of antiderivative = 4.36

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
1/288*(12*(3*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt
(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*sq
rt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*(9*(sqrt(2)*sqrt(pi)
*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)*b^2) + 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)
*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/
d^2 + (sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*I*sqrt(6
)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 6*I*(2*I*(d*x + c)^(3/2)
*b*d - 4*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^(-3*(-I*(d*x + c)*b
+ I*b*c - I*a*d)/d)/b^2)/d^2 + 9*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d
- 3*d^2)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
 + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^...
```

### 3.119.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2} \cos^2(a+bx) \sin(a+bx) \, dx = \int \cos\left(a+b\,x\right)^2 \,\sin\left(a+b\,x\right)\,\left(c+d\,x\right)^{3/2} \,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(3/2), x)
```

# 3.120 $\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx$

3.120.1 Optimal result . . . . . 991
3.120.2 Mathematica [C] (verified) . . . . . 992
3.120.3 Rubi [A] (verified) . . . . . 992
3.120.4 Maple [A] (verified) . . . . . 994
3.120.5 Fricas [A] (verification not implemented) . . . . . 994
3.120.6 Sympy [F] . . . . . 995
3.120.7 Maxima [C] (verification not implemented) . . . . . 995
3.120.8 Giac [C] (verification not implemented) . . . . . 996
3.120.9 Mupad [F(-1)] . . . . . 997

## 3.120.1 Optimal result

Integrand size = 24, antiderivative size = 304

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{\sqrt{c+dx}\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{12b^{3/2}}-\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{4b^{3/2}}$$

output
```
1/72*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1/72*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2
)+1/8*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2)
)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/4*co
s(b*x+a)*(d*x+c)^(1/2)/b-1/12*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b
```

### 3.120.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.24 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.83

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{e^{-\frac{3i(bc+ad)}{d}}\sqrt{c+dx}\left(9e^{2i\left(2a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+9e^{2ia+\frac{4ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)+\sqrt{3}\left(e^{6\cdots}\right.\right.}{72b\sqrt{\frac{b^2(c+dx)^2}{d^2}}}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/72*(Sqrt[c + d*x]*(9*E^((2*I)*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*
Gamma[3/2, ((-I)*b*(c + d*x))/d] + 9*E^((2*I)*a + ((4*I)*b*c)/d)*Sqrt[((-I
)*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d] + Sqrt[3]*(E^((6*I)*a)*Sqr
t[(I*b*(c + d*x))/d]*Gamma[3/2, ((-3*I)*b*(c + d*x))/d] + E^(((6*I)*b*c)/d
)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((3*I)*b*(c + d*x))/d])))/(b*E^(((
3*I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

### 3.120.3 Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin(a+bx)\cos^2(a+bx)\,dx$$
$$\Big\downarrow \text{4906}$$
$$\int \left(\frac{1}{4}\sqrt{c+dx}\sin(a+bx)+\frac{1}{4}\sqrt{c+dx}\sin(3a+3bx)\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{c+dx}\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/4*(Sqrt[c + d*x]*Cos[a + b*x])/b - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(12
*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]])/(4*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)
/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) - (
Sqrt[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Si
n[3*a - (3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*S
qrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2))
```

### 3.120.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.120.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| derivativedivides | $-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{}}{}$ |
| default | $-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{}}{}$ |

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+1/16/b*d*2^(1/2)*
Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d))-1/24/b*d*(d*x+c)^(1/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c
)/d)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fr
esnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-
b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

### 3.120.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.77

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx$$
$$=\frac{\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+9\,\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\,\sqrt{}}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6
)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c
- a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi
*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(
b*c - a*d)/d) - sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x +
 c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*sqrt(d*x + c)*b*cos(b*x + a
)^3)/b^2
```

### 3.120.6 Sympy [F]

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx = \int \sqrt{c+dx}\sin(a+bx)\cos^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)*cos(a + b*x)**2, x)
```

### 3.120.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.39

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{\left(\frac{24\sqrt{dx+c}b^2\cos\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{72\sqrt{dx+c}b^2\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+\left((i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/288*(24*sqrt(d*x + c)*b^2*cos(3*((d*x + c)*b - b*c + a*d)/d)/d + 72*sqr
t(d*x + c)*b^2*cos(((d*x + c)*b - b*c + a*d)/d)/d + ((I - 1)*9^(1/4)*sqrt(
2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt
(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(3*I*b/d)) - 9*(-(I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c -
a*d)/d) - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*
erf(sqrt(d*x + c)*sqrt(I*b/d)) - 9*((I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(
1/4)*cos(-(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(
-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(
2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt
(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(-3*I*b/d)))*d/b^3
```

### 3.120.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 844, normalized size of antiderivative = 2.78

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/144*(9*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqr
t(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*
(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*
I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*
c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)
*(2*b*c - I*d)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b) + sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*
(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*(3*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*I*s
qrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c -
 I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(2)*sqrt(pi)*d*er
f(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi
)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c + 18
*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 6*sqrt(d*x + c)
*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((...
```

## 3.120.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx=\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(1/2), x)
```

# 3.121 $\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx$

3.121.1 Optimal result . . . . . . . . . . 998
3.121.2 Mathematica [C] (verified) . . . . . . . . . . 999
3.121.3 Rubi [A] (verified) . . . . . . . . . . 999
3.121.4 Maple [A] (verified) . . . . . . . . . . 1001
3.121.5 Fricas [A] (verification not implemented) . . . . . . . . . . 1001
3.121.6 Sympy [F] . . . . . . . . . . 1002
3.121.7 Maxima [C] (verification not implemented) . . . . . . . . . . 1002
3.121.8 Giac [C] (verification not implemented) . . . . . . . . . . 1003
3.121.9 Mupad [F(-1)] . . . . . . . . . . 1004

## 3.121.1 Optimal result

Integrand size = 24, antiderivative size = 304

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{\sqrt{c+dx}\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{12b^{3/2}}-\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{4b^{3/2}}$$

output
```
1/72*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1/72*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2
)+1/8*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2)
)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/4*co
s(b*x+a)*(d*x+c)^(1/2)/b-1/12*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b
```

### 3.121.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.83

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{e^{-\frac{3i(bc+ad)}{d}}\sqrt{c+dx}\left(9e^{2i\left(2a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+9e^{2ia+\frac{4ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)+\sqrt{3}\left(e^{6}\right.\right.}{72b\sqrt{\frac{b^2(c+dx)^2}{d^2}}}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/72*(Sqrt[c + d*x]*(9*E^((2*I)*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*
Gamma[3/2, ((-I)*b*(c + d*x))/d] + 9*E^((2*I)*a + ((4*I)*b*c)/d)*Sqrt[((-I
)*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d] + Sqrt[3]*(E^((6*I)*a)*Sqr
t[(I*b*(c + d*x))/d]*Gamma[3/2, ((-3*I)*b*(c + d*x))/d] + E^(((6*I)*b*c)/d
)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((3*I)*b*(c + d*x))/d])))/(b*E^(((
3*I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

### 3.121.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin(a+bx)\cos^2(a+bx)\,dx$$
$$\Big\downarrow \text{4906}$$
$$\int \left(\frac{1}{4}\sqrt{c+dx}\sin(a+bx)+\frac{1}{4}\sqrt{c+dx}\sin(3a+3bx)\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{c+dx}\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/4*(Sqrt[c + d*x]*Cos[a + b*x])/b - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(12
*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]])/(4*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)
/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) - (
Sqrt[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Si
n[3*a - (3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*S
qrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2))
```

### 3.121.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.121.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| derivativedivides | $-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{}}{}$ |
| default | $-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-cb}{d}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{}}{}$ |

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+1/16/b*d*2^(1/2)*
Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d))-1/24/b*d*(d*x+c)^(1/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c
)/d)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fr
esnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-
b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

### 3.121.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.77

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx$$

$$=\frac{\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+9\,\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\,\sqrt{}}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6
)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c
- a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi
*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(
b*c - a*d)/d) - sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x +
 c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*sqrt(d*x + c)*b*cos(b*x + a
)^3)/b^2
```

### 3.121.6 Sympy [F]

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx = \int \sqrt{c+dx}\sin(a+bx)\cos^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)*cos(a + b*x)**2, x)
```

### 3.121.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 424, normalized size of antiderivative = 1.39

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{\left(\frac{24\sqrt{dx+c}b^2\cos\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d} + \frac{72\sqrt{dx+c}b^2\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d} + \left((i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/288*(24*sqrt(d*x + c)*b^2*cos(3*((d*x + c)*b - b*c + a*d)/d)/d + 72*sqr
t(d*x + c)*b^2*cos(((d*x + c)*b - b*c + a*d)/d)/d + ((I - 1)*9^(1/4)*sqrt(
2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt
(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(3*I*b/d)) - 9*(-(I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c -
a*d)/d) - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*
erf(sqrt(d*x + c)*sqrt(I*b/d)) - 9*((I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(
1/4)*cos(-(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(
-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(
2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt
(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(-3*I*b/d)))*d/b^3
```

### 3.121.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 844, normalized size of antiderivative = 2.78

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/144*(9*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqr
t(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*
(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*
I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*
c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(2)*sqrt(pi)
*(2*b*c - I*d)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b) + sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*
(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*(3*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*I*s
qrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c -
 I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(2)*sqrt(pi)*d*er
f(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi
)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c + 18
*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 6*sqrt(d*x + c)
*d*e^(-3*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((...
```

### 3.121.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx=\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(1/2), x)
```

# 3.122 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx$

3.122.1 Optimal result . . . 1005
3.122.2 Mathematica [C] (verified) . . . 1006
3.122.3 Rubi [A] (verified) . . . 1007
3.122.4 Maple [A] (verified) . . . 1009
3.122.5 Fricas [A] (verification not implemented) . . . 1009
3.122.6 Sympy [F] . . . 1010
3.122.7 Maxima [C] (verification not implemented) . . . 1010
3.122.8 Giac [C] (verification not implemented) . . . 1011
3.122.9 Mupad [F(-1)] . . . 1012

## 3.122.1 Optimal result

Integrand size = 24, antiderivative size = 353

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx=-\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{d^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{d^{3/2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{24b^{5/2}}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{8b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(a+bx)}{8b^2}+\frac{d\sqrt{c+dx}\sin(3a+3bx)}{24b^2}$$

output
```
-1/4*(d*x+c)^(3/2)*cos(b*x+a)/b-1/12*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b-1/144*
d^(3/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d
^(1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1/144*d^(3/2)*FresnelC(b^(1/2)*6^(1/2)/Pi
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/
16*d^(3/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)-3/16*d^(3/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(
1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/8*d*si
n(b*x+a)*(d*x+c)^(1/2)/b^2+1/24*d*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b^2
```

### 3.122.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.35 (sec) , antiderivative size = 751, normalized size of antiderivative = 2.13

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = \frac{\sqrt{d}e^{-\frac{3i(ad+b(c+dx))}{d}}\left(-12\sqrt{b}\sqrt{d}e^{\frac{3ibc}{d}}\sqrt{c+dx}\left(-i+2bx+e^{6i(a+bx)}(i+2bx)\right)-(1-i)(2bc+id)e^{\frac{3ib(2}{}}\cdots\right.}{576b^{5/2}}$$
$$+\frac{icde^{-\frac{i(bc+ad)}{d}}\left(-e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)\right)}{8b^2\sqrt{c+dx}}$$
$$+\frac{1}{4}c\left(-\frac{e^{3i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\,\Gamma\left(\frac{3}{2},-\frac{3ib(c+dx)}{d}\right)}{6\sqrt{3}b\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{-3i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\,\Gamma\left(\frac{3}{2},\frac{3ib(c+dx)}{d}\right)}{6\sqrt{3}b\sqrt{\frac{ib(c+dx)}{d}}}\right)$$
$$+\frac{\sqrt{d}\left(e^{i\left(a-\frac{bc}{d}\right)}\left(-2\sqrt{b}\sqrt{d}e^{\frac{ib(c+dx)}{d}}(3i+2bx)\sqrt{c+dx}+\sqrt[4]{-1}(2ibc+3d)\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)\right)+i\left(2\sqrt{b}\sqrt{\cdots}\right.\right.}{}$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
(Sqrt[d]*(-12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(-I + 2*b*x
+ E^((6*I)*(a + b*x))*(I + 2*b*x)) - (1 - I)*(2*b*c + I*d)*E^(((3*I)*b*(2*
c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]] + (1 + I)*((2*I)*b*c + d)*E^((3*I)*(2*a + b*x))*Sqrt[6*Pi]*Erfi[((1 +
 I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(576*b^(5/2)*E^(((3*I)*(a*
d + b*(c + d*x)))/d)) + ((I/8)*c*d*(-(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/
d]*Gamma[3/2, ((-I)*b*(c + d*x))/d]) + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*
x))/d]*Gamma[3/2, (I*b*(c + d*x))/d]))/(b^2*E^((I*(b*c + a*d))/d)*Sqrt[c +
 d*x]) + (c*(-1/6*(E^((3*I)*(a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-3*I
)*b*(c + d*x))/d])/(Sqrt[3]*b*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]
*Gamma[3/2, ((3*I)*b*(c + d*x))/d])/(6*Sqrt[3]*b*E^((3*I)*(a - (b*c)/d))*S
qrt[(I*b*(c + d*x))/d])))/4 + (Sqrt[d]*(E^(I*(a - (b*c)/d))*(-2*Sqrt[b]*Sq
rt[d]*E^((I*b*(c + d*x))/d)*(3*I + 2*b*x)*Sqrt[c + d*x] + (-1)^(1/4)*((2*I
)*b*c + 3*d)*Sqrt[Pi]*Erfi[((-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]) +
I*(2*Sqrt[b]*Sqrt[d]*(3 + (2*I)*b*x)*Sqrt[c + d*x] + (1 + I)*(2*b*c + (3*I
)*d)*Sqrt[Pi/2]*Erf[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[2]*Sqrt[d])]*(Co
s[b*(c/d + x)] + I*Sin[b*(c/d + x)]))*(Cos[a + b*x] - I*Sin[a + b*x])))/(3
2*b^(5/2))
```

## 3.122.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{4}(c+dx)^{3/2}\sin(a+bx)+\frac{1}{4}(c+dx)^{3/2}\sin(3a+3bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(a+bx)}{8b^2}+\frac{d\sqrt{c+dx}\sin(3a+3bx)}{24b^2}-\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
-1/4*((c + d*x)^(3/2)*Cos[a + b*x])/b - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])
/(12*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (
3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/2
)) - (d^(3/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[
d]]*Sin[3*a - (3*b*c)/d])/(24*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(S
qrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) +
(3*d*Sqrt[c + d*x]*Sin[a + b*x])/(8*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*
x])/(24*b^2)
```

### 3.122.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.122.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.09

| method | result |
|---|---|
| derivativedivides | $-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\cdots\right.\right.}{4b\sqrt{\frac{b}{d}}}\cdots\right.}{4b}\cdots$ |
| default | $-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\cdots\right.\right.}{4b\sqrt{\frac{b}{d}}}\cdots\right.}{4b}\cdots$ |

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+3/8/b*d*(1/2/b*d*
(d*x+c)^(1/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^
(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)))-1/24/b*d*(d*x+c)^(3/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/8/b*d*(
1/6/b*d*(d*x+c)^(1/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi
^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3
^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

### 3.122.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.79

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+27\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+2\cdots}{\cdots}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sq
rt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos
(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sq
rt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-(b*c - a*d)/d) + sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6
)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(2*(b^2*d*x + b
^2*c)*cos(b*x + a)^3 - (b*d*cos(b*x + a)^2 + 2*b*d)*sin(b*x + a))*sqrt(d*x
 + c))/b^3
```

## 3.122.6 Sympy [F]

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin{(a+bx)}\cos^2{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)*cos(a + b*x)**2, x)
```

## 3.122.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.41

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{\left(\frac{48\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)}{d} + \frac{144\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d} - 24\sqrt{dx+c}b^2\sin\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right) - 216 \cdots\right.}{\cdots}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/576*(48*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d)/d + 144*
(d*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/d - 24*sqrt(d*x + c)*
b^2*sin(3*((d*x + c)*b - b*c + a*d)/d) - 216*sqrt(d*x + c)*b^2*sin(((d*x +
 c)*b - b*c + a*d)/d) - (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(
1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2
)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + 27*((I +
 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqr
t(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(I*b/d)) + 27*(-(I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c
- a*d)/d) + (I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/
d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*
d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)
*b*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/
d)))*d/b^4
```

### 3.122.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 1540, normalized size of antiderivative = 4.36

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
1/288*(12*(3*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt
(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*sq
rt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*(9*(sqrt(2)*sqrt(pi)
*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)*b^2) + 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)
*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/
d^2 + (sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*I*sqrt(6
)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 6*I*(2*I*(d*x + c)^(3/2)
*b*d - 4*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^(-3*(-I*(d*x + c)*b
+ I*b*c - I*a*d)/d)/b^2)/d^2 + 9*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d
- 3*d^2)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
 + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^...
```

### 3.122.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx=\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\left(c+d\,x\right)^{3/2}\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(3/2), x)
```

# 3.123 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx$

3.123.1 Optimal result . . . 1013
3.123.2 Mathematica [C] (verified) . . . 1014
3.123.3 Rubi [A] (verified) . . . 1015
3.123.4 Maple [A] (verified) . . . 1017
3.123.5 Fricas [A] (verification not implemented) . . . 1018
3.123.6 Sympy [F(-1)] . . . 1018
3.123.7 Maxima [C] (verification not implemented) . . . 1019
3.123.8 Giac [C] (verification not implemented) . . . 1019
3.123.9 Mupad [F(-1)] . . . 1020

## 3.123.1 Optimal result

Integrand size = 24, antiderivative size = 406

$$\begin{aligned}\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx &= \frac{15d^2\sqrt{c+dx}\cos(a+bx)}{16b^3} \\ &- \frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\cos(3a+3bx)}{144b^3} \\ &- \frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b}-\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} \\ &- \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}} \\ &+ \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{144b^{7/2}} \\ &+ \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{16b^{7/2}} \\ &+ \frac{5d(c+dx)^{3/2}\sin(a+bx)}{8b^2}+\frac{5d(c+dx)^{3/2}\sin(3a+3bx)}{72b^2}\end{aligned}$$

output
```
-1/4*(d*x+c)^(5/2)*cos(b*x+a)/b-1/12*(d*x+c)^(5/2)*cos(3*b*x+3*a)/b+5/8*d*
(d*x+c)^(3/2)*sin(b*x+a)/b^2+5/72*d*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b^2-5/864
*d^(5/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/
d^(1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)+5/864*d^(5/2)*FresnelS(b^(1/2)*6^(1/2)/P
i^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(7/2)-1
5/32*d^(5/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/
d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/32*d^(5/2)*FresnelS(b^(1/2)*2^(1/2)/P
i^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/16
*d^2*cos(b*x+a)*(d*x+c)^(1/2)/b^3+5/144*d^2*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b
^3
```

### 3.123.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.01 (sec) , antiderivative size = 1445, normalized size of antiderivative = 3.56

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
(c*Sqrt[d]*(-12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(-I + 2*b*
x + E^((6*I)*(a + b*x))*(I + 2*b*x)) - (1 - I)*(2*b*c + I*d)*E^(((3*I)*b*(
2*c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sq
rt[d]] + (1 + I)*((2*I)*b*c + d)*E^((3*I)*(2*a + b*x))*Sqrt[6*Pi]*Erfi[((1
 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(288*b^(5/2)*E^(((3*I)*(
a*d + b*(c + d*x)))/d)) + ((I/8)*c^2*d*(-(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*
x))/d]*Gamma[3/2, ((-I)*b*(c + d*x))/d]) + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c
+ d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d]))/(b^2*E^((I*(b*c + a*d))/d)*Sqrt
[c + d*x]) + (c^2*(-1/6*(E^((3*I)*(a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2,
((-3*I)*b*(c + d*x))/d])/(Sqrt[3]*b*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c
+ d*x]*Gamma[3/2, ((3*I)*b*(c + d*x))/d])/(6*Sqrt[3]*b*E^((3*I)*(a - (b*c)
/d))*Sqrt[(I*b*(c + d*x))/d])))/4 + (c*Sqrt[d]*(E^(I*(a - (b*c)/d))*(-2*Sq
rt[b]*Sqrt[d]*E^((I*b*(c + d*x))/d)*(3*I + 2*b*x)*Sqrt[c + d*x] + (-1)^(1/
4)*((2*I)*b*c + 3*d)*Sqrt[Pi]*Erfi[((-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]]) + I*(2*Sqrt[b]*Sqrt[d]*(3 + (2*I)*b*x)*Sqrt[c + d*x] + (1 + I)*(2*b*
c + (3*I)*d)*Sqrt[Pi/2]*Erf[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[2]*Sqrt[
d])]*(Cos[b*(c/d + x)] + I*Sin[b*(c/d + x)]))*(Cos[a + b*x] - I*Sin[a + b*
x])))/(16*b^(5/2)) - ((I/64)*Sqrt[d]*((Cos[a - (b*c)/d] + I*Sin[a - (b*c)/
d])*((1 + I)*(4*b^2*c^2 - (12*I)*b*c*d - 15*d^2)*Sqrt[Pi/2]*Erfi[((1 + I)*
Sqrt[b]*Sqrt[c + d*x])/(Sqrt[2]*Sqrt[d])] + 2*Sqrt[b]*Sqrt[d]*Sqrt[c + ...
```

### 3.123.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{4}(c+dx)^{5/2}\sin(a+bx)+\frac{1}{4}(c+dx)^{5/2}\sin(3a+3bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(a+bx)}{16b^3}+\frac{5d^2\sqrt{c+dx}\cos(3a+3bx)}{144b^3}+\frac{5d(c+dx)^{3/2}\sin(a+bx)}{8b^2}+\frac{5d(c+dx)^{3/2}\sin(3a+3bx)}{72b^2}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

output
```
(15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^3) - ((c + d*x)^(5/2)*Cos[a + b*
x])/(4*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(144*b^3) - ((c + d*x)^
(5/2)*Cos[3*a + 3*b*x])/(12*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*F
resnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(
5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(144*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*S
qrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (1
5*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*
Sin[a - (b*c)/d])/(16*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[a + b*x])/(8*b^2
) + (5*d*(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(72*b^2)
```

### 3.123.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.123.4 Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.17

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\right.}{}\right.}{}\right.}{4b}}{}$ |
| default | $\frac{-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\right.}{}\right.}{}\right.}{4b}}{}$ |

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+5/8/b*d*(1/2/b*d*
(d*x+c)^(3/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(1/2)
*cos(b/d*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((a
*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin((a
*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))-1/2
4/b*d*(d*x+c)^(5/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+5/24/b*d*(1/6/b*d*(d*
x+c)^(3/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1/2
)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)
^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b
*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/
d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

### 3.123.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.84

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{5\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+405\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(
sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 405*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 40
5*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(
pi*d)))*sin(-(b*c - a*d)/d) - 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(
sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(30*b*d^2
*cos(b*x + a) - (12*b^3*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos
(b*x + a)^3 + 10*(2*b^2*d^2*x + 2*b^2*c*d + (b^2*d^2*x + b^2*c*d)*cos(b*x
+ a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^4
```

### 3.123.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

output
```
Timed out
```

### 3.123.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 547, normalized size of antiderivative = 1.35

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \frac{\left(240\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)+2160\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)-24\left(\frac{12\,(dx+c)^{\frac{5}{2}}b^4}{d}-\right.\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/3456*(240*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d) + 2160*
(d*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d) - 24*(12*(d*x + c)^(5
/2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*cos(3*((d*x + c)*b - b*c + a*d)/d) - 21
6*(4*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(((d*x + c)*b - b*
c + a*d)/d) - 5*(-(I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*c
os(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1
/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 405*(-(I - 1
)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqr
t(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)
*sqrt(I*b/d)) - 405*((I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(
b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c -
 a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - 5*((I + 1)*9^(1/4)*sqrt(2)*sqr
t(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2
)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*
sqrt(-3*I*b/d)))*d/b^5
```

### 3.123.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 2468, normalized size of antiderivative = 6.08

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

output
```
1/1728*(72*(3*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d
*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(
I*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*I*sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*s
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d
)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2*(9*(sqrt(2)*sq
rt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*
x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)
/b^2)/d^2 + (sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*I*
sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c
- I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 6*I*(2*I*(d*x + c)
^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^(-3*(-I*(d*x +
 c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 9*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*
b*c*d - 3*d^2)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +...
```

### 3.123.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx=\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\left(c+d\,x\right)^{5/2}\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(5/2), x)
```

# 3.124 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx$

3.124.1 Optimal result . . . 1021
3.124.2 Mathematica [C] (verified) . . . 1022
3.124.3 Rubi [A] (verified) . . . 1022
3.124.4 Maple [A] (verified) . . . 1024
3.124.5 Fricas [A] (verification not implemented) . . . 1024
3.124.6 Sympy [F(-1)] . . . 1025
3.124.7 Maxima [C] (verification not implemented) . . . 1025
3.124.8 Giac [C] (verification not implemented) . . . 1026
3.124.9 Mupad [F(-1)] . . . 1027

## 3.124.1 Optimal result

Integrand size = 26, antiderivative size = 228

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} - \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}} - \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{4096b^{7/2}} + \frac{15d^2\sqrt{c+dx}\sin(4a+4bx)}{2048b^3} - \frac{(c+dx)^{5/2}\sin(4a+4bx)}{32b}$$

output
```
1/28*(d*x+c)^(7/2)/d-5/256*d*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b^2-1/32*(d*x+c)
^(5/2)*sin(4*b*x+4*a)/b-15/8192*d^(5/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2
)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)-15/8192
*d^(5/2)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*
a-4*b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/2048*d^2*sin(4*b*x+4*a)*(d*x+c)^(1/
2)/b^3
```

### 3.124.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.61

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx=\frac{512(c+dx)^4+\frac{7d^4e^{4i\left(a-\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{4ib(c+dx)}{d}\right)}{b^4}+\frac{7d^4e^{-4i\left(a-\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{4ib(c+dx)}{d}\right)}{b^4}}{14336d\sqrt{c+dx}}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(512*(c + d*x)^4 + (7*d^4*E^((4*I)*(a - (b*c)/d))*Sqrt[((-I)*b*(c + d*x))/
d]*Gamma[7/2, ((-4*I)*b*(c + d*x))/d])/b^4 + (7*d^4*Sqrt[(I*b*(c + d*x))/d
]*Gamma[7/2, ((4*I)*b*(c + d*x))/d])/(b^4*E^((4*I)*(a - (b*c)/d))))/(14336
*d*Sqrt[c + d*x])
```

### 3.124.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin^2(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}(c+dx)^{5/2}-\frac{1}{8}(c+dx)^{5/2}\cos(4a+4bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15d^2\sqrt{c+dx}\sin(4a+4bx)}{2048b^3}-\frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2}-\frac{(c+dx)^{5/2}\sin(4a+4bx)}{32b}+\frac{(c+dx)^{7/2}}{28d}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^(7/2)/(28*d) - (5*d*(c + d*x)^(3/2)*Cos[4*a + 4*b*x])/(256*b^2)
- (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[2/P
i]*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2]*Fresne
lC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(40
96*b^(7/2)) + (15*d^2*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(2048*b^3) - ((c + d
*x)^(5/2)*Sin[4*a + 4*b*x])/(32*b)
```

### 3.124.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.124.4 Maple [A] (verified)

Time = 3.81 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.10

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{(dx+c)^{\frac{7}{2}}}{28}-\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad}{}\right.}{8b}\right.}{}\right.}{}}{d}$ |
| default | $\frac{\frac{(dx+c)^{\frac{7}{2}}}{28}-\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad}{}\right.}{8b}\right.}{}\right.}{}}{d}$ |

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/56*(d*x+c)^(7/2)-1/64/b*d*(d*x+c)^(5/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*
c)/d)+5/64/b*d*(-1/8/b*d*(d*x+c)^(3/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+3/
8/b*d*(1/8/b*d*(d*x+c)^(1/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-1/32/b*d*2^(
1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/
(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1
/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

## 3.124.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.52

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{105\sqrt{2}\pi d^4\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+105\sqrt{2}\pi d^4\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4}{}\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/57344*(105*sqrt(2)*pi*d^4*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_
sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 105*sqrt(2)*pi*d^4*sqrt(b/(p
i*d))*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*
d)/d) - 16*(128*b^4*d^3*x^3 + 384*b^4*c*d^2*x^2 + 128*b^4*c^3 - 70*b^2*c*d
^2 - 560*(b^2*d^3*x + b^2*c*d^2)*cos(b*x + a)^4 + 560*(b^2*d^3*x + b^2*c*d
^2)*cos(b*x + a)^2 + 2*(192*b^4*c^2*d - 35*b^2*d^3)*x - 7*(2*(64*b^3*d^3*x
^2 + 128*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a)^3 - (64*b^3*d
^3*x^2 + 128*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a))*sin(b*x
+ a))*sqrt(d*x + c))/(b^4*d)
```

## 3.124.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

output
```
Timed out
```

## 3.124.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.25

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{\sqrt{2}\left(\frac{4096\sqrt{2}(dx+c)^{\frac{7}{2}}b^4}{d} - 2240\sqrt{2}(dx+c)^{\frac{3}{2}}b^2d\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right) + 105\left(-(i+1)\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\text{c}\right.\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/229376*sqrt(2)*(4096*sqrt(2)*(d*x + c)^(7/2)*b^4/d - 2240*sqrt(2)*(d*x +
 c)^(3/2)*b^2*d*cos(4*((d*x + c)*b - b*c + a*d)/d) + 105*(-(I + 1)*sqrt(pi
)*d^3*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(pi)*d^3*(b^2/d^
2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + 105*((I
 - 1)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I + 1)*sqrt(pi
)*d^3*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b
/d)) - 56*(64*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d*x + c)*b*d^2
)*sin(4*((d*x + c)*b - b*c + a*d)/d))/b^4
```

### 3.124.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.05 (sec) , antiderivative size = 1379, normalized size of antiderivative = 6.05

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/573440*(17920*(I*sqrt(2)*sqrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 8*sqrt(d*x + c))*c^3 - 56*c*d^2*(512*(3*(d
*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)/d^2 - 15*(I*s
qrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sq
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 4*(-8*I*(d*x + c)^(3/2)*b*d + 16*
I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-4*(-I*(d*x + c)*b + I*b*c
 - I*a*d)/d)/b^2)/d^2 - 15*(-I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*b*c*d -
 3*d^2)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) +
 4*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d
^2)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) - d^3*(4096*(5*(d*x
 + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x
+ c)*c^3)/d^3 - 35*(-I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 - 192*I*b^2*c^2*d - 7
2*b*c*d^2 + 15*I*d^3)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt
(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2)
 + 1)*b^3) + 4*(-64*I*(d*x + c)^(5/2)*b^2*d + 192*I*(d*x + c)^(3/2)*b^2...
```

### 3.124.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx=\int \cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^2\left(c+d\,x\right)^{5/2}\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

# 3.125 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx$

3.125.1 Optimal result . . . . . . . . . . 1028
3.125.2 Mathematica [C] (verified) . . . . . . . . . . 1029
3.125.3 Rubi [A] (verified) . . . . . . . . . . 1029
3.125.4 Maple [A] (verified) . . . . . . . . . . 1031
3.125.5 Fricas [A] (verification not implemented) . . . . . . . . . . 1031
3.125.6 Sympy [F] . . . . . . . . . . 1032
3.125.7 Maxima [C] (verification not implemented) . . . . . . . . . . 1032
3.125.8 Giac [C] (verification not implemented) . . . . . . . . . . 1033
3.125.9 Mupad [F(-1)] . . . . . . . . . . 1033

## 3.125.1 Optimal result

Integrand size = 26, antiderivative size = 200

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx=\frac{(c+dx)^{5/2}}{20d}-\frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2}+\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{512b^{5/2}}-\frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b}$$

output
```
1/20*(d*x+c)^(5/2)/d-1/32*(d*x+c)^(3/2)*sin(4*b*x+4*a)/b+3/1024*d^(3/2)*co
s(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*
2^(1/2)*Pi^(1/2)/b^(5/2)-3/1024*d^(3/2)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2
)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)-3/256*d
*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b^2
```

### 3.125.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.99 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.70

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{\sqrt{c+dx}\left(128(c+dx)^2-\frac{5d^2e^{4i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{5}{2},-\frac{4ib(c+dx)}{d}\right)}{b^2\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{5d^2e^{-4i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{5}{2},\frac{4ib(c+dx)}{d}\right)}{b^2\sqrt{\frac{ib(c+dx)}{d}}}\right)}{2560d}$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(Sqrt[c + d*x]*(128*(c + d*x)^2 - (5*d^2*E^((4*I)*(a - (b*c)/d))*Gamma[5/2
, ((-4*I)*b*(c + d*x))/d])/(b^2*Sqrt[((-I)*b*(c + d*x))/d]) - (5*d^2*Gamma
[5/2, ((4*I)*b*(c + d*x))/d])/(b^2*E^((4*I)*(a - (b*c)/d))*Sqrt[(I*b*(c +
d*x))/d])))/(2560*d)
```

### 3.125.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin^2(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}(c+dx)^{3/2}-\frac{1}{8}(c+dx)^{3/2}\cos(4a+4bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2}-\frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b}+\frac{(c+dx)^{5/2}}{20d}$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^(5/2)/(20*d) - (3*d*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(256*b^2) +
(3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*
Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(2
*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(
5/2)) - ((c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(32*b)
```

### 3.125.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.125.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.03

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{(dx+c)^{\frac{5}{2}}}{20}-\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{8b}+\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{4ad-4cb}{d}\right)\text{FresnelC}\left(\right.\right.}{32b}}{d}$ |
| default | $\frac{\frac{(dx+c)^{\frac{5}{2}}}{20}-\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{8b}+\frac{d\sqrt{2}\sqrt{\pi}\left(\cos\left(\frac{4ad-4cb}{d}\right)\text{FresnelC}\left(\right.\right.}{32b}}{d}$ |

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/40*(d*x+c)^(5/2)-1/64/b*d*(d*x+c)^(3/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*
c)/d)+3/64/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+1/
32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)
/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(
1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

### 3.125.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.24

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\text{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\,\text{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\ldots}{\ldots}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/5120*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d)
)*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d
) + 4*(64*b^3*d^2*x^2 - 120*b*d^2*cos(b*x + a)^4 + 128*b^3*c*d*x + 64*b^3*
c^2 + 120*b*d^2*cos(b*x + a)^2 - 15*b*d^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*c
os(b*x + a)^3 - (b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x
 + c))/(b^3*d)
```

### 3.125.6 Sympy [F]

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx=\int (c+dx)^{\frac{3}{2}}\sin^2(a+bx)\cos^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x)**2, x)
```

### 3.125.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.32

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx=\frac{\sqrt{2}\left(\frac{512\sqrt{2}(dx+c)^{\frac{5}{2}}b^3}{d}-320\sqrt{2}(dx+c)^{\frac{3}{2}}b^2\sin\left(\frac{4((dx+c)b-bc+ad)}{d}\right)-120\sqrt{2}\sqrt{dx+c}bd\cos\left(\frac{4((dx+c)b-}{d}\right.\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/20480*sqrt(2)*(512*sqrt(2)*(d*x + c)^(5/2)*b^3/d - 320*sqrt(2)*(d*x + c)
^(3/2)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) - 120*sqrt(2)*sqrt(d*x + c)*
b*d*cos(4*((d*x + c)*b - b*c + a*d)/d) + 15*(-(I - 1)*sqrt(pi)*d^2*(b^2/d^
2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I + 1)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(
-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + 15*((I + 1)*sqrt(pi)
*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(pi)*d^2*(b^2/d^2
)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)))/b^3
```

### 3.125.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 857, normalized size of antiderivative = 4.28

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/30720*(960*(I*sqrt(2)*sqrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(
-I*b*d/sqrt(b^2*d^2) + 1)) - 8*sqrt(d*x + c))*c^2 - d^2*(512*(3*(d*x + c)^
(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)/d^2 - 15*(I*sqrt(2)*s
qrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*
(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 4*(-8*I*(d*x + c)^(3/2)*b*d + 16*I*sqrt(d
*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d
)/d)/b^2)/d^2 - 15*(-I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)*
d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 4*(8*I*
(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-
4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) + 80*(-3*I*sqrt(2)*sqrt(pi)
*(8*b*c - I*d)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b
) + 3*I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)*b) - 64*(d*x + c)^(3/2) + 192*sqrt(d*x + c)*c ...
```

### 3.125.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)^2\sin(a+b\,x)^2\,(c+d\,x)^{3/2}\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

# 3.126 $\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx$

3.126.1 Optimal result . . . . . . . . . . 1034
3.126.2 Mathematica [C] (verified) . . . . . . . . . . 1035
3.126.3 Rubi [A] (verified) . . . . . . . . . . 1035
3.126.4 Maple [A] (verified) . . . . . . . . . . 1036
3.126.5 Fricas [A] (verification not implemented) . . . . . . . . . . 1037
3.126.6 Sympy [F] . . . . . . . . . . 1037
3.126.7 Maxima [C] (verification not implemented) . . . . . . . . . . 1037
3.126.8 Giac [C] (verification not implemented) . . . . . . . . . . 1038
3.126.9 Mupad [F(-1)] . . . . . . . . . . 1039

## 3.126.1 Optimal result

Integrand size = 26, antiderivative size = 174

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{(c+dx)^{3/2}}{12d} + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{64b^{3/2}} - \frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}$$

output
```
1/12*(d*x+c)^(3/2)/d+1/128*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^
(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/128*Fresne
lC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*d^(1
/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/32*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b
```

### 3.126.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.69

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{(c+dx)^{3/2}\left(32+\frac{3e^{4i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{3}{2},-\frac{4ib(c+dx)}{d}\right)}{\left(-\frac{ib(c+dx)}{d}\right)^{3/2}}+\frac{3e^{-4i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{3}{2},\frac{4ib(c+dx)}{d}\right)}{\left(\frac{ib(c+dx)}{d}\right)^{3/2}}\right)}{384d}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
((c + d*x)^(3/2)*(32 + (3*E^((4*I)*(a - (b*c)/d))*Gamma[3/2, ((-4*I)*b*(c
+ d*x))/d])/(((-I)*b*(c + d*x))/d)^(3/2) + (3*Gamma[3/2, ((4*I)*b*(c + d*x
))/d])/(E^((4*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^(3/2))))/(384*d)
```

### 3.126.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^2(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \; 4906$$

$$\int \left(\frac{1}{8}\sqrt{c+dx}-\frac{1}{8}\sqrt{c+dx}\cos(4a+4bx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{(c+dx)^{3/2}}{12d}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^(3/2)/(12*d) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS
[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sq
rt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a -
(4*b*c)/d])/(64*b^(3/2)) - (Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(32*b)
```

### 3.126.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.126.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.91

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{(dx+c)^{\frac{3}{2}}}{12}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{4ad-4cb}{d}\right)\mathrm{FresnelS}\left(\frac{2\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{4ad-4cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{\cdots}}{\cdots}\right.\right.}{128b\sqrt{\frac{b}{d}}}}{d}$ |
| default | $\frac{\frac{(dx+c)^{\frac{3}{2}}}{12}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{4ad-4cb}{d}\right)\mathrm{FresnelS}\left(\frac{2\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{4ad-4cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{\cdots}}{\cdots}\right.\right.}{128b\sqrt{\frac{b}{d}}}}{d}$ |

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/24*(d*x+c)^(3/2)-1/64/b*d*(d*x+c)^(1/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*
c)/d)+1/256/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(
2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(4*(a*d-b*c)/d)*Fresn
elC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))
```

### 3.126.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.01

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-a}{d}\right.}{384\,b^2d}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/384*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(2
*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*f
resnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) +
 16*(2*b^2*d*x + 2*b^2*c - 3*(2*b*d*cos(b*x + a)^3 - b*d*cos(b*x + a))*sin
(b*x + a))*sqrt(d*x + c))/(b^2*d)
```

### 3.126.6 Sympy [F]

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \sqrt{c+dx}\sin^2(a+bx)\cos^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x)**2, x)
```

### 3.126.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.26

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{\sqrt{2}\left(\frac{64\sqrt{2}(dx+c)^{\frac{3}{2}}b^2}{d}-24\sqrt{2}\sqrt{dx+c}b\sin\left(\frac{4((dx+c)b-bc+ad)}{d}\right)+3\left((i+1)\sqrt{\pi}d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right)-(i-\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/1536*sqrt(2)*(64*sqrt(2)*(d*x + c)^(3/2)*b^2/d - 24*sqrt(2)*sqrt(d*x + c
)*b*sin(4*((d*x + c)*b - b*c + a*d)/d) + 3*((I + 1)*sqrt(pi)*d*(b^2/d^2)^(
1/4)*cos(-4*(b*c - a*d)/d) - (I - 1)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-4*(b*
c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + 3*(-(I - 1)*sqrt(pi)*d*(b^
2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I + 1)*sqrt(pi)*d*(b^2/d^2)^(1/4)*si
n(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)))/b^2
```

## 3.126.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.55 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.64

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{-\frac{3i\sqrt{2}\sqrt{\pi}(8bc-id)d\,\mathrm{erf}\left(-\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{4(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+\frac{3i\sqrt{2}\sqrt{\pi}(8bc+id)d\,\mathrm{erf}\left(\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-4\right.}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/768*(-3*I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqr
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 3*I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(
I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*
b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 24*(I*sqrt(2)*s
qrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*sqr
t(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1))
 - 8*sqrt(d*x + c))*c - 64*(d*x + c)^(3/2) + 192*sqrt(d*x + c)*c + 12*I*sq
rt(d*x + c)*d*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 12*I*sqrt(d*x +
 c)*d*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)/d
```

### 3.126.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)^2\sin(a+b\,x)^2\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

# 3.127 $\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx$

3.127.1 Optimal result . . . . . 1040
3.127.2 Mathematica [C] (verified) . . . . . 1041
3.127.3 Rubi [A] (verified) . . . . . 1041
3.127.4 Maple [A] (verified) . . . . . 1042
3.127.5 Fricas [A] (verification not implemented) . . . . . 1043
3.127.6 Sympy [F] . . . . . 1043
3.127.7 Maxima [C] (verification not implemented) . . . . . 1043
3.127.8 Giac [C] (verification not implemented) . . . . . 1044
3.127.9 Mupad [F(-1)] . . . . . 1045

## 3.127.1 Optimal result

Integrand size = 26, antiderivative size = 174

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{(c+dx)^{3/2}}{12d} + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{64b^{3/2}} - \frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}$$

output
```
1/12*(d*x+c)^(3/2)/d+1/128*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^
(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/128*Fresne
lC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*d^(1
/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/32*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b
```

### 3.127.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.69

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{(c+dx)^{3/2}\left(32+\frac{3e^{4i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{3}{2},-\frac{4ib(c+dx)}{d}\right)}{\left(-\frac{ib(c+dx)}{d}\right)^{3/2}}+\frac{3e^{-4i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{3}{2},\frac{4ib(c+dx)}{d}\right)}{\left(\frac{ib(c+dx)}{d}\right)^{3/2}}\right)}{384d}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
((c + d*x)^(3/2)*(32 + (3*E^((4*I)*(a - (b*c)/d))*Gamma[3/2, ((-4*I)*b*(c
+ d*x))/d])/(((-I)*b*(c + d*x))/d)^(3/2) + (3*Gamma[3/2, ((4*I)*b*(c + d*x
))/d])/(E^((4*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^(3/2))))/(384*d)
```

### 3.127.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^2(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}\sqrt{c+dx}-\frac{1}{8}\sqrt{c+dx}\cos(4a+4bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{(c+dx)^{3/2}}{12d}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^(3/2)/(12*d) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS
[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sq
rt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a -
(4*b*c)/d])/(64*b^(3/2)) - (Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(32*b)
```

### 3.127.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.127.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.91

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{(dx+c)^{\frac{3}{2}}}{12}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{4ad-4cb}{d}\right)\mathrm{FresnelS}\left(\frac{2\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{4ad-4cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{\cdot}}{\sqrt{\cdot}}\right)\right)}{128b\sqrt{\frac{b}{d}}}}{d}$ |
| default | $\frac{\frac{(dx+c)^{\frac{3}{2}}}{12}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{4ad-4cb}{d}\right)\mathrm{FresnelS}\left(\frac{2\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{4ad-4cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{\cdot}}{\sqrt{\cdot}}\right)\right)}{128b\sqrt{\frac{b}{d}}}}{d}$ |

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/24*(d*x+c)^(3/2)-1/64/b*d*(d*x+c)^(1/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*
c)/d)+1/256/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(
2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(4*(a*d-b*c)/d)*Fresn
elC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))
```

### 3.127.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.01

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-a}{d}\right.}{384\,b^2d}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/384*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(2
*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*f
resnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) +
 16*(2*b^2*d*x + 2*b^2*c - 3*(2*b*d*cos(b*x + a)^3 - b*d*cos(b*x + a))*sin
(b*x + a))*sqrt(d*x + c))/(b^2*d)
```

### 3.127.6 Sympy [F]

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \sqrt{c+dx}\sin^2{(a+bx)}\cos^2{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x)**2, x)
```

### 3.127.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.26

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{\sqrt{2}\left(\frac{64\sqrt{2}(dx+c)^{\frac{3}{2}}b^2}{d}-24\sqrt{2}\sqrt{dx+c}b\sin\left(\frac{4((dx+c)b-bc+ad)}{d}\right)+3\left((i+1)\sqrt{\pi}d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4(bc-ad)}{d}\right)-(i-\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/1536*sqrt(2)*(64*sqrt(2)*(d*x + c)^(3/2)*b^2/d - 24*sqrt(2)*sqrt(d*x + c
)*b*sin(4*((d*x + c)*b - b*c + a*d)/d) + 3*((I + 1)*sqrt(pi)*d*(b^2/d^2)^(
1/4)*cos(-4*(b*c - a*d)/d) - (I - 1)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-4*(b*
c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + 3*(-(I - 1)*sqrt(pi)*d*(b^
2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I + 1)*sqrt(pi)*d*(b^2/d^2)^(1/4)*si
n(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)))/b^2
```

### 3.127.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 460, normalized size of antiderivative = 2.64

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{-\frac{3i\sqrt{2}\sqrt{\pi}(8bc-id)d\,\mathrm{erf}\left(-\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-\frac{4(ibc-iad)}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+\frac{3i\sqrt{2}\sqrt{\pi}(8bc+id)d\,\mathrm{erf}\left(\frac{i\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(-4\right.}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/768*(-3*I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqr
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 3*I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(
I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*
b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 24*(I*sqrt(2)*s
qrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*sqr
t(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1))
 - 8*sqrt(d*x + c))*c - 64*(d*x + c)^(3/2) + 192*sqrt(d*x + c)*c + 12*I*sq
rt(d*x + c)*d*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 12*I*sqrt(d*x +
 c)*d*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b)/d
```

### 3.127.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^2\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

# 3.128 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx$


3.128.1 Optimal result . . . 1046
3.128.2 Mathematica [C] (verified) . . . 1047
3.128.3 Rubi [A] (verified) . . . 1047
3.128.4 Maple [A] (verified) . . . 1049
3.128.5 Fricas [A] (verification not implemented) . . . 1049
3.128.6 Sympy [F] . . . 1050
3.128.7 Maxima [C] (verification not implemented) . . . 1050
3.128.8 Giac [C] (verification not implemented) . . . 1051
3.128.9 Mupad [F(-1)] . . . 1051


## 3.128.1 Optimal result

Integrand size = 26, antiderivative size = 200

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{512b^{5/2}} - \frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b}$$

output
```
1/20*(d*x+c)^(5/2)/d-1/32*(d*x+c)^(3/2)*sin(4*b*x+4*a)/b+3/1024*d^(3/2)*co
s(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*
2^(1/2)*Pi^(1/2)/b^(5/2)-3/1024*d^(3/2)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2
)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)-3/256*d
*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b^2
```

### 3.128.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.70

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{\sqrt{c+dx}\left(128(c+dx)^2-\frac{5d^2e^{4i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{5}{2},-\frac{4ib(c+dx)}{d}\right)}{b^2\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{5d^2e^{-4i\left(a-\frac{bc}{d}\right)}\Gamma\left(\frac{5}{2},\frac{4ib(c+dx)}{d}\right)}{b^2\sqrt{\frac{ib(c+dx)}{d}}}\right)}{2560d}$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(Sqrt[c + d*x]*(128*(c + d*x)^2 - (5*d^2*E^((4*I)*(a - (b*c)/d))*Gamma[5/2
, ((-4*I)*b*(c + d*x))/d])/(b^2*Sqrt[((-I)*b*(c + d*x))/d]) - (5*d^2*Gamma
[5/2, ((4*I)*b*(c + d*x))/d])/(b^2*E^((4*I)*(a - (b*c)/d))*Sqrt[(I*b*(c +
d*x))/d])))/(2560*d)
```

### 3.128.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin^2(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}(c+dx)^{3/2}-\frac{1}{8}(c+dx)^{3/2}\cos(4a+4bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3d\sqrt{c+dx}\cos(4a+4bx)}{256b^2}-\frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b}+\frac{(c+dx)^{5/2}}{20d}$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^(5/2)/(20*d) - (3*d*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(256*b^2) +
(3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*
Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(2
*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(
5/2)) - ((c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(32*b)
```

### 3.128.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.128.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.03

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{(dx+c)^{\frac{5}{2}}}{20}-\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{8b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{4ad-4cb}{d}\right)\mathrm{FresnelC}\left(\right.\right.}{}\right.}{32b}}{d}$ |
| default | $\frac{\frac{(dx+c)^{\frac{5}{2}}}{20}-\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{8b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{4ad-4cb}{d}\right)\mathrm{FresnelC}\left(\right.\right.}{}\right.}{32b}}{d}$ |

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/40*(d*x+c)^(5/2)-1/64/b*d*(d*x+c)^(3/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*
c)/d)+3/64/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+1/
32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)
/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(
1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

### 3.128.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.24

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\cdots}{\cdots}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/5120*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d)
)*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d
) + 4*(64*b^3*d^2*x^2 - 120*b*d^2*cos(b*x + a)^4 + 128*b^3*c*d*x + 64*b^3*
c^2 + 120*b*d^2*cos(b*x + a)^2 - 15*b*d^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*c
os(b*x + a)^3 - (b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x
 + c))/(b^3*d)
```

## 3.128.6 Sympy [F]

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx=\int (c+dx)^{\frac{3}{2}}\sin^2(a+bx)\cos^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x)**2, x)
```

## 3.128.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.32

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx=\frac{\sqrt{2}\left(\frac{512\sqrt{2}(dx+c)^{\frac{5}{2}}b^3}{d}-320\sqrt{2}(dx+c)^{\frac{3}{2}}b^2\sin\left(\frac{4((dx+c)b-bc+ad)}{d}\right)-120\sqrt{2}\sqrt{dx+c}bd\cos\left(\frac{4((dx+c)b-$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/20480*sqrt(2)*(512*sqrt(2)*(d*x + c)^(5/2)*b^3/d - 320*sqrt(2)*(d*x + c)
^(3/2)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) - 120*sqrt(2)*sqrt(d*x + c)*
b*d*cos(4*((d*x + c)*b - b*c + a*d)/d) + 15*(-(I - 1)*sqrt(pi)*d^2*(b^2/d^
2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I + 1)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(
-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + 15*((I + 1)*sqrt(pi)
*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(pi)*d^2*(b^2/d^2
)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)))/b^3
```

### 3.128.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 857, normalized size of antiderivative = 4.28

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/30720*(960*(I*sqrt(2)*sqrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(
-I*b*d/sqrt(b^2*d^2) + 1)) - 8*sqrt(d*x + c))*c^2 - d^2*(512*(3*(d*x + c)^
(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)/d^2 - 15*(I*sqrt(2)*s
qrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*
(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 4*(-8*I*(d*x + c)^(3/2)*b*d + 16*I*sqrt(d
*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-4*(-I*(d*x + c)*b + I*b*c - I*a*d
)/d)/b^2)/d^2 - 15*(-I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)*
d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 4*(8*I*
(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-
4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) + 80*(-3*I*sqrt(2)*sqrt(pi)
*(8*b*c - I*d)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b
) + 3*I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)*b) - 64*(d*x + c)^(3/2) + 192*sqrt(d*x + c)*c ...
```

### 3.128.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)^2\sin(a+b\,x)^2\,(c+d\,x)^{3/2}\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

# 3.129 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx$


3.129.1 Optimal result . . . 1052
3.129.2 Mathematica [C] (verified) . . . 1053
3.129.3 Rubi [A] (verified) . . . 1053
3.129.4 Maple [A] (verified) . . . 1055
3.129.5 Fricas [A] (verification not implemented) . . . 1055
3.129.6 Sympy [F(-1)] . . . 1056
3.129.7 Maxima [C] (verification not implemented) . . . 1056
3.129.8 Giac [C] (verification not implemented) . . . 1057
3.129.9 Mupad [F(-1)] . . . 1058


## 3.129.1 Optimal result

Integrand size = 26, antiderivative size = 228

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx=\frac{(c+dx)^{7/2}}{28d}-\frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2}-\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{4096b^{7/2}}+\frac{15d^2\sqrt{c+dx}\sin(4a+4bx)}{2048b^3}-\frac{(c+dx)^{5/2}\sin(4a+4bx)}{32b}$$

output
```
1/28*(d*x+c)^(7/2)/d-5/256*d*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b^2-1/32*(d*x+c)
^(5/2)*sin(4*b*x+4*a)/b-15/8192*d^(5/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2
)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)-15/8192
*d^(5/2)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*
a-4*b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/2048*d^2*sin(4*b*x+4*a)*(d*x+c)^(1/
2)/b^3
```

### 3.129.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.61

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{512(c+dx)^4+\frac{7d^4e^{4i\left(a-\frac{bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{4ib(c+dx)}{d}\right)}{b^4}+\frac{7d^4e^{-4i\left(a-\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{4ib(c+dx)}{d}\right)}{b^4}}{14336d\sqrt{c+dx}}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(512*(c + d*x)^4 + (7*d^4*E^((4*I)*(a - (b*c)/d))*Sqrt[((-I)*b*(c + d*x))/
d]*Gamma[7/2, ((-4*I)*b*(c + d*x))/d])/b^4 + (7*d^4*Sqrt[(I*b*(c + d*x))/d
]*Gamma[7/2, ((4*I)*b*(c + d*x))/d])/(b^4*E^((4*I)*(a - (b*c)/d))))/(14336
*d*Sqrt[c + d*x])
```

### 3.129.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin^2(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}(c+dx)^{5/2}-\frac{1}{8}(c+dx)^{5/2}\cos(4a+4bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15d^2\sqrt{c+dx}\sin(4a+4bx)}{2048b^3}-\frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2}-\frac{(c+dx)^{5/2}\sin(4a+4bx)}{32b}+\frac{(c+dx)^{7/2}}{28d}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

output
```
(c + d*x)^(7/2)/(28*d) - (5*d*(c + d*x)^(3/2)*Cos[4*a + 4*b*x])/(256*b^2)
- (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[2/P
i]*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2]*Fresne
lC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(40
96*b^(7/2)) + (15*d^2*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(2048*b^3) - ((c + d
*x)^(5/2)*Sin[4*a + 4*b*x])/(32*b)
```

### 3.129.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.129.4 Maple [A] (verified)

Time = 3.53 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.10

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{(dx+c)^{\frac{7}{2}}}{28}-\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad}{}\right.}{8b}\right.}{}\right.}{}}{d}$ |
| default | $\frac{\frac{(dx+c)^{\frac{7}{2}}}{28}-\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{32b}+\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{4b(dx+c)}{d}+\frac{4ad-4cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{4b(dx+c)}{d}+\frac{4ad}{}\right.}{8b}\right.}{}\right.}{}}{d}$ |

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/56*(d*x+c)^(7/2)-1/64/b*d*(d*x+c)^(5/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*
c)/d)+5/64/b*d*(-1/8/b*d*(d*x+c)^(3/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+3/
8/b*d*(1/8/b*d*(d*x+c)^(1/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-1/32/b*d*2^(
1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/
(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1
/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

### 3.129.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.52

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx =$$
$$-\frac{105\sqrt{2}\pi d^4\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+105\sqrt{2}\pi d^4\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4}{}\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/57344*(105*sqrt(2)*pi*d^4*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_
sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 105*sqrt(2)*pi*d^4*sqrt(b/(p
i*d))*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*
d)/d) - 16*(128*b^4*d^3*x^3 + 384*b^4*c*d^2*x^2 + 128*b^4*c^3 - 70*b^2*c*d
^2 - 560*(b^2*d^3*x + b^2*c*d^2)*cos(b*x + a)^4 + 560*(b^2*d^3*x + b^2*c*d
^2)*cos(b*x + a)^2 + 2*(192*b^4*c^2*d - 35*b^2*d^3)*x - 7*(2*(64*b^3*d^3*x
^2 + 128*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a)^3 - (64*b^3*d
^3*x^2 + 128*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a))*sin(b*x
+ a))*sqrt(d*x + c))/(b^4*d)
```

### 3.129.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

output
```
Timed out
```

### 3.129.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.25

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \frac{\sqrt{2}\left(\frac{4096\sqrt{2}(dx+c)^{\frac{7}{2}}b^4}{d} - 2240\sqrt{2}(dx+c)^{\frac{3}{2}}b^2d\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right) + 105\left(-(i+1)\sqrt{\pi}d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}c\right.\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/229376*sqrt(2)*(4096*sqrt(2)*(d*x + c)^(7/2)*b^4/d - 2240*sqrt(2)*(d*x +
 c)^(3/2)*b^2*d*cos(4*((d*x + c)*b - b*c + a*d)/d) + 105*(-(I + 1)*sqrt(pi
)*d^3*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(pi)*d^3*(b^2/d^
2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + 105*((I
 - 1)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I + 1)*sqrt(pi
)*d^3*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b
/d)) - 56*(64*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d*x + c)*b*d^2
)*sin(4*((d*x + c)*b - b*c + a*d)/d))/b^4
```

### 3.129.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 1379, normalized size of antiderivative = 6.05

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/573440*(17920*(I*sqrt(2)*sqrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 8*sqrt(d*x + c))*c^3 - 56*c*d^2*(512*(3*(d
*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)/d^2 - 15*(I*s
qrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sq
rt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 4*(-8*I*(d*x + c)^(3/2)*b*d + 16*
I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^(-4*(-I*(d*x + c)*b + I*b*c
 - I*a*d)/d)/b^2)/d^2 - 15*(-I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*b*c*d -
 3*d^2)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) +
 4*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d
^2)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2) - d^3*(4096*(5*(d*x
 + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x
+ c)*c^3)/d^3 - 35*(-I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 - 192*I*b^2*c^2*d - 7
2*b*c*d^2 + 15*I*d^3)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt
(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2)
 + 1)*b^3) + 4*(-64*I*(d*x + c)^(5/2)*b^2*d + 192*I*(d*x + c)^(3/2)*b^2...
```

### 3.129.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx=\int \cos(a+b\,x)^2\sin(a+b\,x)^2\,(c+d\,x)^{5/2}\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

# 3.130 $\int (c+dx)^{5/2} \cos^2(a+bx) \sin^3(a+bx)\, dx$

3.130.1 Optimal result . . . . . . . . . . 1060
3.130.2 Mathematica [C] (verified) . . . . . . . . . . 1061
3.130.3 Rubi [A] (verified) . . . . . . . . . . 1062
3.130.4 Maple [A] (verified) . . . . . . . . . . 1064
3.130.5 Fricas [A] (verification not implemented) . . . . . . . . . . 1066
3.130.6 Sympy [F(-1)] . . . . . . . . . . 1066
3.130.7 Maxima [C] (verification not implemented) . . . . . . . . . . 1067
3.130.8 Giac [C] (verification not implemented) . . . . . . . . . . 1067
3.130.9 Mupad [F(-1)] . . . . . . . . . . 1068

### 3.130.1 Optimal result

Integrand size = 26, antiderivative size = 615

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{15d^2\sqrt{c+dx}\cos(a+bx)}{32b^3} - \frac{(c+dx)^{5/2}\cos(a+bx)}{8b} + \frac{5d^2\sqrt{c+dx}\cos(3a+3bx)}{576b^3} - \frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b} - \frac{3d^2\sqrt{c+dx}\cos(5a+5bx)}{1600b^3} + \frac{(c+dx)^{5/2}\cos(5a+5bx)}{80b} - \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}} - \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}} + \frac{3d^{5/2}\sqrt{\frac{\pi}{10}}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}} - \frac{3d^{5/2}\sqrt{\frac{\pi}{10}}\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(5a-\frac{5bc}{d}\right)}{1600b^{7/2}} + \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{576b^{7/2}} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{32b^{7/2}} + \frac{5d(c+dx)^{3/2}\sin(a+bx)}{16b^2} + \frac{5d(c+dx)^{3/2}\sin(3a+3bx)}{288b^2} - \frac{d(c+dx)^{3/2}\sin(5a+5bx)}{160b^2}$$

output
```
-1/8*(d*x+c)^(5/2)*cos(b*x+a)/b-1/48*(d*x+c)^(5/2)*cos(3*b*x+3*a)/b+1/80*(
d*x+c)^(5/2)*cos(5*b*x+5*a)/b+5/16*d*(d*x+c)^(3/2)*sin(b*x+a)/b^2+5/288*d*
(d*x+c)^(3/2)*sin(3*b*x+3*a)/b^2-1/160*d*(d*x+c)^(3/2)*sin(5*b*x+5*a)/b^2+
3/16000*d^(5/2)*cos(5*a-5*b*c/d)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(7/2)-3/16000*d^(5/2)*FresnelS(b^(1/2
)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*10^(1/2)*Pi^(1
/2)/b^(7/2)-5/3456*d^(5/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)+5/3456*d^(5/2)*Fresnel
S(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)
*Pi^(1/2)/b^(7/2)-15/64*d^(5/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(
1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/64*d^(5/2)*Fresnel
S(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^
(1/2)/b^(7/2)+15/32*d^2*cos(b*x+a)*(d*x+c)^(1/2)/b^3+5/576*d^2*cos(3*b*x+3
*a)*(d*x+c)^(1/2)/b^3-3/1600*d^2*cos(5*b*x+5*a)*(d*x+c)^(1/2)/b^3
```

### 3.130.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 17.08 (sec) , antiderivative size = 2177, normalized size of antiderivative = 3.54

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
(c*Sqrt[d]*(-12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(-I + 2*b*
x + E^((6*I)*(a + b*x))*(I + 2*b*x)) - (1 - I)*(2*b*c + I*d)*E^(((3*I)*b*(
2*c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sq
rt[d]] + (1 + I)*((2*I)*b*c + d)*E^((3*I)*(2*a + b*x))*Sqrt[6*Pi]*Erfi[((1
 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(1152*b^(5/2)*E^(((3*I)*
(a*d + b*(c + d*x)))/d)) - (c*Sqrt[d]*(-20*Sqrt[b]*Sqrt[d]*E^(((5*I)*b*c)/
d)*Sqrt[c + d*x]*(-3*I + 10*b*x + E^((10*I)*(a + b*x))*(3*I + 10*b*x)) - (
1 - I)*(10*b*c + (3*I)*d)*E^(((5*I)*b*(2*c + d*x))/d)*Sqrt[10*Pi]*Erf[((1
+ I)*Sqrt[5/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + (1 + I)*((10*I)*b*c + 3*d
)*E^((5*I)*(2*a + b*x))*Sqrt[10*Pi]*Erfi[((1 + I)*Sqrt[5/2]*Sqrt[b]*Sqrt[c
 + d*x])/Sqrt[d]]))/(16000*b^(5/2)*E^(((5*I)*(a*d + b*(c + d*x)))/d)) + ((
I/16)*c^2*d*(-(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-I)*b*(
c + d*x))/d]) + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, (I*b*
(c + d*x))/d]))/(b^2*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x]) + (c^2*(-1/6*(E^
((3*I)*(a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-3*I)*b*(c + d*x))/d])/(S
qrt[3]*b*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]*Gamma[3/2, ((3*I)*b*
(c + d*x))/d])/(6*Sqrt[3]*b*E^((3*I)*(a - (b*c)/d))*Sqrt[(I*b*(c + d*x))/d
])))/16 - (c^2*(-1/10*(E^((5*I)*(a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((
-5*I)*b*(c + d*x))/d])/(Sqrt[5]*b*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c +
d*x]*Gamma[3/2, ((5*I)*b*(c + d*x))/d])/(10*Sqrt[5]*b*E^((5*I)*(a - (b*...
```

### 3.130.3 Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin^3(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}(c+dx)^{5/2}\sin(a+bx)+\frac{1}{16}(c+dx)^{5/2}\sin(3a+3bx)-\frac{1}{16}(c+dx)^{5/2}\sin(5a+5bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(a+bx)}{32b^3}+\frac{5d^2\sqrt{c+dx}\cos(3a+3bx)}{576b^3}-\frac{3d^2\sqrt{c+dx}\cos(5a+5bx)}{1600b^3}+\frac{5d(c+dx)^{3/2}\sin(a+bx)}{16b^2}+\frac{5d(c+dx)^{3/2}\sin(3a+3bx)}{288b^2}-\frac{d(c+dx)^{3/2}\sin(5a+5bx)}{160b^2}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{5/2}\cos(5a+5bx)}{80b}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

3.130. $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

output
```
(15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(32*b^3) - ((c + d*x)^(5/2)*Cos[a + b*
x])/(8*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^
(5/2)*Cos[3*a + 3*b*x])/(48*b) - (3*d^2*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(1
600*b^3) + ((c + d*x)^(5/2)*Cos[5*a + 5*b*x])/(80*b) - (15*d^(5/2)*Sqrt[Pi
/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])
/(32*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[
b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) + (3*d^(5/2)*Sqrt[Pi/
10]*Cos[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt
[d]])/(1600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi
]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) + (5*d^(5/2
)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a
- (3*b*c)/d])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqr
t[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) + (5*d*(c +
 d*x)^(3/2)*Sin[a + b*x])/(16*b^2) + (5*d*(c + d*x)^(3/2)*Sin[3*a + 3*b*x]
)/(288*b^2) - (d*(c + d*x)^(3/2)*Sin[5*a + 5*b*x])/(160*b^2)
```

### 3.130.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.130.4 Maple [A] (verified)

Time = 12.01 (sec) , antiderivative size = 719, normalized size of antiderivative = 1.17

| method | result |
|---|---|
| derivativedivides | $-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,(\cdots}{\cdots}\right.}{\cdots}\right.}{8b}$ |
| default | $-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,(\cdots}{\cdots}\right.}{\cdots}\right.}{8b}$ |

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+5/16/b*d*(1/2/b*
d*(d*x+c)^(3/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(1/
2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(
(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1
/96/b*d*(d*x+c)^(5/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+5/96/b*d*(1/6/b*d*(
d*x+c)^(3/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1
/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/
d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)
*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(
b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))+1/160/b*d*(d*x+c)^(5/2)*cos(5*b/d*(d*x+c)
+5*(a*d-b*c)/d)-1/32/b*d*(1/10/b*d*(d*x+c)^(3/2)*sin(5*b/d*(d*x+c)+5*(a*d-
b*c)/d)-3/10/b*d*(-1/10/b*d*(d*x+c)^(1/2)*cos(5*b/d*(d*x+c)+5*(a*d-b*c)/d)
+1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*Fresne
lC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(5*(a*d-b*c)
/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

## 3.130. $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

### 3.130.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 521, normalized size of antiderivative = 0.85

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{81\sqrt{10}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-625\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_
cos(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 625*sqrt(6)*pi*d^3*sqrt(b/(pi
*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)
)) - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(
sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d
))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) +
 625*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(
b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fresn
el_sin(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*
(9*(20*b^3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2 - 3*b*d^2)*cos(b*x + a)^5 +
 390*b*d^2*cos(b*x + a) - 5*(60*b^3*d^2*x^2 + 120*b^3*c*d*x + 60*b^3*c^2 -
 13*b*d^2)*cos(b*x + a)^3 + 10*(26*b^2*d^2*x - 9*(b^2*d^2*x + b^2*c*d)*cos
(b*x + a)^4 + 26*b^2*c*d + 13*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*sin(b*
x + a))*sqrt(d*x + c))/b^4
```

### 3.130.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

output
```
Timed out
```

### 3.130.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 826, normalized size of antiderivative = 1.34

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/1728000*sqrt(2)*(5400*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(5*((d*x + c)*b -
b*c + a*d)/d)/d - 15000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(3*((d*x + c)*b - b
*c + a*d)/d)/d - 270000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(((d*x + c)*b - b*c
 + a*d)/d)/d - 540*(20*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 3*sqrt(2)*sqrt(d*
x + c)*b^3)*cos(5*((d*x + c)*b - b*c + a*d)/d) + 1500*(12*sqrt(2)*(d*x + c
)^(5/2)*b^5/d^2 - 5*sqrt(2)*sqrt(d*x + c)*b^3)*cos(3*((d*x + c)*b - b*c +
a*d)/d) + 27000*(4*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 15*sqrt(2)*sqrt(d*x +
 c)*b^3)*cos(((d*x + c)*b - b*c + a*d)/d) - 81*(-(I - 1)*25^(1/4)*sqrt(pi)
*b^2*d*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) - (I + 1)*25^(1/4)*sqrt(pi)*b
^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d
)) - 625*((I - 1)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d
)/d) + (I + 1)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d
))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 101250*((I - 1)*sqrt(pi)*b^2*d*(b^2/
d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*si
n(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 101250*(-(I + 1)*sqrt(
pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqrt(pi)*b^2*d*(b^
2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - 625*(-
(I + 1)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I
- 1)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(-3*I*b/d)) - 81*((I + 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d...
```

### 3.130.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.74 (sec) , antiderivative size = 3706, normalized size of antiderivative = 6.03

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/864000*(1800*(30*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)
*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt
(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(10)*sqrt(pi)*d*erf(-1/2*I*sqrt(1
0)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a
*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 30*sqrt(2)*sqrt(pi)*d*erf(-
1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*
b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)
*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt
(10)*sqrt(pi)*d*erf(1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)))*c^3 + 18*c*d^2*(2250*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d
^2)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*I
*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)
*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 125*(sqrt(6)*sqrt(pi)*(
12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*...
```

### 3.130.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx=\int \cos(a+b\,x)^2\sin(a+b\,x)^3\,(c+d\,x)^{5/2}\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

# 3.131 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

3.131.1 Optimal result . . . 1069
3.131.2 Mathematica [C] (verified) . . . 1070
3.131.3 Rubi [A] (verified) . . . 1071
3.131.4 Maple [A] (verified) . . . 1073
3.131.5 Fricas [A] (verification not implemented) . . . 1074
3.131.6 Sympy [F] . . . 1075
3.131.7 Maxima [C] (verification not implemented) . . . 1075
3.131.8 Giac [C] (verification not implemented) . . . 1076
3.131.9 Mupad [F(-1)] . . . 1076

## 3.131.1 Optimal result

Integrand size = 26, antiderivative size = 534

$$\begin{aligned}\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx &= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b} \\ &- \frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b} + \frac{(c+dx)^{3/2}\cos(5a+5bx)}{80b} \\ &- \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}} \\ &- \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}} \\ &+ \frac{3d^{3/2}\sqrt{\frac{\pi}{10}}\cos\left(5a-\frac{5bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}} \\ &+ \frac{3d^{3/2}\sqrt{\frac{\pi}{10}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(5a-\frac{5bc}{d}\right)}{800b^{5/2}} \\ &- \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{96b^{5/2}} \\ &- \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{16b^{5/2}} + \frac{3d\sqrt{c+dx}\sin(a+bx)}{16b^2} \\ &+ \frac{d\sqrt{c+dx}\sin(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\sin(5a+5bx)}{800b^2}\end{aligned}$$

output
```
-1/8*(d*x+c)^(3/2)*cos(b*x+a)/b-1/48*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b+1/80*(
d*x+c)^(3/2)*cos(5*b*x+5*a)/b+3/8000*d^(3/2)*cos(5*a-5*b*c/d)*FresnelS(b^(
1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(5/2)+3/
8000*d^(3/2)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin
(5*a-5*b*c/d)*10^(1/2)*Pi^(1/2)/b^(5/2)-1/576*d^(3/2)*cos(3*a-3*b*c/d)*Fre
snelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(
5/2)-1/576*d^(3/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2)
)*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/32*d^(3/2)*cos(a-b*c/d)*Fres
nelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5
/2)-3/32*d^(3/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*
sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d*sin(b*x+a)*(d*x+c)^(1/2)/b^2+
1/96*d*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b^2-3/800*d*sin(5*b*x+5*a)*(d*x+c)^(1/
2)/b^2
```

## 3.131.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.54 (sec) , antiderivative size = 1126, normalized size of antiderivative = 2.11

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{\sqrt{d}e^{-\frac{3i(ad+b(c+dx))}{d}}\left(-12\sqrt{b}\sqrt{d}e^{\frac{3ibc}{d}}\sqrt{c+dx}\left(-i+2bx+e^{6i(a+bx)}(i+2bx)\right)-(1-i)(2bc+id)e^{\frac{3ib(2\cdots}{}}\cdots\right.}{2304b^{5/2}} - \frac{\sqrt{d}e^{-\frac{5i(ad+b(c+dx))}{d}}\left(-20\sqrt{b}\sqrt{d}e^{\frac{5ibc}{d}}\sqrt{c+dx}\left(-3i+10bx+e^{10i(a+bx)}(3i+10bx)\right)-(1-i)(10bc+3id)e^{\frac{5ib(2\cdots}{}}\cdots\right.}{32000b^{5/2}} + \frac{icde^{-\frac{i(bc+ad)}{d}}\left(-e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)\right)}{16b^2\sqrt{c+dx}} + \frac{1}{16}c\left(-\frac{e^{3i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\Gamma\left(\frac{3}{2},-\frac{3ib(c+dx)}{d}\right)}{6\sqrt{3}b\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{-3i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\Gamma\left(\frac{3}{2},\frac{3ib(c+dx)}{d}\right)}{6\sqrt{3}b\sqrt{\frac{ib(c+dx)}{d}}}\right)-\frac{1}{16}c\left(-\frac{e^{5i\left(a-\frac{bc}{d}\right)}\sqrt{c+d\cdots}}{10\sqrt{5}b\sqrt{\cdots}}\cdots\right.$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
(Sqrt[d]*(-12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(-I + 2*b*x
+ E^((6*I)*(a + b*x))*(I + 2*b*x)) - (1 - I)*(2*b*c + I*d)*E^(((3*I)*b*(2*
c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]] + (1 + I)*((2*I)*b*c + d)*E^((3*I)*(2*a + b*x))*Sqrt[6*Pi]*Erfi[((1 +
 I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(2304*b^(5/2)*E^(((3*I)*(a
*d + b*(c + d*x)))/d)) - (Sqrt[d]*(-20*Sqrt[b]*Sqrt[d]*E^(((5*I)*b*c)/d)*S
qrt[c + d*x]*(-3*I + 10*b*x + E^((10*I)*(a + b*x))*(3*I + 10*b*x)) - (1 -
I)*(10*b*c + (3*I)*d)*E^(((5*I)*b*(2*c + d*x))/d)*Sqrt[10*Pi]*Erf[((1 + I)
*Sqrt[5/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + (1 + I)*((10*I)*b*c + 3*d)*E^
((5*I)*(2*a + b*x))*Sqrt[10*Pi]*Erfi[((1 + I)*Sqrt[5/2]*Sqrt[b]*Sqrt[c + d
*x])/Sqrt[d]]))/(32000*b^(5/2)*E^(((5*I)*(a*d + b*(c + d*x)))/d)) + ((I/16
)*c*d*(-(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-I)*b*(c + d*
x))/d]) + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d
*x))/d]))/(b^2*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x]) + (c*(-1/6*(E^((3*I)*(
a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-3*I)*b*(c + d*x))/d])/(Sqrt[3]*b
*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]*Gamma[3/2, ((3*I)*b*(c + d*x
))/d])/(6*Sqrt[3]*b*E^((3*I)*(a - (b*c)/d))*Sqrt[(I*b*(c + d*x))/d])))/16
- (c*(-1/10*(E^((5*I)*(a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-5*I)*b*(c
 + d*x))/d])/(Sqrt[5]*b*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]*Gamma
[3/2, ((5*I)*b*(c + d*x))/d])/(10*Sqrt[5]*b*E^((5*I)*(a - (b*c)/d))*Sqr...
```

### 3.131.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin^3(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}(c+dx)^{3/2}\sin(a+bx)+\frac{1}{16}(c+dx)^{3/2}\sin(3a+3bx)-\frac{1}{16}(c+dx)^{3/2}\sin(5a+5bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(a+bx)}{16b^2}+\frac{d\sqrt{c+dx}\sin(3a+3bx)}{96b^2}-\frac{3d\sqrt{c+dx}\sin(5a+5bx)}{800b^2}-\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}\cos(5a+5bx)}{80b}$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
-1/8*((c + d*x)^(3/2)*Cos[a + b*x])/b - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])
/(48*b) + ((c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(80*b) - (3*d^(3/2)*Sqrt[Pi/2
]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(
16*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*S
qrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*C
os[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])
/(800*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt
[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt[Pi
/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)
/d])/(96*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqr
t[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + (3*d*Sqrt[c + d*x]*S
in[a + b*x])/(16*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(96*b^2) - (3*d
*Sqrt[c + d*x]*Sin[5*a + 5*b*x])/(800*b^2)
```

## 3.131. $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

### 3.131.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.131.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.09

| method | result |
|---|---|
| derivativedivides | $-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-cb}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\cdots\right)\right)}{4b\sqrt{\frac{b}{d}}}\cdots\right)}{8b}\cdots$ |
| default | $-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-cb}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\cdots\right)\right)}{4b\sqrt{\frac{b}{d}}}\cdots\right)}{8b}\cdots$ |

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+3/16/b*d*(1/2/b*
d*(d*x+c)^(1/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^
(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^
(1/2)/d)))-1/96/b*d*(d*x+c)^(3/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/32/b*
d*(1/6/b*d*(d*x+c)^(1/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)
*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2
)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/
2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))+1/160/b*d*(d*x+c)^(3/
2)*cos(5*b/d*(d*x+c)+5*(a*d-b*c)/d)-3/160/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(
5*b/d*(d*x+c)+5*(a*d-b*c)/d)-1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2
)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x
+c)^(1/2)/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d)))
```

### 3.131.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 427, normalized size of antiderivative = 0.80

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{27\sqrt{10}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-125\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_s
in(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 125*sqrt(6)*pi*d^2*sqrt(b/(pi*
d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))
) - 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fr
esnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*
sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi
*d)))*sin(-3*(b*c - a*d)/d) + 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_co
s(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(30*(
b^2*d*x + b^2*c)*cos(b*x + a)^5 - 50*(b^2*d*x + b^2*c)*cos(b*x + a)^3 - (9
*b*d*cos(b*x + a)^4 - 13*b*d*cos(b*x + a)^2 - 26*b*d)*sin(b*x + a))*sqrt(d
*x + c))/b^3
```

### 3.131.6 Sympy [F]

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin^3(a+bx)\cos^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)**3*cos(a + b*x)**2, x)
```

### 3.131.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 760, normalized size of antiderivative = 1.42

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/288000*sqrt(2)*(1800*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(5*((d*x + c)*b - b*
c + a*d)/d)/d^2 - 3000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(3*((d*x + c)*b - b*
c + a*d)/d)/d^2 - 18000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(((d*x + c)*b - b*c
 + a*d)/d)/d^2 - 540*sqrt(2)*sqrt(d*x + c)*b^3*sin(5*((d*x + c)*b - b*c +
a*d)/d)/d + 1500*sqrt(2)*sqrt(d*x + c)*b^3*sin(3*((d*x + c)*b - b*c + a*d)
/d)/d + 27000*sqrt(2)*sqrt(d*x + c)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d
 + 27*((I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)
 - (I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*er
f(sqrt(d*x + c)*sqrt(5*I*b/d)) + 125*(-(I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d
^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(
1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + 6750*(-(I +
 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I - 1)*sqrt(pi)*b^
2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + 67
50*((I - 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqr
t(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b
/d)) + 125*((I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d
)/d) - (I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))
*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + 27*(-(I - 1)*25^(1/4)*sqrt(pi)*b^2*(b
^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) + (I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d
^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2...
```

### 3.131.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 2314, normalized size of antiderivative = 4.33

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/144000*(300*(30*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*
d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*
sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(
b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(10)*sqrt(pi)*d*erf(-1/2*I*sqrt(10
)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 30*sqrt(2)*sqrt(pi)*d*erf(-1
/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b
*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*
d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(
10)*sqrt(pi)*d*erf(1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)))*c^2 + d^2*(2250*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*
erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*I*(2*I*
(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-
I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 125*(sqrt(6)*sqrt(pi)*(12*b^2
*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt...
```

### 3.131.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \cos(a+b\,x)^2\sin(a+b\,x)^3\,(c+d\,x)^{3/2}\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

# 3.132 $\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx$

3.132.1 Optimal result . . . 1077
3.132.2 Mathematica [C] (verified) . . . 1078
3.132.3 Rubi [A] (verified) . . . 1079
3.132.4 Maple [A] (verified) . . . 1080
3.132.5 Fricas [A] (verification not implemented) . . . 1081
3.132.6 Sympy [F] . . . 1082
3.132.7 Maxima [C] (verification not implemented) . . . 1082
3.132.8 Giac [C] (verification not implemented) . . . 1083
3.132.9 Mupad [F(-1)] . . . 1084

## 3.132.1 Optimal result

Integrand size = 26, antiderivative size = 459

$$\begin{aligned}
\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx &= -\frac{\sqrt{c+dx}\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{48b} \\
&+\frac{\sqrt{c+dx}\cos(5a+5bx)}{80b} \\
&+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}} \\
&+\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}} \\
&-\frac{\sqrt{d}\sqrt{\frac{\pi}{10}}\cos\left(5a-\frac{5bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}} \\
&+\frac{\sqrt{d}\sqrt{\frac{\pi}{10}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(5a-\frac{5bc}{d}\right)}{80b^{3/2}} \\
&-\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{48b^{3/2}} \\
&-\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{8b^{3/2}}
\end{aligned}$$

output
```
-1/800*cos(5*a-5*b*c/d)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d
^(1/2))*d^(1/2)*10^(1/2)*Pi^(1/2)/b^(3/2)+1/800*FresnelS(b^(1/2)*10^(1/2)/
Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*d^(1/2)*10^(1/2)*Pi^(1/2)
/b^(3/2)+1/288*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1/288*FresnelS(b^(1/2)*6^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(
1/2)/b^(3/2)+1/16*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(
1/2)/d^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelS(b^(1/2)*2^(1/
2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b
^(3/2)-1/8*cos(b*x+a)*(d*x+c)^(1/2)/b-1/48*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b+
1/80*cos(5*b*x+5*a)*(d*x+c)^(1/2)/b
```

### 3.132.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.81

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{e^{-\frac{5i(bc+ad)}{d}}\sqrt{c+dx}\left(450e^{6ia+\frac{4ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+450e^{4ia+\frac{6ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)+25\sqrt{}\right.}{}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
-1/7200*(Sqrt[c + d*x]*(450*E^((6*I)*a + ((4*I)*b*c)/d)*Sqrt[(I*b*(c + d*x
))/d]*Gamma[3/2, ((-I)*b*(c + d*x))/d] + 450*E^((4*I)*a + ((6*I)*b*c)/d)*S
qrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d] + 25*Sqrt[3]*E^((2
*I)*(4*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-3*I)*b*(c + d*x
))/d] + 25*Sqrt[3]*E^((2*I)*a + ((8*I)*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*
Gamma[3/2, ((3*I)*b*(c + d*x))/d] - 9*Sqrt[5]*E^((10*I)*a)*Sqrt[(I*b*(c +
d*x))/d]*Gamma[3/2, ((-5*I)*b*(c + d*x))/d] - 9*Sqrt[5]*E^(((10*I)*b*c)/d)
*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((5*I)*b*(c + d*x))/d]))/(b*E^(((5*
I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

### 3.132.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^3(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}\sqrt{c+dx}\sin(a+bx)+\frac{1}{16}\sqrt{c+dx}\sin(3a+3bx)-\frac{1}{16}\sqrt{c+dx}\sin(5a+5bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\cos(5a+5bx)}{80b}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
-1/8*(Sqrt[c + d*x]*Cos[a + b*x])/b - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(48
*b) + (Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(80*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a
- (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(3/2
)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]
*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/10]*Cos[5*a - (5
*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(80*b^(3/2
)) + (Sqrt[d]*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[5*a - (5*b*c)/d])/(80*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*FresnelS[(S
qrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(48*b^(3/2
)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[
d]]*Sin[a - (b*c)/d])/(8*b^(3/2))
```

### 3.132.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.132.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 447, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-cb}{d}\right)\text{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-cb}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{c}}{}}{}$ |
| default | $\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-cb}{d}\right)\text{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-cb}{d}\right)\text{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{c}}{}}{}$ |

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+1/32/b*d*2^(1/2)
*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d))-1/96/b*d*(d*x+c)^(1/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*
c)/d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*F
resnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d
-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))
+1/160/b*d*(d*x+c)^(1/2)*cos(5*b/d*(d*x+c)+5*(a*d-b*c)/d)-1/1600/b*d*2^(1/
2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1
/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(
1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

### 3.132.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.78

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{9\sqrt{10}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-25\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\cdots}\right.}{\cdots}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_cos(
sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*co
s(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 45
0*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt
(d*x + c)*sqrt(b/(pi*d))) + 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sq
rt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*
sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b
*c - a*d)/d) - 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(10)*sqrt(d*
x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^5 - 5
*b*cos(b*x + a)^3)*sqrt(d*x + c))/b^2
```

### 3.132.6 Sympy [F]

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \sqrt{c+dx}\sin^3\left(a+bx\right)\cos^2\left(a+bx\right)dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x)**2, x)
```

### 3.132.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.48

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{\sqrt{2}\left(\frac{180\sqrt{2}\sqrt{dx+c}b^3\cos\left(\frac{5((dx+c)b-bc+ad)}{d}\right)}{d^2} - \frac{300\sqrt{2}\sqrt{dx+c}b^3\cos\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d^2} - \frac{1800\sqrt{2}\sqrt{dx+c}b^3\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d^2} - 9\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/28800*sqrt(2)*(180*sqrt(2)*sqrt(d*x + c)*b^3*cos(5*((d*x + c)*b - b*c +
a*d)/d)/d^2 - 300*sqrt(2)*sqrt(d*x + c)*b^3*cos(3*((d*x + c)*b - b*c + a*d
)/d)/d^2 - 1800*sqrt(2)*sqrt(d*x + c)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)
/d^2 - 9*(-(I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d
)/d)/d - (I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/
d)/d)*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - 25*((I - 1)*9^(1/4)*sqrt(pi)*b^2*
(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d + (I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^
2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 4
50*((I - 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d + (I + 1)*s
qrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(
I*b/d)) - 450*(-(I + 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d
 - (I - 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*
x + c)*sqrt(-I*b/d)) - 25*(-(I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*c
os(-3*(b*c - a*d)/d)/d - (I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(
-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - 9*((I + 1)*25^(1/
4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)/d + (I - 1)*25^(1/4)
*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*s
qrt(-5*I*b/d)))*d^2/b^4
```

### 3.132.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 1270, normalized size of antiderivative = 2.77

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/14400*(450*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b
*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*er
f(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(10)*
sqrt(pi)*(10*b*c - I*d)*d*erf(-1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)*b) + 450*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*I*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d
)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c + I
*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9
*sqrt(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)*b) - 30*(30*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*I
*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(10)*sqrt(pi)*d
*erf(-1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/...
```

## 3.132.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx=\int \cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^3\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

# 3.133 $\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx$


3.133.1 Optimal result . . . 1085
3.133.2 Mathematica [C] (verified) . . . 1086
3.133.3 Rubi [A] (verified) . . . 1087
3.133.4 Maple [A] (verified) . . . 1088
3.133.5 Fricas [A] (verification not implemented) . . . 1089
3.133.6 Sympy [F] . . . 1090
3.133.7 Maxima [C] (verification not implemented) . . . 1090
3.133.8 Giac [C] (verification not implemented) . . . 1091
3.133.9 Mupad [F(-1)] . . . 1092


## 3.133.1 Optimal result

Integrand size = 26, antiderivative size = 459

$$\begin{aligned}\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx = &-\frac{\sqrt{c+dx}\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{48b}\\
&+\frac{\sqrt{c+dx}\cos(5a+5bx)}{80b}\\
&+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}\\
&+\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}\\
&-\frac{\sqrt{d}\sqrt{\frac{\pi}{10}}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}\\
&+\frac{\sqrt{d}\sqrt{\frac{\pi}{10}}\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(5a-\frac{5bc}{d}\right)}{80b^{3/2}}\\
&-\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{48b^{3/2}}\\
&-\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{8b^{3/2}}\end{aligned}$$

output
```
-1/800*cos(5*a-5*b*c/d)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d
^(1/2))*d^(1/2)*10^(1/2)*Pi^(1/2)/b^(3/2)+1/800*FresnelS(b^(1/2)*10^(1/2)/
Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*d^(1/2)*10^(1/2)*Pi^(1/2)
/b^(3/2)+1/288*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1/288*FresnelS(b^(1/2)*6^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(
1/2)/b^(3/2)+1/16*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(
1/2)/d^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelS(b^(1/2)*2^(1/
2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b
^(3/2)-1/8*cos(b*x+a)*(d*x+c)^(1/2)/b-1/48*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b+
1/80*cos(5*b*x+5*a)*(d*x+c)^(1/2)/b
```

### 3.133.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.81

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{e^{-\frac{5i(bc+ad)}{d}}\sqrt{c+dx}\left(450e^{6ia+\frac{4ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+450e^{4ia+\frac{6ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)+25\sqrt{\phantom{x}}\right.}{}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
-1/7200*(Sqrt[c + d*x]*(450*E^((6*I)*a + ((4*I)*b*c)/d)*Sqrt[(I*b*(c + d*x
))/d]*Gamma[3/2, ((-I)*b*(c + d*x))/d] + 450*E^((4*I)*a + ((6*I)*b*c)/d)*S
qrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d] + 25*Sqrt[3]*E^((2
*I)*(4*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-3*I)*b*(c + d*x
))/d] + 25*Sqrt[3]*E^((2*I)*a + ((8*I)*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*
Gamma[3/2, ((3*I)*b*(c + d*x))/d] - 9*Sqrt[5]*E^((10*I)*a)*Sqrt[(I*b*(c +
d*x))/d]*Gamma[3/2, ((-5*I)*b*(c + d*x))/d] - 9*Sqrt[5]*E^(((10*I)*b*c)/d)
*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((5*I)*b*(c + d*x))/d]))/(b*E^(((5*
I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

### 3.133.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^3(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}\sqrt{c+dx}\sin(a+bx)+\frac{1}{16}\sqrt{c+dx}\sin(3a+3bx)-\frac{1}{16}\sqrt{c+dx}\sin(5a+5bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\cos(5a+5bx)}{80b}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
-1/8*(Sqrt[c + d*x]*Cos[a + b*x])/b - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(48
*b) + (Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(80*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a
- (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(3/2
)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]
*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/10]*Cos[5*a - (5
*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(80*b^(3/2
)) + (Sqrt[d]*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[5*a - (5*b*c)/d])/(80*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*FresnelS[(S
qrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(48*b^(3/2
)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[
d]]*Sin[a - (b*c)/d])/(8*b^(3/2))
```

### 3.133.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.133.4 Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 447, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| derivativedivides | $-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-cb}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-cb}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-$ |
| default | $-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-cb}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{ad-cb}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-$ |

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+1/32/b*d*2^(1/2)
*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d))-1/96/b*d*(d*x+c)^(1/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*
c)/d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*F
resnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d
-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))
+1/160/b*d*(d*x+c)^(1/2)*cos(5*b/d*(d*x+c)+5*(a*d-b*c)/d)-1/1600/b*d*2^(1/
2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1
/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(
1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

### 3.133.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.78

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{9\sqrt{10}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-25\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\right.}\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_cos(
sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*co
s(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 45
0*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt
(d*x + c)*sqrt(b/(pi*d))) + 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sq
rt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*
sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b
*c - a*d)/d) - 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(10)*sqrt(d*
x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^5 - 5
*b*cos(b*x + a)^3)*sqrt(d*x + c))/b^2
```

### 3.133.6 Sympy [F]

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \sqrt{c+dx}\sin^3(a+bx)\cos^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x)**2, x)
```

### 3.133.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.48

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{\sqrt{2}\left(\frac{180\sqrt{2}\sqrt{dx+c}b^3\cos\left(\frac{5((dx+c)b-bc+ad)}{d}\right)}{d^2} - \frac{300\sqrt{2}\sqrt{dx+c}b^3\cos\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d^2} - \frac{1800\sqrt{2}\sqrt{dx+c}b^3\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d^2} - 9\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/28800*sqrt(2)*(180*sqrt(2)*sqrt(d*x + c)*b^3*cos(5*((d*x + c)*b - b*c +
a*d)/d)/d^2 - 300*sqrt(2)*sqrt(d*x + c)*b^3*cos(3*((d*x + c)*b - b*c + a*d
)/d)/d^2 - 1800*sqrt(2)*sqrt(d*x + c)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)
/d^2 - 9*(-(I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d
)/d)/d - (I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/
d)/d)*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - 25*((I - 1)*9^(1/4)*sqrt(pi)*b^2*
(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d + (I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^
2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 4
50*((I - 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d + (I + 1)*s
qrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(
I*b/d)) - 450*(-(I + 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d
 - (I - 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*
x + c)*sqrt(-I*b/d)) - 25*(-(I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*c
os(-3*(b*c - a*d)/d)/d - (I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(
-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - 9*((I + 1)*25^(1/
4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)/d + (I - 1)*25^(1/4)
*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*s
qrt(-5*I*b/d)))*d^2/b^4
```

### 3.133.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 1270, normalized size of antiderivative = 2.77

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/14400*(450*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b
*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*er
f(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(10)*
sqrt(pi)*(10*b*c - I*d)*d*erf(-1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)*b) + 450*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*I*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d
)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c + I
*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9
*sqrt(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)*b) - 30*(30*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*I
*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(10)*sqrt(pi)*d
*erf(-1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/...
```

### 3.133.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx=\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3\,\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

# 3.134 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

3.134.1 Optimal result . . . 1093
3.134.2 Mathematica [C] (verified) . . . 1094
3.134.3 Rubi [A] (verified) . . . 1095
3.134.4 Maple [A] (verified) . . . 1097
3.134.5 Fricas [A] (verification not implemented) . . . 1098
3.134.6 Sympy [F] . . . 1099
3.134.7 Maxima [C] (verification not implemented) . . . 1099
3.134.8 Giac [C] (verification not implemented) . . . 1100
3.134.9 Mupad [F(-1)] . . . 1100

## 3.134.1 Optimal result

Integrand size = 26, antiderivative size = 534

$$\begin{aligned}
\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx &= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b} \\
&- \frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b} + \frac{(c+dx)^{3/2}\cos(5a+5bx)}{80b} \\
&- \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}} \\
&- \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}} \\
&+ \frac{3d^{3/2}\sqrt{\frac{\pi}{10}}\cos\left(5a-\frac{5bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}} \\
&+ \frac{3d^{3/2}\sqrt{\frac{\pi}{10}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(5a-\frac{5bc}{d}\right)}{800b^{5/2}} \\
&- \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{96b^{5/2}} \\
&- \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{16b^{5/2}} + \frac{3d\sqrt{c+dx}\sin(a+bx)}{16b^2} \\
&+ \frac{d\sqrt{c+dx}\sin(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\sin(5a+5bx)}{800b^2}
\end{aligned}$$

output
```
-1/8*(d*x+c)^(3/2)*cos(b*x+a)/b-1/48*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b+1/80*(
d*x+c)^(3/2)*cos(5*b*x+5*a)/b+3/8000*d^(3/2)*cos(5*a-5*b*c/d)*FresnelS(b^(
1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(5/2)+3/
8000*d^(3/2)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin
(5*a-5*b*c/d)*10^(1/2)*Pi^(1/2)/b^(5/2)-1/576*d^(3/2)*cos(3*a-3*b*c/d)*Fre
snelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(
5/2)-1/576*d^(3/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2)
)*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/32*d^(3/2)*cos(a-b*c/d)*Fres
nelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5
/2)-3/32*d^(3/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*
sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d*sin(b*x+a)*(d*x+c)^(1/2)/b^2+
1/96*d*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b^2-3/800*d*sin(5*b*x+5*a)*(d*x+c)^(1/
2)/b^2
```

## 3.134.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.46 (sec) , antiderivative size = 1126, normalized size of antiderivative = 2.11

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{\sqrt{d}e^{-\frac{3i(ad+b(c+dx))}{d}}\left(-12\sqrt{b}\sqrt{d}e^{\frac{3ibc}{d}}\sqrt{c+dx}\left(-i+2bx+e^{6i(a+bx)}(i+2bx)\right)-(1-i)(2bc+id)e^{\frac{3ib(2}{}}\cdots\right.}{2304b^{5/2}} - \frac{\sqrt{d}e^{-\frac{5i(ad+b(c+dx))}{d}}\left(-20\sqrt{b}\sqrt{d}e^{\frac{5ibc}{d}}\sqrt{c+dx}\left(-3i+10bx+e^{10i(a+bx)}(3i+10bx)\right)-(1-i)(10bc+3id)e^{\frac{5ib(2}{}}\cdots\right.}{32000b^{5/2}} + \frac{icde^{-\frac{i(bc+ad)}{d}}\left(-e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)\right)}{16b^2\sqrt{c+dx}} + \frac{1}{16}c\left(-\frac{e^{3i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\Gamma\left(\frac{3}{2},-\frac{3ib(c+dx)}{d}\right)}{6\sqrt{3}b\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{-3i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\Gamma\left(\frac{3}{2},\frac{3ib(c+dx)}{d}\right)}{6\sqrt{3}b\sqrt{\frac{ib(c+dx)}{d}}}\right)-\frac{1}{16}c\left(-\frac{e^{5i\left(a-\frac{bc}{d}\right)}\sqrt{c+d}\cdots}{10\sqrt{5}b\sqrt{\cdots}}\right.$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
(Sqrt[d]*(-12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(-I + 2*b*x
+ E^((6*I)*(a + b*x))*(I + 2*b*x)) - (1 - I)*(2*b*c + I*d)*E^(((3*I)*b*(2*
c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]] + (1 + I)*((2*I)*b*c + d)*E^((3*I)*(2*a + b*x))*Sqrt[6*Pi]*Erfi[((1 +
 I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(2304*b^(5/2)*E^(((3*I)*(a
*d + b*(c + d*x)))/d)) - (Sqrt[d]*(-20*Sqrt[b]*Sqrt[d]*E^(((5*I)*b*c)/d)*S
qrt[c + d*x]*(-3*I + 10*b*x + E^((10*I)*(a + b*x))*(3*I + 10*b*x)) - (1 -
I)*(10*b*c + (3*I)*d)*E^(((5*I)*b*(2*c + d*x))/d)*Sqrt[10*Pi]*Erf[((1 + I)
*Sqrt[5/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + (1 + I)*((10*I)*b*c + 3*d)*E^
((5*I)*(2*a + b*x))*Sqrt[10*Pi]*Erfi[((1 + I)*Sqrt[5/2]*Sqrt[b]*Sqrt[c + d
*x])/Sqrt[d]]))/(32000*b^(5/2)*E^(((5*I)*(a*d + b*(c + d*x)))/d)) + ((I/16
)*c*d*(-(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-I)*b*(c + d*
x))/d]) + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d
*x))/d]))/(b^2*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x]) + (c*(-1/6*(E^((3*I)*(
a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-3*I)*b*(c + d*x))/d])/(Sqrt[3]*b
*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]*Gamma[3/2, ((3*I)*b*(c + d*x
))/d])/(6*Sqrt[3]*b*E^((3*I)*(a - (b*c)/d))*Sqrt[(I*b*(c + d*x))/d])))/16
- (c*(-1/10*(E^((5*I)*(a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-5*I)*b*(c
 + d*x))/d])/(Sqrt[5]*b*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]*Gamma
[3/2, ((5*I)*b*(c + d*x))/d])/(10*Sqrt[5]*b*E^((5*I)*(a - (b*c)/d))*Sqr...
```

### 3.134.3 Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin^3(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}(c+dx)^{3/2}\sin(a+bx)+\frac{1}{16}(c+dx)^{3/2}\sin(3a+3bx)-\frac{1}{16}(c+dx)^{3/2}\sin(5a+5bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(a+bx)}{16b^2}+\frac{d\sqrt{c+dx}\sin(3a+3bx)}{96b^2}-\frac{3d\sqrt{c+dx}\sin(5a+5bx)}{800b^2}-\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}\cos(5a+5bx)}{80b}$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
-1/8*((c + d*x)^(3/2)*Cos[a + b*x])/b - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])
/(48*b) + ((c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(80*b) - (3*d^(3/2)*Sqrt[Pi/2
]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(
16*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*S
qrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*C
os[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])
/(800*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt
[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt[Pi
/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)
/d])/(96*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqr
t[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + (3*d*Sqrt[c + d*x]*S
in[a + b*x])/(16*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(96*b^2) - (3*d
*Sqrt[c + d*x]*Sin[5*a + 5*b*x])/(800*b^2)
```

## 3.134. $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

### 3.134.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.134.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.09

| method | result |
|---|---|
| derivativedivides | $-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\right.\right.}{4b\sqrt{\frac{b}{d}}}\right.}{8b}$ |
| default | $-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\right.\right.}{4b\sqrt{\frac{b}{d}}}\right.}{8b}$ |

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+3/16/b*d*(1/2/b*
d*(d*x+c)^(1/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^
(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^
(1/2)/d)))-1/96/b*d*(d*x+c)^(3/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/32/b*
d*(1/6/b*d*(d*x+c)^(1/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)
*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2
)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/
2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))+1/160/b*d*(d*x+c)^(3/
2)*cos(5*b/d*(d*x+c)+5*(a*d-b*c)/d)-3/160/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(
5*b/d*(d*x+c)+5*(a*d-b*c)/d)-1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2
)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x
+c)^(1/2)/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d)))
```

### 3.134.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 427, normalized size of antiderivative = 0.80

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{27\sqrt{10}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)S\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-125\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)S\left(\sqrt{6}\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_s
in(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 125*sqrt(6)*pi*d^2*sqrt(b/(pi*
d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))
) - 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fr
esnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*
sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi
*d)))*sin(-3*(b*c - a*d)/d) + 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_co
s(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(30*(
b^2*d*x + b^2*c)*cos(b*x + a)^5 - 50*(b^2*d*x + b^2*c)*cos(b*x + a)^3 - (9
*b*d*cos(b*x + a)^4 - 13*b*d*cos(b*x + a)^2 - 26*b*d)*sin(b*x + a))*sqrt(d
*x + c))/b^3
```

### 3.134.6 Sympy [F]

$$\int (c+dx)^{3/2} \cos^2(a+bx) \sin^3(a+bx) \, dx = \int (c+dx)^{\frac{3}{2}} \sin^3(a+bx) \cos^2(a+bx) \, dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)**3*cos(a + b*x)**2, x)
```

### 3.134.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 760, normalized size of antiderivative = 1.42

$$\int (c+dx)^{3/2} \cos^2(a+bx) \sin^3(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/288000*sqrt(2)*(1800*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(5*((d*x + c)*b - b*
c + a*d)/d)/d^2 - 3000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(3*((d*x + c)*b - b*
c + a*d)/d)/d^2 - 18000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(((d*x + c)*b - b*c
 + a*d)/d)/d^2 - 540*sqrt(2)*sqrt(d*x + c)*b^3*sin(5*((d*x + c)*b - b*c +
a*d)/d)/d + 1500*sqrt(2)*sqrt(d*x + c)*b^3*sin(3*((d*x + c)*b - b*c + a*d)
/d)/d + 27000*sqrt(2)*sqrt(d*x + c)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d
 + 27*((I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)
 - (I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*er
f(sqrt(d*x + c)*sqrt(5*I*b/d)) + 125*(-(I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d
^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(
1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + 6750*(-(I +
 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I - 1)*sqrt(pi)*b^
2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + 67
50*((I - 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I + 1)*sqr
t(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b
/d)) + 125*((I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d
)/d) - (I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))
*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + 27*(-(I - 1)*25^(1/4)*sqrt(pi)*b^2*(b
^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) + (I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d
^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2...
```

### 3.134.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 2314, normalized size of antiderivative = 4.33

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/144000*(300*(30*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*
d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*
sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(
b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(10)*sqrt(pi)*d*erf(-1/2*I*sqrt(10
)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 30*sqrt(2)*sqrt(pi)*d*erf(-1
/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b
*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*
d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(
10)*sqrt(pi)*d*erf(1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)))*c^2 + d^2*(2250*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*
erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*I*(2*I*
(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-
I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 125*(sqrt(6)*sqrt(pi)*(12*b^2
*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt...
```

### 3.134.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \cos(a+b\,x)^2\sin(a+b\,x)^3\,(c+d\,x)^{3/2}\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

# 3.135 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

3.135.1 Optimal result . . . 1102
3.135.2 Mathematica [C] (verified) . . . 1103
3.135.3 Rubi [A] (verified) . . . 1104
3.135.4 Maple [A] (verified) . . . 1106
3.135.5 Fricas [A] (verification not implemented) . . . 1108
3.135.6 Sympy [F(-1)] . . . 1108
3.135.7 Maxima [C] (verification not implemented) . . . 1109
3.135.8 Giac [C] (verification not implemented) . . . 1109
3.135.9 Mupad [F(-1)] . . . 1110

### 3.135.1 Optimal result

Integrand size = 26, antiderivative size = 615

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{15d^2\sqrt{c+dx}\cos(a+bx)}{32b^3} - \frac{(c+dx)^{5/2}\cos(a+bx)}{8b} + \frac{5d^2\sqrt{c+dx}\cos(3a+3bx)}{576b^3} - \frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b} - \frac{3d^2\sqrt{c+dx}\cos(5a+5bx)}{1600b^3} + \frac{(c+dx)^{5/2}\cos(5a+5bx)}{80b} - \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}} - \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}} + \frac{3d^{5/2}\sqrt{\frac{\pi}{10}}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}} - \frac{3d^{5/2}\sqrt{\frac{\pi}{10}}\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(5a-\frac{5bc}{d}\right)}{1600b^{7/2}} + \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{576b^{7/2}} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{32b^{7/2}} + \frac{5d(c+dx)^{3/2}\sin(a+bx)}{16b^2} + \frac{5d(c+dx)^{3/2}\sin(3a+3bx)}{288b^2} - \frac{d(c+dx)^{3/2}\sin(5a+5bx)}{160b^2}$$

output
```
-1/8*(d*x+c)^(5/2)*cos(b*x+a)/b-1/48*(d*x+c)^(5/2)*cos(3*b*x+3*a)/b+1/80*(
d*x+c)^(5/2)*cos(5*b*x+5*a)/b+5/16*d*(d*x+c)^(3/2)*sin(b*x+a)/b^2+5/288*d*
(d*x+c)^(3/2)*sin(3*b*x+3*a)/b^2-1/160*d*(d*x+c)^(3/2)*sin(5*b*x+5*a)/b^2+
3/16000*d^(5/2)*cos(5*a-5*b*c/d)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(7/2)-3/16000*d^(5/2)*FresnelS(b^(1/2
)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*10^(1/2)*Pi^(1
/2)/b^(7/2)-5/3456*d^(5/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)+5/3456*d^(5/2)*Fresnel
S(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)
*Pi^(1/2)/b^(7/2)-15/64*d^(5/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(
1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/64*d^(5/2)*Fresnel
S(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^
(1/2)/b^(7/2)+15/32*d^2*cos(b*x+a)*(d*x+c)^(1/2)/b^3+5/576*d^2*cos(3*b*x+3
*a)*(d*x+c)^(1/2)/b^3-3/1600*d^2*cos(5*b*x+5*a)*(d*x+c)^(1/2)/b^3
```

### 3.135.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.02 (sec) , antiderivative size = 2177, normalized size of antiderivative = 3.54

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

output
```
(c*Sqrt[d]*(-12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(-I + 2*b*
x + E^((6*I)*(a + b*x))*(I + 2*b*x)) - (1 - I)*(2*b*c + I*d)*E^(((3*I)*b*(
2*c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sq
rt[d]] + (1 + I)*((2*I)*b*c + d)*E^((3*I)*(2*a + b*x))*Sqrt[6*Pi]*Erfi[((1
 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(1152*b^(5/2)*E^(((3*I)*
(a*d + b*(c + d*x)))/d)) - (c*Sqrt[d]*(-20*Sqrt[b]*Sqrt[d]*E^(((5*I)*b*c)/
d)*Sqrt[c + d*x]*(-3*I + 10*b*x + E^((10*I)*(a + b*x))*(3*I + 10*b*x)) - (
1 - I)*(10*b*c + (3*I)*d)*E^(((5*I)*b*(2*c + d*x))/d)*Sqrt[10*Pi]*Erf[((1
+ I)*Sqrt[5/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + (1 + I)*((10*I)*b*c + 3*d
)*E^((5*I)*(2*a + b*x))*Sqrt[10*Pi]*Erfi[((1 + I)*Sqrt[5/2]*Sqrt[b]*Sqrt[c
 + d*x])/Sqrt[d]]))/(16000*b^(5/2)*E^(((5*I)*(a*d + b*(c + d*x)))/d)) + ((
I/16)*c^2*d*(-(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-I)*b*(
c + d*x))/d]) + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, (I*b*
(c + d*x))/d]))/(b^2*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x]) + (c^2*(-1/6*(E^
((3*I)*(a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-3*I)*b*(c + d*x))/d])/(S
qrt[3]*b*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]*Gamma[3/2, ((3*I)*b*
(c + d*x))/d])/(6*Sqrt[3]*b*E^((3*I)*(a - (b*c)/d))*Sqrt[(I*b*(c + d*x))/d
])))/16 - (c^2*(-1/10*(E^((5*I)*(a - (b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((
-5*I)*b*(c + d*x))/d])/(Sqrt[5]*b*Sqrt[((-I)*b*(c + d*x))/d]) - (Sqrt[c +
d*x]*Gamma[3/2, ((5*I)*b*(c + d*x))/d])/(10*Sqrt[5]*b*E^((5*I)*(a - (b*...
```

## 3.135.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin^3(a+bx)\cos^2(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}(c+dx)^{5/2}\sin(a+bx)+\frac{1}{16}(c+dx)^{5/2}\sin(3a+3bx)-\frac{1}{16}(c+dx)^{5/2}\sin(5a+5bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\cos\left(5a-\frac{5bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\sin\left(5a-\frac{5bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(a+bx)}{32b^3}+\frac{5d^2\sqrt{c+dx}\cos(3a+3bx)}{576b^3}-\frac{3d^2\sqrt{c+dx}\cos(5a+5bx)}{1600b^3}+\frac{5d(c+dx)^{3/2}\sin(a+bx)}{16b^2}+\frac{5d(c+dx)^{3/2}\sin(3a+3bx)}{288b^2}-\frac{d(c+dx)^{3/2}\sin(5a+5bx)}{160b^2}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{5/2}\cos(5a+5bx)}{80b}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

3.135. $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

output
```
(15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(32*b^3) - ((c + d*x)^(5/2)*Cos[a + b*
x])/(8*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^
(5/2)*Cos[3*a + 3*b*x])/(48*b) - (3*d^2*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(1
600*b^3) + ((c + d*x)^(5/2)*Cos[5*a + 5*b*x])/(80*b) - (15*d^(5/2)*Sqrt[Pi
/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])
/(32*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[
b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) + (3*d^(5/2)*Sqrt[Pi/
10]*Cos[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt
[d]])/(1600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi
]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) + (5*d^(5/2
)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a
- (3*b*c)/d])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqr
t[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) + (5*d*(c +
 d*x)^(3/2)*Sin[a + b*x])/(16*b^2) + (5*d*(c + d*x)^(3/2)*Sin[3*a + 3*b*x]
)/(288*b^2) - (d*(c + d*x)^(3/2)*Sin[5*a + 5*b*x])/(160*b^2)
```

### 3.135.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.135.4 Maple [A] (verified)

Time = 11.52 (sec) , antiderivative size = 719, normalized size of antiderivative = 1.17

| method | result |
|---|---|
| derivativedivides | $-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{8b}\cdots$ |
| default | $-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{8b}\cdots$ |

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+5/16/b*d*(1/2/b*
d*(d*x+c)^(3/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(1/
2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(
(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1
/96/b*d*(d*x+c)^(5/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+5/96/b*d*(1/6/b*d*(
d*x+c)^(3/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1
/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/
d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)
*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(
b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))+1/160/b*d*(d*x+c)^(5/2)*cos(5*b/d*(d*x+c)
+5*(a*d-b*c)/d)-1/32/b*d*(1/10/b*d*(d*x+c)^(3/2)*sin(5*b/d*(d*x+c)+5*(a*d-
b*c)/d)-3/10/b*d*(-1/10/b*d*(d*x+c)^(1/2)*cos(5*b/d*(d*x+c)+5*(a*d-b*c)/d)
+1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*Fresne
lC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(5*(a*d-b*c)
/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

## 3.135. $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

### 3.135.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 521, normalized size of antiderivative = 0.85

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \frac{81\sqrt{10}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-625\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_
cos(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 625*sqrt(6)*pi*d^3*sqrt(b/(pi
*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)
)) - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(
sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d
))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) +
 625*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(
b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fresn
el_sin(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*
(9*(20*b^3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2 - 3*b*d^2)*cos(b*x + a)^5 +
 390*b*d^2*cos(b*x + a) - 5*(60*b^3*d^2*x^2 + 120*b^3*c*d*x + 60*b^3*c^2 -
 13*b*d^2)*cos(b*x + a)^3 + 10*(26*b^2*d^2*x - 9*(b^2*d^2*x + b^2*c*d)*cos
(b*x + a)^4 + 26*b^2*c*d + 13*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*sin(b*
x + a))*sqrt(d*x + c))/b^4
```

### 3.135.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

output
```
Timed out
```

### 3.135.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 826, normalized size of antiderivative = 1.34

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/1728000*sqrt(2)*(5400*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(5*((d*x + c)*b -
b*c + a*d)/d)/d - 15000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(3*((d*x + c)*b - b
*c + a*d)/d)/d - 270000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(((d*x + c)*b - b*c
 + a*d)/d)/d - 540*(20*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 3*sqrt(2)*sqrt(d*
x + c)*b^3)*cos(5*((d*x + c)*b - b*c + a*d)/d) + 1500*(12*sqrt(2)*(d*x + c
)^(5/2)*b^5/d^2 - 5*sqrt(2)*sqrt(d*x + c)*b^3)*cos(3*((d*x + c)*b - b*c +
a*d)/d) + 27000*(4*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 15*sqrt(2)*sqrt(d*x +
 c)*b^3)*cos(((d*x + c)*b - b*c + a*d)/d) - 81*(-(I - 1)*25^(1/4)*sqrt(pi)
*b^2*d*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) - (I + 1)*25^(1/4)*sqrt(pi)*b
^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d
)) - 625*((I - 1)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d
)/d) + (I + 1)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d
))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 101250*((I - 1)*sqrt(pi)*b^2*d*(b^2/
d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*si
n(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 101250*(-(I + 1)*sqrt(
pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*sqrt(pi)*b^2*d*(b^
2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - 625*(-
(I + 1)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I
- 1)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(-3*I*b/d)) - 81*((I + 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d...
```

### 3.135.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.71 (sec) , antiderivative size = 3706, normalized size of antiderivative = 6.03

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/864000*(1800*(30*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)
*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt
(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(10)*sqrt(pi)*d*erf(-1/2*I*sqrt(1
0)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a
*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 30*sqrt(2)*sqrt(pi)*d*erf(-
1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*
b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)
*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt
(10)*sqrt(pi)*d*erf(1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)))*c^3 + 18*c*d^2*(2250*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d
^2)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*I
*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)
*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 125*(sqrt(6)*sqrt(pi)*(
12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*...
```

### 3.135.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx=\int \cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^3\left(c+d\,x\right)^{5/2}\,dx$$

input
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

# 3.136 $\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\,dx$

3.136.1 Optimal result . . . 1111
3.136.2 Mathematica [A] (verified) . . . 1112
3.136.3 Rubi [A] (verified) . . . 1112
3.136.4 Maple [F] . . . 1114
3.136.5 Fricas [A] (verification not implemented) . . . 1114
3.136.6 Sympy [F(-2)] . . . 1114
3.136.7 Maxima [F] . . . 1115
3.136.8 Giac [F] . . . 1115
3.136.9 Mupad [F(-1)] . . . 1115

## 3.136.1 Optimal result

Integrand size = 22, antiderivative size = 273

$$\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\,dx = -\frac{2^{-4-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} - \frac{2^{-4-m}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2ib(c+dx)}{d}\right)}{b} - \frac{2^{-2(3+m)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{4ib(c+dx)}{d}\right)}{b} - \frac{2^{-2(3+m)}e^{-4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{4ib(c+dx)}{d}\right)}{b}$$

output
```
-2^(-4-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*
b*(d*x+c)/d)^m)-2^(-4-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a
-b*c/d))/((I*b*(d*x+c)/d)^m)-exp(4*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-4*I*b
*(d*x+c)/d)/(2^(6+2*m))/b/((-I*b*(d*x+c)/d)^m)-(d*x+c)^m*GAMMA(1+m,4*I*b*(
d*x+c)/d)/(2^(6+2*m))/b/exp(4*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

### 3.136.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.90

$$\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{4^{-3-m}e^{-\frac{4i(bc+ad)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(2^{2+m}e^{2i\left(3a+\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^m\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)+2^{2+m}e^{2i\left(a\right.}\right.}{}$$

input
```
Integrate[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
-((4^(-3 - m)*(c + d*x)^m*(2^(2 + m)*E^((2*I)*(3*a + (b*c)/d))*((I*b*(c +
d*x))/d)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d] + 2^(2 + m)*E^((2*I)*(a +
(3*b*c)/d))*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d] +
 E^((8*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-4*I)*b*(c + d*x))/d] +
E^(((8*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((4*I)*b*(c + d*x)
)/d]))/(b*E^(((4*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^m))
```

### 3.136.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\cos^3(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{4}\sin(2a+2bx)(c+dx)^m+\frac{1}{8}\sin(4a+4bx)(c+dx)^m\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2^{-m-4}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{4ib(c+dx)}{d}\right)}{b}-\frac{2^{-m-4}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-2(m+3)}e^{-4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{4ib(c+dx)}{d}\right)}{b}$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
-((2^(-4 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(
c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)) - (2^(-4 - m)*(c + d*x)^m*Gamm
a[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x
))/d)^m) - (E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-4*I)*b*(c
+ d*x))/d])/(2^(2*(3 + m))*b*(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamm
a[1 + m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*E^((4*I)*(a - (b*c)/d))*
((I*b*(c + d*x))/d)^m)
```

### 3.136.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.136.4 Maple [F]

$$\int (dx+c)^m \cos(xb+a)^3 \sin(xb+a)\, dx$$

input
```
int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x)
```

output
```
int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x)
```

### 3.136.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.69

$$\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\, dx =$$
$$-\frac{4\, e^{\left(-\frac{dm\log\left(-\frac{2i\,b}{d}\right)+2i\,bc-2i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{2\,(i\,bdx+i\,bc)}{d}\right)+e^{\left(-\frac{dm\log\left(-\frac{4i\,b}{d}\right)+4i\,bc-4i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{4\,(i\,bdx+i\,bc)}{d}\right)+4}{64\,b}$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/64*(4*e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)/d)*gamma(m + 1, -2*(I
*b*d*x + I*b*c)/d) + e^(-(d*m*log(-4*I*b/d) + 4*I*b*c - 4*I*a*d)/d)*gamma(
m + 1, -4*(I*b*d*x + I*b*c)/d) + 4*e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a
*d)/d)*gamma(m + 1, -2*(-I*b*d*x - I*b*c)/d) + e^(-(d*m*log(4*I*b/d) - 4*I
*b*c + 4*I*a*d)/d)*gamma(m + 1, -4*(-I*b*d*x - I*b*c)/d))/b
```

### 3.136.6 Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d*x+c)**m*cos(b*x+a)**3*sin(b*x+a),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.136.7 Maxima [F]

$$\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a), x)
```

### 3.136.8 Giac [F]

$$\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a), x)
```

### 3.136.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\,dx = \int \cos(a+b\,x)^3 \sin(a+b\,x)\,(c+d\,x)^m\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^m, x)
```

# 3.137 $\int (c+dx)^4 \cos^3(a+bx) \sin(a+bx)\,dx$


3.137.1 Optimal result . . . 1116
3.137.2 Mathematica [A] (verified) . . . 1117
3.137.3 Rubi [A] (verified) . . . 1117
3.137.4 Maple [A] (verified) . . . 1121
3.137.5 Fricas [A] (verification not implemented) . . . 1121
3.137.6 Sympy [B] (verification not implemented) . . . 1122
3.137.7 Maxima [B] (verification not implemented) . . . 1123
3.137.8 Giac [A] (verification not implemented) . . . 1124
3.137.9 Mupad [B] (verification not implemented) . . . 1124


## 3.137.1 Optimal result

Integrand size = 22, antiderivative size = 260

$$\int (c+dx)^4 \cos^3(a+bx) \sin(a+bx)\,dx = -\frac{45cd^3x}{64b^3} - \frac{45d^4x^2}{128b^3} + \frac{3(c+dx)^4}{32b} - \frac{45d^4\cos^2(a+bx)}{128b^5} + \frac{9d^2(c+dx)^2\cos^2(a+bx)}{16b^3} - \frac{3d^4\cos^4(a+bx)}{128b^5} + \frac{3d^2(c+dx)^2\cos^4(a+bx)}{16b^3} - \frac{(c+dx)^4\cos^4(a+bx)}{4b} - \frac{45d^3(c+dx)\cos(a+bx)\sin(a+bx)}{64b^4} + \frac{3d(c+dx)^3\cos(a+bx)\sin(a+bx)}{8b^2} - \frac{3d^3(c+dx)\cos^3(a+bx)\sin(a+bx)}{32b^4} + \frac{d(c+dx)^3\cos^3(a+bx)\sin(a+bx)}{4b^2}$$

output
```
-45/64*c*d^3*x/b^3-45/128*d^4*x^2/b^3+3/32*(d*x+c)^4/b-45/128*d^4*cos(b*x+
a)^2/b^5+9/16*d^2*(d*x+c)^2*cos(b*x+a)^2/b^3-3/128*d^4*cos(b*x+a)^4/b^5+3/
16*d^2*(d*x+c)^2*cos(b*x+a)^4/b^3-1/4*(d*x+c)^4*cos(b*x+a)^4/b-45/64*d^3*(
d*x+c)*cos(b*x+a)*sin(b*x+a)/b^4+3/8*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2
-3/32*d^3*(d*x+c)*cos(b*x+a)^3*sin(b*x+a)/b^4+1/4*d*(d*x+c)^3*cos(b*x+a)^3
*sin(b*x+a)/b^2
```

### 3.137.2 Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.61

$$\int (c+dx)^4 \cos^3(a+bx) \sin(a+bx) \, dx =$$
$$-\frac{64(3d^4-6b^2d^2(c+dx)^2+2b^4(c+dx)^4)\cos(2(a+bx))+(3d^4-24b^2d^2(c+dx)^2+32b^4(c+dx)^4)\cos}{1}$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
-1/1024*(64*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a +
 b*x)] + (3*d^4 - 24*b^2*d^2*(c + d*x)^2 + 32*b^4*(c + d*x)^4)*Cos[4*(a +
b*x)] - 8*b*d*(c + d*x)*(16*(-3*d^2 + 2*b^2*(c + d*x)^2) + (-3*d^2 + 8*b^2
*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*x)])/b^5
```

### 3.137.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {4905, 3042, 3792, 3042, 3791, 3042, 3791, 17, 3792, 17, 3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sin(a+bx) \cos^3(a+bx) \, dx$$
$$\Big\downarrow \text{4905}$$
$$\frac{d\int (c+dx)^3 \cos^4(a+bx)dx}{b} - \frac{(c+dx)^4 \cos^4(a+bx)}{4b}$$
$$\Big\downarrow \text{3042}$$
$$\frac{d\int (c+dx)^3 \sin\left(a+bx+\frac{\pi}{2}\right)^4 dx}{b} - \frac{(c+dx)^4 \cos^4(a+bx)}{4b}$$
$$\Big\downarrow \text{3792}$$
$$\frac{d\left(-\frac{3d^2\int (c+dx)\cos^4(a+bx)dx}{8b^2}+\frac{3}{4}\int (c+dx)^3\cos^2(a+bx)dx+\frac{3d(c+dx)^2\cos^4(a+bx)}{16b^2}+\frac{(c+dx)^3\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{b} - \frac{(c+dx)^4 \cos^4(a+bx)}{4b}$$

$$\Big\downarrow \; 3042$$

$$\frac{d\left(-\frac{3d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^4dx}{8b^2}+\frac{3}{4}\int(c+dx)^3\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{3d(c+dx)^2\cos^4(a+bx)}{16b^2}+\frac{(c+dx)^3\sin(a+bx)\cos^3(a+bx)}{4b}\right.}{b}$$
$$\frac{(c+dx)^4\cos^4(a+bx)}{4b}$$

$$\Big\downarrow \; 3791$$

$$\frac{d\left(-\frac{3d^2\left(\frac{3}{4}\int(c+dx)\cos^2(a+bx)dx+\frac{d\cos^4(a+bx)}{16b^2}+\frac{(c+dx)\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int(c+dx)^3\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{3d(c+dx)^2\cos}{16b^2}\right.}{b}$$
$$\frac{(c+dx)^4\cos^4(a+bx)}{4b}$$

$$\Big\downarrow \; 3042$$

$$\frac{d\left(-\frac{3d^2\left(\frac{3}{4}\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{d\cos^4(a+bx)}{16b^2}+\frac{(c+dx)\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int(c+dx)^3\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{3d(c+dx)}{}\right.}{b}$$
$$\frac{(c+dx)^4\cos^4(a+bx)}{4b}$$

$$\Big\downarrow \; 3791$$

$$\frac{d\left(-\frac{3d^2\left(\frac{3}{4}\left(\frac{1}{2}\int(c+dx)dx+\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{d\cos^4(a+bx)}{16b^2}+\frac{(c+dx)\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int(c+dx)^3\sin(a\right.}{b}$$
$$\frac{(c+dx)^4\cos^4(a+bx)}{4b}$$

$$\Big\downarrow \; 17$$

$$\frac{d\left(\frac{3}{4}\int(c+dx)^3\sin\left(a+bx+\frac{\pi}{2}\right)^2dx-\frac{3d^2\left(\frac{3}{4}\left(\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)+\frac{d\cos^4(a+bx)}{16b^2}+\frac{(c+dx)\sin(a+bx)\cos}{4b}\right.}{8b^2}\right.}{b}$$
$$\frac{(c+dx)^4\cos^4(a+bx)}{4b}$$

$$\Big\downarrow \; 3792$$

3.137. $\int(c+dx)^4\cos^3(a+bx)\sin(a+bx)\,dx$

$$\frac{d\left(\frac{3}{4}\left(-\frac{3d^2\int(c+dx)\cos^2(a+bx)dx}{2b^2}+\frac{1}{2}\int(c+dx)^3dx+\frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}\right)-\frac{3d^2\left(\frac{3}{4}\left(\frac{d\cos^2(a+\cdots}{4b^2}\right.\right.}{\cdots}\right.}{\cdots}$$

$$\frac{(c+dx)^4\cos^4(a+bx)}{4b}$$

$\downarrow$ 17

$$\frac{d\left(\frac{3}{4}\left(-\frac{3d^2\int(c+dx)\cos^2(a+bx)dx}{2b^2}+\frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)-\frac{3d^2\left(\frac{3}{4}\left(\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx}{\cdots}\right.\right.}{b}\right.}{\cdots}$$

$$\frac{(c+dx)^4\cos^4(a+bx)}{4b}$$

$\downarrow$ 3042

$$\frac{d\left(\frac{3}{4}\left(-\frac{3d^2\int(c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^2dx}{2b^2}+\frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)-\frac{3d^2\left(\frac{3}{4}\left(\frac{d\cos^2(a+bx)}{4b^2}+\cdots\right.\right.}{b}\right.}{\cdots}$$

$$\frac{(c+dx)^4\cos^4(a+bx)}{4b}$$

$\downarrow$ 3791

$$\frac{d\left(\frac{3}{4}\left(-\frac{3d^2\left(\frac{1}{2}\int(c+dx)dx+\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)}{8d}\cdots\right.\right.}{\cdots}$$

$$\frac{(c+dx)^4\cos^4(a+bx)}{4b}$$

$\downarrow$ 17

$$\frac{d\left(\frac{3}{4}\left(-\frac{3d^2\left(\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{2b^2}+\frac{3d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)\cdots\right.}{\cdots}$$

$$\frac{(c+dx)^4\cos^4(a+bx)}{4b}$$

input

```
Int[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x],x]
```

## 3.137. $\int(c+dx)^4\cos^3(a+bx)\sin(a+bx)\,dx$

output
```
-1/4*((c + d*x)^4*Cos[a + b*x]^4)/b + (d*((3*d*(c + d*x)^2*Cos[a + b*x]^4)
/(16*b^2) + ((c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x])/(4*b) - (3*d^2*((d*C
os[a + b*x]^4)/(16*b^2) + ((c + d*x)*Cos[a + b*x]^3*Sin[a + b*x])/(4*b) +
(3*((c + d*x)^2/(4*d) + (d*Cos[a + b*x]^2)/(4*b^2) + ((c + d*x)*Cos[a + b*
x]*Sin[a + b*x])/(2*b)))/4))/(8*b^2) + (3*((c + d*x)^4/(8*d) + (3*d*(c + d
*x)^2*Cos[a + b*x]^2)/(4*b^2) + ((c + d*x)^3*Cos[a + b*x]*Sin[a + b*x])/(2
*b) - (3*d^2*((c + d*x)^2/(4*d) + (d*Cos[a + b*x]^2)/(4*b^2) + ((c + d*x)*
Cos[a + b*x]*Sin[a + b*x])/(2*b)))/(2*b^2)))/4))/b
```

### 3.137.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

### 3.137.4 Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.72

| method | result |
|---|---|
| parallelrisch | $\frac{\left(-128b^4(dx+c)^4+384d^2(dx+c)^2b^2-192d^4\right)\cos(2xb+2a)+\left(-32b^4(dx+c)^4+24d^2(dx+c)^2b^2-3d^4\right)\cos(4xb+4a)+256b}{1024}$ |
| risch | $-\frac{(32d^4x^4b^4+128b^4c\,d^3x^3+192b^4c^2d^2x^2+128b^4c^3dx+32b^4c^4-24b^2d^4x^2-48b^2c\,d^3x-24b^2c^2d^2+3d^4)\cos(4xb+4a)}{1024b^5}+$ |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |

input
```
int((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/1024*((-128*b^4*(d*x+c)^4+384*d^2*(d*x+c)^2*b^2-192*d^4)*cos(2*b*x+2*a)+
(-32*b^4*(d*x+c)^4+24*d^2*(d*x+c)^2*b^2-3*d^4)*cos(4*b*x+4*a)+256*b*((d*x+
c)^2*b^2-3/2*d^2)*d*(d*x+c)*sin(2*b*x+2*a)+32*((d*x+c)^2*b^2-3/8*d^2)*d*(d
*x+c)*b*sin(4*b*x+4*a)+160*b^4*c^4-408*b^2*c^2*d^2+195*d^4)/b^5
```

### 3.137.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.45

$$\int (c+dx)^4\cos^3(a+bx)\sin(a+bx)\,dx$$
$$=\frac{12\,b^4d^4x^4+48\,b^4cd^3x^3-(32\,b^4d^4x^4+128\,b^4cd^3x^3+32\,b^4c^4-24\,b^2c^2d^2+3\,d^4+24\,(8\,b^4c^2d^2-b^2d^4)x^2+}{}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/128*(12*b^4*d^4*x^4 + 48*b^4*c*d^3*x^3 - (32*b^4*d^4*x^4 + 128*b^4*c*d^3
*x^3 + 32*b^4*c^4 - 24*b^2*c^2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*
x^2 + 16*(8*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^4 + 9*(8*b^4*c^2*d^2
- 5*b^2*d^4)*x^2 + 9*(8*b^2*d^4*x^2 + 16*b^2*c*d^3*x + 8*b^2*c^2*d^2 - 5*d
^4)*cos(b*x + a)^2 + 6*(8*b^4*c^3*d - 15*b^2*c*d^3)*x + 2*(2*(8*b^3*d^4*x^
3 + 24*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(8*b^3*c^2*d^2 - b*d^4)
*x)*cos(b*x + a)^3 + 3*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 1
5*b*c*d^3 + 3*(8*b^3*c^2*d^2 - 5*b*d^4)*x)*cos(b*x + a))*sin(b*x + a))/b^5
```

### 3.137.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(262) = 524$.

Time = 0.88 (sec) , antiderivative size = 935, normalized size of antiderivative = 3.60

$$\int (c+dx)^4\cos^3(a+bx)\sin(a+bx)\,dx$$
$$= \begin{cases} -\frac{c^4\cos^4(a+bx)}{4b} + \frac{3c^3dx\sin^4(a+bx)}{8b} + \frac{3c^3dx\sin^2(a+bx)\cos^2(a+bx)}{4b} - \frac{5c^3dx\cos^4(a+bx)}{8b} + \frac{9c^2d^2x^2\sin^4(a+bx)}{16b} + \frac{9c^2d^2x^2\sin^2(a\ldots \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\sin(a)\cos^3(a) \end{cases}$$

input
```
integrate((d*x+c)**4*cos(b*x+a)**3*sin(b*x+a),x)
```

output
```
Piecewise((-c**4*cos(a + b*x)**4/(4*b) + 3*c**3*d*x*sin(a + b*x)**4/(8*b)
+ 3*c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(4*b) - 5*c**3*d*x*cos(a + b*
x)**4/(8*b) + 9*c**2*d**2*x**2*sin(a + b*x)**4/(16*b) + 9*c**2*d**2*x**2*s
in(a + b*x)**2*cos(a + b*x)**2/(8*b) - 15*c**2*d**2*x**2*cos(a + b*x)**4/(
16*b) + 3*c*d**3*x**3*sin(a + b*x)**4/(8*b) + 3*c*d**3*x**3*sin(a + b*x)**
2*cos(a + b*x)**2/(4*b) - 5*c*d**3*x**3*cos(a + b*x)**4/(8*b) + 3*d**4*x**
4*sin(a + b*x)**4/(32*b) + 3*d**4*x**4*sin(a + b*x)**2*cos(a + b*x)**2/(16
*b) - 5*d**4*x**4*cos(a + b*x)**4/(32*b) + 3*c**3*d*sin(a + b*x)**3*cos(a
+ b*x)/(8*b**2) + 5*c**3*d*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 9*c**2*
d**2*x*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 15*c**2*d**2*x*sin(a + b*x)
*cos(a + b*x)**3/(8*b**2) + 9*c*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)/(8*
b**2) + 15*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 3*d**4*x**3
*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 5*d**4*x**3*sin(a + b*x)*cos(a +
b*x)**3/(8*b**2) - 9*c**2*d**2*sin(a + b*x)**4/(32*b**3) + 15*c**2*d**2*co
s(a + b*x)**4/(32*b**3) - 45*c*d**3*x*sin(a + b*x)**4/(64*b**3) - 9*c*d**3
*x*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**3) + 51*c*d**3*x*cos(a + b*x)**4
/(64*b**3) - 45*d**4*x**2*sin(a + b*x)**4/(128*b**3) - 9*d**4*x**2*sin(a +
 b*x)**2*cos(a + b*x)**2/(64*b**3) + 51*d**4*x**2*cos(a + b*x)**4/(128*b**
3) - 45*c*d**3*sin(a + b*x)**3*cos(a + b*x)/(64*b**4) - 51*c*d**3*sin(a +
b*x)*cos(a + b*x)**3/(64*b**4) - 45*d**4*x*sin(a + b*x)**3*cos(a + b*x)...
```

### 3.137.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 967 vs. $2(236) = 472$.

Time = 0.27 (sec) , antiderivative size = 967, normalized size of antiderivative = 3.72

$$\int (c+dx)^4 \cos^3(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/1024*(256*c^4*cos(b*x + a)^4 - 1024*a*c^3*d*cos(b*x + a)^4/b + 1536*a^2
*c^2*d^2*cos(b*x + a)^4/b^2 - 1024*a^3*c*d^3*cos(b*x + a)^4/b^3 + 256*a^4*
d^4*cos(b*x + a)^4/b^4 + 32*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*c
os(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*c^3*d/b - 96*(4*(
b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*
a) - 8*sin(2*b*x + 2*a))*a*c^2*d^2/b^2 + 96*(4*(b*x + a)*cos(4*b*x + 4*a)
+ 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*a
^2*c*d^3/b^3 - 32*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x +
 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*a^3*d^4/b^4 + 24*((8*(b*x +
 a)^2 - 1)*cos(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*
(b*x + a)*sin(4*b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 -
48*((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*
x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*a
*c*d^3/b^3 + 24*((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) + 16*(2*(b*x + a)^2
- 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) - 32*(b*x + a)*sin(2*
b*x + 2*a))*a^2*d^4/b^4 + 4*(4*(8*(b*x + a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4
*a) + 64*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3*(8*(b*x + a)^2
 - 1)*sin(4*b*x + 4*a) - 96*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*c*d^3/b^
3 - 4*(4*(8*(b*x + a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) + 64*(2*(b*x + a)^
3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4...
```

### 3.137.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.39

$$\int (c+dx)^4\cos^3(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{(32\,b^4d^4x^4+128\,b^4cd^3x^3+192\,b^4c^2d^2x^2+128\,b^4c^3dx+32\,b^4c^4-24\,b^2d^4x^2-48\,b^2cd^3x-24\,b^2c^2d^2+3\ldots}{1024\,b^5}$$
$$-\frac{(2\,b^4d^4x^4+8\,b^4cd^3x^3+12\,b^4c^2d^2x^2+8\,b^4c^3dx+2\,b^4c^4-6\,b^2d^4x^2-12\,b^2cd^3x-6\,b^2c^2d^2+3\,d^4)\cos(2\,b\ldots}{16\,b^5}$$
$$+\frac{(8\,b^3d^4x^3+24\,b^3cd^3x^2+24\,b^3c^2d^2x+8\,b^3c^3d-3\,bd^4x-3\,bcd^3)\sin(4\,bx+4\,a)}{256\,b^5}$$
$$+\frac{(2\,b^3d^4x^3+6\,b^3cd^3x^2+6\,b^3c^2d^2x+2\,b^3c^3d-3\,bd^4x-3\,bcd^3)\sin(2\,bx+2\,a)}{8\,b^5}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/1024*(32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 192*b^4*c^2*d^2*x^2 + 128*b^
4*c^3*d*x + 32*b^4*c^4 - 24*b^2*d^4*x^2 - 48*b^2*c*d^3*x - 24*b^2*c^2*d^2
+ 3*d^4)*cos(4*b*x + 4*a)/b^5 - 1/16*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12
*b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^
3*x - 6*b^2*c^2*d^2 + 3*d^4)*cos(2*b*x + 2*a)/b^5 + 1/256*(8*b^3*d^4*x^3 +
 24*b^3*c*d^3*x^2 + 24*b^3*c^2*d^2*x + 8*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3
)*sin(4*b*x + 4*a)/b^5 + 1/8*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*
d^2*x + 2*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

### 3.137.9 Mupad [B] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 576, normalized size of antiderivative = 2.22

$$\int (c+dx)^4\cos^3(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{192\,d^4\,\cos(2\,a+2\,b\,x)+3\,d^4\,\cos(4\,a+4\,b\,x)+128\,b^4\,c^4\,\cos(2\,a+2\,b\,x)+32\,b^4\,c^4\,\cos(4\,a+4\,b\,x)-2\ldots}{}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^4,x)
```

output
```
-(192*d^4*cos(2*a + 2*b*x) + 3*d^4*cos(4*a + 4*b*x) + 128*b^4*c^4*cos(2*a
+ 2*b*x) + 32*b^4*c^4*cos(4*a + 4*b*x) - 256*b^3*c^3*d*sin(2*a + 2*b*x) -
32*b^3*c^3*d*sin(4*a + 4*b*x) - 384*b^2*c^2*d^2*cos(2*a + 2*b*x) - 24*b^2*
c^2*d^2*cos(4*a + 4*b*x) - 384*b^2*d^4*x^2*cos(2*a + 2*b*x) - 24*b^2*d^4*x
^2*cos(4*a + 4*b*x) + 128*b^4*d^4*x^4*cos(2*a + 2*b*x) + 32*b^4*d^4*x^4*co
s(4*a + 4*b*x) - 256*b^3*d^4*x^3*sin(2*a + 2*b*x) - 32*b^3*d^4*x^3*sin(4*a
 + 4*b*x) + 384*b*c*d^3*sin(2*a + 2*b*x) + 12*b*c*d^3*sin(4*a + 4*b*x) + 3
84*b*d^4*x*sin(2*a + 2*b*x) + 12*b*d^4*x*sin(4*a + 4*b*x) + 768*b^4*c^2*d^
2*x^2*cos(2*a + 2*b*x) + 192*b^4*c^2*d^2*x^2*cos(4*a + 4*b*x) - 768*b^2*c*
d^3*x*cos(2*a + 2*b*x) + 512*b^4*c^3*d*x*cos(2*a + 2*b*x) - 48*b^2*c*d^3*x
*cos(4*a + 4*b*x) + 128*b^4*c^3*d*x*cos(4*a + 4*b*x) + 512*b^4*c*d^3*x^3*c
os(2*a + 2*b*x) + 128*b^4*c*d^3*x^3*cos(4*a + 4*b*x) - 768*b^3*c^2*d^2*x*s
in(2*a + 2*b*x) - 768*b^3*c*d^3*x^2*sin(2*a + 2*b*x) - 96*b^3*c^2*d^2*x*si
n(4*a + 4*b*x) - 96*b^3*c*d^3*x^2*sin(4*a + 4*b*x))/(1024*b^5)
```

3.137. $\int (c+dx)^4 \cos^3(a+bx)\sin(a+bx)\,dx$

# 3.138 $\int (c+dx)^3 \cos^3(a+bx) \sin(a+bx) \, dx$

3.138.1 Optimal result . . . . . 1126
3.138.2 Mathematica [A] (verified) . . . . . 1127
3.138.3 Rubi [A] (verified) . . . . . 1127
3.138.4 Maple [A] (verified) . . . . . 1131
3.138.5 Fricas [A] (verification not implemented) . . . . . 1131
3.138.6 Sympy [B] (verification not implemented) . . . . . 1132
3.138.7 Maxima [B] (verification not implemented) . . . . . 1133
3.138.8 Giac [A] (verification not implemented) . . . . . 1133
3.138.9 Mupad [B] (verification not implemented) . . . . . 1134

## 3.138.1 Optimal result

Integrand size = 22, antiderivative size = 196

$$\int (c+dx)^3 \cos^3(a+bx) \sin(a+bx) \, dx = -\frac{45d^3x}{256b^3} + \frac{3(c+dx)^3}{32b} + \frac{9d^2(c+dx)\cos^2(a+bx)}{32b^3} + \frac{3d^2(c+dx)\cos^4(a+bx)}{32b^3} - \frac{(c+dx)^3\cos^4(a+bx)}{4b} - \frac{45d^3\cos(a+bx)\sin(a+bx)}{256b^4} + \frac{9d(c+dx)^2\cos(a+bx)\sin(a+bx)}{32b^2} - \frac{3d^3\cos^3(a+bx)\sin(a+bx)}{128b^4} + \frac{3d(c+dx)^2\cos^3(a+bx)\sin(a+bx)}{16b^2}$$

output
```
-45/256*d^3*x/b^3+3/32*(d*x+c)^3/b+9/32*d^2*(d*x+c)*cos(b*x+a)^2/b^3+3/32*
d^2*(d*x+c)*cos(b*x+a)^4/b^3-1/4*(d*x+c)^3*cos(b*x+a)^4/b-45/256*d^3*cos(b
*x+a)*sin(b*x+a)/b^4+9/32*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^2-3/128*d^3*
cos(b*x+a)^3*sin(b*x+a)/b^4+3/16*d*(d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)/b^2
```

### 3.138.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.69

$$\int (c+dx)^3\cos^3(a+bx)\sin(a+bx)\,dx = \frac{-64b(c+dx)\left(-3d^2+2b^2(c+dx)^2\right)\cos(2(a+bx))-4b(c+dx)\left(-3d^2+8b^2(c+dx)^2\right)\cos(4(a+bx))+}{1024b^4}$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
(-64*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] - 4*b*(c +
d*x)*(-3*d^2 + 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] + 6*d*(16*(-d^2 + 2*b^2
*(c + d*x)^2) + (-d^2 + 8*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*
x)])/(1024*b^4)
```

### 3.138.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.24, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {4905, 3042, 3792, 3042, 3115, 3042, 3115, 24, 3792, 17, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\sin(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{4905}$$

$$\frac{3d\int (c+dx)^2\cos^4(a+bx)dx}{4b}-\frac{(c+dx)^3\cos^4(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d\int (c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^4dx}{4b}-\frac{(c+dx)^3\cos^4(a+bx)}{4b}$$

$$\Big\downarrow \text{3792}$$

$$\frac{3d\left(-\frac{d^2\int\cos^4(a+bx)dx}{8b^2}+\frac{3}{4}\int (c+dx)^2\cos^2(a+bx)dx+\frac{d(c+dx)\cos^4(a+bx)}{8b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{4b}-\frac{(c+dx)^3\cos^4(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d\left(-\frac{d^2\int \sin\left(a+bx+\frac{\pi}{2}\right)^4dx}{8b^2}+\frac{3}{4}\int (c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{d(c+dx)\cos^4(a+bx)}{8b^2}+\frac{(c+dx)^2\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{4b}-\frac{(c+dx)^3\cos^4(a+bx)}{4b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{3d\left(-\frac{d^2\left(\frac{3}{4}\int \cos^2(a+bx)dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int (c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{d(c+dx)\cos^4(a+bx)}{8b^2}+\frac{(c+dx)^2\sin(a+\ldots}{4\ldots}\right.}{4b}$$
$$\frac{(c+dx)^3\cos^4(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d\left(-\frac{d^2\left(\frac{3}{4}\int \sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int (c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{d(c+dx)\cos^4(a+bx)}{8b^2}+\frac{(c+dx)^2\sin\ldots}{\ldots}\right.}{4b}$$
$$\frac{(c+dx)^3\cos^4(a+bx)}{4b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{3d\left(-\frac{d^2\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int (c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{d(c+dx)\cos^4(a+bx)}{8b^2}+(c+\ldots\right.}{4b}$$
$$\frac{(c+dx)^3\cos^4(a+bx)}{4b}$$

$$\Big\downarrow \text{24}$$

$$\frac{3d\left(\frac{3}{4}\int (c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{d(c+dx)\cos^4(a+bx)}{8b^2}-\frac{d^2\left(\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)\right)}{8b^2}+\frac{(c+dx)^2\ldots}{\ldots}\right.}{4b}$$
$$\frac{(c+dx)^3\cos^4(a+bx)}{4b}$$

$$\Big\downarrow \text{3792}$$

3.138. $\int (c+dx)^3\cos^3(a+bx)\sin(a+bx)\,dx$

$$3d\left(\frac{3}{4}\left(-\frac{d^2\int\cos^2(a+bx)dx}{2b^2}+\frac{1}{2}\int(c+dx)^2dx+\frac{d(c+dx)\cos^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{d(c+dx)\cos^4(a+bx)}{8b^2}-\frac{d^2\cdots}{\cdots}\right.$$
$$\overline{\qquad 4b}$$
$$\frac{(c+dx)^3\cos^4(a+bx)}{4b}$$

$\big\downarrow$ 17

$$3d\left(\frac{3}{4}\left(-\frac{d^2\int\cos^2(a+bx)dx}{2b^2}+\frac{d(c+dx)\cos^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)+\frac{d(c+dx)\cos^4(a+bx)}{8b^2}-\frac{d^2\left(\frac{\sin(a+bx)\cdots}{\cdots}\right.}{\cdots}\right.$$
$$\overline{\qquad 4b}$$
$$\frac{(c+dx)^3\cos^4(a+bx)}{4b}$$

$\big\downarrow$ 3042

$$3d\left(\frac{3}{4}\left(-\frac{d^2\int\sin\left(a+bx+\frac{\pi}{2}\right)^2dx}{2b^2}+\frac{d(c+dx)\cos^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)+\frac{d(c+dx)\cos^4(a+bx)}{8b^2}-\frac{d^2\left(\frac{\sin(\cdots}{\cdots}\right.}{\cdots}\right.$$
$$\overline{\qquad 4b}$$
$$\frac{(c+dx)^3\cos^4(a+bx)}{4b}$$

$\big\downarrow$ 3115

$$3d\left(\frac{3}{4}\left(-\frac{d^2\left(\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{d(c+dx)\cos^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)+\frac{d(c+dx)\cos^4(a+bx)}{8b^2}-\cdots\right.$$
$$\overline{\qquad 4b}$$
$$\frac{(c+dx)^3\cos^4(a+bx)}{4b}$$

$\big\downarrow$ 24

$$3d\left(\frac{3}{4}\left(\frac{d(c+dx)\cos^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)}{2b^2}+\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)+\frac{d(c+dx)\cos^4(a+bx)}{8b^2}-\frac{d^2\left(\frac{s\cdots}{\cdots}\right.}{\cdots}\right.$$
$$\overline{\qquad 4b}$$
$$\frac{(c+dx)^3\cos^4(a+bx)}{4b}$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x],x]
```

## 3.138. $\int(c+dx)^3\cos^3(a+bx)\sin(a+bx)\,dx$

output
```
-1/4*((c + d*x)^3*Cos[a + b*x]^4)/b + (3*d*((d*(c + d*x)*Cos[a + b*x]^4)/(
8*b^2) + ((c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x])/(4*b) - (d^2*((Cos[a +
b*x]^3*Sin[a + b*x])/(4*b) + (3*(x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)))
/4))/(8*b^2) + (3*((c + d*x)^3/(6*d) + (d*(c + d*x)*Cos[a + b*x]^2)/(2*b^2
) + ((c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(2*b) - (d^2*(x/2 + (Cos[a + b
*x]*Sin[a + b*x])/(2*b)))/(2*b^2)))/4))/(4*b)
```

### 3.138.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

## 3.138.4 Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.76

| method | result |
|---|---|
| parallelrisch | $\frac{-32b\left((dx+c)^2b^2-\frac{3d^2}{2}\right)(dx+c)\cos(2xb+2a)-8b\left((dx+c)^2b^2-\frac{3d^2}{8}\right)(dx+c)\cos(4xb+4a)+48d\left((dx+c)^2b^2-\frac{d^2}{2}\right)\sin(2xb \cdots}{256b^4}$ |
| risch | $-\frac{(8b^2d^3x^3+24b^2c\,d^2x^2+24b^2c^2dx+8b^2c^3-3d^3x-3c\,d^2)\cos(4xb+4a)}{256b^3}+\frac{3d(8x^2d^2b^2+16b^2cdx+8b^2c^2-d^2)\sin(4xb+ \cdots}{1024b^4}$ |
| derivativedivides | $\frac{a^3d^3\cos(xb+a)^4}{4b^3}-\frac{3a^2c\,d^2\cos(xb+a)^4}{4b^2}+\frac{3a^2d^3\left(-\frac{(xb+a)\cos(xb+a)^4}{4}+\frac{\left(\cos(xb+a)^3+\frac{3\cos(xb+a)}{2}\right)\sin(xb+a)}{16}+\frac{3xb}{32}+\frac{3a}{32}\right)}{b^3}+\frac{3a\,c \cdots}{}$ |
| default | $\frac{a^3d^3\cos(xb+a)^4}{4b^3}-\frac{3a^2c\,d^2\cos(xb+a)^4}{4b^2}+\frac{3a^2d^3\left(-\frac{(xb+a)\cos(xb+a)^4}{4}+\frac{\left(\cos(xb+a)^3+\frac{3\cos(xb+a)}{2}\right)\sin(xb+a)}{16}+\frac{3xb}{32}+\frac{3a}{32}\right)}{b^3}+\frac{3a\,c \cdots}{}$ |

input
```
int((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/256*(-32*b*((d*x+c)^2*b^2-3/2*d^2)*(d*x+c)*cos(2*b*x+2*a)-8*b*((d*x+c)^2
*b^2-3/8*d^2)*(d*x+c)*cos(4*b*x+4*a)+48*d*((d*x+c)^2*b^2-1/2*d^2)*sin(2*b*
x+2*a)+6*((d*x+c)^2*b^2-1/8*d^2)*d*sin(4*b*x+4*a)+40*b^3*c^3-51*c*d^2*b)/b
^4
```

## 3.138.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.21

$$\int (c+dx)^3\cos^3(a+bx)\sin(a+bx)\,dx$$
$$=\frac{24\,b^3d^3x^3+72\,b^3cd^2x^2-8\,(8\,b^3d^3x^3+24\,b^3cd^2x^2+8\,b^3c^3-3\,bcd^2+3\,(8\,b^3c^2d-bd^3)x)\cos{(bx+a)}^4+7\cdots}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/256*(24*b^3*d^3*x^3 + 72*b^3*c*d^2*x^2 - 8*(8*b^3*d^3*x^3 + 24*b^3*c*d^2
*x^2 + 8*b^3*c^3 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^4 +
 72*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2 + 9*(8*b^3*c^2*d - 5*b*d^3)*x + 3*(
2*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^3 + 3*
(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - 5*d^3)*cos(b*x + a))*sin(b
*x + a))/b^4
```

### 3.138.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(197) = 394$.

Time = 0.63 (sec) , antiderivative size = 602, normalized size of antiderivative = 3.07

$$\int (c+dx)^3\cos^3(a+bx)\sin(a+bx)\,dx$$
$$= \begin{cases} -\frac{c^3\cos^4(a+bx)}{4b} + \frac{9c^2dx\sin^4(a+bx)}{32b} + \frac{9c^2dx\sin^2(a+bx)\cos^2(a+bx)}{16b} - \frac{15c^2dx\cos^4(a+bx)}{32b} + \frac{9cd^2x^2\sin^4(a+bx)}{32b} + \frac{9cd^2x^2\sin^2(a\cdots}{} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin(a)\cos^3(a) \end{cases}$$

input
```
integrate((d*x+c)**3*cos(b*x+a)**3*sin(b*x+a),x)
```

output
```
Piecewise((-c**3*cos(a + b*x)**4/(4*b) + 9*c**2*d*x*sin(a + b*x)**4/(32*b)
 + 9*c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 15*c**2*d*x*cos(a +
 b*x)**4/(32*b) + 9*c*d**2*x**2*sin(a + b*x)**4/(32*b) + 9*c*d**2*x**2*sin
(a + b*x)**2*cos(a + b*x)**2/(16*b) - 15*c*d**2*x**2*cos(a + b*x)**4/(32*b
) + 3*d**3*x**3*sin(a + b*x)**4/(32*b) + 3*d**3*x**3*sin(a + b*x)**2*cos(a
 + b*x)**2/(16*b) - 5*d**3*x**3*cos(a + b*x)**4/(32*b) + 9*c**2*d*sin(a +
b*x)**3*cos(a + b*x)/(32*b**2) + 15*c**2*d*sin(a + b*x)*cos(a + b*x)**3/(3
2*b**2) + 9*c*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 15*c*d**2*x*
sin(a + b*x)*cos(a + b*x)**3/(16*b**2) + 9*d**3*x**2*sin(a + b*x)**3*cos(a
 + b*x)/(32*b**2) + 15*d**3*x**2*sin(a + b*x)*cos(a + b*x)**3/(32*b**2) -
9*c*d**2*sin(a + b*x)**4/(64*b**3) + 15*c*d**2*cos(a + b*x)**4/(64*b**3) -
 45*d**3*x*sin(a + b*x)**4/(256*b**3) - 9*d**3*x*sin(a + b*x)**2*cos(a + b
*x)**2/(128*b**3) + 51*d**3*x*cos(a + b*x)**4/(256*b**3) - 45*d**3*sin(a +
 b*x)**3*cos(a + b*x)/(256*b**4) - 51*d**3*sin(a + b*x)*cos(a + b*x)**3/(2
56*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/
4)*sin(a)*cos(a)**3, True))
```

### 3.138.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(178) = 356$.

Time = 0.25 (sec) , antiderivative size = 549, normalized size of antiderivative = 2.80

$$\int (c+dx)^3\cos^3(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{256\,c^3\cos\left(bx+a\right)^4-\frac{768\,ac^2d\cos(bx+a)^4}{b}+\frac{768\,a^2cd^2\cos(bx+a)^4}{b^2}-\frac{256\,a^3d^3\cos(bx+a)^4}{b^3}+\frac{24\,(4\,(bx+a)\cos(4\,bx+4\,a)+16\,(ba}{}}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/1024*(256*c^3*cos(b*x + a)^4 - 768*a*c^2*d*cos(b*x + a)^4/b + 768*a^2*c
*d^2*cos(b*x + a)^4/b^2 - 256*a^3*d^3*cos(b*x + a)^4/b^3 + 24*(4*(b*x + a)
*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*s
in(2*b*x + 2*a))*c^2*d/b - 48*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)
*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*a*c*d^2/b^2 + 2
4*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*
x + 4*a) - 8*sin(2*b*x + 2*a))*a^2*d^3/b^3 + 12*((8*(b*x + a)^2 - 1)*cos(4
*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*
b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*c*d^2/b^2 - 12*((8*(b*x + a)^2
 - 1)*cos(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x
+ a)*sin(4*b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*a*d^3/b^3 + (4*(8*(
b*x + a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) + 64*(2*(b*x + a)^3 - 3*b*x - 3
*a)*cos(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) - 96*(2*(b*x
 + a)^2 - 1)*sin(2*b*x + 2*a))*d^3/b^3)/b
```

### 3.138.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.23

$$\begin{aligned}\int (c+dx)^3\cos^3(a+bx)\sin(a+bx)\,dx &= -\frac{\left(8\,b^3d^3x^3+24\,b^3cd^2x^2+24\,b^3c^2dx+8\,b^3c^3-3\,bd^3x-3\,bcd^2\right)\cos\left(4\,bx+4\,a\right)}{256\,b^4}\\ &\quad-\frac{\left(2\,b^3d^3x^3+6\,b^3cd^2x^2+6\,b^3c^2dx+2\,b^3c^3-3\,bd^3x-3\,bcd^2\right)\cos\left(2\,bx+2\,a\right)}{16\,b^4}\\ &\quad+\frac{3\left(8\,b^2d^3x^2+16\,b^2cd^2x+8\,b^2c^2d-d^3\right)\sin\left(4\,bx+4\,a\right)}{1024\,b^4}\\ &\quad+\frac{3\left(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-d^3\right)\sin\left(2\,bx+2\,a\right)}{32\,b^4}\end{aligned}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/256*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 - 3*
b*d^3*x - 3*b*c*d^2)*cos(4*b*x + 4*a)/b^4 - 1/16*(2*b^3*d^3*x^3 + 6*b^3*c*
d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^2)*cos(2*b*x + 2
*a)/b^4 + 3/1024*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*sin(
4*b*x + 4*a)/b^4 + 3/32*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3
)*sin(2*b*x + 2*a)/b^4
```

### 3.138.9 Mupad [B] (verification not implemented)

Time = 25.02 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.87

$$\int (c+dx)^3\cos^3(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{24\,d^3\,\sin\left(2\,a+2\,b\,x\right)+\frac{3\,d^3\,\sin\left(4\,a+4\,b\,x\right)}{4}+32\,b^3\,c^3\,\cos\left(2\,a+2\,b\,x\right)+8\,b^3\,c^3\,\cos\left(4\,a+4\,b\,x\right)-48\,b^2\,c^2\,d\,\mathrm{s}}{}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^3,x)
```

output
```
-(24*d^3*sin(2*a + 2*b*x) + (3*d^3*sin(4*a + 4*b*x))/4 + 32*b^3*c^3*cos(2*
a + 2*b*x) + 8*b^3*c^3*cos(4*a + 4*b*x) - 48*b^2*c^2*d*sin(2*a + 2*b*x) -
6*b^2*c^2*d*sin(4*a + 4*b*x) + 32*b^3*d^3*x^3*cos(2*a + 2*b*x) + 8*b^3*d^3
*x^3*cos(4*a + 4*b*x) - 48*b^2*d^3*x^2*sin(2*a + 2*b*x) - 6*b^2*d^3*x^2*si
n(4*a + 4*b*x) - 48*b*c*d^2*cos(2*a + 2*b*x) - 3*b*c*d^2*cos(4*a + 4*b*x)
- 48*b*d^3*x*cos(2*a + 2*b*x) - 3*b*d^3*x*cos(4*a + 4*b*x) + 96*b^3*c^2*d*
x*cos(2*a + 2*b*x) + 24*b^3*c^2*d*x*cos(4*a + 4*b*x) - 96*b^2*c*d^2*x*sin(
2*a + 2*b*x) - 12*b^2*c*d^2*x*sin(4*a + 4*b*x) + 96*b^3*c*d^2*x^2*cos(2*a
+ 2*b*x) + 24*b^3*c*d^2*x^2*cos(4*a + 4*b*x))/(256*b^4)
```

# 3.139 $\int (c+dx)^2 \cos^3(a+bx) \sin(a+bx) \, dx$


3.139.1 Optimal result . . . . . . . . . . 1135
3.139.2 Mathematica [A] (verified) . . . . . . . . . . 1135
3.139.3 Rubi [A] (verified) . . . . . . . . . . 1136
3.139.4 Maple [A] (verified) . . . . . . . . . . 1138
3.139.5 Fricas [A] (verification not implemented) . . . . . . . . . . 1138
3.139.6 Sympy [B] (verification not implemented) . . . . . . . . . . 1139
3.139.7 Maxima [B] (verification not implemented) . . . . . . . . . . 1139
3.139.8 Giac [A] (verification not implemented) . . . . . . . . . . 1140
3.139.9 Mupad [B] (verification not implemented) . . . . . . . . . . 1140


## 3.139.1 Optimal result

Integrand size = 22, antiderivative size = 134

$$\begin{aligned}\int (c+dx)^2 \cos^3(a+bx) \sin(a+bx) \, dx = {} & \frac{3cdx}{16b} + \frac{3d^2x^2}{32b} + \frac{3d^2\cos^2(a+bx)}{32b^3} \\ & + \frac{d^2\cos^4(a+bx)}{32b^3} - \frac{(c+dx)^2\cos^4(a+bx)}{4b} \\ & + \frac{3d(c+dx)\cos(a+bx)\sin(a+bx)}{16b^2} \\ & + \frac{d(c+dx)\cos^3(a+bx)\sin(a+bx)}{8b^2}\end{aligned}$$

output
```
3/16*c*d*x/b+3/32*d^2*x^2/b+3/32*d^2*cos(b*x+a)^2/b^3+1/32*d^2*cos(b*x+a)^
4/b^3-1/4*(d*x+c)^2*cos(b*x+a)^4/b+3/16*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^
2+1/8*d*(d*x+c)*cos(b*x+a)^3*sin(b*x+a)/b^2
```

## 3.139.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.66

$$\int (c+dx)^2 \cos^3(a+bx) \sin(a+bx) \, dx$$
$$= \frac{-16(-d^2+2b^2(c+dx)^2)\cos(2(a+bx)) + (d^2-8b^2(c+dx)^2)\cos(4(a+bx)) + 4bd(c+dx)(8\sin(2(a+}{256b^3}$$

input
```
Integrate[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
(-16*(-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + (d^2 - 8*b^2*(c + d*x)^
2)*Cos[4*(a + b*x)] + 4*b*d*(c + d*x)*(8*Sin[2*(a + b*x)] + Sin[4*(a + b*x
)]))/(256*b^3)
```

### 3.139.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4905, 3042, 3791, 3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{4905}$$

$$\frac{d\int (c+dx)\cos^4(a+bx)dx}{2b}-\frac{(c+dx)^2\cos^4(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^4dx}{2b}-\frac{(c+dx)^2\cos^4(a+bx)}{4b}$$

$$\Big\downarrow \text{3791}$$

$$\frac{d\left(\frac{3}{4}\int (c+dx)\cos^2(a+bx)dx+\frac{d\cos^4(a+bx)}{16b^2}+\frac{(c+dx)\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{2b}-\frac{(c+dx)^2\cos^4(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d\left(\frac{3}{4}\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{d\cos^4(a+bx)}{16b^2}+\frac{(c+dx)\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{2b}-\frac{(c+dx)^2\cos^4(a+bx)}{4b}$$

$$\Big\downarrow \text{3791}$$

$$\frac{d\left(\frac{3}{4}\left(\frac{1}{2}\int (c+dx)dx+\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{d\cos^4(a+bx)}{16b^2}+\frac{(c+dx)\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{2b}-\frac{(c+dx)^2\cos^4(a+bx)}{4b}$$

$$\Big\downarrow \text{17}$$

$$\frac{d\left(\frac{3}{4}\left(\frac{d\cos^2(a+bx)}{4b^2}+\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)+\frac{d\cos^4(a+bx)}{16b^2}+\frac{(c+dx)\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{2b}-\frac{(c+dx)^2\cos^4(a+bx)}{4b}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
-1/4*((c + d*x)^2*Cos[a + b*x]^4)/b + (d*((d*Cos[a + b*x]^4)/(16*b^2) + ((
c + d*x)*Cos[a + b*x]^3*Sin[a + b*x])/(4*b) + (3*((c + d*x)^2/(4*d) + (d*C
os[a + b*x]^2)/(4*b^2) + ((c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b)))/4))
/(2*b)
```

### 3.139.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

### 3.139.4 Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.81

| method | result |
|---|---|
| parallelrisch | $\frac{16\left(-2(dx+c)^2b^2+d^2\right)\cos(2xb+2a)+\left(-8(dx+c)^2b^2+d^2\right)\cos(4xb+4a)+32bd(dx+c)\sin(2xb+2a)+4d(dx+c)b\sin(4xb-}{256b^3}$ |
| risch | $-\frac{(8x^2d^2b^2+16b^2cdx+8b^2c^2-d^2)\cos(4xb+4a)}{256b^3}+\frac{d(dx+c)\sin(4xb+4a)}{64b^2}-\frac{(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2)\cos(2xb+}{16b^3}$ |
| derivativedivides | $\frac{-\frac{a^2d^2\cos(xb+a)^4}{4b^2}+\frac{acd\cos(xb+a)^4}{2b}-\frac{2a\,d^2\left(-\frac{(xb+a)\cos(xb+a)^4}{4}+\frac{\left(\cos(xb+a)^3+\frac{3\cos(xb+a)}{2}\right)\sin(xb+a)}{16}+\frac{3xb}{32}+\frac{3a}{32}\right)}{b^2}-\frac{c^2\cos(}{4}}{}$ |
| default | $\frac{-\frac{a^2d^2\cos(xb+a)^4}{4b^2}+\frac{acd\cos(xb+a)^4}{2b}-\frac{2a\,d^2\left(-\frac{(xb+a)\cos(xb+a)^4}{4}+\frac{\left(\cos(xb+a)^3+\frac{3\cos(xb+a)}{2}\right)\sin(xb+a)}{16}+\frac{3xb}{32}+\frac{3a}{32}\right)}{b^2}-\frac{c^2\cos(}{4}}{}$ |
| norman | $\frac{-\frac{5d^2x^2}{32b}-\frac{3d^2\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^4}{4b^3}+\frac{\left(16b^2c^2-5d^2\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}{8b^3}+\frac{\left(16b^2c^2-5d^2\right)\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^6}{8b^3}+\frac{5cd\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{8b^2}-\frac{3cd\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3}{8b^2}}{}$ |

input
```
int((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/256*(16*(-2*(d*x+c)^2*b^2+d^2)*cos(2*b*x+2*a)+(-8*(d*x+c)^2*b^2+d^2)*cos
(4*b*x+4*a)+32*b*d*(d*x+c)*sin(2*b*x+2*a)+4*d*(d*x+c)*b*sin(4*b*x+4*a)+40*
b^2*c^2-17*d^2)/b^3
```

### 3.139.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.97

$$\int (c+dx)^2\cos^3(a+bx)\sin(a+bx)\,dx$$
$$=\frac{3\,b^2d^2x^2+6\,b^2cdx-(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-d^2)\cos(bx+a)^4+3\,d^2\cos(bx+a)^2+2\,(2\,(bd^2x+b}{32\,b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/32*(3*b^2*d^2*x^2 + 6*b^2*c*d*x - (8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*
c^2 - d^2)*cos(b*x + a)^4 + 3*d^2*cos(b*x + a)^2 + 2*(2*(b*d^2*x + b*c*d)*
cos(b*x + a)^3 + 3*(b*d^2*x + b*c*d)*cos(b*x + a))*sin(b*x + a))/b^3
```

## 3.139.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(129) = 258$.

Time = 0.43 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.39

$$\int (c+dx)^2\cos^3(a+bx)\sin(a+bx)\,dx$$
$$= \begin{cases} -\frac{c^2\cos^4(a+bx)}{4b} + \frac{3cdx\sin^4(a+bx)}{16b} + \frac{3cdx\sin^2(a+bx)\cos^2(a+bx)}{8b} - \frac{5cdx\cos^4(a+bx)}{16b} + \frac{3d^2x^2\sin^4(a+bx)}{32b} + \frac{3d^2x^2\sin^2(a+bx)\cos^2(a+bx)}{16b} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin(a)\cos^3(a) \end{cases}$$

input
```
integrate((d*x+c)**2*cos(b*x+a)**3*sin(b*x+a),x)
```

output
```
Piecewise((-c**2*cos(a + b*x)**4/(4*b) + 3*c*d*x*sin(a + b*x)**4/(16*b) +
3*c*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(8*b) - 5*c*d*x*cos(a + b*x)**4/(1
6*b) + 3*d**2*x**2*sin(a + b*x)**4/(32*b) + 3*d**2*x**2*sin(a + b*x)**2*co
s(a + b*x)**2/(16*b) - 5*d**2*x**2*cos(a + b*x)**4/(32*b) + 3*c*d*sin(a +
b*x)**3*cos(a + b*x)/(16*b**2) + 5*c*d*sin(a + b*x)*cos(a + b*x)**3/(16*b*
*2) + 3*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 5*d**2*x*sin(a + b
*x)*cos(a + b*x)**3/(16*b**2) - 3*d**2*sin(a + b*x)**4/(64*b**3) + 5*d**2*
cos(a + b*x)**4/(64*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*s
in(a)*cos(a)**3, True))
```

## 3.139.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(120) = 240$.

Time = 0.24 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.96

$$\int (c+dx)^2\cos^3(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{64c^2\cos(bx+a)^4 - \frac{128acd\cos(bx+a)^4}{b} + \frac{64a^2d^2\cos(bx+a)^4}{b^2} + \frac{4(4(bx+a)\cos(4bx+4a)+16(bx+a)\cos(2bx+2a)-\sin(4bx+4}{b}}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/256*(64*c^2*cos(b*x + a)^4 - 128*a*c*d*cos(b*x + a)^4/b + 64*a^2*d^2*co
s(b*x + a)^4/b^2 + 4*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*
x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*c*d/b - 4*(4*(b*x + a)*c
os(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin
(2*b*x + 2*a))*a*d^2/b^2 + ((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) + 16*(2*(
b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) - 32*(b*x
+ a)*sin(2*b*x + 2*a))*d^2/b^2)/b
```

### 3.139.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.08

$$\int (c+dx)^2\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-d^2)\cos{(4\,bx+4\,a)}}{256\,b^3} - \frac{(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2)\cos{(2\,bx+2\,a)}}{16\,b^3} + \frac{(bd^2x+bcd)\sin{(4\,bx+4\,a)}}{64\,b^3} + \frac{(bd^2x+bcd)\sin{(2\,bx+2\,a)}}{8\,b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/256*(8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(4*b*x + 4*a)/b
^3 - 1/16*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2*a)
/b^3 + 1/64*(b*d^2*x + b*c*d)*sin(4*b*x + 4*a)/b^3 + 1/8*(b*d^2*x + b*c*d)
*sin(2*b*x + 2*a)/b^3
```

### 3.139.9 Mupad [B] (verification not implemented)

Time = 23.75 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.51

$$\int (c+dx)^2\cos^3(a+bx)\sin(a+bx)\,dx = \frac{8\,d^2\,\cos{(2\,a+2\,b\,x)}+\frac{d^2\,\cos(4\,a+4\,b\,x)}{2}-16\,b^2\,c^2\,\cos{(2\,a+2\,b\,x)}-4\,b^2\,c^2\,\cos{(4\,a+4\,b\,x)}+16\,b\,c\,d\,\sin{(2\,a}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^2,x)
```

output
```
(8*d^2*cos(2*a + 2*b*x) + (d^2*cos(4*a + 4*b*x))/2 - 16*b^2*c^2*cos(2*a +
2*b*x) - 4*b^2*c^2*cos(4*a + 4*b*x) + 16*b*c*d*sin(2*a + 2*b*x) + 2*b*c*d*
sin(4*a + 4*b*x) - 16*b^2*d^2*x^2*cos(2*a + 2*b*x) - 4*b^2*d^2*x^2*cos(4*a
 + 4*b*x) + 16*b*d^2*x*sin(2*a + 2*b*x) + 2*b*d^2*x*sin(4*a + 4*b*x) - 32*
b^2*c*d*x*cos(2*a + 2*b*x) - 8*b^2*c*d*x*cos(4*a + 4*b*x))/(128*b^3)
```

## 3.139. $\int (c+dx)^2 \cos^3(a+bx) \sin(a+bx)\,dx$

# 3.140 $\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx$

3.140.1 Optimal result . . . . . . . . . . 1142
3.140.2 Mathematica [A] (verified) . . . . . . . . . . 1142
3.140.3 Rubi [A] (verified) . . . . . . . . . . 1143
3.140.4 Maple [A] (verified) . . . . . . . . . . 1144
3.140.5 Fricas [A] (verification not implemented) . . . . . . . . . . 1145
3.140.6 Sympy [A] (verification not implemented) . . . . . . . . . . 1146
3.140.7 Maxima [A] (verification not implemented) . . . . . . . . . . 1146
3.140.8 Giac [A] (verification not implemented) . . . . . . . . . . 1147
3.140.9 Mupad [B] (verification not implemented) . . . . . . . . . . 1147

## 3.140.1 Optimal result

Integrand size = 20, antiderivative size = 72

$$\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx = \frac{3dx}{32b}-\frac{(c+dx)\cos^4(a+bx)}{4b}+\frac{3d\cos(a+bx)\sin(a+bx)}{32b^2}+\frac{d\cos^3(a+bx)\sin(a+bx)}{16b^2}$$

output
```
3/32*d*x/b-1/4*(d*x+c)*cos(b*x+a)^4/b+3/32*d*cos(b*x+a)*sin(b*x+a)/b^2+1/1
6*d*cos(b*x+a)^3*sin(b*x+a)/b^2
```

## 3.140.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{c\cos^4(a+bx)}{4b}+\frac{d(-2bx\cos(2(a+bx))+\sin(2(a+bx)))}{16b^2}+\frac{d(-4bx\cos(4(a+bx))+\sin(4(a+bx)))}{128b^2}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
-1/4*(c*Cos[a + b*x]^4)/b + (d*(-2*b*x*Cos[2*(a + b*x)] + Sin[2*(a + b*x)]
))/(16*b^2) + (d*(-4*b*x*Cos[4*(a + b*x)] + Sin[4*(a + b*x)]))/(128*b^2)
```

## 3.140.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4905, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \ 4905$$

$$\frac{d\int \cos^4(a+bx)dx}{4b}-\frac{(c+dx)\cos^4(a+bx)}{4b}$$

$$\Big\downarrow \ 3042$$

$$\frac{d\int \sin\left(a+bx+\frac{\pi}{2}\right)^4dx}{4b}-\frac{(c+dx)\cos^4(a+bx)}{4b}$$

$$\Big\downarrow \ 3115$$

$$\frac{d\left(\frac{3}{4}\int \cos^2(a+bx)dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{4b}-\frac{(c+dx)\cos^4(a+bx)}{4b}$$

$$\Big\downarrow \ 3042$$

$$\frac{d\left(\frac{3}{4}\int \sin\left(a+bx+\frac{\pi}{2}\right)^2dx+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{4b}-\frac{(c+dx)\cos^4(a+bx)}{4b}$$

$$\Big\downarrow \ 3115$$

$$\frac{d\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)+\frac{\sin(a+bx)\cos^3(a+bx)}{4b}\right)}{4b}-\frac{(c+dx)\cos^4(a+bx)}{4b}$$

$$\Big\downarrow \ 24$$

$$\frac{d\left(\frac{\sin(a+bx)\cos^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}\right)\right)}{4b}-\frac{(c+dx)\cos^4(a+bx)}{4b}$$

input
```
Int[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

3.140. $\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx$

output
```
-1/4*((c + d*x)*Cos[a + b*x]^4)/b + (d*((Cos[a + b*x]^3*Sin[a + b*x])/(4*b
) + (3*(x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)))/4))/(4*b)
```

### 3.140.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4905
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[a + b*x]^(n + 1)/(b*(n + 1
))), x] + Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

## 3.140.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

| method | result |
|---|---|
| parallelrisch | $\frac{-16b(dx+c)\cos(2xb+2a)-4b(dx+c)\cos(4xb+4a)+20cb+8d\sin(2xb+2a)+d\sin(4xb+4a)}{128b^{2}}$ |
| risch | $-\frac{(dx+c)\cos(4xb+4a)}{32b}+\frac{d\sin(4xb+4a)}{128b^{2}}-\frac{(dx+c)\cos(2xb+2a)}{8b}+\frac{d\sin(2xb+2a)}{16b^{2}}$ |
| derivativedivides | $\frac{\frac{da\cos(xb+a)^{4}}{4b}-\frac{c\cos(xb+a)^{4}}{4}+\frac{d\left(-\frac{(xb+a)\cos(xb+a)^{4}}{4}+\frac{\left(\cos(xb+a)^{3}+\frac{3\cos(xb+a)}{2}\right)\sin(xb+a)}{16}+\frac{3xb}{32}+\frac{3a}{32}\right)}{b}}{b}$ |
| default | $\frac{\frac{da\cos(xb+a)^{4}}{4b}-\frac{c\cos(xb+a)^{4}}{4}+\frac{d\left(-\frac{(xb+a)\cos(xb+a)^{4}}{4}+\frac{\left(\cos(xb+a)^{3}+\frac{3\cos(xb+a)}{2}\right)\sin(xb+a)}{16}+\frac{3xb}{32}+\frac{3a}{32}\right)}{b}}{b}$ |
| norman | $\frac{\frac{5d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{16b^{2}}-\frac{3d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{3}}{16b^{2}}+\frac{3d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{5}}{16b^{2}}-\frac{5d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{7}}{16b^{2}}-\frac{5dx}{32b}+\frac{2c\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}}{b}+\frac{2c\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{6}}{b}+\frac{11dx\tan\left(\frac{a}{2}\right.}{8b}}{\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)^{4}}$ |

input
```
int((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/128*(-16*b*(d*x+c)*cos(2*b*x+2*a)-4*b*(d*x+c)*cos(4*b*x+4*a)+20*c*b+8*d*
sin(2*b*x+2*a)+d*sin(4*b*x+4*a))/b^2
```

## 3.140.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{8\,(bdx+bc)\cos{(bx+a)}^4-3\,bdx-\left(2\,d\cos{(bx+a)}^3+3\,d\cos{(bx+a)}\right)\sin{(bx+a)}}{32\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/32*(8*(b*d*x + b*c)*cos(b*x + a)^4 - 3*b*d*x - (2*d*cos(b*x + a)^3 + 3*
d*cos(b*x + a))*sin(b*x + a))/b^2
```

## 3.140.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.92

$$\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx$$
$$= \begin{cases} -\frac{c\cos^4(a+bx)}{4b}+\frac{3dx\sin^4(a+bx)}{32b}+\frac{3dx\sin^2(a+bx)\cos^2(a+bx)}{16b}-\frac{5dx\cos^4(a+bx)}{32b}+\frac{3d\sin^3(a+bx)\cos(a+bx)}{32b^2}+\frac{5d\sin(a+bx)\cos}{32b^2} \\ \left(cx+\frac{dx^2}{2}\right)\sin(a)\cos^3(a) \end{cases}$$

input
```
integrate((d*x+c)*cos(b*x+a)**3*sin(b*x+a),x)
```

output
```
Piecewise((-c*cos(a + b*x)**4/(4*b) + 3*d*x*sin(a + b*x)**4/(32*b) + 3*d*x
*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 5*d*x*cos(a + b*x)**4/(32*b) + 3
*d*sin(a + b*x)**3*cos(a + b*x)/(32*b**2) + 5*d*sin(a + b*x)*cos(a + b*x)*
*3/(32*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*sin(a)*cos(a)**3, True))
```

## 3.140.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.28

$$\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{32\,c\cos(bx+a)^4-\frac{32\,ad\cos(bx+a)^4}{b}+\frac{(4\,(bx+a)\cos(4\,bx+4\,a)+16\,(bx+a)\cos(2\,bx+2\,a)-\sin(4\,bx+4\,a)-8\,\sin(2\,bx+2\,a))d}{b}}{128\,b}$$

input
```
integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/128*(32*c*cos(b*x + a)^4 - 32*a*d*cos(b*x + a)^4/b + (4*(b*x + a)*cos(4
*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b
*x + 2*a))*d/b)/b
```

## 3.140.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04

$$\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{(bdx+bc)\cos(4\,bx+4\,a)}{32\,b^2} - \frac{(bdx+bc)\cos(2\,bx+2\,a)}{8\,b^2} + \frac{d\sin(4\,bx+4\,a)}{128\,b^2} + \frac{d\sin(2\,bx+2\,a)}{16\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/32*(b*d*x + b*c)*cos(4*b*x + 4*a)/b^2 - 1/8*(b*d*x + b*c)*cos(2*b*x + 2
*a)/b^2 + 1/128*d*sin(4*b*x + 4*a)/b^2 + 1/16*d*sin(2*b*x + 2*a)/b^2
```

## 3.140.9 Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.31

$$\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx$$

$$= \frac{4\,d\,\sin(2\,a+2\,b\,x) + \frac{d\,\sin(4\,a+4\,b\,x)}{2} + 4\,b\,c\sin(2\,a+2\,b\,x)^2 + 16\,b\,c\sin(a+b\,x)^2 + 8\,b\,d\,x\,\left(2\sin(a+b\,x)^2 -\right.}{64\,b^2}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x),x)
```

output
```
(4*d*sin(2*a + 2*b*x) + (d*sin(4*a + 4*b*x))/2 + 4*b*c*sin(2*a + 2*b*x)^2
+ 16*b*c*sin(a + b*x)^2 + 8*b*d*x*(2*sin(a + b*x)^2 - 1) + 2*b*d*x*(2*sin(
2*a + 2*b*x)^2 - 1))/(64*b^2)
```

# 3.141 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx$


3.141.1 Optimal result . . . . . . . . . . 1148
3.141.2 Mathematica [A] (verified) . . . . . . . . . . 1148
3.141.3 Rubi [A] (verified) . . . . . . . . . . 1149
3.141.4 Maple [A] (verified) . . . . . . . . . . 1150
3.141.5 Fricas [A] (verification not implemented) . . . . . . . . . . 1151
3.141.6 Sympy [F] . . . . . . . . . . 1151
3.141.7 Maxima [C] (verification not implemented) . . . . . . . . . . 1151
3.141.8 Giac [C] (verification not implemented) . . . . . . . . . . 1152
3.141.9 Mupad [F(-1)] . . . . . . . . . . 1153


## 3.141.1 Optimal result

Integrand size = 22, antiderivative size = 129

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx = \frac{\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{8d} + \frac{\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{4d} + \frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{4d} + \frac{\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{8d}$$

output
```
1/4*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d+1/8*cos(4*a-4*b*c/d)*Si(4*b*c/d+4
*b*x)/d+1/8*Ci(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d+1/4*Ci(2*b*c/d+2*b*x)*sin
(2*a-2*b*c/d)/d
```

## 3.141.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.85

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx = \frac{\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)\sin\left(4a-\frac{4bc}{d}\right)+2\,\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right)+2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+d}{d}\right.}{8d}$$

input
```
Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x),x]
```

output
```
(CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + 2*CosIntegral[(2*b*
(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + 2*Cos[2*a - (2*b*c)/d]*SinIntegral[(2
*b*(c + d*x))/d] + Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(8
*d)
```

### 3.141.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\cos^3(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{\sin(2a+2bx)}{4(c+dx)}+\frac{\sin(4a+4bx)}{8(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{4d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{4d}+\frac{\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{8d}$$

input
```
Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x),x]
```

output
```
(CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/(8*d) + (CosIntegral
[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(4*d) + (Cos[2*a - (2*b*c)/d]*Si
nIntegral[(2*b*c)/d + 2*b*x])/(4*d) + (Cos[4*a - (4*b*c)/d]*SinIntegral[(4
*b*c)/d + 4*b*x])/(8*d)
```

### 3.141.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.141.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.38

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{b\left(-\frac{2\,\mathrm{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2xb+2a+\frac{-2ad+2cb}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d}\right)}{8}+\frac{b\left(-\frac{4\,\mathrm{Si}\left(-4xb-4a-\frac{4(-ad\cdots}{d}\right)\cdots}{d}\cdots\right)}{\cdots}}{b}$ |
| default | $\frac{\frac{b\left(-\frac{2\,\mathrm{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2xb+2a+\frac{-2ad+2cb}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d}\right)}{8}+\frac{b\left(-\frac{4\,\mathrm{Si}\left(-4xb-4a-\frac{4(-ad\cdots}{d}\right)\cdots}{d}\cdots\right)}{\cdots}}{b}$ |
| risch | $-\frac{i e^{-\frac{4i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{16d}-\frac{i e^{-\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{8d}+\frac{i e^{\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(-2ibx-2\cdots\right)}{8d}$ |

input
```
int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*b*(-2*Si(-2*x*b-2*a-2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*
x*b+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)+1/32*b*(-4*Si(-4*x*b-4*a-4*
(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*x*b+4*a+4*(-a*d+b*c)/d)*sin(4*(
-a*d+b*c)/d)/d))
```

### 3.141.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.96

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx$$
$$= \frac{2\,\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+\mathrm{Ci}\left(\frac{4\,(bdx+bc)}{d}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)+\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{4\,(bdx+bc)}{d}\right)+2\cos}{8\,d}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
1/8*(2*cos_integral(2*(b*d*x + b*c)/d)*sin(-2*(b*c - a*d)/d) + cos_integra
l(4*(b*d*x + b*c)/d)*sin(-4*(b*c - a*d)/d) + cos(-4*(b*c - a*d)/d)*sin_int
egral(4*(b*d*x + b*c)/d) + 2*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x +
 b*c)/d))/d
```

### 3.141.6 Sympy [F]

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx = \int \frac{\sin(a+bx)\cos^3(a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x), x)
```

### 3.141.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.18

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx$$
$$= \frac{2\,b\left(i\,E_1\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_1\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b\left(-i\,E_1\left(\frac{4\,(-i\,bc-i\,(bx+a)d-}{d}\right.\right.}{}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
1/16*(2*b*(I*exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*e
xp_integral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a*d
)/d) - b*(-I*exp_integral_e(1, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*e
xp_integral_e(1, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c - a*d
)/d) - 2*b*(exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_
integral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)/d
) - b*(exp_integral_e(1, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integ
ral_e(1, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d)/d))/(b
*d)
```

### 3.141.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.54 (sec) , antiderivative size = 6046, normalized size of antiderivative = 46.87

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
1/16*(imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b
*c/d)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)
^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(co
s_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d
)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2
*tan(b*c/d)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(
2*b*c/d)^2*tan(b*c/d)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2
*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 4*real_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 2*real_part(c
os_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^
2 + 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*
b*c/d)*tan(b*c/d)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^
2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*real_part(cos_integral(-2*b*x - 2
*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_i
ntegral(4*b*x + 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 -
2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)
^2*tan(b*c/d)^2 + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(
a)^2*tan(2*b*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^
2*tan(a)^2*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))...
```

### 3.141.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx = \int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)}{c+d\,x}\,dx$$

input
```
int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x),x)
```

output
```
int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x), x)
```

# 3.142 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2} \, dx$

3.142.1 Optimal result . . . 1154
3.142.2 Mathematica [A] (verified) . . . 1155
3.142.3 Rubi [A] (verified) . . . 1155
3.142.4 Maple [A] (verified) . . . 1156
3.142.5 Fricas [A] (verification not implemented) . . . 1157
3.142.6 Sympy [F] . . . 1157
3.142.7 Maxima [C] (verification not implemented) . . . 1158
3.142.8 Giac [C] (verification not implemented) . . . 1158
3.142.9 Mupad [F(-1)] . . . 1159

## 3.142.1 Optimal result

Integrand size = 22, antiderivative size = 179

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2} \, dx = \frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^2} + \frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2} - \frac{\sin(2a+2bx)}{4d(c+dx)} - \frac{\sin(4a+4bx)}{8d(c+dx)} - \frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^2} - \frac{b\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{2d^2}$$

output
```
1/2*b*Ci(4*b*c/d+4*b*x)*cos(4*a-4*b*c/d)/d^2+1/2*b*Ci(2*b*c/d+2*b*x)*cos(2
*a-2*b*c/d)/d^2-1/2*b*Si(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^2-1/2*b*Si(2*b*
c/d+2*b*x)*sin(2*a-2*b*c/d)/d^2-1/4*sin(2*b*x+2*a)/d/(d*x+c)-1/8*sin(4*b*x
+4*a)/d/(d*x+c)
```

### 3.142.2 Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.84

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{-4b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)-4b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)+\frac{2d\sin(2(a+bx))}{c+dx}+\frac{d}{}}{8d^2}$$

input
```
Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^2,x]
```

output
```
-1/8*(-4*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - 4*b*Cos[4
*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] + (2*d*Sin[2*(a + b*x)])/(c
 + d*x) + (d*Sin[4*(a + b*x)])/(c + d*x) + 4*b*Sin[2*a - (2*b*c)/d]*SinInt
egral[(2*b*(c + d*x))/d] + 4*b*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c +
d*x))/d])/d^2
```

### 3.142.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\cos^3(a+bx)}{(c+dx)^2}dx$$
$$\Big\downarrow \ 4906$$
$$\int \left(\frac{\sin(2a+2bx)}{4(c+dx)^2}+\frac{\sin(4a+4bx)}{8(c+dx)^2}\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^2}-\frac{b\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{\sin(2a+2bx)}{4d(c+dx)}-\frac{\sin(4a+4bx)}{8d(c+dx)}$$

input
```
Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^2,x]
```

output
```
(b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) + (b*Cos[4
*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(2*d^2) - Sin[2*a + 2*b*x]
/(4*d*(c + d*x)) - Sin[4*a + 4*b*x]/(8*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)
/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) - (b*Sin[4*a - (4*b*c)/d]*SinI
ntegral[(4*b*c)/d + 4*b*x])/(2*d^2)
```

### 3.142.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.142.4 Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.43

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{b^2\left(-\frac{2\sin(2xb+2a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{4\,\mathrm{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(2xb+2a+\frac{-2ad+2cb}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}}{d}\right)}{8}+\frac{b^2}{}}{b}$ |
| default | $\frac{\frac{b^2\left(-\frac{2\sin(2xb+2a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{4\,\mathrm{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(2xb+2a+\frac{-2ad+2cb}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}}{d}\right)}{8}+\frac{b^2}{}}{b}$ |
| risch | $-\frac{b\,\mathrm{e}^{-\frac{4i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{4d^2}-\frac{b\,\mathrm{e}^{-\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{4d^2}-\frac{b\,\mathrm{e}^{\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(-2ibx\right.}{4d^2}$ |

input
```
int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*b^2*(-2*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d+2*(-2*Si(-2*x*b-2*a
-2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*x*b+2*a+2*(-a*d+b*c)/d)*cos(
2*(-a*d+b*c)/d)/d)/d)+1/32*b^2*(-4*sin(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))/d+4
*(-4*Si(-4*x*b-4*a-4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4*x*b+4*a+4*
(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d)/d))
```

## 3.142.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.02

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{2\,d\cos\left(bx+a\right)^3\sin\left(bx+a\right)-\left(bdx+bc\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{4\,(bdx+bc)}{d}\right)-\left(bdx+bc\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}}{2\left(d^3x+\right.}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*d*cos(b*x + a)^3*sin(b*x + a) - (b*d*x + b*c)*cos(-4*(b*c - a*d)/d
)*cos_integral(4*(b*d*x + b*c)/d) - (b*d*x + b*c)*cos(-2*(b*c - a*d)/d)*co
s_integral(2*(b*d*x + b*c)/d) + (b*d*x + b*c)*sin(-4*(b*c - a*d)/d)*sin_in
tegral(4*(b*d*x + b*c)/d) + (b*d*x + b*c)*sin(-2*(b*c - a*d)/d)*sin_integr
al(2*(b*d*x + b*c)/d))/(d^3*x + c*d^2)
```

## 3.142.6 Sympy [F]

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin\left(a+bx\right)\cos^3\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x)**2, x)
```

### 3.142.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.72

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{2\,b^2\left(i\,E_2\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_2\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^2\left(-i\,E_2\left(\frac{4\,(-i\,bc-i\,(bx+a}{d}\right.\right.}{}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/16*(2*b^2*(I*exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I
*exp_integral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a
*d)/d) - b^2*(-I*exp_integral_e(2, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 I*exp_integral_e(2, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c -
 a*d)/d) - 2*b^2*(exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d)
+ exp_integral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c -
a*d)/d) - b^2*(exp_integral_e(2, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + e
xp_integral_e(2, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d
)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

### 3.142.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.63 (sec) , antiderivative size = 63510, normalized size of antiderivative = 354.80

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/4*(b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^
2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + b*d*x*real_part(cos_in
tegral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2
*b*c/d)^2*tan(b*c/d)^2 + b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + b
*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan
(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*b*d*x*imag_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*
c/d)^2*tan(b*c/d) + 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 4*b*d*
x*sin_integral(2*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a
)^2*tan(2*b*c/d)^2*tan(b*c/d) - 2*b*d*x*imag_part(cos_integral(4*b*x + 4*b
*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)
^2 + 2*b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*
x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 4*b*d*x*sin_integral(
4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d
)*tan(b*c/d)^2 + 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*
x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*b*d*x*im
ag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2
*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*b*d*x*sin_integral(2*(b*d*x + b...
```

### 3.142.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)}{\left(c+d\,x\right)^2}\,dx$$

input
```
int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^2,x)
```

output
```
int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^2, x)
```

# 3.143 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$


3.143.1 Optimal result . . . . . 1160
3.143.2 Mathematica [A] (verified) . . . . . 1161
3.143.3 Rubi [A] (verified) . . . . . 1161
3.143.4 Maple [A] (verified) . . . . . 1162
3.143.5 Fricas [A] (verification not implemented) . . . . . 1163
3.143.6 Sympy [F] . . . . . 1163
3.143.7 Maxima [C] (verification not implemented) . . . . . 1164
3.143.8 Giac [C] (verification not implemented) . . . . . 1164
3.143.9 Mupad [F(-1)] . . . . . 1165


## 3.143.1 Optimal result

Integrand size = 22, antiderivative size = 231

$$\begin{aligned}\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx =& -\frac{b\cos(2a+2bx)}{4d^2(c+dx)}-\frac{b\cos(4a+4bx)}{4d^2(c+dx)}\\ &-\frac{b^2\,\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{d^3}\\ &-\frac{b^2\,\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d^3}-\frac{\sin(2a+2bx)}{8d(c+dx)^2}\\ &-\frac{\sin(4a+4bx)}{16d(c+dx)^2}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^3}\\ &-\frac{b^2\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{d^3}\end{aligned}$$

output
```
-1/4*b*cos(2*b*x+2*a)/d^2/(d*x+c)-1/4*b*cos(4*b*x+4*a)/d^2/(d*x+c)-1/2*b^2
*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^3-b^2*cos(4*a-4*b*c/d)*Si(4*b*c/d+4*
b*x)/d^3-b^2*Ci(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^3-1/2*b^2*Ci(2*b*c/d+2*b
*x)*sin(2*a-2*b*c/d)/d^3-1/8*sin(2*b*x+2*a)/d/(d*x+c)^2-1/16*sin(4*b*x+4*a
)/d/(d*x+c)^2
```

### 3.143.2 Mathematica [A] (verified)

Time = 4.01 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.85

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx =$$

$$-\frac{16b^2\,\text{CosIntegral}\left(\frac{4b(c+dx)}{d}\right)\sin\left(4a-\frac{4bc}{d}\right)+8b^2\,\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right)+\frac{2d(2b(c+dx)\cos(2(a\cdots}{(c+\cdots}}{\cdots}$$

input
```
Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^3,x]
```

output
```
-1/16*(16*b^2*CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + 8*b^2*
CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (2*d*(2*b*(c + d*x)*
Cos[2*(a + b*x)] + d*Sin[2*(a + b*x)]))/(c + d*x)^2 + (d*(4*b*(c + d*x)*Co
s[4*(a + b*x)] + d*Sin[4*(a + b*x)]))/(c + d*x)^2 + 8*b^2*Cos[2*a - (2*b*c
)/d]*SinIntegral[(2*b*(c + d*x))/d] + 16*b^2*Cos[4*a - (4*b*c)/d]*SinInteg
ral[(4*b*(c + d*x))/d])/d^3
```

### 3.143.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\cos^3(a+bx)}{(c+dx)^3}\,dx$$

$$\Big\downarrow \; 4906$$

$$\int \left(\frac{\sin(2a+2bx)}{4(c+dx)^3}+\frac{\sin(4a+4bx)}{8(c+dx)^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^3}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^3}-\frac{b^2\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b\cos(2a+2bx)}{4d^2(c+dx)}-\frac{b\cos(4a+4bx)}{4d^2(c+dx)}-\frac{\sin(2a+2bx)}{8d(c+dx)^2}-\frac{\sin(4a+4bx)}{16d(c+dx)^2}$$

input
```
Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^3,x]
```

output
```
-1/4*(b*Cos[2*a + 2*b*x])/(d^2*(c + d*x)) - (b*Cos[4*a + 4*b*x])/(4*d^2*(c
 + d*x)) - (b^2*CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/d^3 -
 (b^2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d^3) - Sin[2
*a + 2*b*x]/(8*d*(c + d*x)^2) - Sin[4*a + 4*b*x]/(16*d*(c + d*x)^2) - (b^2
*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^3) - (b^2*Cos[4
*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/d^3
```

### 3.143.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.143.4 Maple [A] (verified)

Time = 1.69 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.42

| method | result |
|---|---|
| derivativedivides | $\frac{b^3\left(-\frac{\sin(2xb+2a)}{(-ad+cb+d(xb+a))^2 d}+\frac{-\frac{2\cos(2xb+2a)}{(-ad+cb+d(xb+a))d}-\frac{2\left(-\frac{2\,\mathrm{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2xb+2a+\frac{-2ad}{d}\right.}{}\right.}{d}}{d}\right.}{8}$ |
| default | $\frac{b^3\left(-\frac{\sin(2xb+2a)}{(-ad+cb+d(xb+a))^2 d}+\frac{-\frac{2\cos(2xb+2a)}{(-ad+cb+d(xb+a))d}-\frac{2\left(-\frac{2\,\mathrm{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2xb+2a+\frac{-2ad}{d}\right.}{}\right.}{d}}{d}\right.}{8}$ |
| risch | $\frac{ib^2 e^{-\frac{4i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{2d^3}+\frac{ib^2 e^{-\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{4d^3}-\frac{ib^2 e^{\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(-2ib\right.}{4d^3}$ |

3.143. $\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$

input
```
int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*b^3*(-sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^2/d+(-2*cos(2*b*x+2*a)/
(-a*d+c*b+d*(b*x+a))/d-2*(-2*Si(-2*x*b-2*a-2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c
)/d)/d-2*Ci(2*x*b+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d)+1/32*b^
3*(-2*sin(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))^2/d+2*(-4*cos(4*b*x+4*a)/(-a*d+c
*b+d*(b*x+a))/d-4*(-4*Si(-4*x*b-4*a-4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-
4*Ci(4*x*b+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)/d)/d))
```

### 3.143.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.34

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx =$$

$$-\frac{d^2\cos{(bx+a)}^3\sin{(bx+a)}+4\,(bd^2x+bcd)\cos{(bx+a)}^4-3\,(bd^2x+bcd)\cos{(bx+a)}^2+(b^2d^2x^2+2}{}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/2*(d^2*cos(b*x + a)^3*sin(b*x + a) + 4*(b*d^2*x + b*c*d)*cos(b*x + a)^4
 - 3*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2)*cos_integral(2*(b*d*x + b*c)/d)*sin(-2*(b*c - a*d)/d) + 2*(b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(4*(b*d*x + b*c)/d)*sin(-4*(b*c - a*
d)/d) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-4*(b*c - a*d)/d)*sin_
integral(4*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-2
*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d))/(d^5*x^2 + 2*c*d^4*x + c^
2*d^3)
```

### 3.143.6 Sympy [F]

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin{(a+bx)}\cos^3{(a+bx)}}{(c+dx)^3}\,dx$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c)**3,x)
```

output
```
Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x)**3, x)
```

### 3.143.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.48

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \frac{2\,b^3\left(i\,E_3\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_3\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^3\left(-i\,E_3\left(\frac{4\,(-i\,bc-i\,(bx+a}{d}\right.\right.}{}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/16*(2*b^3*(I*exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I
*exp_integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a
*d)/d) - b^3*(-I*exp_integral_e(3, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 I*exp_integral_e(3, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c -
 a*d)/d) - 2*b^3*(exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d)
+ exp_integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c -
a*d)/d) - b^3*(exp_integral_e(3, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + e
xp_integral_e(3, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d
)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 -
 a*d^3)*(b*x + a))*b)
```

### 3.143.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.42 (sec) , antiderivative size = 111694, normalized size of antiderivative = 483.52

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/8*(4*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*
tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2
*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)
^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*b^2*d^2*x^2*imag_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*
b*c/d)^2*tan(b*c/d)^2 - 4*b^2*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*
c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d
)^2 + 8*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^
2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*b^2*d^2*x^2*sin_inte
gral(2*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*
b*c/d)^2*tan(b*c/d)^2 + 4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)
 + 4*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*ta
n(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 8*b^2*d^2*x^2*rea
l_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*t
an(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 8*b^2*d^2*x^2*real_part(cos_integral(-
4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)
*tan(b*c/d)^2 - 4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan
(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*b^2
*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x...
```

### 3.143.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)}{\left(c+d\,x\right)^3}\,dx$$

input
```
int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^3,x)
```

output
```
int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^3, x)
```

# 3.144 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$

3.144.1 Optimal result . . . 1166
3.144.2 Mathematica [A] (verified) . . . 1167
3.144.3 Rubi [A] (verified) . . . 1167
3.144.4 Maple [A] (verified) . . . 1169
3.144.5 Fricas [A] (verification not implemented) . . . 1170
3.144.6 Sympy [F] . . . 1170
3.144.7 Maxima [C] (verification not implemented) . . . 1171
3.144.8 Giac [C] (verification not implemented) . . . 1171
3.144.9 Mupad [F(-1)] . . . 1172

## 3.144.1 Optimal result

Integrand size = 22, antiderivative size = 287

$$\begin{aligned}\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx =& -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2}-\frac{b\cos(4a+4bx)}{12d^2(c+dx)^2}\\ &-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}\\ &-\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4}\\ &-\frac{\sin(2a+2bx)}{12d(c+dx)^3}+\frac{b^2\sin(2a+2bx)}{6d^3(c+dx)}-\frac{\sin(4a+4bx)}{24d(c+dx)^3}\\ &+\frac{b^2\sin(4a+4bx)}{3d^3(c+dx)}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}\\ &+\frac{4b^3\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{3d^4}\end{aligned}$$

output
```
-4/3*b^3*Ci(4*b*c/d+4*b*x)*cos(4*a-4*b*c/d)/d^4-1/3*b^3*Ci(2*b*c/d+2*b*x)*
cos(2*a-2*b*c/d)/d^4-1/12*b*cos(2*b*x+2*a)/d^2/(d*x+c)^2-1/12*b*cos(4*b*x+
4*a)/d^2/(d*x+c)^2+4/3*b^3*Si(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^4+1/3*b^3*
Si(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^4-1/12*sin(2*b*x+2*a)/d/(d*x+c)^3+1/6
*b^2*sin(2*b*x+2*a)/d^3/(d*x+c)-1/24*sin(4*b*x+4*a)/d/(d*x+c)^3+1/3*b^2*si
n(4*b*x+4*a)/d^3/(d*x+c)
```

### 3.144.2 Mathematica [A] (verified)

Time = 2.30 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.10

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx =$$

$$-\frac{2d\cos(2bx)\left(bd(c+dx)\cos(2a)+\left(d^2-2b^2(c+dx)^2\right)\sin(2a)\right)+d\cos(4bx)\left(2bd(c+dx)\cos(4a)+(d^2\right.}{}$$

input
```
Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^4,x]
```

output
```
-1/24*(2*d*Cos[2*b*x]*(b*d*(c + d*x)*Cos[2*a] + (d^2 - 2*b^2*(c + d*x)^2)*
Sin[2*a]) + d*Cos[4*b*x]*(2*b*d*(c + d*x)*Cos[4*a] + (d^2 - 8*b^2*(c + d*x
)^2)*Sin[4*a]) - 2*d*((-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*a] + b*d*(c + d*x)*
Sin[2*a])*Sin[2*b*x] - d*((-d^2 + 8*b^2*(c + d*x)^2)*Cos[4*a] + 2*b*d*(c +
 d*x)*Sin[4*a])*Sin[4*b*x] + 8*b^3*(c + d*x)^3*(Cos[2*a - (2*b*c)/d]*CosIn
tegral[(2*b*(c + d*x))/d] - Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x
))/d]) + 32*b^3*(c + d*x)^3*(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*
x))/d] - Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d]))/(d^4*(c + d
*x)^3)
```

### 3.144.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\cos^3(a+bx)}{(c+dx)^4}\,dx$$

$$\Big\downarrow \; 4906$$

$$\int \left(\frac{\sin(2a+2bx)}{4(c+dx)^4}+\frac{\sin(4a+4bx)}{8(c+dx)^4}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{4b^3\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{3d^4}+\frac{b^2\sin(2a+2bx)}{6d^3(c+dx)}+\frac{b^2\sin(4a+4bx)}{3d^3(c+dx)}-\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2}-\frac{b\cos(4a+4bx)}{12d^2(c+dx)^2}-\frac{\sin(2a+2bx)}{12d(c+dx)^3}-\frac{\sin(4a+4bx)}{24d(c+dx)^3}$$

input
```
Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^4,x]
```

output
```
-1/12*(b*Cos[2*a + 2*b*x])/(d^2*(c + d*x)^2) - (b*Cos[4*a + 4*b*x])/(12*d^
2*(c + d*x)^2) - (b^3*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])
/(3*d^4) - (4*b^3*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(3*
d^4) - Sin[2*a + 2*b*x]/(12*d*(c + d*x)^3) + (b^2*Sin[2*a + 2*b*x])/(6*d^3
*(c + d*x)) - Sin[4*a + 4*b*x]/(24*d*(c + d*x)^3) + (b^2*Sin[4*a + 4*b*x])
/(3*d^3*(c + d*x)) + (b^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b
*x])/(3*d^4) + (4*b^3*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])
/(3*d^4)
```

### 3.144.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.144.4 Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.41

| method | result |
|---|---|
| derivativedivides | $\frac{b^4\left(-\frac{2\sin(2xb+2a)}{3(-ad+cb+d(xb+a))^3 d}+\frac{-\frac{2\cos(2xb+2a)}{3(-ad+cb+d(xb+a))^2 d}-\frac{2\left(-\frac{2\sin(2xb+2a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{4\,\mathrm{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\sin\left(\frac{-2ad+\cdots}{d}\right)}{d}\cdots}{3d}\right)}{d}\cdots}{}\right)}{8}\cdots$ |
| default | $\frac{b^4\left(-\frac{2\sin(2xb+2a)}{3(-ad+cb+d(xb+a))^3 d}+\frac{-\frac{2\cos(2xb+2a)}{3(-ad+cb+d(xb+a))^2 d}-\frac{2\left(-\frac{2\sin(2xb+2a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{4\,\mathrm{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\sin\left(\frac{-2ad+\cdots}{d}\right)}{d}\cdots}{3d}\right)}{d}\cdots}{}\right)}{8}\cdots$ |
| risch | $\frac{2b^3 e^{-\frac{4i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(4ibx+4ia-\frac{4i(ad-cb)}{d}\right)}{3d^4}+\frac{b^3 e^{-\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{6d^4}+\frac{b^3 e^{\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(-2ibx\cdots\right)}{6d^4}\cdots$ |

input
```
int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*b^4*(-2/3*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^3/d+2/3*(-cos(2*b*x
+2*a)/(-a*d+c*b+d*(b*x+a))^2/d-(-2*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d+2
*(-2*Si(-2*x*b-2*a-2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*x*b+2*a+2*
(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)/d)/d)+1/32*b^4*(-4/3*sin(4*b*x+4*a
)/(-a*d+c*b+d*(b*x+a))^3/d+4/3*(-2*cos(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))^2/d
-2*(-4*sin(4*b*x+4*a)/(-a*d+c*b+d*(b*x+a))/d+4*(-4*Si(-4*x*b-4*a-4*(-a*d+b
*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4*x*b+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*
c)/d)/d)/d)/d)/d))
```

### 3.144.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.59

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx =$$
$$-\frac{4\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^4-3\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^2+8\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)}{}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/6*(4*(b*d^3*x + b*c*d^2)*cos(b*x + a)^4 - 3*(b*d^3*x + b*c*d^2)*cos(b*x
 + a)^2 + 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(
-4*(b*c - a*d)/d)*cos_integral(4*(b*d*x + b*c)/d) + 2*(b^3*d^3*x^3 + 3*b^3
*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-2*(b*c - a*d)/d)*cos_integral(2
*(b*d*x + b*c)/d) - 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3
*c^3)*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) - 2*(b^3*d^3*x
^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-2*(b*c - a*d)/d)*sin_
integral(2*(b*d*x + b*c)/d) - 2*((8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c
^2*d - d^3)*cos(b*x + a)^3 - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*c
os(b*x + a))*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

### 3.144.6 Sympy [F]

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sin\left(a+bx\right)\cos^3\left(a+bx\right)}{\left(c+dx\right)^4}\,dx$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c)**4,x)
```

output
```
Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x)**4, x)
```

### 3.144.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.37

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$$
$$= \frac{2\,b^4\left(i\,E_4\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)-i\,E_4\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^4\left(-i\,E_4\left(\frac{4\,(-i\,bc-i\,(bx+a}{d}\right.\right.}{16\,(b^3}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/16*(2*b^4*(I*exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I
*exp_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a
*d)/d) - b^4*(-I*exp_integral_e(4, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 I*exp_integral_e(4, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-4*(b*c -
 a*d)/d) - 2*b^4*(exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d)
+ exp_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c -
a*d)/d) - b^4*(exp_integral_e(4, 4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + e
xp_integral_e(4, -4*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-4*(b*c - a*d
)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^
3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a
^2*d^4)*(b*x + a))*b)
```

### 3.144.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.64 (sec) , antiderivative size = 157526, normalized size of antiderivative = 548.87

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

output
```
-1/12*(8*b^3*d^3*x^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2
*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^
3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a
)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2
*b*c/d)^2*tan(b*c/d)^2 + 8*b^3*d^3*x^3*real_part(cos_integral(-4*b*x - 4*b
*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/
d)^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*x)^2
*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 4*b^3*d^3*x^3*
imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)
^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 8*b^3*d^3*x^3*sin_integral(2*(b*d*
x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan
(b*c/d) - 16*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*
x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 16*b^3*d^3
*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(b*x)^2*tan
(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 32*b^3*d^3*x^3*sin_integral(4
*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)
*tan(b*c/d)^2 + 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan
(2*b*x)^2*tan(b*x)^2*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*b^3
*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*x)^2*tan(b*x...
```

### 3.144.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)}{\left(c+d\,x\right)^4}\,dx$$

input
```
int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^4,x)
```

output
```
int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^4, x)
```

# 3.145 $\int (c+dx)^m \cos^3(a+bx)\sin^2(a+bx)\,dx$


3.145.1 Optimal result . . . 1173
3.145.2 Mathematica [A] (verified) . . . 1174
3.145.3 Rubi [A] (verified) . . . 1175
3.145.4 Maple [F] . . . 1176
3.145.5 Fricas [A] (verification not implemented) . . . 1177
3.145.6 Sympy [F(-2)] . . . 1177
3.145.7 Maxima [F] . . . 1178
3.145.8 Giac [F] . . . 1178
3.145.9 Mupad [F(-1)] . . . 1178


## 3.145.1 Optimal result

Integrand size = 24, antiderivative size = 419

$$\begin{aligned}
&\int (c+dx)^m \cos^3(a+bx)\sin^2(a+bx)\,dx \\
&= -\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{16b} \\
&\quad+\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)}{16b} \\
&\quad+\frac{i3^{-1-m}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3ib(c+dx)}{d}\right)}{32b} \\
&\quad-\frac{i3^{-1-m}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3ib(c+dx)}{d}\right)}{32b} \\
&\quad+\frac{i5^{-1-m}e^{5i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{5ib(c+dx)}{d}\right)}{32b} \\
&\quad-\frac{i5^{-1-m}e^{-5i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{5ib(c+dx)}{d}\right)}{32b}
\end{aligned}$$

output
```
-1/16*I*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x
+c)/d)^m)+1/16*I*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I
*b*(d*x+c)/d)^m)+1/32*I*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3
*I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)-1/32*I*3^(-1-m)*(d*x+c)^m*GAMMA(1+m
,3*I*b*(d*x+c)/d)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)+1/32*I*5^(-1-m)
*exp(5*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-5*I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)
/d)^m)-1/32*I*5^(-1-m)*(d*x+c)^m*GAMMA(1+m,5*I*b*(d*x+c)/d)/b/exp(5*I*(a-b
*c/d))/((I*b*(d*x+c)/d)^m)
```

### 3.145.2 Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.98

$$\int (c+dx)^m \cos^3(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{ie^{-\frac{i(bc+ad)}{d}}(c+dx)^m\left(e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)-e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)\right)}{16b}$$
$$-\frac{i3^{-1-m}e^{-\frac{3i(bc+ad)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(-e^{6ia}\left(\frac{ib(c+dx)}{d}\right)^m\Gamma\left(1+m,-\frac{3ib(c+dx)}{d}\right)+e^{\frac{6ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m\cdots}{32b}$$
$$-\frac{i5^{-1-m}e^{-\frac{5i(bc+ad)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(-e^{10ia}\left(\frac{ib(c+dx)}{d}\right)^m\Gamma\left(1+m,-\frac{5ib(c+dx)}{d}\right)+e^{\frac{10ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m\cdots}{32b}$$

input
```
Integrate[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
((-1/16*I)*(c + d*x)^m*((E^((2*I)*a)*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(
((-I)*b*(c + d*x))/d)^m - (E^(((2*I)*b*c)/d)*Gamma[1 + m, (I*b*(c + d*x))/
d])/((I*b*(c + d*x))/d)^m))/(b*E^((I*(b*c + a*d))/d)) - ((I/32)*3^(-1 - m)
*(c + d*x)^m*(-(E^((6*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-3*I)*b*(
c + d*x))/d]) + E^(((6*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((
3*I)*b*(c + d*x))/d]))/(b*E^(((3*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2
)^m) - ((I/32)*5^(-1 - m)*(c + d*x)^m*(-(E^((10*I)*a)*((I*b*(c + d*x))/d)^
m*Gamma[1 + m, ((-5*I)*b*(c + d*x))/d]) + E^(((10*I)*b*c)/d)*(((-I)*b*(c +
 d*x))/d)^m*Gamma[1 + m, ((5*I)*b*(c + d*x))/d]))/(b*E^(((5*I)*(b*c + a*d)
)/d)*((b^2*(c + d*x)^2)/d^2)^m)
```

### 3.145.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\cos^3(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}\cos(a+bx)(c+dx)^m-\frac{1}{16}\cos(3a+3bx)(c+dx)^m-\frac{1}{16}\cos(5a+5bx)(c+dx)^m\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{16b}+\frac{i3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{32b}+\frac{i5^{-m-1}e^{5i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{5ib(c+dx)}{d}\right)}{32b}+\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{16b}-\frac{i3^{-m-1}e^{-3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{3ib(c+dx)}{d}\right)}{32b}-\frac{i5^{-m-1}e^{-5i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{5ib(c+dx)}{d}\right)}{32b}$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
((-1/16*I)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))
/d])/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/16)*(c + d*x)^m*Gamma[1 + m, (I*b*
(c + d*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + ((I/32)*3^(
-1 - m)*E^((3*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*
x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) - ((I/32)*3^(-1 - m)*(c + d*x)^m*Gamm
a[1 + m, ((3*I)*b*(c + d*x))/d])/(b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x
))/d)^m) + ((I/32)*5^(-1 - m)*E^((5*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1
+ m, ((-5*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) - ((I/32)*5^(-1
 - m)*(c + d*x)^m*Gamma[1 + m, ((5*I)*b*(c + d*x))/d])/(b*E^((5*I)*(a - (b
*c)/d))*((I*b*(c + d*x))/d)^m)
```

### 3.145.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.145.4 Maple [F]

$$\int (dx+c)^m \cos(xb+a)^3 \sin(xb+a)^2 \, dx$$

input
```
int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

### 3.145.5 Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.67

$$\int (c+dx)^m \cos^3(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{30i\,e^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-i\,bc+i\,ad}{d}\right)}\Gamma\left(m+1,\frac{i\,bdx+i\,bc}{d}\right)+5i\,e^{\left(-\frac{dm\log\left(-\frac{3i\,b}{d}\right)+3i\,bc-3i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{3\,(i\,bdx+i\,bc)}{d}\right)+3i\,e^{\left(-\frac{d}{}\right.}}{}$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/480*(30*I*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I*b*d*x
+ I*b*c)/d) + 5*I*e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d)*gamma(m +
 1, -3*(I*b*d*x + I*b*c)/d) + 3*I*e^(-(d*m*log(-5*I*b/d) + 5*I*b*c - 5*I*a
*d)/d)*gamma(m + 1, -5*(I*b*d*x + I*b*c)/d) - 30*I*e^(-(d*m*log(-I*b/d) +
I*b*c - I*a*d)/d)*gamma(m + 1, (-I*b*d*x - I*b*c)/d) - 5*I*e^(-(d*m*log(3*
I*b/d) - 3*I*b*c + 3*I*a*d)/d)*gamma(m + 1, -3*(-I*b*d*x - I*b*c)/d) - 3*I
*e^(-(d*m*log(5*I*b/d) - 5*I*b*c + 5*I*a*d)/d)*gamma(m + 1, -5*(-I*b*d*x -
 I*b*c)/d))/b
```

### 3.145.6 Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m \cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d*x+c)**m*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.145.7 Maxima [F]

$$\int (c+dx)^m \cos^3(a+bx) \sin^2(a+bx) \, dx = \int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)^2 \, dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^2, x)
```

### 3.145.8 Giac [F]

$$\int (c+dx)^m \cos^3(a+bx) \sin^2(a+bx) \, dx = \int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)^2 \, dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^2, x)
```

### 3.145.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cos^3(a+bx) \sin^2(a+bx) \, dx = \int \cos(a+b\,x)^3 \sin(a+b\,x)^2 (c+d\,x)^m \, dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^m, x)
```

# 3.146 $\int (c+dx)^4 \cos^3(a+bx) \sin^2(a+bx) \, dx$

3.146.1 Optimal result . . . 1179
3.146.2 Mathematica [A] (verified) . . . 1180
3.146.3 Rubi [A] (verified) . . . 1181
3.146.4 Maple [A] (verified) . . . 1182
3.146.5 Fricas [A] (verification not implemented) . . . 1183
3.146.6 Sympy [B] (verification not implemented) . . . 1183
3.146.7 Maxima [B] (verification not implemented) . . . 1184
3.146.8 Giac [A] (verification not implemented) . . . 1185
3.146.9 Mupad [B] (verification not implemented) . . . 1186

## 3.146.1 Optimal result

Integrand size = 24, antiderivative size = 330

$$\begin{aligned}\int (c+dx)^4 \cos^3(a+bx) \sin^2(a+bx) \, dx = & -\frac{3d^3(c+dx)\cos(a+bx)}{b^4} \\ & +\frac{d(c+dx)^3\cos(a+bx)}{2b^2} \\ & +\frac{d^3(c+dx)\cos(3a+3bx)}{54b^4} \\ & -\frac{d(c+dx)^3\cos(3a+3bx)}{36b^2} \\ & +\frac{3d^3(c+dx)\cos(5a+5bx)}{1250b^4} \\ & -\frac{d(c+dx)^3\cos(5a+5bx)}{100b^2} \\ & +\frac{3d^4\sin(a+bx)}{b^5}-\frac{3d^2(c+dx)^2\sin(a+bx)}{2b^3} \\ & +\frac{(c+dx)^4\sin(a+bx)}{8b}-\frac{d^4\sin(3a+3bx)}{162b^5} \\ & +\frac{d^2(c+dx)^2\sin(3a+3bx)}{36b^3} \\ & -\frac{(c+dx)^4\sin(3a+3bx)}{48b}-\frac{3d^4\sin(5a+5bx)}{6250b^5} \\ & +\frac{3d^2(c+dx)^2\sin(5a+5bx)}{500b^3} \\ & -\frac{(c+dx)^4\sin(5a+5bx)}{80b}\end{aligned}$$

output
```
-3*d^3*(d*x+c)*cos(b*x+a)/b^4+1/2*d*(d*x+c)^3*cos(b*x+a)/b^2+1/54*d^3*(d*x
+c)*cos(3*b*x+3*a)/b^4-1/36*d*(d*x+c)^3*cos(3*b*x+3*a)/b^2+3/1250*d^3*(d*x
+c)*cos(5*b*x+5*a)/b^4-1/100*d*(d*x+c)^3*cos(5*b*x+5*a)/b^2+3*d^4*sin(b*x+
a)/b^5-3/2*d^2*(d*x+c)^2*sin(b*x+a)/b^3+1/8*(d*x+c)^4*sin(b*x+a)/b-1/162*d
^4*sin(3*b*x+3*a)/b^5+1/36*d^2*(d*x+c)^2*sin(3*b*x+3*a)/b^3-1/48*(d*x+c)^4
*sin(3*b*x+3*a)/b-3/6250*d^4*sin(5*b*x+5*a)/b^5+3/500*d^2*(d*x+c)^2*sin(5*
b*x+5*a)/b^3-1/80*(d*x+c)^4*sin(5*b*x+5*a)/b
```

## 3.146.2 Mathematica [A] (verified)

Time = 3.71 (sec) , antiderivative size = 563, normalized size of antiderivative = 1.71

$$\int (c+dx)^4 \cos^3(a+bx)\sin^2(a+bx)\,dx =$$
$$-\frac{-506250b^4c^4\sin(a+bx) - 2025000b^3c^3d(\cos(a+bx)+bx\sin(a+bx)) - 2025000bcd^3(3(-2+b^2x^2)\,\mathrm{co}}{}$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
-1/4050000*(-506250*b^4*c^4*Sin[a + b*x] - 2025000*b^3*c^3*d*(Cos[a + b*x]
 + b*x*Sin[a + b*x]) - 2025000*b*c*d^3*(3*(-2 + b^2*x^2)*Cos[a + b*x] + b*
x*(-6 + b^2*x^2)*Sin[a + b*x]) - 3037500*b^2*c^2*d^2*(2*b*x*Cos[a + b*x] +
 (-2 + b^2*x^2)*Sin[a + b*x]) - 506250*d^4*(4*b*x*(-6 + b^2*x^2)*Cos[a + b
*x] + (24 - 12*b^2*x^2 + b^4*x^4)*Sin[a + b*x]) + 84375*b^4*c^4*Sin[3*(a +
 b*x)] + 112500*b^3*c^3*d*(Cos[3*(a + b*x)] + 3*b*x*Sin[3*(a + b*x)]) + 37
500*b*c*d^3*((-2 + 9*b^2*x^2)*Cos[3*(a + b*x)] + 3*b*x*(-2 + 3*b^2*x^2)*Si
n[3*(a + b*x)]) + 56250*b^2*c^2*d^2*(6*b*x*Cos[3*(a + b*x)] + (-2 + 9*b^2*
x^2)*Sin[3*(a + b*x)]) + 3125*d^4*(12*b*x*(-2 + 3*b^2*x^2)*Cos[3*(a + b*x)
] + (8 - 36*b^2*x^2 + 27*b^4*x^4)*Sin[3*(a + b*x)]) + 50625*b^4*c^4*Sin[5*
(a + b*x)] + 40500*b^3*c^3*d*(Cos[5*(a + b*x)] + 5*b*x*Sin[5*(a + b*x)]) +
 1620*b*c*d^3*((-6 + 75*b^2*x^2)*Cos[5*(a + b*x)] + 5*b*x*(-6 + 25*b^2*x^2
)*Sin[5*(a + b*x)]) + 12150*b^2*c^2*d^2*(10*b*x*Cos[5*(a + b*x)] + (-2 + 2
5*b^2*x^2)*Sin[5*(a + b*x)]) + 81*d^4*(20*b*x*(-6 + 25*b^2*x^2)*Cos[5*(a +
 b*x)] + (24 - 300*b^2*x^2 + 625*b^4*x^4)*Sin[5*(a + b*x)]))/b^5
```

### 3.146.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sin^2(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}(c+dx)^4\cos(a+bx)-\frac{1}{16}(c+dx)^4\cos(3a+3bx)-\frac{1}{16}(c+dx)^4\cos(5a+5bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3d^4\sin(a+bx)}{b^5}-\frac{d^4\sin(3a+3bx)}{162b^5}-\frac{3d^4\sin(5a+5bx)}{6250b^5}-\frac{3d^3(c+dx)\cos(a+bx)}{b^4}+\frac{d^3(c+dx)\cos(3a+3bx)}{54b^4}+\frac{3d^3(c+dx)\cos(5a+5bx)}{1250b^4}-\frac{3d^2(c+dx)^2\sin(a+bx)}{2b^3}+\frac{d^2(c+dx)^2\sin(3a+3bx)}{36b^3}+\frac{3d^2(c+dx)^2\sin(5a+5bx)}{500b^3}+\frac{d(c+dx)^3\cos(a+bx)}{2b^2}-\frac{d(c+dx)^3\cos(3a+3bx)}{36b^2}-\frac{d(c+dx)^3\cos(5a+5bx)}{100b^2}+\frac{(c+dx)^4\sin(a+bx)}{8b}-\frac{(c+dx)^4\sin(3a+3bx)}{48b}-\frac{(c+dx)^4\sin(5a+5bx)}{80b}$$

input
```
Int[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
(-3*d^3*(c + d*x)*Cos[a + b*x])/b^4 + (d*(c + d*x)^3*Cos[a + b*x])/(2*b^2)
 + (d^3*(c + d*x)*Cos[3*a + 3*b*x])/(54*b^4) - (d*(c + d*x)^3*Cos[3*a + 3*
b*x])/(36*b^2) + (3*d^3*(c + d*x)*Cos[5*a + 5*b*x])/(1250*b^4) - (d*(c + d
*x)^3*Cos[5*a + 5*b*x])/(100*b^2) + (3*d^4*Sin[a + b*x])/b^5 - (3*d^2*(c +
 d*x)^2*Sin[a + b*x])/(2*b^3) + ((c + d*x)^4*Sin[a + b*x])/(8*b) - (d^4*Si
n[3*a + 3*b*x])/(162*b^5) + (d^2*(c + d*x)^2*Sin[3*a + 3*b*x])/(36*b^3) -
((c + d*x)^4*Sin[3*a + 3*b*x])/(48*b) - (3*d^4*Sin[5*a + 5*b*x])/(6250*b^5
) + (3*d^2*(c + d*x)^2*Sin[5*a + 5*b*x])/(500*b^3) - ((c + d*x)^4*Sin[5*a
+ 5*b*x])/(80*b)
```

#### 3.146.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.146.4 Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.77

| method | result |
|---|---|
| parallelrisch | $\frac{\left(-84375b^4(dx+c)^4+112500d^2(dx+c)^2b^2-25000d^4\right)\sin(3xb+3a)+\left(-50625b^4(dx+c)^4+24300d^2(dx+c)^2b^2-1944d^4\right)\text{s}}{}$ |
| risch | $\frac{d(b^2d^3x^3+3b^2c\,d^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6c\,d^2)\cos(xb+a)}{2b^4}+\frac{(d^4x^4b^4+4b^4c\,d^3x^3+6b^4c^2d^2x^2+4b^4c^3dx+b^4c^4-12}{8b^5}$ |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |

input
```
int((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4050000*((-84375*b^4*(d*x+c)^4+112500*d^2*(d*x+c)^2*b^2-25000*d^4)*sin(3
*b*x+3*a)+(-50625*b^4*(d*x+c)^4+24300*d^2*(d*x+c)^2*b^2-1944*d^4)*sin(5*b*
x+5*a)-112500*b*((d*x+c)^2*b^2-2/3*d^2)*d*(d*x+c)*cos(3*b*x+3*a)-40500*b*(
(d*x+c)^2*b^2-6/25*d^2)*d*(d*x+c)*cos(5*b*x+5*a)+(506250*b^4*(d*x+c)^4-607
5000*d^2*(d*x+c)^2*b^2+12150000*d^4)*sin(b*x+a)+2025000*b*d*(((d*x+c)^2*b^
2-6*d^2)*(d*x+c)*cos(b*x+a)-208/225*c*(b^2*c^2-6284/975*d^2)))/b^5
```

### 3.146.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 527, normalized size of antiderivative = 1.60

$$\int (c+dx)^4\cos^3(a+bx)\sin^2(a+bx)\,dx =$$
$$-\frac{1620\,(25\,b^3d^4x^3+75\,b^3cd^3x^2+25\,b^3c^3d-6\,bcd^3+3\,(25\,b^3c^2d^2-2\,bd^4)x)\cos(bx+a)^5-300\,(75\,b^3d^4x^3}{}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/253125*(1620*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 25*b^3*c^3*d - 6*b*c*
d^3 + 3*(25*b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^5 - 300*(75*b^3*d^4*x^3
 + 225*b^3*c*d^3*x^2 + 75*b^3*c^3*d + 22*b*c*d^3 + (225*b^3*c^2*d^2 + 22*b
*d^4)*x)*cos(b*x + a)^3 - 1800*(75*b^3*d^4*x^3 + 225*b^3*c*d^3*x^2 + 75*b^
3*c^3*d - 428*b*c*d^3 + (225*b^3*c^2*d^2 - 428*b*d^4)*x)*cos(b*x + a) - (3
3750*b^4*d^4*x^4 + 135000*b^4*c*d^3*x^3 + 33750*b^4*c^4 - 385200*b^2*c^2*d
^2 - 81*(625*b^4*d^4*x^4 + 2500*b^4*c*d^3*x^3 + 625*b^4*c^4 - 300*b^2*c^2*
d^2 + 24*d^4 + 150*(25*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 100*(25*b^4*c^3*d -
6*b^2*c*d^3)*x)*cos(b*x + a)^4 + 760816*d^4 + 900*(225*b^4*c^2*d^2 - 428*b
^2*d^4)*x^2 + (16875*b^4*d^4*x^4 + 67500*b^4*c*d^3*x^3 + 16875*b^4*c^4 + 9
900*b^2*c^2*d^2 - 4792*d^4 + 450*(225*b^4*c^2*d^2 + 22*b^2*d^4)*x^2 + 900*
(75*b^4*c^3*d + 22*b^2*c*d^3)*x)*cos(b*x + a)^2 + 1800*(75*b^4*c^3*d - 428
*b^2*c*d^3)*x)*sin(b*x + a))/b^5
```

### 3.146.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1098 vs. 2(325) = 650.

Time = 1.15 (sec) , antiderivative size = 1098, normalized size of antiderivative = 3.33

$$\int (c+dx)^4\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**4*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

output
```
Piecewise((2*c**4*sin(a + b*x)**5/(15*b) + c**4*sin(a + b*x)**3*cos(a + b*
x)**2/(3*b) + 8*c**3*d*x*sin(a + b*x)**5/(15*b) + 4*c**3*d*x*sin(a + b*x)*
*3*cos(a + b*x)**2/(3*b) + 4*c**2*d**2*x**2*sin(a + b*x)**5/(5*b) + 2*c**2
*d**2*x**2*sin(a + b*x)**3*cos(a + b*x)**2/b + 8*c*d**3*x**3*sin(a + b*x)*
*5/(15*b) + 4*c*d**3*x**3*sin(a + b*x)**3*cos(a + b*x)**2/(3*b) + 2*d**4*x
**4*sin(a + b*x)**5/(15*b) + d**4*x**4*sin(a + b*x)**3*cos(a + b*x)**2/(3*
b) + 8*c**3*d*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 52*c**3*d*sin(a + b
*x)**2*cos(a + b*x)**3/(45*b**2) + 104*c**3*d*cos(a + b*x)**5/(225*b**2) +
 8*c**2*d**2*x*sin(a + b*x)**4*cos(a + b*x)/(5*b**2) + 52*c**2*d**2*x*sin(
a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 104*c**2*d**2*x*cos(a + b*x)**5/(7
5*b**2) + 8*c*d**3*x**2*sin(a + b*x)**4*cos(a + b*x)/(5*b**2) + 52*c*d**3*
x**2*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 104*c*d**3*x**2*cos(a + b
*x)**5/(75*b**2) + 8*d**4*x**3*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 52
*d**4*x**3*sin(a + b*x)**2*cos(a + b*x)**3/(45*b**2) + 104*d**4*x**3*cos(a
 + b*x)**5/(225*b**2) - 1712*c**2*d**2*sin(a + b*x)**5/(1125*b**3) - 676*c
**2*d**2*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 104*c**2*d**2*sin(a
+ b*x)*cos(a + b*x)**4/(75*b**3) - 3424*c*d**3*x*sin(a + b*x)**5/(1125*b**
3) - 1352*c*d**3*x*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 208*c*d**3
*x*sin(a + b*x)*cos(a + b*x)**4/(75*b**3) - 1712*d**4*x**2*sin(a + b*x)**5
/(1125*b**3) - 676*d**4*x**2*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3)...
```

### 3.146.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1339 vs. 2(304) = 608.

Time = 0.32 (sec) , antiderivative size = 1339, normalized size of antiderivative = 4.06

$$\int (c+dx)^4 \cos^3(a+bx) \sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4050000*(270000*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*c^4 - 1080000*(3*
sin(b*x + a)^5 - 5*sin(b*x + a)^3)*a*c^3*d/b + 1620000*(3*sin(b*x + a)^5 -
 5*sin(b*x + a)^3)*a^2*c^2*d^2/b^2 - 1080000*(3*sin(b*x + a)^5 - 5*sin(b*x
 + a)^3)*a^3*c*d^3/b^3 + 270000*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*a^4*
d^4/b^4 + 4500*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3
*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a
) - 450*cos(b*x + a))*c^3*d/b - 13500*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*
(b*x + a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*
a) + 25*cos(3*b*x + 3*a) - 450*cos(b*x + a))*a*c^2*d^2/b^2 + 13500*(45*(b*
x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a) - 450*(b*x + a)*si
n(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) - 450*cos(b*x + a))*
a^2*c*d^3/b^3 - 4500*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b
*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x
 + 3*a) - 450*cos(b*x + a))*a^3*d^4/b^4 + 450*(270*(b*x + a)*cos(5*b*x + 5
*a) + 750*(b*x + a)*cos(3*b*x + 3*a) - 13500*(b*x + a)*cos(b*x + a) + 27*(
25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2 - 2)*sin(3*b*x +
 3*a) - 6750*((b*x + a)^2 - 2)*sin(b*x + a))*c^2*d^2/b^2 - 900*(270*(b*x +
 a)*cos(5*b*x + 5*a) + 750*(b*x + a)*cos(3*b*x + 3*a) - 13500*(b*x + a)*co
s(b*x + a) + 27*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2
 - 2)*sin(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*sin(b*x + a))*a*c*d^3/b...
```

### 3.146.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.61

$$\int (c+dx)^4\cos^3(a+bx)\sin^2(a+bx)\,dx$$
$$= -\frac{(25\,b^3d^4x^3+75\,b^3cd^3x^2+75\,b^3c^2d^2x+25\,b^3c^3d-6\,bd^4x-6\,bcd^3)\cos(5\,bx+5\,a)}{2500\,b^5}$$
$$-\frac{(3\,b^3d^4x^3+9\,b^3cd^3x^2+9\,b^3c^2d^2x+3\,b^3c^3d-2\,bd^4x-2\,bcd^3)\cos(3\,bx+3\,a)}{108\,b^5}$$
$$+\frac{(b^3d^4x^3+3\,b^3cd^3x^2+3\,b^3c^2d^2x+b^3c^3d-6\,bd^4x-6\,bcd^3)\cos(bx+a)}{2\,b^5}$$
$$-\frac{(625\,b^4d^4x^4+2500\,b^4cd^3x^3+3750\,b^4c^2d^2x^2+2500\,b^4c^3dx+625\,b^4c^4-300\,b^2d^4x^2-600\,b^2cd^3x-300\,b}{50000\,b^5}$$
$$-\frac{(27\,b^4d^4x^4+108\,b^4cd^3x^3+162\,b^4c^2d^2x^2+108\,b^4c^3dx+27\,b^4c^4-36\,b^2d^4x^2-72\,b^2cd^3x-36\,b^2c^2d^2+8}{1296\,b^5}$$
$$+\frac{(b^4d^4x^4+4\,b^4cd^3x^3+6\,b^4c^2d^2x^2+4\,b^4c^3dx+b^4c^4-12\,b^2d^4x^2-24\,b^2cd^3x-12\,b^2c^2d^2+24\,d^4)\sin(bx}{8\,b^5}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2500*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 75*b^3*c^2*d^2*x + 25*b^3*c^3
*d - 6*b*d^4*x - 6*b*c*d^3)*cos(5*b*x + 5*a)/b^5 - 1/108*(3*b^3*d^4*x^3 +
9*b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*b*c*d^3)*c
os(3*b*x + 3*a)/b^5 + 1/2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x
 + b^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*cos(b*x + a)/b^5 - 1/50000*(625*b^4*
d^4*x^4 + 2500*b^4*c*d^3*x^3 + 3750*b^4*c^2*d^2*x^2 + 2500*b^4*c^3*d*x + 6
25*b^4*c^4 - 300*b^2*d^4*x^2 - 600*b^2*c*d^3*x - 300*b^2*c^2*d^2 + 24*d^4)
*sin(5*b*x + 5*a)/b^5 - 1/1296*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b
^4*c^2*d^2*x^2 + 108*b^4*c^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*
d^3*x - 36*b^2*c^2*d^2 + 8*d^4)*sin(3*b*x + 3*a)/b^5 + 1/8*(b^4*d^4*x^4 +
4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4
*x^2 - 24*b^2*c*d^3*x - 12*b^2*c^2*d^2 + 24*d^4)*sin(b*x + a)/b^5
```

### 3.146.9 Mupad [B] (verification not implemented)

Time = 28.90 (sec) , antiderivative size = 816, normalized size of antiderivative = 2.47

$$\int (c+dx)^4 \cos^3(a+bx) \sin^2(a+bx)\, dx =$$

$$-\frac{\frac{d^4\,\sin(3\,a+3\,b\,x)}{162}-3\,d^4\,\sin\left(a+b\,x\right)+\frac{3\,d^4\,\sin(5\,a+5\,b\,x)}{6250}-\frac{b^4\,c^4\,\sin(a+b\,x)}{8}+\frac{b^4\,c^4\,\sin(3\,a+3\,b\,x)}{48}+\frac{b^4\,c^4\,\sin(5\,a+5\,b\,x)}{80}+\ldots}{}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^4,x)
```

output

```
-((d^4*sin(3*a + 3*b*x))/162 - 3*d^4*sin(a + b*x) + (3*d^4*sin(5*a + 5*b*x
))/6250 - (b^4*c^4*sin(a + b*x))/8 + (b^4*c^4*sin(3*a + 3*b*x))/48 + (b^4*
c^4*sin(5*a + 5*b*x))/80 + (b^3*c^3*d*cos(3*a + 3*b*x))/36 + (b^3*c^3*d*co
s(5*a + 5*b*x))/100 + (3*b^2*c^2*d^2*sin(a + b*x))/2 - (b^3*d^4*x^3*cos(a
+ b*x))/2 + (3*b^2*d^4*x^2*sin(a + b*x))/2 - (b^4*d^4*x^4*sin(a + b*x))/8
+ 3*b*c*d^3*cos(a + b*x) + 3*b*d^4*x*cos(a + b*x) - (b^2*c^2*d^2*sin(3*a +
 3*b*x))/36 - (3*b^2*c^2*d^2*sin(5*a + 5*b*x))/500 + (b^3*d^4*x^3*cos(3*a
+ 3*b*x))/36 + (b^3*d^4*x^3*cos(5*a + 5*b*x))/100 - (b^2*d^4*x^2*sin(3*a +
 3*b*x))/36 - (3*b^2*d^4*x^2*sin(5*a + 5*b*x))/500 + (b^4*d^4*x^4*sin(3*a
+ 3*b*x))/48 + (b^4*d^4*x^4*sin(5*a + 5*b*x))/80 - (b*c*d^3*cos(3*a + 3*b*
x))/54 - (3*b*c*d^3*cos(5*a + 5*b*x))/1250 - (b^3*c^3*d*cos(a + b*x))/2 -
(b*d^4*x*cos(3*a + 3*b*x))/54 - (3*b*d^4*x*cos(5*a + 5*b*x))/1250 + 3*b^2*
c*d^3*x*sin(a + b*x) - (b^4*c^3*d*x*sin(a + b*x))/2 + (b^4*c^2*d^2*x^2*sin
(3*a + 3*b*x))/8 + (3*b^4*c^2*d^2*x^2*sin(5*a + 5*b*x))/40 - (3*b^3*c^2*d^
2*x*cos(a + b*x))/2 - (3*b^3*c*d^3*x^2*cos(a + b*x))/2 - (b^2*c*d^3*x*sin(
3*a + 3*b*x))/18 + (b^4*c^3*d*x*sin(3*a + 3*b*x))/12 - (3*b^2*c*d^3*x*sin(
5*a + 5*b*x))/250 + (b^4*c^3*d*x*sin(5*a + 5*b*x))/20 - (b^4*c*d^3*x^3*sin
(a + b*x))/2 + (b^3*c^2*d^2*x*cos(3*a + 3*b*x))/12 + (b^3*c*d^3*x^2*cos(3*
a + 3*b*x))/12 + (3*b^3*c^2*d^2*x*cos(5*a + 5*b*x))/100 + (3*b^3*c*d^3*x^2
*cos(5*a + 5*b*x))/100 + (b^4*c*d^3*x^3*sin(3*a + 3*b*x))/12 + (b^4*c*d...
```

3.146. $\int (c+dx)^4 \cos^3(a+bx) \sin^2(a+bx)\, dx$

# 3.147 $\int (c+dx)^3 \cos^3(a+bx)\sin^2(a+bx)\,dx$

3.147.1 Optimal result . . . 1188
3.147.2 Mathematica [A] (verified) . . . 1189
3.147.3 Rubi [A] (verified) . . . 1189
3.147.4 Maple [A] (verified) . . . 1190
3.147.5 Fricas [A] (verification not implemented) . . . 1191
3.147.6 Sympy [B] (verification not implemented) . . . 1191
3.147.7 Maxima [B] (verification not implemented) . . . 1192
3.147.8 Giac [A] (verification not implemented) . . . 1193
3.147.9 Mupad [B] (verification not implemented) . . . 1194

## 3.147.1 Optimal result

Integrand size = 24, antiderivative size = 259

$$\begin{aligned}\int (c+dx)^3\cos^3(a+bx)\sin^2(a+bx)\,dx = &-\frac{3d^3\cos(a+bx)}{4b^4}+\frac{3d(c+dx)^2\cos(a+bx)}{8b^2}\\&+\frac{d^3\cos(3a+3bx)}{216b^4}-\frac{d(c+dx)^2\cos(3a+3bx)}{48b^2}\\&+\frac{3d^3\cos(5a+5bx)}{5000b^4}-\frac{3d(c+dx)^2\cos(5a+5bx)}{400b^2}\\&-\frac{3d^2(c+dx)\sin(a+bx)}{4b^3}+\frac{(c+dx)^3\sin(a+bx)}{8b}\\&+\frac{d^2(c+dx)\sin(3a+3bx)}{72b^3}\\&-\frac{(c+dx)^3\sin(3a+3bx)}{48b}\\&+\frac{3d^2(c+dx)\sin(5a+5bx)}{1000b^3}\\&-\frac{(c+dx)^3\sin(5a+5bx)}{80b}\end{aligned}$$

output
```
-3/4*d^3*cos(b*x+a)/b^4+3/8*d*(d*x+c)^2*cos(b*x+a)/b^2+1/216*d^3*cos(3*b*x
+3*a)/b^4-1/48*d*(d*x+c)^2*cos(3*b*x+3*a)/b^2+3/5000*d^3*cos(5*b*x+5*a)/b^
4-3/400*d*(d*x+c)^2*cos(5*b*x+5*a)/b^2-3/4*d^2*(d*x+c)*sin(b*x+a)/b^3+1/8*
(d*x+c)^3*sin(b*x+a)/b+1/72*d^2*(d*x+c)*sin(3*b*x+3*a)/b^3-1/48*(d*x+c)^3*
sin(3*b*x+3*a)/b+3/1000*d^2*(d*x+c)*sin(5*b*x+5*a)/b^3-1/80*(d*x+c)^3*sin(
5*b*x+5*a)/b
```

### 3.147.2 Mathematica [A] (verified)

Time = 2.25 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.75

$$\int (c+dx)^3\cos^3(a+bx)\sin^2(a+bx)\,dx = -\frac{-101250d(-2d^2+b^2(c+dx)^2)\cos(a+bx)+625d(-2d^2+9b^2(c+dx)^2)\cos(3(a+bx))+81d(-2d^2+\cdots}{\cdots}$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
-1/270000*(-101250*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + 625*d*(-2*d
^2 + 9*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 81*d*(-2*d^2 + 25*b^2*(c + d*x)
^2)*Cos[5*(a + b*x)] + 30*b*(c + d*x)*(-825*b^2*c^2 + 6598*d^2 - 1650*b^2*
c*d*x - 825*b^2*d^2*x^2 + 8*(-38*d^2 + 75*b^2*(c + d*x)^2)*Cos[2*(a + b*x)
] + 9*(-6*d^2 + 25*b^2*(c + d*x)^2)*Cos[4*(a + b*x)])*Sin[a + b*x])/b^4
```

### 3.147.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\sin^2(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \; 4906$$

$$\int \left(\frac{1}{8}(c+dx)^3\cos(a+bx)-\frac{1}{16}(c+dx)^3\cos(3a+3bx)-\frac{1}{16}(c+dx)^3\cos(5a+5bx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{3d^3\cos(a+bx)}{4b^4}+\frac{d^3\cos(3a+3bx)}{216b^4}+\frac{3d^3\cos(5a+5bx)}{5000b^4}-\frac{3d^2(c+dx)\sin(a+bx)}{4b^3}+\frac{d^2(c+dx)\sin(3a+3bx)}{72b^3}+\frac{3d^2(c+dx)\sin(5a+5bx)}{1000b^3}+\frac{3d(c+dx)^2\cos(a+bx)}{8b^2}-\frac{d(c+dx)^2\cos(3a+3bx)}{48b^2}-\frac{3d(c+dx)^2\cos(5a+5bx)}{400b^2}+\frac{(c+dx)^3\sin(a+bx)}{8b}-\frac{(c+dx)^3\sin(3a+3bx)}{48b}-\frac{(c+dx)^3\sin(5a+5bx)}{80b}$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
(-3*d^3*Cos[a + b*x])/(4*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x])/(8*b^2) + (
d^3*Cos[3*a + 3*b*x])/(216*b^4) - (d*(c + d*x)^2*Cos[3*a + 3*b*x])/(48*b^2
) + (3*d^3*Cos[5*a + 5*b*x])/(5000*b^4) - (3*d*(c + d*x)^2*Cos[5*a + 5*b*x
])/(400*b^2) - (3*d^2*(c + d*x)*Sin[a + b*x])/(4*b^3) + ((c + d*x)^3*Sin[a
 + b*x])/(8*b) + (d^2*(c + d*x)*Sin[3*a + 3*b*x])/(72*b^3) - ((c + d*x)^3*
Sin[3*a + 3*b*x])/(48*b) + (3*d^2*(c + d*x)*Sin[5*a + 5*b*x])/(1000*b^3) -
 ((c + d*x)^3*Sin[5*a + 5*b*x])/(80*b)
```

### 3.147.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.147.4 Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| parallelrisch | $\frac{-25b\left((dx+c)^2b^2-\frac{2d^2}{3}\right)(dx+c)\sin(3xb+3a)-15\left((dx+c)^2b^2-\frac{6d^2}{25}\right)b(dx+c)\sin(5xb+5a)-25d\left((dx+c)^2b^2-\frac{2d^2}{9}\right)\cos(3}{}$ |
| risch | $\frac{3d(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2)\cos(xb+a)}{8b^4}+\frac{(b^2d^3x^3+3b^2c\,d^2x^2+3b^2c^2dx+b^2c^3-6d^3x-6c\,d^2)\sin(xb+a)}{8b^3}-\frac{3d(25}{}$ |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |

input
```
int((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/1200*(-25*b*((d*x+c)^2*b^2-2/3*d^2)*(d*x+c)*sin(3*b*x+3*a)-15*((d*x+c)^2
*b^2-6/25*d^2)*b*(d*x+c)*sin(5*b*x+5*a)-25*d*((d*x+c)^2*b^2-2/9*d^2)*cos(3
*b*x+3*a)-9*d*((d*x+c)^2*b^2-2/25*d^2)*cos(5*b*x+5*a)+150*((d*x+c)^2*b^2-6
*d^2)*(d*x+c)*b*sin(b*x+a)+450*(((d*x+c)^2*b^2-2*d^2)*cos(b*x+a)+208/225*b
^2*c^2-100544/50625*d^2)*d)/b^4
```

## 3.147.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.32

$$\int (c+dx)^3\cos^3(a+bx)\sin^2(a+bx)\,dx =$$
$$-\frac{81\,(25\,b^2d^3x^2+50\,b^2cd^2x+25\,b^2c^2d-2\,d^3)\cos{(bx+a)}^5-5\,(225\,b^2d^3x^2+450\,b^2cd^2x+225\,b^2c^2d+22}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/16875*(81*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*cos(
b*x + a)^5 - 5*(225*b^2*d^3*x^2 + 450*b^2*c*d^2*x + 225*b^2*c^2*d + 22*d^3
)*cos(b*x + a)^3 - 30*(225*b^2*d^3*x^2 + 450*b^2*c*d^2*x + 225*b^2*c^2*d -
 428*d^3)*cos(b*x + a) - 15*(150*b^3*d^3*x^3 + 450*b^3*c*d^2*x^2 + 150*b^3
*c^3 - 9*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 25*b^3*c^3 - 6*b*c*d^2 + 3*(
25*b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^4 - 856*b*c*d^2 + (75*b^3*d^3*x^3
+ 225*b^3*c*d^2*x^2 + 75*b^3*c^3 + 22*b*c*d^2 + (225*b^3*c^2*d + 22*b*d^3)
*x)*cos(b*x + a)^2 + 2*(225*b^3*c^2*d - 428*b*d^3)*x)*sin(b*x + a))/b^4
```

## 3.147.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 690 vs. 2(255) = 510.

Time = 0.83 (sec) , antiderivative size = 690, normalized size of antiderivative = 2.66

$$\int (c+dx)^3\cos^3(a+bx)\sin^2(a+bx)\,dx$$
$$=\begin{cases}\frac{2c^3\sin^5{(a+bx)}}{15b}+\frac{c^3\sin^3{(a+bx)}\cos^2{(a+bx)}}{3b}+\frac{2c^2dx\sin^5{(a+bx)}}{5b}+\frac{c^2dx\sin^3{(a+bx)}\cos^2{(a+bx)}}{b}+\frac{2cd^2x^2\sin^5{(a+bx)}}{5b}+\frac{cd^2x^2\sin^3}{} \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\sin^2{(a)}\cos^3{(a)}\end{cases}$$

input
```
integrate((d*x+c)**3*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

output
```
Piecewise((2*c**3*sin(a + b*x)**5/(15*b) + c**3*sin(a + b*x)**3*cos(a + b*
x)**2/(3*b) + 2*c**2*d*x*sin(a + b*x)**5/(5*b) + c**2*d*x*sin(a + b*x)**3*
cos(a + b*x)**2/b + 2*c*d**2*x**2*sin(a + b*x)**5/(5*b) + c*d**2*x**2*sin(
a + b*x)**3*cos(a + b*x)**2/b + 2*d**3*x**3*sin(a + b*x)**5/(15*b) + d**3*
x**3*sin(a + b*x)**3*cos(a + b*x)**2/(3*b) + 2*c**2*d*sin(a + b*x)**4*cos(
a + b*x)/(5*b**2) + 13*c**2*d*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) +
26*c**2*d*cos(a + b*x)**5/(75*b**2) + 4*c*d**2*x*sin(a + b*x)**4*cos(a + b
*x)/(5*b**2) + 26*c*d**2*x*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 52*
c*d**2*x*cos(a + b*x)**5/(75*b**2) + 2*d**3*x**2*sin(a + b*x)**4*cos(a + b
*x)/(5*b**2) + 13*d**3*x**2*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 26
*d**3*x**2*cos(a + b*x)**5/(75*b**2) - 856*c*d**2*sin(a + b*x)**5/(1125*b*
*3) - 338*c*d**2*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 52*c*d**2*si
n(a + b*x)*cos(a + b*x)**4/(75*b**3) - 856*d**3*x*sin(a + b*x)**5/(1125*b*
*3) - 338*d**3*x*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 52*d**3*x*si
n(a + b*x)*cos(a + b*x)**4/(75*b**3) - 856*d**3*sin(a + b*x)**4*cos(a + b*
x)/(1125*b**4) - 5114*d**3*sin(a + b*x)**2*cos(a + b*x)**3/(3375*b**4) - 1
2568*d**3*cos(a + b*x)**5/(16875*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**
2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**2*cos(a)**3, True))
```

### 3.147.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 766 vs. $2(235) = 470$.

Time = 0.27 (sec) , antiderivative size = 766, normalized size of antiderivative = 2.96

$$\int (c+dx)^3\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/270000*(18000*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*c^3 - 54000*(3*sin(
b*x + a)^5 - 5*sin(b*x + a)^3)*a*c^2*d/b + 54000*(3*sin(b*x + a)^5 - 5*sin
(b*x + a)^3)*a^2*c*d^2/b^2 - 18000*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*a
^3*d^3/b^3 + 225*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x +
 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3
*a) - 450*cos(b*x + a))*c^2*d/b - 450*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*
(b*x + a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*
a) + 25*cos(3*b*x + 3*a) - 450*cos(b*x + a))*a*c*d^2/b^2 + 225*(45*(b*x +
a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*
x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) - 450*cos(b*x + a))*a^2*
d^3/b^3 + 15*(270*(b*x + a)*cos(5*b*x + 5*a) + 750*(b*x + a)*cos(3*b*x + 3
*a) - 13500*(b*x + a)*cos(b*x + a) + 27*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5
*a) + 125*(9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*si
n(b*x + a))*c*d^2/b^2 - 15*(270*(b*x + a)*cos(5*b*x + 5*a) + 750*(b*x + a)
*cos(3*b*x + 3*a) - 13500*(b*x + a)*cos(b*x + a) + 27*(25*(b*x + a)^2 - 2)
*sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 6750*((b*x
+ a)^2 - 2)*sin(b*x + a))*a*d^3/b^3 + (81*(25*(b*x + a)^2 - 2)*cos(5*b*x +
 5*a) + 625*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 101250*((b*x + a)^2 - 2
)*cos(b*x + a) + 135*(25*(b*x + a)^3 - 6*b*x - 6*a)*sin(5*b*x + 5*a) + 187
5*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) - 33750*((b*x + a)^3 -...
```

### 3.147.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.36

$$
\begin{aligned}
&\int (c+dx)^3\cos^3(a+bx)\sin^2(a+bx)\,dx \\
&= -\frac{3\,(25\,b^2d^3x^2+50\,b^2cd^2x+25\,b^2c^2d-2\,d^3)\cos(5\,bx+5\,a)}{10000\,b^4} \\
&\quad -\frac{(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3)\cos(3\,bx+3\,a)}{432\,b^4} \\
&\quad +\frac{3\,(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d-2\,d^3)\cos(bx+a)}{8\,b^4} \\
&\quad -\frac{(25\,b^3d^3x^3+75\,b^3cd^2x^2+75\,b^3c^2dx+25\,b^3c^3-6\,bd^3x-6\,bcd^2)\sin(5\,bx+5\,a)}{2000\,b^4} \\
&\quad -\frac{(3\,b^3d^3x^3+9\,b^3cd^2x^2+9\,b^3c^2dx+3\,b^3c^3-2\,bd^3x-2\,bcd^2)\sin(3\,bx+3\,a)}{144\,b^4} \\
&\quad +\frac{(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3-6\,bd^3x-6\,bcd^2)\sin(bx+a)}{8\,b^4}
\end{aligned}
$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-3/10000*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*cos(5*b*
x + 5*a)/b^4 - 1/432*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3
)*cos(3*b*x + 3*a)/b^4 + 3/8*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*
d^3)*cos(b*x + a)/b^4 - 1/2000*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 75*b^3
*c^2*d*x + 25*b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(5*b*x + 5*a)/b^4 - 1/14
4*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d^3*x
 - 2*b*c*d^2)*sin(3*b*x + 3*a)/b^4 + 1/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(b*x + a)/b^4
```

## 3.147.9 Mupad [B] (verification not implemented)

Time = 2.95 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.99

$$\int (c+dx)^3\cos^3(a+bx)\sin^2(a+bx)\,dx = -\frac{\frac{3\,d^3\,\cos(a+b\,x)}{4}-\frac{d^3\,\cos(3\,a+3\,b\,x)}{216}-\frac{3\,d^3\,\cos(5\,a+5\,b\,x)}{5000}-\frac{b^3\,c^3\,\sin(a+b\,x)}{8}+\frac{b^3\,c^3\,\sin(3\,a+3\,b\,x)}{48}+\frac{b^3\,c^3\,\sin(5\,a+5\,b\,x)}{80}+\frac{b^2\,c^2\,d}{}}{}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^3,x)
```

output
```
-((3*d^3*cos(a + b*x))/4 - (d^3*cos(3*a + 3*b*x))/216 - (3*d^3*cos(5*a + 5
*b*x))/5000 - (b^3*c^3*sin(a + b*x))/8 + (b^3*c^3*sin(3*a + 3*b*x))/48 + (
b^3*c^3*sin(5*a + 5*b*x))/80 + (b^2*c^2*d*cos(3*a + 3*b*x))/48 + (3*b^2*c^
2*d*cos(5*a + 5*b*x))/400 - (3*b^2*d^3*x^2*cos(a + b*x))/8 - (b^3*d^3*x^3*
sin(a + b*x))/8 + (3*b*c*d^2*sin(a + b*x))/4 + (3*b*d^3*x*sin(a + b*x))/4
+ (b^2*d^3*x^2*cos(3*a + 3*b*x))/48 + (3*b^2*d^3*x^2*cos(5*a + 5*b*x))/400
 + (b^3*d^3*x^3*sin(3*a + 3*b*x))/48 + (b^3*d^3*x^3*sin(5*a + 5*b*x))/80 -
 (3*b^2*c^2*d*cos(a + b*x))/8 - (b*c*d^2*sin(3*a + 3*b*x))/72 - (3*b*c*d^2
*sin(5*a + 5*b*x))/1000 - (b*d^3*x*sin(3*a + 3*b*x))/72 - (3*b*d^3*x*sin(5
*a + 5*b*x))/1000 - (3*b^2*c*d^2*x*cos(a + b*x))/4 - (3*b^3*c^2*d*x*sin(a
+ b*x))/8 + (b^2*c*d^2*x*cos(3*a + 3*b*x))/24 + (3*b^2*c*d^2*x*cos(5*a + 5
*b*x))/200 + (b^3*c^2*d*x*sin(3*a + 3*b*x))/16 + (3*b^3*c^2*d*x*sin(5*a +
5*b*x))/80 - (3*b^3*c*d^2*x^2*sin(a + b*x))/8 + (b^3*c*d^2*x^2*sin(3*a + 3
*b*x))/16 + (3*b^3*c*d^2*x^2*sin(5*a + 5*b*x))/80)/b^4
```

# 3.148 $\int (c+dx)^2 \cos^3(a+bx) \sin^2(a+bx)\,dx$

3.148.1 Optimal result . . . 1195
3.148.2 Mathematica [A] (verified) . . . 1196
3.148.3 Rubi [A] (verified) . . . 1196
3.148.4 Maple [A] (verified) . . . 1197
3.148.5 Fricas [A] (verification not implemented) . . . 1198
3.148.6 Sympy [B] (verification not implemented) . . . 1198
3.148.7 Maxima [B] (verification not implemented) . . . 1199
3.148.8 Giac [A] (verification not implemented) . . . 1200
3.148.9 Mupad [B] (verification not implemented) . . . 1200

## 3.148.1 Optimal result

Integrand size = 24, antiderivative size = 184

$$\int (c+dx)^2 \cos^3(a+bx) \sin^2(a+bx)\,dx = \frac{d(c+dx)\cos(a+bx)}{4b^2} - \frac{d(c+dx)\cos(3a+3bx)}{72b^2} - \frac{d(c+dx)\cos(5a+5bx)}{200b^2} - \frac{d^2\sin(a+bx)}{4b^3} + \frac{(c+dx)^2\sin(a+bx)}{8b} + \frac{d^2\sin(3a+3bx)}{216b^3} - \frac{(c+dx)^2\sin(3a+3bx)}{48b} + \frac{d^2\sin(5a+5bx)}{1000b^3} - \frac{(c+dx)^2\sin(5a+5bx)}{80b}$$

output
```
1/4*d*(d*x+c)*cos(b*x+a)/b^2-1/72*d*(d*x+c)*cos(3*b*x+3*a)/b^2-1/200*d*(d*
x+c)*cos(5*b*x+5*a)/b^2-1/4*d^2*sin(b*x+a)/b^3+1/8*(d*x+c)^2*sin(b*x+a)/b+
1/216*d^2*sin(3*b*x+3*a)/b^3-1/48*(d*x+c)^2*sin(3*b*x+3*a)/b+1/1000*d^2*si
n(5*b*x+5*a)/b^3-1/80*(d*x+c)^2*sin(5*b*x+5*a)/b
```

### 3.148.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.37

$$\int (c+dx)^2\cos^3(a+bx)\sin^2(a+bx)\,dx = -\frac{-13500bd(c+dx)\cos(a+bx)+750bd(c+dx)\cos(3(a+bx))+270bcd\cos(5(a+bx))+270bd^2x\cos(5}{}$$

input
```
Integrate[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
-1/54000*(-13500*b*d*(c + d*x)*Cos[a + b*x] + 750*b*d*(c + d*x)*Cos[3*(a +
 b*x)] + 270*b*c*d*Cos[5*(a + b*x)] + 270*b*d^2*x*Cos[5*(a + b*x)] - 6750*
b^2*c^2*Sin[a + b*x] + 13500*d^2*Sin[a + b*x] - 13500*b^2*c*d*x*Sin[a + b*
x] - 6750*b^2*d^2*x^2*Sin[a + b*x] + 1125*b^2*c^2*Sin[3*(a + b*x)] - 250*d
^2*Sin[3*(a + b*x)] + 2250*b^2*c*d*x*Sin[3*(a + b*x)] + 1125*b^2*d^2*x^2*S
in[3*(a + b*x)] + 675*b^2*c^2*Sin[5*(a + b*x)] - 54*d^2*Sin[5*(a + b*x)] +
 1350*b^2*c*d*x*Sin[5*(a + b*x)] + 675*b^2*d^2*x^2*Sin[5*(a + b*x)])/b^3
```

### 3.148.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\sin^2(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}(c+dx)^2\cos(a+bx)-\frac{1}{16}(c+dx)^2\cos(3a+3bx)-\frac{1}{16}(c+dx)^2\cos(5a+5bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2\sin(a+bx)}{4b^3}+\frac{d^2\sin(3a+3bx)}{216b^3}+\frac{d^2\sin(5a+5bx)}{1000b^3}+\frac{d(c+dx)\cos(a+bx)}{4b^2}-\frac{d(c+dx)\cos(3a+3bx)}{72b^2}-\frac{d(c+dx)\cos(5a+5bx)}{200b^2}+\frac{(c+dx)^2\sin(a+bx)}{8b}-\frac{(c+dx)^2\sin(3a+3bx)}{48b}-\frac{(c+dx)^2\sin(5a+5bx)}{80b}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
(d*(c + d*x)*Cos[a + b*x])/(4*b^2) - (d*(c + d*x)*Cos[3*a + 3*b*x])/(72*b^
2) - (d*(c + d*x)*Cos[5*a + 5*b*x])/(200*b^2) - (d^2*Sin[a + b*x])/(4*b^3)
 + ((c + d*x)^2*Sin[a + b*x])/(8*b) + (d^2*Sin[3*a + 3*b*x])/(216*b^3) - (
(c + d*x)^2*Sin[3*a + 3*b*x])/(48*b) + (d^2*Sin[5*a + 5*b*x])/(1000*b^3) -
 ((c + d*x)^2*Sin[5*a + 5*b*x])/(80*b)
```

### 3.148.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.148.4 Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.78

| method | result |
|---|---|
| parallelrisch | $\frac{\left(-1125(dx+c)^2b^2+250d^2\right)\sin(3xb+3a)+\left(-675(dx+c)^2b^2+54d^2\right)\sin(5xb+5a)-750bd(dx+c)\cos(3xb+3a)-270bd(dx}{54000b^3}$ |
| risch | $\frac{d(dx+c)\cos(xb+a)}{4b^2}+\frac{\left(x^2d^2b^2+2b^2cdx+b^2c^2-2d^2\right)\sin(xb+a)}{8b^3}-\frac{d(dx+c)\cos(5xb+5a)}{200b^2}-\frac{\left(25x^2d^2b^2+50b^2cdx+25\right.}{200}$ |
| derivativedivides | $\frac{\frac{a^2d^2\left(-\frac{\cos(xb+a)^4\sin(xb+a)}{5}+\frac{\left(2+\cos(xb+a)^2\right)\sin(xb+a)}{15}\right)}{b^2}-\frac{2acd\left(-\frac{\cos(xb+a)^4\sin(xb+a)}{5}+\frac{\left(2+\cos(xb+a)^2\right)\sin(xb+a)}{15}\right)}{b}-\frac{2a}{}}{}$ |
| default | $\frac{\frac{a^2d^2\left(-\frac{\cos(xb+a)^4\sin(xb+a)}{5}+\frac{\left(2+\cos(xb+a)^2\right)\sin(xb+a)}{15}\right)}{b^2}-\frac{2acd\left(-\frac{\cos(xb+a)^4\sin(xb+a)}{5}+\frac{\left(2+\cos(xb+a)^2\right)\sin(xb+a)}{15}\right)}{b}-\frac{2a}{}}{}$ |

input
```
int((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/54000*((-1125*(d*x+c)^2*b^2+250*d^2)*sin(3*b*x+3*a)+(-675*(d*x+c)^2*b^2+
54*d^2)*sin(5*b*x+5*a)-750*b*d*(d*x+c)*cos(3*b*x+3*a)-270*b*d*(d*x+c)*cos(
5*b*x+5*a)+(6750*(d*x+c)^2*b^2-13500*d^2)*sin(b*x+a)+13500*b*d*((d*x+c)*co
s(b*x+a)+208/225*c))/b^3
```

### 3.148.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.05

$$\int (c+dx)^2\cos^3(a+bx)\sin^2(a+bx)\,dx =$$
$$-\frac{270\left(bd^2x+bcd\right)\cos\left(bx+a\right)^5-150\left(bd^2x+bcd\right)\cos\left(bx+a\right)^3-900\left(bd^2x+bcd\right)\cos\left(bx+a\right)-\left(450\,b\right.}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/3375*(270*(b*d^2*x + b*c*d)*cos(b*x + a)^5 - 150*(b*d^2*x + b*c*d)*cos(
b*x + a)^3 - 900*(b*d^2*x + b*c*d)*cos(b*x + a) - (450*b^2*d^2*x^2 + 900*b
^2*c*d*x - 27*(25*b^2*d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*cos(b*x
 + a)^4 + 450*b^2*c^2 + (225*b^2*d^2*x^2 + 450*b^2*c*d*x + 225*b^2*c^2 + 2
2*d^2)*cos(b*x + a)^2 - 856*d^2)*sin(b*x + a))/b^3
```

### 3.148.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. 2(172) = 344.

Time = 0.57 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.08

$$\int (c+dx)^2\cos^3(a+bx)\sin^2(a+bx)\,dx$$
$$=\begin{cases}\frac{2c^2\sin^5(a+bx)}{15b}+\frac{c^2\sin^3(a+bx)\cos^2(a+bx)}{3b}+\frac{4cdx\sin^5(a+bx)}{15b}+\frac{2cdx\sin^3(a+bx)\cos^2(a+bx)}{3b}+\frac{2d^2x^2\sin^5(a+bx)}{15b}+\frac{d^2x^2\sin^3(a\cdots}{} \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sin^2(a)\cos^3(a)\end{cases}$$

input
```
integrate((d*x+c)**2*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

output
```
Piecewise((2*c**2*sin(a + b*x)**5/(15*b) + c**2*sin(a + b*x)**3*cos(a + b*
x)**2/(3*b) + 4*c*d*x*sin(a + b*x)**5/(15*b) + 2*c*d*x*sin(a + b*x)**3*cos
(a + b*x)**2/(3*b) + 2*d**2*x**2*sin(a + b*x)**5/(15*b) + d**2*x**2*sin(a
+ b*x)**3*cos(a + b*x)**2/(3*b) + 4*c*d*sin(a + b*x)**4*cos(a + b*x)/(15*b
**2) + 26*c*d*sin(a + b*x)**2*cos(a + b*x)**3/(45*b**2) + 52*c*d*cos(a + b
*x)**5/(225*b**2) + 4*d**2*x*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 26*d
**2*x*sin(a + b*x)**2*cos(a + b*x)**3/(45*b**2) + 52*d**2*x*cos(a + b*x)**
5/(225*b**2) - 856*d**2*sin(a + b*x)**5/(3375*b**3) - 338*d**2*sin(a + b*x
)**3*cos(a + b*x)**2/(675*b**3) - 52*d**2*sin(a + b*x)*cos(a + b*x)**4/(22
5*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**2*cos(a)**3
, True))
```

## 3.148.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(166) = 332$.

Time = 0.25 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.04

$$\int (c+dx)^2 \cos^3(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{3600\left(3\sin\left(bx+a\right)^5-5\sin\left(bx+a\right)^3\right)c^2-\frac{7200\left(3\sin(bx+a)^5-5\sin(bx+a)^3\right)acd}{b}+\frac{3600\left(3\sin(bx+a)^5-5\sin(bx+a)^3\right)}{b^2}}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/54000*(3600*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*c^2 - 7200*(3*sin(b*x
 + a)^5 - 5*sin(b*x + a)^3)*a*c*d/b + 3600*(3*sin(b*x + a)^5 - 5*sin(b*x +
 a)^3)*a^2*d^2/b^2 + 30*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(
3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*
b*x + 3*a) - 450*cos(b*x + a))*c*d/b - 30*(45*(b*x + a)*sin(5*b*x + 5*a) +
 75*(b*x + a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x
+ 5*a) + 25*cos(3*b*x + 3*a) - 450*cos(b*x + a))*a*d^2/b^2 + (270*(b*x + a
)*cos(5*b*x + 5*a) + 750*(b*x + a)*cos(3*b*x + 3*a) - 13500*(b*x + a)*cos(
b*x + a) + 27*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2 -
 2)*sin(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*sin(b*x + a))*d^2/b^2)/b
```

### 3.148.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.14

$$\int (c+dx)^2\cos^3(a+bx)\sin^2(a+bx)\,dx$$
$$= -\frac{(bd^2x+bcd)\cos(5\,bx+5\,a)}{200\,b^3} - \frac{(bd^2x+bcd)\cos(3\,bx+3\,a)}{72\,b^3} + \frac{(bd^2x+bcd)\cos(bx+a)}{4\,b^3} - \frac{(25\,b^2d^2x^2+50\,b^2cdx+25\,b^2c^2-2\,d^2)\sin(5\,bx+5\,a)}{2000\,b^3} - \frac{(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2)\sin(3\,bx+3\,a)}{432\,b^3} + \frac{(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,d^2)\sin(bx+a)}{8\,b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/200*(b*d^2*x + b*c*d)*cos(5*b*x + 5*a)/b^3 - 1/72*(b*d^2*x + b*c*d)*cos
(3*b*x + 3*a)/b^3 + 1/4*(b*d^2*x + b*c*d)*cos(b*x + a)/b^3 - 1/2000*(25*b^
2*d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*sin(5*b*x + 5*a)/b^3 - 1/43
2*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*sin(3*b*x + 3*a)/b^3
+ 1/8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(b*x + a)/b^3
```

### 3.148.9 Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.60

$$\int (c+dx)^2\cos^3(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{52\,d^2\,x\cos(a+b\,x)^5}{225\,b^2} - \frac{52\,d^2\cos(a+b\,x)^4\,\sin(a+b\,x)}{225\,b^3} - \frac{\cos(a+b\,x)^2\sin(a+b\,x)^3\,(338\,d^2-225\,b^2\,c^2)}{675\,b^3} - \frac{2\sin(a+b\,x)^5\,(428\,d^2-225\,b^2\,c^2)}{3375\,b^3} + \frac{2\,d^2\,x^2\sin(a+b\,x)^5}{15\,b} + \frac{52\,c\,d\cos(a+b\,x)^5}{225\,b^2} + \frac{4\,c\,d\,\cos(a+b\,x)\,\sin(a+b\,x)^4}{15\,b^2} + \frac{4\,c\,d\,x\sin(a+b\,x)^5}{15\,b} + \frac{d^2\,x^2\cos(a+b\,x)^2\sin(a+b\,x)^3}{3\,b} + \frac{26\,c\,d\cos(a+b\,x)^3\sin(a+b\,x)^2}{45\,b^2} + \frac{4\,d^2\,x\,\cos(a+b\,x)\,\sin(a+b\,x)^4}{15\,b^2} + \frac{26\,d^2\,x\cos(a+b\,x)^3\sin(a+b\,x)^2}{45\,b^2} + \frac{2\,c\,d\,x\cos(a+b\,x)^2\sin(a+b\,x)^3}{3\,b}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^2,x)
```

output
```
(52*d^2*x*cos(a + b*x)^5)/(225*b^2) - (52*d^2*cos(a + b*x)^4*sin(a + b*x))
/(225*b^3) - (cos(a + b*x)^2*sin(a + b*x)^3*(338*d^2 - 225*b^2*c^2))/(675*
b^3) - (2*sin(a + b*x)^5*(428*d^2 - 225*b^2*c^2))/(3375*b^3) + (2*d^2*x^2*
sin(a + b*x)^5)/(15*b) + (52*c*d*cos(a + b*x)^5)/(225*b^2) + (4*c*d*cos(a
+ b*x)*sin(a + b*x)^4)/(15*b^2) + (4*c*d*x*sin(a + b*x)^5)/(15*b) + (d^2*x
^2*cos(a + b*x)^2*sin(a + b*x)^3)/(3*b) + (26*c*d*cos(a + b*x)^3*sin(a + b
*x)^2)/(45*b^2) + (4*d^2*x*cos(a + b*x)*sin(a + b*x)^4)/(15*b^2) + (26*d^2
*x*cos(a + b*x)^3*sin(a + b*x)^2)/(45*b^2) + (2*c*d*x*cos(a + b*x)^2*sin(a
 + b*x)^3)/(3*b)
```

3.148. $\int (c+dx)^2 \cos^3(a+bx) \sin^2(a+bx) \, dx$

# 3.149 $\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx$

3.149.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1202
3.149.2 Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 1202
3.149.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1203
3.149.4 Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1204
3.149.5 Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . 1205
3.149.6 Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . 1205
3.149.7 Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . 1206
3.149.8 Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 1206
3.149.9 Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . 1207

## 3.149.1 Optimal result

Integrand size = 22, antiderivative size = 109

$$\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx = \frac{d\cos(a+bx)}{8b^2}-\frac{d\cos(3a+3bx)}{144b^2}-\frac{d\cos(5a+5bx)}{400b^2}+\frac{(c+dx)\sin(a+bx)}{8b}-\frac{(c+dx)\sin(3a+3bx)}{48b}-\frac{(c+dx)\sin(5a+5bx)}{80b}$$

output
```
1/8*d*cos(b*x+a)/b^2-1/144*d*cos(3*b*x+3*a)/b^2-1/400*d*cos(5*b*x+5*a)/b^2
+1/8*(d*x+c)*sin(b*x+a)/b-1/48*(d*x+c)*sin(3*b*x+3*a)/b-1/80*(d*x+c)*sin(5
*b*x+5*a)/b
```

## 3.149.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01

$$\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx = -\frac{-450d\cos(a+bx)+25d\cos(3(a+bx))+9d\cos(5(a+bx))-450bc\sin(a+bx)-450bdx\sin(a+bx)}{3600b^2}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
-1/3600*(-450*d*Cos[a + b*x] + 25*d*Cos[3*(a + b*x)] + 9*d*Cos[5*(a + b*x)
] - 450*b*c*Sin[a + b*x] - 450*b*d*x*Sin[a + b*x] + 75*b*c*Sin[3*(a + b*x)
] + 75*b*d*x*Sin[3*(a + b*x)] + 45*b*c*Sin[5*(a + b*x)] + 45*b*d*x*Sin[5*(
a + b*x)])/b^2
```

### 3.149.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin^2(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}(c+dx)\cos(a+bx)-\frac{1}{16}(c+dx)\cos(3a+3bx)-\frac{1}{16}(c+dx)\cos(5a+5bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\cos(a+bx)}{8b^2}-\frac{d\cos(3a+3bx)}{144b^2}-\frac{d\cos(5a+5bx)}{400b^2}+\frac{(c+dx)\sin(a+bx)}{8b}-\frac{(c+dx)\sin(3a+3bx)}{48b}-\frac{(c+dx)\sin(5a+5bx)}{80b}$$

input
```
Int[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
(d*Cos[a + b*x])/(8*b^2) - (d*Cos[3*a + 3*b*x])/(144*b^2) - (d*Cos[5*a + 5
*b*x])/(400*b^2) + ((c + d*x)*Sin[a + b*x])/(8*b) - ((c + d*x)*Sin[3*a + 3
*b*x])/(48*b) - ((c + d*x)*Sin[5*a + 5*b*x])/(80*b)
```

### 3.149.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.149.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

| method | result |
|---|---|
| parallelrisch | $\frac{-75b(dx+c)\sin(3xb+3a)-45b(dx+c)\sin(5xb+5a)-25\cos(3xb+3a)d-9\cos(5xb+5a)d+450(dx+c)b\sin(xb+a)+450\cos}{3600b^2}$ |
| risch | $\frac{d\cos(xb+a)}{8b^2}-\frac{d\cos(3xb+3a)}{144b^2}-\frac{d\cos(5xb+5a)}{400b^2}+\frac{(dx+c)\sin(xb+a)}{8b}-\frac{(dx+c)\sin(3xb+3a)}{48b}-\frac{(dx+c)\sin(5xb+}{80b}$ |
| derivativedivides | $\frac{-\frac{da\left(-\frac{\cos(xb+a)^4\sin(xb+a)}{5}+\frac{\left(2+\cos(xb+a)^2\right)\sin(xb+a)}{15}\right)}{b}+c\left(-\frac{\cos(xb+a)^4\sin(xb+a)}{5}+\frac{\left(2+\cos(xb+a)^2\right)\sin(xb+a)}{15}\right)+\frac{d\left(\frac{(x}{}\right.}{}}{b}$ |
| default | $\frac{-\frac{da\left(-\frac{\cos(xb+a)^4\sin(xb+a)}{5}+\frac{\left(2+\cos(xb+a)^2\right)\sin(xb+a)}{15}\right)}{b}+c\left(-\frac{\cos(xb+a)^4\sin(xb+a)}{5}+\frac{\left(2+\cos(xb+a)^2\right)\sin(xb+a)}{15}\right)+\frac{d\left(\frac{(x}{}\right.}{}}{b}$ |
| norman | $\frac{\frac{52d}{225b^2}+\frac{8c\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^3}{3b}-\frac{16c\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^5}{15b}+\frac{8c\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^7}{3b}+\frac{4d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^6}{3b^2}+\frac{44d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^4}{45b^2}+\frac{52d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2}{45b^2}+8dx\tan}{\left(1+\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^2\right)^5}$ |

input
```
int((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3600*(-75*b*(d*x+c)*sin(3*b*x+3*a)-45*b*(d*x+c)*sin(5*b*x+5*a)-25*cos(3*
b*x+3*a)*d-9*cos(5*b*x+5*a)*d+450*(d*x+c)*b*sin(b*x+a)+450*cos(b*x+a)*d+41
6*d)/b^2
```

### 3.149.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx = -\frac{9\,d\cos(bx+a)^5-5\,d\cos(bx+a)^3-30\,d\cos(bx+a)+15\left(3\,(bdx+bc)\cos(bx+a)^4-2\,bdx-(bdx\right.}{225\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/225*(9*d*cos(b*x + a)^5 - 5*d*cos(b*x + a)^3 - 30*d*cos(b*x + a) + 15*(
3*(b*d*x + b*c)*cos(b*x + a)^4 - 2*b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 -
2*b*c)*sin(b*x + a))/b^2
```

### 3.149.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.50

$$\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx$$
$$= \begin{cases} \frac{2c\sin^5(a+bx)}{15b}+\frac{c\sin^3(a+bx)\cos^2(a+bx)}{3b}+\frac{2dx\sin^5(a+bx)}{15b}+\frac{dx\sin^3(a+bx)\cos^2(a+bx)}{3b}+\frac{2d\sin^4(a+bx)\cos(a+bx)}{15b^2}+\frac{13d\sin^2(a\cdots \\ \left(cx+\frac{dx^2}{2}\right)\sin^2(a)\cos^3(a) \end{cases}$$

input
```
integrate((d*x+c)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

output
```
Piecewise((2*c*sin(a + b*x)**5/(15*b) + c*sin(a + b*x)**3*cos(a + b*x)**2/
(3*b) + 2*d*x*sin(a + b*x)**5/(15*b) + d*x*sin(a + b*x)**3*cos(a + b*x)**2
/(3*b) + 2*d*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 13*d*sin(a + b*x)**2
*cos(a + b*x)**3/(45*b**2) + 26*d*cos(a + b*x)**5/(225*b**2), Ne(b, 0)), (
(c*x + d*x**2/2)*sin(a)**2*cos(a)**3, True))
```

## 3.149.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.28

$$\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx = -\frac{240\left(3\sin(bx+a)^5-5\sin(bx+a)^3\right)c-\frac{240\left(3\sin(bx+a)^5-5\sin(bx+a)^3\right)ad}{b}+\frac{(45(bx+a)\sin(5bx+5a)+75(bx+a)\sin(}{}}{3600\,b}$$

input
```
integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/3600*(240*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*c - 240*(3*sin(b*x + a)
^5 - 5*sin(b*x + a)^3)*a*d/b + (45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x +
a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25
*cos(3*b*x + 3*a) - 450*cos(b*x + a))*d/b)/b
```

## 3.149.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.97

$$\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx = -\frac{d\cos(5\,bx+5\,a)}{400\,b^2}-\frac{d\cos(3\,bx+3\,a)}{144\,b^2}+\frac{d\cos(bx+a)}{8\,b^2}-\frac{(bdx+bc)\sin(5\,bx+5\,a)}{80\,b^2}-\frac{(bdx+bc)\sin(3\,bx+3\,a)}{48\,b^2}+\frac{(bdx+bc)\sin(bx+a)}{8\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/400*d*cos(5*b*x + 5*a)/b^2 - 1/144*d*cos(3*b*x + 3*a)/b^2 + 1/8*d*cos(b
*x + a)/b^2 - 1/80*(b*d*x + b*c)*sin(5*b*x + 5*a)/b^2 - 1/48*(b*d*x + b*c)
*sin(3*b*x + 3*a)/b^2 + 1/8*(b*d*x + b*c)*sin(b*x + a)/b^2
```

### 3.149.9 Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.09

$$\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{26\,d\cos(a+b\,x)^5+65\,d\cos(a+b\,x)^3\sin(a+b\,x)^2+30\,d\,\cos(a+b\,x)\,\sin(a+b\,x)^4+30\,b\,c\sin(a+b\,x}{225\,b^2}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x),x)
```

output
```
(26*d*cos(a + b*x)^5 + 65*d*cos(a + b*x)^3*sin(a + b*x)^2 + 30*d*cos(a + b
*x)*sin(a + b*x)^4 + 30*b*c*sin(a + b*x)^5 + 30*b*d*x*sin(a + b*x)^5 + 75*
b*c*cos(a + b*x)^2*sin(a + b*x)^3 + 75*b*d*x*cos(a + b*x)^2*sin(a + b*x)^3
)/(225*b^2)
```

# 3.150 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx$

3.150.1 Optimal result . . . . . 1208
3.150.2 Mathematica [A] (verified) . . . . . 1209
3.150.3 Rubi [A] (verified) . . . . . 1209
3.150.4 Maple [A] (verified) . . . . . 1210
3.150.5 Fricas [A] (verification not implemented) . . . . . 1211
3.150.6 Sympy [F] . . . . . 1211
3.150.7 Maxima [C] (verification not implemented) . . . . . 1212
3.150.8 Giac [C] (verification not implemented) . . . . . 1212
3.150.9 Mupad [F(-1)] . . . . . 1213

## 3.150.1 Optimal result

Integrand size = 24, antiderivative size = 185

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d} - \frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d} - \frac{\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{16d} - \frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d} + \frac{\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{16d} + \frac{\sin\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{16d}$$

output
```
-1/16*Ci(5*b*c/d+5*b*x)*cos(5*a-5*b*c/d)/d-1/16*Ci(3*b*c/d+3*b*x)*cos(3*a-
3*b*c/d)/d+1/8*Ci(b*c/d+b*x)*cos(a-b*c/d)/d+1/16*Si(5*b*c/d+5*b*x)*sin(5*a
-5*b*c/d)/d+1/16*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d-1/8*Si(b*c/d+b*x)*si
n(a-b*c/d)/d
```

## 3.150.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.83

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx$$
$$= \frac{2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)-\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)-\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegra}}{16d}$$

input
```
Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x),x]
```

output
```
(2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] - Cos[3*a - (3*b*c)/d]*CosInt
egral[(3*b*(c + d*x))/d] - Cos[5*a - (5*b*c)/d]*CosIntegral[(5*b*(c + d*x)
)/d] - 2*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Sin[3*a - (3*b*c)/d]*
SinIntegral[(3*b*(c + d*x))/d] + Sin[5*a - (5*b*c)/d]*SinIntegral[(5*b*(c
+ d*x))/d])/(16*d)
```

## 3.150.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\cos^3(a+bx)}{c+dx}\,dx$$
$$\Big\downarrow \; 4906$$
$$\int \left(\frac{\cos(a+bx)}{8(c+dx)}-\frac{\cos(3a+3bx)}{16(c+dx)}-\frac{\cos(5a+5bx)}{16(c+dx)}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d}-\frac{\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{16d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d}+\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{16d}+\frac{\sin\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{16d}$$

input
```
Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x),x]
```

output
```
(Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d) - (Cos[3*a - (3*b*c)/d
]*CosIntegral[(3*b*c)/d + 3*b*x])/(16*d) - (Cos[5*a - (5*b*c)/d]*CosIntegr
al[(5*b*c)/d + 5*b*x])/(16*d) - (Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*
x])/(8*d) + (Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d) +
 (Sin[5*a - (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d)
```

### 3.150.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.150.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.39

| method | result |
|---|---|
| derivativedivides | $\frac{b\left(-\frac{\mathrm{Si}\left(-xb-a-\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}+\frac{\mathrm{Ci}\left(xb+a+\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}\right)}{8}-\frac{b\left(-\frac{3\ \mathrm{Si}\left(-3xb-3a-\frac{3(-ad+cb)}{d}\right)\sin\left(\frac{-3ad+3}{d}\right.}{d}\right.}{}$ |
| default | $\frac{b\left(-\frac{\mathrm{Si}\left(-xb-a-\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}+\frac{\mathrm{Ci}\left(xb+a+\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}\right)}{8}-\frac{b\left(-\frac{3\ \mathrm{Si}\left(-3xb-3a-\frac{3(-ad+cb)}{d}\right)\sin\left(\frac{-3ad+3}{d}\right.}{d}\right.}{}$ |
| risch | $\frac{\mathrm{e}^{-\frac{5i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(5ibx+5ia-\frac{5i(ad-cb)}{d}\right)}{32d}+\frac{\mathrm{e}^{-\frac{3i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{32d}-\frac{\mathrm{e}^{-\frac{i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(ibx+ia-\frac{i(ad}{}\right.}{16d}$ |

input
```
int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*b*(-Si(-x*b-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(x*b+a+(-a*d+b*
c)/d)*cos((-a*d+b*c)/d)/d)-1/48*b*(-3*Si(-3*x*b-3*a-3*(-a*d+b*c)/d)*sin(3*
(-a*d+b*c)/d)/d+3*Ci(3*x*b+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d)-1/80
*b*(-5*Si(-5*x*b-5*a-5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d+5*Ci(5*x*b+5*a+
5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d))
```

### 3.150.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.98

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx =$$
$$-\frac{\cos\left(-\frac{5\,(bc-ad)}{d}\right)\operatorname{Ci}\left(\frac{5\,(bdx+bc)}{d}\right)+\cos\left(-\frac{3\,(bc-ad)}{d}\right)\operatorname{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)-2\,\cos\left(-\frac{bc-ad}{d}\right)\operatorname{Ci}\left(\frac{bdx+bc}{d}\right)-\sin\left(-\frac{5}{}\right.}{16\,d}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
-1/16*(cos(-5*(b*c - a*d)/d)*cos_integral(5*(b*d*x + b*c)/d) + cos(-3*(b*c
 - a*d)/d)*cos_integral(3*(b*d*x + b*c)/d) - 2*cos(-(b*c - a*d)/d)*cos_int
egral((b*d*x + b*c)/d) - sin(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c
)/d) - sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) + 2*sin(-(b*c
 - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

### 3.150.6 Sympy [F]

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \int \frac{\sin^2\left(a+bx\right)\cos^3\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x), x)
```

## 3.150.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.23

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx =$$
$$-\frac{2b\left(E_1\left(\frac{ibc+i(bx+a)d-iad}{d}\right)+E_1\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b\left(E_1\left(\frac{3(-ibc-i(bx+a)d+iad)}{d}\right)+E_1\left(-\right.\right.}{}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
-1/32*(2*b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_int
egral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b*(e
xp_integral_e(1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(1,
 -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b*(exp_in
tegral_e(1, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(1, -5*(
-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-5*(b*c - a*d)/d) - 2*b*(I*exp_int
egral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(1, -(I*b*
c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b*(-I*exp_integral_e(
1, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integral_e(1, -3*(-I*b*c
- I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d) + b*(-I*exp_integral_e(
1, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I*exp_integral_e(1, -5*(-I*b*c
- I*(b*x + a)*d + I*a*d)/d))*sin(-5*(b*c - a*d)/d))/(b*d)
```

## 3.150.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.72 (sec) , antiderivative size = 44961, normalized size of antiderivative = 243.03

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
-1/32*(real_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*
tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + real_par
t(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_integra
l(b*x + b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-b*x - b*c/d))*
tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*a)^2*t
an(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^
2 + real_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*imag_par
t(cos_integral(b*x + b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/
2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*imag_part(cos_integral(-b*x
 - b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2
*b*c/d)^2*tan(1/2*b*c/d) + 8*sin_integral((b*d*x + b*c)/d)*tan(5/2*a)^2*ta
n(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) -
 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(
1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*imag_part(co
s_integral(-3*b*x - 3*b*c/d))*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5
/2*b*c/d)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 4*sin_integral(3*(b*d*x +...
```

## 3.150.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^2}{c+d\,x}\,dx$$

input
```
int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x),x)
```

output
```
int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x), x)
```

# 3.151 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$

3.151.1 Optimal result . . . . . . . . . . 1214
3.151.2 Mathematica [A] (verified) . . . . . . . . . . 1215
3.151.3 Rubi [A] (verified) . . . . . . . . . . 1215
3.151.4 Maple [A] (verified) . . . . . . . . . . 1216
3.151.5 Fricas [A] (verification not implemented) . . . . . . . . . . 1217
3.151.6 Sympy [F] . . . . . . . . . . 1217
3.151.7 Maxima [C] (verification not implemented) . . . . . . . . . . 1218
3.151.8 Giac [C] (verification not implemented) . . . . . . . . . . 1218
3.151.9 Mupad [F(-1)] . . . . . . . . . . 1219

## 3.151.1 Optimal result

Integrand size = 24, antiderivative size = 257

$$\begin{aligned}\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = &-\frac{\cos(a+bx)}{8d(c+dx)}+\frac{\cos(3a+3bx)}{16d(c+dx)}+\frac{\cos(5a+5bx)}{16d(c+dx)}\\ &+\frac{5b\,\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)\sin\left(5a-\frac{5bc}{d}\right)}{16d^2}\\ &+\frac{3b\,\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{16d^2}\\ &-\frac{b\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{8d^2}\\ &-\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^2}\\ &+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{16d^2}\\ &+\frac{5b\cos\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{16d^2}\end{aligned}$$

output
```
-1/8*cos(b*x+a)/d/(d*x+c)+1/16*cos(3*b*x+3*a)/d/(d*x+c)+1/16*cos(5*b*x+5*a
)/d/(d*x+c)-1/8*b*cos(a-b*c/d)*Si(b*c/d+b*x)/d^2+3/16*b*cos(3*a-3*b*c/d)*S
i(3*b*c/d+3*b*x)/d^2+5/16*b*cos(5*a-5*b*c/d)*Si(5*b*c/d+5*b*x)/d^2+5/16*b*
Ci(5*b*c/d+5*b*x)*sin(5*a-5*b*c/d)/d^2+3/16*b*Ci(3*b*c/d+3*b*x)*sin(3*a-3*
b*c/d)/d^2-1/8*b*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^2
```

### 3.151.2 Mathematica [A] (verified)

Time = 2.01 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.82

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{\frac{d\cos(3(a+bx))}{c+dx} + \frac{d\cos(5(a+bx))}{c+dx} + 5b\,\text{CosIntegral}\left(\frac{5b(c+dx)}{d}\right)\sin\left(5a-\frac{5bc}{d}\right) + 3b\,\text{CosIntegral}\left(\frac{3b(c+dx)}{d}\right)\sin\left(3a-\right.}{}$$

input
```
Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
((d*Cos[3*(a + b*x)])/(c + d*x) + (d*Cos[5*(a + b*x)])/(c + d*x) + 5*b*Cos
Integral[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d] + 3*b*CosIntegral[(3*b*(c
 + d*x))/d]*Sin[3*a - (3*b*c)/d] - 2*((d*Cos[a + b*x])/(c + d*x) + b*CosIn
tegral[b*(c/d + x)]*Sin[a - (b*c)/d] + b*Cos[a - (b*c)/d]*SinIntegral[b*(c
/d + x)]) + 3*b*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d] + 5*b*
Cos[5*a - (5*b*c)/d]*SinIntegral[(5*b*(c + d*x))/d])/(16*d^2)
```

### 3.151.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\cos^3(a+bx)}{(c+dx)^2}\,dx$$
$$\Big\downarrow \text{4906}$$
$$\int \left(\frac{\cos(a+bx)}{8(c+dx)^2}-\frac{\cos(3a+3bx)}{16(c+dx)^2}-\frac{\cos(5a+5bx)}{16(c+dx)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{5b\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{16d^2}+\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^2}-\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{16d^2}+\frac{5b\cos\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{16d^2}-\frac{\cos(a+bx)}{8d(c+dx)}+\frac{\cos(3a+3bx)}{16d(c+dx)}+\frac{\cos(5a+5bx)}{16d(c+dx)}$$

input
```
Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
-1/8*Cos[a + b*x]/(d*(c + d*x)) + Cos[3*a + 3*b*x]/(16*d*(c + d*x)) + Cos[
5*a + 5*b*x]/(16*d*(c + d*x)) + (5*b*CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*
a - (5*b*c)/d])/(16*d^2) + (3*b*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (
3*b*c)/d])/(16*d^2) - (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(8*d
^2) - (b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^2) + (3*b*Cos[3
*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) + (5*b*Cos[5*a -
(5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d^2)
```

### 3.151.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.151.4 Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.45

| method | result |
|---|---|
| derivativedivides | $\frac{b^2\left(-\frac{\cos(xb+a)}{(-ad+cb+d(xb+a))d}-\frac{-\frac{\operatorname{Si}\left(-xb-a-\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}-\frac{\operatorname{Ci}\left(xb+a+\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}}{d}\right)}{8}-\frac{b^2\left(-\frac{5\cos(5xb+\cdots}{(-ad+cb+d(x\cdots}\cdots\right.}{\cdots}$ |
| default | $\frac{b^2\left(-\frac{\cos(xb+a)}{(-ad+cb+d(xb+a))d}-\frac{-\frac{\operatorname{Si}\left(-xb-a-\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}-\frac{\operatorname{Ci}\left(xb+a+\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}}{d}\right)}{8}-\frac{b^2\left(-\frac{5\cos(5xb+\cdots}{(-ad+cb+d(x\cdots}\cdots\right.}{\cdots}$ |
| risch | $-\frac{5ib\,e^{-\frac{5i(ad-cb)}{d}}\operatorname{Ei}_1\left(5ibx+5ia-\frac{5i(ad-cb)}{d}\right)}{32d^2}-\frac{3ib\,e^{-\frac{3i(ad-cb)}{d}}\operatorname{Ei}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{32d^2}+\frac{ib\,e^{-\frac{i(ad-cb)}{d}}\operatorname{Ei}_1\left(ib\cdots\right.}{16d^{\cdots}}$ |

input
```
int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*b^2*(-cos(b*x+a)/(-a*d+c*b+d*(b*x+a))/d-(-Si(-x*b-a-(-a*d+b*c)/d)
*cos((-a*d+b*c)/d)/d-Ci(x*b+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)-1/80*b
^2*(-5*cos(5*b*x+5*a)/(-a*d+c*b+d*(b*x+a))/d-5*(-5*Si(-5*x*b-5*a-5*(-a*d+b
*c)/d)*cos(5*(-a*d+b*c)/d)/d-5*Ci(5*x*b+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*
c)/d)/d)/d)-1/48*b^2*(-3*cos(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d-3*(-3*Si(-3
*x*b-3*a-3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*x*b+3*a+3*(-a*d+b*c)
/d)*sin(3*(-a*d+b*c)/d)/d)/d))
```

### 3.151.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.03

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{16\,d\cos\left(bx+a\right)^5 - 16\,d\cos\left(bx+a\right)^3 - 2\left(bdx+bc\right)\operatorname{Ci}\left(\frac{bdx+bc}{d}\right)\sin\left(-\frac{bc-ad}{d}\right) + 3\left(bdx+bc\right)\operatorname{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)}{}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/16*(16*d*cos(b*x + a)^5 - 16*d*cos(b*x + a)^3 - 2*(b*d*x + b*c)*cos_inte
gral((b*d*x + b*c)/d)*sin(-(b*c - a*d)/d) + 3*(b*d*x + b*c)*cos_integral(3
*(b*d*x + b*c)/d)*sin(-3*(b*c - a*d)/d) + 5*(b*d*x + b*c)*cos_integral(5*(
b*d*x + b*c)/d)*sin(-5*(b*c - a*d)/d) + 5*(b*d*x + b*c)*cos(-5*(b*c - a*d)
/d)*sin_integral(5*(b*d*x + b*c)/d) + 3*(b*d*x + b*c)*cos(-3*(b*c - a*d)/d
)*sin_integral(3*(b*d*x + b*c)/d) - 2*(b*d*x + b*c)*cos(-(b*c - a*d)/d)*si
n_integral((b*d*x + b*c)/d))/(d^3*x + c*d^2)
```

### 3.151.6 Sympy [F]

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin^2\left(a+bx\right)\cos^3\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x)**2, x)
```

### 3.151.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 446, normalized size of antiderivative = 1.74

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{2\,b^2\left(E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^2\left(E_2\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_2}{}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/32*(2*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_i
ntegral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b^
2*(exp_integral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_
e(2, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b^2*(
exp_integral_e(2, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(2
, -5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-5*(b*c - a*d)/d) - 2*b^2*(I
*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(2
, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b^2*(I*exp_in
tegral_e(2, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(2, -3
*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d) - b^2*(I*exp_i
ntegral_e(2, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(2, -
5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-5*(b*c - a*d)/d))/((b*c*d + (b
*x + a)*d^2 - a*d^2)*b)
```

### 3.151.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 49.03 (sec) , antiderivative size = 1022022, normalized size of antiderivative = 3976.74

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/32*(5*b*d*x*imag_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*x)^2*tan(
3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b
*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 3*b*d*x*imag_part(cos_integral
(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)
^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d)^2 - 2*b*d*x*imag_part(cos_integral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(3/
2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c
/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*b*d*x*imag_part(cos_integral(-
b*x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*ta
n(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 - 3*b*d*x*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(5/2*b*x)^2*tan(3/
2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c
/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 5*b*d*x*imag_part(cos_integral(-
5*b*x - 5*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^
2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 + 10*b*d*x*sin_integral(5*(b*d*x + b*c)/d)*tan(5/2*b*x)^2*tan(3/2*b*x
)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 6*b*d*x*sin_integral(3*(b*d*x + b*c)/
d)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*
tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 4*b*d...
```

### 3.151.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(a+b\,x)^3\,\sin(a+b\,x)^2}{(c+d\,x)^2}\,dx$$

input
```
int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^2,x)
```

output
```
int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^2, x)
```

# 3.152 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$

3.152.1 Optimal result . . . . . . . . . . 1220
3.152.2 Mathematica [A] (verified) . . . . . . . . . . 1221
3.152.3 Rubi [A] (verified) . . . . . . . . . . 1221
3.152.4 Maple [A] (verified) . . . . . . . . . . 1223
3.152.5 Fricas [A] (verification not implemented) . . . . . . . . . . 1224
3.152.6 Sympy [F] . . . . . . . . . . 1224
3.152.7 Maxima [C] (verification not implemented) . . . . . . . . . . 1225
3.152.8 Giac [C] (verification not implemented) . . . . . . . . . . 1225
3.152.9 Mupad [F(-1)] . . . . . . . . . . 1226

## 3.152.1 Optimal result

Integrand size = 24, antiderivative size = 338

$$\begin{aligned}\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = &-\frac{\cos(a+bx)}{16d(c+dx)^2}+\frac{\cos(3a+3bx)}{32d(c+dx)^2}+\frac{\cos(5a+5bx)}{32d(c+dx)^2}\\
&-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{16d^3}\\
&+\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^3}\\
&+\frac{25b^2\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{32d^3}\\
&+\frac{b\sin(a+bx)}{16d^2(c+dx)}-\frac{3b\sin(3a+3bx)}{32d^2(c+dx)}\\
&-\frac{5b\sin(5a+5bx)}{32d^2(c+dx)}+\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{16d^3}\\
&-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{32d^3}\\
&-\frac{25b^2\sin\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{32d^3}\end{aligned}$$

output
```
25/32*b^2*Ci(5*b*c/d+5*b*x)*cos(5*a-5*b*c/d)/d^3+9/32*b^2*Ci(3*b*c/d+3*b*x
)*cos(3*a-3*b*c/d)/d^3-1/16*b^2*Ci(b*c/d+b*x)*cos(a-b*c/d)/d^3-1/16*cos(b*
x+a)/d/(d*x+c)^2+1/32*cos(3*b*x+3*a)/d/(d*x+c)^2+1/32*cos(5*b*x+5*a)/d/(d*
x+c)^2-25/32*b^2*Si(5*b*c/d+5*b*x)*sin(5*a-5*b*c/d)/d^3-9/32*b^2*Si(3*b*c/
d+3*b*x)*sin(3*a-3*b*c/d)/d^3+1/16*b^2*Si(b*c/d+b*x)*sin(a-b*c/d)/d^3+1/16
*b*sin(b*x+a)/d^2/(d*x+c)-3/32*b*sin(3*b*x+3*a)/d^2/(d*x+c)-5/32*b*sin(5*b
*x+5*a)/d^2/(d*x+c)
```

### 3.152.2 Mathematica [A] (verified)

Time = 2.96 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.84

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \frac{\frac{d^2\cos(3(a+bx))}{(c+dx)^2}+\frac{d^2\cos(5(a+bx))}{(c+dx)^2}-2b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(b\left(\frac{c}{d}+x\right)\right)+9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3}{}\right.}{}$$

input
```
Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

output
```
((d^2*Cos[3*(a + b*x)])/(c + d*x)^2 + (d^2*Cos[5*(a + b*x)])/(c + d*x)^2 -
 2*b^2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] + 9*b^2*Cos[3*a - (3*b*c)
/d]*CosIntegral[(3*b*(c + d*x))/d] + 25*b^2*Cos[5*a - (5*b*c)/d]*CosIntegr
al[(5*b*(c + d*x))/d] + (2*d*(-(d*Cos[a + b*x]) + b*(c + d*x)*Sin[a + b*x]
))/(c + d*x)^2 - (3*b*d*Sin[3*(a + b*x)])/(c + d*x) - (5*b*d*Sin[5*(a + b*
x)])/(c + d*x) + 2*b^2*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - 9*b^2*S
in[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d] - 25*b^2*Sin[5*a - (5*b
*c)/d]*SinIntegral[(5*b*(c + d*x))/d])/(32*d^3)
```

### 3.152.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\cos^3(a+bx)}{(c+dx)^3}\,dx$$

$$\Big\downarrow \; 4906$$

$$\int \left(\frac{\cos(a+bx)}{8(c+dx)^3}-\frac{\cos(3a+3bx)}{16(c+dx)^3}-\frac{\cos(5a+5bx)}{16(c+dx)^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{16d^3}+\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^3}+\frac{25b^2\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{32d^3}+\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{16d^3}-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{32d^3}-\frac{25b^2\sin\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{32d^3}+\frac{b\sin(a+bx)}{16d^2(c+dx)}-\frac{3b\sin(3a+3bx)}{32d^2(c+dx)}-\frac{5b\sin(5a+5bx)}{32d^2(c+dx)}-\frac{\cos(a+bx)}{16d(c+dx)^2}+\frac{\cos(3a+3bx)}{32d(c+dx)^2}+\frac{\cos(5a+5bx)}{32d(c+dx)^2}$$

input
```
Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

output
```
-1/16*Cos[a + b*x]/(d*(c + d*x)^2) + Cos[3*a + 3*b*x]/(32*d*(c + d*x)^2) +
 Cos[5*a + 5*b*x]/(32*d*(c + d*x)^2) - (b^2*Cos[a - (b*c)/d]*CosIntegral[(
b*c)/d + b*x])/(16*d^3) + (9*b^2*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/
d + 3*b*x])/(32*d^3) + (25*b^2*Cos[5*a - (5*b*c)/d]*CosIntegral[(5*b*c)/d
+ 5*b*x])/(32*d^3) + (b*Sin[a + b*x])/(16*d^2*(c + d*x)) - (3*b*Sin[3*a +
3*b*x])/(32*d^2*(c + d*x)) - (5*b*Sin[5*a + 5*b*x])/(32*d^2*(c + d*x)) + (
b^2*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(16*d^3) - (9*b^2*Sin[3*a
 - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(32*d^3) - (25*b^2*Sin[5*a -
 (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(32*d^3)
```

### 3.152.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.152.4 Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.41

| method | result |
|---|---|
| derivativedivides | $\frac{b^3\left(-\frac{\cos(xb+a)}{2(-ad+cb+d(xb+a))^2 d}-\frac{-\frac{\sin(xb+a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{\mathrm{Si}\left(-xb-a-\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}+\frac{\mathrm{Ci}\left(xb+a+\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right.}{d}}{d}}{2d}\right.}{8}$ |
| default | $\frac{b^3\left(-\frac{\cos(xb+a)}{2(-ad+cb+d(xb+a))^2 d}-\frac{-\frac{\sin(xb+a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{\mathrm{Si}\left(-xb-a-\frac{-ad+cb}{d}\right)\sin\left(\frac{-ad+cb}{d}\right)}{d}+\frac{\mathrm{Ci}\left(xb+a+\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right.}{d}}{d}}{2d}\right.}{8}$ |
| risch | $-\frac{25b^2\mathrm{e}^{-\frac{5i(ad-cb)}{d}}\mathrm{Ei}_1\left(5ibx+5ia-\frac{5i(ad-cb)}{d}\right)}{64d^3}-\frac{9b^2\mathrm{e}^{-\frac{3i(ad-cb)}{d}}\mathrm{Ei}_1\left(3ibx+3ia-\frac{3i(ad-cb)}{d}\right)}{64d^3}+\frac{b^2\mathrm{e}^{-\frac{i(ad-cb)}{d}}\mathrm{Ei}_1\left(\right.}{32}$ |

input
```
int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*b^3*(-1/2*cos(b*x+a)/(-a*d+c*b+d*(b*x+a))^2/d-1/2*(-sin(b*x+a)/(-
a*d+c*b+d*(b*x+a))/d+(-Si(-x*b-a-(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(x*b+
a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d)-1/80*b^3*(-5/2*cos(5*b*x+5*a)/(
-a*d+c*b+d*(b*x+a))^2/d-5/2*(-5*sin(5*b*x+5*a)/(-a*d+c*b+d*(b*x+a))/d+5*(-
5*Si(-5*x*b-5*a-5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d+5*Ci(5*x*b+5*a+5*(-a
*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d)/d)/d)-1/48*b^3*(-3/2*cos(3*b*x+3*a)/(-a*
d+c*b+d*(b*x+a))^2/d-3/2*(-3*sin(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d+3*(-3*S
i(-3*x*b-3*a-3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*x*b+3*a+3*(-a*d+
b*c)/d)*cos(3*(-a*d+b*c)/d)/d)/d)/d))
```

### 3.152.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.30

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{16\,d^2\cos{(bx+a)}^5 - 16\,d^2\cos{(bx+a)}^3 + 25\,(b^2d^2x^2 + 2\,b^2cdx + b^2c^2)\cos\left(-\frac{5\,(bc-ad)}{d}\right)\operatorname{Ci}\left(\frac{5\,(bdx+bc)}{d}\right) + 9}{}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/32*(16*d^2*cos(b*x + a)^5 - 16*d^2*cos(b*x + a)^3 + 25*(b^2*d^2*x^2 + 2*
b^2*c*d*x + b^2*c^2)*cos(-5*(b*c - a*d)/d)*cos_integral(5*(b*d*x + b*c)/d)
 + 9*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-3*(b*c - a*d)/d)*cos_integ
ral(3*(b*d*x + b*c)/d) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-(b*c
 - a*d)/d)*cos_integral((b*d*x + b*c)/d) - 25*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*sin(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d) - 9*(b^2*d
^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*
x + b*c)/d) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-(b*c - a*d)/d)*
sin_integral((b*d*x + b*c)/d) - 16*(5*(b*d^2*x + b*c*d)*cos(b*x + a)^4 - 3
*(b*d^2*x + b*c*d)*cos(b*x + a)^2)*sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^
2*d^3)
```

### 3.152.6 Sympy [F]

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin^2{(a+bx)}\cos^3{(a+bx)}}{(c+dx)^3}\,dx$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c)**3,x)
```

output
```
Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x)**3, x)
```

### 3.152.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.42

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx =$$
$$-\frac{2\,b^3\left(E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^3\left(E_3\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_3\right.}{}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/32*(2*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_i
ntegral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b^
3*(exp_integral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_
e(3, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b^3*(
exp_integral_e(3, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(3
, -5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-5*(b*c - a*d)/d) - 2*b^3*(I
*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(3
, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b^3*(I*exp_in
tegral_e(3, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(3, -3
*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d) - b^3*(I*exp_i
ntegral_e(3, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(3, -
5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-5*(b*c - a*d)/d))/((b^2*c^2*d
- 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))
*b)
```

### 3.152.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 98.98 (sec) , antiderivative size = 1708998, normalized size of antiderivative = 5056.21

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/64*(25*b^2*d^2*x^2*real_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b*x)
^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part
(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)
^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^
2*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2*real_part(cos_integral(b*x + b*c/d))*ta
n(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1
/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*b^2*d^2*x^2
*real_part(cos_integral(-b*x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1
/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*
b*c/d)^2*tan(1/2*b*c/d)^2 + 9*b^2*d^2*x^2*real_part(cos_integral(-3*b*x -
3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/
2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2
5*b^2*d^2*x^2*real_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*b*x)^2*tan
(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*
b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*b^2*d^2*x^2*imag_part(cos_i
ntegral(b*x + b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2
*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d) - 4*b^2*d^2*x^2*imag_part(cos_integral(-b*x - b*c/d))*tan(5/2*b*x)^
2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*...
```

### 3.152.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cos\left(a+b\,x\right)^3\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^3}\,dx$$

input
```
int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^3,x)
```

output
```
int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^3, x)
```

# 3.153 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx$

3.153.1 Optimal result . . . . . 1227
3.153.2 Mathematica [A] (verified) . . . . . 1228
3.153.3 Rubi [A] (verified) . . . . . 1229
3.153.4 Maple [A] (verified) . . . . . 1230
3.153.5 Fricas [A] (verification not implemented) . . . . . 1231
3.153.6 Sympy [F] . . . . . 1232
3.153.7 Maxima [C] (verification not implemented) . . . . . 1232
3.153.8 Giac [C] (verification not implemented) . . . . . 1233
3.153.9 Mupad [F(-1)] . . . . . 1233

## 3.153.1 Optimal result

Integrand size = 24, antiderivative size = 413

$$\begin{aligned}\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = &-\frac{\cos(a+bx)}{24d(c+dx)^3}+\frac{b^2\cos(a+bx)}{48d^3(c+dx)}+\frac{\cos(3a+3bx)}{48d(c+dx)^3} \\ &-\frac{3b^2\cos(3a+3bx)}{32d^3(c+dx)}+\frac{\cos(5a+5bx)}{48d(c+dx)^3}-\frac{25b^2\cos(5a+5bx)}{96d^3(c+dx)} \\ &-\frac{125b^3\,\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)\sin\left(5a-\frac{5bc}{d}\right)}{96d^4} \\ &-\frac{9b^3\,\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{32d^4} \\ &+\frac{b^3\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{48d^4} \\ &+\frac{b\sin(a+bx)}{48d^2(c+dx)^2}-\frac{b\sin(3a+3bx)}{32d^2(c+dx)^2} \\ &-\frac{5b\sin(5a+5bx)}{96d^2(c+dx)^2}+\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{48d^4} \\ &-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{32d^4} \\ &-\frac{125b^3\cos\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{96d^4}\end{aligned}$$

output
```
-1/24*cos(b*x+a)/d/(d*x+c)^3+1/48*b^2*cos(b*x+a)/d^3/(d*x+c)+1/48*cos(3*b*
x+3*a)/d/(d*x+c)^3-3/32*b^2*cos(3*b*x+3*a)/d^3/(d*x+c)+1/48*cos(5*b*x+5*a)
/d/(d*x+c)^3-25/96*b^2*cos(5*b*x+5*a)/d^3/(d*x+c)+1/48*b^3*cos(a-b*c/d)*Si
(b*c/d+b*x)/d^4-9/32*b^3*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^4-125/96*b^3
*cos(5*a-5*b*c/d)*Si(5*b*c/d+5*b*x)/d^4-125/96*b^3*Ci(5*b*c/d+5*b*x)*sin(5
*a-5*b*c/d)/d^4-9/32*b^3*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^4+1/48*b^3*C
i(b*c/d+b*x)*sin(a-b*c/d)/d^4+1/48*b*sin(b*x+a)/d^2/(d*x+c)^2-1/32*b*sin(3
*b*x+3*a)/d^2/(d*x+c)^2-5/96*b*sin(5*b*x+5*a)/d^2/(d*x+c)^2
```

### 3.153.2 Mathematica [A] (verified)

Time = 2.49 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.09

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx =$$

$$-\frac{d\cos(3bx)\left(\left(-2d^2+9b^2(c+dx)^2\right)\cos(3a)+3bd(c+dx)\sin(3a)\right)+d\cos(5bx)\left(\left(-2d^2+25b^2(c+dx)^2\right)\right.}{}$$

input
```
Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

output
```
-1/96*(d*Cos[3*b*x]*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*
x)*Sin[3*a]) + d*Cos[5*b*x]*((-2*d^2 + 25*b^2*(c + d*x)^2)*Cos[5*a] + 5*b*
d*(c + d*x)*Sin[5*a]) + d*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c +
 d*x)^2)*Sin[3*a])*Sin[3*b*x] + d*(5*b*d*(c + d*x)*Cos[5*a] - (-2*d^2 + 25
*b^2*(c + d*x)^2)*Sin[5*a])*Sin[5*b*x] - 2*(d*Cos[b*x]*((-2*d^2 + b^2*(c +
 d*x)^2)*Cos[a] + b*d*(c + d*x)*Sin[a]) + d*(b*d*(c + d*x)*Cos[a] - (-2*d^
2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x] + b^3*(c + d*x)^3*(CosIntegral[b*(c/
d + x)]*Sin[a - (b*c)/d] + Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)])) + 2
7*b^3*(c + d*x)^3*(CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + C
os[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d]) + 125*b^3*(c + d*x)^3*
(CosIntegral[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d] + Cos[5*a - (5*b*c)/d
]*SinIntegral[(5*b*(c + d*x))/d]))/(d^4*(c + d*x)^3)
```

### 3.153.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\cos^3(a+bx)}{(c+dx)^4}\,dx$$

$$\Big\downarrow \ 4906$$

$$\int \left(\frac{\cos(a+bx)}{8(c+dx)^4}-\frac{\cos(3a+3bx)}{16(c+dx)^4}-\frac{\cos(5a+5bx)}{16(c+dx)^4}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{125b^3\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{96d^4}-\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{48d^4}+\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{48d^4}-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{32d^4}-\frac{125b^3\cos\left(5a-\frac{5bc}{d}\right)\text{Si}\left(\frac{5bc}{d}+5bx\right)}{96d^4}+\frac{b^2\cos(a+bx)}{48d^3(c+dx)}-\frac{3b^2\cos(3a+3bx)}{32d^3(c+dx)}-\frac{25b^2\cos(5a+5bx)}{96d^3(c+dx)}+\frac{b\sin(a+bx)}{48d^2(c+dx)^2}-\frac{b\sin(3a+3bx)}{32d^2(c+dx)^2}-\frac{5b\sin(5a+5bx)}{96d^2(c+dx)^2}-\frac{\cos(a+bx)}{24d(c+dx)^3}+\frac{\cos(3a+3bx)}{48d(c+dx)^3}+\frac{\cos(5a+5bx)}{48d(c+dx)^3}$$

input
```
Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

output
```
-1/24*Cos[a + b*x]/(d*(c + d*x)^3) + (b^2*Cos[a + b*x])/(48*d^3*(c + d*x))
 + Cos[3*a + 3*b*x]/(48*d*(c + d*x)^3) - (3*b^2*Cos[3*a + 3*b*x])/(32*d^3*
(c + d*x)) + Cos[5*a + 5*b*x]/(48*d*(c + d*x)^3) - (25*b^2*Cos[5*a + 5*b*x
])/(96*d^3*(c + d*x)) - (125*b^3*CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*a -
(5*b*c)/d])/(96*d^4) - (9*b^3*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*
b*c)/d])/(32*d^4) + (b^3*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(48*
d^4) + (b*Sin[a + b*x])/(48*d^2*(c + d*x)^2) - (b*Sin[3*a + 3*b*x])/(32*d^
2*(c + d*x)^2) - (5*b*Sin[5*a + 5*b*x])/(96*d^2*(c + d*x)^2) + (b^3*Cos[a
- (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(48*d^4) - (9*b^3*Cos[3*a - (3*b*c)
/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(32*d^4) - (125*b^3*Cos[5*a - (5*b*c)/
d]*SinIntegral[(5*b*c)/d + 5*b*x])/(96*d^4)
```

#### 3.153.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.153.4 Maple [A] (verified)

Time = 2.90 (sec) , antiderivative size = 588, normalized size of antiderivative = 1.42

| method | result |
|---|---|
| derivativedivides | $\frac{b^4\left(-\frac{\cos(xb+a)}{3(-ad+cb+d(xb+a))^3d}-\frac{-\frac{\sin(xb+a)}{2(-ad+cb+d(xb+a))^2d}+\frac{-\frac{\cos(xb+a)}{(-ad+cb+d(xb+a))d}-\frac{-\frac{\mathrm{Si}\left(-xb-a-\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}-\frac{\mathrm{Ci}\left(x\right.}{d}}{2d}}{3d}\right.}{8}$ |
| default | $\frac{b^4\left(-\frac{\cos(xb+a)}{3(-ad+cb+d(xb+a))^3d}-\frac{-\frac{\sin(xb+a)}{2(-ad+cb+d(xb+a))^2d}+\frac{-\frac{\cos(xb+a)}{(-ad+cb+d(xb+a))d}-\frac{-\frac{\mathrm{Si}\left(-xb-a-\frac{-ad+cb}{d}\right)\cos\left(\frac{-ad+cb}{d}\right)}{d}-\frac{\mathrm{Ci}\left(x\right.}{d}}{2d}}{3d}\right.}{8}$ |
| risch | Expression too large to display |

input
```
int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/8*b^4*(-1/3*cos(b*x+a)/(-a*d+c*b+d*(b*x+a))^3/d-1/3*(-1/2*sin(b*x+a
)/(-a*d+c*b+d*(b*x+a))^2/d+1/2*(-cos(b*x+a)/(-a*d+c*b+d*(b*x+a))/d-(-Si(-x
*b-a-(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(x*b+a+(-a*d+b*c)/d)*sin((-a*d+b*
c)/d)/d)/d)/d)/d)-1/80*b^4*(-5/3*cos(5*b*x+5*a)/(-a*d+c*b+d*(b*x+a))^3/d-5
/3*(-5/2*sin(5*b*x+5*a)/(-a*d+c*b+d*(b*x+a))^2/d+5/2*(-5*cos(5*b*x+5*a)/(-
a*d+c*b+d*(b*x+a))/d-5*(-5*Si(-5*x*b-5*a-5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/
d)/d-5*Ci(5*x*b+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d)/d)/d)/d)-1/48*b
^4*(-cos(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))^3/d-(-3/2*sin(3*b*x+3*a)/(-a*d+c*
b+d*(b*x+a))^2/d+3/2*(-3*cos(3*b*x+3*a)/(-a*d+c*b+d*(b*x+a))/d-3*(-3*Si(-3
*x*b-3*a-3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*x*b+3*a+3*(-a*d+b*c)
/d)*sin(3*(-a*d+b*c)/d)/d)/d)/d)/d))
```

### 3.153.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.55

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx =$$
$$-\frac{16\,(25\,b^2d^3x^2+50\,b^2cd^2x+25\,b^2c^2d-2\,d^3)\cos{(bx+a)}^5-16\,(29\,b^2d^3x^2+58\,b^2cd^2x+29\,b^2c^2d-2\,d^3)}{}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/96*(16*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*cos(b*x
 + a)^5 - 16*(29*b^2*d^3*x^2 + 58*b^2*c*d^2*x + 29*b^2*c^2*d - 2*d^3)*cos(
b*x + a)^3 - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*c
os_integral((b*d*x + b*c)/d)*sin(-(b*c - a*d)/d) + 27*(b^3*d^3*x^3 + 3*b^3
*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(3*(b*d*x + b*c)/d)*sin(
-3*(b*c - a*d)/d) + 125*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b
^3*c^3)*cos_integral(5*(b*d*x + b*c)/d)*sin(-5*(b*c - a*d)/d) + 125*(b^3*d
^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-5*(b*c - a*d)/d)*
sin_integral(5*(b*d*x + b*c)/d) + 27*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + b^3*c^3)*cos(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d)
 - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-(b*c -
 a*d)/d)*sin_integral((b*d*x + b*c)/d) + 96*(b^2*d^3*x^2 + 2*b^2*c*d^2*x +
 b^2*c^2*d)*cos(b*x + a) + 16*(5*(b*d^3*x + b*c*d^2)*cos(b*x + a)^4 - 3*(b
*d^3*x + b*c*d^2)*cos(b*x + a)^2)*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3
*c^2*d^5*x + c^3*d^4)
```

### 3.153.6 Sympy [F]

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sin^2(a+bx)\cos^3(a+bx)}{(c+dx)^4}\,dx$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c)**4,x)
```

output
```
Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x)**4, x)
```

### 3.153.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.29

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = -\frac{2\,b^4\left(E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b^4\left(E_4\left(\frac{3\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+E_4\right.}{}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/32*(2*b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_i
ntegral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b^
4*(exp_integral_e(4, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_
e(4, -3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b^4*(
exp_integral_e(4, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_integral_e(4
, -5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-5*(b*c - a*d)/d) - 2*b^4*(I
*exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_integral_e(4
, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b^4*(I*exp_in
tegral_e(4, 3*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(4, -3
*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-3*(b*c - a*d)/d) - b^4*(I*exp_i
ntegral_e(4, 5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) - I*exp_integral_e(4, -
5*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-5*(b*c - a*d)/d))/((b^3*c^3*d
 - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3
 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))
*b)
```

### 3.153.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 198.19 (sec) , antiderivative size = 2478918, normalized size of antiderivative = 6002.22

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

output
```
-1/192*(125*b^3*d^3*x^3*imag_part(cos_integral(5*b*x + 5*b*c/d))*tan(5/2*b
*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2
*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 27*b^3*d^3*x^3*imag_
part(cos_integral(3*b*x + 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*
b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c
/d)^2*tan(1/2*b*c/d)^2 - 2*b^3*d^3*x^3*imag_part(cos_integral(b*x + b*c/d)
)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*t
an(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*b^3*d^3
*x^3*imag_part(cos_integral(-b*x - b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*t
an(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(
3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 27*b^3*d^3*x^3*imag_part(cos_integral(-3*b
*x - 3*b*c/d))*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*t
an(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^
2 - 125*b^3*d^3*x^3*imag_part(cos_integral(-5*b*x - 5*b*c/d))*tan(5/2*b*x)
^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 250*b^3*d^3*x^3*sin_int
egral(5*(b*d*x + b*c)/d)*tan(5/2*b*x)^2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(
5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*tan(5/2*b*c/d)^2*tan(3/2*b*c/d)^2*tan(1
/2*b*c/d)^2 + 54*b^3*d^3*x^3*sin_integral(3*(b*d*x + b*c)/d)*tan(5/2*b*x)^
2*tan(3/2*b*x)^2*tan(1/2*b*x)^2*tan(5/2*a)^2*tan(3/2*a)^2*tan(1/2*a)^2*...
```

### 3.153.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos\left(a+b\,x\right)^3\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^4}\,dx$$

input
```
int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^4,x)
```

output
```
int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^4, x)
```

3.153. $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx$

# 3.154 $\int (c+dx)^m \cos^3(a+bx)\sin^3(a+bx)\,dx$

3.154.1 Optimal result . . . 1235
3.154.2 Mathematica [A] (verified) . . . 1236
3.154.3 Rubi [A] (verified) . . . 1236
3.154.4 Maple [F] . . . 1238
3.154.5 Fricas [A] (verification not implemented) . . . 1238
3.154.6 Sympy [F(-2)] . . . 1238
3.154.7 Maxima [F] . . . 1239
3.154.8 Giac [F] . . . 1239
3.154.9 Mupad [F(-1)] . . . 1239

## 3.154.1 Optimal result

Integrand size = 24, antiderivative size = 285

$$\begin{aligned}\int (c+dx)^m \cos^3(a+bx)\sin^3(a+bx)\,dx &= -\frac{3\,2^{-7-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b}\\&-\frac{3\,2^{-7-m}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2ib(c+dx)}{d}\right)}{b}\\&+\frac{2^{-7-m}3^{-1-m}e^{6i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{6ib(c+dx)}{d}\right)}{b}\\&+\frac{2^{-7-m}3^{-1-m}e^{-6i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{6ib(c+dx)}{d}\right)}{b}\end{aligned}$$

output

```
-3*2^(-7-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-
I*b*(d*x+c)/d)^m)-3*2^(-7-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*
I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)+2^(-7-m)*3^(-1-m)*exp(6*I*(a-b*c/d))*(d*x
+c)^m*GAMMA(1+m,-6*I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)+2^(-7-m)*3^(-1-m)
*(d*x+c)^m*GAMMA(1+m,6*I*b*(d*x+c)/d)/b/exp(6*I*(a-b*c/d))/((I*b*(d*x+c)/d
)^m)
```

### 3.154.2 Mathematica [A] (verified)

Time = 2.27 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.89

$$\int (c+dx)^m \cos^3(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{2^{-7-m}3^{-1-m}e^{-\frac{6i(bc+ad)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(-3^{2+m}e^{4i\left(2a+\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^m\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)-3^{2+m}\cdots\right.}{\cdots}$$

input
```
Integrate[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(2^(-7 - m)*3^(-1 - m)*(c + d*x)^m*(-(3^(2 + m)*E^((4*I)*(2*a + (b*c)/d))*
((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d]) - 3^(2 + m)*E^
((4*I)*a + ((8*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((2*I)*b*(
c + d*x))/d] + E^((12*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-6*I)*b*(
c + d*x))/d] + E^(((12*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((
6*I)*b*(c + d*x))/d]))/(b*E^(((6*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2
)^m)
```

### 3.154.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^3(a+bx)\cos^3(a+bx)(c+dx)^m\,dx$$
$$\Big\downarrow \text{4906}$$
$$\int \left(\frac{3}{32}\sin(2a+2bx)(c+dx)^m-\frac{1}{32}\sin(6a+6bx)(c+dx)^m\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{3\,2^{-m-7}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-m-7}3^{-m-1}e^{6i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{6ib(c+dx)}{d}\right)}{b}-\frac{3\,2^{-m-7}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-m-7}3^{-m-1}e^{-6i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{6ib(c+dx)}{d}\right)}{b}$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(-3*2^(-7 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*
(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) - (3*2^(-7 - m)*(c + d*x)^m*Ga
mma[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d
*x))/d)^m) + (2^(-7 - m)*3^(-1 - m)*E^((6*I)*(a - (b*c)/d))*(c + d*x)^m*Ga
mma[1 + m, ((-6*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) + (2^(-7
- m)*3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((6*I)*b*(c + d*x))/d])/(b*E^((6*
I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

### 3.154.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.154.4 Maple [F]

$$\int (dx+c)^m \cos(xb+a)^3 \sin(xb+a)^3 \, dx$$

input
```
int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

output
```
int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

### 3.154.5 Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.67

$$\int (c+dx)^m \cos^3(a+bx)\sin^3(a+bx)\,dx = -\frac{9\,e^{\left(-\frac{dm\log\left(-\frac{2i\,b}{d}\right)+2i\,bc-2i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{2\,(i\,bdx+i\,bc)}{d}\right)-e^{\left(-\frac{dm\log\left(-\frac{6i\,b}{d}\right)+6i\,bc-6i\,ad}{d}\right)}\Gamma\left(m+1,-\frac{6\,(i\,bdx+i\,bc)}{d}\right)+9}{384\,b}$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/384*(9*e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)/d)*gamma(m + 1, -2*(
I*b*d*x + I*b*c)/d) - e^(-(d*m*log(-6*I*b/d) + 6*I*b*c - 6*I*a*d)/d)*gamma
(m + 1, -6*(I*b*d*x + I*b*c)/d) + 9*e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*
a*d)/d)*gamma(m + 1, -2*(-I*b*d*x - I*b*c)/d) - e^(-(d*m*log(6*I*b/d) - 6*
I*b*c + 6*I*a*d)/d)*gamma(m + 1, -6*(-I*b*d*x - I*b*c)/d))/b
```

### 3.154.6 Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m \cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d*x+c)**m*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.154.7 Maxima [F]

$$\int (c+dx)^m \cos^3(a+bx) \sin^3(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^3, x)
```

### 3.154.8 Giac [F]

$$\int (c+dx)^m \cos^3(a+bx) \sin^3(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^3, x)
```

### 3.154.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cos^3(a+bx) \sin^3(a+bx)\,dx = \int \cos(a+b\,x)^3 \sin(a+b\,x)^3 (c+d\,x)^m\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^m, x)
```

# 3.155 $\int (c+dx)^4 \cos^3(a+bx) \sin^3(a+bx) \, dx$

3.155.1 Optimal result . . . 1240
3.155.2 Mathematica [A] (verified) . . . 1241
3.155.3 Rubi [A] (verified) . . . 1241
3.155.4 Maple [A] (verified) . . . 1242
3.155.5 Fricas [B] (verification not implemented) . . . 1243
3.155.6 Sympy [B] (verification not implemented) . . . 1243
3.155.7 Maxima [B] (verification not implemented) . . . 1244
3.155.8 Giac [A] (verification not implemented) . . . 1245
3.155.9 Mupad [B] (verification not implemented) . . . 1246

## 3.155.1 Optimal result

Integrand size = 24, antiderivative size = 233

$$\begin{aligned}\int (c+dx)^4 \cos^3(a+bx) \sin^3(a+bx) \, dx = &-\frac{9d^4\cos(2a+2bx)}{128b^5} + \frac{9d^2(c+dx)^2\cos(2a+2bx)}{64b^3} \\ &- \frac{3(c+dx)^4\cos(2a+2bx)}{64b} + \frac{d^4\cos(6a+6bx)}{10368b^5} \\ &- \frac{d^2(c+dx)^2\cos(6a+6bx)}{576b^3} \\ &+ \frac{(c+dx)^4\cos(6a+6bx)}{192b} \\ &- \frac{9d^3(c+dx)\sin(2a+2bx)}{64b^4} \\ &+ \frac{3d(c+dx)^3\sin(2a+2bx)}{32b^2} \\ &+ \frac{d^3(c+dx)\sin(6a+6bx)}{1728b^4} \\ &- \frac{d(c+dx)^3\sin(6a+6bx)}{288b^2}\end{aligned}$$

output
```
-9/128*d^4*cos(2*b*x+2*a)/b^5+9/64*d^2*(d*x+c)^2*cos(2*b*x+2*a)/b^3-3/64*(
d*x+c)^4*cos(2*b*x+2*a)/b+1/10368*d^4*cos(6*b*x+6*a)/b^5-1/576*d^2*(d*x+c)
^2*cos(6*b*x+6*a)/b^3+1/192*(d*x+c)^4*cos(6*b*x+6*a)/b-9/64*d^3*(d*x+c)*si
n(2*b*x+2*a)/b^4+3/32*d*(d*x+c)^3*sin(2*b*x+2*a)/b^2+1/1728*d^3*(d*x+c)*si
n(6*b*x+6*a)/b^4-1/288*d*(d*x+c)^3*sin(6*b*x+6*a)/b^2
```

### 3.155.2 Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.66

$$\int (c+dx)^4 \cos^3(a+bx) \sin^3(a+bx)\, dx$$
$$= \frac{-243(3d^4 - 6b^2d^2(c+dx)^2 + 2b^4(c+dx)^4)\cos(2(a+bx)) + (d^4 - 18b^2d^2(c+dx)^2 + 54b^4(c+dx)^4)\cos(}{1036}$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(-243*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a + b*x)]
 + (d^4 - 18*b^2*d^2*(c + d*x)^2 + 54*b^4*(c + d*x)^4)*Cos[6*(a + b*x)] -
12*b*d*(c + d*x)*(121*d^2 - 78*b^2*(c + d*x)^2 + (-d^2 + 6*b^2*(c + d*x)^2
)*Cos[4*(a + b*x)])*Sin[2*(a + b*x)])/(10368*b^5)
```

### 3.155.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sin^3(a+bx) \cos^3(a+bx)\, dx$$
$$\Big\downarrow \text{4906}$$
$$\int \left(\frac{3}{32}(c+dx)^4 \sin(2a+2bx) - \frac{1}{32}(c+dx)^4 \sin(6a+6bx)\right) dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{9d^4\cos(2a+2bx)}{128b^5} + \frac{d^4\cos(6a+6bx)}{10368b^5} - \frac{9d^3(c+dx)\sin(2a+2bx)}{64b^4} + \frac{d^3(c+dx)\sin(6a+6bx)}{1728b^4} + \frac{9d^2(c+dx)^2\cos(2a+2bx)}{64b^3} - \frac{d^2(c+dx)^2\cos(6a+6bx)}{576b^3} + \frac{3d(c+dx)^3\sin(2a+2bx)}{32b^2} - \frac{d(c+dx)^3\sin(6a+6bx)}{288b^2} - \frac{3(c+dx)^4\cos(2a+2bx)}{64b} + \frac{(c+dx)^4\cos(6a+6bx)}{192b}$$

input
```
Int[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(-9*d^4*Cos[2*a + 2*b*x])/(128*b^5) + (9*d^2*(c + d*x)^2*Cos[2*a + 2*b*x])
/(64*b^3) - (3*(c + d*x)^4*Cos[2*a + 2*b*x])/(64*b) + (d^4*Cos[6*a + 6*b*x
])/(10368*b^5) - (d^2*(c + d*x)^2*Cos[6*a + 6*b*x])/(576*b^3) + ((c + d*x)
^4*Cos[6*a + 6*b*x])/(192*b) - (9*d^3*(c + d*x)*Sin[2*a + 2*b*x])/(64*b^4)
 + (3*d*(c + d*x)^3*Sin[2*a + 2*b*x])/(32*b^2) + (d^3*(c + d*x)*Sin[6*a +
6*b*x])/(1728*b^4) - (d*(c + d*x)^3*Sin[6*a + 6*b*x])/(288*b^2)
```

### 3.155.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.155.4 Maple [A] (verified)

Time = 3.44 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| parallelrisch | $\frac{\left(-486b^4(dx+c)^4+1458d^2(dx+c)^2b^2-729d^4\right)\cos(2xb+2a)+\left(54b^4(dx+c)^4-18d^2(dx+c)^2b^2+d^4\right)\cos(6xb+6a)+972b(}{10368b}$ |
| risch | $\frac{(54d^4x^4b^4+216b^4c\,d^3x^3+324b^4c^2d^2x^2+216b^4c^3dx+54b^4c^4-18b^2d^4x^2-36b^2c\,d^3x-18b^2c^2d^2+d^4)\cos(6xb+6a)}{10368b^5}-\frac{d(}{}$ |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |

input
```
int((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10368*((-486*b^4*(d*x+c)^4+1458*d^2*(d*x+c)^2*b^2-729*d^4)*cos(2*b*x+2*a
)+(54*b^4*(d*x+c)^4-18*d^2*(d*x+c)^2*b^2+d^4)*cos(6*b*x+6*a)+972*b*((d*x+c
)^2*b^2-3/2*d^2)*d*(d*x+c)*sin(2*b*x+2*a)-36*((d*x+c)^2*b^2-1/6*d^2)*b*d*(
d*x+c)*sin(6*b*x+6*a)+432*b^4*c^4-1440*b^2*c^2*d^2+728*d^4)/b^5
```

### 3.155.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 546 vs. 2(213) = 426.

Time = 0.27 (sec) , antiderivative size = 546, normalized size of antiderivative = 2.34

$$\int (c+dx)^4\cos^3(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{27\,b^4d^4x^4+108\,b^4cd^3x^3+2\,(54\,b^4d^4x^4+216\,b^4cd^3x^3+54\,b^4c^4-18\,b^2c^2d^2+d^4+18\,(18\,b^4c^2d^2-b^2d^4)x^2}{}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/648*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 2*(54*b^4*d^4*x^4 + 216*b^4*c*
d^3*x^3 + 54*b^4*c^4 - 18*b^2*c^2*d^2 + d^4 + 18*(18*b^4*c^2*d^2 - b^2*d^4
)*x^2 + 36*(6*b^4*c^3*d - b^2*c*d^3)*x)*cos(b*x + a)^6 - 3*(54*b^4*d^4*x^4
 + 216*b^4*c*d^3*x^3 + 54*b^4*c^4 - 18*b^2*c^2*d^2 + d^4 + 18*(18*b^4*c^2*
d^2 - b^2*d^4)*x^2 + 36*(6*b^4*c^3*d - b^2*c*d^3)*x)*cos(b*x + a)^4 + 18*(
9*b^4*c^2*d^2 - 5*b^2*d^4)*x^2 + 18*(9*b^2*d^4*x^2 + 18*b^2*c*d^3*x + 9*b^
2*c^2*d^2 - 5*d^4)*cos(b*x + a)^2 + 36*(3*b^4*c^3*d - 5*b^2*c*d^3)*x - 12*
((6*b^3*d^4*x^3 + 18*b^3*c*d^3*x^2 + 6*b^3*c^3*d - b*c*d^3 + (18*b^3*c^2*d
^2 - b*d^4)*x)*cos(b*x + a)^5 - (6*b^3*d^4*x^3 + 18*b^3*c*d^3*x^2 + 6*b^3*
c^3*d - b*c*d^3 + (18*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^3 - 3*(3*b^3*d^
4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d - 5*b*c*d^3 + (9*b^3*c^2*d^2 - 5*b*d
^4)*x)*cos(b*x + a))*sin(b*x + a))/b^5
```

### 3.155.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1334 vs. 2(231) = 462.

Time = 1.52 (sec) , antiderivative size = 1334, normalized size of antiderivative = 5.73

$$\int (c+dx)^4\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**4*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

output
```
Piecewise((-c**4*sin(a + b*x)**2*cos(a + b*x)**4/(4*b) - c**4*cos(a + b*x)
**6/(12*b) + c**3*d*x*sin(a + b*x)**6/(6*b) + c**3*d*x*sin(a + b*x)**4*cos
(a + b*x)**2/(2*b) - c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**4/(2*b) - c**3
*d*x*cos(a + b*x)**6/(6*b) + c**2*d**2*x**2*sin(a + b*x)**6/(4*b) + 3*c**2
*d**2*x**2*sin(a + b*x)**4*cos(a + b*x)**2/(4*b) - 3*c**2*d**2*x**2*sin(a
+ b*x)**2*cos(a + b*x)**4/(4*b) - c**2*d**2*x**2*cos(a + b*x)**6/(4*b) + c
*d**3*x**3*sin(a + b*x)**6/(6*b) + c*d**3*x**3*sin(a + b*x)**4*cos(a + b*x
)**2/(2*b) - c*d**3*x**3*sin(a + b*x)**2*cos(a + b*x)**4/(2*b) - c*d**3*x*
*3*cos(a + b*x)**6/(6*b) + d**4*x**4*sin(a + b*x)**6/(24*b) + d**4*x**4*si
n(a + b*x)**4*cos(a + b*x)**2/(8*b) - d**4*x**4*sin(a + b*x)**2*cos(a + b*
x)**4/(8*b) - d**4*x**4*cos(a + b*x)**6/(24*b) + c**3*d*sin(a + b*x)**5*co
s(a + b*x)/(6*b**2) + 4*c**3*d*sin(a + b*x)**3*cos(a + b*x)**3/(9*b**2) +
c**3*d*sin(a + b*x)*cos(a + b*x)**5/(6*b**2) + c**2*d**2*x*sin(a + b*x)**5
*cos(a + b*x)/(2*b**2) + 4*c**2*d**2*x*sin(a + b*x)**3*cos(a + b*x)**3/(3*
b**2) + c**2*d**2*x*sin(a + b*x)*cos(a + b*x)**5/(2*b**2) + c*d**3*x**2*si
n(a + b*x)**5*cos(a + b*x)/(2*b**2) + 4*c*d**3*x**2*sin(a + b*x)**3*cos(a
+ b*x)**3/(3*b**2) + c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**5/(2*b**2) + d
**4*x**3*sin(a + b*x)**5*cos(a + b*x)/(6*b**2) + 4*d**4*x**3*sin(a + b*x)*
*3*cos(a + b*x)**3/(9*b**2) + d**4*x**3*sin(a + b*x)*cos(a + b*x)**5/(6*b*
*2) - c**2*d**2*sin(a + b*x)**6/(12*b**3) + c**2*d**2*sin(a + b*x)**2*c...
```

### 3.155.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1033 vs. $2(213) = 426$.

Time = 0.30 (sec) , antiderivative size = 1033, normalized size of antiderivative = 4.43

$$\int (c+dx)^4 \cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/10368*(864*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c^4 - 3456*(2*sin(b*x
+ a)^6 - 3*sin(b*x + a)^4)*a*c^3*d/b + 5184*(2*sin(b*x + a)^6 - 3*sin(b*x
+ a)^4)*a^2*c^2*d^2/b^2 - 3456*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*a^3*c
*d^3/b^3 + 864*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*a^4*d^4/b^4 - 36*(6*(
b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*
a) + 27*sin(2*b*x + 2*a))*c^3*d/b + 108*(6*(b*x + a)*cos(6*b*x + 6*a) - 54
*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*a*c^
2*d^2/b^2 - 108*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2
*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*a^2*c*d^3/b^3 + 36*(6*(b*x +
 a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) +
27*sin(2*b*x + 2*a))*a^3*d^4/b^4 - 18*((18*(b*x + a)^2 - 1)*cos(6*b*x + 6*
a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b*x + a)*sin(6*b*x + 6*a
) + 162*(b*x + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 + 36*((18*(b*x + a)^2 - 1)
*cos(6*b*x + 6*a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b*x + a)*
sin(6*b*x + 6*a) + 162*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3/b^3 - 18*((18*(
b*x + a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a)
 - 6*(b*x + a)*sin(6*b*x + 6*a) + 162*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^4/
b^4 - 6*(6*(6*(b*x + a)^3 - b*x - a)*cos(6*b*x + 6*a) - 162*(2*(b*x + a)^3
 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - (18*(b*x + a)^2 - 1)*sin(6*b*x + 6*a) +
 243*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*c*d^3/b^3 + 6*(6*(6*(b*x + a...
```

### 3.155.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.54

$$\int (c+dx)^4\cos^3(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{(54\,b^4d^4x^4 + 216\,b^4cd^3x^3 + 324\,b^4c^2d^2x^2 + 216\,b^4c^3dx + 54\,b^4c^4 - 18\,b^2d^4x^2 - 36\,b^2cd^3x - 18\,b^2c^2d^2 + d^4)}{10368\,b^5}$$
$$- \frac{3\,(2\,b^4d^4x^4 + 8\,b^4cd^3x^3 + 12\,b^4c^2d^2x^2 + 8\,b^4c^3dx + 2\,b^4c^4 - 6\,b^2d^4x^2 - 12\,b^2cd^3x - 6\,b^2c^2d^2 + 3\,d^4)\cos(2}{128\,b^5}$$
$$- \frac{(6\,b^3d^4x^3 + 18\,b^3cd^3x^2 + 18\,b^3c^2d^2x + 6\,b^3c^3d - bd^4x - bcd^3)\sin(6\,bx + 6\,a)}{1728\,b^5}$$
$$+ \frac{3\,(2\,b^3d^4x^3 + 6\,b^3cd^3x^2 + 6\,b^3c^2d^2x + 2\,b^3c^3d - 3\,bd^4x - 3\,bcd^3)\sin(2\,bx + 2\,a)}{64\,b^5}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/10368*(54*b^4*d^4*x^4 + 216*b^4*c*d^3*x^3 + 324*b^4*c^2*d^2*x^2 + 216*b^
4*c^3*d*x + 54*b^4*c^4 - 18*b^2*d^4*x^2 - 36*b^2*c*d^3*x - 18*b^2*c^2*d^2
+ d^4)*cos(6*b*x + 6*a)/b^5 - 3/128*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*
b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3
*x - 6*b^2*c^2*d^2 + 3*d^4)*cos(2*b*x + 2*a)/b^5 - 1/1728*(6*b^3*d^4*x^3 +
 18*b^3*c*d^3*x^2 + 18*b^3*c^2*d^2*x + 6*b^3*c^3*d - b*d^4*x - b*c*d^3)*si
n(6*b*x + 6*a)/b^5 + 3/64*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2
*x + 2*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

### 3.155.9 Mupad [B] (verification not implemented)

Time = 26.37 (sec) , antiderivative size = 576, normalized size of antiderivative = 2.47

$$\int (c+dx)^4 \cos^3(a+bx) \sin^3(a+bx) \, dx = -\frac{729\, d^4\, \cos(2\,a+2\,b\,x) - d^4\, \cos(6\,a+6\,b\,x) + 486\,b^4\,c^4\, \cos(2\,a+2\,b\,x) - 54\,b^4\,c^4\, \cos(6\,a+6\,b\,x) - 97$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^4,x)
```

output
```
-(729*d^4*cos(2*a + 2*b*x) - d^4*cos(6*a + 6*b*x) + 486*b^4*c^4*cos(2*a +
2*b*x) - 54*b^4*c^4*cos(6*a + 6*b*x) - 972*b^3*c^3*d*sin(2*a + 2*b*x) + 36
*b^3*c^3*d*sin(6*a + 6*b*x) - 1458*b^2*c^2*d^2*cos(2*a + 2*b*x) + 18*b^2*c
^2*d^2*cos(6*a + 6*b*x) - 1458*b^2*d^4*x^2*cos(2*a + 2*b*x) + 486*b^4*d^4*
x^4*cos(2*a + 2*b*x) + 18*b^2*d^4*x^2*cos(6*a + 6*b*x) - 54*b^4*d^4*x^4*co
s(6*a + 6*b*x) - 972*b^3*d^4*x^3*sin(2*a + 2*b*x) + 36*b^3*d^4*x^3*sin(6*a
 + 6*b*x) + 1458*b*c*d^3*sin(2*a + 2*b*x) - 6*b*c*d^3*sin(6*a + 6*b*x) + 1
458*b*d^4*x*sin(2*a + 2*b*x) - 6*b*d^4*x*sin(6*a + 6*b*x) + 2916*b^4*c^2*d
^2*x^2*cos(2*a + 2*b*x) - 324*b^4*c^2*d^2*x^2*cos(6*a + 6*b*x) - 2916*b^2*
c*d^3*x*cos(2*a + 2*b*x) + 1944*b^4*c^3*d*x*cos(2*a + 2*b*x) + 36*b^2*c*d^
3*x*cos(6*a + 6*b*x) - 216*b^4*c^3*d*x*cos(6*a + 6*b*x) + 1944*b^4*c*d^3*x
^3*cos(2*a + 2*b*x) - 216*b^4*c*d^3*x^3*cos(6*a + 6*b*x) - 2916*b^3*c^2*d^
2*x*sin(2*a + 2*b*x) - 2916*b^3*c*d^3*x^2*sin(2*a + 2*b*x) + 108*b^3*c^2*d
^2*x*sin(6*a + 6*b*x) + 108*b^3*c*d^3*x^2*sin(6*a + 6*b*x))/(10368*b^5)
```

# 3.156 $\int (c+dx)^3 \cos^3(a+bx) \sin^3(a+bx)\,dx$

3.156.1 Optimal result . . . 1247
3.156.2 Mathematica [A] (verified) . . . 1247
3.156.3 Rubi [A] (verified) . . . 1248
3.156.4 Maple [A] (verified) . . . 1249
3.156.5 Fricas [B] (verification not implemented) . . . 1249
3.156.6 Sympy [B] (verification not implemented) . . . 1250
3.156.7 Maxima [B] (verification not implemented) . . . 1251
3.156.8 Giac [A] (verification not implemented) . . . 1252
3.156.9 Mupad [B] (verification not implemented) . . . 1253

## 3.156.1 Optimal result

Integrand size = 24, antiderivative size = 181

$$\int (c+dx)^3 \cos^3(a+bx) \sin^3(a+bx)\,dx = \frac{9d^2(c+dx)\cos(2a+2bx)}{128b^3} - \frac{3(c+dx)^3\cos(2a+2bx)}{64b} - \frac{d^2(c+dx)\cos(6a+6bx)}{1152b^3} + \frac{(c+dx)^3\cos(6a+6bx)}{192b} - \frac{9d^3\sin(2a+2bx)}{256b^4} + \frac{9d(c+dx)^2\sin(2a+2bx)}{128b^2} + \frac{d^3\sin(6a+6bx)}{6912b^4} - \frac{d(c+dx)^2\sin(6a+6bx)}{384b^2}$$

output
```
9/128*d^2*(d*x+c)*cos(2*b*x+2*a)/b^3-3/64*(d*x+c)^3*cos(2*b*x+2*a)/b-1/115
2*d^2*(d*x+c)*cos(6*b*x+6*a)/b^3+1/192*(d*x+c)^3*cos(6*b*x+6*a)/b-9/256*d^
3*sin(2*b*x+2*a)/b^4+9/128*d*(d*x+c)^2*sin(2*b*x+2*a)/b^2+1/6912*d^3*sin(6
*b*x+6*a)/b^4-1/384*d*(d*x+c)^2*sin(6*b*x+6*a)/b^2
```

## 3.156.2 Mathematica [A] (verified)

Time = 2.45 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.73

$$\int (c+dx)^3 \cos^3(a+bx) \sin^3(a+bx)\,dx = \frac{-324b(c+dx)\left(-3d^2+2b^2(c+dx)^2\right)\cos(2(a+bx)) + 12b(c+dx)\left(-d^2+6b^2(c+dx)^2\right)\cos(6(a+bx)) -}{13824b^4}$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(-324*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 12*b*(c
+ d*x)*(-d^2 + 6*b^2*(c + d*x)^2)*Cos[6*(a + b*x)] - 4*d*(121*d^2 - 234*b^
2*(c + d*x)^2 + (-d^2 + 18*b^2*(c + d*x)^2)*Cos[4*(a + b*x)])*Sin[2*(a + b
*x)])/(13824*b^4)
```

### 3.156.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin^3(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{3}{32}(c+dx)^3\sin(2a+2bx)-\frac{1}{32}(c+dx)^3\sin(6a+6bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{9d^3\sin(2a+2bx)}{256b^4}+\frac{d^3\sin(6a+6bx)}{6912b^4}+\frac{9d^2(c+dx)\cos(2a+2bx)}{128b^3}-\frac{d^2(c+dx)\cos(6a+6bx)}{1152b^3}+\frac{9d(c+dx)^2\sin(2a+2bx)}{128b^2}-\frac{d(c+dx)^2\sin(6a+6bx)}{384b^2}-\frac{3(c+dx)^3\cos(2a+2bx)}{64b}+\frac{(c+dx)^3\cos(6a+6bx)}{192b}$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(9*d^2*(c + d*x)*Cos[2*a + 2*b*x])/(128*b^3) - (3*(c + d*x)^3*Cos[2*a + 2*
b*x])/(64*b) - (d^2*(c + d*x)*Cos[6*a + 6*b*x])/(1152*b^3) + ((c + d*x)^3*
Cos[6*a + 6*b*x])/(192*b) - (9*d^3*Sin[2*a + 2*b*x])/(256*b^4) + (9*d*(c +
 d*x)^2*Sin[2*a + 2*b*x])/(128*b^2) + (d^3*Sin[6*a + 6*b*x])/(6912*b^4) -
(d*(c + d*x)^2*Sin[6*a + 6*b*x])/(384*b^2)
```

### 3.156.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.156.4 Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.82

| method | result |
|---|---|
| parallelrisch | $\frac{-54b\left((dx+c)^2b^2-\frac{3d^2}{2}\right)(dx+c)\cos(2xb+2a)+6b\left((dx+c)^2b^2-\frac{d^2}{6}\right)(dx+c)\cos(6xb+6a)+81d\left((dx+c)^2b^2-\frac{d^2}{2}\right)\sin(2xb-}{1152b^4}$ |
| risch | $\frac{(6b^2d^3x^3+18b^2c\,d^2x^2+18b^2c^2dx+6b^2c^3-d^3x-c\,d^2)\cos(6xb+6a)}{1152b^3}-\frac{d(18x^2d^2b^2+36b^2cdx+18b^2c^2-d^2)\sin(6xb+6a)}{6912b^4}$ |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |

input
```
int((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/1152*(-54*b*((d*x+c)^2*b^2-3/2*d^2)*(d*x+c)*cos(2*b*x+2*a)+6*b*((d*x+c)^
2*b^2-1/6*d^2)*(d*x+c)*cos(6*b*x+6*a)+81*d*((d*x+c)^2*b^2-1/2*d^2)*sin(2*b
*x+2*a)-3*((d*x+c)^2*b^2-1/18*d^2)*d*sin(6*b*x+6*a)+48*b^3*c^3-80*c*d^2*b)
/b^4
```

## 3.156.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(165) = 330.

Time = 0.25 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.93

$$\int (c+dx)^3\cos^3(a+bx)\sin^3(a+bx)\,dx$$
$$=\frac{9\,b^3d^3x^3+27\,b^3cd^2x^2+6\,(6\,b^3d^3x^3+18\,b^3cd^2x^2+6\,b^3c^3-bcd^2+(18\,b^3c^2d-bd^3)x)\cos{(bx+a)}^6-9\,(6\,b}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/216*(9*b^3*d^3*x^3 + 27*b^3*c*d^2*x^2 + 6*(6*b^3*d^3*x^3 + 18*b^3*c*d^2*
x^2 + 6*b^3*c^3 - b*c*d^2 + (18*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^6 - 9*(
6*b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 + 6*b^3*c^3 - b*c*d^2 + (18*b^3*c^2*d - b
*d^3)*x)*cos(b*x + a)^4 + 27*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2 + 3*(9*b^3
*c^2*d - 5*b*d^3)*x - ((18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d
^3)*cos(b*x + a)^5 - (18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3
)*cos(b*x + a)^3 - 3*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 5*d^3
)*cos(b*x + a))*sin(b*x + a))/b^4
```

### 3.156.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(178) = 356$.

Time = 1.13 (sec) , antiderivative size = 857, normalized size of antiderivative = 4.73

$$\int (c+dx)^3\cos^3(a+bx)\sin^3(a+bx)\,dx$$
$$=\begin{cases} -\frac{c^3\sin^2(a+bx)\cos^4(a+bx)}{4b}-\frac{c^3\cos^6(a+bx)}{12b}+\frac{c^2dx\sin^6(a+bx)}{8b}+\frac{3c^2dx\sin^4(a+bx)\cos^2(a+bx)}{8b}-\frac{3c^2dx\sin^2(a+bx)\cos^4(a+bx)}{8b} - \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\sin^3(a)\cos^3(a) \end{cases}$$

input
```
integrate((d*x+c)**3*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

output
```
Piecewise((-c**3*sin(a + b*x)**2*cos(a + b*x)**4/(4*b) - c**3*cos(a + b*x)
**6/(12*b) + c**2*d*x*sin(a + b*x)**6/(8*b) + 3*c**2*d*x*sin(a + b*x)**4*c
os(a + b*x)**2/(8*b) - 3*c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) -
c**2*d*x*cos(a + b*x)**6/(8*b) + c*d**2*x**2*sin(a + b*x)**6/(8*b) + 3*c*d
**2*x**2*sin(a + b*x)**4*cos(a + b*x)**2/(8*b) - 3*c*d**2*x**2*sin(a + b*x
)**2*cos(a + b*x)**4/(8*b) - c*d**2*x**2*cos(a + b*x)**6/(8*b) + d**3*x**3
*sin(a + b*x)**6/(24*b) + d**3*x**3*sin(a + b*x)**4*cos(a + b*x)**2/(8*b)
- d**3*x**3*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) - d**3*x**3*cos(a + b*x)
**6/(24*b) + c**2*d*sin(a + b*x)**5*cos(a + b*x)/(8*b**2) + c**2*d*sin(a +
 b*x)**3*cos(a + b*x)**3/(3*b**2) + c**2*d*sin(a + b*x)*cos(a + b*x)**5/(8
*b**2) + c*d**2*x*sin(a + b*x)**5*cos(a + b*x)/(4*b**2) + 2*c*d**2*x*sin(a
 + b*x)**3*cos(a + b*x)**3/(3*b**2) + c*d**2*x*sin(a + b*x)*cos(a + b*x)**
5/(4*b**2) + d**3*x**2*sin(a + b*x)**5*cos(a + b*x)/(8*b**2) + d**3*x**2*s
in(a + b*x)**3*cos(a + b*x)**3/(3*b**2) + d**3*x**2*sin(a + b*x)*cos(a + b
*x)**5/(8*b**2) - c*d**2*sin(a + b*x)**6/(24*b**3) + c*d**2*sin(a + b*x)**
2*cos(a + b*x)**4/(6*b**3) + 7*c*d**2*cos(a + b*x)**6/(72*b**3) - 5*d**3*x
*sin(a + b*x)**6/(72*b**3) - d**3*x*sin(a + b*x)**4*cos(a + b*x)**2/(12*b*
*3) + d**3*x*sin(a + b*x)**2*cos(a + b*x)**4/(12*b**3) + 5*d**3*x*cos(a +
b*x)**6/(72*b**3) - 5*d**3*sin(a + b*x)**5*cos(a + b*x)/(72*b**4) - 31*d**
3*sin(a + b*x)**3*cos(a + b*x)**3/(216*b**4) - 5*d**3*sin(a + b*x)*cos(...
```

### 3.156.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(165) = 330$.

Time = 0.26 (sec) , antiderivative size = 602, normalized size of antiderivative = 3.33

$$\int (c+dx)^3\cos^3(a+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{576\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)c^3-\frac{1728\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)ac^2d}{b}+\frac{1728\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)}{b^2}}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/6912*(576*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c^3 - 1728*(2*sin(b*x +
 a)^6 - 3*sin(b*x + a)^4)*a*c^2*d/b + 1728*(2*sin(b*x + a)^6 - 3*sin(b*x +
 a)^4)*a^2*c*d^2/b^2 - 576*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*a^3*d^3/b
^3 - 18*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - si
n(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*c^2*d/b + 36*(6*(b*x + a)*cos(6*b*x
+ 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x +
 2*a))*a*c*d^2/b^2 - 18*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2
*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*a^2*d^3/b^3 - 6*((18
*(b*x + a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*
a) - 6*(b*x + a)*sin(6*b*x + 6*a) + 162*(b*x + a)*sin(2*b*x + 2*a))*c*d^2/
b^2 + 6*((18*(b*x + a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*(b*x + a)^2 - 1)*co
s(2*b*x + 2*a) - 6*(b*x + a)*sin(6*b*x + 6*a) + 162*(b*x + a)*sin(2*b*x +
2*a))*a*d^3/b^3 - (6*(6*(b*x + a)^3 - b*x - a)*cos(6*b*x + 6*a) - 162*(2*(
b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - (18*(b*x + a)^2 - 1)*sin(6*b*
x + 6*a) + 243*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*d^3/b^3)/b
```

### 3.156.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.33

$$\int (c+dx)^3\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{(6\,b^3d^3x^3+18\,b^3cd^2x^2+18\,b^3c^2dx+6\,b^3c^3-bd^3x-bcd^2)\cos(6\,bx+6\,a)}{1152\,b^4} - \frac{3\,(2\,b^3d^3x^3+6\,b^3cd^2x^2+6\,b^3c^2dx+2\,b^3c^3-3\,bd^3x-3\,bcd^2)\cos(2\,bx+2\,a)}{128\,b^4} - \frac{(18\,b^2d^3x^2+36\,b^2cd^2x+18\,b^2c^2d-d^3)\sin(6\,bx+6\,a)}{6912\,b^4} + \frac{9\,(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-d^3)\sin(2\,bx+2\,a)}{256\,b^4}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/1152*(6*b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 + 18*b^3*c^2*d*x + 6*b^3*c^3 - b*
d^3*x - b*c*d^2)*cos(6*b*x + 6*a)/b^4 - 3/128*(2*b^3*d^3*x^3 + 6*b^3*c*d^2
*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^2)*cos(2*b*x + 2*a)
/b^4 - 1/6912*(18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3)*sin(6
*b*x + 6*a)/b^4 + 9/256*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3
)*sin(2*b*x + 2*a)/b^4
```

### 3.156.9 Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.02

$$\int (c+dx)^3 \cos^3(a+bx) \sin^3(a+bx)\, dx =$$
$$-\frac{243\, d^3 \sin(2\,a+2\,b\,x) - d^3 \sin(6\,a+6\,b\,x) + 324\, b^3\, c^3 \cos(2\,a+2\,b\,x) - 36\, b^3\, c^3 \cos(6\,a+6\,b\,x) - 48\cdots}{\cdots}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^3,x)
```

output
```
-(243*d^3*sin(2*a + 2*b*x) - d^3*sin(6*a + 6*b*x) + 324*b^3*c^3*cos(2*a +
2*b*x) - 36*b^3*c^3*cos(6*a + 6*b*x) - 486*b^2*c^2*d*sin(2*a + 2*b*x) + 18
*b^2*c^2*d*sin(6*a + 6*b*x) + 324*b^3*d^3*x^3*cos(2*a + 2*b*x) - 36*b^3*d^
3*x^3*cos(6*a + 6*b*x) - 486*b^2*d^3*x^2*sin(2*a + 2*b*x) + 18*b^2*d^3*x^2
*sin(6*a + 6*b*x) - 486*b*c*d^2*cos(2*a + 2*b*x) + 6*b*c*d^2*cos(6*a + 6*b
*x) - 486*b*d^3*x*cos(2*a + 2*b*x) + 6*b*d^3*x*cos(6*a + 6*b*x) + 972*b^3*
c^2*d*x*cos(2*a + 2*b*x) - 108*b^3*c^2*d*x*cos(6*a + 6*b*x) - 972*b^2*c*d^
2*x*sin(2*a + 2*b*x) + 36*b^2*c*d^2*x*sin(6*a + 6*b*x) + 972*b^3*c*d^2*x^2
*cos(2*a + 2*b*x) - 108*b^3*c*d^2*x^2*cos(6*a + 6*b*x))/(6912*b^4)
```

# 3.157 $\int (c+dx)^2 \cos^3(a+bx) \sin^3(a+bx) \, dx$

3.157.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1254
3.157.2 Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 1254
3.157.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1255
3.157.4 Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1256
3.157.5 Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . 1257
3.157.6 Sympy [B] (verification not implemented) . . . . . . . . . . . . . . . . . 1257
3.157.7 Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . . 1258
3.157.8 Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . . 1259
3.157.9 Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . 1259

## 3.157.1 Optimal result

Integrand size = 24, antiderivative size = 129

$$\int (c+dx)^2 \cos^3(a+bx) \sin^3(a+bx) \, dx = \frac{3d^2 \cos(2a+2bx)}{128b^3} - \frac{3(c+dx)^2 \cos(2a+2bx)}{64b} - \frac{d^2 \cos(6a+6bx)}{3456b^3} + \frac{(c+dx)^2 \cos(6a+6bx)}{192b} + \frac{3d(c+dx) \sin(2a+2bx)}{64b^2} - \frac{d(c+dx) \sin(6a+6bx)}{576b^2}$$

output
```
3/128*d^2*cos(2*b*x+2*a)/b^3-3/64*(d*x+c)^2*cos(2*b*x+2*a)/b-1/3456*d^2*co
s(6*b*x+6*a)/b^3+1/192*(d*x+c)^2*cos(6*b*x+6*a)/b+3/64*d*(d*x+c)*sin(2*b*x
+2*a)/b^2-1/576*d*(d*x+c)*sin(6*b*x+6*a)/b^2
```

## 3.157.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.71

$$\int (c+dx)^2 \cos^3(a+bx) \sin^3(a+bx) \, dx = \frac{-81(-d^2+2b^2(c+dx)^2) \cos(2(a+bx)) + (-d^2+18b^2(c+dx)^2) \cos(6(a+bx)) - 6bd(c+dx)(-27 \sin(2}{3456b^3}$$

input
```
Integrate[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(-81*(-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + (-d^2 + 18*b^2*(c + d*x
)^2)*Cos[6*(a + b*x)] - 6*b*d*(c + d*x)*(-27*Sin[2*(a + b*x)] + Sin[6*(a +
 b*x)]))/(3456*b^3)
```

### 3.157.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin^3(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{3}{32}(c+dx)^2\sin(2a+2bx)-\frac{1}{32}(c+dx)^2\sin(6a+6bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3d^2\cos(2a+2bx)}{128b^3}-\frac{d^2\cos(6a+6bx)}{3456b^3}+\frac{3d(c+dx)\sin(2a+2bx)}{64b^2}-\frac{d(c+dx)\sin(6a+6bx)}{576b^2}-\frac{3(c+dx)^2\cos(2a+2bx)}{64b}+\frac{(c+dx)^2\cos(6a+6bx)}{192b}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(3*d^2*Cos[2*a + 2*b*x])/(128*b^3) - (3*(c + d*x)^2*Cos[2*a + 2*b*x])/(64*
b) - (d^2*Cos[6*a + 6*b*x])/(3456*b^3) + ((c + d*x)^2*Cos[6*a + 6*b*x])/(1
92*b) + (3*d*(c + d*x)*Sin[2*a + 2*b*x])/(64*b^2) - (d*(c + d*x)*Sin[6*a +
 6*b*x])/(576*b^2)
```

### 3.157.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.157.4 Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.86

| method | result |
|---|---|
| parallelrisch | $\frac{81\left(-2(dx+c)^2b^2+d^2\right)\cos(2xb+2a)+\left(18(dx+c)^2b^2-d^2\right)\cos(6xb+6a)+162bd(dx+c)\sin(2xb+2a)-6bd(dx+c)\sin(6xb}{3456b^3}$ |
| risch | $\frac{(18x^2d^2b^2+36b^2cdx+18b^2c^2-d^2)\cos(6xb+6a)}{3456b^3}-\frac{d(dx+c)\sin(6xb+6a)}{576b^2}-\frac{3(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2)\cos(2xb+}{128b^3}$ |
| derivativedivides | $\frac{\frac{a^2d^2\left(-\frac{\sin(xb+a)^2\cos(xb+a)^4}{6}-\frac{\cos(xb+a)^4}{12}\right)}{b^2}-\frac{2acd\left(-\frac{\sin(xb+a)^2\cos(xb+a)^4}{6}-\frac{\cos(xb+a)^4}{12}\right)}{b}-\frac{2a\,d^2\left(\frac{(xb+a)\sin(xb+a)^4}{4}+\frac{(\mathrm{si}}{}\right.}{}}{}$ |
| default | $\frac{\frac{a^2d^2\left(-\frac{\sin(xb+a)^2\cos(xb+a)^4}{6}-\frac{\cos(xb+a)^4}{12}\right)}{b^2}-\frac{2acd\left(-\frac{\sin(xb+a)^2\cos(xb+a)^4}{6}-\frac{\cos(xb+a)^4}{12}\right)}{b}-\frac{2a\,d^2\left(\frac{(xb+a)\sin(xb+a)^4}{4}+\frac{(\mathrm{si}}{}\right.}{}}{}$ |

input
```
int((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3456*(81*(-2*(d*x+c)^2*b^2+d^2)*cos(2*b*x+2*a)+(18*(d*x+c)^2*b^2-d^2)*co
s(6*b*x+6*a)+162*b*d*(d*x+c)*sin(2*b*x+2*a)-6*b*d*(d*x+c)*sin(6*b*x+6*a)+1
44*b^2*c^2-80*d^2)/b^3
```

### 3.157.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.50

$$\int (c+dx)^2 \cos^3(a+bx)\sin^3(a+bx)\,dx$$
$$= \frac{2\left(18\,b^2d^2x^2+36\,b^2cdx+18\,b^2c^2-d^2\right)\cos\left(bx+a\right)^6+9\,b^2d^2x^2+18\,b^2cdx-3\left(18\,b^2d^2x^2+36\,b^2cdx+18\right.}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/216*(2*(18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^2*c^2 - d^2)*cos(b*x + a)^6
 + 9*b^2*d^2*x^2 + 18*b^2*c*d*x - 3*(18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^
2*c^2 - d^2)*cos(b*x + a)^4 + 9*d^2*cos(b*x + a)^2 - 6*(2*(b*d^2*x + b*c*d
)*cos(b*x + a)^5 - 2*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - 3*(b*d^2*x + b*c*d
)*cos(b*x + a))*sin(b*x + a))/b^3
```

### 3.157.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 461 vs. 2(126) = 252.

Time = 0.81 (sec) , antiderivative size = 461, normalized size of antiderivative = 3.57

$$\int (c+dx)^2 \cos^3(a+bx)\sin^3(a+bx)\,dx$$
$$= \begin{cases} -\frac{c^2\sin^2(a+bx)\cos^4(a+bx)}{4b} - \frac{c^2\cos^6(a+bx)}{12b} + \frac{cdx\sin^6(a+bx)}{12b} + \frac{cdx\sin^4(a+bx)\cos^2(a+bx)}{4b} - \frac{cdx\sin^2(a+bx)\cos^4(a+bx)}{4b} - \frac{cdx}{} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin^3(a)\cos^3(a) \end{cases}$$

input
```
integrate((d*x+c)**2*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

output
```
Piecewise((-c**2*sin(a + b*x)**2*cos(a + b*x)**4/(4*b) - c**2*cos(a + b*x)
**6/(12*b) + c*d*x*sin(a + b*x)**6/(12*b) + c*d*x*sin(a + b*x)**4*cos(a +
b*x)**2/(4*b) - c*d*x*sin(a + b*x)**2*cos(a + b*x)**4/(4*b) - c*d*x*cos(a
+ b*x)**6/(12*b) + d**2*x**2*sin(a + b*x)**6/(24*b) + d**2*x**2*sin(a + b*
x)**4*cos(a + b*x)**2/(8*b) - d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**4/(8
*b) - d**2*x**2*cos(a + b*x)**6/(24*b) + c*d*sin(a + b*x)**5*cos(a + b*x)/
(12*b**2) + 2*c*d*sin(a + b*x)**3*cos(a + b*x)**3/(9*b**2) + c*d*sin(a + b
*x)*cos(a + b*x)**5/(12*b**2) + d**2*x*sin(a + b*x)**5*cos(a + b*x)/(12*b*
*2) + 2*d**2*x*sin(a + b*x)**3*cos(a + b*x)**3/(9*b**2) + d**2*x*sin(a + b
*x)*cos(a + b*x)**5/(12*b**2) - d**2*sin(a + b*x)**6/(72*b**3) + d**2*sin(
a + b*x)**2*cos(a + b*x)**4/(18*b**3) + 7*d**2*cos(a + b*x)**6/(216*b**3),
 Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**3*cos(a)**3, True))
```

### 3.157.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(117) = 234$.

Time = 0.26 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.35

$$\int (c+dx)^2\cos^3(a+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{288\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)c^2-\frac{576\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)acd}{b}+\frac{288\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)a^2d}{b^2}}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/3456*(288*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c^2 - 576*(2*sin(b*x +
a)^6 - 3*sin(b*x + a)^4)*a*c*d/b + 288*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^
4)*a^2*d^2/b^2 - 6*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x
+ 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*c*d/b + 6*(6*(b*x + a)*co
s(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin
(2*b*x + 2*a))*a*d^2/b^2 - ((18*(b*x + a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*
(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b*x + a)*sin(6*b*x + 6*a) + 162*(b*
x + a)*sin(2*b*x + 2*a))*d^2/b^2)/b
```

### 3.157.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12

$$\int (c+dx)^2\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{(18\,b^2d^2x^2+36\,b^2cdx+18\,b^2c^2-d^2)\cos(6\,bx+6\,a)}{3456\,b^3} - \frac{3\,(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2)\cos(2\,bx+2\,a)}{128\,b^3} - \frac{(bd^2x+bcd)\sin(6\,bx+6\,a)}{576\,b^3} + \frac{3\,(bd^2x+bcd)\sin(2\,bx+2\,a)}{64\,b^3}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/3456*(18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^2*c^2 - d^2)*cos(6*b*x + 6*a)
/b^3 - 3/128*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2
*a)/b^3 - 1/576*(b*d^2*x + b*c*d)*sin(6*b*x + 6*a)/b^3 + 3/64*(b*d^2*x + b
*c*d)*sin(2*b*x + 2*a)/b^3
```

### 3.157.9 Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.57

$$\int (c+dx)^2\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{81\,d^2\,\cos(2\,a+2\,b\,x)-d^2\,\cos(6\,a+6\,b\,x)-162\,b^2\,c^2\,\cos(2\,a+2\,b\,x)+18\,b^2\,c^2\,\cos(6\,a+6\,b\,x)+162\,b\,c}{}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^2,x)
```

output
```
(81*d^2*cos(2*a + 2*b*x) - d^2*cos(6*a + 6*b*x) - 162*b^2*c^2*cos(2*a + 2*
b*x) + 18*b^2*c^2*cos(6*a + 6*b*x) + 162*b*c*d*sin(2*a + 2*b*x) - 6*b*c*d*
sin(6*a + 6*b*x) - 162*b^2*d^2*x^2*cos(2*a + 2*b*x) + 18*b^2*d^2*x^2*cos(6
*a + 6*b*x) + 162*b*d^2*x*sin(2*a + 2*b*x) - 6*b*d^2*x*sin(6*a + 6*b*x) -
324*b^2*c*d*x*cos(2*a + 2*b*x) + 36*b^2*c*d*x*cos(6*a + 6*b*x))/(3456*b^3)
```

# 3.158 $\int (c+dx)\cos^3(a+bx)\sin^3(a+bx)\,dx$

3.158.1 Optimal result . . . . . 1260
3.158.2 Mathematica [A] (verified) . . . . . 1260
3.158.3 Rubi [A] (verified) . . . . . 1261
3.158.4 Maple [A] (verified) . . . . . 1262
3.158.5 Fricas [A] (verification not implemented) . . . . . 1262
3.158.6 Sympy [B] (verification not implemented) . . . . . 1263
3.158.7 Maxima [A] (verification not implemented) . . . . . 1263
3.158.8 Giac [A] (verification not implemented) . . . . . 1264
3.158.9 Mupad [B] (verification not implemented) . . . . . 1264

## 3.158.1 Optimal result

Integrand size = 22, antiderivative size = 77

$$\int (c+dx)\cos^3(a+bx)\sin^3(a+bx)\,dx = -\frac{3(c+dx)\cos(2a+2bx)}{64b} + \frac{(c+dx)\cos(6a+6bx)}{192b} + \frac{3d\sin(2a+2bx)}{128b^2} - \frac{d\sin(6a+6bx)}{1152b^2}$$

output
```
-3/64*(d*x+c)*cos(2*b*x+2*a)/b+1/192*(d*x+c)*cos(6*b*x+6*a)/b+3/128*d*sin(
2*b*x+2*a)/b^2-1/1152*d*sin(6*b*x+6*a)/b^2
```

## 3.158.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.82

$$\int (c+dx)\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{-54b(c+dx)\cos(2(a+bx)) + 6b(c+dx)\cos(6(a+bx)) + d(27\sin(2(a+bx)) - \sin(6(a+bx)))}{1152b^2}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(-54*b*(c + d*x)*Cos[2*(a + b*x)] + 6*b*(c + d*x)*Cos[6*(a + b*x)] + d*(27
*Sin[2*(a + b*x)] - Sin[6*(a + b*x)]))/(1152*b^2)
```

### 3.158.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin^3(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{3}{32}(c+dx)\sin(2a+2bx)-\frac{1}{32}(c+dx)\sin(6a+6bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3d\sin(2a+2bx)}{128b^2}-\frac{d\sin(6a+6bx)}{1152b^2}-\frac{3(c+dx)\cos(2a+2bx)}{64b}+\frac{(c+dx)\cos(6a+6bx)}{192b}$$

input
```
Int[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(-3*(c + d*x)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)*Cos[6*a + 6*b*x])/(192
*b) + (3*d*Sin[2*a + 2*b*x])/(128*b^2) - (d*Sin[6*a + 6*b*x])/(1152*b^2)
```

#### 3.158.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.158.4 Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90

| method | result |
|---|---|
| parallelrisch | $\frac{-54b(dx+c)\cos(2xb+2a)+6b(dx+c)\cos(6xb+6a)+48cb+27d\sin(2xb+2a)-d\sin(6xb+6a)}{1152b^{2}}$ |
| risch | $-\frac{3(dx+c)\cos(2xb+2a)}{64b}+\frac{(dx+c)\cos(6xb+6a)}{192b}+\frac{3d\sin(2xb+2a)}{128b^{2}}-\frac{d\sin(6xb+6a)}{1152b^{2}}$ |
| derivativedivides | $\frac{-\frac{da\left(-\frac{\sin(xb+a)^{2}\cos(xb+a)^{4}}{6}-\frac{\cos(xb+a)^{4}}{12}\right)}{b}+c\left(-\frac{\sin(xb+a)^{2}\cos(xb+a)^{4}}{6}-\frac{\cos(xb+a)^{4}}{12}\right)+\frac{d\left(\frac{(xb+a)\sin(xb+a)^{4}}{4}+\frac{(\sin(xb+a}{}\right.}{}}{b}$ |
| default | $\frac{-\frac{da\left(-\frac{\sin(xb+a)^{2}\cos(xb+a)^{4}}{6}-\frac{\cos(xb+a)^{4}}{12}\right)}{b}+c\left(-\frac{\sin(xb+a)^{2}\cos(xb+a)^{4}}{6}-\frac{\cos(xb+a)^{4}}{12}\right)+\frac{d\left(\frac{(xb+a)\sin(xb+a)^{4}}{4}+\frac{(\sin(xb+a}{}\right.}{}}{b}$ |
| norman | $\frac{\frac{4c\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{4}}{b}+\frac{4c\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{8}}{b}+\frac{d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)}{12b^{2}}+\frac{17d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{3}}{36b^{2}}-\frac{d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{5}}{2b^{2}}+\frac{d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{7}}{2b^{2}}-\frac{17d\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{9}}{36b^{2}}-}{}$ |

input
```
int((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/1152*(-54*b*(d*x+c)*cos(2*b*x+2*a)+6*b*(d*x+c)*cos(6*b*x+6*a)+48*c*b+27*
d*sin(2*b*x+2*a)-d*sin(6*b*x+6*a))/b^2
```

### 3.158.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.13

$$\int (c+dx)\cos^{3}(a+bx)\sin^{3}(a+bx)\,dx$$
$$=\frac{12\,(bdx+bc)\cos\left(bx+a\right)^{6}-18\,(bdx+bc)\cos\left(bx+a\right)^{4}+3\,bdx-\left(2\,d\cos\left(bx+a\right)^{5}-2\,d\cos\left(bx+a\right)^{3}-\right.}{72\,b^{2}}$$

input
```
integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/72*(12*(b*d*x + b*c)*cos(b*x + a)^6 - 18*(b*d*x + b*c)*cos(b*x + a)^4 +
3*b*d*x - (2*d*cos(b*x + a)^5 - 2*d*cos(b*x + a)^3 - 3*d*cos(b*x + a))*sin
(b*x + a))/b^2
```

### 3.158.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(80) = 160.

Time = 0.55 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.61

$$\int (c+dx)\cos^3(a+bx)\sin^3(a+bx)\,dx = \begin{cases} -\frac{c\sin^2(a+bx)\cos^4(a+bx)}{4b} - \frac{c\cos^6(a+bx)}{12b} + \frac{dx\sin^6(a+bx)}{24b} + \frac{dx\sin^4(a+bx)\cos^2(a+bx)}{8b} - \frac{dx\sin^2(a+bx)\cos^4(a+bx)}{8b} - \frac{dx\cos^6(\cdots}{24\cdots} \\ \left(cx+\frac{dx^2}{2}\right)\sin^3(a)\cos^3(a) \end{cases}$$

input
```
integrate((d*x+c)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

output
```
Piecewise((-c*sin(a + b*x)**2*cos(a + b*x)**4/(4*b) - c*cos(a + b*x)**6/(1
2*b) + d*x*sin(a + b*x)**6/(24*b) + d*x*sin(a + b*x)**4*cos(a + b*x)**2/(8
*b) - d*x*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) - d*x*cos(a + b*x)**6/(24*
b) + d*sin(a + b*x)**5*cos(a + b*x)/(24*b**2) + d*sin(a + b*x)**3*cos(a +
b*x)**3/(9*b**2) + d*sin(a + b*x)*cos(a + b*x)**5/(24*b**2), Ne(b, 0)), ((
c*x + d*x**2/2)*sin(a)**3*cos(a)**3, True))
```

### 3.158.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.55

$$\int (c+dx)\cos^3(a+bx)\sin^3(a+bx)\,dx = -\frac{96\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)c - \frac{96\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)ad}{b} - \frac{(6(bx+a)\cos(6bx+6a)-54(bx+a)\cos(2b\cdots}{b}}{1152b}$$

input
```
integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/1152*(96*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c - 96*(2*sin(b*x + a)^6
 - 3*sin(b*x + a)^4)*a*d/b - (6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*
cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*d/b)/b
```

### 3.158.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int (c+dx)\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{(bdx+bc)\cos(6\,bx+6\,a)}{192\,b^2} - \frac{3\,(bdx+bc)\cos(2\,bx+2\,a)}{64\,b^2} - \frac{d\sin(6\,bx+6\,a)}{1152\,b^2} + \frac{3\,d\sin(2\,bx+2\,a)}{128\,b^2}$$

input
```
integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/192*(b*d*x + b*c)*cos(6*b*x + 6*a)/b^2 - 3/64*(b*d*x + b*c)*cos(2*b*x +
2*a)/b^2 - 1/1152*d*sin(6*b*x + 6*a)/b^2 + 3/128*d*sin(2*b*x + 2*a)/b^2
```

### 3.158.9 Mupad [B] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09

$$\int (c+dx)\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{\frac{27\,d\,\sin(2\,a+2\,b\,x)}{4} - \frac{d\,\sin(6\,a+6\,b\,x)}{4} - \frac{27\,b\,c\,\cos(2\,a+2\,b\,x)}{2} + \frac{3\,b\,c\,\cos(6\,a+6\,b\,x)}{2} - \frac{27\,b\,d\,x\,\cos(2\,a+2\,b\,x)}{2} + \frac{3\,b\,d\,x\,\cos(6\,a+6\,b\,x)}{2}}{288\,b^2}$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x),x)
```

output
```
((27*d*sin(2*a + 2*b*x))/4 - (d*sin(6*a + 6*b*x))/4 - (27*b*c*cos(2*a + 2*
b*x))/2 + (3*b*c*cos(6*a + 6*b*x))/2 - (27*b*d*x*cos(2*a + 2*b*x))/2 + (3*
b*d*x*cos(6*a + 6*b*x))/2)/(288*b^2)
```

# 3.159 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx$

3.159.1 Optimal result . . . 1265
3.159.2 Mathematica [A] (verified) . . . 1265
3.159.3 Rubi [A] (verified) . . . 1266
3.159.4 Maple [A] (verified) . . . 1267
3.159.5 Fricas [A] (verification not implemented) . . . 1268
3.159.6 Sympy [F] . . . 1268
3.159.7 Maxima [C] (verification not implemented) . . . 1268
3.159.8 Giac [C] (verification not implemented) . . . 1269
3.159.9 Mupad [F(-1)] . . . 1270

## 3.159.1 Optimal result

Integrand size = 24, antiderivative size = 129

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx = -\frac{\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)\sin\left(6a-\frac{6bc}{d}\right)}{32d} + \frac{3\,\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{32d} + \frac{3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{32d} - \frac{\cos\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6bc}{d}+6bx\right)}{32d}$$

output
```
3/32*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d-1/32*cos(6*a-6*b*c/d)*Si(6*b*c/d
+6*b*x)/d-1/32*Ci(6*b*c/d+6*b*x)*sin(6*a-6*b*c/d)/d+3/32*Ci(2*b*c/d+2*b*x)
*sin(2*a-2*b*c/d)/d
```

## 3.159.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.85

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx = -\frac{\text{CosIntegral}\left(\frac{6b(c+dx)}{d}\right)\sin\left(6a-\frac{6bc}{d}\right) - 3\,\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right) - 3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(\right.}{}}{32d}$$

input
```
Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x),x]
```

output
```
-1/32*(CosIntegral[(6*b*(c + d*x))/d]*Sin[6*a - (6*b*c)/d] - 3*CosIntegral
[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] - 3*Cos[2*a - (2*b*c)/d]*SinInteg
ral[(2*b*(c + d*x))/d] + Cos[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c + d*x))/
d])/d
```

### 3.159.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)\cos^3(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{3\sin(2a+2bx)}{32(c+dx)}-\frac{\sin(6a+6bx)}{32(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sin\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{32d}+\frac{3\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{32d}+\frac{3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{32d}-\frac{\cos\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6bc}{d}+6bx\right)}{32d}$$

input
```
Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x),x]
```

output
```
-1/32*(CosIntegral[(6*b*c)/d + 6*b*x]*Sin[6*a - (6*b*c)/d])/d + (3*CosInte
gral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(32*d) + (3*Cos[2*a - (2*b*c
)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(32*d) - (Cos[6*a - (6*b*c)/d]*SinInt
egral[(6*b*c)/d + 6*b*x])/(32*d)
```

### 3.159.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.159.4 Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.38

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{3b\left(-\frac{2\,\mathrm{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2xb+2a+\frac{-2ad+2cb}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d}\right)}{64}-\frac{b\left(-\frac{6\,\mathrm{Si}\left(-6xb-6a-\frac{6(-ad}{d}\right.}{}\right.}{}}{b}$ |
| default | $\frac{\frac{3b\left(-\frac{2\,\mathrm{Si}\left(-2xb-2a-\frac{2(-ad+cb)}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2xb+2a+\frac{-2ad+2cb}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d}\right)}{64}-\frac{b\left(-\frac{6\,\mathrm{Si}\left(-6xb-6a-\frac{6(-ad}{d}\right.}{}\right.}{}}{b}$ |
| risch | $\frac{i e^{-\frac{6i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(6ibx+6ia-\frac{6i(ad-cb)}{d}\right)}{64d}-\frac{3i e^{-\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{64d}+\frac{3i e^{\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(-2ibx-2\right.}{64d}$ |

input
```
int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/b*(3/64*b*(-2*Si(-2*x*b-2*a-2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2
*x*b+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)-1/192*b*(-6*Si(-6*x*b-6*a-
6*(-a*d+b*c)/d)*cos(6*(-a*d+b*c)/d)/d-6*Ci(6*x*b+6*a+6*(-a*d+b*c)/d)*sin(6
*(-a*d+b*c)/d)/d))
```

### 3.159.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.98

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx$$
$$= \frac{3\,\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-\mathrm{Ci}\left(\frac{6\,(bdx+bc)}{d}\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right)-\cos\left(-\frac{6\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{6\,(bdx+bc)}{d}\right)+3\cos}{32\,d}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
1/32*(3*cos_integral(2*(b*d*x + b*c)/d)*sin(-2*(b*c - a*d)/d) - cos_integr
al(6*(b*d*x + b*c)/d)*sin(-6*(b*c - a*d)/d) - cos(-6*(b*c - a*d)/d)*sin_in
tegral(6*(b*d*x + b*c)/d) + 3*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x
+ b*c)/d))/d
```

### 3.159.6 Sympy [F]

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \int \frac{\sin^3(a+bx)\cos^3(a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)**3*cos(a + b*x)**3/(c + d*x), x)
```

### 3.159.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.18

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx =$$
$$-\frac{3\,b\left(-i\,E_1\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_1\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b\left(-i\,E_1\left(\frac{6\,(-i\,bc-i\,(bx}{d}\right.\right.}{}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output
```
-1/64*(3*b*(-I*exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I
*exp_integral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c - a
*d)/d) - b*(-I*exp_integral_e(1, 6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + I
*exp_integral_e(1, -6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-6*(b*c - a
*d)/d) + 3*b*(exp_integral_e(1, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + ex
p_integral_e(1, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c - a*d)
/d) - b*(exp_integral_e(1, 6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) + exp_int
egral_e(1, -6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-6*(b*c - a*d)/d))/
(b*d)
```

### 3.159.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.58 (sec) , antiderivative size = 6046, normalized size of antiderivative = 46.87

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output
```
-1/64*(imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*
b*c/d)^2*tan(b*c/d)^2 - 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a
)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 3*imag_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - imag_part(c
os_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/
d)^2 + 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^
2*tan(b*c/d)^2 - 6*sin_integral(2*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan
(3*b*c/d)^2*tan(b*c/d)^2 - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(
3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) - 6*real_part(cos_integral(-2*b*
x - 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) + 2*real_part(
cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)
^2 + 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3
*b*c/d)*tan(b*c/d)^2 + 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)
^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d)^2 + 6*real_part(cos_integral(-2*b*x -
2*b*c/d))*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_
integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 -
 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d
)^2*tan(b*c/d)^2 + imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan
(a)^2*tan(3*b*c/d)^2 + 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)
^2*tan(a)^2*tan(3*b*c/d)^2 - 3*imag_part(cos_integral(-2*b*x - 2*b*c/d)...
```

## 3.159.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^3}{c+d\,x}\,dx$$

input
```
int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x),x)
```

output
```
int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x), x)
```

# 3.160 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$

3.160.1 Optimal result . . . . . . . . . . 1271
3.160.2 Mathematica [A] (verified) . . . . . . . . . . 1272
3.160.3 Rubi [A] (verified) . . . . . . . . . . 1272
3.160.4 Maple [A] (verified) . . . . . . . . . . 1273
3.160.5 Fricas [A] (verification not implemented) . . . . . . . . . . 1274
3.160.6 Sympy [F] . . . . . . . . . . 1274
3.160.7 Maxima [C] (verification not implemented) . . . . . . . . . . 1275
3.160.8 Giac [C] (verification not implemented) . . . . . . . . . . 1275
3.160.9 Mupad [F(-1)] . . . . . . . . . . 1276

## 3.160.1 Optimal result

Integrand size = 24, antiderivative size = 179

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \frac{3b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{16d^2} - \frac{3b\cos\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{16d^2} - \frac{3\sin(2a+2bx)}{32d(c+dx)} + \frac{\sin(6a+6bx)}{32d(c+dx)} - \frac{3b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{16d^2} + \frac{3b\sin\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6bc}{d}+6bx\right)}{16d^2}$$

output
```
-3/16*b*Ci(6*b*c/d+6*b*x)*cos(6*a-6*b*c/d)/d^2+3/16*b*Ci(2*b*c/d+2*b*x)*co
s(2*a-2*b*c/d)/d^2+3/16*b*Si(6*b*c/d+6*b*x)*sin(6*a-6*b*c/d)/d^2-3/16*b*Si
(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^2-3/32*sin(2*b*x+2*a)/d/(d*x+c)+1/32*si
n(6*b*x+6*a)/d/(d*x+c)
```

### 3.160.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.06

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{6b(c+dx)\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)-6b(c+dx)\cos\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6b(c+dx)}{d}\right)-3d\,c}{}$$

input
```
Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

output
```
(6*b*(c + d*x)*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - 6*b*(
c + d*x)*Cos[6*a - (6*b*c)/d]*CosIntegral[(6*b*(c + d*x))/d] - 3*d*Cos[2*b
*x]*Sin[2*a] + d*Cos[6*b*x]*Sin[6*a] - 3*d*Cos[2*a]*Sin[2*b*x] + d*Cos[6*a
]*Sin[6*b*x] - 6*b*(c + d*x)*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*
x))/d] + 6*b*(c + d*x)*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c + d*x))/d]
)/(32*d^2*(c + d*x))
```

### 3.160.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)\cos^3(a+bx)}{(c+dx)^2}\,dx$$
$$\Big\downarrow \; 4906$$
$$\int \left(\frac{3\sin(2a+2bx)}{32(c+dx)^2}-\frac{\sin(6a+6bx)}{32(c+dx)^2}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{3b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{16d^2}-\frac{3b\cos\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{16d^2}-\frac{3b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{16d^2}+\frac{3b\sin\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6bc}{d}+6bx\right)}{16d^2}-\frac{3\sin(2a+2bx)}{32d(c+dx)}+\frac{\sin(6a+6bx)}{32d(c+dx)}$$

input
```
Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

output
```
(3*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(16*d^2) - (3*b*
Cos[6*a - (6*b*c)/d]*CosIntegral[(6*b*c)/d + 6*b*x])/(16*d^2) - (3*Sin[2*a
 + 2*b*x])/(32*d*(c + d*x)) + Sin[6*a + 6*b*x]/(32*d*(c + d*x)) - (3*b*Sin
[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(16*d^2) + (3*b*Sin[6*a
- (6*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b*x])/(16*d^2)
```

### 3.160.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.160.4 Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.43

| method | result |
|---|---|
| derivativedivides | $-\frac{\frac{b^2\left(-\frac{6\sin(6xb+6a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{36\,\mathrm{Si}\left(-6xb-6a-\frac{6(-ad+cb)}{d}\right)\sin\left(\frac{-6ad+6cb}{d}\right)}{d}+\frac{36\,\mathrm{Ci}\left(6xb+6a+\frac{-6ad+6cb}{d}\right)\cos\left(\frac{-6ad+6cb}{d}\right)}{d}}{d}\right)}{192}}{b}$ |
| default | $-\frac{\frac{b^2\left(-\frac{6\sin(6xb+6a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{36\,\mathrm{Si}\left(-6xb-6a-\frac{6(-ad+cb)}{d}\right)\sin\left(\frac{-6ad+6cb}{d}\right)}{d}+\frac{36\,\mathrm{Ci}\left(6xb+6a+\frac{-6ad+6cb}{d}\right)\cos\left(\frac{-6ad+6cb}{d}\right)}{d}}{d}\right)}{192}}{b}$ |
| risch | $\frac{3b\,\mathrm{e}^{-\frac{6i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(6ibx+6ia-\frac{6i(ad-cb)}{d}\right)}{32d^2}-\frac{3b\,\mathrm{e}^{-\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{32d^2}-\frac{3b\,\mathrm{e}^{\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(-2ibx\right.}{32d^2}$ |

input
```
int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/192*b^2*(-6*sin(6*b*x+6*a)/(-a*d+c*b+d*(b*x+a))/d+6*(-6*Si(-6*x*b-
6*a-6*(-a*d+b*c)/d)*sin(6*(-a*d+b*c)/d)/d+6*Ci(6*x*b+6*a+6*(-a*d+b*c)/d)*c
os(6*(-a*d+b*c)/d)/d)/d)+3/64*b^2*(-2*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/
d+2*(-2*Si(-2*x*b-2*a-2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*x*b+2*a
+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d))
```

### 3.160.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.11

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx =$$

$$-\frac{3\,(bdx+bc)\cos\left(-\frac{6\,(bc-ad)}{d}\right)\operatorname{Ci}\left(\frac{6\,(bdx+bc)}{d}\right)-3\,(bdx+bc)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)-3\,(bdx+bc)}{}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/16*(3*(b*d*x + b*c)*cos(-6*(b*c - a*d)/d)*cos_integral(6*(b*d*x + b*c)/
d) - 3*(b*d*x + b*c)*cos(-2*(b*c - a*d)/d)*cos_integral(2*(b*d*x + b*c)/d)
 - 3*(b*d*x + b*c)*sin(-6*(b*c - a*d)/d)*sin_integral(6*(b*d*x + b*c)/d) +
 3*(b*d*x + b*c)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) - 1
6*(d*cos(b*x + a)^5 - d*cos(b*x + a)^3)*sin(b*x + a))/(d^3*x + c*d^2)
```

### 3.160.6 Sympy [F]

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin^3{(a+bx)}\cos^3{(a+bx)}}{(c+dx)^2}\,dx$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)**3*cos(a + b*x)**3/(c + d*x)**2, x)
```

### 3.160.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.72

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{3\,b^2\left(-i\,E_2\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_2\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^2\left(-i\,E_2\left(\frac{6\,(-i\,bc-i\,(\cdots}{\cdots}\right.\right.}{\cdots}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/64*(3*b^2*(-I*exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 I*exp_integral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c -
 a*d)/d) - b^2*(-I*exp_integral_e(2, 6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d)
 + I*exp_integral_e(2, -6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-6*(b*c
 - a*d)/d) + 3*b^2*(exp_integral_e(2, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d
) + exp_integral_e(2, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c
- a*d)/d) - b^2*(exp_integral_e(2, 6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 exp_integral_e(2, -6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-6*(b*c - a
*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

### 3.160.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.43 (sec) , antiderivative size = 63798, normalized size of antiderivative = 356.41

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/32*(3*b*d*x*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*tan(b
*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 3*b*d*x*real_part(
cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2
*tan(3*b*c/d)^2*tan(b*c/d)^2 - 3*b*d*x*real_part(cos_integral(-2*b*x - 2*b
*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/
d)^2 + 3*b*d*x*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*x)^2*tan(
b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 6*b*d*x*imag_part
(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^
2*tan(3*b*c/d)^2*tan(b*c/d) - 6*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d
) + 12*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(3*b*x)^2*tan(b*x)^2*tan(3
*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) - 6*b*d*x*imag_part(cos_integral(
6*b*x + 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)
*tan(b*c/d)^2 + 6*b*d*x*imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*
x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 - 12*b*d*x*s
in_integral(6*(b*d*x + b*c)/d)*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2
*tan(3*b*c/d)*tan(b*c/d)^2 - 6*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/
d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d)^2
+ 6*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^
2*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d)^2 - 12*b*d*x*sin_integral...
```

### 3.160.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(a+b\,x\right)^3\sin\left(a+b\,x\right)^3}{\left(c+d\,x\right)^2}\,dx$$

input
```
int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^2,x)
```

output
```
int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^2, x)
```

# 3.161 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$

3.161.1 Optimal result . . . 1277
3.161.2 Mathematica [A] (verified) . . . 1278
3.161.3 Rubi [A] (verified) . . . 1278
3.161.4 Maple [A] (verified) . . . 1280
3.161.5 Fricas [A] (verification not implemented) . . . 1280
3.161.6 Sympy [F] . . . 1281
3.161.7 Maxima [C] (verification not implemented) . . . 1281
3.161.8 Giac [C] (verification not implemented) . . . 1282
3.161.9 Mupad [F(-1)] . . . 1283

## 3.161.1 Optimal result

Integrand size = 24, antiderivative size = 235

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = -\frac{3b\cos(2a+2bx)}{32d^2(c+dx)} + \frac{3b\cos(6a+6bx)}{32d^2(c+dx)} + \frac{9b^2\,\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)\sin\left(6a-\frac{6bc}{d}\right)}{16d^3} - \frac{3b^2\,\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{16d^3} - \frac{3\sin(2a+2bx)}{64d(c+dx)^2} + \frac{\sin(6a+6bx)}{64d(c+dx)^2} - \frac{3b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{16d^3} + \frac{9b^2\cos\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6bc}{d}+6bx\right)}{16d^3}$$

output
```
-3/32*b*cos(2*b*x+2*a)/d^2/(d*x+c)+3/32*b*cos(6*b*x+6*a)/d^2/(d*x+c)-3/16*
b^2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^3+9/16*b^2*cos(6*a-6*b*c/d)*Si(6*
b*c/d+6*b*x)/d^3+9/16*b^2*Ci(6*b*c/d+6*b*x)*sin(6*a-6*b*c/d)/d^3-3/16*b^2*
Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^3-3/64*sin(2*b*x+2*a)/d/(d*x+c)^2+1/6
4*sin(6*b*x+6*a)/d/(d*x+c)^2
```

### 3.161.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.02

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{-3d\cos(2bx)(2b(c+dx)\cos(2a)+d\sin(2a))+d\cos(6bx)(6b(c+dx)\cos(6a)+d\sin(6a))+3d(-d\cos(2}{}$$

input
```
Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

output
```
(-3*d*Cos[2*b*x]*(2*b*(c + d*x)*Cos[2*a] + d*Sin[2*a]) + d*Cos[6*b*x]*(6*b
*(c + d*x)*Cos[6*a] + d*Sin[6*a]) + 3*d*(-(d*Cos[2*a]) + 2*b*(c + d*x)*Sin
[2*a])*Sin[2*b*x] + d*(d*Cos[6*a] - 6*b*(c + d*x)*Sin[6*a])*Sin[6*b*x] + 6
*b^2*(c + d*x)^2*(6*CosIntegral[(6*b*(c + d*x))/d]*Sin[6*a - (6*b*c)/d] -
2*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] - 2*Cos[2*a - (2*b*c
)/d]*SinIntegral[(2*b*(c + d*x))/d] + 6*Cos[6*a - (6*b*c)/d]*SinIntegral[(
6*b*(c + d*x))/d]))/(64*d^3*(c + d*x)^2)
```

### 3.161.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)\cos^3(a+bx)}{(c+dx)^3}\,dx$$
$$\Big\downarrow \text{4906}$$
$$\int \left(\frac{3\sin(2a+2bx)}{32(c+dx)^3}-\frac{\sin(6a+6bx)}{32(c+dx)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{9b^2\sin\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{16d^3}-\frac{3b^2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{16d^3}-\frac{3b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{16d^3}+\frac{9b^2\cos\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6bc}{d}+6bx\right)}{16d^3}-\frac{3b\cos(2a+2bx)}{32d^2(c+dx)}+\frac{3b\cos(6a+6bx)}{32d^2(c+dx)}-\frac{3\sin(2a+2bx)}{64d(c+dx)^2}+\frac{\sin(6a+6bx)}{64d(c+dx)^2}$$

input
```
Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

output
```
(-3*b*Cos[2*a + 2*b*x])/(32*d^2*(c + d*x)) + (3*b*Cos[6*a + 6*b*x])/(32*d^
2*(c + d*x)) + (9*b^2*CosIntegral[(6*b*c)/d + 6*b*x]*Sin[6*a - (6*b*c)/d])
/(16*d^3) - (3*b^2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(1
6*d^3) - (3*Sin[2*a + 2*b*x])/(64*d*(c + d*x)^2) + Sin[6*a + 6*b*x]/(64*d*
(c + d*x)^2) - (3*b^2*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])
/(16*d^3) + (9*b^2*Cos[6*a - (6*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b*x])/(1
6*d^3)
```

### 3.161.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.161.4 Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.40

| method | result |
|---|---|
| derivativedivides | $-\frac{b^3\left(-\frac{3\sin(6xb+6a)}{(-ad+cb+d(xb+a))^2 d}+\frac{-\frac{18\cos(6xb+6a)}{(-ad+cb+d(xb+a))d}-\frac{18\left(-\frac{6\,\mathrm{Si}\left(-6xb-6a-\frac{6(-ad+cb)}{d}\right)\cos\left(\frac{-6ad+6cb}{d}\right)}{d}-\frac{6\,\mathrm{Ci}\left(6xb+6a+\frac{-6}{}\right.}{}\right.}{d}}{d}\right.}{192}$ |
| default | $-\frac{b^3\left(-\frac{3\sin(6xb+6a)}{(-ad+cb+d(xb+a))^2 d}+\frac{-\frac{18\cos(6xb+6a)}{(-ad+cb+d(xb+a))d}-\frac{18\left(-\frac{6\,\mathrm{Si}\left(-6xb-6a-\frac{6(-ad+cb)}{d}\right)\cos\left(\frac{-6ad+6cb}{d}\right)}{d}-\frac{6\,\mathrm{Ci}\left(6xb+6a+\frac{-6}{}\right.}{}\right.}{d}}{d}\right.}{192}$ |
| risch | $-\frac{9ib^2 e^{-\frac{6i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(6ibx+6ia-\frac{6i(ad-cb)}{d}\right)}{32d^3}+\frac{3ib^2 e^{-\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{32d^3}-\frac{3ib^2 e^{\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1}{}$ |

input
```
int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/192*b^3*(-3*sin(6*b*x+6*a)/(-a*d+c*b+d*(b*x+a))^2/d+3*(-6*cos(6*b*
x+6*a)/(-a*d+c*b+d*(b*x+a))/d-6*(-6*Si(-6*x*b-6*a-6*(-a*d+b*c)/d)*cos(6*(-
a*d+b*c)/d)/d-6*Ci(6*x*b+6*a+6*(-a*d+b*c)/d)*sin(6*(-a*d+b*c)/d)/d)/d)/d)+
3/64*b^3*(-sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^2/d+(-2*cos(2*b*x+2*a)/(-a*
d+c*b+d*(b*x+a))/d-2*(-2*Si(-2*x*b-2*a-2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)
/d-2*Ci(2*x*b+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d))
```

### 3.161.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.48

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$$

$$=\frac{48\left(bd^2x+bcd\right)\cos\left(bx+a\right)^6-72\left(bd^2x+bcd\right)\cos\left(bx+a\right)^4+24\left(bd^2x+bcd\right)\cos\left(bx+a\right)^2-3\left(b^2d^2x^2+\right.}{}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/16*(48*(b*d^2*x + b*c*d)*cos(b*x + a)^6 - 72*(b*d^2*x + b*c*d)*cos(b*x +
 a)^4 + 24*(b*d^2*x + b*c*d)*cos(b*x + a)^2 - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*cos_integral(2*(b*d*x + b*c)/d)*sin(-2*(b*c - a*d)/d) + 9*(b^2
*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(6*(b*d*x + b*c)/d)*sin(-6*(
b*c - a*d)/d) + 9*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-6*(b*c - a*d)
/d)*sin_integral(6*(b*d*x + b*c)/d) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) + 8*(d^2*cos(b*x
 + a)^5 - d^2*cos(b*x + a)^3)*sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3
)
```

## 3.161.6 Sympy [F]

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin^3{(a+bx)}\cos^3{(a+bx)}}{(c+dx)^3}\,dx$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c)**3,x)
```

output
```
Integral(sin(a + b*x)**3*cos(a + b*x)**3/(c + d*x)**3, x)
```

## 3.161.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.46

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx =$$
$$-\frac{3\,b^3\left(-i\,E_3\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_3\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^3\left(-i\,E_3\left(\frac{6\,(-i\,bc-i\,\cdots}{}\right.\right.}{}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/64*(3*b^3*(-I*exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 I*exp_integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c -
 a*d)/d) - b^3*(-I*exp_integral_e(3, 6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d)
 + I*exp_integral_e(3, -6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-6*(b*c
 - a*d)/d) + 3*b^3*(exp_integral_e(3, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d
) + exp_integral_e(3, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c
- a*d)/d) - b^3*(exp_integral_e(3, 6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 exp_integral_e(3, -6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-6*(b*c - a
*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2
 - a*d^3)*(b*x + a))*b)
```

### 3.161.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.47 (sec) , antiderivative size = 111694, normalized size of antiderivative = 475.29

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/32*(9*b^2*d^2*x^2*imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*
tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 3*b^2*d^2*x^2
*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)
^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 3*b^2*d^2*x^2*imag_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*
b*c/d)^2*tan(b*c/d)^2 - 9*b^2*d^2*x^2*imag_part(cos_integral(-6*b*x - 6*b*
c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d
)^2 + 18*b^2*d^2*x^2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*b*x)^2*tan(b*x)
^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 6*b^2*d^2*x^2*sin_int
egral(2*(b*d*x + b*c)/d)*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3
*b*c/d)^2*tan(b*c/d)^2 - 6*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*
c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d
) - 6*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*t
an(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) + 18*b^2*d^2*x^2*r
eal_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2
*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 + 18*b^2*d^2*x^2*real_part(cos_integra
l(-6*b*x - 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c
/d)*tan(b*c/d)^2 + 6*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*
tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d)^2 + 6*
b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(...
```

## 3.161.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx=\int \frac{{\cos\left(a+b\,x\right)}^3\,{\sin\left(a+b\,x\right)}^3}{{\left(c+d\,x\right)}^3}\,dx$$

input
```
int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^3,x)
```

output
```
int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^3, x)
```

# 3.162 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$

3.162.1 Optimal result . . . 1284
3.162.2 Mathematica [A] (verified) . . . 1285
3.162.3 Rubi [A] (verified) . . . 1285
3.162.4 Maple [A] (verified) . . . 1287
3.162.5 Fricas [A] (verification not implemented) . . . 1288
3.162.6 Sympy [F(-1)] . . . 1288
3.162.7 Maxima [C] (verification not implemented) . . . 1289
3.162.8 Giac [C] (verification not implemented) . . . 1289
3.162.9 Mupad [F(-1)] . . . 1290

## 3.162.1 Optimal result

Integrand size = 24, antiderivative size = 287

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = -\frac{b\cos(2a+2bx)}{32d^2(c+dx)^2}+\frac{b\cos(6a+6bx)}{32d^2(c+dx)^2} - \frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{8d^4} + \frac{9b^3\cos\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{8d^4} - \frac{\sin(2a+2bx)}{32d(c+dx)^3}+\frac{b^2\sin(2a+2bx)}{16d^3(c+dx)}+\frac{\sin(6a+6bx)}{96d(c+dx)^3} - \frac{3b^2\sin(6a+6bx)}{16d^3(c+dx)}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{8d^4} - \frac{9b^3\sin\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6bc}{d}+6bx\right)}{8d^4}$$

output
```
9/8*b^3*Ci(6*b*c/d+6*b*x)*cos(6*a-6*b*c/d)/d^4-1/8*b^3*Ci(2*b*c/d+2*b*x)*c
os(2*a-2*b*c/d)/d^4-1/32*b*cos(2*b*x+2*a)/d^2/(d*x+c)^2+1/32*b*cos(6*b*x+6
*a)/d^2/(d*x+c)^2-9/8*b^3*Si(6*b*c/d+6*b*x)*sin(6*a-6*b*c/d)/d^4+1/8*b^3*S
i(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^4-1/32*sin(2*b*x+2*a)/d/(d*x+c)^3+1/16
*b^2*sin(2*b*x+2*a)/d^3/(d*x+c)+1/96*sin(6*b*x+6*a)/d/(d*x+c)^3-3/16*b^2*s
in(6*b*x+6*a)/d^3/(d*x+c)
```

### 3.162.2 Mathematica [A] (verified)

Time = 5.30 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.93

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$$
$$= \frac{-3bcd^2\cos(2(a+bx)) - 3bd^3x\cos(2(a+bx)) + 3bcd^2\cos(6(a+bx)) + 3bd^3x\cos(6(a+bx)) - 12b^3(c+}{}$$

input
```
Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

output
```
(-3*b*c*d^2*Cos[2*(a + b*x)] - 3*b*d^3*x*Cos[2*(a + b*x)] + 3*b*c*d^2*Cos[
6*(a + b*x)] + 3*b*d^3*x*Cos[6*(a + b*x)] - 12*b^3*(c + d*x)^3*Cos[2*a - (
2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + 108*b^3*(c + d*x)^3*Cos[6*a - (
6*b*c)/d]*CosIntegral[(6*b*(c + d*x))/d] + 6*b^2*c^2*d*Sin[2*(a + b*x)] -
3*d^3*Sin[2*(a + b*x)] + 12*b^2*c*d^2*x*Sin[2*(a + b*x)] + 6*b^2*d^3*x^2*S
in[2*(a + b*x)] - 18*b^2*c^2*d*Sin[6*(a + b*x)] + d^3*Sin[6*(a + b*x)] - 3
6*b^2*c*d^2*x*Sin[6*(a + b*x)] - 18*b^2*d^3*x^2*Sin[6*(a + b*x)] + 12*b^3*
c^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 36*b^3*c^2*d*x*S
in[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 36*b^3*c*d^2*x^2*Sin[
2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 12*b^3*d^3*x^3*Sin[2*a -
 (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] - 108*b^3*c^3*Sin[6*a - (6*b*c)
/d]*SinIntegral[(6*b*(c + d*x))/d] - 324*b^3*c^2*d*x*Sin[6*a - (6*b*c)/d]*
SinIntegral[(6*b*(c + d*x))/d] - 324*b^3*c*d^2*x^2*Sin[6*a - (6*b*c)/d]*Si
nIntegral[(6*b*(c + d*x))/d] - 108*b^3*d^3*x^3*Sin[6*a - (6*b*c)/d]*SinInt
egral[(6*b*(c + d*x))/d])/(96*d^4*(c + d*x)^3)
```

### 3.162.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^3(a+bx)\cos^3(a+bx)}{(c+dx)^4}\,dx$$
$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{3\sin(2a+2bx)}{32(c+dx)^4}-\frac{\sin(6a+6bx)}{32(c+dx)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{8d^4}+\frac{9b^3\cos\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{8d^4}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{8d^4}-\frac{9b^3\sin\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\frac{6bc}{d}+6bx\right)}{8d^4}+\frac{b^2\sin(2a+2bx)}{16d^3(c+dx)}-\frac{3b^2\sin(6a+6bx)}{16d^3(c+dx)}-\frac{b\cos(2a+2bx)}{32d^2(c+dx)^2}+\frac{b\cos(6a+6bx)}{32d^2(c+dx)^2}-\frac{\sin(2a+2bx)}{32d(c+dx)^3}+\frac{\sin(6a+6bx)}{96d(c+dx)^3}$$

input
```
Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

output
```
-1/32*(b*Cos[2*a + 2*b*x])/(d^2*(c + d*x)^2) + (b*Cos[6*a + 6*b*x])/(32*d^
2*(c + d*x)^2) - (b^3*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])
/(8*d^4) + (9*b^3*Cos[6*a - (6*b*c)/d]*CosIntegral[(6*b*c)/d + 6*b*x])/(8*
d^4) - Sin[2*a + 2*b*x]/(32*d*(c + d*x)^3) + (b^2*Sin[2*a + 2*b*x])/(16*d^
3*(c + d*x)) + Sin[6*a + 6*b*x]/(96*d*(c + d*x)^3) - (3*b^2*Sin[6*a + 6*b*
x])/(16*d^3*(c + d*x)) + (b^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d +
 2*b*x])/(8*d^4) - (9*b^3*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b
*x])/(8*d^4)
```

### 3.162.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.162.4 Maple [A] (verified)

Time = 3.59 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.41

| method | result |
|---|---|
| derivativedivides | $-\frac{b^4\left(-\frac{2\sin(6xb+6a)}{(-ad+cb+d(xb+a))^3 d}+\frac{-\frac{6\cos(6xb+6a)}{(-ad+cb+d(xb+a))^2 d}-\frac{6\left(-\frac{6\sin(6xb+6a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{36\,\mathrm{Si}\left(-6xb-6a-\frac{6(-ad+cb)}{d}\right)\sin\left(\frac{-6ad}{d}\cdots\right)}{d}\cdots}{d}\right)}{d}}{d}\right)}{192}\cdots$ |
| default | $-\frac{b^4\left(-\frac{2\sin(6xb+6a)}{(-ad+cb+d(xb+a))^3 d}+\frac{-\frac{6\cos(6xb+6a)}{(-ad+cb+d(xb+a))^2 d}-\frac{6\left(-\frac{6\sin(6xb+6a)}{(-ad+cb+d(xb+a))d}+\frac{-\frac{36\,\mathrm{Si}\left(-6xb-6a-\frac{6(-ad+cb)}{d}\right)\sin\left(\frac{-6ad}{d}\cdots\right)}{d}\cdots}{d}\right)}{d}}{d}\right)}{192}\cdots$ |
| risch | $-\frac{9b^3 e^{-\frac{6i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(6ibx+6ia-\frac{6i(ad-cb)}{d}\right)}{16d^4}+\frac{b^3 e^{-\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{16d^4}+\frac{b^3 e^{\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(-2\cdots\right.}{16\cdots}$ |

input
```
int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/192*b^4*(-2*sin(6*b*x+6*a)/(-a*d+c*b+d*(b*x+a))^3/d+2*(-3*cos(6*b*
x+6*a)/(-a*d+c*b+d*(b*x+a))^2/d-3*(-6*sin(6*b*x+6*a)/(-a*d+c*b+d*(b*x+a))/
d+6*(-6*Si(-6*x*b-6*a-6*(-a*d+b*c)/d)*sin(6*(-a*d+b*c)/d)/d+6*Ci(6*x*b+6*a
+6*(-a*d+b*c)/d)*cos(6*(-a*d+b*c)/d)/d)/d)/d)/d)+3/64*b^4*(-2/3*sin(2*b*x+
2*a)/(-a*d+c*b+d*(b*x+a))^3/d+2/3*(-cos(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^2/
d-(-2*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d+2*(-2*Si(-2*x*b-2*a-2*(-a*d+b*
c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*x*b+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c
)/d)/d)/d)/d)/d))
```

### 3.162.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.83

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$$
$$= \frac{24\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^6 - 36\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^4 + 12\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^2 + 27\left(b^3d^3\right.}{}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/24*(24*(b*d^3*x + b*c*d^2)*cos(b*x + a)^6 - 36*(b*d^3*x + b*c*d^2)*cos(b
*x + a)^4 + 12*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2 + 27*(b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-6*(b*c - a*d)/d)*cos_integral(
6*(b*d*x + b*c)/d) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^
3*c^3)*cos(-2*(b*c - a*d)/d)*cos_integral(2*(b*d*x + b*c)/d) - 27*(b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-6*(b*c - a*d)/d)*si
n_integral(6*(b*d*x + b*c)/d) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c
^2*d*x + b^3*c^3)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) -
8*((18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3)*cos(b*x + a)^5 -
 (18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3)*cos(b*x + a)^3 + 3
*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a))*sin(b*x + a))/(d^
7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

### 3.162.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \text{Timed out}$$

input
```
integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c)**4,x)
```

output
```
Timed out
```

### 3.162.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.37

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx =$$
$$-\frac{3\,b^4\left(-i\,E_4\left(\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)+i\,E_4\left(-\frac{2\,(-i\,bc-i\,(bx+a)d+i\,ad)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-b^4\left(-i\,E_4\left(\frac{6\,(-i\,bc-i\,(}{}\right.\right.}{64}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/64*(3*b^4*(-I*exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 I*exp_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-2*(b*c -
 a*d)/d) - b^4*(-I*exp_integral_e(4, 6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d)
 + I*exp_integral_e(4, -6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*cos(-6*(b*c
 - a*d)/d) + 3*b^4*(exp_integral_e(4, 2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d
) + exp_integral_e(4, -2*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-2*(b*c
- a*d)/d) - b^4*(exp_integral_e(4, 6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d) +
 exp_integral_e(4, -6*(-I*b*c - I*(b*x + a)*d + I*a*d)/d))*sin(-6*(b*c - a
*d)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 -
a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 +
 a^2*d^4)*(b*x + a))*b)
```

### 3.162.8 Giac [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.84 (sec) , antiderivative size = 157526, normalized size of antiderivative = 548.87

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")
```

output
```
1/48*(27*b^3*d^3*x^3*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*x)^2
*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 3*b^3*d^3*x^
3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a
)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 3*b^3*d^3*x^3*real_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3
*b*c/d)^2*tan(b*c/d)^2 + 27*b^3*d^3*x^3*real_part(cos_integral(-6*b*x - 6*
b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c
/d)^2 + 6*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*x)^
2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) - 6*b^3*d^3*x^3
*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*tan(3*a
)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) + 12*b^3*d^3*x^3*sin_integral(2*(b*
d*x + b*c)/d)*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*t
an(b*c/d) - 54*b^3*d^3*x^3*imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*
b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 + 54*b^3*d
^3*x^3*imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*x)^2*tan(b*x)^2*t
an(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 - 108*b^3*d^3*x^3*sin_integra
l(6*(b*d*x + b*c)/d)*tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)^2*tan(3*b*c
/d)*tan(b*c/d)^2 - 6*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*
tan(3*b*x)^2*tan(b*x)^2*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d)^2 + 6*
b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*x)^2*tan(...
```

### 3.162.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cos\left(a+b\,x\right)^3\sin\left(a+b\,x\right)^3}{\left(c+d\,x\right)^4}\,dx$$

input
```
int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^4,x)
```

output
```
int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^4, x)
```

# 3.163 $\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx$

3.163.1 Optimal result . . . . . 1291
3.163.2 Mathematica [N/A] . . . . . 1291
3.163.3 Rubi [N/A] . . . . . 1292
3.163.4 Maple [N/A] (verified) . . . . . 1294
3.163.5 Fricas [N/A] . . . . . 1295
3.163.6 Sympy [N/A] . . . . . 1295
3.163.7 Maxima [N/A] . . . . . 1295
3.163.8 Giac [N/A] . . . . . 1296
3.163.9 Mupad [N/A] . . . . . 1296

## 3.163.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx = \frac{2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} + \frac{2^{-3-m}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2ib(c+dx)}{d}\right)}{b} + \operatorname{Int}((c+dx)^m\cot(a+bx),x)$$

output
```
2^(-3-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*b
*(d*x+c)/d)^m)+2^(-3-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a-
b*c/d))/((I*b*(d*x+c)/d)^m)+Unintegrable((d*x+c)^m*cot(b*x+a),x)
```

## 3.163.2 Mathematica [N/A]

Not integrable

Time = 12.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx = \int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Cos[a + b*x]^2*Cot[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Cos[a + b*x]^2*Cot[a + b*x], x]
```

### 3.163.3 Rubi [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4908, 3042, 25, 4222, 4906, 27, 3042, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos^2(a+bx)\cot(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{4908}$$

$$\int (c+dx)^m\cot(a+bx)dx-\int (c+dx)^m\cos(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int -(c+dx)^m\tan\left(a+bx+\frac{\pi}{2}\right)dx-\int (c+dx)^m\cos(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)^m\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx-\int (c+dx)^m\cos(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{4222}$$

$$-\int (c+dx)^m\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx-\int (c+dx)^m\cos(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{4906}$$

$$-\int \frac{1}{2}(c+dx)^m\sin(2a+2bx)dx-\int (c+dx)^m\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}\int (c+dx)^m\sin(2a+2bx)dx-\int (c+dx)^m\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}\int (c+dx)^m \sin(2a+2bx)dx - \int (c+dx)^m \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

$$\Big\downarrow \; 3789$$

$$-\int (c+dx)^m \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx + \frac{1}{2}\left(\frac{1}{2}i\int e^{2i(a+bx)}(c+dx)^m dx - \frac{1}{2}i\int e^{-2i(a+bx)}(c+dx)^m dx\right)$$

$$\Big\downarrow \; 2612$$

$$-\int (c+dx)^m \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx + \frac{1}{2}\left(\frac{2^{-m-2}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b} + \frac{2^{-m-2}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma}{b}\right.$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]^2*Cot[a + b*x],x]
```

output
```
$Aborted
```

### 3.163.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.163. $\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx$

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

### 3.163.4 Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \cos(xb+a)^2 \cot(xb+a)\,dx$$

input
```
int((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x)
```

output
```
int((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x)
```

### 3.163.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx = \int (dx+c)^m \cos\left(bx+a\right)^2 \cot\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*cos(b*x + a)^2*cot(b*x + a), x)
```

### 3.163.6 Sympy [N/A]

Not integrable

Time = 24.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx = \int (c+dx)^m \cos^2\left(a+bx\right)\cot\left(a+bx\right)dx$$

input
```
integrate((d*x+c)**m*cos(b*x+a)**2*cot(b*x+a),x)
```

output
```
Integral((c + d*x)**m*cos(a + b*x)**2*cot(a + b*x), x)
```

### 3.163.7 Maxima [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx = \int (dx+c)^m \cos\left(bx+a\right)^2 \cot\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^2*cot(b*x + a), x)
```

### 3.163.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cos^2(a+bx) \cot(a+bx) \, dx = \int (dx+c)^m \cos(bx+a)^2 \cot(bx+a) \, dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)^2*cot(b*x + a), x)
```

### 3.163.9 Mupad [N/A]

Not integrable

Time = 25.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cos^2(a+bx) \cot(a+bx) \, dx = \int \cos(a+b\,x)^2 \cot(a+b\,x) \, (c+d\,x)^m \, dx$$

input
```
int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^m, x)
```

# 3.164 $\int (c+dx)^4 \cos^2(a+bx) \cot(a+bx)\,dx$

3.164.1 Optimal result . . . . . . . . . . 1297
3.164.2 Mathematica [B] (verified) . . . . . . . . . . 1298
3.164.3 Rubi [A] (verified) . . . . . . . . . . 1298
3.164.4 Maple [B] (verified) . . . . . . . . . . 1305
3.164.5 Fricas [B] (verification not implemented) . . . . . . . . . . 1306
3.164.6 Sympy [F] . . . . . . . . . . 1307
3.164.7 Maxima [B] (verification not implemented) . . . . . . . . . . 1308
3.164.8 Giac [F] . . . . . . . . . . 1308
3.164.9 Mupad [F(-1)] . . . . . . . . . . 1309

## 3.164.1 Optimal result

Integrand size = 22, antiderivative size = 307

$$\begin{aligned}
&\int (c+dx)^4 \cos^2(a+bx)\cot(a+bx)\,dx \\
&= -\frac{3cd^3x}{2b^3} - \frac{3d^4x^2}{4b^3} + \frac{(c+dx)^4}{4b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} \\
&\quad - \frac{2id(c+dx)^3\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2} + \frac{3d^2(c+dx)^2\operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3} \\
&\quad + \frac{3id^3(c+dx)\operatorname{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4} - \frac{3d^4\operatorname{PolyLog}\left(5,e^{2i(a+bx)}\right)}{2b^5} \\
&\quad + \frac{3d^3(c+dx)\cos(a+bx)\sin(a+bx)}{2b^4} - \frac{d(c+dx)^3\cos(a+bx)\sin(a+bx)}{b^2} \\
&\quad - \frac{3d^4\sin^2(a+bx)}{4b^5} + \frac{3d^2(c+dx)^2\sin^2(a+bx)}{2b^3} - \frac{(c+dx)^4\sin^2(a+bx)}{2b}
\end{aligned}$$

output
```
-3/2*c*d^3*x/b^3-3/4*d^4*x^2/b^3+1/4*(d*x+c)^4/b-1/5*I*(d*x+c)^5/d+(d*x+c)
^4*ln(1-exp(2*I*(b*x+a)))/b-2*I*d*(d*x+c)^3*polylog(2,exp(2*I*(b*x+a)))/b^
2+3*d^2*(d*x+c)^2*polylog(3,exp(2*I*(b*x+a)))/b^3+3*I*d^3*(d*x+c)*polylog(
4,exp(2*I*(b*x+a)))/b^4-3/2*d^4*polylog(5,exp(2*I*(b*x+a)))/b^5+3/2*d^3*(d
*x+c)*cos(b*x+a)*sin(b*x+a)/b^4-d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2-3/4*
d^4*sin(b*x+a)^2/b^5+3/2*d^2*(d*x+c)^2*sin(b*x+a)^2/b^3-1/2*(d*x+c)^4*sin(
b*x+a)^2/b
```

### 3.164.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2918 vs. $2(307) = 614$.

Time = 6.45 (sec) , antiderivative size = 2918, normalized size of antiderivative = 9.50

$$\int (c+dx)^4 \cos^2(a+bx)\cot(a+bx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x]^2*Cot[a + b*x],x]
```

output
```
-((c^2*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2
*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2
*Log[1 + E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, -E^((-I
)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, E^((-I)*(a + b*x))] +
(6*I)*(1 - E^((-2*I)*a))*PolyLog[3, -E^((-I)*(a + b*x))] + (6*I)*(1 - E^((
-2*I)*a))*PolyLog[3, E^((-I)*(a + b*x))]))/b^3) - (c*d^3*E^(I*a)*Csc[a]*((
b^4*x^4)/E^((2*I)*a) + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a
 + b*x))] + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] -
 6*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - 6*b^2*(1 -
 E^((-2*I)*a))*x^2*PolyLog[2, E^((-I)*(a + b*x))] + (12*I)*b*(1 - E^((-2*I
)*a))*x*PolyLog[3, -E^((-I)*(a + b*x))] + (12*I)*b*(1 - E^((-2*I)*a))*x*Po
lyLog[3, E^((-I)*(a + b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, -E^((-I)*(
a + b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, E^((-I)*(a + b*x))]))/b^4 -
(d^4*E^(I*a)*Csc[a]*((2*b^5*x^5)/E^((2*I)*a) + (5*I)*b^4*(1 - E^((-2*I)*a)
)*x^4*Log[1 - E^((-I)*(a + b*x))] + (5*I)*b^4*(1 - E^((-2*I)*a))*x^4*Log[1
 + E^((-I)*(a + b*x))] - 20*b^3*(1 - E^((-2*I)*a))*x^3*PolyLog[2, -E^((-I)
*(a + b*x))] - 20*b^3*(1 - E^((-2*I)*a))*x^3*PolyLog[2, E^((-I)*(a + b*x))
] + (60*I)*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[3, -E^((-I)*(a + b*x))] + (6
0*I)*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[3, E^((-I)*(a + b*x))] + 120*b*(1
- E^((-2*I)*a))*x*PolyLog[4, -E^((-I)*(a + b*x))] + 120*b*(1 - E^((-2*I...
```

### 3.164.3 Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.21, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {4908, 3042, 25, 4202, 2620, 3011, 4904, 3042, 3792, 17, 3042, 3791, 17, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \cos^2(a+bx)\cot(a+bx)\,dx$$

$$\Big\downarrow \text{4908}$$

$$\int (c+dx)^4 \cot(a+bx)dx-\int (c+dx)^4 \cos(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int -(c+dx)^4 \tan\left(a+bx+\frac{\pi}{2}\right)dx-\int (c+dx)^4 \cos(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)^4 \tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx-\int (c+dx)^4 \cos(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{4202}$$

$$2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)^4}{1+e^{i(2a+2bx+\pi)}}dx-\int (c+dx)^4 \cos(a+bx)\sin(a+bx)dx-\frac{i(c+dx)^5}{5d}$$

$$\Big\downarrow \text{2620}$$

$$2i\left(\frac{2id\int (c+dx)^3 \log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{b}-\frac{i(c+dx)^4 \log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\int (c+dx)^4 \cos(a+bx)\sin(a+bx)dx-\frac{i(c+dx)^5}{5d}$$

$$\Big\downarrow \text{3011}$$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3 \text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int (c+dx)^2 \text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4 \log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$
$$\int (c+dx)^4 \cos(a+bx)\sin(a+bx)dx-\frac{i(c+dx)^5}{5d}$$

$$\Big\downarrow \text{4904}$$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3 \text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int (c+dx)^2 \text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4 \log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$
$$\frac{2d\int (c+dx)^3 \sin^2(a+bx)dx}{b}-\frac{(c+dx)^4 \sin^2(a+bx)}{2b}-\frac{i(c+dx)^5}{5d}$$

$$\Big\downarrow \text{3042}$$

3.164. $\int (c+dx)^4 \cos^2(a+bx)\cot(a+bx)\,dx$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3 \operatorname{PolyLog}(2,-e^{i(2a+2bx+\pi)})}{2b}-\frac{3id\int (c+dx)^2 \operatorname{PolyLog}(2,-e^{i(2a+2bx+\pi)})dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$

$$\frac{2d\int (c+dx)^3\sin(a+bx)^2dx}{b}-\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{i(c+dx)^5}{5d}$$

$$\Big\downarrow \text{3792}$$

$$\frac{2d\left(-\frac{3d^2\int (c+dx)\sin^2(a+bx)dx}{2b^2}+\frac{1}{2}\int (c+dx)^3dx+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}\right)}{b}+$$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3 \operatorname{PolyLog}(2,-e^{i(2a+2bx+\pi)})}{2b}-\frac{3id\int (c+dx)^2 \operatorname{PolyLog}(2,-e^{i(2a+2bx+\pi)})dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$

$$\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{i(c+dx)^5}{5d}$$

$$\Big\downarrow \text{17}$$

$$\frac{2d\left(-\frac{3d^2\int (c+dx)\sin^2(a+bx)dx}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)}{b}+$$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3 \operatorname{PolyLog}(2,-e^{i(2a+2bx+\pi)})}{2b}-\frac{3id\int (c+dx)^2 \operatorname{PolyLog}(2,-e^{i(2a+2bx+\pi)})dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$

$$\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{i(c+dx)^5}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\left(-\frac{3d^2\int (c+dx)\sin(a+bx)^2dx}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)}{b}+$$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3 \operatorname{PolyLog}(2,-e^{i(2a+2bx+\pi)})}{2b}-\frac{3id\int (c+dx)^2 \operatorname{PolyLog}(2,-e^{i(2a+2bx+\pi)})dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$

$$\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{i(c+dx)^5}{5d}$$

$$\Big\downarrow \text{3791}$$

3.164. $\int (c+dx)^4\cos^2(a+bx)\cot(a+bx)\,dx$

$$\frac{2d\left(-\frac{3d^2\left(\frac{1}{2}\int(c+dx)dx+\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)}{b}$$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$

$$\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{i(c+dx)^5}{5d}$$

$$\Big\downarrow 17$$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$

$$\frac{2d\left(-\frac{3d^2\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)}{b}-$$

$$\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{i(c+dx)^5}{5d}$$

$$\Big\downarrow 7163$$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\left(\frac{id\int(c+dx)\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)dx}{b}-\frac{i(c+dx)^2\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{2b}\right)}{b}-\frac{i(\ldots}{}\right.$$

$$\frac{2d\left(-\frac{3d^2\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)}{b}-$$

$$\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{i(c+dx)^5}{5d}$$

$$\Big\downarrow 7163$$

3.164. $\int (c+dx)^4\cos^2(a+bx)\cot(a+bx)\,dx$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\left(\frac{id\left(\frac{id\int\operatorname{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}-\frac{i(c+dx)^2\operatorname{Pol}\cdots}{\cdots}\right)}{2b}\cdots\right)}{b}\cdots\right.$$

$$\frac{2d\left(-\frac{3d^2\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)}{b}-$$

$$\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{i(c+dx)^5}{5d}$$

$\big\downarrow$ 2720

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\left(\frac{id\left(\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,-e^{i(2\cdots}\right)}{2b}\cdots\right)}{b}\cdots\right)}{2b}\cdots\right)}{b}\cdots\right.$$

$$\frac{2d\left(-\frac{3d^2\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)}{b}-$$

$$\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{i(c+dx)^5}{5d}$$

$\big\downarrow$ 7143

## 3.164. $\int(c+dx)^4\cos^2(a+bx)\cot(a+bx)\,dx$

$$\frac{2d\left(-\frac{3d^2\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{2b^2}+\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}-\frac{(c+dx)^3\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^4}{8d}\right)}{b}+$$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}(2,-e^{i(2a+2bx+\pi)})}{2b}-\frac{3id\left(\frac{id\left(\frac{d\operatorname{PolyLog}(5,-e^{i(2a+2bx+\pi)})}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}(4,-e^{i(2a+2bx+\pi)})}{2b}\right)}{b}-\frac{i(c+dx)^2\operatorname{PolyLog}}{}\right.}{2b}\right.}{b}\right.$$

$$\frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{i(c+dx)^5}{5d}$$

input
```
Int[(c + d*x)^4*Cos[a + b*x]^2*Cot[a + b*x],x]
```

output
```
((-1/5*I)*(c + d*x)^5)/d + (2*I)*(((-1/2*I)*(c + d*x)^4*Log[1 + E^(I*(2*a
+ Pi + 2*b*x))])/b + ((2*I)*d*(((I/2)*(c + d*x)^3*PolyLog[2, -E^(I*(2*a +
Pi + 2*b*x))])/b - (((3*I)/2)*d*(((-1/2*I)*(c + d*x)^2*PolyLog[3, -E^(I*(2
*a + Pi + 2*b*x))])/b + (I*d*(((-1/2*I)*(c + d*x)*PolyLog[4, -E^(I*(2*a +
Pi + 2*b*x))])/b + (d*PolyLog[5, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2)))/b))
/b))/b) - ((c + d*x)^4*Sin[a + b*x]^2)/(2*b) + (2*d*((c + d*x)^4/(8*d) - (
(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (3*d*(c + d*x)^2*Sin[a + b*
x]^2)/(4*b^2) - (3*d^2*((c + d*x)^2/(4*d) - ((c + d*x)*Cos[a + b*x]*Sin[a
+ b*x])/(2*b) + (d*Sin[a + b*x]^2)/(4*b^2)))/(2*b^2)))/b
```

### 3.164.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

## 3.164. $\int (c+dx)^4 \cos^2(a+bx) \cot(a+bx)\,dx$

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

### 3.164.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1334 vs. $2(276) = 552$.

Time = 3.24 (sec) , antiderivative size = 1335, normalized size of antiderivative = 4.35

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1335 |

input
```
int((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^4*d*(2*b^2*d^3*x^3+6*b^2*c*d^2*x^2+6*b^2*c^2*d*x+2*b^2*c^3-3*d^3*x-
3*c*d^2)*sin(2*b*x+2*a)+I*c^4*x+1/5*I/d*c^5-12*I/b^2*d^2*c^2*polylog(2,-ex
p(I*(b*x+a)))*x-12*I/b^2*d^2*c^2*polylog(2,exp(I*(b*x+a)))*x-8*I/b*d*c^3*x
*a+12*I/b^2*d^2*c^2*a^2*x-1/5*I*d^4*x^5-8*I/b^3*d^3*c*a^3*x+12/b^3*d^2*c^2
*polylog(3,-exp(I*(b*x+a)))-1/b^5*d^4*ln(1-exp(I*(b*x+a)))*a^4+12/b^3*d^4*
polylog(3,exp(I*(b*x+a)))*x^2+12/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2+12
/b^3*d^2*c^2*polylog(3,exp(I*(b*x+a)))-2/b^5*d^4*a^4*ln(exp(I*(b*x+a)))+1/
b^5*d^4*a^4*ln(exp(I*(b*x+a))-1)+1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4+1/b*d^4*
ln(exp(I*(b*x+a))+1)*x^4+8/5*I/b^5*d^4*a^5-24*d^4*polylog(5,-exp(I*(b*x+a)
))/b^5-24*d^4*polylog(5,exp(I*(b*x+a)))/b^5-I*d^3*c*x^4-2*I*d^2*c^2*x^3-2*
I*d*c^3*x^2+1/8*(2*b^4*d^4*x^4+8*b^4*c*d^3*x^3+12*b^4*c^2*d^2*x^2+8*b^4*c^
3*d*x+2*b^4*c^4-6*b^2*d^4*x^2-12*b^2*c*d^3*x-6*b^2*c^2*d^2+3*d^4)/b^5*cos(
2*b*x+2*a)+1/b*c^4*ln(exp(I*(b*x+a))+1)-2/b*c^4*ln(exp(I*(b*x+a)))+1/b*c^4
*ln(exp(I*(b*x+a))-1)-12*I/b^2*d^3*c*polylog(2,exp(I*(b*x+a)))*x^2-12*I/b^
2*d^3*c*polylog(2,-exp(I*(b*x+a)))*x^2+8/b^2*c^3*d*a*ln(exp(I*(b*x+a)))-4/
b^2*c^3*d*a*ln(exp(I*(b*x+a))-1)-4/b^4*c*d^3*a^3*ln(exp(I*(b*x+a))-1)+24*I
/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x+24*I/b^4*d^4*polylog(4,-exp(I*(b*x+a)
))*x+2*I/b^4*d^4*a^4*x+24*I/b^4*d^3*c*polylog(4,exp(I*(b*x+a)))+24*I/b^4*d
^3*c*polylog(4,-exp(I*(b*x+a)))-4*I/b^2*d*c^3*a^2-4*I/b^2*d^4*polylog(2,ex
p(I*(b*x+a)))*x^3-4*I/b^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3+8*I/b^3*d^...
```

### 3.164.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1453 vs. $2(272) = 544$.

Time = 0.35 (sec) , antiderivative size = 1453, normalized size of antiderivative = 4.73

$$\int (c+dx)^4 \cos^2(a+bx)\cot(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 48*d^4*polylog(5, cos(b*x + a) + I*s
in(b*x + a)) + 48*d^4*polylog(5, cos(b*x + a) - I*sin(b*x + a)) + 48*d^4*p
olylog(5, -cos(b*x + a) + I*sin(b*x + a)) + 48*d^4*polylog(5, -cos(b*x + a
) - I*sin(b*x + a)) + 3*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 - (2*b^4*d^4*x^4 + 8
*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 - b^
2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^2 + 2*(2*b^3*d^
4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d
^4)*x)*cos(b*x + a)*sin(b*x + a) + 2*(2*b^4*c^3*d - 3*b^2*c*d^3)*x + 8*(I*
b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(c
os(b*x + a) + I*sin(b*x + a)) + 8*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*
I*b^3*c^2*d^2*x - I*b^3*c^3*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) + 8*(-
I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*d^2*x - I*b^3*c^3*d)*dilog
(-cos(b*x + a) + I*sin(b*x + a)) + 8*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 +
3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 2
*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*
c^4)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b^4*d^4*x^4 + 4*b^4*c*d^3
*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(cos(b*x + a) - I*s
in(b*x + a) + 1) - 2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 2*(
b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*...
```

### 3.164.6 Sympy [F]

$$\int (c+dx)^4\cos^2(a+bx)\cot(a+bx)\,dx = \int (c+dx)^4\cos^2\left(a+bx\right)\cot\left(a+bx\right)dx$$

input
```
integrate((d*x+c)**4*cos(b*x+a)**2*cot(b*x+a),x)
```

output
```
Integral((c + d*x)**4*cos(a + b*x)**2*cot(a + b*x), x)
```

### 3.164.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1654 vs. $2(272) = 544$.

Time = 0.55 (sec) , antiderivative size = 1654, normalized size of antiderivative = 5.39

$$\int (c+dx)^4 \cos^2(a+bx)\cot(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

output
```
-1/40*(20*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*c^4 - 80*(sin(b*x + a)^2
- log(sin(b*x + a)^2))*a*c^3*d/b + 120*(sin(b*x + a)^2 - log(sin(b*x + a)^
2))*a^2*c^2*d^2/b^2 - 80*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*a^3*c*d^3/
b^3 + 20*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*a^4*d^4/b^4 - (-8*I*(b*x +
 a)^5*d^4 - 40*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^4 - 960*d^4*polylog(5, -e^(
I*b*x + I*a)) - 960*d^4*polylog(5, e^(I*b*x + I*a)) - 80*(I*b^2*c^2*d^2 -
2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a)^3 - 80*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d
^2 + 3*I*a^2*b*c*d^3 - I*a^3*d^4)*(b*x + a)^2 - 40*(-I*(b*x + a)^4*d^4 + 4
*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 6*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 -
I*a^2*d^4)*(b*x + a)^2 + 4*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 - 3*I*a^2*b*c
*d^3 + I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 40*
(I*(b*x + a)^4*d^4 + 4*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^3 + 6*(I*b^2*c^2*d^
2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a)^2 + 4*(I*b^3*c^3*d - 3*I*a*b^2*c^
2*d^2 + 3*I*a^2*b*c*d^3 - I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a), -cos
(b*x + a) + 1) + 5*(2*(b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 3*(
2*a^2 - 1)*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(2*b^2*c^2*d^2 - 4*a*
b*c*d^3 + (2*a^2 - 1)*d^4)*(b*x + a)^2 + 4*(2*b^3*c^3*d - 6*a*b^2*c^2*d^2
+ 3*(2*a^2 - 1)*b*c*d^3 - (2*a^3 - 3*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) -
 160*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 + I*(b*x + a)^3*d^
4 - I*a^3*d^4 + 3*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 ...
```

### 3.164.8 Giac [F]

$$\int (c+dx)^4 \cos^2(a+bx)\cot(a+bx)\,dx = \int (dx+c)^4 \cos(bx+a)^2 \cot(bx+a)\,dx$$

input
```
integrate((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*cos(b*x + a)^2*cot(b*x + a), x)
```

### 3.164.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \cos^2(a+bx)\cot(a+bx)\,dx = \int \cos(a+b\,x)^2 \cot(a+b\,x)\,\left(c+d\,x\right)^4 dx$$

input
```
int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^4,x)
```

output
```
int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^4, x)
```

# 3.165 $\int (c+dx)^3 \cos^2(a+bx) \cot(a+bx)\,dx$

3.165.1 Optimal result . . . 1310
3.165.2 Mathematica [B] (verified) . . . 1311
3.165.3 Rubi [A] (verified) . . . 1311
3.165.4 Maple [B] (verified) . . . 1317
3.165.5 Fricas [B] (verification not implemented) . . . 1318
3.165.6 Sympy [F] . . . 1319
3.165.7 Maxima [B] (verification not implemented) . . . 1320
3.165.8 Giac [F] . . . 1320
3.165.9 Mupad [F(-1)] . . . 1321

## 3.165.1 Optimal result

Integrand size = 22, antiderivative size = 246

$$\int (c+dx)^3 \cos^2(a+bx) \cot(a+bx)\,dx = -\frac{3d^3x}{8b^3}+\frac{(c+dx)^3}{4b}-\frac{i(c+dx)^4}{4d}+\frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{3id(c+dx)^2\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}+\frac{3d^2(c+dx)\operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}+\frac{3id^3\operatorname{PolyLog}\left(4,e^{2i(a+bx)}\right)}{4b^4}+\frac{3d^3\cos(a+bx)\sin(a+bx)}{8b^4}-\frac{3d(c+dx)^2\cos(a+bx)\sin(a+bx)}{4b^2}+\frac{3d^2(c+dx)\sin^2(a+bx)}{4b^3}-\frac{(c+dx)^3\sin^2(a+bx)}{2b}$$

output
```
-3/8*d^3*x/b^3+1/4*(d*x+c)^3/b-1/4*I*(d*x+c)^4/d+(d*x+c)^3*ln(1-exp(2*I*(b
*x+a)))/b-3/2*I*d*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2+3/2*d^2*(d*x+c
)*polylog(3,exp(2*I*(b*x+a)))/b^3+3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^
4+3/8*d^3*cos(b*x+a)*sin(b*x+a)/b^4-3/4*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/
b^2+3/4*d^2*(d*x+c)*sin(b*x+a)^2/b^3-1/2*(d*x+c)^3*sin(b*x+a)^2/b
```

### 3.165.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1956 vs. 2(246) = 492.

Time = 6.41 (sec) , antiderivative size = 1956, normalized size of antiderivative = 7.95

$$\int (c+dx)^3 \cos^2(a+bx)\cot(a+bx)\,dx = \text{Too large to display}$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]^2*Cot[a + b*x],x]
```

output
```
-1/2*(c*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-
2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^
2*Log[1 + E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, -E^((-
I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, E^((-I)*(a + b*x))] +
 (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, -E^((-I)*(a + b*x))] + (6*I)*(1 - E^(
(-2*I)*a))*PolyLog[3, E^((-I)*(a + b*x))]))/b^3 - (d^3*E^(I*a)*Csc[a]*((b^
4*x^4)/E^((2*I)*a) + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a +
 b*x))] + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - 6
*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - 6*b^2*(1 - E
^((-2*I)*a))*x^2*PolyLog[2, E^((-I)*(a + b*x))] + (12*I)*b*(1 - E^((-2*I)*
a))*x*PolyLog[3, -E^((-I)*(a + b*x))] + (12*I)*b*(1 - E^((-2*I)*a))*x*Poly
Log[3, E^((-I)*(a + b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, -E^((-I)*(a
+ b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, E^((-I)*(a + b*x))]))/(4*b^4)
+ (c^3*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[
a]))/(b*(Cos[a]^2 + Sin[a]^2)) + Csc[a]*(Cos[2*a + 2*b*x]/(64*b^4) - ((I/6
4)*Sin[2*a + 2*b*x])/b^4)*(32*b^4*c^3*x*Cos[a + 2*b*x] + 48*b^4*c^2*d*x^2*
Cos[a + 2*b*x] + 32*b^4*c*d^2*x^3*Cos[a + 2*b*x] + 8*b^4*d^3*x^4*Cos[a + 2
*b*x] + 32*b^4*c^3*x*Cos[3*a + 2*b*x] + 48*b^4*c^2*d*x^2*Cos[3*a + 2*b*x]
+ 32*b^4*c*d^2*x^3*Cos[3*a + 2*b*x] + 8*b^4*d^3*x^4*Cos[3*a + 2*b*x] + (4*
I)*b^3*c^3*Cos[3*a + 4*b*x] - 6*b^2*c^2*d*Cos[3*a + 4*b*x] - (6*I)*b*c*...
```

### 3.165.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.21, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {4908, 3042, 25, 4202, 2620, 3011, 4904, 3042, 3792, 17, 3042, 3115, 24, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \cos^2(a+bx) \cot(a+bx)\, dx$$

$\downarrow$ 4908

$$\int (c+dx)^3 \cot(a+bx)dx - \int (c+dx)^3 \cos(a+bx) \sin(a+bx)dx$$

$\downarrow$ 3042

$$\int -(c+dx)^3 \tan\left(a+bx+\frac{\pi}{2}\right) dx - \int (c+dx)^3 \cos(a+bx) \sin(a+bx)dx$$

$\downarrow$ 25

$$-\int (c+dx)^3 \tan\left(\frac{1}{2}(2a+\pi)+bx\right) dx - \int (c+dx)^3 \cos(a+bx) \sin(a+bx)dx$$

$\downarrow$ 4202

$$2i \int \frac{e^{i(2a+2bx+\pi)}(c+dx)^3}{1+e^{i(2a+2bx+\pi)}}dx - \int (c+dx)^3 \cos(a+bx) \sin(a+bx)dx - \frac{i(c+dx)^4}{4d}$$

$\downarrow$ 2620

$$2i\left(\frac{3id\int (c+dx)^2 \log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{2b} - \frac{i(c+dx)^3 \log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right) - \int (c+dx)^3 \cos(a+bx) \sin(a+bx)dx - \frac{i(c+dx)^4}{4d}$$

$\downarrow$ 3011

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b} - \frac{id\int (c+dx) \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b} - \frac{i(c+dx)^3 \log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right) - \int (c+dx)^3 \cos(a+bx) \sin(a+bx)dx - \frac{i(c+dx)^4}{4d}$$

$\downarrow$ 4904

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b} - \frac{id\int (c+dx) \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b} - \frac{i(c+dx)^3 \log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right) - \frac{3d\int (c+dx)^2 \sin^2(a+bx)dx}{2b} - \frac{(c+dx)^3 \sin^2(a+bx)}{2b} - \frac{i(c+dx)^4}{4d}$$

$\downarrow$ 3042

3.165. $\int (c+dx)^3 \cos^2(a+bx) \cot(a+bx)\, dx$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-$$
$$\frac{3d\int(c+dx)^2\sin(a+bx)^2dx}{2b}-\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\Big\downarrow \text{3792}$$

$$\frac{3d\left(-\frac{d^2\int\sin^2(a+bx)dx}{2b^2}+\frac{1}{2}\int(c+dx)^2dx+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b}+$$
$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-$$
$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\Big\downarrow \text{17}$$

$$\frac{3d\left(-\frac{d^2\int\sin^2(a+bx)dx}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}+$$
$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-$$
$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d\left(-\frac{d^2\int\sin(a+bx)^2dx}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}+$$
$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-$$
$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{3d\left(-\frac{d^2\left(\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}+$$
$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-$$
$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

---

3.165. $\int(c+dx)^3\cos^2(a+bx)\cot(a+bx)\,dx$

$\Big\downarrow$ 24

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-$$
$$\frac{3d\left(\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}-$$
$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$\Big\downarrow$ 7163

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\left(\frac{id\int\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i(c+dx)\ldots}{}\right.$$
$$\frac{3d\left(\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}-$$
$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$\Big\downarrow$ 2720

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\left(\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(2a+2b\ldots}\right.}{2b}\right.}{b}\right.}{2b}\right.$$
$$\frac{3d\left(\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}-$$
$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$\Big\downarrow$ 7143

3.165. $\int(c+dx)^3\cos^2(a+bx)\cot(a+bx)\,dx$

$$\frac{3d\left(\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}+$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\left(\frac{d\operatorname{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i(c+dx)^3\operatorname{lo}\right.$$

$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]^2*Cot[a + b*x],x]
```

output
```
((-1/4*I)*(c + d*x)^4)/d + (2*I)*(((-1/2*I)*(c + d*x)^3*Log[1 + E^(I*(2*a
+ Pi + 2*b*x))])/b + (((3*I)/2)*d*(((I/2)*(c + d*x)^2*PolyLog[2, -E^(I*(2*
a + Pi + 2*b*x))])/b - (I*d*(((-1/2*I)*(c + d*x)*PolyLog[3, -E^(I*(2*a + P
i + 2*b*x))])/b + (d*PolyLog[4, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2)))/b))/
b) - ((c + d*x)^3*Sin[a + b*x]^2)/(2*b) + (3*d*((c + d*x)^3/(6*d) - ((c +
d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(2*b) + (d*(c + d*x)*Sin[a + b*x]^2)/(2*
b^2) - (d^2*(x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)))/(2*b^2)))/(2*b)
```

### 3.165.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

## 3.165. $\int (c+dx)^3 \cos^2(a+bx)\cot(a+bx)\,dx$

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

### 3.165.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 907 vs. $2(215) = 430$.

Time = 2.37 (sec) , antiderivative size = 908, normalized size of antiderivative = 3.69

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 908 |

input
```
int((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-6*I/b*d*c^2*x*a+6*I/b^2*c*d^2*a^2*x-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)
))*x-6*I/b^2*c*d^2*polylog(2,-exp(I*(b*x+a)))*x-I*d^2*c*x^3-3/2*I*d*c^2*x^
2-6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a)))+3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)+
6/b^2*c^2*d*a*ln(exp(I*(b*x+a)))-3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)+3/b^2*
d*c^2*ln(1-exp(I*(b*x+a)))*a-3/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2-3/16*d*(
2*b^2*d^2*x^2+4*b^2*c*d*x+2*b^2*c^2-d^2)/b^4*sin(2*b*x+2*a)+1/b*c^3*ln(exp
(I*(b*x+a))+1)-2/b*c^3*ln(exp(I*(b*x+a)))+1/b*c^3*ln(exp(I*(b*x+a))-1)-1/4
*I*d^3*x^4+1/8/b^3*(2*b^2*d^3*x^3+6*b^2*c*d^2*x^2+6*b^2*c^2*d*x+2*b^2*c^3-
3*d^3*x-3*c*d^2)*cos(2*b*x+2*a)+3/b*d*c^2*ln(1-exp(I*(b*x+a)))*x+3/b*d*c^2
*ln(exp(I*(b*x+a))+1)*x+3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2+3/b*c*d^2*ln(ex
p(I*(b*x+a))+1)*x^2-3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-3*I/b^2*d*c
^2*a^2-3*I/b^2*d*c^2*polylog(2,exp(I*(b*x+a)))-3*I/b^2*d*c^2*polylog(2,-ex
p(I*(b*x+a)))+4*I/b^3*c*d^2*a^3-2*I/b^3*d^3*a^3*x-3*I/b^2*d^3*polylog(2,ex
p(I*(b*x+a)))*x^2+2/b^4*d^3*a^3*ln(exp(I*(b*x+a)))-1/b^4*d^3*a^3*ln(exp(I*
(b*x+a))-1)+1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+6/b^3*d^3*polylog(3,exp(I*(
b*x+a)))*x+6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+1/b*d^3*ln(1-exp(I*(b*x+
a)))*x^3+1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3+6/b^3*c*d^2*polylog(3,exp(I*(b*x
+a)))+6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))-3/2*I/b^4*d^3*a^4+6*I/b^4*d^3
*polylog(4,-exp(I*(b*x+a)))+I*c^3*x+1/4*I/d*c^4+6*I*d^3*polylog(4,exp(I*(b
*x+a)))/b^4
```

### 3.165.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 984 vs. $2(211) = 422$.

Time = 0.33 (sec) , antiderivative size = 984, normalized size of antiderivative = 4.00

$$\int (c+dx)^3 \cos^2(a+bx)\cot(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

output
```
-1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 24*I*d^3*polylog(4, cos(b*x + a) +
 I*sin(b*x + a)) + 24*I*d^3*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + 24
*I*d^3*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) - 24*I*d^3*polylog(4, -c
os(b*x + a) - I*sin(b*x + a)) - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3
*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2*b^2*d^
3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 3*(
2*b^3*c^2*d - b*d^3)*x + 12*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d
)*dilog(cos(b*x + a) + I*sin(b*x + a)) + 12*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^
2*x - I*b^2*c^2*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) + 12*(-I*b^2*d^3*x
^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-cos(b*x + a) + I*sin(b*x + a))
+ 12*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(-cos(b*x + a) -
 I*sin(b*x + a)) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*
c^3)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2
*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 4
*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a)
 + 1/2*I*sin(b*x + a) + 1/2) - 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 4*(b^3*d^3*
x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^
3*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 4*(b^3*d^3*x^3 + 3*b^3*c*
d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(...
```

### 3.165.6 Sympy [F]

$$\int (c+dx)^3\cos^2(a+bx)\cot(a+bx)\,dx = \int (c+dx)^3\cos^2{(a+bx)}\cot{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**3*cos(b*x+a)**2*cot(b*x+a),x)
```

output
```
Integral((c + d*x)**3*cos(a + b*x)**2*cot(a + b*x), x)
```

### 3.165.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 979 vs. 2(211) = 422.

Time = 0.42 (sec) , antiderivative size = 979, normalized size of antiderivative = 3.98

$$\int (c+dx)^3 \cos^2(a+bx) \cot(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

output
```
-1/16*(8*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*c^3 - 24*(sin(b*x + a)^2 -
 log(sin(b*x + a)^2))*a*c^2*d/b + 24*(sin(b*x + a)^2 - log(sin(b*x + a)^2)
)*a^2*c*d^2/b^2 - 8*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*a^3*d^3/b^3 - (
-4*I*(b*x + a)^4*d^3 - 16*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^3 + 96*I*d^3*pol
ylog(4, -e^(I*b*x + I*a)) + 96*I*d^3*polylog(4, e^(I*b*x + I*a)) - 24*(I*b
^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2*d^3)*(b*x + a)^2 - 16*(-I*(b*x + a)^3*d^3
 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2
- I*a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 16*(I*(b
*x + a)^3*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I
*a*b*c*d^2 + I*a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1
) + 2*(2*(b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x
+ a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 - 1)*d^3)*(b*x + a))*cos(2*
b*x + 2*a) - 48*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d
^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*dilog(-e^(I*b*x + I*a)) - 48*(I*b^
2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I
*a*d^3)*(b*x + a))*dilog(e^(I*b*x + I*a)) + 8*((b*x + a)^3*d^3 + 3*(b*c*d^
2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*
log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + 8*((b*x + a)^3
*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + ...
```

### 3.165.8 Giac [F]

$$\int (c+dx)^3 \cos^2(a+bx) \cot(a+bx) \, dx = \int (dx+c)^3 \cos(bx+a)^2 \cot(bx+a) \, dx$$

input
```
integrate((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*cos(b*x + a)^2*cot(b*x + a), x)
```

### 3.165.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \cos^2(a+bx)\cot(a+bx)\,dx = \int \cos\left(a+b\,x\right)^2 \cot(a+b\,x)\,\left(c+d\,x\right)^3 dx$$

input
```
int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^3,x)
```

output
```
int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^3, x)
```

# 3.166 $\int (c+dx)^2 \cos^2(a+bx) \cot(a+bx)\,dx$


3.166.1 Optimal result . . . . . . . . . . 1322
3.166.2 Mathematica [B] (verified) . . . . . . . . . . 1323
3.166.3 Rubi [A] (verified) . . . . . . . . . . 1323
3.166.4 Maple [B] (verified) . . . . . . . . . . 1327
3.166.5 Fricas [B] (verification not implemented) . . . . . . . . . . 1328
3.166.6 Sympy [F] . . . . . . . . . . 1329
3.166.7 Maxima [B] (verification not implemented) . . . . . . . . . . 1329
3.166.8 Giac [F] . . . . . . . . . . 1330
3.166.9 Mupad [F(-1)] . . . . . . . . . . 1330


## 3.166.1 Optimal result

Integrand size = 22, antiderivative size = 181

$$\int (c+dx)^2 \cos^2(a+bx) \cot(a+bx)\,dx = \frac{cdx}{2b} + \frac{d^2x^2}{4b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2 \log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id(c+dx)\,\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2} + \frac{d^2\,\mathrm{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3} - \frac{d(c+dx)\cos(a+bx)\sin(a+bx)}{2b^2} + \frac{d^2\sin^2(a+bx)}{4b^3} - \frac{(c+dx)^2\sin^2(a+bx)}{2b}$$

output
```
1/2*c*d*x/b+1/4*d^2*x^2/b-1/3*I*(d*x+c)^3/d+(d*x+c)^2*ln(1-exp(2*I*(b*x+a)
))/b-I*d*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/b^2+1/2*d^2*polylog(3,exp(2*I
*(b*x+a)))/b^3-1/2*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2+1/4*d^2*sin(b*x+a)^
2/b^3-1/2*(d*x+c)^2*sin(b*x+a)^2/b
```

### 3.166.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 564 vs. 2(181) = 362.

Time = 3.18 (sec) , antiderivative size = 564, normalized size of antiderivative = 3.12

$$\int (c+dx)^2 \cos^2(a+bx)\cot(a+bx)\,dx$$
$$= \frac{48ib^2cd\pi x + 16ib^3d^2x^3 - 96ib^2cdx\arctan(\tan(a)) + 48b^3cdx^2\cot(a) - 6bcd\cos(a+2bx)\csc(a) - 6bd^2x\,\mathrm{c}}{}$$

input
```
Integrate[(c + d*x)^2*Cos[a + b*x]^2*Cot[a + b*x],x]
```

output
```
((48*I)*b^2*c*d*Pi*x + (16*I)*b^3*d^2*x^3 - (96*I)*b^2*c*d*x*ArcTan[Tan[a]
] + 48*b^3*c*d*x^2*Cot[a] - 6*b*c*d*Cos[a + 2*b*x]*Csc[a] - 6*b*d^2*x*Cos[
a + 2*b*x]*Csc[a] + 6*b*c*d*Cos[3*a + 2*b*x]*Csc[a] + 6*b*d^2*x*Cos[3*a +
2*b*x]*Csc[a] + 48*b*c*d*Pi*Log[1 + E^((-2*I)*b*x)] + 48*b^2*d^2*x^2*Log[1
 - E^((-I)*(a + b*x))] + 48*b^2*d^2*x^2*Log[1 + E^((-I)*(a + b*x))] + 96*b
^2*c*d*x*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + 96*b*c*d*ArcTan[Tan[a
]]*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] - 48*b*c*d*Pi*Log[Cos[b*x]] +
 48*b^2*c^2*Log[Sin[a + b*x]] - 96*b*c*d*ArcTan[Tan[a]]*Log[Sin[b*x + ArcT
an[Tan[a]]]] + (96*I)*b*d^2*x*PolyLog[2, -E^((-I)*(a + b*x))] + (96*I)*b*d
^2*x*PolyLog[2, E^((-I)*(a + b*x))] - (48*I)*b*c*d*PolyLog[2, E^((2*I)*(b*
x + ArcTan[Tan[a]]))] + 96*d^2*PolyLog[3, -E^((-I)*(a + b*x))] + 96*d^2*Po
lyLog[3, E^((-I)*(a + b*x))] - 48*b^3*c*d*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*
Sqrt[Sec[a]^2] - 6*b^2*c^2*Csc[a]*Sin[a + 2*b*x] + 3*d^2*Csc[a]*Sin[a + 2*
b*x] - 12*b^2*c*d*x*Csc[a]*Sin[a + 2*b*x] - 6*b^2*d^2*x^2*Csc[a]*Sin[a + 2
*b*x] + 6*b^2*c^2*Csc[a]*Sin[3*a + 2*b*x] - 3*d^2*Csc[a]*Sin[3*a + 2*b*x]
+ 12*b^2*c*d*x*Csc[a]*Sin[3*a + 2*b*x] + 6*b^2*d^2*x^2*Csc[a]*Sin[3*a + 2*
b*x])/(48*b^3)
```

### 3.166.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {4908, 3042, 25, 4202, 2620, 3011, 2720, 4904, 3042, 3791, 17, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\cos^2(a+bx)\cot(a+bx)\,dx$$

$$\Big\downarrow \text{4908}$$

$$\int (c+dx)^2\cot(a+bx)dx-\int (c+dx)^2\cos(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int -(c+dx)^2\tan\left(a+bx+\frac{\pi}{2}\right)dx-\int (c+dx)^2\cos(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)^2\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx-\int (c+dx)^2\cos(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{4202}$$

$$2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)^2}{1+e^{i(2a+2bx+\pi)}}dx-\int (c+dx)^2\cos(a+bx)\sin(a+bx)dx-\frac{i(c+dx)^3}{3d}$$

$$\Big\downarrow \text{2620}$$

$$2i\left(\frac{id\int (c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\int (c+dx)^2\cos(a+bx)\sin(a+bx)dx-\frac{i(c+dx)^3}{3d}$$

$$\Big\downarrow \text{3011}$$

$$2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\int (c+dx)^2\cos(a+bx)\sin(a+bx)dx-\frac{i(c+dx)^3}{3d}$$

$$\Big\downarrow \text{2720}$$

$$2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2\right.}}{2b}\right.$$
$$\int (c+dx)^2\cos(a+bx)\sin(a+bx)dx-\frac{i(c+dx)^3}{3d}$$

$$\Big\downarrow \text{4904}$$

3.166. $\int (c+dx)^2\cos^2(a+bx)\cot(a+bx)\,dx$

$$2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2\right.}}{2b}\right.$$
$$\frac{d\int(c+dx)\sin^2(a+bx)dx}{b}-\frac{(c+dx)^2\sin^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

$\big\downarrow$ 3042

$$2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2\right.}}{2b}\right.$$
$$\frac{d\int(c+dx)\sin(a+bx)^2dx}{b}-\frac{(c+dx)^2\sin^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

$\big\downarrow$ 3791

$$2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2\right.}}{2b}\right.$$
$$\frac{d\left(\frac{1}{2}\int(c+dx)dx+\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}\right)}{b}-\frac{(c+dx)^2\sin^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

$\big\downarrow$ 17

$$2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2\right.}}{2b}\right.$$
$$\frac{d\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{b}-\frac{(c+dx)^2\sin^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

$\big\downarrow$ 7143

$$2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)+$$
$$\frac{d\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{b}-\frac{(c+dx)^2\sin^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x]^2*Cot[a + b*x],x]
```

## 3.166. $\int(c+dx)^2\cos^2(a+bx)\cot(a+bx)\,dx$

output
```
((-1/3*I)*(c + d*x)^3)/d + (2*I)*(((-1/2*I)*(c + d*x)^2*Log[1 + E^(I*(2*a
+ Pi + 2*b*x))])/b + (I*d*(((I/2)*(c + d*x)*PolyLog[2, -E^(I*(2*a + Pi + 2
*b*x))])/b - (d*PolyLog[3, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2)))/b) - ((c
+ d*x)^2*Sin[a + b*x]^2)/(2*b) + (d*((c + d*x)^2/(4*d) - ((c + d*x)*Cos[a
+ b*x]*Sin[a + b*x])/(2*b) + (d*Sin[a + b*x]^2)/(4*b^2)))/b
```

### 3.166.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.166.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(159) = 318$.

Time = 1.87 (sec) , antiderivative size = 544, normalized size of antiderivative = 3.01

| method | result |
|---|---|
| risch | $-\frac{d(dx+c)\sin(2xb+2a)}{4b^2}+\frac{(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2)\cos(2xb+2a)}{8b^3}-\frac{4idcxa}{b}-\frac{2id^2\,\mathrm{polylog}(2,e^{i(xb+a)})x}{b^2}-\frac{2id^2\,\mathrm{polylog}(}{}$ |

input
```
int((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))+1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+4/3*I
/b^3*d^2*a^3-I*d*c*x^2-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x-2*I/b^2*d^2
*polylog(2,-exp(I*(b*x+a)))*x+1/b*c^2*ln(exp(I*(b*x+a))+1)-2/b*c^2*ln(exp(
I*(b*x+a)))+1/b*c^2*ln(exp(I*(b*x+a))-1)-1/4*d*(d*x+c)*sin(2*b*x+2*a)/b^2-
1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2+1/b*d^
2*ln(exp(I*(b*x+a))+1)*x^2+I*c^2*x+1/3*I/d*c^3+2/b*d*c*ln(exp(I*(b*x+a))+1
)*x+2/b^2*d*c*ln(1-exp(I*(b*x+a)))*a+4/b^2*c*d*a*ln(exp(I*(b*x+a)))-2/b^2*
c*d*a*ln(exp(I*(b*x+a))-1)+2/b*d*c*ln(1-exp(I*(b*x+a)))*x-2*I/b^2*d*c*a^2-
2*I/b^2*d*c*polylog(2,exp(I*(b*x+a)))-2*I/b^2*d*c*polylog(2,-exp(I*(b*x+a)
))+2*I/b^2*d^2*a^2*x-1/3*I*d^2*x^3-4*I/b*d*c*x*a+1/8*(2*b^2*d^2*x^2+4*b^2*
c*d*x+2*b^2*c^2-d^2)/b^3*cos(2*b*x+2*a)+2*d^2*polylog(3,-exp(I*(b*x+a)))/b
^3+2*d^2*polylog(3,exp(I*(b*x+a)))/b^3
```

### 3.166.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(156) = 312$.

Time = 0.29 (sec) , antiderivative size = 594, normalized size of antiderivative = 3.28

$$\int (c+dx)^2\cos^2(a+bx)\cot(a+bx)\,dx =$$
$$-\frac{b^2d^2x^2+2\,b^2cdx-\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2\right)\cos\left(bx+a\right)^2-4\,d^2\mathrm{polylog}(3,\cos\left(bx+a\right)+i\,\sin$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2
 - d^2)*cos(b*x + a)^2 - 4*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) -
 4*d^2*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 4*d^2*polylog(3, -cos(b
*x + a) + I*sin(b*x + a)) - 4*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a
)) + 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) + 4*(I*b*d^2*x + I*b*c*
d)*dilog(cos(b*x + a) + I*sin(b*x + a)) + 4*(-I*b*d^2*x - I*b*c*d)*dilog(c
os(b*x + a) - I*sin(b*x + a)) + 4*(-I*b*d^2*x - I*b*c*d)*dilog(-cos(b*x +
a) + I*sin(b*x + a)) + 4*(I*b*d^2*x + I*b*c*d)*dilog(-cos(b*x + a) - I*sin
(b*x + a)) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) + I*
sin(b*x + a) + 1) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x +
a) - I*sin(b*x + a) + 1) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(
b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*l
og(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 2*(b^2*d^2*x^2 + 2*b^2*
c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(
b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin
(b*x + a) + 1))/b^3
```

## 3.166.6 Sympy [F]

$$\int (c+dx)^2\cos^2(a+bx)\cot(a+bx)\,dx=\int (c+dx)^2\cos^2(a+bx)\cot(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*cos(b*x+a)**2*cot(b*x+a),x)
```

output
```
Integral((c + d*x)**2*cos(a + b*x)**2*cot(a + b*x), x)
```

## 3.166.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(156) = 312$.

Time = 0.36 (sec) , antiderivative size = 529, normalized size of antiderivative = 2.92

$$\int (c+dx)^2\cos^2(a+bx)\cot(a+bx)\,dx=$$
$$-\frac{12\left(\sin(bx+a)^2-\log\left(\sin(bx+a)^2\right)\right)c^2-\frac{24\left(\sin(bx+a)^2-\log\left(\sin(bx+a)^2\right)\right)acd}{b}+\frac{12\left(\sin(bx+a)^2-\log\left(\sin(bx+a)^2\right)\right)a}{b^2}}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

output
```
-1/24*(12*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*c^2 - 24*(sin(b*x + a)^2
- log(sin(b*x + a)^2))*a*c*d/b + 12*(sin(b*x + a)^2 - log(sin(b*x + a)^2))
*a^2*d^2/b^2 - (-8*I*(b*x + a)^3*d^2 - 24*(I*b*c*d - I*a*d^2)*(b*x + a)^2
+ 48*d^2*polylog(3, -e^(I*b*x + I*a)) + 48*d^2*polylog(3, e^(I*b*x + I*a))
 - 24*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*arctan2(sin(
b*x + a), cos(b*x + a) + 1) - 24*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2
)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 3*(2*(b*x + a)^2*d
^2 + 4*(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(2*b*x + 2*a) - 48*(I*b*c*d + I
*(b*x + a)*d^2 - I*a*d^2)*dilog(-e^(I*b*x + I*a)) - 48*(I*b*c*d + I*(b*x +
 a)*d^2 - I*a*d^2)*dilog(e^(I*b*x + I*a)) + 12*((b*x + a)^2*d^2 + 2*(b*c*d
 - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a)
+ 1) + 12*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 6*(b*c*d + (b*x + a)*d^2 - a*d
^2)*sin(2*b*x + 2*a))/b^2)/b
```

### 3.166.8 Giac [F]

$$\int (c+dx)^2 \cos^2(a+bx)\cot(a+bx)\,dx = \int (dx+c)^2 \cos(bx+a)^2 \cot(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*cos(b*x + a)^2*cot(b*x + a), x)
```

### 3.166.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \cos^2(a+bx)\cot(a+bx)\,dx = \int \cos(a+b\,x)^2 \cot(a+b\,x)\,(c+d\,x)^2\,dx$$

input
```
int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^2,x)
```

output
```
int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^2, x)
```

# 3.167 $\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx$

3.167.1 Optimal result . . . 1331
3.167.2 Mathematica [A] (verified) . . . 1331
3.167.3 Rubi [A] (verified) . . . 1332
3.167.4 Maple [B] (verified) . . . 1335
3.167.5 Fricas [B] (verification not implemented) . . . 1336
3.167.6 Sympy [F] . . . 1336
3.167.7 Maxima [B] (verification not implemented) . . . 1337
3.167.8 Giac [F] . . . 1337
3.167.9 Mupad [F(-1)] . . . 1338

## 3.167.1 Optimal result

Integrand size = 20, antiderivative size = 114

$$\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx = \frac{dx}{4b}-\frac{i(c+dx)^2}{2d}+\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{id\,\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{d\cos(a+bx)\sin(a+bx)}{4b^2}-\frac{(c+dx)\sin^2(a+bx)}{2b}$$

output
```
1/4*d*x/b-1/2*I*(d*x+c)^2/d+(d*x+c)*ln(1-exp(2*I*(b*x+a)))/b-1/2*I*d*polyl
og(2,exp(2*I*(b*x+a)))/b^2-1/4*d*cos(b*x+a)*sin(b*x+a)/b^2-1/2*(d*x+c)*sin
(b*x+a)^2/b
```

## 3.167.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.22

$$\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx = \frac{dx\cos(2(a+bx))}{4b}+\frac{c\log(\sin(a+bx))}{b}-\frac{ad(\log(\cos(a+bx))+\log(\tan(a+bx)))}{b^2}+\frac{d\left((a+bx)\log\left(1-e^{2i(a+bx)}\right)-\frac{1}{2}i\left((a+bx)^2+\text{PolyLog}\left(2,e^{2i(a+bx)}\right)\right)\right)}{b^2}-\frac{c\sin^2(a+bx)}{2b}-\frac{d\sin(2(a+bx))}{8b^2}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x]^2*Cot[a + b*x],x]
```

output
```
(d*x*Cos[2*(a + b*x)])/(4*b) + (c*Log[Sin[a + b*x]])/b - (a*d*(Log[Cos[a +
 b*x]] + Log[Tan[a + b*x]]))/b^2 + (d*((a + b*x)*Log[1 - E^((2*I)*(a + b*x
))] - (I/2)*((a + b*x)^2 + PolyLog[2, E^((2*I)*(a + b*x))])))/b^2 - (c*Sin
[a + b*x]^2)/(2*b) - (d*Sin[2*(a + b*x)])/(8*b^2)
```

### 3.167.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {4908, 3042, 25, 4202, 2620, 2715, 2838, 4904, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx$$

$$\Big\downarrow \text{4908}$$

$$\int (c+dx)\cot(a+bx)dx-\int (c+dx)\cos(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\left((c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)\right)dx-\int (c+dx)\cos(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx-\int (c+dx)\cos(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{4202}$$

$$2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)}{1+e^{i(2a+2bx+\pi)}}dx-\int (c+dx)\cos(a+bx)\sin(a+bx)dx-\frac{i(c+dx)^2}{2d}$$

$$\Big\downarrow \text{2620}$$

$$2i\left(\frac{id\int \log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\int (c+dx)\cos(a+bx)\sin(a+bx)dx-\frac{i(c+dx)^2}{2d}$$

$$\Big\downarrow \text{2715}$$

$$2i\left(\frac{d\int e^{-i(2a+2bx+\pi)}\log\left(1+e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\int(c+dx)\cos(a+bx)\sin(a+bx)dx-\frac{i(c+dx)^2}{2d}$$

$$\Big\downarrow \text{ 2838}$$

$$-\int(c+dx)\cos(a+bx)\sin(a+bx)dx+2i\left(-\frac{d\,\text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{i(c+dx)^2}{2d}$$

$$\Big\downarrow \text{ 4904}$$

$$\frac{d\int\sin^2(a+bx)dx}{2b}+2i\left(-\frac{d\,\text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{(c+dx)\sin^2(a+bx)}{2b}-\frac{i(c+dx)^2}{2d}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{d\int\sin(a+bx)^2dx}{2b}+2i\left(-\frac{d\,\text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{(c+dx)\sin^2(a+bx)}{2b}-\frac{i(c+dx)^2}{2d}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{d\left(\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b}+2i\left(-\frac{d\,\text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{(c+dx)\sin^2(a+bx)}{2b}-\frac{i(c+dx)^2}{2d}$$

$$\Big\downarrow \text{ 24}$$

$$2i\left(-\frac{d\,\text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{(c+dx)\sin^2(a+bx)}{2b}+\frac{d\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b}-\frac{i(c+dx)^2}{2d}$$

input
```
Int[(c + d*x)*Cos[a + b*x]^2*Cot[a + b*x],x]
```

## 3.167. $\int(c+dx)\cos^2(a+bx)\cot(a+bx)\,dx$

output
```
((-1/2*I)*(c + d*x)^2)/d + (2*I)*(((-1/2*I)*(c + d*x)*Log[1 + E^(I*(2*a +
Pi + 2*b*x))])/b - (d*PolyLog[2, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2)) - ((
c + d*x)*Sin[a + b*x]^2)/(2*b) + (d*(x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*
b)))/(2*b)
```

### 3.167.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

### 3.167.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 248 vs. $2(98) = 196$.

Time = 1.68 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.18

| method | result |
|---|---|
| risch | $-\frac{id\,a^2}{b^2}-\frac{id\,\mathrm{polylog}\left(2,\mathrm{e}^{i(xb+a)}\right)}{b^2}+\frac{c\ln\left(\mathrm{e}^{i(xb+a)}+1\right)}{b}-\frac{2c\ln\left(\mathrm{e}^{i(xb+a)}\right)}{b}+\frac{c\ln\left(\mathrm{e}^{i(xb+a)}-1\right)}{b}-\frac{id\,\mathrm{polylog}\left(2,-\mathrm{e}^{i(xb+a)}\right)}{b^2}-$ |

input
```
int((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-I/b^2*d*a^2-I*d*polylog(2,exp(I*(b*x+a)))/b^2+1/b*c*ln(exp(I*(b*x+a))+1)-
2/b*c*ln(exp(I*(b*x+a)))+1/b*c*ln(exp(I*(b*x+a))-1)-I*d*polylog(2,-exp(I*(
b*x+a)))/b^2-1/2*I*d*x^2+I*c*x+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-e
xp(I*(b*x+a)))*a-2*I/b*d*x*a+1/b*d*ln(exp(I*(b*x+a))+1)*x+2/b^2*d*a*ln(exp
(I*(b*x+a)))-1/b^2*d*a*ln(exp(I*(b*x+a))-1)+1/4*(d*x+c)*cos(2*b*x+2*a)/b-1
/8*d*sin(2*b*x+2*a)/b^2
```

### 3.167.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(95) = 190$.

Time = 0.27 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.56

$$\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx =$$
$$-\frac{bdx - 2\,(bdx + bc)\cos{(bx+a)}^2 + d\cos{(bx+a)}\sin{(bx+a)} + 2i\,d\mathrm{Li}_2(\cos{(bx+a)} + i\,\sin{(bx+a)}) -}{}$$

input
```
integrate((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*x + a)
 + 2*I*d*dilog(cos(b*x + a) + I*sin(b*x + a)) - 2*I*d*dilog(cos(b*x + a) -
 I*sin(b*x + a)) - 2*I*d*dilog(-cos(b*x + a) + I*sin(b*x + a)) + 2*I*d*dil
og(-cos(b*x + a) - I*sin(b*x + a)) - 2*(b*d*x + b*c)*log(cos(b*x + a) + I*
sin(b*x + a) + 1) - 2*(b*d*x + b*c)*log(cos(b*x + a) - I*sin(b*x + a) + 1)
 - 2*(b*c - a*d)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 2*(b*
c - a*d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 2*(b*d*x + a*
d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b*d*x + a*d)*log(-cos(b*x
+ a) - I*sin(b*x + a) + 1))/b^2
```

### 3.167.6 Sympy [F]

$$\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx = \int (c+dx)\cos^2{(a+bx)}\cot{(a+bx)}\,dx$$

input
```
integrate((d*x+c)*cos(b*x+a)**2*cot(b*x+a),x)
```

output
```
Integral((c + d*x)*cos(a + b*x)**2*cot(a + b*x), x)
```

### 3.167.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(95) = 190$.

Time = 0.30 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.96

$$\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx$$
$$= \frac{-4i\,b^2dx^2 - 8i\,b^2cx - 8i\,bdx\arctan\left(\sin\left(bx+a\right), -\cos\left(bx+a\right)+1\right) + 8i\,bc\arctan\left(\sin\left(bx+a\right), \cos\left(bx\right.\right.}{}$$

input
```
integrate((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

output
```
1/8*(-4*I*b^2*d*x^2 - 8*I*b^2*c*x - 8*I*b*d*x*arctan2(sin(b*x + a), -cos(b
*x + a) + 1) + 8*I*b*c*arctan2(sin(b*x + a), cos(b*x + a) - 1) - 8*(-I*b*d
*x - I*b*c)*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 2*(b*d*x + b*c)*cos(
2*b*x + 2*a) - 8*I*d*dilog(-e^(I*b*x + I*a)) - 8*I*d*dilog(e^(I*b*x + I*a)
) + 4*(b*d*x + b*c)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) +
 1) + 4*(b*d*x + b*c)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a)
 + 1) - d*sin(2*b*x + 2*a))/b^2
```

### 3.167.8 Giac [F]

$$\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx = \int (dx+c)\cos\left(bx+a\right)^2\cot\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*cos(b*x + a)^2*cot(b*x + a), x)
```

### 3.167.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx = \int \cos\left(a+b\,x\right)^2\cot(a+b\,x)\,\left(c+d\,x\right)\,dx$$

input
```
int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x),x)
```

output
```
int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x), x)
```

# 3.168 $\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx$

3.168.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1339
3.168.2 Mathematica [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . 1339
3.168.3 Rubi [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1340
3.168.4 Maple [N/A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 1343
3.168.5 Fricas [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1343
3.168.6 Sympy [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1343
3.168.7 Maxima [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1344
3.168.8 Giac [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1344
3.168.9 Mupad [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1345

## 3.168.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx = -\frac{\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d} - \frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d} + \text{Int}\left(\frac{\cot(a+bx)}{c+dx},x\right)$$

output
```
-1/2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d-1/2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*
b*c/d)/d+Unintegrable(cot(b*x+a)/(d*x+c),x)
```

## 3.168.2 Mathematica [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x),x]
```

output
```
Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x), x]
```

### 3.168.3 Rubi [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4908, 3042, 25, 4222, 4906, 27, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{4908}$$

$$\int \frac{\cot(a+bx)}{c+dx}dx-\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx-\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{c+dx}dx-\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{4222}$$

$$-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{c+dx}dx-\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{4906}$$

$$-\int \frac{\sin(2a+2bx)}{2(c+dx)}dx-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{c+dx}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}dx-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{c+dx}dx$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}dx-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{c+dx}dx$$

$$\Big\downarrow \text{3784}$$

$$\frac{1}{2}\left(-\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx-\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)-\int\frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{c+dx}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(-\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx-\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)-\int\frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{c+dx}dx$$

$$\Big\downarrow \text{3780}$$

$$\frac{1}{2}\left(-\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx-\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)-\int\frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{c+dx}dx$$

$$\Big\downarrow \text{3783}$$

$$\frac{1}{2}\left(-\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}-\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)-\int\frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{c+dx}dx$$

input
```
Int[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x),x]
```

output
```
$Aborted
```

### 3.168.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

## 3.168. $\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx$

### 3.168.4 Maple [N/A] (verified)

Not integrable

Time = 0.87 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cos (xb+a)^2 \cot (xb+a)}{dx+c}dx$$

input
```
int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x)
```

output
```
int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x)
```

### 3.168.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos (bx+a)^2 \cot (bx+a)}{dx+c}\,dx$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^2*cot(b*x + a)/(d*x + c), x)
```

### 3.168.6 Sympy [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos^2 (a+bx) \cot (a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)**2*cot(b*x+a)/(d*x+c),x)
```

output
```
Integral(cos(a + b*x)**2*cot(a + b*x)/(c + d*x), x)
```

### 3.168.7 Maxima [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 231, normalized size of antiderivative = 10.50

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)^2\cot(bx+a)}{dx+c}\,dx$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
-1/4*((I*exp_integral_e(1, 2*(-I*b*d*x - I*b*c)/d) - I*exp_integral_e(1, -
2*(-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 4*d*integrate(sin(b*x + a
)/((d*x + c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)
*cos(b*x + a) + c), x) - 4*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a
)^2 + (d*x + c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) -
 (exp_integral_e(1, 2*(-I*b*d*x - I*b*c)/d) + exp_integral_e(1, -2*(-I*b*d
*x - I*b*c)/d))*sin(-2*(b*c - a*d)/d))/d
```

### 3.168.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)^2\cot(bx+a)}{dx+c}\,dx$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2*cot(b*x + a)/(d*x + c), x)
```

### 3.168.9 Mupad [N/A]

Not integrable

Time = 26.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx = \int \frac{\cos{(a+b\,x)}^2\cot(a+b\,x)}{c+d\,x}\,dx$$

input
```
int((cos(a + b*x)^2*cot(a + b*x))/(c + d*x),x)
```

output
```
int((cos(a + b*x)^2*cot(a + b*x))/(c + d*x), x)
```

# 3.169 $\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$

3.169.1 Optimal result . . . . . . . . . . 1346
3.169.2 Mathematica [N/A] . . . . . . . . . . 1346
3.169.3 Rubi [N/A] . . . . . . . . . . 1347
3.169.4 Maple [N/A] (verified) . . . . . . . . . . 1350
3.169.5 Fricas [N/A] . . . . . . . . . . 1351
3.169.6 Sympy [N/A] . . . . . . . . . . 1351
3.169.7 Maxima [N/A] . . . . . . . . . . 1351
3.169.8 Giac [N/A] . . . . . . . . . . 1352
3.169.9 Mupad [N/A] . . . . . . . . . . 1352

## 3.169.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = -\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{\sin(2a+2bx)}{2d(c+dx)}+\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\text{Int}\left(\frac{\cot(a+bx)}{(c+dx)^2},x\right)$$

output
```
-b*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^2+b*Si(2*b*c/d+2*b*x)*sin(2*a-2*b*
c/d)/d^2+1/2*sin(2*b*x+2*a)/d/(d*x+c)+Unintegrable(cot(b*x+a)/(d*x+c)^2,x)
```

## 3.169.2 Mathematica [N/A]

Not integrable

Time = 2.88 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x)^2, x]
```

### 3.169.3 Rubi [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4908, 3042, 25, 4222, 4906, 27, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{4908}$$

$$\int \frac{\cot(a+bx)}{(c+dx)^2}dx-\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx-\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx-\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{4222}$$

$$-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx-\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{4906}$$

$$-\int \frac{\sin(2a+2bx)}{2(c+dx)^2}dx-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}dx-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}dx-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3778}$$

$$\frac{1}{2}\left(\frac{\sin(2a+2bx)}{d(c+dx)}-\frac{2b\int \frac{\cos(2a+2bx)}{c+dx}dx}{d}\right)-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx$$

$\big\downarrow$ 3042

$$\frac{1}{2}\left(\frac{\sin(2a+2bx)}{d(c+dx)}-\frac{2b\int \frac{\sin\left(2a+2bx+\frac{\pi}{2}\right)}{c+dx}dx}{d}\right)-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx$$

$\big\downarrow$ 3784

$$\frac{1}{2}\left(\frac{\sin(2a+2bx)}{d(c+dx)}-\frac{2b\left(\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx-\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}\right)-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx$$

$\big\downarrow$ 3042

$$\frac{1}{2}\left(\frac{\sin(2a+2bx)}{d(c+dx)}-\frac{2b\left(\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx-\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}\right)-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx$$

$\big\downarrow$ 3780

$$\frac{1}{2}\left(\frac{\sin(2a+2bx)}{d(c+dx)}-\frac{2b\left(\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx-\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}\right)-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx$$

$\big\downarrow$ 3783

$$\frac{1}{2}\left(\frac{\sin(2a+2bx)}{d(c+dx)}-\frac{2b\left(\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}-\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}\right)-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx$$

---

3.169. $\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$

input
```
Int[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

### 3.169.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

## 3.169. $\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

### 3.169.4 Maple [N/A] (verified)

Not integrable

Time = 1.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cos \left(xb+a\right)^{2} \cot \left(xb+a\right)}{\left(dx+c\right)^{2}}dx$$

input
```
int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x)
```

output
```
int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x)
```

### 3.169.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos{(bx+a)}^2\cot{(bx+a)}}{(dx+c)^2}\,dx$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)^2*cot(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.169.6 Sympy [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos^2{(a+bx)}\cot{(a+bx)}}{(c+dx)^2}\,dx$$

input
```
integrate(cos(b*x+a)**2*cot(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(cos(a + b*x)**2*cot(a + b*x)/(c + d*x)**2, x)
```

### 3.169.7 Maxima [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 343, normalized size of antiderivative = 15.59

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos{(bx+a)}^2\cot{(bx+a)}}{(dx+c)^2}\,dx$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/4*((I*exp_integral_e(2, 2*(-I*b*d*x - I*b*c)/d) - I*exp_integral_e(2, -
2*(-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 4*(d^2*x + c*d)*integrate
(sin(b*x + a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^
2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(b*x + a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x
+ c^2)*cos(b*x + a)), x) - 4*(d^2*x + c*d)*integrate(sin(b*x + a)/(d^2*x^2
 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2 + 2*c*d*x
 + c^2)*sin(b*x + a)^2 + c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)),
x) - (exp_integral_e(2, 2*(-I*b*d*x - I*b*c)/d) + exp_integral_e(2, -2*(-I
*b*d*x - I*b*c)/d))*sin(-2*(b*c - a*d)/d))/(d^2*x + c*d)
```

## 3.169.8 Giac [N/A]

Not integrable

Time = 2.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos{(bx+a)}^2\cot{(bx+a)}}{(dx+c)^2}\,dx$$

input
```
integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)^2*cot(b*x + a)/(d*x + c)^2, x)
```

## 3.169.9 Mupad [N/A]

Not integrable

Time = 26.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos{(a+b\,x)}^2\cot(a+b\,x)}{(c+d\,x)^2}\,dx$$

input
```
int((cos(a + b*x)^2*cot(a + b*x))/(c + d*x)^2,x)
```

output
```
int((cos(a + b*x)^2*cot(a + b*x))/(c + d*x)^2, x)
```

# 3.170 $\int (c+dx)^m \cos(a+bx)\cot^2(a+bx)\,dx$

3.170.1 Optimal result . . . . . 1353
3.170.2 Mathematica [N/A] . . . . . 1353
3.170.3 Rubi [N/A] . . . . . 1354
3.170.4 Maple [N/A] (verified) . . . . . 1356
3.170.5 Fricas [N/A] . . . . . 1356
3.170.6 Sympy [N/A] . . . . . 1356
3.170.7 Maxima [N/A] . . . . . 1357
3.170.8 Giac [N/A] . . . . . 1357
3.170.9 Mupad [N/A] . . . . . 1357

## 3.170.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (c+dx)^m \cos(a+bx)\cot^2(a+bx)\,dx = \frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b} - \frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)}{2b} + \text{Int}((c+dx)^m\cot(a+bx)\csc(a+bx),x)$$

output
```
CannotIntegrate((d*x+c)^m*cot(b*x+a)*csc(b*x+a),x)+1/2*I*exp(I*(a-b*c/d))*
(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)-1/2*I*(d*x+c)^m
*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

## 3.170.2 Mathematica [N/A]

Not integrable

Time = 13.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cos(a+bx)\cot^2(a+bx)\,dx = \int (c+dx)^m \cos(a+bx)\cot^2(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x]^2, x]
```

## 3.170.3 Rubi [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4908, 3042, 3788, 26, 2612, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cos(a+bx)\cot^2(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{4908}$$

$$\int (c+dx)^m\cot(a+bx)\csc(a+bx)dx-\int (c+dx)^m\cos(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m\cot(a+bx)\csc(a+bx)dx-\int (c+dx)^m\sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3788}$$

$$-\frac{1}{2}i\int -ie^{-i(a+bx)}(c+dx)^mdx+\frac{1}{2}i\int ie^{i(a+bx)}(c+dx)^mdx+\int (c+dx)^m\cot(a+bx)\csc(a+bx)dx$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{2}\int e^{-i(a+bx)}(c+dx)^mdx-\frac{1}{2}\int e^{i(a+bx)}(c+dx)^mdx+\int (c+dx)^m\cot(a+bx)\csc(a+bx)dx$$

$$\Big\downarrow \text{2612}$$

$$\int (c+dx)^m\cot(a+bx)\csc(a+bx)dx+\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}-\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}$$

$$\Big\downarrow \text{7299}$$

$$\int (c+dx)^m \cot(a+bx)\csc(a+bx)dx + \frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b} - \frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}$$

input
```
Int[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

output
```
$Aborted
```

### 3.170.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

### 3.170.4 Maple [N/A] (verified)

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \cos(xb+a)\cot(xb+a)^2\,dx$$

input
```
int((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x)
```

### 3.170.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cos(a+bx)\cot^2(a+bx)\,dx = \int (dx+c)^m \cos(bx+a)\cot(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*cos(b*x + a)*cot(b*x + a)^2, x)
```

### 3.170.6 Sympy [N/A]

Not integrable

Time = 22.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \cos(a+bx)\cot^2(a+bx)\,dx = \int (c+dx)^m \cos(a+bx)\cot^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*cos(b*x+a)*cot(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*cos(a + b*x)*cot(a + b*x)**2, x)
```

### 3.170.7 Maxima [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cos(a+bx) \cot^2(a+bx)\,dx = \int (dx+c)^m \cos(bx+a) \cot(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a)^2, x)
```

### 3.170.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cos(a+bx) \cot^2(a+bx)\,dx = \int (dx+c)^m \cos(bx+a) \cot(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a)^2, x)
```

### 3.170.9 Mupad [N/A]

Not integrable

Time = 25.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \cos(a+bx) \cot^2(a+bx)\,dx = \int \cos(a+b\,x)\,\cot(a+b\,x)^2\,(c+d\,x)^m\,dx$$

input
```
int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^m,x)
```

output
```
int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^m, x)
```

# 3.171 $\int (c+dx)^4 \cos(a+bx) \cot^2(a+bx) \, dx$

3.171.1 Optimal result . . . 1358
3.171.2 Mathematica [B] (verified) . . . 1359
3.171.3 Rubi [A] (verified) . . . 1360
3.171.4 Maple [B] (verified) . . . 1367
3.171.5 Fricas [B] (verification not implemented) . . . 1368
3.171.6 Sympy [F] . . . 1368
3.171.7 Maxima [B] (verification not implemented) . . . 1369
3.171.8 Giac [F] . . . 1369
3.171.9 Mupad [F(-1)] . . . 1370

## 3.171.1 Optimal result

Integrand size = 22, antiderivative size = 299

$$\begin{aligned}\int (c+dx)^4 \cos(a+bx) \cot^2(a+bx) \, dx = & -\frac{8d(c+dx)^3 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b^2} \\ & +\frac{24d^3(c+dx)\cos(a+bx)}{b^4} \\ & -\frac{4d(c+dx)^3\cos(a+bx)}{b^2}-\frac{(c+dx)^4\csc(a+bx)}{b} \\ & +\frac{12id^2(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3} \\ & -\frac{12id^2(c+dx)^2 \operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3} \\ & -\frac{24d^3(c+dx) \operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^4} \\ & +\frac{24d^3(c+dx) \operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^4} \\ & -\frac{24id^4 \operatorname{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^5} \\ & +\frac{24id^4 \operatorname{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^5}-\frac{24d^4\sin(a+bx)}{b^5} \\ & +\frac{12d^2(c+dx)^2\sin(a+bx)}{b^3} \\ & -\frac{(c+dx)^4\sin(a+bx)}{b}\end{aligned}$$

output
```
-8*d*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b^2+24*d^3*(d*x+c)*cos(b*x+a)/b^4-4
*d*(d*x+c)^3*cos(b*x+a)/b^2-(d*x+c)^4*csc(b*x+a)/b+12*I*d^2*(d*x+c)^2*poly
log(2,-exp(I*(b*x+a)))/b^3-12*I*d^2*(d*x+c)^2*polylog(2,exp(I*(b*x+a)))/b^
3-24*d^3*(d*x+c)*polylog(3,-exp(I*(b*x+a)))/b^4+24*d^3*(d*x+c)*polylog(3,e
xp(I*(b*x+a)))/b^4-24*I*d^4*polylog(4,-exp(I*(b*x+a)))/b^5+24*I*d^4*polylo
g(4,exp(I*(b*x+a)))/b^5-24*d^4*sin(b*x+a)/b^5+12*d^2*(d*x+c)^2*sin(b*x+a)/
b^3-(d*x+c)^4*sin(b*x+a)/b
```

### 3.171.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 798 vs. 2(299) = 598.

Time = 1.94 (sec) , antiderivative size = 798, normalized size of antiderivative = 2.67

$$\int (c+dx)^4 \cos(a+bx)\cot^2(a+bx)\,dx$$
$$= \frac{\csc(a+bx)\left(-3b^4c^4+12b^2c^2d^2-24d^4-12b^4c^3dx+24b^2cd^3x-18b^4c^2d^2x^2+12b^2d^4x^2-12b^4cd^3x^3-3b\right.}{}$$

input
```
Integrate[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

output
```
(Csc[a + b*x]*(-3*b^4*c^4 + 12*b^2*c^2*d^2 - 24*d^4 - 12*b^4*c^3*d*x + 24*
b^2*c*d^3*x - 18*b^4*c^2*d^2*x^2 + 12*b^2*d^4*x^2 - 12*b^4*c*d^3*x^3 - 3*b
^4*d^4*x^4 + b^4*c^4*Cos[2*(a + b*x)] - 12*b^2*c^2*d^2*Cos[2*(a + b*x)] +
24*d^4*Cos[2*(a + b*x)] + 4*b^4*c^3*d*x*Cos[2*(a + b*x)] - 24*b^2*c*d^3*x*
Cos[2*(a + b*x)] + 6*b^4*c^2*d^2*x^2*Cos[2*(a + b*x)] - 12*b^2*d^4*x^2*Cos
[2*(a + b*x)] + 4*b^4*c*d^3*x^3*Cos[2*(a + b*x)] + b^4*d^4*x^4*Cos[2*(a +
b*x)] - 16*b^3*c^3*d*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] -
 48*b^3*c^2*d^2*x*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] - 48
*b^3*c*d^3*x^2*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] - 16*b^
3*d^4*x^3*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] + (24*I)*b^2
*d^2*(c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]]*Sin[a + b*x] -
 (24*I)*b^2*d^2*(c + d*x)^2*PolyLog[2, Cos[a + b*x] + I*Sin[a + b*x]]*Sin[
a + b*x] - 48*b*c*d^3*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]]*Sin[a + b
*x] - 48*b*d^4*x*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]]*Sin[a + b*x] +
 48*b*c*d^3*PolyLog[3, Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] + 48*b*
d^4*x*PolyLog[3, Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] - (48*I)*d^4*
PolyLog[4, -Cos[a + b*x] - I*Sin[a + b*x]]*Sin[a + b*x] + (48*I)*d^4*PolyL
og[4, Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] - 4*b^3*c^3*d*Sin[2*(a +
 b*x)] + 24*b*c*d^3*Sin[2*(a + b*x)] - 12*b^3*c^2*d^2*x*Sin[2*(a + b*x)] +
 24*b*d^4*x*Sin[2*(a + b*x)] - 12*b^3*c*d^3*x^2*Sin[2*(a + b*x)] - 4*b^...
```

### 3.171.3 Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.10, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {4908, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117, 4910, 3042, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \cos(a+bx)\cot^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 4908}$$

$$\int (c+dx)^4 \cot(a+bx)\csc(a+bx)dx-\int (c+dx)^4\cos(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^4 \cot(a+bx)\csc(a+bx)dx-\int (c+dx)^4\sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 3777}$$

$$-\frac{4d\int -(c+dx)^3\sin(a+bx)dx}{b}+\int (c+dx)^4\cot(a+bx)\csc(a+bx)dx-\frac{(c+dx)^4\sin(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{4d\int (c+dx)^3\sin(a+bx)dx}{b}+\int (c+dx)^4\cot(a+bx)\csc(a+bx)dx-\frac{(c+dx)^4\sin(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4d\int (c+dx)^3\sin(a+bx)dx}{b}+\int (c+dx)^4\cot(a+bx)\csc(a+bx)dx-\frac{(c+dx)^4\sin(a+bx)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{4d\left(\frac{3d\int (c+dx)^2\cos(a+bx)dx}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}+\int (c+dx)^4\cot(a+bx)\csc(a+bx)dx-\frac{(c+dx)^4\sin(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4d\left(\frac{3d\int (c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}+\int (c+dx)^4\cot(a+bx)\csc(a+bx)dx-\frac{(c+dx)^4\sin(a+bx)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{4d\left(\frac{3d\left(\frac{2d\int -((c+dx)\sin(a+bx))dx}{b}+\frac{(c+dx)^2\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}+\int (c+dx)^4\cot(a+bx)\csc(a+bx)dx-\frac{(c+dx)^4\sin(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int (c+dx)\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}+\int (c+dx)^4\cot(a+bx)\csc(a+bx)dx-\frac{(c+dx)^4\sin(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

3.171. $\int (c+dx)^4\cos(a+bx)\cot^2(a+bx)\,dx$

$$\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\int(c+dx)\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}+\int(c+dx)^4\cot(a+bx)\csc(a+bx)dx-\frac{(c+dx)^4\sin(a+bx)}{b}$$

$$\Big\downarrow \textbf{3777}$$

$$\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\cos(a+bx)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}+\int(c+dx)^4\cot(a+bx)\csc(a+bx)dx-\frac{(c+dx)^4\sin(a+bx)}{b}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}+\int(c+dx)^4\cot(a+bx)\csc(a+bx)dx-\frac{(c+dx)^4\sin(a+bx)}{b}$$

$$\Big\downarrow \textbf{3117}$$

$$\int(c+dx)^4\cot(a+bx)\csc(a+bx)dx+\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^4\sin(a+bx)}{b}$$

$$\Big\downarrow \textbf{4910}$$

3.171. $\int(c+dx)^4\cos(a+bx)\cot^2(a+bx)\,dx$

$$\frac{4d\int (c+dx)^3\csc(a+bx)dx}{b}+\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{(c+dx)^4\csc(a+bx)}{b}$$

$\Big\downarrow$ 3042

$$\frac{4d\int (c+dx)^3\csc(a+bx)dx}{b}+\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{(c+dx)^4\csc(a+bx)}{b}$$

$\Big\downarrow$ 4671

$$\frac{4d\left(-\frac{3d\int (c+dx)^2\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{3d\int (c+dx)^2\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)}{b}+\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{(c+dx)^4\csc(a+bx)}{b}$$

$\Big\downarrow$ 3011

$$\frac{4d\left(\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\int (c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{2id\int (c+dx)\text{PolyLog}\left(2,e^{i(a+b}{b}\right.}{b}\right.}{b}+\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{(c+dx)^4\csc(a+bx)}{b}$$

3.171. $\int (c+dx)^4\cos(a+bx)\cot^2(a+bx)\,dx$

$$\Big\downarrow \text{7163}$$

$$\frac{4d\left(\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{id\int \operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}\cdots\right.}{\cdots}\cdots\right.}{b}\cdots$$

$$\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{(c+dx)^4\csc(a+bx)}{b}$$

$$\Big\downarrow \text{2720}$$

$$\frac{4d\left(\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{Po}\cdots}{\cdots}\right.}{\cdots}\cdots\right.}{b}\cdots$$

$$\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{(c+dx)^4\csc(a+bx)}{b}$$

$$\Big\downarrow \text{7143}$$

---

3.171. $\int (c+dx)^4\cos(a+bx)\cot^2(a+bx)\,dx$

$$\frac{4d\left(-\frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\operatorname{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\right.}{b}$$

$$\frac{4d\left(\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}-\frac{(c+dx)^3\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^4\sin(a+bx)}{b}-\frac{(c+dx)^4\csc(a+bx)}{b}$$

input
```
Int[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

output
```
-(((c + d*x)^4*Csc[a + b*x])/b) + (4*d*((-2*(c + d*x)^3*ArcTanh[E^(I*(a +
b*x))])/b + (3*d*((I*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b - ((2*I)*
d*(((-I)*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b + (d*PolyLog[4, -E^(I*(
a + b*x))])/b^2))/b))/b - (3*d*((I*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))]
)/b - ((2*I)*d*(((-I)*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b + (d*PolyLo
g[4, E^(I*(a + b*x))])/b^2))/b))/b))/b - ((c + d*x)^4*Sin[a + b*x])/b + (4
*d*(-(((c + d*x)^3*Cos[a + b*x])/b) + (3*d*(((c + d*x)^2*Sin[a + b*x])/b -
 (2*d*(-(((c + d*x)*Cos[a + b*x])/b) + (d*Sin[a + b*x])/b^2))/b))/b))/b
```

### 3.171.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.171. $\int (c+dx)^4 \cos(a+bx) \cot^2(a+bx)\,dx$

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

## 3.171.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1055 vs. 2(281) = 562.

Time = 4.40 (sec) , antiderivative size = 1056, normalized size of antiderivative = 3.53

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1056 |

input
```
int((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
24*I*d^4*polylog(4,exp(I*(b*x+a)))/b^5-24*I*d^4*polylog(4,-exp(I*(b*x+a)))
/b^5-1/2*I*(d^4*x^4*b^4+4*b^4*c*d^3*x^3+6*b^4*c^2*d^2*x^2+4*b^4*c^3*d*x-4*
I*b^3*d^4*x^3+b^4*c^4-12*b^2*d^4*x^2-12*I*b^3*c*d^3*x^2-24*b^2*c*d^3*x-12*
I*b^3*c^2*d^2*x-12*b^2*c^2*d^2-4*I*b^3*c^3*d+24*I*b*d^4*x+24*d^4+24*I*b*c*
d^3)/b^5*exp(-I*(b*x+a))-24*I*d^3/b^3*c*polylog(2,exp(I*(b*x+a)))*x+4*d^4/
b^2*ln(1-exp(I*(b*x+a)))*x^3-4*d^4/b^2*ln(exp(I*(b*x+a))+1)*x^3-24*d^3/b^4
*c*polylog(3,-exp(I*(b*x+a)))-12*I*d^2/b^3*c^2*polylog(2,exp(I*(b*x+a)))+1
2*I*d^2/b^3*c^2*polylog(2,-exp(I*(b*x+a)))-12*d^3/b^2*c*ln(exp(I*(b*x+a))+
1)*x^2+12*d^2/b^2*c^2*ln(1-exp(I*(b*x+a)))*x-12*d^2/b^2*c^2*ln(exp(I*(b*x+
a))+1)*x+12*d^3/b^4*c*ln(exp(I*(b*x+a))+1)*a^2-12*d^3/b^4*c*ln(1-exp(I*(b*
x+a)))*a^2+12*d^2/b^3*c^2*ln(1-exp(I*(b*x+a)))*a-12*d^2/b^3*c^2*ln(exp(I*(
b*x+a))+1)*a+12*d^3/b^2*c*ln(1-exp(I*(b*x+a)))*x^2-24*d^3/b^4*c*a^2*arctan
h(exp(I*(b*x+a)))+24*d^2/b^3*c^2*a*arctanh(exp(I*(b*x+a)))-12*I*d^4/b^3*po
lylog(2,exp(I*(b*x+a)))*x^2+12*I*d^4/b^3*polylog(2,-exp(I*(b*x+a)))*x^2-24
*d^4/b^4*polylog(3,-exp(I*(b*x+a)))*x+24*d^4/b^4*polylog(3,exp(I*(b*x+a)))
*x-8*d/b^2*c^3*arctanh(exp(I*(b*x+a)))+4*d^4/b^5*ln(1-exp(I*(b*x+a)))*a^3-
4*d^4/b^5*ln(exp(I*(b*x+a))+1)*a^3-2*I*(d^4*x^4+4*c*d^3*x^3+6*c^2*d^2*x^2+
4*c^3*d*x+c^4)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)+1/2*I*(d^4*x^4*b^4+4*
b^4*c*d^3*x^3+6*b^4*c^2*d^2*x^2+4*b^4*c^3*d*x+4*I*b^3*d^4*x^3+b^4*c^4-12*b
^2*d^4*x^2+12*I*b^3*c*d^3*x^2-24*b^2*c*d^3*x+12*I*b^3*c^2*d^2*x-12*b^2*...
```

### 3.171.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1233 vs. $2(275) = 550$.

Time = 0.34 (sec) , antiderivative size = 1233, normalized size of antiderivative = 4.12

$$\int (c+dx)^4 \cos(a+bx)\cot^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 12*b^2*c^2*d^2 - 12*I*d^4*
polylog(4, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*I*d^4*polylog(
4, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - 12*I*d^4*polylog(4, -cos(
b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*I*d^4*polylog(4, -cos(b*x + a
) - I*sin(b*x + a))*sin(b*x + a) + 24*d^4 + 12*(b^4*c^2*d^2 - b^2*d^4)*x^2
 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 24*d^4 + 6*
(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a
)^2 + 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d - 6*b*c*d^3 + 3*(b^3*c^
2*d^2 - 2*b*d^4)*x)*cos(b*x + a)*sin(b*x + a) + 6*(I*b^2*d^4*x^2 + 2*I*b^2
*c*d^3*x + I*b^2*c^2*d^2)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a
) + 6*(-I*b^2*d^4*x^2 - 2*I*b^2*c*d^3*x - I*b^2*c^2*d^2)*dilog(cos(b*x + a
) - I*sin(b*x + a))*sin(b*x + a) + 6*(I*b^2*d^4*x^2 + 2*I*b^2*c*d^3*x + I*
b^2*c^2*d^2)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*(-I*b^
2*d^4*x^2 - 2*I*b^2*c*d^3*x - I*b^2*c^2*d^2)*dilog(-cos(b*x + a) - I*sin(b
*x + a))*sin(b*x + a) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x
 + b^3*c^3*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 2*(b^3
*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*log(cos(b*x + a)
 - I*sin(b*x + a) + 1)*sin(b*x + a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a
^2*b*c*d^3 - a^3*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*si
n(b*x + a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*...
```

### 3.171.6 Sympy [F]

$$\int (c+dx)^4 \cos(a+bx)\cot^2(a+bx)\,dx = \int (c+dx)^4 \cos(a+bx)\cot^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**4*cos(b*x+a)*cot(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**4*cos(a + b*x)*cot(a + b*x)**2, x)
```

### 3.171.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 17589 vs. 2(275) = 550.

Time = 4.51 (sec) , antiderivative size = 17589, normalized size of antiderivative = 58.83

$$\int (c+dx)^4 \cos(a+bx) \cot^2(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*c^4*(1/sin(b*x + a) + sin(b*x + a)) - 8*a*c^3*d*(1/sin(b*x + a) +
sin(b*x + a))/b + 12*a^2*c^2*d^2*(1/sin(b*x + a) + sin(b*x + a))/b^2 - 8*a
^3*c*d^3*(1/sin(b*x + a) + sin(b*x + a))/b^3 + 2*a^4*d^4*(1/sin(b*x + a) +
 sin(b*x + a))/b^4 - 4*(((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1
)*cos(3*b*x + 3*a)^3 + (b*x - (b*x + a)*cos(2*b*x + 2*a) + a - sin(2*b*x +
 2*a))*sin(3*b*x + 3*a)^3 - 6*(b*x + a)*sin(b*x + a)^3 - 2*(4*(b*x + a)*co
s(b*x + a)*sin(2*b*x + 2*a) - (3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*co
s(2*b*x + 2*a) + 3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a)
^2 - ((b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a)^2 + (8*(b*x
+ a)*cos(2*b*x + 2*a)*sin(b*x + a) + ((b*x + a)*sin(2*b*x + 2*a) - cos(2*b
*x + 2*a) + 1)*cos(3*b*x + 3*a) - 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x +
a))*sin(2*b*x + 2*a) - 8*(b*x + a)*sin(b*x + a))*sin(3*b*x + 3*a)^2 - ((b*
x + a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a)^2 + (12*(b*x + a)*cos
(b*x + a)*sin(b*x + a) - (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x
 + a)^2 + sin(b*x + a)^2 + 2)*cos(2*b*x + 2*a) + cos(2*b*x + 2*a)^2 + cos(
b*x + a)^2 + (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2)*sin(
2*b*x + 2*a) + sin(2*b*x + 2*a)^2 + sin(b*x + a)^2 + 1)*cos(3*b*x + 3*a) +
 2*(3*(b*x + a)*sin(b*x + a)^3 + (3*(b*x + a)*cos(b*x + a)^2 + b*x + a)*si
n(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a) - ((cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x +...
```

### 3.171.8 Giac [F]

$$\int (c+dx)^4 \cos(a+bx) \cot^2(a+bx) \, dx = \int (dx+c)^4 \cos(bx+a) \cot(bx+a)^2 \, dx$$

input
```
integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*cos(b*x + a)*cot(b*x + a)^2, x)
```

### 3.171.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \cos(a+bx)\cot^2(a+bx)\,dx = \int \cos(a+b\,x)\,\cot(a+b\,x)^2\,(c+d\,x)^4\,dx$$

input
```
int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^4,x)
```

output
```
int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^4, x)
```

# 3.172 $\int (c+dx)^3 \cos(a+bx) \cot^2(a+bx) \, dx$

3.172.1 Optimal result . . . 1371
3.172.2 Mathematica [B] (verified) . . . 1372
3.172.3 Rubi [A] (verified) . . . 1372
3.172.4 Maple [B] (verified) . . . 1377
3.172.5 Fricas [B] (verification not implemented) . . . 1378
3.172.6 Sympy [F] . . . 1379
3.172.7 Maxima [B] (verification not implemented) . . . 1379
3.172.8 Giac [F] . . . 1380
3.172.9 Mupad [F(-1)] . . . 1380

## 3.172.1 Optimal result

Integrand size = 22, antiderivative size = 216

$$\int (c+dx)^3 \cos(a+bx) \cot^2(a+bx) \, dx = -\frac{6d(c+dx)^2 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b^2} + \frac{6d^3 \cos(a+bx)}{b^4} - \frac{3d(c+dx)^2 \cos(a+bx)}{b^2} - \frac{(c+dx)^3 \csc(a+bx)}{b} + \frac{6id^2(c+dx) \operatorname{PolyLog}\left(2, -e^{i(a+bx)}\right)}{b^3} - \frac{6id^2(c+dx) \operatorname{PolyLog}\left(2, e^{i(a+bx)}\right)}{b^3} - \frac{6d^3 \operatorname{PolyLog}\left(3, -e^{i(a+bx)}\right)}{b^4} + \frac{6d^3 \operatorname{PolyLog}\left(3, e^{i(a+bx)}\right)}{b^4} + \frac{6d^2(c+dx) \sin(a+bx)}{b^3} - \frac{(c+dx)^3 \sin(a+bx)}{b}$$

output
```
-6*d*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b^2+6*d^3*cos(b*x+a)/b^4-3*d*(d*x+c
)^2*cos(b*x+a)/b^2-(d*x+c)^3*csc(b*x+a)/b+6*I*d^2*(d*x+c)*polylog(2,-exp(I
*(b*x+a)))/b^3-6*I*d^2*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^3-6*d^3*polylog
(3,-exp(I*(b*x+a)))/b^4+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4+6*d^2*(d*x+c)*
sin(b*x+a)/b^3-(d*x+c)^3*sin(b*x+a)/b
```

### 3.172.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 539 vs. 2(216) = 432.

Time = 1.49 (sec) , antiderivative size = 539, normalized size of antiderivative = 2.50

$$\int (c+dx)^3\cos(a+bx)\cot^2(a+bx)\,dx$$
$$= \frac{\csc(a+bx)\left(-3b^3c^3+6bcd^2-9b^3c^2dx+6bd^3x-9b^3cd^2x^2-3b^3d^3x^3+b^3c^3\cos(2(a+bx))-6bcd^2\cos(2\right.}{}$$

input
```
Integrate[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

output
```
(Csc[a + b*x]*(-3*b^3*c^3 + 6*b*c*d^2 - 9*b^3*c^2*d*x + 6*b*d^3*x - 9*b^3*
c*d^2*x^2 - 3*b^3*d^3*x^3 + b^3*c^3*Cos[2*(a + b*x)] - 6*b*c*d^2*Cos[2*(a
+ b*x)] + 3*b^3*c^2*d*x*Cos[2*(a + b*x)] - 6*b*d^3*x*Cos[2*(a + b*x)] + 3*
b^3*c*d^2*x^2*Cos[2*(a + b*x)] + b^3*d^3*x^3*Cos[2*(a + b*x)] + 6*b^2*c^2*
d*Log[1 - E^(I*(a + b*x))]*Sin[a + b*x] + 12*b^2*c*d^2*x*Log[1 - E^(I*(a +
 b*x))]*Sin[a + b*x] + 6*b^2*d^3*x^2*Log[1 - E^(I*(a + b*x))]*Sin[a + b*x]
 - 6*b^2*c^2*d*Log[1 + E^(I*(a + b*x))]*Sin[a + b*x] - 12*b^2*c*d^2*x*Log[
1 + E^(I*(a + b*x))]*Sin[a + b*x] - 6*b^2*d^3*x^2*Log[1 + E^(I*(a + b*x))]
*Sin[a + b*x] + (12*I)*b*d^2*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))]*Sin[a
+ b*x] - (12*I)*b*d^2*(c + d*x)*PolyLog[2, E^(I*(a + b*x))]*Sin[a + b*x] -
 12*d^3*PolyLog[3, -E^(I*(a + b*x))]*Sin[a + b*x] + 12*d^3*PolyLog[3, E^(I
*(a + b*x))]*Sin[a + b*x] - 3*b^2*c^2*d*Sin[2*(a + b*x)] + 6*d^3*Sin[2*(a
+ b*x)] - 6*b^2*c*d^2*x*Sin[2*(a + b*x)] - 3*b^2*d^3*x^2*Sin[2*(a + b*x)])
)/(2*b^4)
```

### 3.172.3 Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.07, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {4908, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118, 4910, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\cos(a+bx)\cot^2(a+bx)\,dx$$

$$\downarrow \text{4908}$$

$$\int (c+dx)^3 \cot(a+bx)\csc(a+bx)dx - \int (c+dx)^3 \cos(a+bx)dx$$

$$\downarrow \text{3042}$$

$$\int (c+dx)^3 \cot(a+bx)\csc(a+bx)dx - \int (c+dx)^3 \sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\downarrow \text{3777}$$

$$-\frac{3d\int -(c+dx)^2 \sin(a+bx)dx}{b} + \int (c+dx)^3 \cot(a+bx)\csc(a+bx)dx - \frac{(c+dx)^3 \sin(a+bx)}{b}$$

$$\downarrow \text{25}$$

$$\frac{3d\int (c+dx)^2 \sin(a+bx)dx}{b} + \int (c+dx)^3 \cot(a+bx)\csc(a+bx)dx - \frac{(c+dx)^3 \sin(a+bx)}{b}$$

$$\downarrow \text{3042}$$

$$\frac{3d\int (c+dx)^2 \sin(a+bx)dx}{b} + \int (c+dx)^3 \cot(a+bx)\csc(a+bx)dx - \frac{(c+dx)^3 \sin(a+bx)}{b}$$

$$\downarrow \text{3777}$$

$$\frac{3d\left(\frac{2d\int (c+dx)\cos(a+bx)dx}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b} + \int (c+dx)^3 \cot(a+bx)\csc(a+bx)dx - \frac{(c+dx)^3 \sin(a+bx)}{b}$$

$$\downarrow \text{3042}$$

$$\frac{3d\left(\frac{2d\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b} + \int (c+dx)^3 \cot(a+bx)\csc(a+bx)dx - \frac{(c+dx)^3 \sin(a+bx)}{b}$$

$$\downarrow \text{3777}$$

$$\frac{3d\left(\frac{2d\left(\frac{d\int -\sin(a+bx)dx}{b} + \frac{(c+dx)\sin(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b} + \int (c+dx)^3 \cot(a+bx)\csc(a+bx)dx - \frac{(c+dx)^3 \sin(a+bx)}{b}$$

$$\downarrow \text{25}$$

3.172. $\int (c+dx)^3 \cos(a+bx)\cot^2(a+bx)\,dx$

$$\frac{3d\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}+\int(c+dx)^3\cot(a+bx)\csc(a+bx)dx-\frac{(c+dx)^3\sin(a+bx)}{b}$$

$\big\downarrow$ 3042

$$\frac{3d\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int\sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}+\int(c+dx)^3\cot(a+bx)\csc(a+bx)dx-\frac{(c+dx)^3\sin(a+bx)}{b}$$

$\big\downarrow$ 3118

$$\int(c+dx)^3\cot(a+bx)\csc(a+bx)dx+\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\sin(a+bx)}{b}$$

$\big\downarrow$ 4910

$$\frac{3d\int(c+dx)^2\csc(a+bx)dx}{b}+\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

$\big\downarrow$ 3042

$$\frac{3d\int(c+dx)^2\csc(a+bx)dx}{b}+\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

$\big\downarrow$ 4671

$$\frac{3d\left(-\frac{2d\int(c+dx)\log\left(1-e^{i(a+bx)}\right)dx}{b}+\frac{2d\int(c+dx)\log\left(1+e^{i(a+bx)}\right)dx}{b}-\frac{2(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}\right)}{b}+\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

$\big\downarrow$ 3011

3.172. $\int(c+dx)^3\cos(a+bx)\cot^2(a+bx)\,dx$

$$\frac{3d\left(\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{id\int \operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{id\int \operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2(c+\cdots\right)}{b}$$

$$\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

$\downarrow$ 2720

$$\frac{3d\left(\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\cdots}{b\cdots}\right)}{b}\cdots\right)}{b}$$

$$\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

$\downarrow$ 7143

$$\frac{3d\left(-\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b}-\frac{d\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b}-\frac{d\operatorname{Poly}\cdots}{\cdots}\right)}{b}\cdots\right)}{b}$$

$$\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\sin(a+bx)}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

input
```
Int[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

output
```
-(((c + d*x)^3*Csc[a + b*x])/b) + (3*d*((-2*(c + d*x)^2*ArcTanh[E^(I*(a +
b*x))])/b + (2*d*((I*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b - (d*PolyLo
g[3, -E^(I*(a + b*x))])/b^2))/b - (2*d*((I*(c + d*x)*PolyLog[2, E^(I*(a +
b*x))])/b - (d*PolyLog[3, E^(I*(a + b*x))])/b^2))/b))/b - ((c + d*x)^3*Sin
[a + b*x])/b + (3*d*(-(((c + d*x)^2*Cos[a + b*x])/b) + (2*d*((d*Cos[a + b*
x])/b^2 + ((c + d*x)*Sin[a + b*x])/b))/b))/b
```

## 3.172. $\int (c+dx)^3\cos(a+bx)\cot^2(a+bx)\,dx$

### 3.172.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.172.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(204) = 408$.

Time = 2.56 (sec) , antiderivative size = 649, normalized size of antiderivative = 3.00

| method | result |
|---|---|
| risch | $-\frac{6id^2c\,\mathrm{polylog}(2,\mathrm{e}^{i(xb+a)})}{b^3}-\frac{2i(d^3x^3+3c\,d^2x^2+3c^2dx+c^3)\mathrm{e}^{i(xb+a)}}{b(\mathrm{e}^{2i(xb+a)}-1)}+\frac{6id^2c\,\mathrm{polylog}(2,-\mathrm{e}^{i(xb+a)})}{b^3}-\frac{i(d^3x^3b^3+3b^3c\,d^2x^2-3i}{}$ |

input
```
int((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-6*I*d^2/b^3*c*polylog(2,exp(I*(b*x+a)))-2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*
x+c^3)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)+6*I*d^2/b^3*c*polylog(2,-exp(
I*(b*x+a)))-1/2*I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b
^2*d^3*x^2-6*b*d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*
exp(-I*(b*x+a))+1/2*I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3
*I*b^2*d^3*x^2-6*b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/
b^4*exp(I*(b*x+a))+12*d^2/b^3*c*a*arctanh(exp(I*(b*x+a)))-6*d^3/b^4*a^2*ar
ctanh(exp(I*(b*x+a)))+3*d^3/b^2*ln(1-exp(I*(b*x+a)))*x^2+6*d^3*polylog(3,e
xp(I*(b*x+a)))/b^4-3*d^3/b^2*ln(exp(I*(b*x+a))+1)*x^2-6*d^3*polylog(3,-exp
(I*(b*x+a)))/b^4-3*d^3/b^4*ln(1-exp(I*(b*x+a)))*a^2+3*d^3/b^4*ln(exp(I*(b*
x+a))+1)*a^2+6*d^2/b^2*c*ln(1-exp(I*(b*x+a)))*x-6*d^2/b^2*c*ln(exp(I*(b*x+
a))+1)*x+6*d^2/b^3*c*ln(1-exp(I*(b*x+a)))*a-6*d^2/b^3*c*ln(exp(I*(b*x+a))+
1)*a+6*I*d^3/b^3*polylog(2,-exp(I*(b*x+a)))*x-6*I*d^3/b^3*polylog(2,exp(I*
(b*x+a)))*x-6*d/b^2*c^2*arctanh(exp(I*(b*x+a)))
```

### 3.172.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 797 vs. $2(200) = 400$.

Time = 0.32 (sec) , antiderivative size = 797, normalized size of antiderivative = 3.69

$$\int (c+dx)^3\cos(a+bx)\cot^2(a+bx)\,dx =$$
$$-\frac{4\,b^3d^3x^3+12\,b^3cd^2x^2+4\,b^3c^3-6\,d^3\text{polylog}(3,\cos{(bx+a)}+i\,\sin{(bx+a)})\sin{(bx+a)}-6\,d^3\text{polylog}(}{}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 4*b^3*c^3 - 6*d^3*polylog(3, cos(
b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 6*d^3*polylog(3, cos(b*x + a) -
I*sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a) + I*sin(b*x
+ a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(
b*x + a) - 12*b*c*d^2 - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c
*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 6*(b^2*d^3*x^2 + 2*b^2*
c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a)*sin(b*x + a) + 6*(I*b*d^3*x + I*
b*c*d^2)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*(-I*b*d^3*x
 - I*b*c*d^2)*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*(I*b*d
^3*x + I*b*c*d^2)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*(
-I*b*d^3*x - I*b*c*d^2)*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a)
 + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a) + I*sin(b*
x + a) + 1)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log
(cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x +
 a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*
sin(b*x + a) + 1/2)*sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*
c*d^2 - a^2*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - 3*
(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(b*x + a) -
I*sin(b*x + a) + 1)*sin(b*x + a) + 12*(b^3*c^2*d - b*d^3)*x)/(b^4*sin(b...
```

### 3.172.6 Sympy [F]

$$\int (c+dx)^3 \cos(a+bx)\cot^2(a+bx)\,dx = \int (c+dx)^3 \cos\left(a+bx\right)\cot^2\left(a+bx\right)dx$$

input
```
integrate((d*x+c)**3*cos(b*x+a)*cot(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**3*cos(a + b*x)*cot(a + b*x)**2, x)
```

### 3.172.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 10994 vs. $2(200) = 400$.

Time = 1.31 (sec) , antiderivative size = 10994, normalized size of antiderivative = 50.90

$$\int (c+dx)^3 \cos(a+bx)\cot^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*c^3*(1/sin(b*x + a) + sin(b*x + a)) - 6*a*c^2*d*(1/sin(b*x + a) +
sin(b*x + a))/b + 6*a^2*c*d^2*(1/sin(b*x + a) + sin(b*x + a))/b^2 - 2*a^3*
d^3*(1/sin(b*x + a) + sin(b*x + a))/b^3 - 3*(((b*x + a)*sin(2*b*x + 2*a) -
 cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^3 + (b*x - (b*x + a)*cos(2*b*x + 2
*a) + a - sin(2*b*x + 2*a))*sin(3*b*x + 3*a)^3 - 6*(b*x + a)*sin(b*x + a)^
3 - 2*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - (3*(b*x + a)*sin(b*x +
a) + cos(b*x + a))*cos(2*b*x + 2*a) + 3*(b*x + a)*sin(b*x + a) + cos(b*x +
 a))*cos(3*b*x + 3*a)^2 - ((b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*
x + 2*a)^2 + (8*(b*x + a)*cos(2*b*x + 2*a)*sin(b*x + a) + ((b*x + a)*sin(2
*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a) - 2*(3*(b*x + a)*cos(
b*x + a) - sin(b*x + a))*sin(2*b*x + 2*a) - 8*(b*x + a)*sin(b*x + a))*sin(
3*b*x + 3*a)^2 - ((b*x + a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a)^
2 + (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) - (12*(b*x + a)*cos(b*x + a)*s
in(b*x + a) + cos(b*x + a)^2 + sin(b*x + a)^2 + 2)*cos(2*b*x + 2*a) + cos(
2*b*x + 2*a)^2 + cos(b*x + a)^2 + (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)
*sin(b*x + a)^2)*sin(2*b*x + 2*a) + sin(2*b*x + 2*a)^2 + sin(b*x + a)^2 +
1)*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*sin(b*x + a)^3 + (3*(b*x + a)*cos(b*x
 + a)^2 + b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a) - ((cos(2
*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3
*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b...
```

### 3.172.8 Giac [F]

$$\int (c+dx)^3 \cos(a+bx) \cot^2(a+bx) \, dx = \int (dx+c)^3 \cos(bx+a) \cot(bx+a)^2 \, dx$$

input
```
integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*cos(b*x + a)*cot(b*x + a)^2, x)
```

### 3.172.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \cos(a+bx) \cot^2(a+bx) \, dx = \int \cos(a+b\,x)\, \cot(a+b\,x)^2\, (c+d\,x)^3 \, dx$$

input
```
int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^3,x)
```

output
```
int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^3, x)
```

# 3.173 $\int (c+dx)^2 \cos(a+bx) \cot^2(a+bx)\,dx$

3.173.1 Optimal result . . . . . 1381
3.173.2 Mathematica [B] (verified) . . . . . 1382
3.173.3 Rubi [A] (verified) . . . . . 1382
3.173.4 Maple [B] (verified) . . . . . 1385
3.173.5 Fricas [B] (verification not implemented) . . . . . 1386
3.173.6 Sympy [F] . . . . . 1387
3.173.7 Maxima [B] (verification not implemented) . . . . . 1387
3.173.8 Giac [F] . . . . . 1388
3.173.9 Mupad [F(-1)] . . . . . 1388

## 3.173.1 Optimal result

Integrand size = 22, antiderivative size = 139

$$\int (c+dx)^2 \cos(a+bx) \cot^2(a+bx)\,dx = -\frac{4d(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2d(c+dx)\cos(a+bx)}{b^2} - \frac{(c+dx)^2\csc(a+bx)}{b} + \frac{2id^2\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3} - \frac{2id^2\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3} + \frac{2d^2\sin(a+bx)}{b^3} - \frac{(c+dx)^2\sin(a+bx)}{b}$$

output
```
-4*d*(d*x+c)*arctanh(exp(I*(b*x+a)))/b^2-2*d*(d*x+c)*cos(b*x+a)/b^2-(d*x+c
)^2*csc(b*x+a)/b+2*I*d^2*polylog(2,-exp(I*(b*x+a)))/b^3-2*I*d^2*polylog(2,
exp(I*(b*x+a)))/b^3+2*d^2*sin(b*x+a)/b^3-(d*x+c)^2*sin(b*x+a)/b
```

### 3.173.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 310 vs. 2(139) = 278.

Time = 4.43 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.23

$$\int (c+dx)^2 \cos(a+bx)\cot^2(a+bx)\,dx =$$

$$-\frac{8bcd\text{arctanh}\left(\cos(a)-\sin(a)\tan\left(\frac{bx}{2}\right)\right)+2b^2(c+dx)^2\csc(a)-4d^2\Big(2\arctan(\tan(a))\text{arctanh}\left(\cos(a)-\right.}{}$$

input
```
Integrate[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

output
```
-1/2*(8*b*c*d*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] + 2*b^2*(c + d*x)^2*Cs
c[a] - 4*d^2*(2*ArcTan[Tan[a]]*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] + (((
b*x + ArcTan[Tan[a]])*(Log[1 - E^(I*(b*x + ArcTan[Tan[a]]))] - Log[1 + E^(
I*(b*x + ArcTan[Tan[a]]))]) + I*PolyLog[2, -E^(I*(b*x + ArcTan[Tan[a]]))]
- I*PolyLog[2, E^(I*(b*x + ArcTan[Tan[a]]))])*Sec[a])/Sqrt[Sec[a]^2]) + 2*
Cos[b*x]*(2*b*d*(c + d*x)*Cos[a] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a]) - b^
2*(c + d*x)^2*Csc[a/2]*Csc[(a + b*x)/2]*Sin[(b*x)/2] + b^2*(c + d*x)^2*Sec
[a/2]*Sec[(a + b*x)/2]*Sin[(b*x)/2] + 2*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a]
 - 2*b*d*(c + d*x)*Sin[a])*Sin[b*x])/b^3
```

### 3.173.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {4908, 3042, 3777, 25, 3042, 3777, 3042, 3117, 4910, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \cos(a+bx)\cot^2(a+bx)\,dx$$

$$\Big\downarrow \text{4908}$$

$$\int (c+dx)^2 \cot(a+bx)\csc(a+bx)dx - \int (c+dx)^2 \cos(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2 \cot(a+bx)\csc(a+bx)dx - \int (c+dx)^2 \sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$-\frac{2d\int -((c+dx)\sin(a+bx))dx}{b} + \int (c+dx)^2 \cot(a+bx)\csc(a+bx)dx - \frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{2d\int (c+dx)\sin(a+bx)dx}{b} + \int (c+dx)^2 \cot(a+bx)\csc(a+bx)dx - \frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\int (c+dx)\sin(a+bx)dx}{b} + \int (c+dx)^2 \cot(a+bx)\csc(a+bx)dx - \frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{2d\left(\frac{d\int \cos(a+bx)dx}{b} - \frac{(c+dx)\cos(a+bx)}{b}\right)}{b} + \int (c+dx)^2 \cot(a+bx)\csc(a+bx)dx - \frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\left(\frac{d\int \sin\left(a+bx+\frac{\pi}{2}\right)dx}{b} - \frac{(c+dx)\cos(a+bx)}{b}\right)}{b} + \int (c+dx)^2 \cot(a+bx)\csc(a+bx)dx - \frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\Big\downarrow \text{3117}$$

$$\int (c+dx)^2 \cot(a+bx)\csc(a+bx)dx + \frac{2d\left(\frac{d\sin(a+bx)}{b^2} - \frac{(c+dx)\cos(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\Big\downarrow \text{4910}$$

$$\frac{2d\int (c+dx)\csc(a+bx)dx}{b} + \frac{2d\left(\frac{d\sin(a+bx)}{b^2} - \frac{(c+dx)\cos(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2\sin(a+bx)}{b} - \frac{(c+dx)^2\csc(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\int (c+dx)\csc(a+bx)dx}{b} + \frac{2d\left(\frac{d\sin(a+bx)}{b^2} - \frac{(c+dx)\cos(a+bx)}{b}\right)}{b} - \frac{(c+dx)^2\sin(a+bx)}{b} - \frac{(c+dx)^2\csc(a+bx)}{b}$$

$$\Big\downarrow \text{4671}$$

3.173. $\int (c+dx)^2 \cos(a+bx)\cot^2(a+bx)\,dx$

$$\frac{2d\left(-\frac{d\int\log(1-e^{i(a+bx)})dx}{b}+\frac{d\int\log(1+e^{i(a+bx)})dx}{b}-\frac{2(c+dx)\text{arctanh}(e^{i(a+bx)})}{b}\right)}{b}+\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

$$\Big\downarrow \text{2715}$$

$$\frac{2d\left(\frac{id\int e^{-i(a+bx)}\log(1-e^{i(a+bx)})de^{i(a+bx)}}{b^2}-\frac{id\int e^{-i(a+bx)}\log(1+e^{i(a+bx)})de^{i(a+bx)}}{b^2}-\frac{2(c+dx)\text{arctanh}(e^{i(a+bx)})}{b}\right)}{b}+\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

$$\Big\downarrow \text{2838}$$

$$\frac{2d\left(-\frac{2(c+dx)\text{arctanh}(e^{i(a+bx)})}{b}+\frac{id\,\text{PolyLog}(2,-e^{i(a+bx)})}{b^2}-\frac{id\,\text{PolyLog}(2,e^{i(a+bx)})}{b^2}\right)}{b}+\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

input
```
Int[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

output
```
-(((c + d*x)^2*Csc[a + b*x])/b) + (2*d*((-2*(c + d*x)*ArcTanh[E^(I*(a + b*
x))])/b + (I*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, E^(I*(a
 + b*x))])/b^2))/b - ((c + d*x)^2*Sin[a + b*x])/b + (2*d*(-(((c + d*x)*Cos
[a + b*x])/b) + (d*Sin[a + b*x])/b^2))/b
```

### 3.173.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.173. $\int (c+dx)^2\cos(a+bx)\cot^2(a+bx)\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

## 3.173.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(131) = 262$.

Time = 1.78 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.39

| method | result |
|---|---|
| risch | $\frac{i(x^2d^2b^2+2b^2cdx+2ib\,d^2x+b^2c^2+2ibcd-2d^2)\mathrm{e}^{i(xb+a)}}{2b^3} - \frac{i(x^2d^2b^2+2b^2cdx-2ib\,d^2x+b^2c^2-2ibcd-2d^2)\mathrm{e}^{-i(xb+a)}}{2b^3} - \frac{2i(x^2d^2+2c}{b(e^{2i(}}$ |

input
```
int((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*ex
p(I*(b*x+a))-1/2*I*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*
b*c*d)/b^3*exp(-I*(b*x+a))-2*I*(d^2*x^2+2*c*d*x+c^2)*exp(I*(b*x+a))/b/(exp
(2*I*(b*x+a))-1)-4*d/b^2*c*arctanh(exp(I*(b*x+a)))+2*d^2/b^2*ln(1-exp(I*(b
*x+a)))*x+2*d^2/b^3*ln(1-exp(I*(b*x+a)))*a-2*I*d^2*polylog(2,exp(I*(b*x+a)
))/b^3-2*d^2/b^2*ln(exp(I*(b*x+a))+1)*x-2*d^2/b^3*ln(exp(I*(b*x+a))+1)*a+2
*I*d^2*polylog(2,-exp(I*(b*x+a)))/b^3+4*d^2/b^3*a*arctanh(exp(I*(b*x+a)))
```

### 3.173.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(127) = 254$.

Time = 0.28 (sec) , antiderivative size = 448, normalized size of antiderivative = 3.22

$$\int (c+dx)^2 \cos(a+bx)\cot^2(a+bx)\,dx =$$
$$-\frac{2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 + i\,d^2\mathrm{Li}_2(\cos(bx+a) + i\,\sin(bx+a))\sin(bx+a) - i\,d^2\mathrm{Li}_2(\cos(bx+a) -}{}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

output
```
-(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + I*d^2*dilog(cos(b*x + a) + I*s
in(b*x + a))*sin(b*x + a) - I*d^2*dilog(cos(b*x + a) - I*sin(b*x + a))*sin
(b*x + a) + I*d^2*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - I*d
^2*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - (b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 + 2*(b*d^2*x + b*c*d)*cos(b*x +
 a)*sin(b*x + a) + (b*d^2*x + b*c*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1
)*sin(b*x + a) + (b*d^2*x + b*c*d)*log(cos(b*x + a) - I*sin(b*x + a) + 1)*
sin(b*x + a) - (b*c*d - a*d^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a)
+ 1/2)*sin(b*x + a) - (b*c*d - a*d^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*
x + a) + 1/2)*sin(b*x + a) - (b*d^2*x + a*d^2)*log(-cos(b*x + a) + I*sin(b
*x + a) + 1)*sin(b*x + a) - (b*d^2*x + a*d^2)*log(-cos(b*x + a) - I*sin(b*
x + a) + 1)*sin(b*x + a) - 2*d^2)/(b^3*sin(b*x + a))
```

### 3.173.6 Sympy [F]

$$\int (c+dx)^2\cos(a+bx)\cot^2(a+bx)\,dx=\int (c+dx)^2\cos{(a+bx)}\cot^2{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**2*cos(b*x+a)*cot(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*cos(a + b*x)*cot(a + b*x)**2, x)
```

### 3.173.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3199 vs. $2(127) = 254$.

Time = 1.14 (sec) , antiderivative size = 3199, normalized size of antiderivative = 23.01

$$\int (c+dx)^2\cos(a+bx)\cot^2(a+bx)\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(b^2*d^2*x^2*(-I*cos(a) + sin(a)) + b^2*c^2*(-I*cos(a) + sin(a)) - 2*b
*c*d*(cos(a) + I*sin(a)) - 2*d^2*(-I*cos(a) + sin(a)) - 2*(b^2*c*d*(I*cos(
a) - sin(a)) + b*d^2*(cos(a) + I*sin(a)))*x - 4*((b*d^2*x*(-I*cos(a) + sin
(a)) + b*c*d*(-I*cos(a) + sin(a)) + (I*b*d^2*x + I*b*c*d)*cos(2*b*x + 3*a)
 - (b*d^2*x + b*c*d)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + ((-I*b*d^2*x - I
*b*c*d)*cos(b*x + a) + (b*d^2*x + b*c*d)*sin(b*x + a))*cos(2*b*x + 3*a) +
(b*d^2*x*(I*cos(a) - sin(a)) + b*c*d*(I*cos(a) - sin(a)))*cos(b*x + a) + (
b*d^2*x*(cos(a) + I*sin(a)) + b*c*d*(cos(a) + I*sin(a)) - (b*d^2*x + b*c*d
)*cos(2*b*x + 3*a) + (-I*b*d^2*x - I*b*c*d)*sin(2*b*x + 3*a))*sin(3*b*x +
3*a) + ((b*d^2*x + b*c*d)*cos(b*x + a) + (I*b*d^2*x + I*b*c*d)*sin(b*x + a
))*sin(2*b*x + 3*a) - (b*d^2*x*(cos(a) + I*sin(a)) + b*c*d*(cos(a) + I*sin
(a)))*sin(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 4*(b*c*d*(-I
*cos(a) + sin(a))*cos(b*x + a) + b*c*d*(cos(a) + I*sin(a))*sin(b*x + a) +
(b*c*d*(I*cos(a) - sin(a)) - I*b*c*d*cos(2*b*x + 3*a) + b*c*d*sin(2*b*x +
3*a))*cos(3*b*x + 3*a) + (I*b*c*d*cos(b*x + a) - b*c*d*sin(b*x + a))*cos(2
*b*x + 3*a) - (b*c*d*(cos(a) + I*sin(a)) - b*c*d*cos(2*b*x + 3*a) - I*b*c*
d*sin(2*b*x + 3*a))*sin(3*b*x + 3*a) - (b*c*d*cos(b*x + a) + I*b*c*d*sin(b
*x + a))*sin(2*b*x + 3*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) - 4*(b*
d^2*x*(I*cos(a) - sin(a))*cos(b*x + a) - b*d^2*x*(cos(a) + I*sin(a))*sin(b
*x + a) + (b*d^2*x*(-I*cos(a) + sin(a)) + I*b*d^2*x*cos(2*b*x + 3*a) - ...
```

### 3.173.8 Giac [F]

$$\int (c+dx)^2 \cos(a+bx) \cot^2(a+bx) \, dx = \int (dx+c)^2 \cos(bx+a) \cot(bx+a)^2 \, dx$$

input
```
integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*cos(b*x + a)*cot(b*x + a)^2, x)
```

### 3.173.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \cos(a+bx) \cot^2(a+bx) \, dx = \int \cos(a+b\,x)\, \cot(a+b\,x)^2 \,(c+d\,x)^2 \, dx$$

input
```
int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^2,x)
```

output
```
int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^2, x)
```

# 3.174 $\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx$

3.174.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1389
3.174.2 Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 1389
3.174.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1390
3.174.4 Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1392
3.174.5 Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . 1392
3.174.6 Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1393
3.174.7 Maxima [B] (verification not implemented) . . . . . . . . . . . . . . 1393
3.174.8 Giac [B] (verification not implemented) . . . . . . . . . . . . . . . . 1394
3.174.9 Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . 1395

## 3.174.1 Optimal result

Integrand size = 20, antiderivative size = 58

$$\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx = -\frac{d\operatorname{arctanh}(\cos(a+bx))}{b^2}-\frac{d\cos(a+bx)}{b^2} -\frac{(c+dx)\csc(a+bx)}{b}-\frac{(c+dx)\sin(a+bx)}{b}$$

output
```
-d*arctanh(cos(b*x+a))/b^2-d*cos(b*x+a)/b^2-(d*x+c)*csc(b*x+a)/b-(d*x+c)*s
in(b*x+a)/b
```

## 3.174.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.79

$$\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx = -\frac{2d\cos(a+bx)+bdx\cot\left(\frac{1}{2}(a+bx)\right)+2bc\csc(a+bx)+2d\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)-2d\log\left(\sin\left(\frac{1}{2}(a+\right.\right.}{2b^2}$$

input
```
Integrate[(c + d*x)*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

output
```
-1/2*(2*d*Cos[a + b*x] + b*d*x*Cot[(a + b*x)/2] + 2*b*c*Csc[a + b*x] + 2*d
*Log[Cos[(a + b*x)/2]] - 2*d*Log[Sin[(a + b*x)/2]] + 2*b*c*Sin[a + b*x] +
2*b*d*x*Sin[a + b*x] + b*d*x*Tan[(a + b*x)/2])/b^2
```

### 3.174.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {4908, 3042, 3777, 25, 3042, 3118, 4910, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx$$

$$\Big\downarrow \textbf{4908}$$

$$\int (c+dx)\cot(a+bx)\csc(a+bx)dx-\int (c+dx)\cos(a+bx)dx$$

$$\Big\downarrow \textbf{3042}$$

$$\int (c+dx)\cot(a+bx)\csc(a+bx)dx-\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \textbf{3777}$$

$$\int (c+dx)\cot(a+bx)\csc(a+bx)dx-\frac{d\int -\sin(a+bx)dx}{b}-\frac{(c+dx)\sin(a+bx)}{b}$$

$$\Big\downarrow \textbf{25}$$

$$\int (c+dx)\cot(a+bx)\csc(a+bx)dx+\frac{d\int \sin(a+bx)dx}{b}-\frac{(c+dx)\sin(a+bx)}{b}$$

$$\Big\downarrow \textbf{3042}$$

$$\int (c+dx)\cot(a+bx)\csc(a+bx)dx+\frac{d\int \sin(a+bx)dx}{b}-\frac{(c+dx)\sin(a+bx)}{b}$$

$$\Big\downarrow \textbf{3118}$$

$$\int (c+dx)\cot(a+bx)\csc(a+bx)dx-\frac{d\cos(a+bx)}{b^2}-\frac{(c+dx)\sin(a+bx)}{b}$$

$$\Big\downarrow \textbf{4910}$$

$$\frac{d\int \csc(a+bx)dx}{b}-\frac{d\cos(a+bx)}{b^2}-\frac{(c+dx)\sin(a+bx)}{b}-\frac{(c+dx)\csc(a+bx)}{b}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{d\int \csc(a+bx)dx}{b}-\frac{d\cos(a+bx)}{b^2}-\frac{(c+dx)\sin(a+bx)}{b}-\frac{(c+dx)\csc(a+bx)}{b}$$

$$\Big\downarrow \textbf{4257}$$

$$-\frac{d\operatorname{arctanh}(\cos(a+bx))}{b^2}-\frac{d\cos(a+bx)}{b^2}-\frac{(c+dx)\sin(a+bx)}{b}-\frac{(c+dx)\csc(a+bx)}{b}$$

input
```
Int[(c + d*x)*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

output
```
-((d*ArcTanh[Cos[a + b*x]])/b^2) - (d*Cos[a + b*x])/b^2 - ((c + d*x)*Csc[a
 + b*x])/b - ((c + d*x)*Sin[a + b*x])/b
```

### 3.174.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 4910
```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csc[a + b*x]^n/(b*n)), x
] + Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; Free
Q[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

### 3.174.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.31 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.14

| method | result | size |
|---|---|---|
| risch | $\frac{i(dxb+cb+id)e^{i(xb+a)}}{2b^2}-\frac{i(dxb+cb-id)e^{-i(xb+a)}}{2b^2}-\frac{2i(dx+c)e^{i(xb+a)}}{b(e^{2i(xb+a)}-1)}-\frac{d\ln(e^{i(xb+a)}+1)}{b^2}+\frac{d\ln(e^{i(xb+a)}-1)}{b^2}$ | 124 |

input
```
int((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*(d*x*b+c*b+I*d)/b^2*exp(I*(b*x+a))-1/2*I*(d*x*b+c*b-I*d)/b^2*exp(-I*
(b*x+a))-2*I*(d*x+c)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)-d/b^2*ln(exp(I*
(b*x+a))+1)+d/b^2*ln(exp(I*(b*x+a))-1)
```

### 3.174.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.64

$$\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx =$$
$$-\frac{4\,bdx-2\,(bdx+bc)\cos{(bx+a)}^2+2\,d\cos{(bx+a)}\sin{(bx+a)}+d\log{\left(\frac{1}{2}\,\cos{(bx+a)}+\frac{1}{2}\right)}\sin{(bx+a}}{2\,b^2\sin{(bx+a)}}$$

input
```
integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(4*b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + 2*d*cos(b*x + a)*sin(b*x
+ a) + d*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - d*log(-1/2*cos(b*x + a
) + 1/2)*sin(b*x + a) + 4*b*c)/(b^2*sin(b*x + a))
```

### 3.174.6 Sympy [F]

$$\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx = \int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx$$

input
```
integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*cos(a + b*x)*cot(a + b*x)**2, x)
```

### 3.174.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2110 vs. $2(58) = 116$.

Time = 0.30 (sec) , antiderivative size = 2110, normalized size of antiderivative = 36.38

$$\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*c*(1/sin(b*x + a) + sin(b*x + a)) - 2*a*d*(1/sin(b*x + a) + sin(b*
x + a))/b - (((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*cos(3*b*x
 + 3*a)^3 + (b*x - (b*x + a)*cos(2*b*x + 2*a) + a - sin(2*b*x + 2*a))*sin(
3*b*x + 3*a)^3 - 6*(b*x + a)*sin(b*x + a)^3 - 2*(4*(b*x + a)*cos(b*x + a)*
sin(2*b*x + 2*a) - (3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2
*a) + 3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a)^2 - ((b*x
+ a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a)^2 + (8*(b*x + a)*cos(2*
b*x + 2*a)*sin(b*x + a) + ((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) +
 1)*cos(3*b*x + 3*a) - 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x + a))*sin(2*b
*x + 2*a) - 8*(b*x + a)*sin(b*x + a))*sin(3*b*x + 3*a)^2 - ((b*x + a)*sin(
b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a)^2 + (12*(b*x + a)*cos(b*x + a)*s
in(b*x + a) - (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x + a)^2 + s
in(b*x + a)^2 + 2)*cos(2*b*x + 2*a) + cos(2*b*x + 2*a)^2 + cos(b*x + a)^2
+ (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2)*sin(2*b*x + 2*a
) + sin(2*b*x + 2*a)^2 + sin(b*x + a)^2 + 1)*cos(3*b*x + 3*a) + 2*(3*(b*x
+ a)*sin(b*x + a)^3 + (3*(b*x + a)*cos(b*x + a)^2 + b*x + a)*sin(b*x + a)
+ cos(b*x + a))*cos(2*b*x + 2*a) - ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)
^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*
x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 -
2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x ...
```

### 3.174.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1759 vs. $2(58) = 116$.

Time = 0.86 (sec) , antiderivative size = 1759, normalized size of antiderivative = 30.33

$$\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4*tan(1/2*a)^4 +
 6*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*b*d*x*tan(1/2*b*x)^3*tan(1/2*a)^3
 - d*log(4*(tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(
tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 + 1))*tan(1/2*
b*x)^4*tan(1/2*a)^3 + d*log(4*(tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a)
+ tan(1/2*a)^2)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)
^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a)^3 + 6*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4
 - d*log(4*(tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(
tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 + 1))*tan(1/2*
b*x)^3*tan(1/2*a)^4 + d*log(4*(tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a)
+ tan(1/2*a)^2)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)
^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a)^4 + 6*b*c*tan(1/2*b*x)^4*tan(1/2*a)^2 +
 8*b*c*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + 6*b
*c*tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + b*d*x*t
an(1/2*b*x)^4 - 8*b*d*x*tan(1/2*b*x)^3*tan(1/2*a) - d*log(4*(tan(1/2*b*x)^
2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*b*x)^2*tan(1/2*a)
^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a) + d*log
(4*(tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*b*
x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan
(1/2*a) - 12*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - 8*b*d*x*tan(1/2*b*x)*t...
```

### 3.174.9 Mupad [B] (verification not implemented)

Time = 26.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.79

$$\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx = \mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\left(\frac{(b\,c+d\,1\mathrm{i})\ 1\mathrm{i}}{2\,b^2}+\frac{d\,x\,1\mathrm{i}}{2\,b}\right) + \mathrm{e}^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}\left(\frac{(-b\,c+d\,1\mathrm{i})\ 1\mathrm{i}}{2\,b^2}-\frac{d\,x\,1\mathrm{i}}{2\,b}\right) - \frac{d\,\ln\left(\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}{b^2} + \frac{d\,\ln\left(d\,2\mathrm{i}-d\,\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,1\mathrm{i}}\,2\mathrm{i}\right)}{b^2} + \frac{2\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,(c+d\,x)}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}-\mathrm{i}\right)}$$

input
```
int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x),x)
```

output
```
exp(a*1i + b*x*1i)*(((d*1i + b*c)*1i)/(2*b^2) + (d*x*1i)/(2*b)) + exp(- a*
1i - b*x*1i)*(((d*1i - b*c)*1i)/(2*b^2) - (d*x*1i)/(2*b)) - (d*log(exp(a*1
i + b*x*1i)*1i + 1i))/b^2 + (d*log(d*2i - d*exp(a*1i)*exp(b*x*1i)*2i))/b^2
 + (2*exp(a*1i + b*x*1i)*(c + d*x))/(b*(exp(a*2i + b*x*2i)*1i - 1i))
```

# 3.175 $\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx$

3.175.1 Optimal result . . . 1396
3.175.2 Mathematica [N/A] . . . 1396
3.175.3 Rubi [N/A] . . . 1397
3.175.4 Maple [N/A] (verified) . . . 1399
3.175.5 Fricas [N/A] . . . 1399
3.175.6 Sympy [N/A] . . . 1399
3.175.7 Maxima [N/A] . . . 1400
3.175.8 Giac [N/A] . . . 1400
3.175.9 Mupad [N/A] . . . 1401

## 3.175.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx = -\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d} + \frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d} + \text{Int}\left(\frac{\cot(a+bx)\csc(a+bx)}{c+dx},x\right)$$

output
```
CannotIntegrate(cot(b*x+a)*csc(b*x+a)/(d*x+c),x)-Ci(b*c/d+b*x)*cos(a-b*c/d
)/d+Si(b*c/d+b*x)*sin(a-b*c/d)/d
```

## 3.175.2 Mathematica [N/A]

Not integrable

Time = 3.79 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx = \int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x),x]
```

output
```
Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x), x]
```

### 3.175.3 Rubi [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4908, 3042, 3784, 3042, 3780, 3783, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{4908}$$

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}dx-\int \frac{\cos(a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}dx-\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx$$

$$\Big\downarrow \text{3784}$$

$$\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx-\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{3042}$$

$$\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx-\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx+\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{3780}$$

$$-\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx+\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}dx+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

$$\Big\downarrow \text{3783}$$

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}dx-\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}dx-\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

input
```
Int[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x),x]
```

output
```
$Aborted
```

### 3.175.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

### 3.175.4 Maple [N/A] (verified)

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cos(xb+a)\cot(xb+a)^2}{dx+c}dx$$

input
```
int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x)
```

output
```
int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x)
```

### 3.175.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx = \int \frac{\cos(bx+a)\cot(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)*cot(b*x + a)^2/(d*x + c), x)
```

### 3.175.6 Sympy [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx = \int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(cos(a + b*x)*cot(a + b*x)**2/(c + d*x), x)
```

### 3.175.7 Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 1438, normalized size of antiderivative = 65.36

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx=\int \frac{\cos(bx+a)\cot(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
1/2*(b*c*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b
*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(I*exp_integral_e(1, (I*b*d*x
+ I*b*c)/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/
d) + (b*c*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*
b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(I*exp_integral_e(1, (I*b*d*x
 + I*b*c)/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)
/d) + (b*d*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I
*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(I*exp_integral_e(1, (I*b*d*
x + I*b*c)/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d
)/d))*x)*cos(2*b*x + 2*a)^2 - 4*d*cos(b*x + a)*sin(2*b*x + 2*a) + (b*c*(ex
p_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c
)/d))*cos(-(b*c - a*d)/d) - b*c*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/d)
- I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d) + (b*d*(e
xp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*
c)/d))*cos(-(b*c - a*d)/d) - b*d*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/d)
 - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x)*sin(
2*b*x + 2*a)^2 + (b*d*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integr
al_e(1, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(I*exp_integral_e
(1, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(
-(b*c - a*d)/d))*x - 2*(b*c*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + e...
```

### 3.175.8 Giac [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx=\int \frac{\cos(bx+a)\cot(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*cot(b*x + a)^2/(d*x + c), x)
```

### 3.175.9 Mupad [N/A]

Not integrable

Time = 26.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx = \int \frac{\cos\left(a+b\,x\right)\,\cot(a+b\,x)^2}{c+d\,x}\,dx$$

input
```
int((cos(a + b*x)*cot(a + b*x)^2)/(c + d*x),x)
```

output
```
int((cos(a + b*x)*cot(a + b*x)^2)/(c + d*x), x)
```

# 3.176 $\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx$


3.176.1 Optimal result . . . 1402
3.176.2 Mathematica [N/A] . . . 1402
3.176.3 Rubi [N/A] . . . 1403
3.176.4 Maple [N/A] (verified) . . . 1405
3.176.5 Fricas [N/A] . . . 1406
3.176.6 Sympy [N/A] . . . 1406
3.176.7 Maxima [N/A] . . . 1406
3.176.8 Giac [N/A] . . . 1407
3.176.9 Mupad [N/A] . . . 1408


## 3.176.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx = \frac{\cos(a+bx)}{d(c+dx)} + \frac{b\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^2} + \frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2} + \text{Int}\left(\frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2},x\right)$$

output
```
CannotIntegrate(cot(b*x+a)*csc(b*x+a)/(d*x+c)^2,x)+cos(b*x+a)/d/(d*x+c)+b*
cos(a-b*c/d)*Si(b*c/d+b*x)/d^2+b*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^2
```

## 3.176.2 Mathematica [N/A]

Not integrable

Time = 4.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x)^2, x]
```

### 3.176.3 Rubi [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4908, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{4908}$$

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}dx-\int \frac{\cos(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}dx-\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3778}$$

$$-\frac{b\int -\frac{\sin(a+bx)}{c+dx}dx}{d}+\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}dx+\frac{\cos(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\int \frac{\sin(a+bx)}{c+dx}dx}{d}+\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}dx+\frac{\cos(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\int \frac{\sin(a+bx)}{c+dx}dx}{d}+\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}dx+\frac{\cos(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3784}$$

$$\frac{b\left(\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}+\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}dx+\frac{\cos(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\left(\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}+\int\frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}dx+\frac{\cos(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{ 3780}$$

$$\frac{b\left(\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}+\int\frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}dx+\frac{\cos(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{ 3783}$$

$$\int\frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}dx+\frac{b\left(\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}+\frac{\cos(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{ 7299}$$

$$\int\frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}dx+\frac{b\left(\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}+\frac{\cos(a+bx)}{d(c+dx)}$$

input
```
Int[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
$Aborted
```

### 3.176.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

### 3.176.4 Maple [N/A] (verified)

Not integrable

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cos (xb+a) \cot (xb+a)^2}{(dx+c)^2} dx$$

input
```
int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x)
```

### 3.176.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(bx+a)\cot(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(cos(b*x + a)*cot(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.176.6 Sympy [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(cos(a + b*x)*cot(a + b*x)**2/(c + d*x)**2, x)
```

### 3.176.7 Maxima [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 1678, normalized size of antiderivative = 76.27

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos(bx+a)\cot(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*(b*c*(exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b
*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(I*exp_integral_e(2, (I*b*d*x
+ I*b*c)/d) - I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/
d) + (b*c*(exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*
b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(I*exp_integral_e(2, (I*b*d*x
 + I*b*c)/d) - I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)
/d) + (b*d*(exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I
*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(I*exp_integral_e(2, (I*b*d*
x + I*b*c)/d) - I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d
)/d))*x)*cos(2*b*x + 2*a)^2 - 4*d*cos(b*x + a)*sin(2*b*x + 2*a) + (b*c*(ex
p_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c
)/d))*cos(-(b*c - a*d)/d) - b*c*(I*exp_integral_e(2, (I*b*d*x + I*b*c)/d)
- I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d) + (b*d*(e
xp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*
c)/d))*cos(-(b*c - a*d)/d) - b*d*(I*exp_integral_e(2, (I*b*d*x + I*b*c)/d)
 - I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x)*sin(
2*b*x + 2*a)^2 + (b*d*(exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integr
al_e(2, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(I*exp_integral_e
(2, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(
-(b*c - a*d)/d))*x - 2*(b*c*(exp_integral_e(2, (I*b*d*x + I*b*c)/d) + e...
```

### 3.176.8 Giac [N/A]

Not integrable

Time = 6.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos{(bx+a)}\cot{(bx+a)}^2}{(dx+c)^2}\,dx$$

input
```
integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(cos(b*x + a)*cot(b*x + a)^2/(d*x + c)^2, x)
```

### 3.176.9 Mupad [N/A]

Not integrable

Time = 25.79 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cos\left(a+b\,x\right)\,\mathrm{cot}(a+b\,x)^2}{{\left(c+d\,x\right)}^2}\,dx$$

input
```
int((cos(a + b*x)*cot(a + b*x)^2)/(c + d*x)^2,x)
```

output
```
int((cos(a + b*x)*cot(a + b*x)^2)/(c + d*x)^2, x)
```

# 3.177 $\int (c+dx)^m \cot^3(a+bx)\,dx$

3.177.1 Optimal result . . . . . . . . . . 1409
3.177.2 Mathematica [N/A] . . . . . . . . . . 1409
3.177.3 Rubi [N/A] . . . . . . . . . . 1410
3.177.4 Maple [N/A] (verified) . . . . . . . . . . 1411
3.177.5 Fricas [N/A] . . . . . . . . . . 1411
3.177.6 Sympy [N/A] . . . . . . . . . . 1412
3.177.7 Maxima [N/A] . . . . . . . . . . 1412
3.177.8 Giac [N/A] . . . . . . . . . . 1412
3.177.9 Mupad [N/A] . . . . . . . . . . 1413

## 3.177.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (c+dx)^m \cot^3(a+bx)\,dx = \operatorname{Int}\left((c+dx)^m \cot^3(a+bx),x\right)$$

output
```
Unintegrable((d*x+c)^m*cot(b*x+a)^3,x)
```

## 3.177.2 Mathematica [N/A]

Not integrable

Time = 14.81 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \cot^3(a+bx)\,dx = \int (c+dx)^m \cot^3(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Cot[a + b*x]^3,x]
```

output
```
Integrate[(c + d*x)^m*Cot[a + b*x]^3, x]
```

### 3.177.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 25, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan\left(a+bx+\frac{\pi}{2}\right)^3(-(c+dx)^m)\,dx$$

$$\Big\downarrow \ 25$$

$$-\int (c+dx)^m\tan\left(\frac{1}{2}(2a+\pi)+bx\right)^3dx$$

$$\Big\downarrow \ 4222$$

$$-\int (c+dx)^m\tan^3\left(\frac{1}{2}(2a+\pi)+bx\right)dx$$

input
```
Int[(c + d*x)^m*Cot[a + b*x]^3,x]
```

output
```
$Aborted
```

#### 3.177.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

### 3.177.4 Maple [N/A] (verified)

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \cot (xb+a)^3 \, dx$$

input
```
int((d*x+c)^m*cot(b*x+a)^3,x)
```

output
```
int((d*x+c)^m*cot(b*x+a)^3,x)
```

### 3.177.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \cot^3(a+bx) \, dx = \int (dx+c)^m \cot (bx+a)^3 \, dx$$

input
```
integrate((d*x+c)^m*cot(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*cot(b*x + a)^3, x)
```

### 3.177.6 Sympy [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (c+dx)^m \cot^3(a+bx)\,dx = \int (c+dx)^m \cot^3{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**m*cot(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**m*cot(a + b*x)**3, x)
```

### 3.177.7 Maxima [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \cot^3(a+bx)\,dx = \int (dx+c)^m \cot{(bx+a)}^3\,dx$$

input
```
integrate((d*x+c)^m*cot(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*cot(b*x + a)^3, x)
```

### 3.177.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \cot^3(a+bx)\,dx = \int (dx+c)^m \cot{(bx+a)}^3\,dx$$

input
```
integrate((d*x+c)^m*cot(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cot(b*x + a)^3, x)
```

### 3.177.9 Mupad [N/A]

Not integrable

Time = 25.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \cot^3(a+bx)\,dx = \int \cot(a+b\,x)^3\,(c+d\,x)^m\,dx$$

input
```
int(cot(a + b*x)^3*(c + d*x)^m,x)
```

output
```
int(cot(a + b*x)^3*(c + d*x)^m, x)
```

# 3.178 $\int (c+dx)^4 \cot^3(a+bx)\,dx$

3.178.1 Optimal result . . . 1414
3.178.2 Mathematica [B] (warning: unable to verify) . . . 1415
3.178.3 Rubi [A] (verified) . . . 1416
3.178.4 Maple [B] (verified) . . . 1424
3.178.5 Fricas [B] (verification not implemented) . . . 1425
3.178.6 Sympy [F] . . . 1426
3.178.7 Maxima [B] (verification not implemented) . . . 1427
3.178.8 Giac [F] . . . 1427
3.178.9 Mupad [F(-1)] . . . 1428

## 3.178.1 Optimal result

Integrand size = 16, antiderivative size = 302

$$\begin{aligned}\int (c+dx)^4 \cot^3(a+bx)\,dx = &-\frac{2id(c+dx)^3}{b^2}-\frac{(c+dx)^4}{2b}+\frac{i(c+dx)^5}{5d}\\&-\frac{2d(c+dx)^3\cot(a+bx)}{b^2}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}\\&+\frac{6d^2(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^3}\\&-\frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b}\\&-\frac{6id^3(c+dx)\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^4}\\&+\frac{2id(c+dx)^3\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}\\&+\frac{3d^4\operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^5}\\&-\frac{3d^2(c+dx)^2\operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3}\\&-\frac{3id^3(c+dx)\operatorname{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4}\\&+\frac{3d^4\operatorname{PolyLog}\left(5,e^{2i(a+bx)}\right)}{2b^5}\end{aligned}$$

output
```
-2*I*d*(d*x+c)^3/b^2-1/2*(d*x+c)^4/b+1/5*I*(d*x+c)^5/d-2*d*(d*x+c)^3*cot(b
*x+a)/b^2-1/2*(d*x+c)^4*cot(b*x+a)^2/b+6*d^2*(d*x+c)^2*ln(1-exp(2*I*(b*x+a
)))/b^3-(d*x+c)^4*ln(1-exp(2*I*(b*x+a)))/b-6*I*d^3*(d*x+c)*polylog(2,exp(2
*I*(b*x+a)))/b^4+2*I*d*(d*x+c)^3*polylog(2,exp(2*I*(b*x+a)))/b^2+3*d^4*pol
ylog(3,exp(2*I*(b*x+a)))/b^5-3*d^2*(d*x+c)^2*polylog(3,exp(2*I*(b*x+a)))/b
^3-3*I*d^3*(d*x+c)*polylog(4,exp(2*I*(b*x+a)))/b^4+3/2*d^4*polylog(5,exp(2
*I*(b*x+a)))/b^5
```

### 3.178.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1632 vs. $2(302) = 604$.

Time = 7.09 (sec) , antiderivative size = 1632, normalized size of antiderivative = 5.40

$$\int (c + dx)^4 \cot^3(a + bx)\, dx = \text{Too large to display}$$

input
```
Integrate[(c + d*x)^4*Cot[a + b*x]^3,x]
```

output
```
-1/5*(x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4)*Cot[
a]) - ((c + d*x)^4*Csc[a + b*x]^2)/(2*b) + (c^2*d^2*E^(I*a)*Csc[a]*((2*b^3
*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a +
b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - 6*
b*(1 - E^((-2*I)*a))*x*PolyLog[2, -E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I
)*a))*x*PolyLog[2, E^((-I)*(a + b*x))] + (6*I)*(1 - E^((-2*I)*a))*PolyLog[
3, -E^((-I)*(a + b*x))] + (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, E^((-I)*(a +
 b*x))]))/b^3 - (d^4*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(
1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*
I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[
2, -E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, E^((-I)*(a +
 b*x))] + (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, -E^((-I)*(a + b*x))] + (6*I)
*(1 - E^((-2*I)*a))*PolyLog[3, E^((-I)*(a + b*x))]))/b^5 + (c*d^3*E^(I*a)*
Csc[a]*((b^4*x^4)/E^((2*I)*a) + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E
^((-I)*(a + b*x))] + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a +
 b*x))] - 6*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - 6
*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[2, E^((-I)*(a + b*x))] + (12*I)*b*(1 -
 E^((-2*I)*a))*x*PolyLog[3, -E^((-I)*(a + b*x))] + (12*I)*b*(1 - E^((-2*I)
*a))*x*PolyLog[3, E^((-I)*(a + b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, -
E^((-I)*(a + b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, E^((-I)*(a + b*x...
```

### 3.178.3 Rubi [A] (verified)

Time = 2.19 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.35, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.438, Rules used = {3042, 25, 4203, 25, 3042, 25, 4202, 2620, 3011, 4203, 17, 25, 3042, 25, 4202, 2620, 3011, 2720, 7143, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \cot^3(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -(c+dx)^4 \tan\left(a+bx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \ 25$$

$$-\int (c+dx)^4 \tan\left(\frac{1}{2}(2a+\pi)+bx\right)^3dx$$

$$\Big\downarrow \ 4203$$

$$\frac{2d\int (c+dx)^3\cot^2(a+bx)dx}{b}+\int -(c+dx)^4\cot(a+bx)dx-\frac{(c+dx)^4\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \ 25$$

$$\frac{2d\int (c+dx)^3\cot^2(a+bx)dx}{b}-\int (c+dx)^4\cot(a+bx)dx-\frac{(c+dx)^4\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \ 3042$$

$$-\int -(c+dx)^4\tan\left(a+bx+\frac{\pi}{2}\right)dx+\frac{2d\int (c+dx)^3\tan\left(a+bx+\frac{\pi}{2}\right)^2dx}{b}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \ 25$$

$$\frac{2d\int (c+dx)^3\tan\left(a+bx+\frac{\pi}{2}\right)^2dx}{b}+\int (c+dx)^4\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx-\frac{(c+dx)^4\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \ 4202$$

$$-2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)^4}{1+e^{i(2a+2bx+\pi)}}dx+\frac{2d\int (c+dx)^3\tan\left(a+bx+\frac{\pi}{2}\right)^2dx}{b}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

$$\Big\downarrow \ 2620$$

$$-2i\left(\frac{2id\int(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)+\frac{2d\int(c+dx)^3\tan\left(a+bx+\frac{\pi}{2}\right)^2dx}{b}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

$\downarrow$ 3011

$$-2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$
$$\frac{2d\int(c+dx)^3\tan\left(a+bx+\frac{\pi}{2}\right)^2dx}{b}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

$\downarrow$ 4203

$$-2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$
$$\frac{2d\left(-\frac{3d\int-(c+dx)^2\cot(a+bx)dx}{b}-\int(c+dx)^3dx-\frac{(c+dx)^3\cot(a+bx)}{b}\right)}{b}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

$\downarrow$ 17

$$-2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$
$$\frac{2d\left(-\frac{3d\int-(c+dx)^2\cot(a+bx)dx}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}\right)}{b}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

$\downarrow$ 25

$$-2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$
$$\frac{2d\left(\frac{3d\int(c+dx)^2\cot(a+bx)dx}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}\right)}{b}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

$\downarrow$ 3042

3.178. $\int(c+dx)^4\cot^3(a+bx)\,dx$

$$-2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$
$$\frac{2d\left(\frac{3d\int-(c+dx)^2\tan\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}\right)}{b}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

$\downarrow$ 25

$$-2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$
$$\frac{2d\left(-\frac{3d\int(c+dx)^2\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}\right)}{b}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}+$$
$$\frac{i(c+dx)^5}{5d}$$

$\downarrow$ 4202

$$-2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$
$$\frac{2d\left(-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\int\frac{e^{i(2a+2bx+\pi)}(c+dx)^2}{1+e^{i(2a+2bx+\pi)}}dx\right)}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}\right)}{b}-\frac{(c+dx)^4\cot^2(a+bx)}{2b}+$$
$$\frac{i(c+dx)^5}{5d}$$

$\downarrow$ 2620

$$-2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$
$$\frac{2d\left(-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\int(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{(c+dx)^4}{4d}\right)}{b}-$$
$$\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

$\downarrow$ 3011

3.178. $\int(c+dx)^4\cot^3(a+bx)\,dx$

$$-2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$

$$2d\left(-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+\ldots}{}\right.$$

$$\overline{\qquad b\qquad}+\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

$\downarrow$ 2720

$$2d\left(-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i\ldots}\right)}{2b}\right.\right.}{b}\right.$$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$

$$\overline{\qquad b\qquad}+\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

$\downarrow$ 7143

$$-2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\int(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$

$$2d\left(-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^3\ldots}{}\right.$$

$$\overline{\qquad b\qquad}+\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

$\downarrow$ 7163

3.178. $\int(c+dx)^4\cot^3(a+bx)\,dx$

$$\frac{-2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\left(\frac{id\int(c+dx)\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)dx}{b}-\frac{i(c+dx)^2\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{2b}\right)}{b}-\ldots\right.}{} \qquad 2d\left(-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^3\ldots}{\ldots}\right.$$

$$\overline{\qquad\qquad b\qquad\qquad}$$

$$\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

$$\Big\downarrow \; 7163$$

$$\frac{-2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\left(\frac{id\left(\frac{id\int\operatorname{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}-\frac{i(c+dx)^2\ldots}{\ldots}\right)}{2b}\ldots\right)}{b}\ldots\right.}{} \qquad 2d\left(-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^3\ldots}{\ldots}\right.$$

$$\overline{\qquad\qquad b\qquad\qquad}$$

$$\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

$$\Big\downarrow \; 2720$$

3.178. $\int (c+dx)^4\cot^3(a+bx)\,dx$

$$-2i\left(\frac{2id\left(\frac{i(c+dx)^3 \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\left(\frac{id\left(\frac{d\int e^{-i(2a+2bx+\pi)} \operatorname{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,-e^{\cdots}\right.}{2b}\right.}{b}\right.}{2b}\right.}{b}\right.$$

$$\frac{2d\left(-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^3\cdots}{}\right.}{b}$$

$$\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

$\downarrow$ 7143

---

3.178. $\int (c+dx)^4 \cot^3(a+bx)\,dx$

$$-2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{3id\left(\frac{id\left(\frac{d\operatorname{PolyLog}\left(5,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}-\frac{i(c+dx)^2\operatorname{PolyL}\cdots}{\cdots}\right)}{2b}\cdots\right)}{b}\cdots\right.$$

$$\frac{2d\left(-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^3\,\cdots}{\cdots}\right)}{b}$$

$$\frac{(c+dx)^4\cot^2(a+bx)}{2b}+\frac{i(c+dx)^5}{5d}$$

input
```
Int[(c + d*x)^4*Cot[a + b*x]^3,x]
```

output
```
((I/5)*(c + d*x)^5)/d - ((c + d*x)^4*Cot[a + b*x]^2)/(2*b) + (2*d*(-1/4*(c
 + d*x)^4/d - ((c + d*x)^3*Cot[a + b*x])/b - (3*d*(((I/3)*(c + d*x)^3)/d -
 (2*I)*(((-1/2*I)*(c + d*x)^2*Log[1 + E^(I*(2*a + Pi + 2*b*x))])/b + (I*d*
(((I/2)*(c + d*x)*PolyLog[2, -E^(I*(2*a + Pi + 2*b*x))])/b - (d*PolyLog[3,
 -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2)))/b)))/b))/b - (2*I)*(((-1/2*I)*(c +
d*x)^4*Log[1 + E^(I*(2*a + Pi + 2*b*x))])/b + ((2*I)*d*(((I/2)*(c + d*x)^3
*PolyLog[2, -E^(I*(2*a + Pi + 2*b*x))])/b - (((3*I)/2)*d*(((-1/2*I)*(c + d
*x)^2*PolyLog[3, -E^(I*(2*a + Pi + 2*b*x))])/b + (I*d*(((-1/2*I)*(c + d*x)
*PolyLog[4, -E^(I*(2*a + Pi + 2*b*x))])/b + (d*PolyLog[5, -E^(I*(2*a + Pi
+ 2*b*x))])/(4*b^2)))/b))/b))/b)
```

3.178. $\int (c+dx)^4\cot^3(a+bx)\,dx$

### 3.178.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

### 3.178.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1876 vs. $2(273) = 546$.

Time = 1.38 (sec) , antiderivative size = 1877, normalized size of antiderivative = 6.22

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1877 |

input
```
int((d*x+c)^4*cot(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(b*d^4*x^4*exp(2*I*(b*x+a))+4*b*c*d^3*x^3*exp(2*I*(b*x+a))+6*b*c^2*d^2*x
^2*exp(2*I*(b*x+a))+4*b*c^3*d*x*exp(2*I*(b*x+a))-2*I*d^4*x^3*exp(2*I*(b*x+
a))+b*c^4*exp(2*I*(b*x+a))-6*I*c*d^3*x^2*exp(2*I*(b*x+a))-6*I*c^2*d^2*x*ex
p(2*I*(b*x+a))+2*I*d^4*x^3-2*I*c^3*d*exp(2*I*(b*x+a))+6*I*c*d^3*x^2+6*I*c^
2*d^2*x+2*I*c^3*d)/b^2/(exp(2*I*(b*x+a))-1)^2-I*c^4*x-1/5*I/d*c^5-12*I/b^2
*d^3*c*x^2-12*I/b^4*d^3*c*a^2-12*I/b^4*d^3*c*polylog(2,exp(I*(b*x+a)))-12*
I/b^4*d^3*c*polylog(2,-exp(I*(b*x+a)))+4*I/b^2*d^4*polylog(2,exp(I*(b*x+a)
))*x^3-24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-24*I/b^4*d^4*polylog(4,-ex
p(I*(b*x+a)))*x-2*I/b^4*d^4*a^4*x-12*I/b^4*d^4*polylog(2,exp(I*(b*x+a)))*x
-8*I/b^3*c^2*d^2*a^3+4*I/b^2*d*c^3*a^2+4*I/b^2*d*c^3*polylog(2,exp(I*(b*x+
a)))+4*I/b^2*d*c^3*polylog(2,-exp(I*(b*x+a)))+4*I/b^2*d^4*polylog(2,-exp(I
*(b*x+a)))*x^3-4/b*d^3*c*ln(1-exp(I*(b*x+a)))*x^3-4/b*d*c^3*ln(1-exp(I*(b*
x+a)))*x-4/b*d*c^3*ln(exp(I*(b*x+a))+1)*x-4/b^2*d*c^3*ln(1-exp(I*(b*x+a)))
*a+12/b^3*a^2*c^2*d^2*ln(exp(I*(b*x+a)))-6/b^3*a^2*c^2*d^2*ln(exp(I*(b*x+a
))-1)+6/b^3*c^2*d^2*ln(1-exp(I*(b*x+a)))*a^2-24/b^3*d^3*c*polylog(3,exp(I*
(b*x+a)))*x-24/b^3*d^3*c*polylog(3,-exp(I*(b*x+a)))*x+12/b^3*d^3*c*ln(1-ex
p(I*(b*x+a)))*x+12/b^3*d^3*c*ln(exp(I*(b*x+a))+1)*x-4/b*d^3*c*ln(exp(I*(b*
x+a))+1)*x^3-6/b*c^2*d^2*ln(1-exp(I*(b*x+a)))*x^2-6/b*c^2*d^2*ln(exp(I*(b*
x+a))+1)*x^2-12*I/b^4*d^4*polylog(2,-exp(I*(b*x+a)))*x+12*I/b^4*d^4*a^2*x+
6*I/b^4*d^3*c*a^4-24*I/b^4*d^3*c*polylog(4,exp(I*(b*x+a)))-24*I/b^4*d^3...
```

### 3.178.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1751 vs. $2(266) = 532$.

Time = 0.30 (sec) , antiderivative size = 1751, normalized size of antiderivative = 5.80

$$\int (c+dx)^4\cot^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cot(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(4*b^4*d^4*x^4 + 16*b^4*c*d^3*x^3 + 24*b^4*c^2*d^2*x^2 + 16*b^4*c^3*d*
x + 4*b^4*c^4 - 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + I*b^3*c^3*d - 3*I*b
*c*d^3 + 3*I*(b^3*c^2*d^2 - b*d^4)*x + (-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2
 - I*b^3*c^3*d + 3*I*b*c*d^3 - 3*I*(b^3*c^2*d^2 - b*d^4)*x)*cos(2*b*x + 2*
a))*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) - 4*(-I*b^3*d^4*x^3 - 3*I
*b^3*c*d^3*x^2 - I*b^3*c^3*d + 3*I*b*c*d^3 - 3*I*(b^3*c^2*d^2 - b*d^4)*x +
 (I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + I*b^3*c^3*d - 3*I*b*c*d^3 + 3*I*(b^3
*c^2*d^2 - b*d^4)*x)*cos(2*b*x + 2*a))*dilog(cos(2*b*x + 2*a) - I*sin(2*b*
x + 2*a)) + 2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 1)*b^2*c^2*d^2 - 4*(a^3
- 3*a)*b*c*d^3 + (a^4 - 6*a^2)*d^4 - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 1
)*b^2*c^2*d^2 - 4*(a^3 - 3*a)*b*c*d^3 + (a^4 - 6*a^2)*d^4)*cos(2*b*x + 2*a
))*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b^4*c^4
- 4*a*b^3*c^3*d + 6*(a^2 - 1)*b^2*c^2*d^2 - 4*(a^3 - 3*a)*b*c*d^3 + (a^4 -
 6*a^2)*d^4 - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 1)*b^2*c^2*d^2 - 4*(a^3
- 3*a)*b*c*d^3 + (a^4 - 6*a^2)*d^4)*cos(2*b*x + 2*a))*log(-1/2*cos(2*b*x +
 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 +
 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 3*a)*b*c*d^3 - (a^4 - 6*a^2)
*d^4 + 6*(b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(b^4*c^3*d - 3*b^2*c*d^3)*x - (b^
4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 -
 3*a)*b*c*d^3 - (a^4 - 6*a^2)*d^4 + 6*(b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(...
```

### 3.178.6 Sympy [F]

$$\int (c+dx)^4 \cot^3(a+bx)\,dx = \int (c+dx)^4 \cot^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**4*cot(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**4*cot(a + b*x)**3, x)
```

### 3.178.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 7158 vs. $2(266) = 532$.

Time = 3.68 (sec) , antiderivative size = 7158, normalized size of antiderivative = 23.70

$$\int (c+dx)^4 \cot^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cot(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(c^4*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2)) - 4*a*c^3*d*(1/sin(b*x
+ a)^2 + log(sin(b*x + a)^2))/b + 6*a^2*c^2*d^2*(1/sin(b*x + a)^2 + log(si
n(b*x + a)^2))/b^2 - 4*a^3*c*d^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2))/
b^3 + a^4*d^4*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2))/b^4 - 2*(2*(b*x + a
)^5*d^4 + 40*b^3*c^3*d - 120*a*b^2*c^2*d^2 + 120*a^2*b*c*d^3 - 40*a^3*d^4
+ 10*(b*c*d^3 - a*d^4)*(b*x + a)^4 + 20*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d
^4)*(b*x + a)^3 + 20*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^
4)*(b*x + a)^2 - 10*((b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 6*a^
2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (
a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b
*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a) + ((b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 1
2*a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2
 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d
^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) -
2*((b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c*d^3
 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x
 + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*
a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^4*d^4 + 6*I*b^2*c^2*d^
2 - 12*I*a*b*c*d^3 + 6*I*a^2*d^4 + 4*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 +
6*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 + (-I*a^2 + I)*d^4)*(b*x + a)^2 + 4*(...
```

### 3.178.8 Giac [F]

$$\int (c+dx)^4 \cot^3(a+bx)\,dx = \int (dx+c)^4 \cot(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^4*cot(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*cot(b*x + a)^3, x)
```

### 3.178.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \cot^3(a+bx)\,dx = \int \cot(a+b\,x)^3\,(c+d\,x)^4\,dx$$

input
```
int(cot(a + b*x)^3*(c + d*x)^4,x)
```

output
```
int(cot(a + b*x)^3*(c + d*x)^4, x)
```

# 3.179 $\int (c+dx)^3 \cot^3(a+bx)\,dx$

3.179.1 Optimal result . . . . . 1429
3.179.2 Mathematica [B] (warning: unable to verify) . . . . . 1430
3.179.3 Rubi [A] (verified) . . . . . 1431
3.179.4 Maple [B] (verified) . . . . . 1438
3.179.5 Fricas [B] (verification not implemented) . . . . . 1439
3.179.6 Sympy [F] . . . . . 1440
3.179.7 Maxima [B] (verification not implemented) . . . . . 1441
3.179.8 Giac [F] . . . . . 1441
3.179.9 Mupad [F(-1)] . . . . . 1442

## 3.179.1 Optimal result

Integrand size = 16, antiderivative size = 256

$$\begin{aligned}\int (c+dx)^3\cot^3(a+bx)\,dx = &-\frac{3id(c+dx)^2}{2b^2}-\frac{(c+dx)^3}{2b}+\frac{i(c+dx)^4}{4d}\\&-\frac{3d(c+dx)^2\cot(a+bx)}{2b^2}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}\\&+\frac{3d^2(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^3}\\&-\frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{3id^3\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^4}\\&+\frac{3id(c+dx)^2\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}\\&-\frac{3d^2(c+dx)\operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}\\&-\frac{3id^3\operatorname{PolyLog}\left(4,e^{2i(a+bx)}\right)}{4b^4}\end{aligned}$$

output
```
-3/2*I*d*(d*x+c)^2/b^2-1/2*(d*x+c)^3/b+1/4*I*(d*x+c)^4/d-3/2*d*(d*x+c)^2*c
ot(b*x+a)/b^2-1/2*(d*x+c)^3*cot(b*x+a)^2/b+3*d^2*(d*x+c)*ln(1-exp(2*I*(b*x
+a)))/b^3-(d*x+c)^3*ln(1-exp(2*I*(b*x+a)))/b-3/2*I*d^3*polylog(2,exp(2*I*(
b*x+a)))/b^4+3/2*I*d*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*
x+c)*polylog(3,exp(2*I*(b*x+a)))/b^3-3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))
/b^4
```

### 3.179.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1032 vs. $2(256) = 512$.

Time = 7.00 (sec) , antiderivative size = 1032, normalized size of antiderivative = 4.03

$$\int (c+dx)^3 \cot^3(a+bx)\,dx =$$

$$-\frac{1}{4}x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)\cot(a)-\frac{(c+dx)^3\csc^2(a+bx)}{2b}$$
$$+\frac{cd^2e^{ia}\csc(a)\left(2b^3e^{-2ia}x^3+3ib^2(1-e^{-2ia})\,x^2\log\left(1-e^{-i(a+bx)}\right)+3ib^2(1-e^{-2ia})\,x^2\log\left(1+e^{-i(a+bx)}\right)\right.}{}$$
$$+\frac{d^3e^{ia}\csc(a)\left(b^4e^{-2ia}x^4+2ib^3(1-e^{-2ia})\,x^3\log\left(1-e^{-i(a+bx)}\right)+2ib^3(1-e^{-2ia})\,x^3\log\left(1+e^{-i(a+bx)}\right)-\right.}{}$$
$$-\frac{c^3\csc(a)(-bx\cos(a)+\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a))}{b\left(\cos^2(a)+\sin^2(a)\right)}$$
$$+\frac{3cd^2\csc(a)(-bx\cos(a)+\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a))}{b^3\left(\cos^2(a)+\sin^2(a)\right)}$$
$$+\frac{3\csc(a)\csc(a+bx)\left(c^2d\sin(bx)+2cd^2x\sin(bx)+d^3x^2\sin(bx)\right)}{2b^2}$$
$$+\frac{3c^2d\csc(a)\sec(a)\left(b^2e^{i\arctan(\tan(a))}x^2+\frac{(ibx(-\pi+2\arctan(\tan(a)))-\pi\log(1+e^{-2ibx})-2(bx+\arctan(\tan(a)))\log(1-e^{2i(bx+\mathrm{arc}}}{}\right.}{2b^2\sqrt{\sec^2(a)}\left(\cos^2(a)+\right.}$$
$$-\frac{3d^3\csc(a)\sec(a)\left(b^2e^{i\arctan(\tan(a))}x^2+\frac{(ibx(-\pi+2\arctan(\tan(a)))-\pi\log(1+e^{-2ibx})-2(bx+\arctan(\tan(a)))\log(1-e^{2i(bx+\mathrm{arcta}}}{}\right.}{2b^4\sqrt{\sec^2(a)}\left(\cos^2(a)+\right.}$$

input
```
Integrate[(c + d*x)^3*Cot[a + b*x]^3,x]
```

output
```
-1/4*(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Cot[a]) - ((c + d*x)^3
*Csc[a + b*x]^2)/(2*b) + (c*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) +
(3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(
1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))
*x*PolyLog[2, -E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, E
^((-I)*(a + b*x))] + (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, -E^((-I)*(a + b*x
))] + (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, E^((-I)*(a + b*x))]))/(2*b^3) +
(d^3*E^(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a) + (2*I)*b^3*(1 - E^((-2*I)*a))*
x^3*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 +
 E^((-I)*(a + b*x))] - 6*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[2, -E^((-I)*(a
 + b*x))] - 6*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[2, E^((-I)*(a + b*x))] +
(12*I)*b*(1 - E^((-2*I)*a))*x*PolyLog[3, -E^((-I)*(a + b*x))] + (12*I)*b*(
1 - E^((-2*I)*a))*x*PolyLog[3, E^((-I)*(a + b*x))] + 12*(1 - E^((-2*I)*a))
*PolyLog[4, -E^((-I)*(a + b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, E^((-I
)*(a + b*x))]))/(4*b^4) - (c^3*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a]
 + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (3*c*d^2*Csc[a]*(
-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos[
a]^2 + Sin[a]^2)) + (3*Csc[a]*Csc[a + b*x]*(c^2*d*Sin[b*x] + 2*c*d^2*x*Sin
[b*x] + d^3*x^2*Sin[b*x]))/(2*b^2) + (3*c^2*d*Csc[a]*Sec[a]*(b^2*E^(I*ArcT
an[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*...
```

### 3.179.3 Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.25, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 1.312$, Rules used = {3042, 25, 4203, 25, 3042, 25, 4202, 2620, 3011, 4203, 17, 25, 3042, 25, 4202, 2620, 2715, 2838, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \cot^3(a+bx) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -(c+dx)^3 \tan\left(a+bx+\frac{\pi}{2}\right)^3 dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)^3 \tan\left(\frac{1}{2}(2a+\pi)+bx\right)^3 dx$$

$$\Big\downarrow \text{4203}$$

$$\frac{3d\int(c+dx)^2\cot^2(a+bx)dx}{2b}+\int-(c+dx)^3\cot(a+bx)dx-\frac{(c+dx)^3\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{3d\int(c+dx)^2\cot^2(a+bx)dx}{2b}-\int(c+dx)^3\cot(a+bx)dx-\frac{(c+dx)^3\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$-\int-(c+dx)^3\tan\left(a+bx+\frac{\pi}{2}\right)dx+\frac{3d\int(c+dx)^2\tan\left(a+bx+\frac{\pi}{2}\right)^2dx}{2b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{3d\int(c+dx)^2\tan\left(a+bx+\frac{\pi}{2}\right)^2dx}{2b}+\int(c+dx)^3\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx-\frac{(c+dx)^3\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \text{4202}$$

$$-2i\int\frac{e^{i(2a+2bx+\pi)}(c+dx)^3}{1+e^{i(2a+2bx+\pi)}}dx+\frac{3d\int(c+dx)^2\tan\left(a+bx+\frac{\pi}{2}\right)^2dx}{2b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$$\Big\downarrow \text{2620}$$

$$-2i\left(\frac{3id\int(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)+\frac{3d\int(c+dx)^2\tan\left(a+bx+\frac{\pi}{2}\right)^2dx}{2b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$$\Big\downarrow \text{3011}$$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)+\frac{3d\int(c+dx)^2\tan\left(a+bx+\frac{\pi}{2}\right)^2dx}{2b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$$\Big\downarrow \text{4203}$$

3.179. $\int(c+dx)^3\cot^3(a+bx)\,dx$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$
$$\frac{3d\left(-\frac{2d\int-((c+dx)\cot(a+bx))dx}{b}-\int(c+dx)^2dx-\frac{(c+dx)^2\cot(a+bx)}{b}\right)}{2b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$\downarrow$ 17

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$
$$\frac{3d\left(-\frac{2d\int-((c+dx)\cot(a+bx))dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$\downarrow$ 25

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$
$$\frac{3d\left(\frac{2d\int(c+dx)\cot(a+bx)dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$\downarrow$ 3042

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$
$$\frac{3d\left(\frac{2d\int-\left((c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)\right)dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$\downarrow$ 25

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$
$$\frac{3d\left(-\frac{2d\int(c+dx)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$\downarrow$ 4202

3.179. $\int(c+dx)^3\cot^3(a+bx)\,dx$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$
$$\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\int\frac{e^{i(2a+2bx+\pi)}(c+dx)}{1+e^{i(2a+2bx+\pi)}}dx\right)}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$\downarrow$ 2620

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)$$
$$\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{id\int\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$\downarrow$ 2715

$$\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{d\int e^{-i(2a+2bx+\pi)}\log\left(1+e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+
}{2b}$$
$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-$$
$$\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$\downarrow$ 2838

3.179. $\int(c+dx)^3\cot^3(a+bx)\,dx$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right.$$

$$\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}-$$

$$\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$\big\downarrow$ 7163

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\left(\frac{id\int\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i(c+d\ldots\right.$$

$$\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}-$$

$$\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$\big\downarrow$ 2720

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\left(\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(2a+\ldots}\right)}{2b}\ldots\right.}{b}\right.}{2b}\right.$$

$$\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}-$$

$$\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$\big\downarrow$ 7143

3.179. $\int(c+dx)^3\cot^3(a+bx)\,dx$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\left(\frac{d\operatorname{PolyLog}\left(4,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{i(2a+2bx+\pi)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i(c+dx)^3}{\phantom{x}}\right.$$

$$\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)\right)}{b}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}-$$

$$\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

input
```
Int[(c + d*x)^3*Cot[a + b*x]^3,x]
```

output
```
((I/4)*(c + d*x)^4)/d - ((c + d*x)^3*Cot[a + b*x]^2)/(2*b) + (3*d*(-1/3*(c
 + d*x)^3/d - ((c + d*x)^2*Cot[a + b*x])/b - (2*d*(((I/2)*(c + d*x)^2)/d -
 (2*I)*(((-1/2*I)*(c + d*x)*Log[1 + E^(I*(2*a + Pi + 2*b*x))])/b - (d*Poly
Log[2, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2))))/b))/(2*b) - (2*I)*(((-1/2*I)
*(c + d*x)^3*Log[1 + E^(I*(2*a + Pi + 2*b*x))])/b + (((3*I)/2)*d*(((I/2)*(
c + d*x)^2*PolyLog[2, -E^(I*(2*a + Pi + 2*b*x))])/b - (I*d*(((-1/2*I)*(c +
 d*x)*PolyLog[3, -E^(I*(2*a + Pi + 2*b*x))])/b + (d*PolyLog[4, -E^(I*(2*a
+ Pi + 2*b*x))])/(4*b^2)))/b))/b)
```

### 3.179.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

## 3.179. $\int (c+dx)^3 \cot^3(a+bx)\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

### 3.179.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1202 vs. 2(221) = 442.

Time = 1.33 (sec) , antiderivative size = 1203, normalized size of antiderivative = 4.70

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1203 |

input
```
int((d*x+c)^3*cot(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
3*d^2/b^3*c*ln(exp(I*(b*x+a))+1)-6*d^2/b^3*c*ln(exp(I*(b*x+a)))+3/2*I/b^4*
d^3*a^4-6*I/b^4*d^3*polylog(4,exp(I*(b*x+a)))+3/2*I*d*c^2*x^2+(2*b*d^3*x^3
*exp(2*I*(b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*exp(2*I*(b*x+
a))-6*I*c*d^2*x*exp(2*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-3*I*c^2*d*ex
p(2*I*(b*x+a))+3*I*d^3*x^2+2*b*c^3*exp(2*I*(b*x+a))+6*I*c*d^2*x+3*I*c^2*d)
/b^2/(exp(2*I*(b*x+a))-1)^2-I*c^3*x-1/4*I/d*c^4+3*d^2/b^3*c*ln(exp(I*(b*x+
a))-1)+3*d^3/b^3*ln(1-exp(I*(b*x+a)))*x+3*d^3/b^4*ln(1-exp(I*(b*x+a)))*a+3
*d^3/b^3*ln(exp(I*(b*x+a))+1)*x+6*d^3/b^4*a*ln(exp(I*(b*x+a)))-3*d^3/b^4*a
*ln(exp(I*(b*x+a))-1)-3*I*d^3/b^2*x^2-3*I*d^3/b^4*a^2-3*I*d^3/b^4*polylog(
2,-exp(I*(b*x+a)))-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4-6*I*d^3*polylog(4
,-exp(I*(b*x+a)))/b^4-6*I*d^3/b^3*x*a+1/4*I*d^3*x^4+I*d^2*c*x^3+6/b^3*c*d^
2*a^2*ln(exp(I*(b*x+a)))-3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)-6/b^2*c^2*d*
a*ln(exp(I*(b*x+a)))+3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)-3/b^2*d*c^2*ln(1-e
xp(I*(b*x+a)))*a+3/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2-1/b*c^3*ln(exp(I*(b*
x+a))+1)+2/b*c^3*ln(exp(I*(b*x+a)))-1/b*c^3*ln(exp(I*(b*x+a))-1)+2*I/b^3*d
^3*a^3*x+3*I/b^2*d*c^2*a^2+3*I/b^2*d*c^2*polylog(2,exp(I*(b*x+a)))+3*I/b^2
*d*c^2*polylog(2,-exp(I*(b*x+a)))-4*I/b^3*c*d^2*a^3+3*I/b^2*d^3*polylog(2,
-exp(I*(b*x+a)))*x^2+3*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2-6*I/b^2*c*d
^2*a^2*x+6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x+6*I/b^2*c*d^2*polylog(2
,-exp(I*(b*x+a)))*x+6*I/b*d*c^2*x*a-3/b*d*c^2*ln(1-exp(I*(b*x+a)))*x-3/...
```

### 3.179.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1139 vs. 2(214) = 428.

Time = 0.27 (sec) , antiderivative size = 1139, normalized size of antiderivative = 4.45

$$\int (c+dx)^3 \cot^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*cot(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/8*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 - 6*(I*
b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d - I*d^3 + (-I*b^2*d^3*x^2 - 2*
I*b^2*c*d^2*x - I*b^2*c^2*d + I*d^3)*cos(2*b*x + 2*a))*dilog(cos(2*b*x + 2
*a) + I*sin(2*b*x + 2*a)) - 6*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^
2*d + I*d^3 + (I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d - I*d^3)*cos(
2*b*x + 2*a))*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 4*(b^3*c^3 -
3*a*b^2*c^2*d + 3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3 - (b^3*c^3 - 3*a*b^2
*c^2*d + 3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3)*cos(2*b*x + 2*a))*log(-1/2
*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 4*(b^3*c^3 - 3*a*b^2*c
^2*d + 3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d +
3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3)*cos(2*b*x + 2*a))*log(-1/2*cos(2*b*
x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^
2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 3*a)*d^3 + 3*(b^3*c^2*d - b*d^3
)*x - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^
3 - 3*a)*d^3 + 3*(b^3*c^2*d - b*d^3)*x)*cos(2*b*x + 2*a))*log(-cos(2*b*x +
 2*a) + I*sin(2*b*x + 2*a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 3*a)*d^3 + 3*(b^3*c^2*d - b*d^3)*x - (b^
3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 3*a)*
d^3 + 3*(b^3*c^2*d - b*d^3)*x)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) - I
*sin(2*b*x + 2*a) + 1) - 3*(I*d^3*cos(2*b*x + 2*a) - I*d^3)*polylog(4, ...
```

### 3.179.6 Sympy [F]

$$\int (c+dx)^3 \cot^3(a+bx)\,dx = \int (c+dx)^3 \cot^3{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**3*cot(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**3*cot(a + b*x)**3, x)
```

### 3.179.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3958 vs. 2(214) = 428.

Time = 1.22 (sec) , antiderivative size = 3958, normalized size of antiderivative = 15.46

$$\int (c+dx)^3 \cot^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*cot(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(c^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2)) - 3*a*c^2*d*(1/sin(b*x
+ a)^2 + log(sin(b*x + a)^2))/b + 3*a^2*c*d^2*(1/sin(b*x + a)^2 + log(sin(
b*x + a)^2))/b^2 - a^3*d^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2))/b^3 -
2*((b*x + a)^4*d^3 + 12*b^2*c^2*d - 24*a*b*c*d^2 + 12*a^2*d^3 + 4*(b*c*d^2
 - a*d^3)*(b*x + a)^3 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a)^2
- 4*((b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)
^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a) + ((b*x + a)^3*
d^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d
 - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 2*((b*x + a)
^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^
2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*x
+ a)^3*d^3 + 3*I*b*c*d^2 - 3*I*a*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^
2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*a^2 + I)*d^3)*(b*x + a))*sin(4*b
*x + 4*a) - 2*(I*(b*x + a)^3*d^3 - 3*I*b*c*d^2 + 3*I*a*d^3 + 3*(I*b*c*d^2
- I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2 - I)*d^3)
*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 12
*(b*c*d^2 - a*d^3 + (b*c*d^2 - a*d^3)*cos(4*b*x + 4*a) - 2*(b*c*d^2 - a*d^
3)*cos(2*b*x + 2*a) + (I*b*c*d^2 - I*a*d^3)*sin(4*b*x + 4*a) + 2*(-I*b*c*d
^2 + I*a*d^3)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) +
4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2...
```

### 3.179.8 Giac [F]

$$\int (c+dx)^3 \cot^3(a+bx)\,dx = \int (dx+c)^3 \cot(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^3*cot(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*cot(b*x + a)^3, x)
```

### 3.179.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \cot^3(a+bx)\,dx = \int \cot(a+b\,x)^3\,(c+d\,x)^3\,dx$$

input
```
int(cot(a + b*x)^3*(c + d*x)^3,x)
```

output
```
int(cot(a + b*x)^3*(c + d*x)^3, x)
```

# 3.180 $\int (c+dx)^2 \cot^3(a+bx)\,dx$


3.180.1 Optimal result . . . 1443
3.180.2 Mathematica [B] (verified) . . . 1443
3.180.3 Rubi [A] (verified) . . . 1445
3.180.4 Maple [B] (verified) . . . 1449
3.180.5 Fricas [B] (verification not implemented) . . . 1450
3.180.6 Sympy [F] . . . 1451
3.180.7 Maxima [B] (verification not implemented) . . . 1451
3.180.8 Giac [F] . . . 1452
3.180.9 Mupad [F(-1)] . . . 1453


## 3.180.1 Optimal result

Integrand size = 16, antiderivative size = 168

$$\int (c+dx)^2 \cot^3(a+bx)\,dx = -\frac{cdx}{b}-\frac{d^2x^2}{2b}+\frac{i(c+dx)^3}{3d}-\frac{d(c+dx)\cot(a+bx)}{b^2}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}-\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}+\frac{d^2\log(\sin(a+bx))}{b^3}+\frac{id(c+dx)\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}$$

output
```
-c*d*x/b-1/2*d^2*x^2/b+1/3*I*(d*x+c)^3/d-d*(d*x+c)*cot(b*x+a)/b^2-1/2*(d*x
+c)^2*cot(b*x+a)^2/b-(d*x+c)^2*ln(1-exp(2*I*(b*x+a)))/b+d^2*ln(sin(b*x+a))
/b^3+I*d*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,exp(2*I
*(b*x+a)))/b^3
```

## 3.180.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 548 vs. 2(168) = 336.

Time = 6.83 (sec) , antiderivative size = 548, normalized size of antiderivative = 3.26

$$\int (c+dx)^2\cot^3(a+bx)\,dx = -\frac{1}{3}x\left(3c^2+3cdx+d^2x^2\right)\cot(a) - \frac{(c+dx)^2\csc^2(a+bx)}{2b}$$
$$+ \frac{d^2e^{ia}\csc(a)\left(2b^3e^{-2ia}x^3+3ib^2\left(1-e^{-2ia}\right)x^2\log\left(1-e^{-i(a+bx)}\right)+3ib^2\left(1-e^{-2ia}\right)x^2\log\left(1+e^{-i(a+bx)}\right)-\cdots\right.}{\cdots}$$
$$- \frac{c^2\csc(a)(-bx\cos(a)+\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a))}{b\left(\cos^2(a)+\sin^2(a)\right)}$$
$$+ \frac{d^2\csc(a)(-bx\cos(a)+\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a))}{b^3\left(\cos^2(a)+\sin^2(a)\right)}$$
$$+ \frac{\csc(a)\csc(a+bx)\left(cd\sin(bx)+d^2x\sin(bx)\right)}{b^2}$$
$$+ \frac{cd\csc(a)\sec(a)\left(b^2e^{i\arctan(\tan(a))}x^2+\frac{(ibx(-\pi+2\arctan(\tan(a)))-\pi\log\left(1+e^{-2ibx}\right)-2(bx+\arctan(\tan(a)))\log\left(1-e^{2i(bx+\arctan\cdots}\right.\cdots}{\cdots}\right.}{b^2\sqrt{\sec^2(a)\left(\cos^2(a)+\cdots\right.}}$$

input
```
Integrate[(c + d*x)^2*Cot[a + b*x]^3,x]
```

output
```
-1/3*(x*(3*c^2 + 3*c*d*x + d^2*x^2)*Cot[a]) - ((c + d*x)^2*Csc[a + b*x]^2)
/(2*b) + (d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^(
(-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*
x^2*Log[1 + E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, -E^(
(-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, E^((-I)*(a + b*x))]
 + (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, -E^((-I)*(a + b*x))] + (6*I)*(1 - E
^((-2*I)*a))*PolyLog[3, E^((-I)*(a + b*x))]))/(6*b^3) - (c^2*Csc[a]*(-(b*x
*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 +
Sin[a]^2)) + (d^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin
[b*x]]*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) + (Csc[a]*Csc[a + b*x]*(c*d*Si
n[b*x] + d^2*x*Sin[b*x]))/b^2 + (c*d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]
])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2
*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log
[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2
, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^2*Sqr
t[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

3.180. $\int (c+dx)^2\cot^3(a+bx)\,dx$

### 3.180.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.17, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.062, Rules used = {3042, 25, 4203, 25, 3042, 25, 4202, 2620, 3011, 2720, 4203, 17, 25, 3042, 25, 3956, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \cot^3(a+bx) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -(c+dx)^2 \tan\left(a+bx+\frac{\pi}{2}\right)^3 dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)^2 \tan\left(\frac{1}{2}(2a+\pi)+bx\right)^3 dx$$

$$\Big\downarrow \text{4203}$$

$$\frac{d\int (c+dx)\cot^2(a+bx)dx}{b}+\int -(c+dx)^2\cot(a+bx)dx-\frac{(c+dx)^2\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{d\int (c+dx)\cot^2(a+bx)dx}{b}-\int (c+dx)^2\cot(a+bx)dx-\frac{(c+dx)^2\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$-\int -(c+dx)^2\tan\left(a+bx+\frac{\pi}{2}\right)dx+\frac{d\int (c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)^2dx}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{d\int (c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)^2dx}{b}+\int (c+dx)^2\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx-\frac{(c+dx)^2\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \text{4202}$$

$$-2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)^2}{1+e^{i(2a+2bx+\pi)}}dx+\frac{d\int (c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)^2dx}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

$$\Big\downarrow \text{2620}$$

$$-2i\left(\frac{id\int (c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)+$$
$$\frac{d\int (c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)^2dx}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

$$\Big\downarrow \ 3011$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{id\int \operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)+$$
$$\frac{d\int (c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)^2dx}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

$$\Big\downarrow \ 2720$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i}\right.}{2b}\right.$$
$$\frac{d\int (c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)^2dx}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

$$\Big\downarrow \ 4203$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i}\right.}{2b}\right.$$
$$\frac{d\left(-\frac{d\int -\cot(a+bx)dx}{b}-\int (c+dx)dx-\frac{(c+dx)\cot(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

$$\Big\downarrow \ 17$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i}\right.}{2b}\right.$$
$$\frac{d\left(-\frac{d\int -\cot(a+bx)dx}{b}-\frac{(c+dx)\cot(a+bx)}{b}-\frac{(c+dx)^2}{2d}\right)}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

$$\Big\downarrow \ 25$$

3.180. $\int (c+dx)^2\cot^3(a+bx)\,dx$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{i(2a+2bx+\pi)})}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}(2,-e^{i(2a+2bx+\pi)})de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i}\right.}{2b}\right.$$

$$\frac{d\left(\frac{d\int\cot(a+bx)dx}{b}-\frac{(c+dx)\cot(a+bx)}{b}-\frac{(c+dx)^2}{2d}\right)}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

$$\downarrow 3042$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{i(2a+2bx+\pi)})}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}(2,-e^{i(2a+2bx+\pi)})de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i}\right.}{2b}\right.$$

$$\frac{d\left(\frac{d\int-\tan\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cot(a+bx)}{b}-\frac{(c+dx)^2}{2d}\right)}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

$$\downarrow 25$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{i(2a+2bx+\pi)})}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}(2,-e^{i(2a+2bx+\pi)})de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i}\right.}{2b}\right.$$

$$\frac{d\left(-\frac{d\int\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx}{b}-\frac{(c+dx)\cot(a+bx)}{b}-\frac{(c+dx)^2}{2d}\right)}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

$$\downarrow 3956$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{i(2a+2bx+\pi)})}{2b}-\frac{d\int e^{-i(2a+2bx+\pi)}\operatorname{PolyLog}(2,-e^{i(2a+2bx+\pi)})de^{i(2a+2bx+\pi)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i}\right.}{2b}\right.$$

$$\frac{d\left(\frac{d\log(-\sin(a+bx))}{b^2}-\frac{(c+dx)\cot(a+bx)}{b}-\frac{(c+dx)^2}{2d}\right)}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

$$\downarrow 7143$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{i(2a+2bx+\pi)})}{2b}-\frac{d\operatorname{PolyLog}(3,-e^{i(2a+2bx+\pi)})}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)+$$

$$\frac{d\left(\frac{d\log(-\sin(a+bx))}{b^2}-\frac{(c+dx)\cot(a+bx)}{b}-\frac{(c+dx)^2}{2d}\right)}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

input
```
Int[(c + d*x)^2*Cot[a + b*x]^3,x]
```

3.180. $\int(c+dx)^2\cot^3(a+bx)\,dx$

output
```
((I/3)*(c + d*x)^3)/d - ((c + d*x)^2*Cot[a + b*x]^2)/(2*b) + (d*(-1/2*(c +
 d*x)^2/d - ((c + d*x)*Cot[a + b*x])/b + (d*Log[-Sin[a + b*x]])/b^2))/b -
(2*I)*(((-1/2*I)*(c + d*x)^2*Log[1 + E^(I*(2*a + Pi + 2*b*x))])/b + (I*d*(
((I/2)*(c + d*x)*PolyLog[2, -E^(I*(2*a + Pi + 2*b*x))])/b - (d*PolyLog[3,
-E^(I*(2*a + Pi + 2*b*x))])/(4*b^2)))/b)
```

### 3.180.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.180.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 643 vs. $2(152) = 304$.

Time = 1.18 (sec) , antiderivative size = 644, normalized size of antiderivative = 3.83

| method | result |
|---|---|
| risch | $\frac{2b\,d^2x^2\mathrm{e}^{2i(xb+a)}+4bcdx\,\mathrm{e}^{2i(xb+a)}+2b\,c^2\mathrm{e}^{2i(xb+a)}-2id^2x\,\mathrm{e}^{2i(xb+a)}-2icd\,\mathrm{e}^{2i(xb+a)}+2id^2x+2idc}{b^2\left(\mathrm{e}^{2i(xb+a)}-1\right)^2}-ic^2x-\frac{ic^3}{3d}-\frac{c^2\ln\left(\mathrm{e}^{i(xb+a)}\right.}{b}$ |

input
```
int((d*x+c)^2*cot(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
4*I/b*c*d*x*a-I*c^2*x-1/3*I/d*c^3+2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))-1/b^3*d
^2*a^2*ln(exp(I*(b*x+a))-1)+2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+2*I/b
^2*c*d*polylog(2,-exp(I*(b*x+a)))+2*(b*d^2*x^2*exp(2*I*(b*x+a))+2*b*c*d*x*
exp(2*I*(b*x+a))+b*c^2*exp(2*I*(b*x+a))-I*d^2*x*exp(2*I*(b*x+a))-I*c*d*exp
(2*I*(b*x+a))+I*d^2*x+I*d*c)/b^2/(exp(2*I*(b*x+a))-1)^2-1/b*c^2*ln(exp(I*(
b*x+a))+1)+2/b*c^2*ln(exp(I*(b*x+a)))-1/b*c^2*ln(exp(I*(b*x+a))-1)+I*d*c*x
^2+1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2-1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b
*d^2*ln(exp(I*(b*x+a))+1)*x^2-2/b*d*c*ln(exp(I*(b*x+a))+1)*x-2/b^2*d*c*ln(
1-exp(I*(b*x+a)))*a-4/b^2*c*d*a*ln(exp(I*(b*x+a)))+2/b^2*c*d*a*ln(exp(I*(b
*x+a))-1)-2/b*d*c*ln(1-exp(I*(b*x+a)))*x-4/3*I/b^3*d^2*a^3+1/3*I*d^2*x^3+1
/b^3*d^2*ln(exp(I*(b*x+a))+1)-2/b^3*d^2*ln(exp(I*(b*x+a)))+1/b^3*d^2*ln(ex
p(I*(b*x+a))-1)-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3-2*d^2*polylog(3,exp(I
*(b*x+a)))/b^3-2*I/b^2*d^2*a^2*x+2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x+2
*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))+2*I/b^2*c*d*a^2
```

### 3.180.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 657 vs. 2(149) = 298.

Time = 0.26 (sec) , antiderivative size = 657, normalized size of antiderivative = 3.91

$$\int (c+dx)^2\cot^3(a+bx)\,dx$$
$$= \frac{4\,b^2d^2x^2+8\,b^2cdx+4\,b^2c^2-2\left(i\,bd^2x+i\,bcd+\left(-i\,bd^2x-i\,bcd\right)\cos\left(2\,bx+2\,a\right)\right)\mathrm{Li}_2(\cos\left(2\,bx+2\,a\right)+i\,s}{}$$

input
```
integrate((d*x+c)^2*cot(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(4*b^2*d^2*x^2 + 8*b^2*c*d*x + 4*b^2*c^2 - 2*(I*b*d^2*x + I*b*c*d + (-
I*b*d^2*x - I*b*c*d)*cos(2*b*x + 2*a))*dilog(cos(2*b*x + 2*a) + I*sin(2*b*
x + 2*a)) - 2*(-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(2*b*x + 2*
a))*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 2*(b^2*c^2 - 2*a*b*c*d
+ (a^2 - 1)*d^2 - (b^2*c^2 - 2*a*b*c*d + (a^2 - 1)*d^2)*cos(2*b*x + 2*a))*
log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b^2*c^2 - 2
*a*b*c*d + (a^2 - 1)*d^2 - (b^2*c^2 - 2*a*b*c*d + (a^2 - 1)*d^2)*cos(2*b*x
 + 2*a))*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b^
2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x
 + 2*a*b*c*d - a^2*d^2)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) + I*sin(2*
b*x + 2*a) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^
2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(2*b*x + 2*a))*log(-cos(
2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1) - (d^2*cos(2*b*x + 2*a) - d^2)*poly
log(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) - (d^2*cos(2*b*x + 2*a) - d^
2)*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 4*(b*d^2*x + b*c*d)
*sin(2*b*x + 2*a))/(b^3*cos(2*b*x + 2*a) - b^3)
```

### 3.180.6 Sympy [F]

$$\int (c+dx)^2 \cot^3(a+bx)\,dx = \int (c+dx)^2 \cot^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*cot(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**2*cot(a + b*x)**3, x)
```

### 3.180.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1966 vs. 2(149) = 298.

Time = 0.54 (sec) , antiderivative size = 1966, normalized size of antiderivative = 11.70

$$\int (c+dx)^2 \cot^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*cot(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(c^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2)) - 2*a*c*d*(1/sin(b*x +
a)^2 + log(sin(b*x + a)^2))/b + a^2*d^2*(1/sin(b*x + a)^2 + log(sin(b*x +
a)^2))/b^2 - 2*(2*(b*x + a)^3*d^2 + 6*(b*c*d - a*d^2)*(b*x + a)^2 + 12*b*c
*d - 12*a*d^2 - 6*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2 + (
(b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(4*b*x + 4*a) - 2*
((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(2*b*x + 2*a) - (
-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a) + I*d^2)*sin(4*b*x +
 4*a) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) - I*d^2)*si
n(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 6*(d^2*cos(4*b*x
 + 4*a) - 2*d^2*cos(2*b*x + 2*a) + I*d^2*sin(4*b*x + 4*a) - 2*I*d^2*sin(2*
b*x + 2*a) + d^2)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 6*((b*x + a)^2
*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*
(b*x + a))*cos(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
+ a))*cos(2*b*x + 2*a) + (I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x +
 a))*sin(4*b*x + 4*a) + 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*
x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 2*((b
*x + a)^3*d^2 + 3*(b*c*d - a*d^2)*(b*x + a)^2 - 6*(b*x + a)*d^2)*cos(4*b*x
 + 4*a) - 4*((b*x + a)^3*d^2 + 3*(b*c*d - (a - I)*d^2)*(b*x + a)^2 + 3*b*c
*d - 3*a*d^2 - 3*(-2*I*b*c*d + (2*I*a + 1)*d^2)*(b*x + a))*cos(2*b*x + 2*a
) + 12*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)...
```

### 3.180.8 Giac [F]

$$\int (c+dx)^2 \cot^3(a+bx)\,dx = \int (dx+c)^2 \cot(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^2*cot(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*cot(b*x + a)^3, x)
```

### 3.180.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \cot^3(a+bx)\,dx = \int \cot(a+b\,x)^3\,(c+d\,x)^2\,dx$$

input
```
int(cot(a + b*x)^3*(c + d*x)^2,x)
```

output
```
int(cot(a + b*x)^3*(c + d*x)^2, x)
```

# 3.181 $\int (c+dx)\cot^3(a+bx)\,dx$


3.181.1 Optimal result . . . . . 1454
3.181.2 Mathematica [B] (verified) . . . . . 1454
3.181.3 Rubi [A] (verified) . . . . . 1455
3.181.4 Maple [B] (verified) . . . . . 1458
3.181.5 Fricas [B] (verification not implemented) . . . . . 1458
3.181.6 Sympy [F] . . . . . 1459
3.181.7 Maxima [B] (verification not implemented) . . . . . 1459
3.181.8 Giac [F] . . . . . 1460
3.181.9 Mupad [F(-1)] . . . . . 1461


## 3.181.1 Optimal result

Integrand size = 14, antiderivative size = 109

$$\int (c+dx)\cot^3(a+bx)\,dx = -\frac{dx}{2b}+\frac{i(c+dx)^2}{2d}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+bx)}{2b} -\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}+\frac{id\,\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}$$

output

```
-1/2*d*x/b+1/2*I*(d*x+c)^2/d-1/2*d*cot(b*x+a)/b^2-1/2*(d*x+c)*cot(b*x+a)^2
/b-(d*x+c)*ln(1-exp(2*I*(b*x+a)))/b+1/2*I*d*polylog(2,exp(2*I*(b*x+a)))/b^
2
```

## 3.181.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 240 vs. 2(109) = 218.

Time = 6.25 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.20

$$\int (c+dx)\cot^3(a+bx)\,dx = -\frac{1}{2}dx^2\cot(a)-\frac{dx\csc^2(a+bx)}{2b} -\frac{c(\cot^2(a+bx)+2\log(\cos(a+bx))+2\log(\tan(a+bx)))}{2b}+\frac{d\csc(a)\csc(a+bx)\sin(bx)}{2b^2} +\frac{d\csc(a)\sec(a)\left(b^2e^{i\arctan(\tan(a))}x^2+\frac{(ibx(-\pi+2\arctan(\tan(a)))-\pi\log(1+e^{-2ibx})-2(bx+\arctan(\tan(a)))\log(1-e^{2i(bx+\arctan(\mathrm{t}}{\cdots}\right.}{2b^2\sqrt{\sec^2(a)\left(\cos^2(a)+\mathrm{s}\cdots\right.}}$$

input
```
Integrate[(c + d*x)*Cot[a + b*x]^3,x]
```

output
```
-1/2*(d*x^2*Cot[a]) - (d*x*Csc[a + b*x]^2)/(2*b) - (c*(Cot[a + b*x]^2 + 2*
Log[Cos[a + b*x]] + 2*Log[Tan[a + b*x]]))/(2*b) + (d*Csc[a]*Csc[a + b*x]*S
in[b*x])/(2*b^2) + (d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*
x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[
Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*
ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x
 + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(C
os[a]^2 + Sin[a]^2)])
```

### 3.181.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 25, 4203, 25, 3042, 25, 3954, 24, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\cot^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\left((c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)^3\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)^3dx$$

$$\Big\downarrow \text{4203}$$

$$\int -((c+dx)\cot(a+bx))dx+\frac{d\int \cot^2(a+bx)dx}{2b}-\frac{(c+dx)\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \text{25}$$

$$-\int (c+dx)\cot(a+bx)dx+\frac{d\int \cot^2(a+bx)dx}{2b}-\frac{(c+dx)\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$-\int -\left((c+dx)\tan\left(a+bx+\frac{\pi}{2}\right)\right)dx+\frac{d\int \tan\left(a+bx+\frac{\pi}{2}\right)^2dx}{2b}-\frac{(c+dx)\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \text{25}$$

$$\int (c+dx)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx+\frac{d\int \tan\left(a+bx+\frac{\pi}{2}\right)^2dx}{2b}-\frac{(c+dx)\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3954}$$

$$\int (c+dx)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx+\frac{d\left(-\int 1dx-\frac{\cot(a+bx)}{b}\right)}{2b}-\frac{(c+dx)\cot^2(a+bx)}{2b}$$

$$\Big\downarrow \text{24}$$

$$\int (c+dx)\tan\left(\frac{1}{2}(2a+\pi)+bx\right)dx-\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{d\left(-\frac{\cot(a+bx)}{b}-x\right)}{2b}$$

$$\Big\downarrow \text{4202}$$

$$-2i\int \frac{e^{i(2a+2bx+\pi)}(c+dx)}{1+e^{i(2a+2bx+\pi)}}dx-\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{d\left(-\frac{\cot(a+bx)}{b}-x\right)}{2b}+\frac{i(c+dx)^2}{2d}$$

$$\Big\downarrow \text{2620}$$

$$-2i\left(\frac{id\int \log\left(1+e^{i(2a+2bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{d\left(-\frac{\cot(a+bx)}{b}-x\right)}{2b}+\frac{i(c+dx)^2}{2d}$$

$$\Big\downarrow \text{2715}$$

$$-2i\left(\frac{d\int e^{-i(2a+2bx+\pi)}\log\left(1+e^{i(2a+2bx+\pi)}\right)de^{i(2a+2bx+\pi)}}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{d\left(-\frac{\cot(a+bx)}{b}-x\right)}{2b}+\frac{i(c+dx)^2}{2d}$$

$$\Big\downarrow \text{2838}$$

$$-2i\left(-\frac{d\,\text{PolyLog}\left(2,-e^{i(2a+2bx+\pi)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{i(2a+2bx+\pi)}\right)}{2b}\right)-\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{d\left(-\frac{\cot(a+bx)}{b}-x\right)}{2b}+\frac{i(c+dx)^2}{2d}$$

input
```
Int[(c + d*x)*Cot[a + b*x]^3,x]
```

3.181. $\int (c+dx)\cot^3(a+bx)\,dx$

output
```
((I/2)*(c + d*x)^2)/d - ((c + d*x)*Cot[a + b*x]^2)/(2*b) + (d*(-x - Cot[a
+ b*x]/b))/(2*b) - (2*I)*(((-1/2*I)*(c + d*x)*Log[1 + E^(I*(2*a + Pi + 2*b
*x))])/b - (d*PolyLog[2, -E^(I*(2*a + Pi + 2*b*x))])/(4*b^2))
```

### 3.181.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

### 3.181.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(93) = 186.

Time = 0.84 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.58

| method | result |
|---|---|
| risch | $\frac{id\,x^2}{2}+\frac{id\,a^2}{b^2}+\frac{2bdx\,\mathrm{e}^{2i(xb+a)}-id\,\mathrm{e}^{2i(xb+a)}+2bc\,\mathrm{e}^{2i(xb+a)}+id}{b^2\left(\mathrm{e}^{2i(xb+a)}-1\right)^2}-\frac{c\ln\left(\mathrm{e}^{i(xb+a)}+1\right)}{b}+\frac{2c\ln\left(\mathrm{e}^{i(xb+a)}\right)}{b}-\frac{c\ln\left(\mathrm{e}^{i(xb+a)}-1\right)}{b}+$ |

input
```
int((d*x+c)*cot(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*d*x^2+I/b^2*d*a^2+(2*b*d*x*exp(2*I*(b*x+a))-I*d*exp(2*I*(b*x+a))+2*b
*c*exp(2*I*(b*x+a))+I*d)/b^2/(exp(2*I*(b*x+a))-1)^2-1/b*c*ln(exp(I*(b*x+a)
)+1)+2/b*c*ln(exp(I*(b*x+a)))-1/b*c*ln(exp(I*(b*x+a))-1)+2*I/b*d*x*a-I*c*x
+I/b^2*d*polylog(2,exp(I*(b*x+a)))-1/b*d*ln(1-exp(I*(b*x+a)))*x-1/b^2*d*ln
(1-exp(I*(b*x+a)))*a+I*d*polylog(2,-exp(I*(b*x+a)))/b^2-1/b*d*ln(exp(I*(b*
x+a))+1)*x-2/b^2*d*a*ln(exp(I*(b*x+a)))+1/b^2*d*a*ln(exp(I*(b*x+a))-1)
```

### 3.181.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(90) = 180.

Time = 0.26 (sec) , antiderivative size = 339, normalized size of antiderivative = 3.11

$$\int (c+dx)\cot^3(a+bx)\,dx$$
$$=\frac{4\,bdx+4\,bc+(i\,d\cos(2\,bx+2\,a)-i\,d)\mathrm{Li}_2(\cos(2\,bx+2\,a)+i\,\sin(2\,bx+2\,a))+(-i\,d\cos(2\,bx+2\,a)+}{}$$

input
```
integrate((d*x+c)*cot(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(4*b*d*x + 4*b*c + (I*d*cos(2*b*x + 2*a) - I*d)*dilog(cos(2*b*x + 2*a)
 + I*sin(2*b*x + 2*a)) + (-I*d*cos(2*b*x + 2*a) + I*d)*dilog(cos(2*b*x + 2
*a) - I*sin(2*b*x + 2*a)) + 2*(b*c - a*d - (b*c - a*d)*cos(2*b*x + 2*a))*l
og(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b*c - a*d -
(b*c - a*d)*cos(2*b*x + 2*a))*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x
+ 2*a) + 1/2) + 2*(b*d*x + a*d - (b*d*x + a*d)*cos(2*b*x + 2*a))*log(-cos(
2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1) + 2*(b*d*x + a*d - (b*d*x + a*d)*co
s(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1) + 2*d*sin(
2*b*x + 2*a))/(b^2*cos(2*b*x + 2*a) - b^2)
```

## 3.181.6 Sympy [F]

$$\int (c+dx)\cot^3(a+bx)\,dx = \int (c+dx)\cot^3{(a+bx)}\,dx$$

input
```
integrate((d*x+c)*cot(b*x+a)**3,x)
```

output
```
Integral((c + d*x)*cot(a + b*x)**3, x)
```

## 3.181.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 830 vs. $2(90) = 180$.

Time = 0.38 (sec) , antiderivative size = 830, normalized size of antiderivative = 7.61

$$\int (c+dx)\cot^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*cot(b*x+a)^3,x, algorithm="maxima")
```

output
```
(b^2*d*x^2 + 2*b^2*c*x - 2*(b*d*x + b*c + (b*d*x + b*c)*cos(4*b*x + 4*a) -
 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (-I*b*d*x - I*b*c)*sin(4*b*x + 4*a) -
2*(I*b*d*x + I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) +
 1) - 2*(b*c*cos(4*b*x + 4*a) - 2*b*c*cos(2*b*x + 2*a) + I*b*c*sin(4*b*x +
 4*a) - 2*I*b*c*sin(2*b*x + 2*a) + b*c)*arctan2(sin(b*x + a), cos(b*x + a)
 - 1) + 2*(b*d*x*cos(4*b*x + 4*a) - 2*b*d*x*cos(2*b*x + 2*a) + I*b*d*x*sin
(4*b*x + 4*a) - 2*I*b*d*x*sin(2*b*x + 2*a) + b*d*x)*arctan2(sin(b*x + a),
-cos(b*x + a) + 1) + (b^2*d*x^2 + 2*b^2*c*x)*cos(4*b*x + 4*a) - 2*(b^2*d*x
^2 + 2*I*b*c + 2*(b^2*c + I*b*d)*x + d)*cos(2*b*x + 2*a) + 2*(d*cos(4*b*x
+ 4*a) - 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) - 2*I*d*sin(2*b*x + 2
*a) + d)*dilog(-e^(I*b*x + I*a)) + 2*(d*cos(4*b*x + 4*a) - 2*d*cos(2*b*x +
 2*a) + I*d*sin(4*b*x + 4*a) - 2*I*d*sin(2*b*x + 2*a) + d)*dilog(e^(I*b*x
+ I*a)) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(4*b*x + 4*a) - 2*(-I*
b*d*x - I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(4*b*x + 4*a) - 2*(b*d*
x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x
 + a) + 1) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(4*b*x + 4*a) - 2*(
-I*b*d*x - I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(4*b*x + 4*a) - 2*(b
*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(
b*x + a) + 1) - (-I*b^2*d*x^2 - 2*I*b^2*c*x)*sin(4*b*x + 4*a) + 2*(-I*b^2*
d*x^2 + 2*b*c + 2*(-I*b^2*c + b*d)*x - I*d)*sin(2*b*x + 2*a) + 2*d)/(-2...
```

### 3.181.8 Giac [F]

$$\int (c+dx)\cot^3(a+bx)\,dx = \int (dx+c)\cot(bx+a)^3\,dx$$

input
```
integrate((d*x+c)*cot(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)*cot(b*x + a)^3, x)
```

### 3.181.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\cot^3(a+bx)\,dx = \int \cot(a+b\,x)^3\,(c+d\,x)\,dx$$

input
```
int(cot(a + b*x)^3*(c + d*x),x)
```

output
```
int(cot(a + b*x)^3*(c + d*x), x)
```

# 3.182 $\int \frac{\cot^3(a+bx)}{c+dx} \, dx$

3.182.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1462
3.182.2 Mathematica [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1462
3.182.3 Rubi [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1463
3.182.4 Maple [N/A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1464
3.182.5 Fricas [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1464
3.182.6 Sympy [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1465
3.182.7 Maxima [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1465
3.182.8 Giac [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1466
3.182.9 Mupad [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1467

## 3.182.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\cot^3(a+bx)}{c+dx} \, dx = \text{Int}\left(\frac{\cot^3(a+bx)}{c+dx}, x\right)$$

output
```
Unintegrable(cot(b*x+a)^3/(d*x+c),x)
```

## 3.182.2 Mathematica [N/A]

Not integrable

Time = 9.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^3(a+bx)}{c+dx} \, dx = \int \frac{\cot^3(a+bx)}{c+dx} \, dx$$

input
```
Integrate[Cot[a + b*x]^3/(c + d*x),x]
```

output
```
Integrate[Cot[a + b*x]^3/(c + d*x), x]
```

### 3.182.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 25, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(a+bx)}{c+dx}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{\tan\left(a+bx+\frac{\pi}{2}\right)^3}{c+dx}dx$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)^3}{c+dx}dx$$
$$\Big\downarrow \text{4222}$$
$$-\int \frac{\tan^3\left(\frac{1}{2}(2a+\pi)+bx\right)}{c+dx}dx$$

input
```
Int[Cot[a + b*x]^3/(c + d*x),x]
```

output
```
$Aborted
```

#### 3.182.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

### 3.182.4 Maple [N/A] (verified)

Not integrable

Time = 0.46 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cot\left(xb+a\right)^{3}}{dx+c}dx$$

input
```
int(cot(b*x+a)^3/(d*x+c),x)
```

output
```
int(cot(b*x+a)^3/(d*x+c),x)
```

### 3.182.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^3(a+bx)}{c+dx}\,dx=\int \frac{\cot\left(bx+a\right)^{3}}{dx+c}\,dx$$

input
```
integrate(cot(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
integral(cot(b*x + a)^3/(d*x + c), x)
```

### 3.182.6 Sympy [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\cot^3(a+bx)}{c+dx}\,dx = \int \frac{\cot^3(a+bx)}{c+dx}\,dx$$

input
```
integrate(cot(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(cot(a + b*x)**3/(c + d*x), x)
```

### 3.182.7 Maxima [N/A]

Not integrable

Time = 2.64 (sec) , antiderivative size = 1636, normalized size of antiderivative = 102.25

$$\int \frac{\cot^3(a+bx)}{c+dx}\,dx = \int \frac{\cot(bx+a)^3}{dx+c}\,dx$$

input
```
integrate(cot(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output
```
-(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2
- (2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
 - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x
^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*
sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate((b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2 - d^2)*sin(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*
x + b^2*c^3)*cos(b*x + a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3)*sin(b*x + a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^
2*d*x + b^2*c^3)*cos(b*x + a)), x) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
+ (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*s
in(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d...
```

### 3.182.8 Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^3(a+bx)}{c+dx} \, dx = \int \frac{\cot(bx+a)^3}{dx+c} \, dx$$

input
```
integrate(cot(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output
```
integrate(cot(b*x + a)^3/(d*x + c), x)
```

### 3.182.9 Mupad [N/A]

Not integrable

Time = 24.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^3(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+b\,x)^3}{c+d\,x}\,dx$$

input
```
int(cot(a + b*x)^3/(c + d*x),x)
```

output
```
int(cot(a + b*x)^3/(c + d*x), x)
```

# 3.183 $\int \frac{\cot^3(a+bx)}{(c+dx)^2} dx$


3.183.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1468
3.183.2 Mathematica [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1468
3.183.3 Rubi [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1469
3.183.4 Maple [N/A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1470
3.183.5 Fricas [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1470
3.183.6 Sympy [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1471
3.183.7 Maxima [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1471
3.183.8 Giac [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1472
3.183.9 Mupad [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1473


## 3.183.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2} dx = \text{Int}\left(\frac{\cot^3(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Unintegrable(cot(b*x+a)^3/(d*x+c)^2,x)
```

## 3.183.2 Mathematica [N/A]

Not integrable

Time = 11.52 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2} dx = \int \frac{\cot^3(a+bx)}{(c+dx)^2} dx$$

input
```
Integrate[Cot[a + b*x]^3/(c + d*x)^2,x]
```

output
```
Integrate[Cot[a + b*x]^3/(c + d*x)^2, x]
```

## 3.183.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 25, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2} \, dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\tan\left(a+bx+\frac{\pi}{2}\right)^3}{(c+dx)^2}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\tan\left(\frac{1}{2}(2a+\pi)+bx\right)^3}{(c+dx)^2}dx$$

$$\Big\downarrow \; 4222$$

$$-\int \frac{\tan^3\left(\frac{1}{2}(2a+\pi)+bx\right)}{(c+dx)^2}dx$$

input
```
Int[Cot[a + b*x]^3/(c + d*x)^2,x]
```

output
```
$Aborted
```

### 3.183.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

## 3.183.4 Maple [N/A] (verified)

Not integrable

Time = 0.47 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\cot\left(xb+a\right)^{3}}{\left(dx+c\right)^{2}}dx$$

input
```
int(cot(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int(cot(b*x+a)^3/(d*x+c)^2,x)
```

## 3.183.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot\left(bx+a\right)^{3}}{\left(dx+c\right)^{2}}\,dx$$

input
```
integrate(cot(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(cot(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.183.6 Sympy [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot^3{(a+bx)}}{(c+dx)^2}\,dx$$

input
```
integrate(cot(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(cot(a + b*x)**3/(c + d*x)**2, x)
```

### 3.183.7 Maxima [N/A]

Not integrable

Time = 10.31 (sec) , antiderivative size = 2124, normalized size of antiderivative = 132.75

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot{(bx+a)}^3}{(dx+c)^2}\,dx$$

input
```
integrate(cot(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2
- 2*((b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
 - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b
^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^
2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*
c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 +
 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x
^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3
*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*
a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x
 + 2*a))*integrate((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 3*d^2)*sin(b*x +
 a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b
^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*
x + b^2*c^4)*cos(b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d
^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(b*x + a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*
c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x)
 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b...
```

## 3.183.8 Giac [N/A]

Not integrable

Time = 2.39 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot{(bx+a)}^3}{(dx+c)^2}\,dx$$

input
```
integrate(cot(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(cot(b*x + a)^3/(d*x + c)^2, x)
```

### 3.183.9 Mupad [N/A]

Not integrable

Time = 24.61 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(a+b\,x)^3}{(c+d\,x)^2}\,dx$$

input
```
int(cot(a + b*x)^3/(c + d*x)^2,x)
```

output
```
int(cot(a + b*x)^3/(c + d*x)^2, x)
```

# 3.184 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx$


3.184.1 Optimal result . . . 1474
3.184.2 Mathematica [C] (verified) . . . 1475
3.184.3 Rubi [A] (verified) . . . 1476
3.184.4 Maple [A] (verified) . . . 1478
3.184.5 Fricas [A] (verification not implemented) . . . 1479
3.184.6 Sympy [F(-1)] . . . 1479
3.184.7 Maxima [C] (verification not implemented) . . . 1480
3.184.8 Giac [C] (verification not implemented) . . . 1480
3.184.9 Mupad [F(-1)] . . . 1481


## 3.184.1 Optimal result

Integrand size = 24, antiderivative size = 407

$$\begin{aligned}\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx &= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}\\
&-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2\sqrt{c+dx}\cos(4a+4bx)}{2048b^3}\\
&-\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}-\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}\\
&-\frac{15d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}\\
&+\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{4096b^{7/2}}\\
&+\frac{15d^{5/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{256b^{7/2}}\\
&+\frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{32b^2}+\frac{5d(c+dx)^{3/2}\sin(4a+4bx)}{256b^2}\end{aligned}$$

output
```
-1/8*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b-1/32*(d*x+c)^(5/2)*cos(4*b*x+4*a)/b+5/
32*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2+5/256*d*(d*x+c)^(3/2)*sin(4*b*x+4*a)
/b^2-15/8192*d^(5/2)*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*
(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/8192*d^(5/2)*FresnelS(2
*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*
Pi^(1/2)/b^(7/2)-15/256*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c
)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+15/256*d^(5/2)*FresnelS(2*b^(1/
2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+15/12
8*d^2*cos(2*b*x+2*a)*(d*x+c)^(1/2)/b^3+15/2048*d^2*cos(4*b*x+4*a)*(d*x+c)^
(1/2)/b^3
```

## 3.184.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.89 (sec) , antiderivative size = 1332, normalized size of antiderivative = 3.27

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \frac{\left(\frac{1}{128}+\frac{i}{128}\right)c\sqrt{d}e^{-\frac{2i(ad+b(c+dx))}{d}}\left((2+2i)\sqrt{b}\sqrt{d}e^{\frac{2ibc}{d}}\sqrt{c+dx}\left(3+4ibx+e^{4i(a+bx)}(-3+4ibx)\right)+i(4\right.}{b^{5/2}}$$
$$+\frac{c\sqrt{d}e^{-\frac{4i(ad+b(c+dx))}{d}}\left(-4\sqrt{b}\sqrt{d}e^{\frac{4ibc}{d}}\sqrt{c+dx}\left(-3i+8bx+e^{8i(a+bx)}(3i+8bx)\right)+(-1)^{3/4}(8bc+3id)e^{\frac{4ib(2c+dx)}{d}}\right.}{1024b^{5/2}}$$
$$+\frac{1}{4}c^2\left(-\frac{e^{2i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\,\Gamma\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{-2i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\,\Gamma\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{\frac{ib(c+dx)}{d}}}\right)+\frac{c^2e^{-\frac{4i(bc+ad)}{d}}\sqrt{c+dx}\left(-\right.}{}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
((1/128 + I/128)*c*Sqrt[d]*((2 + 2*I)*Sqrt[b]*Sqrt[d]*E^(((2*I)*b*c)/d)*Sq
rt[c + d*x]*(3 + (4*I)*b*x + E^((4*I)*(a + b*x))*(-3 + (4*I)*b*x)) + I*(4*
b*c + (3*I)*d)*E^(((2*I)*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[((1 + I)*Sqrt[b]*S
qrt[c + d*x])/Sqrt[d]] + ((4*I)*b*c + 3*d)*E^((2*I)*(2*a + b*x))*Sqrt[Pi]*
Erfi[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(b^(5/2)*E^(((2*I)*(a*d +
b*(c + d*x)))/d)) + (c*Sqrt[d]*(-4*Sqrt[b]*Sqrt[d]*E^(((4*I)*b*c)/d)*Sqrt[
c + d*x]*(-3*I + 8*b*x + E^((8*I)*(a + b*x))*(3*I + 8*b*x)) + (-1)^(3/4)*(
8*b*c + (3*I)*d)*E^(((4*I)*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[(2*(-1)^(1/4)*Sq
rt[b]*Sqrt[c + d*x])/Sqrt[d]] + (-1)^(1/4)*((8*I)*b*c + 3*d)*E^((4*I)*(2*a
 + b*x))*Sqrt[Pi]*Erfi[(2*(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(10
24*b^(5/2)*E^(((4*I)*(a*d + b*(c + d*x)))/d)) + (c^2*(-1/4*(E^((2*I)*(a -
(b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d])/(Sqrt[2]*b*Sqr
t[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]*Gamma[3/2, ((2*I)*b*(c + d*x))/d
])/(4*Sqrt[2]*b*E^((2*I)*(a - (b*c)/d))*Sqrt[(I*b*(c + d*x))/d])))/4 + (c^
2*Sqrt[c + d*x]*(-((E^((8*I)*a)*Gamma[3/2, ((-4*I)*b*(c + d*x))/d])/Sqrt[(
(-I)*b*(c + d*x))/d]) - (E^(((8*I)*b*c)/d)*Gamma[3/2, ((4*I)*b*(c + d*x))/
d])/Sqrt[(I*b*(c + d*x))/d]))/(128*b*E^(((4*I)*(b*c + a*d))/d)) + (Sqrt[d]
*((1 - I)*E^((2*I)*(a - (b*c)/d))*((2 + 2*I)*Sqrt[b]*Sqrt[d]*E^(((2*I)*b*(
c + d*x))/d)*Sqrt[c + d*x]*(15*d - 16*b^2*d*x^2 + (4*I)*b*(c - 5*d*x)) + (
16*b^2*c^2 - (24*I)*b*c*d - 15*d^2)*Sqrt[Pi]*Erfi[((1 + I)*Sqrt[b]*Sqrt...
```

### 3.184.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{4}(c+dx)^{5/2}\sin(2a+2bx)+\frac{1}{8}(c+dx)^{5/2}\sin(4a+4bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15\sqrt{\pi}d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}+\frac{15d^2\sqrt{c+dx}\cos(4a+4bx)}{2048b^3}+\frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{32b^2}+\frac{5d(c+dx)^{3/2}\sin(4a+4bx)}{256b^2}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
(15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2
*a + 2*b*x])/(8*b) + (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) -
((c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a
 - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(409
6*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]
*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2
]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)
/d])/(4096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*
x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*
x)^(3/2)*Sin[2*a + 2*b*x])/(32*b^2) + (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x
])/(256*b^2)
```

### 3.184.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.184.4 Maple [A] (verified)

Time = 3.80 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.15

| method | result |
|---|---|
| derivativedivides | $-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\right.}{}\right.}{8}$ |
| default | $-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\right.}{}\right.}{8}$ |

input

```
int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output

```
2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+5/16/b*d*(1/
4/b*d*(d*x+c)^(3/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*
x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(
cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(
2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))-1/64/
b*d*(d*x+c)^(5/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+5/64/b*d*(1/8/b*d*(d*x+
c)^(3/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*
cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(co
s(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d
)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/
2)/d)))))
```

### 3.184.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.92

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-}{d}\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_co
s(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d
))*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/
d) + 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*
x + c)*sqrt(b/(pi*d))) - 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x
+ c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(192*b^3*d^2*x^2 + 384*b^3*
c*d*x + 192*b^3*c^2 + 360*b*d^2*cos(b*x + a)^2 - 8*(64*b^3*d^2*x^2 + 128*b
^3*c*d*x + 64*b^3*c^2 - 15*b*d^2)*cos(b*x + a)^4 - 225*b*d^2 + 160*(2*(b^2
*d^2*x + b^2*c*d)*cos(b*x + a)^3 + 3*(b^2*d^2*x + b^2*c*d)*cos(b*x + a))*s
in(b*x + a))*sqrt(d*x + c))/b^4
```

### 3.184.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

output
```
Timed out
```

### 3.184.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.35

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \frac{\left(640\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)+5120\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-16\left(\frac{64\,(dx+c)^{\frac{5}{2}}b}{d}\right.\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/32768*(640*(d*x + c)^(3/2)*b^3*sin(4*((d*x + c)*b - b*c + a*d)/d) + 5120
*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 16*(64*(d*x + c)
^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(4*((d*x + c)*b - b*c + a*d)/d)
- 256*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c)
*b - b*c + a*d)/d) - 240*(-(I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2
)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^
2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 15*
(-(I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (
I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2
*sqrt(d*x + c)*sqrt(I*b/d)) - 15*((I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)
^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1
/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - 240*((I + 1
)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (
I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d
))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*d/b^5
```

### 3.184.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 2435, normalized size of antiderivative = 5.98

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
1/16384*(512*(sqrt(2)*sqrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*
d/sqrt(b^2*d^2) + 1)) + 4*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)) + 4*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)))*c^3 + 24*c*d^2*((sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d
*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4
*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 4*I*(8*I*(
d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-4
*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (sqrt(2)*sqrt(pi)*(64*b^2*
c^2 + 16*I*b*c*d - 3*d^2)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^
2*d^2) + 1)*b^2) + 4*I*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d
 + 3*sqrt(d*x + c)*d^2)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2
+ 16*(sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(...
```

### 3.184.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)\,\left(c+d\,x\right)^{5/2}\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(5/2), x)
```

# 3.185 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx$

3.185.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1482
3.185.2 Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1483
3.185.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1483
3.185.4 Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1485
3.185.5 Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1485
3.185.6 Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1486
3.185.7 Maxima [C] (verification not implemented) . . . . . . . . . . . . . . . . . . 1486
3.185.8 Giac [C] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1487
3.185.9 Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1488

## 3.185.1 Optimal result

Integrand size = 24, antiderivative size = 351

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx=-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3d^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{512b^{5/2}}-\frac{3d^{3/2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{64b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(2a+2bx)}{32b^2}+\frac{3d\sqrt{c+dx}\sin(4a+4bx)}{256b^2}$$

output
```
-1/8*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b-1/32*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b-3/
1024*d^(3/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)-3/1024*d^(3/2)*FresnelC(2*b^(1/2)*
2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/
b^(5/2)-3/64*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(
1/2)/Pi^(1/2))*Pi^(1/2)/b^(5/2)-3/64*d^(3/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1
/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)+3/32*d*sin(2*b*x+2
*a)*(d*x+c)^(1/2)/b^2+3/256*d*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b^2
```

### 3.185.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.97

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \frac{e^{-4ia}\left(8e^{2i\left(a-\frac{b(c+dx)}{d}\right)}\left(-4\sqrt{b}de^{\frac{2ibc}{d}}\sqrt{c+dx}\left(-3i+4bx+e^{4i(a+bx)}(3i+4bx)\right)-(1-i)(4bc+3id)\right.\right.}{}$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
(8*E^((2*I)*(a - (b*(c + d*x))/d))*(-4*Sqrt[b]*d*E^(((2*I)*b*c)/d)*Sqrt[c
+ d*x]*(-3*I + 4*b*x + E^((4*I)*(a + b*x))*(3*I + 4*b*x)) - (1 - I)*(4*b*c
 + (3*I)*d)*Sqrt[d]*E^(((2*I)*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[((1 + I)*Sqrt
[b]*Sqrt[c + d*x])/Sqrt[d]] + (1 + I)*Sqrt[d]*((4*I)*b*c + 3*d)*E^((2*I)*(
2*a + b*x))*Sqrt[Pi]*Erfi[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]) + (-4*
Sqrt[b]*d*E^(((4*I)*b*c)/d)*Sqrt[c + d*x]*(-3*I + 8*b*x + E^((8*I)*(a + b*
x))*(3*I + 8*b*x)) + (-1)^(1/4)*((8*I)*b*c - 3*d)*Sqrt[d]*E^(((4*I)*b*(2*c
 + d*x))/d)*Sqrt[Pi]*Erf[(2*(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + (
-1)^(1/4)*Sqrt[d]*((8*I)*b*c + 3*d)*E^((4*I)*(2*a + b*x))*Sqrt[Pi]*Erfi[(2
*(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/E^(((4*I)*b*(c + d*x))/d) - (
64*Sqrt[2]*b^(3/2)*c*E^((2*I)*(a - (b*c)/d))*Sqrt[c + d*x]*(E^((4*I)*a)*Sq
rt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d] + E^(((4*I)*b*c)/
d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((2*I)*b*(c + d*x))/d]))/Sqrt[(b^
2*(c + d*x)^2)/d^2] + (16*b^(3/2)*c*Sqrt[c + d*x]*(-((E^((8*I)*a)*Gamma[3/
2, ((-4*I)*b*(c + d*x))/d])/Sqrt[((-I)*b*(c + d*x))/d]) - (E^(((8*I)*b*c)/
d)*Gamma[3/2, ((4*I)*b*(c + d*x))/d])/Sqrt[(I*b*(c + d*x))/d]))/E^(((4*I)*
b*c)/d))/(2048*b^(5/2)*E^((4*I)*a))
```

### 3.185.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin(a+bx)\cos^3(a+bx)\,dx$$
$$\Big\downarrow \text{4906}$$
$$\int \left(\frac{1}{4}(c+dx)^{3/2}\sin(2a+2bx)+\frac{1}{8}(c+dx)^{3/2}\sin(4a+4bx)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(2a+2bx)}{32b^2}+\frac{3d\sqrt{c+dx}\sin(4a+4bx)}{256b^2}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
-1/8*((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/b - ((c + d*x)^(3/2)*Cos[4*a + 4*b
*x])/(32*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[
b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]
*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi]
)])/(64*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqr
t[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*
b*c)/d])/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) + (3
*d*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

### 3.185.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

3.185. $\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx$

### 3.185.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.07

| method | result |
|---|---|
| derivativedivides | $-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\text{s}\right.}{8b\sqrt{\frac{b}{d}}}\right.}{8b}$ |
| default | $-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\text{s}\right.}{8b\sqrt{\frac{b}{d}}}\right.}{8b}$ |

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+3/16/b*d*(1/
4/b*d*(d*x+c)^(1/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d
)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2
)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)
))-1/64/b*d*(d*x+c)^(3/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+3/64/b*d*(1/8/b
*d*(d*x+c)^(1/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2
)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*
b*(d*x+c)^(1/2)/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d)))
```

### 3.185.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.84

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad}{d}\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))
*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d)
 + 48*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x +
 c)*sqrt(b/(pi*d))) + 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)
*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x
+ a)^4 - 6*b^2*d*x - 6*b^2*c - 3*(2*b*d*cos(b*x + a)^3 + 3*b*d*cos(b*x + a
))*sin(b*x + a))*sqrt(d*x + c))/b^3
```

### 3.185.6 Sympy [F]

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin(a+bx)\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)*cos(a + b*x)**3, x)
```

### 3.185.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.43

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{\left(\frac{128\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)}{d} + \frac{512\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)}{d} - 48\sqrt{dx+c}b^2\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right) - 38\cdots}{\cdots}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/4096*(128*(d*x + c)^(3/2)*b^3*cos(4*((d*x + c)*b - b*c + a*d)/d)/d + 51
2*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 48*sqrt(d*x +
 c)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) - 384*sqrt(d*x + c)*b^2*sin(2*(
(d*x + c)*b - b*c + a*d)/d) + 24*((I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^
2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*
(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) +
3*((I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I
 - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sq
rt(d*x + c)*sqrt(I*b/d)) + 3*(-(I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4
)*cos(-4*(b*c - a*d)/d) + (I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin
(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + 24*(-(I - 1)*4^(1/
4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^
(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt
(d*x + c)*sqrt(-2*I*b/d)))*d/b^4
```

### 3.185.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 1515, normalized size of antiderivative = 4.32

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
1/2048*(64*(sqrt(2)*sqrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)) + 4*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
^2) + 1)) + 4*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
))*c^2 + d^2*((sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-I
*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 4*I*(8*I*(d*x + c
)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-4*(-I*(d
*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 1
6*I*b*c*d - 3*d^2)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)*b^2) + 4*I*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*sq
rt(d*x + c)*d^2)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 16*(s
qrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)*b^2) + 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c...
```

### 3.185.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2} \cos^3(a+bx) \sin(a+bx)\, dx = \int \cos(a+b\,x)^3\, \sin(a+b\,x)\, (c+d\,x)^{3/2}\, dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(3/2), x)
```

# 3.186 $\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx$


3.186.1 Optimal result . . . 1489
3.186.2 Mathematica [C] (verified) . . . 1490
3.186.3 Rubi [A] (verified) . . . 1490
3.186.4 Maple [A] (verified) . . . 1492
3.186.5 Fricas [A] (verification not implemented) . . . 1492
3.186.6 Sympy [F] . . . 1493
3.186.7 Maxima [C] (verification not implemented) . . . 1493
3.186.8 Giac [C] (verification not implemented) . . . 1494
3.186.9 Mupad [F(-1)] . . . 1494


## 3.186.1 Optimal result

Integrand size = 24, antiderivative size = 299

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}}-\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{64b^{3/2}}-\frac{\sqrt{d}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{16b^{3/2}}$$

output
```
1/128*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d
^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/128*FresnelS(2*b^(1/2)*2^(1/2)/
Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/
b^(3/2)+1/16*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^
(1/2))*d^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1
/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/8*cos(2*b*x+2*a)
*(d*x+c)^(1/2)/b-1/32*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b
```

### 3.186.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.84

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{e^{-\frac{4i(bc+ad)}{d}}\sqrt{c+dx}\left(4\sqrt{2}e^{2i\left(3a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)+4\sqrt{2}e^{2i\left(a+\frac{3bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right)\right.}{128b\sqrt{\frac{b^2(c+dx)^2}{d^2}}}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
-1/128*(Sqrt[c + d*x]*(4*Sqrt[2]*E^((2*I)*(3*a + (b*c)/d))*Sqrt[(I*b*(c +
d*x))/d]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d] + 4*Sqrt[2]*E^((2*I)*(a + (3*b
*c)/d))*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((2*I)*b*(c + d*x))/d] + E^(
(8*I)*a)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-4*I)*b*(c + d*x))/d] + E^((
(8*I)*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((4*I)*b*(c + d*x))/d]
))/(b*E^(((4*I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

### 3.186.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{4}\sqrt{c+dx}\sin(2a+2bx)+\frac{1}{8}\sqrt{c+dx}\sin(4a+4bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
-1/8*(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/b - (Sqrt[c + d*x]*Cos[4*a + 4*b*x])
/(32*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqr
t[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a
- (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b
^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x]
)/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS
[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b
^(3/2))
```

### 3.186.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.186.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.96

| method | result |
|---|---|
| derivativedivides | $-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{}}{}$ ... $d$ |
| default | $-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\operatorname{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{}}{}$ ... $d$ |

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/32/b*d*Pi^
(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c
)^(1/2)/d))-1/64/b*d*(d*x+c)^(1/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+1/256/
b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi
^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2
)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))
```

### 3.186.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.78

$$\int\sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx$$
$$=\frac{\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\,S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+\cdots}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_s
in(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 8*pi*d*
sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(p
i*d))) - 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))
*sin(-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 3*b)*sqrt(d*x + c))/b^2
```

### 3.186.6 Sympy [F]

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = \int \sqrt{c+dx}\sin(a+bx)\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)*cos(a + b*x)**3, x)
```

### 3.186.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.43

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{\left(\frac{16\sqrt{dx+c}b^2\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right)}{d} + \frac{64\sqrt{dx+c}b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d} - 4\left(-(i-1)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc}{}\right.\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/512*(16*sqrt(d*x + c)*b^2*cos(4*((d*x + c)*b - b*c + a*d)/d)/d + 64*sqr
t(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 4*(-(I - 1)*4^(1/4)*
sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)
*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x +
c)*sqrt(2*I*b/d)) - (-(I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-4*(b
*c - a*d)/d) - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*
d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - ((I + 1)*sqrt(2)*sqrt(pi)*b*(b^2
/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(
1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - 4*((I + 1)
*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1
)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sq
rt(d*x + c)*sqrt(-2*I*b/d)))*d/b^3
```

### 3.186.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 826, normalized size of antiderivative = 2.76

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/256*(sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(I*sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*(sqrt(2)*sqrt(pi)*d*er
f(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I
*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*
d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 4*sqrt(pi)*
d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 4*sqrt(pi)*d*erf(I*sq
rt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c + 8*sqrt(pi)*(4*b*c - I*d)*d
*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c
- I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*sqrt(pi)*(4*b*c +
I*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(
-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 16*sqrt(d*x
+ c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-
4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^(-2*(-I*(d*x
 + c)*b + I*b*c - I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-4*(-I*(d*x + c)*b...
```

### 3.186.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = \int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)\,\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(1/2), x)
```

# 3.187 $\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx$


3.187.1 Optimal result . . . 1495
3.187.2 Mathematica [C] (verified) . . . 1496
3.187.3 Rubi [A] (verified) . . . 1496
3.187.4 Maple [A] (verified) . . . 1498
3.187.5 Fricas [A] (verification not implemented) . . . 1498
3.187.6 Sympy [F] . . . 1499
3.187.7 Maxima [C] (verification not implemented) . . . 1499
3.187.8 Giac [C] (verification not implemented) . . . 1500
3.187.9 Mupad [F(-1)] . . . 1500


## 3.187.1 Optimal result

Integrand size = 24, antiderivative size = 299

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b} - \frac{\sqrt{c+dx}\cos(4a+4bx)}{32b} + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} + \frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}} - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{64b^{3/2}} - \frac{\sqrt{d}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{16b^{3/2}}$$

output
```
1/128*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d
^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/128*FresnelS(2*b^(1/2)*2^(1/2)/
Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/
b^(3/2)+1/16*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^
(1/2))*d^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1
/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/8*cos(2*b*x+2*a)
*(d*x+c)^(1/2)/b-1/32*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b
```

### 3.187.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.84

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{e^{-\frac{4i(bc+ad)}{d}}\sqrt{c+dx}\left(4\sqrt{2}e^{2i\left(3a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)+4\sqrt{2}e^{2i\left(a+\frac{3bc}{d}\right)}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right)\right.}{128b\sqrt{\frac{b^2(c+dx)^2}{d^2}}}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
-1/128*(Sqrt[c + d*x]*(4*Sqrt[2]*E^((2*I)*(3*a + (b*c)/d))*Sqrt[(I*b*(c +
d*x))/d]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d] + 4*Sqrt[2]*E^((2*I)*(a + (3*b
*c)/d))*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((2*I)*b*(c + d*x))/d] + E^(
(8*I)*a)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-4*I)*b*(c + d*x))/d] + E^((
(8*I)*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((4*I)*b*(c + d*x))/d]
))/(b*E^(((4*I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

### 3.187.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin(a+bx)\cos^3(a+bx)\,dx$$
$$\Big\downarrow \text{4906}$$
$$\int \left(\frac{1}{4}\sqrt{c+dx}\sin(2a+2bx)+\frac{1}{8}\sqrt{c+dx}\sin(4a+4bx)\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
-1/8*(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/b - (Sqrt[c + d*x]*Cos[4*a + 4*b*x])
/(32*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqr
t[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a
- (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b
^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x]
)/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS
[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b
^(3/2))
```

### 3.187.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.187.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.96

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{}}{}}{d}$ |
| default | $\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{}}{}}{d}$ |

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/32/b*d*Pi^
(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c
)^(1/2)/d))-1/64/b*d*(d*x+c)^(1/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+1/256/
b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi
^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2
)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))
```

### 3.187.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.78

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx$$
$$=\frac{\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)+}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_s
in(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 8*pi*d*
sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(p
i*d))) - 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))
*sin(-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 3*b)*sqrt(d*x + c))/b^2
```

## 3.187.6 Sympy [F]

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = \int \sqrt{c+dx}\sin(a+bx)\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)*cos(a + b*x)**3, x)
```

## 3.187.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.43

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{\left(\frac{16\sqrt{dx+c}b^2\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{64\sqrt{dx+c}b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d}-4\left(-(i-1)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc\right.\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/512*(16*sqrt(d*x + c)*b^2*cos(4*((d*x + c)*b - b*c + a*d)/d)/d + 64*sqr
t(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 4*(-(I - 1)*4^(1/4)*
sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)
*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x +
c)*sqrt(2*I*b/d)) - (-(I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-4*(b
*c - a*d)/d) - (I + 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*
d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - ((I + 1)*sqrt(2)*sqrt(pi)*b*(b^2
/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(
1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - 4*((I + 1)
*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I - 1
)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sq
rt(d*x + c)*sqrt(-2*I*b/d)))*d/b^3
```

### 3.187.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 826, normalized size of antiderivative = 2.76

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
-1/256*(sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(I*sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*(sqrt(2)*sqrt(pi)*d*er
f(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I
*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*
d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-
4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 4*sqrt(pi)*
d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 4*sqrt(pi)*d*erf(I*sq
rt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c + 8*sqrt(pi)*(4*b*c - I*d)*d
*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c
- I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*sqrt(pi)*(4*b*c +
I*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(
-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 16*sqrt(d*x
+ c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-
4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^(-2*(-I*(d*x
 + c)*b + I*b*c - I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^(-4*(-I*(d*x + c)*b...
```

### 3.187.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = \int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)\,\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(1/2), x)
```

# 3.188 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx$


3.188.1 Optimal result . . . 1501
3.188.2 Mathematica [C] (verified) . . . 1502
3.188.3 Rubi [A] (verified) . . . 1502
3.188.4 Maple [A] (verified) . . . 1504
3.188.5 Fricas [A] (verification not implemented) . . . 1504
3.188.6 Sympy [F] . . . 1505
3.188.7 Maxima [C] (verification not implemented) . . . 1505
3.188.8 Giac [C] (verification not implemented) . . . 1506
3.188.9 Mupad [F(-1)] . . . 1507


## 3.188.1 Optimal result

Integrand size = 24, antiderivative size = 351

$$\begin{aligned}\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx &= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} \\ &- \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}} \\ &- \frac{3d^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}} \\ &- \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{512b^{5/2}} \\ &- \frac{3d^{3/2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{64b^{5/2}} \\ &+ \frac{3d\sqrt{c+dx}\sin(2a+2bx)}{32b^2} + \frac{3d\sqrt{c+dx}\sin(4a+4bx)}{256b^2}\end{aligned}$$

output

```
-1/8*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b-1/32*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b-3/
1024*d^(3/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)-3/1024*d^(3/2)*FresnelC(2*b^(1/2)*
2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/
b^(5/2)-3/64*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(
1/2)/Pi^(1/2))*Pi^(1/2)/b^(5/2)-3/64*d^(3/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1
/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)+3/32*d*sin(2*b*x+2
*a)*(d*x+c)^(1/2)/b^2+3/256*d*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b^2
```

### 3.188.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.97

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \frac{e^{-4ia}\left(8e^{2i\left(a-\frac{b(c+dx)}{d}\right)}\left(-4\sqrt{b}de^{\frac{2ibc}{d}}\sqrt{c+dx}\left(-3i+4bx+e^{4i(a+bx)}(3i+4bx)\right)-(1-i)(4bc+3id)\right.\right.}{}$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
(8*E^((2*I)*(a - (b*(c + d*x))/d))*(-4*Sqrt[b]*d*E^(((2*I)*b*c)/d)*Sqrt[c
+ d*x]*(-3*I + 4*b*x + E^((4*I)*(a + b*x))*(3*I + 4*b*x)) - (1 - I)*(4*b*c
 + (3*I)*d)*Sqrt[d]*E^(((2*I)*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[((1 + I)*Sqrt
[b]*Sqrt[c + d*x])/Sqrt[d]] + (1 + I)*Sqrt[d]*((4*I)*b*c + 3*d)*E^((2*I)*(
2*a + b*x))*Sqrt[Pi]*Erfi[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]) + (-4*
Sqrt[b]*d*E^(((4*I)*b*c)/d)*Sqrt[c + d*x]*(-3*I + 8*b*x + E^((8*I)*(a + b*
x))*(3*I + 8*b*x)) + (-1)^(1/4)*((8*I)*b*c - 3*d)*Sqrt[d]*E^(((4*I)*b*(2*c
 + d*x))/d)*Sqrt[Pi]*Erf[(2*(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + (
-1)^(1/4)*Sqrt[d]*((8*I)*b*c + 3*d)*E^((4*I)*(2*a + b*x))*Sqrt[Pi]*Erfi[(2
*(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/E^(((4*I)*b*(c + d*x))/d) - (
64*Sqrt[2]*b^(3/2)*c*E^((2*I)*(a - (b*c)/d))*Sqrt[c + d*x]*(E^((4*I)*a)*Sq
rt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d] + E^(((4*I)*b*c)/
d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((2*I)*b*(c + d*x))/d]))/Sqrt[(b^
2*(c + d*x)^2)/d^2] + (16*b^(3/2)*c*Sqrt[c + d*x]*(-((E^((8*I)*a)*Gamma[3/
2, ((-4*I)*b*(c + d*x))/d])/Sqrt[((-I)*b*(c + d*x))/d]) - (E^(((8*I)*b*c)/
d)*Gamma[3/2, ((4*I)*b*(c + d*x))/d])/Sqrt[(I*b*(c + d*x))/d]))/E^(((4*I)*
b*c)/d))/(2048*b^(5/2)*E^((4*I)*a))
```

### 3.188.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{4}(c+dx)^{3/2}\sin(2a+2bx)+\frac{1}{8}(c+dx)^{3/2}\sin(4a+4bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}+\frac{3d\sqrt{c+dx}\sin(2a+2bx)}{32b^2}+\frac{3d\sqrt{c+dx}\sin(4a+4bx)}{256b^2}-\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
-1/8*((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/b - ((c + d*x)^(3/2)*Cos[4*a + 4*b
*x])/(32*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[
b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]
*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi]
)])/(64*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqr
t[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*
b*c)/d])/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) + (3
*d*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

### 3.188.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.188.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.07

| method | result |
|---|---|
| derivativedivides | $-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\text{s}\right.}{8b\sqrt{\frac{b}{d}}}\right.}{8b}$ |
| default | $-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\text{s}\right.}{8b\sqrt{\frac{b}{d}}}\right.}{8b}$ |

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+3/16/b*d*(1/
4/b*d*(d*x+c)^(1/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d
)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2
)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)
))-1/64/b*d*(d*x+c)^(3/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+3/64/b*d*(1/8/b
*d*(d*x+c)^(1/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2
)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*
b*(d*x+c)^(1/2)/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1
/2)*b*(d*x+c)^(1/2)/d))))
```

### 3.188.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.84

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx =$$
$$-\frac{3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\text{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\pi d^2\sqrt{\frac{b}{\pi d}}\,\text{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))
*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d)
 + 48*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x +
 c)*sqrt(b/(pi*d))) + 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)
*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x
+ a)^4 - 6*b^2*d*x - 6*b^2*c - 3*(2*b*d*cos(b*x + a)^3 + 3*b*d*cos(b*x + a
))*sin(b*x + a))*sqrt(d*x + c))/b^3
```

### 3.188.6 Sympy [F]

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin(a+bx)\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)*cos(a + b*x)**3, x)
```

### 3.188.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.43

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{\left(\frac{128\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)}{d} + \frac{512\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)}{d} - 48\sqrt{dx+c}b^2\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right) - 38}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/4096*(128*(d*x + c)^(3/2)*b^3*cos(4*((d*x + c)*b - b*c + a*d)/d)/d + 51
2*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 48*sqrt(d*x +
 c)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) - 384*sqrt(d*x + c)*b^2*sin(2*(
(d*x + c)*b - b*c + a*d)/d) + 24*((I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^
2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*
(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) +
3*((I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (I
 - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sq
rt(d*x + c)*sqrt(I*b/d)) + 3*(-(I - 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4
)*cos(-4*(b*c - a*d)/d) + (I + 1)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin
(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + 24*(-(I - 1)*4^(1/
4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^
(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt
(d*x + c)*sqrt(-2*I*b/d)))*d/b^4
```

### 3.188.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 1515, normalized size of antiderivative = 4.32

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
1/2048*(64*(sqrt(2)*sqrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)) + 4*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
^2) + 1)) + 4*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
))*c^2 + d^2*((sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-I
*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 4*I*(8*I*(d*x + c
)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-4*(-I*(d
*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 1
6*I*b*c*d - 3*d^2)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)*b^2) + 4*I*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*sq
rt(d*x + c)*d^2)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 16*(s
qrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)*b^2) + 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c...
```

### 3.188.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int \cos(a+b\,x)^3\,\sin(a+b\,x)\,(c+d\,x)^{3/2}\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(3/2), x)
```

# 3.189 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx$

3.189.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1508
3.189.2 Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 1509
3.189.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1510
3.189.4 Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 1512
3.189.5 Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . 1513
3.189.6 Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1513
3.189.7 Maxima [C] (verification not implemented) . . . . . . . . . . . . . . . . 1514
3.189.8 Giac [C] (verification not implemented) . . . . . . . . . . . . . . . . . 1514
3.189.9 Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1515

## 3.189.1 Optimal result

Integrand size = 24, antiderivative size = 407

$$\begin{aligned}\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx &= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}\\
&-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2\sqrt{c+dx}\cos(4a+4bx)}{2048b^3}\\
&-\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}-\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}\\
&-\frac{15d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}\\
&+\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(4a-\frac{4bc}{d}\right)}{4096b^{7/2}}\\
&+\frac{15d^{5/2}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{256b^{7/2}}\\
&+\frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{32b^2}+\frac{5d(c+dx)^{3/2}\sin(4a+4bx)}{256b^2}\end{aligned}$$

output
```
-1/8*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b-1/32*(d*x+c)^(5/2)*cos(4*b*x+4*a)/b+5/
32*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2+5/256*d*(d*x+c)^(3/2)*sin(4*b*x+4*a)
/b^2-15/8192*d^(5/2)*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*
(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/8192*d^(5/2)*FresnelS(2
*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*
Pi^(1/2)/b^(7/2)-15/256*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c
)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+15/256*d^(5/2)*FresnelS(2*b^(1/
2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+15/12
8*d^2*cos(2*b*x+2*a)*(d*x+c)^(1/2)/b^3+15/2048*d^2*cos(4*b*x+4*a)*(d*x+c)^
(1/2)/b^3
```

## 3.189.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.02 (sec) , antiderivative size = 1332, normalized size of antiderivative = 3.27

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \frac{\left(\frac{1}{128}+\frac{i}{128}\right)c\sqrt{d}e^{-\frac{2i(ad+b(c+dx))}{d}}\left((2+2i)\sqrt{b}\sqrt{d}e^{\frac{2ibc}{d}}\sqrt{c+dx}\left(3+4ibx+e^{4i(a+bx)}(-3+4ibx)\right)+i(4\right.}{b^{5/2}}$$
$$+\frac{c\sqrt{d}e^{-\frac{4i(ad+b(c+dx))}{d}}\left(-4\sqrt{b}\sqrt{d}e^{\frac{4ibc}{d}}\sqrt{c+dx}\left(-3i+8bx+e^{8i(a+bx)}(3i+8bx)\right)+(-1)^{3/4}(8bc+3id)e^{\frac{4ib(2c+dx)}{d}}\right.}{1024b^{5/2}}$$
$$+\frac{1}{4}c^2\left(-\frac{e^{2i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\,\Gamma\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{-2i\left(a-\frac{bc}{d}\right)}\sqrt{c+dx}\,\Gamma\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{\frac{ib(c+dx)}{d}}}\right)+\frac{c^2e^{-\frac{4i(bc+ad)}{d}}\sqrt{c+dx}\left(-\right.}{}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
((1/128 + I/128)*c*Sqrt[d]*((2 + 2*I)*Sqrt[b]*Sqrt[d]*E^(((2*I)*b*c)/d)*Sq
rt[c + d*x]*(3 + (4*I)*b*x + E^((4*I)*(a + b*x))*(-3 + (4*I)*b*x)) + I*(4*
b*c + (3*I)*d)*E^(((2*I)*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[((1 + I)*Sqrt[b]*S
qrt[c + d*x])/Sqrt[d]] + ((4*I)*b*c + 3*d)*E^((2*I)*(2*a + b*x))*Sqrt[Pi]*
Erfi[((1 + I)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(b^(5/2)*E^(((2*I)*(a*d +
b*(c + d*x)))/d)) + (c*Sqrt[d]*(-4*Sqrt[b]*Sqrt[d]*E^(((4*I)*b*c)/d)*Sqrt[
c + d*x]*(-3*I + 8*b*x + E^((8*I)*(a + b*x))*(3*I + 8*b*x)) + (-1)^(3/4)*(
8*b*c + (3*I)*d)*E^(((4*I)*b*(2*c + d*x))/d)*Sqrt[Pi]*Erf[(2*(-1)^(1/4)*Sq
rt[b]*Sqrt[c + d*x])/Sqrt[d]] + (-1)^(1/4)*((8*I)*b*c + 3*d)*E^((4*I)*(2*a
 + b*x))*Sqrt[Pi]*Erfi[(2*(-1)^(1/4)*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(10
24*b^(5/2)*E^(((4*I)*(a*d + b*(c + d*x)))/d)) + (c^2*(-1/4*(E^((2*I)*(a -
(b*c)/d))*Sqrt[c + d*x]*Gamma[3/2, ((-2*I)*b*(c + d*x))/d])/(Sqrt[2]*b*Sqr
t[((-I)*b*(c + d*x))/d]) - (Sqrt[c + d*x]*Gamma[3/2, ((2*I)*b*(c + d*x))/d
])/(4*Sqrt[2]*b*E^((2*I)*(a - (b*c)/d))*Sqrt[(I*b*(c + d*x))/d])))/4 + (c^
2*Sqrt[c + d*x]*(-((E^((8*I)*a)*Gamma[3/2, ((-4*I)*b*(c + d*x))/d])/Sqrt[(
(-I)*b*(c + d*x))/d]) - (E^(((8*I)*b*c)/d)*Gamma[3/2, ((4*I)*b*(c + d*x))/
d])/Sqrt[(I*b*(c + d*x))/d]))/(128*b*E^(((4*I)*(b*c + a*d))/d)) + (Sqrt[d]
*((1 - I)*E^((2*I)*(a - (b*c)/d))*((2 + 2*I)*Sqrt[b]*Sqrt[d]*E^(((2*I)*b*(
c + d*x))/d)*Sqrt[c + d*x]*(15*d - 16*b^2*d*x^2 + (4*I)*b*(c - 5*d*x)) + (
16*b^2*c^2 - (24*I)*b*c*d - 15*d^2)*Sqrt[Pi]*Erfi[((1 + I)*Sqrt[b]*Sqrt...
```

### 3.189.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{4}(c+dx)^{5/2}\sin(2a+2bx)+\frac{1}{8}(c+dx)^{5/2}\sin(4a+4bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15\sqrt{\pi}d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}+\frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3}+\frac{15d^2\sqrt{c+dx}\cos(4a+4bx)}{2048b^3}+\frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{32b^2}+\frac{5d(c+dx)^{3/2}\sin(4a+4bx)}{256b^2}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

output
```
(15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2
*a + 2*b*x])/(8*b) + (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) -
((c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a
 - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(409
6*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]
*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2
]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)
/d])/(4096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*
x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*
x)^(3/2)*Sin[2*a + 2*b*x])/(32*b^2) + (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x
])/(256*b^2)
```

### 3.189.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

3.189. $\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx$

### 3.189.4 Maple [A] (verified)

Time = 3.58 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.15

| method | result |
|---|---|
| derivativedivides | $-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\cdots\right.}{\cdots}\right.}{8\cdots}$ |
| default | $-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}+\cdots\right.}{\cdots}\right.}{8\cdots}$ |

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+5/16/b*d*(1/
4/b*d*(d*x+c)^(3/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*
x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(
cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(
2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))-1/64/
b*d*(d*x+c)^(5/2)*cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+5/64/b*d*(1/8/b*d*(d*x+
c)^(3/2)*sin(4*b/d*(d*x+c)+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*
cos(4*b/d*(d*x+c)+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(co
s(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d
)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/
2)/d)))))
```

### 3.189.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.92

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx =$$

$$-\frac{15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc\cdots}{d}\right.}{\ }$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_co
s(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d
))*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/
d) + 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*
x + c)*sqrt(b/(pi*d))) - 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x
+ c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(192*b^3*d^2*x^2 + 384*b^3*
c*d*x + 192*b^3*c^2 + 360*b*d^2*cos(b*x + a)^2 - 8*(64*b^3*d^2*x^2 + 128*b
^3*c*d*x + 64*b^3*c^2 - 15*b*d^2)*cos(b*x + a)^4 - 225*b*d^2 + 160*(2*(b^2
*d^2*x + b^2*c*d)*cos(b*x + a)^3 + 3*(b^2*d^2*x + b^2*c*d)*cos(b*x + a))*s
in(b*x + a))*sqrt(d*x + c))/b^4
```

### 3.189.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

output
```
Timed out
```

### 3.189.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.35

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \frac{\left(640\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)+5120\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-16\left(\frac{64\,(dx+c)^{\frac{5}{2}}b}{d}\right.\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

output
```
1/32768*(640*(d*x + c)^(3/2)*b^3*sin(4*((d*x + c)*b - b*c + a*d)/d) + 5120
*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 16*(64*(d*x + c)
^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(4*((d*x + c)*b - b*c + a*d)/d)
- 256*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c)
*b - b*c + a*d)/d) - 240*(-(I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2
)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^
2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 15*
(-(I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (
I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2
*sqrt(d*x + c)*sqrt(I*b/d)) - 15*((I + 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)
^(1/4)*cos(-4*(b*c - a*d)/d) + (I - 1)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1
/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - 240*((I + 1
)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (
I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d
))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*d/b^5
```

### 3.189.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.14 (sec) , antiderivative size = 2435, normalized size of antiderivative = 5.98

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

output
```
1/16384*(512*(sqrt(2)*sqrt(pi)*d*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*
d/sqrt(b^2*d^2) + 1)) + 4*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)) + 4*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)))*c^3 + 24*c*d^2*((sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d
*erf(-I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-4
*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 4*I*(8*I*(
d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^(-4
*(-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (sqrt(2)*sqrt(pi)*(64*b^2*
c^2 + 16*I*b*c*d - 3*d^2)*d*erf(I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^(-4*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^
2*d^2) + 1)*b^2) + 4*I*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d
 + 3*sqrt(d*x + c)*d^2)*e^(-4*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2
+ 16*(sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(...
```

### 3.189.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx=\int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)\,\left(c+d\,x\right)^{5/2}\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(5/2), x)
```

# 3.190 $\int (c+dx)^{5/2} \cos^3(a+bx) \sin^2(a+bx) \, dx$

3.190.1 Optimal result . . . . . 1517
3.190.2 Mathematica [C] (verified) . . . . . 1518
3.190.3 Rubi [A] (verified) . . . . . 1519
3.190.4 Maple [A] (verified) . . . . . 1521
3.190.5 Fricas [A] (verification not implemented) . . . . . 1523
3.190.6 Sympy [F(-1)] . . . . . 1523
3.190.7 Maxima [C] (verification not implemented) . . . . . 1524
3.190.8 Giac [C] (verification not implemented) . . . . . 1524
3.190.9 Mupad [F(-1)] . . . . . 1525

### 3.190.1 Optimal result

Integrand size = 26, antiderivative size = 615

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx)^{3/2}\cos(5a+5bx)}{160b^2} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}} - \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}} - \frac{3d^{5/2}\sqrt{\frac{\pi}{10}}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}} - \frac{3d^{5/2}\sqrt{\frac{\pi}{10}}\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(5a-\frac{5bc}{d}\right)}{1600b^{7/2}} - \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{576b^{7/2}} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{32b^{7/2}} - \frac{15d^2\sqrt{c+dx}\sin(a+bx)}{32b^3} + \frac{(c+dx)^{5/2}\sin(a+bx)}{8b} + \frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{576b^3} - \frac{(c+dx)^{5/2}\sin(3a+3bx)}{48b} + \frac{3d^2\sqrt{c+dx}\sin(5a+5bx)}{1600b^3} - \frac{(c+dx)^{5/2}\sin(5a+5bx)}{80b}$$

output
```
5/16*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2-5/288*d*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b
^2-1/160*d*(d*x+c)^(3/2)*cos(5*b*x+5*a)/b^2+1/8*(d*x+c)^(5/2)*sin(b*x+a)/b
-1/48*(d*x+c)^(5/2)*sin(3*b*x+3*a)/b-1/80*(d*x+c)^(5/2)*sin(5*b*x+5*a)/b-3
/16000*d^(5/2)*cos(5*a-5*b*c/d)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)
^(1/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(7/2)-3/16000*d^(5/2)*FresnelC(b^(1/2)
*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*10^(1/2)*Pi^(1/
2)/b^(7/2)-5/3456*d^(5/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)-5/3456*d^(5/2)*FresnelC
(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*
Pi^(1/2)/b^(7/2)+15/64*d^(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/64*d^(5/2)*FresnelC
(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(
1/2)/b^(7/2)-15/32*d^2*sin(b*x+a)*(d*x+c)^(1/2)/b^3+5/576*d^2*sin(3*b*x+3*
a)*(d*x+c)^(1/2)/b^3+3/1600*d^2*sin(5*b*x+5*a)*(d*x+c)^(1/2)/b^3
```

### 3.190.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 16.71 (sec) , antiderivative size = 2130, normalized size of antiderivative = 3.46

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
-1/1152*(c*Sqrt[d]*(12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(1
+ (2*I)*b*x + E^((6*I)*(a + b*x))*(1 - (2*I)*b*x)) + (1 + I)*(2*b*c + I*d)
*E^(((3*I)*b*(2*c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqr
t[c + d*x])/Sqrt[d]] - (1 + I)*(2*b*c - I*d)*E^((3*I)*(2*a + b*x))*Sqrt[6*
Pi]*Erfi[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(b^(5/2)*E^(
((3*I)*(a*d + b*(c + d*x)))/d)) - (c*Sqrt[d]*(20*Sqrt[b]*Sqrt[d]*E^(((5*I)
*b*c)/d)*Sqrt[c + d*x]*(3 + (10*I)*b*x + E^((10*I)*(a + b*x))*(3 - (10*I)*
b*x)) + (1 + I)*(10*b*c + (3*I)*d)*E^(((5*I)*b*(2*c + d*x))/d)*Sqrt[10*Pi]
*Erf[((1 + I)*Sqrt[5/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] - (1 + I)*(10*b*c
- (3*I)*d)*E^((5*I)*(2*a + b*x))*Sqrt[10*Pi]*Erfi[((1 + I)*Sqrt[5/2]*Sqrt[
b]*Sqrt[c + d*x])/Sqrt[d]]))/(16000*b^(5/2)*E^(((5*I)*(a*d + b*(c + d*x)))
/d)) + (c^2*d*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-I)*b*(
c + d*x))/d] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, (I*b*(
c + d*x))/d]))/(16*b^2*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x]) - (c^2*(c + d*
x)^(3/2)*(-((E^((6*I)*a)*Gamma[3/2, ((-3*I)*b*(c + d*x))/d])/(((-I)*b*(c +
 d*x))/d)^(3/2)) - (E^(((6*I)*b*c)/d)*Gamma[3/2, ((3*I)*b*(c + d*x))/d])/(
(I*b*(c + d*x))/d)^(3/2)))/(96*Sqrt[3]*d*E^(((3*I)*(b*c + a*d))/d)) - (c^2
*(c + d*x)^(3/2)*(-((E^((10*I)*a)*Gamma[3/2, ((-5*I)*b*(c + d*x))/d])/(((-
I)*b*(c + d*x))/d)^(3/2)) - (E^(((10*I)*b*c)/d)*Gamma[3/2, ((5*I)*b*(c + d
*x))/d])/((I*b*(c + d*x))/d)^(3/2)))/(160*Sqrt[5]*d*E^(((5*I)*(b*c + a*...
```

## 3.190.3 Rubi [A] (verified)

Time = 1.66 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin^2(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}(c+dx)^{5/2}\cos(a+bx)-\frac{1}{16}(c+dx)^{5/2}\cos(3a+3bx)-\frac{1}{16}(c+dx)^{5/2}\cos(5a+5bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}-\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-\frac{15d^2\sqrt{c+dx}\sin(a+bx)}{32b^3}+\frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{576b^3}+\frac{3d^2\sqrt{c+dx}\sin(5a+5bx)}{1600b^3}+\frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2}-\frac{d(c+dx)^{3/2}\cos(5a+5bx)}{160b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)}{8b}-\frac{(c+dx)^{5/2}\sin(3a+3bx)}{48b}-\frac{(c+dx)^{5/2}\sin(5a+5bx)}{80b}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

3.190. $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

output
```
(5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(16*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a
 + 3*b*x])/(288*b^2) - (d*(c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(160*b^2) + (1
5*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]])/(32*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d
]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) - (3
*d^(5/2)*Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sq
rt[c + d*x])/Sqrt[d]])/(1600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelC[(S
qrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(
7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/
Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*Fres
nelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(
7/2)) - (15*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(32*b^3) + ((c + d*x)^(5/2)*Si
n[a + b*x])/(8*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(576*b^3) - ((c
 + d*x)^(5/2)*Sin[3*a + 3*b*x])/(48*b) + (3*d^2*Sqrt[c + d*x]*Sin[5*a + 5*
b*x])/(1600*b^3) - ((c + d*x)^(5/2)*Sin[5*a + 5*b*x])/(80*b)
```

### 3.190.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.190.4 Maple [A] (verified)

Time = 11.95 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.16

| method | result |
|---|---|
| derivativedivides | $\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b} - \frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\,(\mathrm{co}\cdots}{\cdots}\right)}{\cdots}\right)}{8b}\cdots$ |
| default | $\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b} - \frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\,(\mathrm{co}\cdots}{\cdots}\right)}{\cdots}\right)}{8b}\cdots$ |

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/16/b*d*(d*x+c)^(5/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-5/16/b*d*(-1/2/b*
d*(d*x+c)^(3/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/2
)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((
a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((
a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1/
96/b*d*(d*x+c)^(5/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+5/96/b*d*(-1/6/b*d*(
d*x+c)^(3/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/
2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d
)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*
b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b
/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1/160/b*d*(d*x+c)^(5/2)*sin(5*b/d*(d*x+c)+
5*(a*d-b*c)/d)+1/32/b*d*(-1/10/b*d*(d*x+c)^(3/2)*cos(5*b/d*(d*x+c)+5*(a*d-
b*c)/d)+3/10/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(5*b/d*(d*x+c)+5*(a*d-b*c)/d)-
1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*Fresnel
S(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(5*(a*d-b*c)/
d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

## 3.190. $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

### 3.190.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 548, normalized size of antiderivative = 0.89

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{81\sqrt{10}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)S\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+625\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)S\left(\sqrt{6}\sqrt{dx+c}\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel
_sin(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 625*sqrt(6)*pi*d^3*sqrt(b/(p
i*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d
))) - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin
(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*
d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d)
+ 625*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt
(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fres
nel_cos(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480
*(90*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^5 - 50*(b^2*d^2*x + b^2*c*d)*cos(b
*x + a)^3 - 300*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) - (120*b^3*d^2*x^2 + 24
0*b^3*c*d*x + 120*b^3*c^2 - 9*(20*b^3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2
- 3*b*d^2)*cos(b*x + a)^4 - 428*b*d^2 + (60*b^3*d^2*x^2 + 120*b^3*c*d*x +
60*b^3*c^2 + 11*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^4
```

### 3.190.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

output
```
Timed out
```

### 3.190.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 826, normalized size of antiderivative = 1.34

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/1728000*sqrt(2)*(5400*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(5*((d*x + c)*b -
b*c + a*d)/d)/d + 15000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(3*((d*x + c)*b - b
*c + a*d)/d)/d - 270000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(((d*x + c)*b - b*c
 + a*d)/d)/d - 81*(-(I + 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5
*(b*c - a*d)/d) + (I - 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-5*(
b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - 625*(-(I + 1)*9^(1/4)*sq
rt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(
pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*
I*b/d)) - 101250*((I + 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/
d) - (I - 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(
d*x + c)*sqrt(I*b/d)) - 101250*(-(I - 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*co
s(-(b*c - a*d)/d) + (I + 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d
)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - 625*((I - 1)*9^(1/4)*sqrt(pi)*b^2*
d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(pi)*b^2*d*(
b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) -
81*((I - 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)
- (I + 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*e
rf(sqrt(d*x + c)*sqrt(-5*I*b/d)) + 540*(20*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2
 - 3*sqrt(2)*sqrt(d*x + c)*b^3)*sin(5*((d*x + c)*b - b*c + a*d)/d) + 1500*
(12*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 5*sqrt(2)*sqrt(d*x + c)*b^3)*sin(...
```

### 3.190.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.71 (sec) , antiderivative size = 3716, normalized size of antiderivative = 6.04

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/864000*(1800*(30*I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt
(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/
(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*I
*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*
c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi
)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 5*I*sqrt(
6)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)) - 3*I*sqrt(10)*sqrt(pi)*d*erf(1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/s
qrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2*(2250*(I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 +
4*I*b*c*d - 3*d^2)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt
(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 125*(I*sq
rt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*I*sqrt(6)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/...
```

### 3.190.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \cos\left(a+b\,x\right)^3\sin\left(a+b\,x\right)^2\left(c+d\,x\right)^{5/2}\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

# 3.191 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$


3.191.1 Optimal result . . . 1526
3.191.2 Mathematica [C] (verified) . . . 1527
3.191.3 Rubi [A] (verified) . . . 1528
3.191.4 Maple [A] (verified) . . . 1530
3.191.5 Fricas [A] (verification not implemented) . . . 1531
3.191.6 Sympy [F] . . . 1532
3.191.7 Maxima [C] (verification not implemented) . . . 1532
3.191.8 Giac [C] (verification not implemented) . . . 1533
3.191.9 Mupad [F(-1)] . . . 1533


## 3.191.1 Optimal result

Integrand size = 26, antiderivative size = 534

$$\begin{aligned}
\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx &= \frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2} \\
&- \frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\cos(5a+5bx)}{800b^2} \\
&- \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}} \\
&+ \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}} \\
&+ \frac{3d^{3/2}\sqrt{\frac{\pi}{10}}\cos\left(5a-\frac{5bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}} \\
&- \frac{3d^{3/2}\sqrt{\frac{\pi}{10}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(5a-\frac{5bc}{d}\right)}{800b^{5/2}} \\
&- \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{96b^{5/2}} \\
&+ \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{16b^{5/2}} + \frac{(c+dx)^{3/2}\sin(a+bx)}{8b} \\
&- \frac{(c+dx)^{3/2}\sin(3a+3bx)}{48b} - \frac{(c+dx)^{3/2}\sin(5a+5bx)}{80b}
\end{aligned}$$

output
```
1/8*(d*x+c)^(3/2)*sin(b*x+a)/b-1/48*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b-1/80*(d
*x+c)^(3/2)*sin(5*b*x+5*a)/b+3/8000*d^(3/2)*cos(5*a-5*b*c/d)*FresnelC(b^(1
/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(5/2)-3/8
000*d^(3/2)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(
5*a-5*b*c/d)*10^(1/2)*Pi^(1/2)/b^(5/2)+1/576*d^(3/2)*cos(3*a-3*b*c/d)*Fres
nelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(5
/2)-1/576*d^(3/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))
*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/32*d^(3/2)*cos(a-b*c/d)*Fresn
elC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/
2)+3/32*d^(3/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*s
in(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d*cos(b*x+a)*(d*x+c)^(1/2)/b^2-1
/96*d*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b^2-3/800*d*cos(5*b*x+5*a)*(d*x+c)^(1/2
)/b^2
```

## 3.191.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.76 (sec) , antiderivative size = 1088, normalized size of antiderivative = 2.04

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{\sqrt{d}e^{-\frac{3i(ad+b(c+dx))}{d}}\left(12\sqrt{b}\sqrt{d}e^{\frac{3ibc}{d}}\sqrt{c+dx}\left(1+2ibx+e^{6i(a+bx)}(1-2ibx)\right)+(1+i)(2bc+id)e^{\frac{3ib(2c+dx)}{d}}\sqrt{6\pi}e\cdots\right.}{2304b^{5/2}}$$

$$-\frac{\sqrt{d}e^{-\frac{5i(ad+b(c+dx))}{d}}\left(20\sqrt{b}\sqrt{d}e^{\frac{5ibc}{d}}\sqrt{c+dx}\left(3+10ibx+e^{10i(a+bx)}(3-10ibx)\right)+(1+i)(10bc+3id)e^{\frac{5ib(2c+dx)}{d}}\cdots\right.}{32000b^{5/2}}$$

$$+\frac{cde^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)\right)}{16b^2\sqrt{c+dx}}$$

$$-\frac{ce^{-\frac{3i(bc+ad)}{d}}(c+dx)^{3/2}\left(-\frac{e^{6ia}\Gamma\left(\frac{3}{2},-\frac{3ib(c+dx)}{d}\right)}{\left(-\frac{ib(c+dx)}{d}\right)^{3/2}}-\frac{e^{\frac{6ibc}{d}}\Gamma\left(\frac{3}{2},\frac{3ib(c+dx)}{d}\right)}{\left(\frac{ib(c+dx)}{d}\right)^{3/2}}\right)}{96\sqrt{3}d}$$

$$-\frac{ce^{-\frac{5i(bc+ad)}{d}}(c+dx)^{3/2}\left(-\frac{e^{10ia}\Gamma\left(\frac{3}{2},-\frac{5ib(c+dx)}{d}\right)}{\left(-\frac{ib(c+dx)}{d}\right)^{3/2}}-\frac{e^{\frac{10ibc}{d}}\Gamma\left(\frac{3}{2},\frac{5ib(c+dx)}{d}\right)}{\left(\frac{ib(c+dx)}{d}\right)^{3/2}}\right)}{160\sqrt{5}d}$$

$$+\frac{\sqrt{d}\left(e^{i\left(a-\frac{bc}{d}\right)}\left(2\sqrt{b}\sqrt{d}e^{\frac{ib(c+dx)}{d}}(3-2ibx)\sqrt{c+dx}+\sqrt[4]{-1}(-2bc+3id)\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)\right)+\left(2\sqrt{b}\sqrt{d}\cdots\right.\right.}{\cdots}$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
-1/2304*(Sqrt[d]*(12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(1 +
(2*I)*b*x + E^((6*I)*(a + b*x))*(1 - (2*I)*b*x)) + (1 + I)*(2*b*c + I*d)*E
^(((3*I)*b*(2*c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[
c + d*x])/Sqrt[d]] - (1 + I)*(2*b*c - I*d)*E^((3*I)*(2*a + b*x))*Sqrt[6*Pi
]*Erfi[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(b^(5/2)*E^(((
3*I)*(a*d + b*(c + d*x)))/d)) - (Sqrt[d]*(20*Sqrt[b]*Sqrt[d]*E^(((5*I)*b*c
)/d)*Sqrt[c + d*x]*(3 + (10*I)*b*x + E^((10*I)*(a + b*x))*(3 - (10*I)*b*x)
) + (1 + I)*(10*b*c + (3*I)*d)*E^(((5*I)*b*(2*c + d*x))/d)*Sqrt[10*Pi]*Erf
[((1 + I)*Sqrt[5/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] - (1 + I)*(10*b*c - (3
*I)*d)*E^((5*I)*(2*a + b*x))*Sqrt[10*Pi]*Erfi[((1 + I)*Sqrt[5/2]*Sqrt[b]*S
qrt[c + d*x])/Sqrt[d]]))/(32000*b^(5/2)*E^(((5*I)*(a*d + b*(c + d*x)))/d))
 + (c*d*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-I)*b*(c + d*
x))/d] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*
x))/d]))/(16*b^2*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x]) - (c*(c + d*x)^(3/2)
*(-((E^((6*I)*a)*Gamma[3/2, ((-3*I)*b*(c + d*x))/d])/(((-I)*b*(c + d*x))/d
)^(3/2)) - (E^(((6*I)*b*c)/d)*Gamma[3/2, ((3*I)*b*(c + d*x))/d])/((I*b*(c
+ d*x))/d)^(3/2)))/(96*Sqrt[3]*d*E^(((3*I)*(b*c + a*d))/d)) - (c*(c + d*x)
^(3/2)*(-((E^((10*I)*a)*Gamma[3/2, ((-5*I)*b*(c + d*x))/d])/(((-I)*b*(c +
d*x))/d)^(3/2)) - (E^(((10*I)*b*c)/d)*Gamma[3/2, ((5*I)*b*(c + d*x))/d])/(
(I*b*(c + d*x))/d)^(3/2)))/(160*Sqrt[5]*d*E^(((5*I)*(b*c + a*d))/d)) + ...
```

## 3.191.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin^2(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}(c+dx)^{3/2}\cos(a+bx)-\frac{1}{16}(c+dx)^{3/2}\cos(3a+3bx)-\frac{1}{16}(c+dx)^{3/2}\cos(5a+5bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}+\frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2}-\frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2}-\frac{3d\sqrt{c+dx}\cos(5a+5bx)}{800b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)}{8b}-\frac{(c+dx)^{3/2}\sin(3a+3bx)}{48b}-\frac{(c+dx)^{3/2}\sin(5a+5bx)}{80b}$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
(3*d*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b
*x])/(96*b^2) - (3*d*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(800*b^2) - (3*d^(3/2
)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/
Sqrt[d]])/(16*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC
[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sq
rt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x]
)/Sqrt[d]])/(800*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[
10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3
/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*
a - (3*b*c)/d])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqr
t[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + ((c + d*x
)^(3/2)*Sin[a + b*x])/(8*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(48*b) -
((c + d*x)^(3/2)*Sin[5*a + 5*b*x])/(80*b)
```

## 3.191. $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

### 3.191.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.191.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.09

| method | result |
|---|---|
| derivativedivides | $\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-cb}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\cdots\right)\right)}{4b\sqrt{\frac{b}{d}}}\right)}{8b}\cdots$ |
| default | $\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-cb}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\cdots\right)\right)}{4b\sqrt{\frac{b}{d}}}\right)}{8b}\cdots$ |

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/16/b*d*(d*x+c)^(3/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-3/16/b*d*(-1/2/b*
d*(d*x+c)^(1/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^
(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^
(1/2)/d)))-1/96/b*d*(d*x+c)^(3/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/32/b*
d*(-1/6/b*d*(d*x+c)^(1/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2
)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/
2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1
/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))-1/160/b*d*(d*x+c)^(3
/2)*sin(5*b/d*(d*x+c)+5*(a*d-b*c)/d)+3/160/b*d*(-1/10/b*d*(d*x+c)^(1/2)*co
s(5*b/d*(d*x+c)+5*(a*d-b*c)/d)+1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1
/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^
(1/2)*b*(d*x+c)^(1/2)/d)))
```

### 3.191.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.84

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \frac{27\sqrt{10}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)C\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+125\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)C\left(\sqrt{6}\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_c
os(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 125*sqrt(6)*pi*d^2*sqrt(b/(pi*
d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))
) - 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fr
esnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*
sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi
*d)))*sin(-3*(b*c - a*d)/d) - 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_si
n(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(9*b*
d*cos(b*x + a)^5 - 5*b*d*cos(b*x + a)^3 - 30*b*d*cos(b*x + a) + 10*(3*(b^2
*d*x + b^2*c)*cos(b*x + a)^4 - 2*b^2*d*x - 2*b^2*c - (b^2*d*x + b^2*c)*cos
(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

### 3.191.6 Sympy [F]

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin^2(a+bx)\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x)**3, x)
```

### 3.191.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 760, normalized size of antiderivative = 1.42

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/288000*sqrt(2)*(1800*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(5*((d*x + c)*b - b
*c + a*d)/d)/d^2 + 3000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(3*((d*x + c)*b - b
*c + a*d)/d)/d^2 - 18000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(((d*x + c)*b - b*
c + a*d)/d)/d^2 + 540*sqrt(2)*sqrt(d*x + c)*b^3*cos(5*((d*x + c)*b - b*c +
 a*d)/d)/d + 1500*sqrt(2)*sqrt(d*x + c)*b^3*cos(3*((d*x + c)*b - b*c + a*d
)/d)/d - 27000*sqrt(2)*sqrt(d*x + c)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/
d - 27*(-(I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/
d) - (I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*
erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - 125*(-(I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2
/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)
^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 6750*((I
- 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(pi)*b
^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 6
750*(-(I + 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*s
qrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I
*b/d)) - 125*((I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a
*d)/d) + (I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d
))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - 27*((I + 1)*25^(1/4)*sqrt(pi)*b^2*(
b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) + (I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/
d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^...
```

### 3.191.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.21 (sec) , antiderivative size = 2319, normalized size of antiderivative = 4.34

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/144000*(300*(30*I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I
*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(
sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*I*
sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi)
*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 5*I*sqrt(6
)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
) - 3*I*sqrt(10)*sqrt(pi)*d*erf(1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)))*c^2 + d^2*(2250*(I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*
c*d - 3*d^2)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b
^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x +
 c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 125*(I*sqrt(6)*
sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqr
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(...
```

### 3.191.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)^3\sin(a+b\,x)^2\,(c+d\,x)^{3/2}\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

# 3.192 $\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx$

3.192.1 Optimal result . . . 1534
3.192.2 Mathematica [C] (verified) . . . 1535
3.192.3 Rubi [A] (verified) . . . 1536
3.192.4 Maple [A] (verified) . . . 1537
3.192.5 Fricas [A] (verification not implemented) . . . 1538
3.192.6 Sympy [F] . . . 1538
3.192.7 Maxima [C] (verification not implemented) . . . 1539
3.192.8 Giac [C] (verification not implemented) . . . 1540
3.192.9 Mupad [F(-1)] . . . 1540

## 3.192.1 Optimal result

Integrand size = 26, antiderivative size = 459

$$\begin{aligned}\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx = &-\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}\\
&+\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}\\
&+\frac{\sqrt{d}\sqrt{\frac{\pi}{10}}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}\\
&+\frac{\sqrt{d}\sqrt{\frac{\pi}{10}}\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(5a-\frac{5bc}{d}\right)}{80b^{3/2}}\\
&+\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{48b^{3/2}}\\
&-\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{8b^{3/2}}\\
&+\frac{\sqrt{c+dx}\sin(a+bx)}{8b}-\frac{\sqrt{c+dx}\sin(3a+3bx)}{48b}\\
&-\frac{\sqrt{c+dx}\sin(5a+5bx)}{80b}\end{aligned}$$

output
```
1/800*cos(5*a-5*b*c/d)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*d^(1/2)*10^(1/2)*Pi^(1/2)/b^(3/2)+1/800*FresnelC(b^(1/2)*10^(1/2)/P
i^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*d^(1/2)*10^(1/2)*Pi^(1/2)/
b^(3/2)+1/288*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(
1/2)/d^(1/2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)+1/288*FresnelC(b^(1/2)*6^(1
/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1
/2)/b^(3/2)-1/16*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1
/2)/d^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelC(b^(1/2)*2^(1/2
)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^
(3/2)+1/8*sin(b*x+a)*(d*x+c)^(1/2)/b-1/48*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b-1
/80*sin(5*b*x+5*a)*(d*x+c)^(1/2)/b
```

### 3.192.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.81

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx = \frac{ie^{-\frac{5i(bc+ad)}{d}}\sqrt{c+dx}\left(-450e^{6ia+\frac{4ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+450e^{4ia+\frac{6ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)+25\sqrt{\ldots}\right.}{\ldots}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
((I/7200)*Sqrt[c + d*x]*(-450*E^((6*I)*a + ((4*I)*b*c)/d)*Sqrt[(I*b*(c + d
*x))/d]*Gamma[3/2, ((-I)*b*(c + d*x))/d] + 450*E^((4*I)*a + ((6*I)*b*c)/d)
*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d] + 25*Sqrt[3]*E^(
(2*I)*(4*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-3*I)*b*(c + d
*x))/d] - 25*Sqrt[3]*E^((2*I)*a + ((8*I)*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d
]*Gamma[3/2, ((3*I)*b*(c + d*x))/d] + 9*Sqrt[5]*E^((10*I)*a)*Sqrt[(I*b*(c
+ d*x))/d]*Gamma[3/2, ((-5*I)*b*(c + d*x))/d] - 9*Sqrt[5]*E^(((10*I)*b*c)/
d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((5*I)*b*(c + d*x))/d]))/(b*E^(((
5*I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

### 3.192.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^2(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}\sqrt{c+dx}\cos(a+bx)-\frac{1}{16}\sqrt{c+dx}\cos(3a+3bx)-\frac{1}{16}\sqrt{c+dx}\cos(5a+5bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}+\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+\frac{\sqrt{c+dx}\sin(a+bx)}{8b}-\frac{\sqrt{c+dx}\sin(3a+3bx)}{48b}-\frac{\sqrt{c+dx}\sin(5a+5bx)}{80b}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
-1/8*(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqr
t[c + d*x])/Sqrt[d]])/b^(3/2) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*F
resnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) + (Sqrt[
d]*Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(80*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqr
t[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(80*b^(3/2)) + (Sqr
t[d]*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3
*a - (3*b*c)/d])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt
[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(3/2)) + (Sqrt[c + d
*x]*Sin[a + b*x])/(8*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(48*b) - (Sqrt[
c + d*x]*Sin[5*a + 5*b*x])/(80*b)
```

#### 3.192.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.192.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 444, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-cb}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx\cdots}}{\cdots}}{\cdots}$ |
| default | $\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-cb}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx\cdots}}{\cdots}}{\cdots}$ |

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/16/b*d*(d*x+c)^(1/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-1/32/b*d*2^(1/2)*
Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d))-1/96/b*d*(d*x+c)^(1/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c
)/d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fr
esnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-
b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))-
1/160/b*d*(d*x+c)^(1/2)*sin(5*b/d*(d*x+c)+5*(a*d-b*c)/d)+1/1600/b*d*2^(1/2
)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/
2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1
/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))
```

### 3.192.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.80

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{9\sqrt{10}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+25\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_sin(s
qrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*cos
(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450
*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(
d*x + c)*sqrt(b/(pi*d))) - 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*s
qrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*
c - a*d)/d) + 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(10)*sqrt(d*x
 + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^4 - b*
cos(b*x + a)^2 - 2*b)*sqrt(d*x + c)*sin(b*x + a))/b^2
```

### 3.192.6 Sympy [F]

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \sqrt{c+dx}\sin^2(a+bx)\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x)**3, x)
```

### 3.192.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.48

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{\sqrt{2}\left(\frac{180\sqrt{2}\sqrt{dx+c}b^3\sin\left(\frac{5((dx+c)b-bc+ad)}{d}\right)}{d^2}+\frac{300\sqrt{2}\sqrt{dx+c}b^3\sin\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d^2}-\frac{1800\sqrt{2}\sqrt{dx+c}b^3\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d^2}-\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/28800*sqrt(2)*(180*sqrt(2)*sqrt(d*x + c)*b^3*sin(5*((d*x + c)*b - b*c +
 a*d)/d)/d^2 + 300*sqrt(2)*sqrt(d*x + c)*b^3*sin(3*((d*x + c)*b - b*c + a*
d)/d)/d^2 - 1800*sqrt(2)*sqrt(d*x + c)*b^3*sin(((d*x + c)*b - b*c + a*d)/d
)/d^2 - 9*((I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d
)/d)/d - (I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/
d)/d)*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - 25*((I + 1)*9^(1/4)*sqrt(pi)*b^2*
(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d - (I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^
2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 4
50*(-(I + 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d + (I - 1)*
sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt
(I*b/d)) - 450*((I - 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d
 - (I + 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*
x + c)*sqrt(-I*b/d)) - 25*(-(I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*c
os(-3*(b*c - a*d)/d)/d + (I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(
-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - 9*(-(I - 1)*25^(1
/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)/d + (I + 1)*25^(1/4
)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*
sqrt(-5*I*b/d)))*d^2/b^4
```

### 3.192.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 1270, normalized size of antiderivative = 2.77

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/14400*(-450*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)
*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sq
rt(10)*sqrt(pi)*(10*b*c - I*d)*d*erf(-1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d
/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*I
*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c +
 I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)
*(6*b*c + I*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)*b) + 9*I*sqrt(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(1/2*I*sqrt(10)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 30*(30*I*sqrt(2)*sqrt(pi)*d*erf
(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I
*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt
(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*
sqrt(10)*sqrt(pi)*d*erf(-1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/...
```

### 3.192.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)^3\sin(a+b\,x)^2\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

# 3.193 $\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx$

3.193.1 Optimal result . . . 1541
3.193.2 Mathematica [C] (verified) . . . 1542
3.193.3 Rubi [A] (verified) . . . 1543
3.193.4 Maple [A] (verified) . . . 1544
3.193.5 Fricas [A] (verification not implemented) . . . 1545
3.193.6 Sympy [F] . . . 1545
3.193.7 Maxima [C] (verification not implemented) . . . 1546
3.193.8 Giac [C] (verification not implemented) . . . 1547
3.193.9 Mupad [F(-1)] . . . 1547

## 3.193.1 Optimal result

Integrand size = 26, antiderivative size = 459

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx = -\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}} + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}} + \frac{\sqrt{d}\sqrt{\frac{\pi}{10}}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}} + \frac{\sqrt{d}\sqrt{\frac{\pi}{10}}\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(5a-\frac{5bc}{d}\right)}{80b^{3/2}} + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{48b^{3/2}} - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{8b^{3/2}} + \frac{\sqrt{c+dx}\sin(a+bx)}{8b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\sin(5a+5bx)}{80b}$$

output
```
1/800*cos(5*a-5*b*c/d)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*d^(1/2)*10^(1/2)*Pi^(1/2)/b^(3/2)+1/800*FresnelC(b^(1/2)*10^(1/2)/P
i^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*d^(1/2)*10^(1/2)*Pi^(1/2)/
b^(3/2)+1/288*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(
1/2)/d^(1/2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)+1/288*FresnelC(b^(1/2)*6^(1
/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1
/2)/b^(3/2)-1/16*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1
/2)/d^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelC(b^(1/2)*2^(1/2
)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^
(3/2)+1/8*sin(b*x+a)*(d*x+c)^(1/2)/b-1/48*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b-1
/80*sin(5*b*x+5*a)*(d*x+c)^(1/2)/b
```

### 3.193.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.81

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{ie^{-\frac{5i(bc+ad)}{d}}\sqrt{c+dx}\left(-450e^{6ia+\frac{4ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+450e^{4ia+\frac{6ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)+25\sqrt{}\right.}{}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
((I/7200)*Sqrt[c + d*x]*(-450*E^((6*I)*a + ((4*I)*b*c)/d)*Sqrt[(I*b*(c + d
*x))/d]*Gamma[3/2, ((-I)*b*(c + d*x))/d] + 450*E^((4*I)*a + ((6*I)*b*c)/d)
*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*x))/d] + 25*Sqrt[3]*E^(
(2*I)*(4*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-3*I)*b*(c + d
*x))/d] - 25*Sqrt[3]*E^((2*I)*a + ((8*I)*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d
]*Gamma[3/2, ((3*I)*b*(c + d*x))/d] + 9*Sqrt[5]*E^((10*I)*a)*Sqrt[(I*b*(c
+ d*x))/d]*Gamma[3/2, ((-5*I)*b*(c + d*x))/d] - 9*Sqrt[5]*E^(((10*I)*b*c)/
d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((5*I)*b*(c + d*x))/d]))/(b*E^(((
5*I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

### 3.193.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^2(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}\sqrt{c+dx}\cos(a+bx)-\frac{1}{16}\sqrt{c+dx}\cos(3a+3bx)-\frac{1}{16}\sqrt{c+dx}\cos(5a+5bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}+\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+\frac{\sqrt{c+dx}\sin(a+bx)}{8b}-\frac{\sqrt{c+dx}\sin(3a+3bx)}{48b}-\frac{\sqrt{c+dx}\sin(5a+5bx)}{80b}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
-1/8*(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqr
t[c + d*x])/Sqrt[d]])/b^(3/2) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*F
resnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) + (Sqrt[
d]*Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(80*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqr
t[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(80*b^(3/2)) + (Sqr
t[d]*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3
*a - (3*b*c)/d])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt
[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(3/2)) + (Sqrt[c + d
*x]*Sin[a + b*x])/(8*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(48*b) - (Sqrt[
c + d*x]*Sin[5*a + 5*b*x])/(80*b)
```

### 3.193.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.193.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 444, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-cb}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx\cdots}}{\cdots}}{\cdots}$ |
| default | $\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{ad-cb}{d}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{ad-cb}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx\cdots}}{\cdots}}{\cdots}$ |

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/16/b*d*(d*x+c)^(1/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-1/32/b*d*2^(1/2)*
Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*b*(d*x+c)^(1/2)/d))-1/96/b*d*(d*x+c)^(1/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c
)/d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fr
esnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(3*(a*d-
b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))-
1/160/b*d*(d*x+c)^(1/2)*sin(5*b/d*(d*x+c)+5*(a*d-b*c)/d)+1/1600/b*d*2^(1/2
)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/
2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1
/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))
```

### 3.193.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.80

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx$$
$$= \frac{9\sqrt{10}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+25\sqrt{6}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_sin(s
qrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*cos
(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450
*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(
d*x + c)*sqrt(b/(pi*d))) - 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*s
qrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*
c - a*d)/d) + 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(10)*sqrt(d*x
 + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^4 - b*
cos(b*x + a)^2 - 2*b)*sqrt(d*x + c)*sin(b*x + a))/b^2
```

### 3.193.6 Sympy [F]

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx=\int \sqrt{c+dx}\sin^2(a+bx)\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x)**3, x)
```

### 3.193.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.48

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{\sqrt{2}\left(\frac{180\sqrt{2}\sqrt{dx+c}b^3\sin\left(\frac{5((dx+c)b-bc+ad)}{d}\right)}{d^2}+\frac{300\sqrt{2}\sqrt{dx+c}b^3\sin\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d^2}-\frac{1800\sqrt{2}\sqrt{dx+c}b^3\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d^2}-\right.}{}$$

input

```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output

```
-1/28800*sqrt(2)*(180*sqrt(2)*sqrt(d*x + c)*b^3*sin(5*((d*x + c)*b - b*c +
 a*d)/d)/d^2 + 300*sqrt(2)*sqrt(d*x + c)*b^3*sin(3*((d*x + c)*b - b*c + a*
d)/d)/d^2 - 1800*sqrt(2)*sqrt(d*x + c)*b^3*sin(((d*x + c)*b - b*c + a*d)/d
)/d^2 - 9*((I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d
)/d)/d - (I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/
d)/d)*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - 25*((I + 1)*9^(1/4)*sqrt(pi)*b^2*
(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d - (I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^
2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 4
50*(-(I + 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d + (I - 1)*
sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt
(I*b/d)) - 450*((I - 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d
 - (I + 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*
x + c)*sqrt(-I*b/d)) - 25*(-(I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*c
os(-3*(b*c - a*d)/d)/d + (I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(
-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - 9*(-(I - 1)*25^(1
/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)/d + (I + 1)*25^(1/4
)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*
sqrt(-5*I*b/d)))*d^2/b^4
```

### 3.193.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.80 (sec) , antiderivative size = 1270, normalized size of antiderivative = 2.77

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/14400*(-450*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(1/2*I*sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)
*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sq
rt(10)*sqrt(pi)*(10*b*c - I*d)*d*erf(-1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d
/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*I
*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c +
 I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)
*(6*b*c + I*d)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)*b) + 9*I*sqrt(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(1/2*I*sqrt(10)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 30*(30*I*sqrt(2)*sqrt(pi)*d*erf
(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I
*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt
(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*
sqrt(10)*sqrt(pi)*d*erf(-1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/...
```

### 3.193.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \cos\left(a+b\,x\right)^3\sin\left(a+b\,x\right)^2\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

# 3.194 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

3.194.1 Optimal result . . . 1548
3.194.2 Mathematica [C] (verified) . . . 1549
3.194.3 Rubi [A] (verified) . . . 1550
3.194.4 Maple [A] (verified) . . . 1552
3.194.5 Fricas [A] (verification not implemented) . . . 1553
3.194.6 Sympy [F] . . . 1554
3.194.7 Maxima [C] (verification not implemented) . . . 1554
3.194.8 Giac [C] (verification not implemented) . . . 1555
3.194.9 Mupad [F(-1)] . . . 1555

## 3.194.1 Optimal result

Integrand size = 26, antiderivative size = 534

$$\begin{aligned}\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx &= \frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2} \\ &- \frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\cos(5a+5bx)}{800b^2} \\ &- \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}} \\ &+ \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}} \\ &+ \frac{3d^{3/2}\sqrt{\frac{\pi}{10}}\cos\left(5a-\frac{5bc}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}} \\ &- \frac{3d^{3/2}\sqrt{\frac{\pi}{10}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(5a-\frac{5bc}{d}\right)}{800b^{5/2}} \\ &- \frac{d^{3/2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{96b^{5/2}} \\ &+ \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{16b^{5/2}} + \frac{(c+dx)^{3/2}\sin(a+bx)}{8b} \\ &- \frac{(c+dx)^{3/2}\sin(3a+3bx)}{48b} - \frac{(c+dx)^{3/2}\sin(5a+5bx)}{80b}\end{aligned}$$

output
```
1/8*(d*x+c)^(3/2)*sin(b*x+a)/b-1/48*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b-1/80*(d
*x+c)^(3/2)*sin(5*b*x+5*a)/b+3/8000*d^(3/2)*cos(5*a-5*b*c/d)*FresnelC(b^(1
/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(5/2)-3/8
000*d^(3/2)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(
5*a-5*b*c/d)*10^(1/2)*Pi^(1/2)/b^(5/2)+1/576*d^(3/2)*cos(3*a-3*b*c/d)*Fres
nelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(5
/2)-1/576*d^(3/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))
*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/32*d^(3/2)*cos(a-b*c/d)*Fresn
elC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/
2)+3/32*d^(3/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*s
in(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d*cos(b*x+a)*(d*x+c)^(1/2)/b^2-1
/96*d*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b^2-3/800*d*cos(5*b*x+5*a)*(d*x+c)^(1/2
)/b^2
```

## 3.194.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.89 (sec) , antiderivative size = 1088, normalized size of antiderivative = 2.04

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{\sqrt{d}e^{-\frac{3i(ad+b(c+dx))}{d}}\left(12\sqrt{b}\sqrt{d}e^{\frac{3ibc}{d}}\sqrt{c+dx}\left(1+2ibx+e^{6i(a+bx)}(1-2ibx)\right)+(1+i)(2bc+id)e^{\frac{3ib(2c+dx)}{d}}\sqrt{6\pi}e\right.}{2304b^{5/2}}$$

$$-\frac{\sqrt{d}e^{-\frac{5i(ad+b(c+dx))}{d}}\left(20\sqrt{b}\sqrt{d}e^{\frac{5ibc}{d}}\sqrt{c+dx}\left(3+10ibx+e^{10i(a+bx)}(3-10ibx)\right)+(1+i)(10bc+3id)e^{\frac{5ib(2c+dx)}{d}}\right.}{32000b^{5/2}}$$

$$+\frac{cde^{-\frac{i(bc+ad)}{d}}\left(e^{2ia}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)+e^{\frac{2ibc}{d}}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)\right)}{16b^2\sqrt{c+dx}}$$

$$-\frac{ce^{-\frac{3i(bc+ad)}{d}}(c+dx)^{3/2}\left(-\frac{e^{6ia}\Gamma\left(\frac{3}{2},-\frac{3ib(c+dx)}{d}\right)}{\left(-\frac{ib(c+dx)}{d}\right)^{3/2}}-\frac{e^{\frac{6ibc}{d}}\Gamma\left(\frac{3}{2},\frac{3ib(c+dx)}{d}\right)}{\left(\frac{ib(c+dx)}{d}\right)^{3/2}}\right)}{96\sqrt{3}d}$$

$$-\frac{ce^{-\frac{5i(bc+ad)}{d}}(c+dx)^{3/2}\left(-\frac{e^{10ia}\Gamma\left(\frac{3}{2},-\frac{5ib(c+dx)}{d}\right)}{\left(-\frac{ib(c+dx)}{d}\right)^{3/2}}-\frac{e^{\frac{10ibc}{d}}\Gamma\left(\frac{3}{2},\frac{5ib(c+dx)}{d}\right)}{\left(\frac{ib(c+dx)}{d}\right)^{3/2}}\right)}{160\sqrt{5}d}$$

$$+\frac{\sqrt{d}\left(e^{i\left(a-\frac{bc}{d}\right)}\left(2\sqrt{b}\sqrt{d}e^{\frac{ib(c+dx)}{d}}(3-2ibx)\sqrt{c+dx}+\sqrt[4]{-1}(-2bc+3id)\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt[4]{-1}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)\right)+\left(2\sqrt{b}\sqrt{d}\right.\right.}{}$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
-1/2304*(Sqrt[d]*(12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(1 +
(2*I)*b*x + E^((6*I)*(a + b*x))*(1 - (2*I)*b*x)) + (1 + I)*(2*b*c + I*d)*E
^(((3*I)*b*(2*c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[
c + d*x])/Sqrt[d]] - (1 + I)*(2*b*c - I*d)*E^((3*I)*(2*a + b*x))*Sqrt[6*Pi
]*Erfi[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(b^(5/2)*E^(((
3*I)*(a*d + b*(c + d*x)))/d)) - (Sqrt[d]*(20*Sqrt[b]*Sqrt[d]*E^(((5*I)*b*c
)/d)*Sqrt[c + d*x]*(3 + (10*I)*b*x + E^((10*I)*(a + b*x))*(3 - (10*I)*b*x)
) + (1 + I)*(10*b*c + (3*I)*d)*E^(((5*I)*b*(2*c + d*x))/d)*Sqrt[10*Pi]*Erf
[((1 + I)*Sqrt[5/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] - (1 + I)*(10*b*c - (3
*I)*d)*E^((5*I)*(2*a + b*x))*Sqrt[10*Pi]*Erfi[((1 + I)*Sqrt[5/2]*Sqrt[b]*S
qrt[c + d*x])/Sqrt[d]]))/(32000*b^(5/2)*E^(((5*I)*(a*d + b*(c + d*x)))/d))
 + (c*d*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-I)*b*(c + d*
x))/d] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, (I*b*(c + d*
x))/d]))/(16*b^2*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x]) - (c*(c + d*x)^(3/2)
*(-((E^((6*I)*a)*Gamma[3/2, ((-3*I)*b*(c + d*x))/d])/(((-I)*b*(c + d*x))/d
)^(3/2)) - (E^(((6*I)*b*c)/d)*Gamma[3/2, ((3*I)*b*(c + d*x))/d])/((I*b*(c
+ d*x))/d)^(3/2)))/(96*Sqrt[3]*d*E^(((3*I)*(b*c + a*d))/d)) - (c*(c + d*x)
^(3/2)*(-((E^((10*I)*a)*Gamma[3/2, ((-5*I)*b*(c + d*x))/d])/(((-I)*b*(c +
d*x))/d)^(3/2)) - (E^(((10*I)*b*c)/d)*Gamma[3/2, ((5*I)*b*(c + d*x))/d])/(
(I*b*(c + d*x))/d)^(3/2)))/(160*Sqrt[5]*d*E^(((5*I)*(b*c + a*d))/d)) + ...
```

### 3.194.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin^2(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \ 4906$$

$$\int \left(\frac{1}{8}(c+dx)^{3/2}\cos(a+bx)-\frac{1}{16}(c+dx)^{3/2}\cos(3a+3bx)-\frac{1}{16}(c+dx)^{3/2}\cos(5a+5bx)\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}+\frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2}-\frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2}-\frac{3d\sqrt{c+dx}\cos(5a+5bx)}{800b^2}+\frac{(c+dx)^{3/2}\sin(a+bx)}{8b}-\frac{(c+dx)^{3/2}\sin(3a+3bx)}{48b}-\frac{(c+dx)^{3/2}\sin(5a+5bx)}{80b}$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
(3*d*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b
*x])/(96*b^2) - (3*d*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(800*b^2) - (3*d^(3/2
)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/
Sqrt[d]])/(16*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC
[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sq
rt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x]
)/Sqrt[d]])/(800*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[
10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3
/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*
a - (3*b*c)/d])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqr
t[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + ((c + d*x
)^(3/2)*Sin[a + b*x])/(8*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(48*b) -
((c + d*x)^(3/2)*Sin[5*a + 5*b*x])/(80*b)
```

3.194. $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

### 3.194.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.194.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.09

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-cb}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\cdots\right.\right.}{4b\sqrt{\frac{b}{d}}}\right.}{8b}\cdots}{\cdots}$ |
| default | $\frac{\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\left(\frac{ad-cb}{d}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,b\sqrt{dx+c}}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\cdots\right.\right.}{4b\sqrt{\frac{b}{d}}}\right.}{8b}\cdots}{\cdots}$ |

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/16/b*d*(d*x+c)^(3/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-3/16/b*d*(-1/2/b*
d*(d*x+c)^(1/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^
(1/2)/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^
(1/2)/d)))-1/96/b*d*(d*x+c)^(3/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/32/b*
d*(-1/6/b*d*(d*x+c)^(1/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2
)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/
2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1
/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))-1/160/b*d*(d*x+c)^(3
/2)*sin(5*b/d*(d*x+c)+5*(a*d-b*c)/d)+3/160/b*d*(-1/10/b*d*(d*x+c)^(1/2)*co
s(5*b/d*(d*x+c)+5*(a*d-b*c)/d)+1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1
/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d
*x+c)^(1/2)/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^
(1/2)*b*(d*x+c)^(1/2)/d)))
```

### 3.194.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 446, normalized size of antiderivative = 0.84

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \frac{27\sqrt{10}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+125\sqrt{6}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_c
os(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 125*sqrt(6)*pi*d^2*sqrt(b/(pi*
d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))
) - 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fr
esnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*
sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi
*d)))*sin(-3*(b*c - a*d)/d) - 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_si
n(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(9*b*
d*cos(b*x + a)^5 - 5*b*d*cos(b*x + a)^3 - 30*b*d*cos(b*x + a) + 10*(3*(b^2
*d*x + b^2*c)*cos(b*x + a)^4 - 2*b^2*d*x - 2*b^2*c - (b^2*d*x + b^2*c)*cos
(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

### 3.194.6 Sympy [F]

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx=\int (c+dx)^{\frac{3}{2}}\sin^2(a+bx)\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x)**3, x)
```

### 3.194.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 760, normalized size of antiderivative = 1.42

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/288000*sqrt(2)*(1800*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(5*((d*x + c)*b - b
*c + a*d)/d)/d^2 + 3000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(3*((d*x + c)*b - b
*c + a*d)/d)/d^2 - 18000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(((d*x + c)*b - b*
c + a*d)/d)/d^2 + 540*sqrt(2)*sqrt(d*x + c)*b^3*cos(5*((d*x + c)*b - b*c +
 a*d)/d)/d + 1500*sqrt(2)*sqrt(d*x + c)*b^3*cos(3*((d*x + c)*b - b*c + a*d
)/d)/d - 27000*sqrt(2)*sqrt(d*x + c)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/
d - 27*(-(I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/
d) - (I + 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*
erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - 125*(-(I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2
/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)
^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - 6750*((I
- 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (I + 1)*sqrt(pi)*b
^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) - 6
750*(-(I + 1)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (I - 1)*s
qrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I
*b/d)) - 125*((I + 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a
*d)/d) + (I - 1)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d
))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - 27*((I + 1)*25^(1/4)*sqrt(pi)*b^2*(
b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) + (I - 1)*25^(1/4)*sqrt(pi)*b^2*(b^2/
d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^...
```

### 3.194.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.24 (sec) , antiderivative size = 2319, normalized size of antiderivative = 4.34

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/144000*(300*(30*I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I
*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(
sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*I*
sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*c
 - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi)
*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 5*I*sqrt(6
)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
) - 3*I*sqrt(10)*sqrt(pi)*d*erf(1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)))*c^2 + d^2*(2250*(I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*
c*d - 3*d^2)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b
^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x +
 c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 125*(I*sqrt(6)*
sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*I*sqrt(6)*sqrt(b*d)*sqr
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/(sqrt(...
```

### 3.194.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)^3\sin(a+b\,x)^2\,(c+d\,x)^{3/2}\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

# 3.195 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

3.195.1 Optimal result . . . . . 1557
3.195.2 Mathematica [C] (verified) . . . . . 1558
3.195.3 Rubi [A] (verified) . . . . . 1559
3.195.4 Maple [A] (verified) . . . . . 1561
3.195.5 Fricas [A] (verification not implemented) . . . . . 1563
3.195.6 Sympy [F(-1)] . . . . . 1563
3.195.7 Maxima [C] (verification not implemented) . . . . . 1564
3.195.8 Giac [C] (verification not implemented) . . . . . 1564
3.195.9 Mupad [F(-1)] . . . . . 1565

### 3.195.1 Optimal result

Integrand size = 26, antiderivative size = 615

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx)^{3/2}\cos(5a+5bx)}{160b^2} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}} - \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}} - \frac{3d^{5/2}\sqrt{\frac{\pi}{10}}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}} - \frac{3d^{5/2}\sqrt{\frac{\pi}{10}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(5a-\frac{5bc}{d}\right)}{1600b^{7/2}} - \frac{5d^{5/2}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(3a-\frac{3bc}{d}\right)}{576b^{7/2}} + \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(a-\frac{bc}{d}\right)}{32b^{7/2}} - \frac{15d^2\sqrt{c+dx}\sin(a+bx)}{32b^3} + \frac{(c+dx)^{5/2}\sin(a+bx)}{8b} + \frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{576b^3} - \frac{(c+dx)^{5/2}\sin(3a+3bx)}{48b} + \frac{3d^2\sqrt{c+dx}\sin(5a+5bx)}{1600b^3} - \frac{(c+dx)^{5/2}\sin(5a+5bx)}{80b}$$

output
```
5/16*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2-5/288*d*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b
^2-1/160*d*(d*x+c)^(3/2)*cos(5*b*x+5*a)/b^2+1/8*(d*x+c)^(5/2)*sin(b*x+a)/b
-1/48*(d*x+c)^(5/2)*sin(3*b*x+3*a)/b-1/80*(d*x+c)^(5/2)*sin(5*b*x+5*a)/b-3
/16000*d^(5/2)*cos(5*a-5*b*c/d)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)
^(1/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(7/2)-3/16000*d^(5/2)*FresnelC(b^(1/2)
*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*10^(1/2)*Pi^(1/
2)/b^(7/2)-5/3456*d^(5/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)-5/3456*d^(5/2)*FresnelC
(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*
Pi^(1/2)/b^(7/2)+15/64*d^(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/64*d^(5/2)*FresnelC
(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(
1/2)/b^(7/2)-15/32*d^2*sin(b*x+a)*(d*x+c)^(1/2)/b^3+5/576*d^2*sin(3*b*x+3*
a)*(d*x+c)^(1/2)/b^3+3/1600*d^2*sin(5*b*x+5*a)*(d*x+c)^(1/2)/b^3
```

### 3.195.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.02 (sec) , antiderivative size = 2130, normalized size of antiderivative = 3.46

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

output
```
-1/1152*(c*Sqrt[d]*(12*Sqrt[b]*Sqrt[d]*E^(((3*I)*b*c)/d)*Sqrt[c + d*x]*(1
+ (2*I)*b*x + E^((6*I)*(a + b*x))*(1 - (2*I)*b*x)) + (1 + I)*(2*b*c + I*d)
*E^(((3*I)*b*(2*c + d*x))/d)*Sqrt[6*Pi]*Erf[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqr
t[c + d*x])/Sqrt[d]] - (1 + I)*(2*b*c - I*d)*E^((3*I)*(2*a + b*x))*Sqrt[6*
Pi]*Erfi[((1 + I)*Sqrt[3/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]))/(b^(5/2)*E^(
((3*I)*(a*d + b*(c + d*x)))/d)) - (c*Sqrt[d]*(20*Sqrt[b]*Sqrt[d]*E^(((5*I)
*b*c)/d)*Sqrt[c + d*x]*(3 + (10*I)*b*x + E^((10*I)*(a + b*x))*(3 - (10*I)*
b*x)) + (1 + I)*(10*b*c + (3*I)*d)*E^(((5*I)*b*(2*c + d*x))/d)*Sqrt[10*Pi]
*Erf[((1 + I)*Sqrt[5/2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] - (1 + I)*(10*b*c
- (3*I)*d)*E^((5*I)*(2*a + b*x))*Sqrt[10*Pi]*Erfi[((1 + I)*Sqrt[5/2]*Sqrt[
b]*Sqrt[c + d*x])/Sqrt[d]]))/(16000*b^(5/2)*E^(((5*I)*(a*d + b*(c + d*x)))
/d)) + (c^2*d*(E^((2*I)*a)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((-I)*b*(
c + d*x))/d] + E^(((2*I)*b*c)/d)*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, (I*b*(
c + d*x))/d]))/(16*b^2*E^((I*(b*c + a*d))/d)*Sqrt[c + d*x]) - (c^2*(c + d*
x)^(3/2)*(-((E^((6*I)*a)*Gamma[3/2, ((-3*I)*b*(c + d*x))/d])/(((-I)*b*(c +
 d*x))/d)^(3/2)) - (E^(((6*I)*b*c)/d)*Gamma[3/2, ((3*I)*b*(c + d*x))/d])/(
(I*b*(c + d*x))/d)^(3/2)))/(96*Sqrt[3]*d*E^(((3*I)*(b*c + a*d))/d)) - (c^2
*(c + d*x)^(3/2)*(-((E^((10*I)*a)*Gamma[3/2, ((-5*I)*b*(c + d*x))/d])/(((-
I)*b*(c + d*x))/d)^(3/2)) - (E^(((10*I)*b*c)/d)*Gamma[3/2, ((5*I)*b*(c + d
*x))/d])/((I*b*(c + d*x))/d)^(3/2)))/(160*Sqrt[5]*d*E^(((5*I)*(b*c + a*...
```

### 3.195.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin^2(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{1}{8}(c+dx)^{5/2}\cos(a+bx)-\frac{1}{16}(c+dx)^{5/2}\cos(3a+3bx)-\frac{1}{16}(c+dx)^{5/2}\cos(5a+5bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}-\frac{3\sqrt{\frac{\pi}{10}}d^{5/2}\cos\left(5a-\frac{5bc}{d}\right)\text{FresnelS}\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-\frac{15d^2\sqrt{c+dx}\sin(a+bx)}{32b^3}+\frac{5d^2\sqrt{c+dx}\sin(3a+3bx)}{576b^3}+\frac{3d^2\sqrt{c+dx}\sin(5a+5bx)}{1600b^3}+\frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2}-\frac{d(c+dx)^{3/2}\cos(5a+5bx)}{160b^2}+\frac{(c+dx)^{5/2}\sin(a+bx)}{8b}-\frac{(c+dx)^{5/2}\sin(3a+3bx)}{48b}-\frac{(c+dx)^{5/2}\sin(5a+5bx)}{80b}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

3.195. $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

output
```
(5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(16*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a
 + 3*b*x])/(288*b^2) - (d*(c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(160*b^2) + (1
5*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]])/(32*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d
]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) - (3
*d^(5/2)*Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sq
rt[c + d*x])/Sqrt[d]])/(1600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelC[(S
qrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(
7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/
Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*Fres
nelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(
7/2)) - (15*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(32*b^3) + ((c + d*x)^(5/2)*Si
n[a + b*x])/(8*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(576*b^3) - ((c
 + d*x)^(5/2)*Sin[3*a + 3*b*x])/(48*b) + (3*d^2*Sqrt[c + d*x]*Sin[5*a + 5*
b*x])/(1600*b^3) - ((c + d*x)^(5/2)*Sin[5*a + 5*b*x])/(80*b)
```

### 3.195.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.195.4 Maple [A] (verified)

Time = 11.66 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.16

| method | result |
|---|---|
| derivativedivides | $\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b} - \frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b} + \frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b} - \frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\right.}{}\right.}{}\right.}{8b}$ |
| default | $\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{8b} - \frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b} + \frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{b(dx+c)}{d}+\frac{ad-cb}{d}\right)}{2b} - \frac{d\sqrt{2}\,\sqrt{\pi}\,\left(\cos\right.}{}\right.}{}\right.}{8b}$ |

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/16/b*d*(d*x+c)^(5/2)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-5/16/b*d*(-1/2/b*
d*(d*x+c)^(3/2)*cos(b/d*(d*x+c)+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/2
)*sin(b/d*(d*x+c)+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos((
a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin((
a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1/
96/b*d*(d*x+c)^(5/2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+5/96/b*d*(-1/6/b*d*(
d*x+c)^(3/2)*cos(3*b/d*(d*x+c)+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/
2)*sin(3*b/d*(d*x+c)+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d
)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*
b*(d*x+c)^(1/2)/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b
/d)^(1/2)*b*(d*x+c)^(1/2)/d))))-1/160/b*d*(d*x+c)^(5/2)*sin(5*b/d*(d*x+c)+
5*(a*d-b*c)/d)+1/32/b*d*(-1/10/b*d*(d*x+c)^(3/2)*cos(5*b/d*(d*x+c)+5*(a*d-
b*c)/d)+3/10/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(5*b/d*(d*x+c)+5*(a*d-b*c)/d)-
1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*Fresnel
S(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(5*(a*d-b*c)/
d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))
```

## 3.195. $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

### 3.195.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 548, normalized size of antiderivative = 0.89

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx =$$

$$-\frac{81\sqrt{10}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+625\sqrt{6}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel
_sin(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 625*sqrt(6)*pi*d^3*sqrt(b/(p
i*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d
))) - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin
(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*
d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d)
+ 625*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt
(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fres
nel_cos(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480
*(90*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^5 - 50*(b^2*d^2*x + b^2*c*d)*cos(b
*x + a)^3 - 300*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) - (120*b^3*d^2*x^2 + 24
0*b^3*c*d*x + 120*b^3*c^2 - 9*(20*b^3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2
- 3*b*d^2)*cos(b*x + a)^4 - 428*b*d^2 + (60*b^3*d^2*x^2 + 120*b^3*c*d*x +
60*b^3*c^2 + 11*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^4
```

### 3.195.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

output
```
Timed out
```

### 3.195.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 826, normalized size of antiderivative = 1.34

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/1728000*sqrt(2)*(5400*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(5*((d*x + c)*b -
b*c + a*d)/d)/d + 15000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(3*((d*x + c)*b - b
*c + a*d)/d)/d - 270000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(((d*x + c)*b - b*c
 + a*d)/d)/d - 81*(-(I + 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5
*(b*c - a*d)/d) + (I - 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-5*(
b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) - 625*(-(I + 1)*9^(1/4)*sq
rt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(
pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*
I*b/d)) - 101250*((I + 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/
d) - (I - 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(
d*x + c)*sqrt(I*b/d)) - 101250*(-(I - 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*co
s(-(b*c - a*d)/d) + (I + 1)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d
)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - 625*((I - 1)*9^(1/4)*sqrt(pi)*b^2*
d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(pi)*b^2*d*(
b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) -
81*((I - 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)
- (I + 1)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*e
rf(sqrt(d*x + c)*sqrt(-5*I*b/d)) + 540*(20*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2
 - 3*sqrt(2)*sqrt(d*x + c)*b^3)*sin(5*((d*x + c)*b - b*c + a*d)/d) + 1500*
(12*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 5*sqrt(2)*sqrt(d*x + c)*b^3)*sin(...
```

### 3.195.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.74 (sec) , antiderivative size = 3716, normalized size of antiderivative = 6.04

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/864000*(1800*(30*I*sqrt(2)*sqrt(pi)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*I*sqrt(6)*sqrt
(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/
(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*I
*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(I*b*
c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi
)*d*erf(-1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 5*I*sqrt(
6)*sqrt(pi)*d*erf(1/2*I*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^(-3*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)) - 3*I*sqrt(10)*sqrt(pi)*d*erf(1/2*I*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-5*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/s
qrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2*(2250*(I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 +
4*I*b*c*d - 3*d^2)*d*erf(1/2*I*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt
(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 125*(I*sq
rt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*I*sqrt(6)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-3*(I*b*c - I*a*d)/d)/...
```

### 3.195.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \cos(a+b\,x)^3\sin(a+b\,x)^2\,(c+d\,x)^{5/2}\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

# 3.196 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$


3.196.1 Optimal result . . . 1566
3.196.2 Mathematica [C] (verified) . . . 1567
3.196.3 Rubi [A] (verified) . . . 1568
3.196.4 Maple [A] (verified) . . . 1569
3.196.5 Fricas [A] (verification not implemented) . . . 1570
3.196.6 Sympy [F(-1)] . . . 1571
3.196.7 Maxima [C] (verification not implemented) . . . 1571
3.196.8 Giac [C] (verification not implemented) . . . 1572
3.196.9 Mupad [F(-1)] . . . 1573


## 3.196.1 Optimal result

Integrand size = 26, antiderivative size = 407

$$
\begin{aligned}
\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx &= \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3} \\
&- \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d^2\sqrt{c+dx}\cos(6a+6bx)}{9216b^3} \\
&+ \frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b} + \frac{5d^{5/2}\sqrt{\frac{\pi}{3}}\cos\left(6a-\frac{6bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}} \\
&- \frac{45d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2048b^{7/2}} \\
&- \frac{5d^{5/2}\sqrt{\frac{\pi}{3}}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(6a-\frac{6bc}{d}\right)}{18432b^{7/2}} \\
&+ \frac{45d^{5/2}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{2048b^{7/2}} \\
&+ \frac{15d(c+dx)^{3/2}\sin(2a+2bx)}{256b^2} - \frac{5d(c+dx)^{3/2}\sin(6a+6bx)}{2304b^2}
\end{aligned}
$$

output
```
-3/64*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b+1/192*(d*x+c)^(5/2)*cos(6*b*x+6*a)/b+
15/256*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2-5/2304*d*(d*x+c)^(3/2)*sin(6*b*x
+6*a)/b^2+5/55296*d^(5/2)*cos(6*a-6*b*c/d)*FresnelC(2*b^(1/2)*3^(1/2)/Pi^(
1/2)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)-5/55296*d^(5/2)*Fresn
elS(2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)*3^(
1/2)*Pi^(1/2)/b^(7/2)-45/2048*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*
(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+45/2048*d^(5/2)*FresnelS(
2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2
)+45/1024*d^2*cos(2*b*x+2*a)*(d*x+c)^(1/2)/b^3-5/9216*d^2*cos(6*b*x+6*a)*(
d*x+c)^(1/2)/b^3
```

### 3.196.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.42 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.63

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{e^{-\frac{6i(bc+ad)}{d}}(c+dx)^{5/2}\left(243e^{4i\left(2a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{2ib(c+dx)}{d}\right)+243e^{4ia+\frac{8ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{2ib(c\cdots}{\cdots}\right.\right.}{41472\sqrt{2}b\left(\frac{b^2(c+dx)^2}{d^2}\cdots\right.}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
((c + d*x)^(5/2)*(243*E^((4*I)*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*Ga
mma[7/2, ((-2*I)*b*(c + d*x))/d] + 243*E^((4*I)*a + ((8*I)*b*c)/d)*Sqrt[((
-I)*b*(c + d*x))/d]*Gamma[7/2, ((2*I)*b*(c + d*x))/d] - Sqrt[3]*(E^((12*I)
*a)*Sqrt[(I*b*(c + d*x))/d]*Gamma[7/2, ((-6*I)*b*(c + d*x))/d] + E^(((12*I
)*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[7/2, ((6*I)*b*(c + d*x))/d])))/
(41472*Sqrt[2]*b*E^(((6*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^(3/2))
```

### 3.196.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin^3(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{3}{32}(c+dx)^{5/2}\sin(2a+2bx)-\frac{1}{32}(c+dx)^{5/2}\sin(6a+6bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{5\sqrt{\frac{\pi}{3}}d^{5/2}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}}-\frac{45\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2048b^{7/2}}-\frac{5\sqrt{\frac{\pi}{3}}d^{5/2}\sin\left(6a-\frac{6bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}}+\frac{45\sqrt{\pi}d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2048b^{7/2}}+\frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3}-\frac{5d^2\sqrt{c+dx}\cos(6a+6bx)}{9216b^3}+\frac{15d(c+dx)^{3/2}\sin(2a+2bx)}{256b^2}-\frac{5d(c+dx)^{3/2}\sin(6a+6bx)}{2304b^2}-\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b}+\frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(45*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(1024*b^3) - (3*(c + d*x)^(5/2)*Co
s[2*a + 2*b*x])/(64*b) - (5*d^2*Sqrt[c + d*x]*Cos[6*a + 6*b*x])/(9216*b^3)
 + ((c + d*x)^(5/2)*Cos[6*a + 6*b*x])/(192*b) + (5*d^(5/2)*Sqrt[Pi/3]*Cos[
6*a - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(
18432*b^(7/2)) - (45*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqr
t[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2048*b^(7/2)) - (5*d^(5/2)*Sqrt[
Pi/3]*FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[6*a - (6*
b*c)/d])/(18432*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c
 + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2048*b^(7/2)) + (15*d*
(c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(256*b^2) - (5*d*(c + d*x)^(3/2)*Sin[6*a
 + 6*b*x])/(2304*b^2)
```

#### 3.196.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.196.4 Maple [A] (verified)

Time = 35.52 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.17

| method | result |
|---|---|
| derivativedivides | $-\frac{3d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{15d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}\right.}{}\right.}{}$ |
| default | $-\frac{3d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{15d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}\right.}{}\right.}{}$ |

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-3/128/b*d*(d*x+c)^(5/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+15/128/b*d*
(1/4/b*d*(d*x+c)^(3/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*
(d*x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2
)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-s
in(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))+1/
384/b*d*(d*x+c)^(5/2)*cos(6*b/d*(d*x+c)+6*(a*d-b*c)/d)-5/384/b*d*(1/12/b*d
*(d*x+c)^(3/2)*sin(6*b/d*(d*x+c)+6*(a*d-b*c)/d)-1/4/b*d*(-1/12/b*d*(d*x+c)
^(1/2)*cos(6*b/d*(d*x+c)+6*(a*d-b*c)/d)+1/144/b*d*2^(1/2)*Pi^(1/2)*6^(1/2)
/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(
1/2)*b*(d*x+c)^(1/2)/d)-sin(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/
2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

### 3.196.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.09

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx=\frac{5\sqrt{3}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-5\sqrt{3}\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/55296*(5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos
(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))
*fresnel_sin(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d)
 - 1215*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 1215*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x
+ c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 96*(24*b^3*d^2*x^2 + 2*(48*b^
3*d^2*x^2 + 96*b^3*c*d*x + 48*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^6 + 48*b^3*c
*d*x + 24*b^3*c^2 + 45*b*d^2*cos(b*x + a)^2 - 3*(48*b^3*d^2*x^2 + 96*b^3*c
*d*x + 48*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^4 - 25*b*d^2 - 20*(2*(b^2*d^2*x
+ b^2*c*d)*cos(b*x + a)^5 - 2*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3 - 3*(b^
2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^4
```

### 3.196.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

output
```
Timed out
```

### 3.196.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.38

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{\left(960\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{6\,((dx+c)b-bc+ad)}{d}\right)-25920\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-48\left(\frac{48\,(dx+c)^{\frac{5}{2}}b^4}{d}-5\sqrt{\right.\right.}}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/442368*(960*(d*x + c)^(3/2)*b^3*sin(6*((d*x + c)*b - b*c + a*d)/d) - 25
920*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 48*(48*(d*x +
 c)^(5/2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*cos(6*((d*x + c)*b - b*c + a*d)/d
) + 1296*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x +
 c)*b - b*c + a*d)/d) - 5*(-(I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d
^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*
(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) -
1215*((I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c -
 a*d)/d) + (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(
b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 1215*(-(I + 1)*4^(1/4)*s
qrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1
/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt
(d*x + c)*sqrt(-2*I*b/d)) - 5*((I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^
2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) + (I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d
^2*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)
))*d/b^5
```

### 3.196.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.64 (sec) , antiderivative size = 2434, normalized size of antiderivative = 5.98

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/110592*(576*(sqrt(3)*sqrt(pi)*d*erf(-I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)) + sqrt(3)*sqrt(pi)*d*erf(I*sqrt(3)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)) - 9*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)) - 9*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)))*c^3 + 36*c*d^2*((sqrt(3)*sqrt(pi)*(48*b^2*c^2 - 8*I*b*c*d - d^2)*d*
erf(-I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*
(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(-4*I*(
d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-6*(-
I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (sqrt(3)*sqrt(pi)*(48*b^2*c^2
 + 8*I*b*c*d - d^2)*d*erf(I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
 + 1)*b^2) - 6*I*(-4*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - sqr
t(d*x + c)*d^2)*e^(-6*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 - 27*(sq
rt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b^2) + 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)...
```

### 3.196.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx=\int \cos(a+b\,x)^3\sin(a+b\,x)^3\,(c+d\,x)^{5/2}\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

# 3.197 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$

3.197.1 Optimal result . . . 1574
3.197.2 Mathematica [C] (verified) . . . 1575
3.197.3 Rubi [A] (verified) . . . 1575
3.197.4 Maple [A] (verified) . . . 1577
3.197.5 Fricas [A] (verification not implemented) . . . 1577
3.197.6 Sympy [F] . . . 1578
3.197.7 Maxima [C] (verification not implemented) . . . 1578
3.197.8 Giac [C] (verification not implemented) . . . 1579
3.197.9 Mupad [F(-1)] . . . 1580

## 3.197.1 Optimal result

Integrand size = 26, antiderivative size = 351

$$\begin{aligned}\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx &= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b}\\ &+\frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b}+\frac{d^{3/2}\sqrt{\frac{\pi}{3}}\cos\left(6a-\frac{6bc}{d}\right)\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}}\\ &-\frac{9d^{3/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{512b^{5/2}}\\ &+\frac{d^{3/2}\sqrt{\frac{\pi}{3}}\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(6a-\frac{6bc}{d}\right)}{1536b^{5/2}}\\ &-\frac{9d^{3/2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{512b^{5/2}}\\ &+\frac{9d\sqrt{c+dx}\sin(2a+2bx)}{256b^2}-\frac{d\sqrt{c+dx}\sin(6a+6bx)}{768b^2}\end{aligned}$$

output
```
-3/64*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b+1/192*(d*x+c)^(3/2)*cos(6*b*x+6*a)/b+
1/4608*d^(3/2)*cos(6*a-6*b*c/d)*FresnelS(2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(5/2)+1/4608*d^(3/2)*FresnelC(2*b^(1/2
)*3^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)*3^(1/2)*Pi^(1/2
)/b^(5/2)-9/512*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/
d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(5/2)-9/512*d^(3/2)*FresnelC(2*b^(1/2)*(d*x+c
)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)+9/256*d*sin(2*
b*x+2*a)*(d*x+c)^(1/2)/b^2-1/768*d*sin(6*b*x+6*a)*(d*x+c)^(1/2)/b^2
```

### 3.197.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.79 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.74

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{ie^{-\frac{6i(bc+ad)}{d}}(c+dx)^{5/2}\left(-81e^{4i\left(2a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{5}{2},-\frac{2ib(c+dx)}{d}\right)+81e^{4ia+\frac{8ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{5}{2},\frac{2ib(c\right.}{}\right.}{6912\sqrt{2}d\left(\frac{b^2(c+dx)^2}{d^2}\right.}$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
((I/6912)*(c + d*x)^(5/2)*(-81*E^((4*I)*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*
x))/d]*Gamma[5/2, ((-2*I)*b*(c + d*x))/d] + 81*E^((4*I)*a + ((8*I)*b*c)/d)
*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[5/2, ((2*I)*b*(c + d*x))/d] + Sqrt[3]*(E
^((12*I)*a)*Sqrt[(I*b*(c + d*x))/d]*Gamma[5/2, ((-6*I)*b*(c + d*x))/d] - E
^(((12*I)*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[5/2, ((6*I)*b*(c + d*x)
)/d])))/(Sqrt[2]*d*E^(((6*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^(3/2)
)
```

### 3.197.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin^3(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{3}{32}(c+dx)^{3/2}\sin(2a+2bx)-\frac{1}{32}(c+dx)^{3/2}\sin(6a+6bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\frac{\pi}{3}}d^{3/2}\sin\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}}-\frac{9\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{512b^{5/2}}+\frac{\sqrt{\frac{\pi}{3}}d^{3/2}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}}-\frac{9\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{512b^{5/2}}+\frac{9d\sqrt{c+dx}\sin(2a+2bx)}{256b^2}-\frac{d\sqrt{c+dx}\sin(6a+6bx)}{768b^2}-\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b}+\frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b}$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(-3*(c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)^(3/2)*Cos[6*a +
6*b*x])/(192*b) + (d^(3/2)*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelS[(2*Sqr
t[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[
Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[
Pi])])/(512*b^(5/2)) + (d^(3/2)*Sqrt[Pi/3]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(1536*b^(5/2)) - (9*d^(3/2)*
Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a -
(2*b*c)/d])/(512*b^(5/2)) + (9*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(256*b^2)
 - (d*Sqrt[c + d*x]*Sin[6*a + 6*b*x])/(768*b^2)
```

### 3.197.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.197.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.09

| method | result |
|---|---|
| derivativedivides | $-\frac{3d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{9d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\right.}{8b\sqrt{\frac{b}{d}}}\right.}{64b}$ |
| default | $-\frac{3d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{9d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\text{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\right.}{8b\sqrt{\frac{b}{d}}}\right.}{64b}$ |

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-3/128/b*d*(d*x+c)^(3/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+9/128/b*d*(
1/4/b*d*(d*x+c)^(1/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b
/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/
d)))+1/384/b*d*(d*x+c)^(3/2)*cos(6*b/d*(d*x+c)+6*(a*d-b*c)/d)-1/128/b*d*(1
/12/b*d*(d*x+c)^(1/2)*sin(6*b/d*(d*x+c)+6*(a*d-b*c)/d)-1/144/b*d*2^(1/2)*P
i^(1/2)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*
6^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(6*(a*d-b*c)/d)*FresnelC(2^(1/2)
/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

### 3.197.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.93

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx=\frac{\sqrt{3}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6\,(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+\sqrt{3}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6}{\right.}}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4608*(sqrt(3)*pi*d^2*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_sin(2*
sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(3)*pi*d^2*sqrt(b/(pi*d))*fres
nel_cos(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 81
*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*s
qrt(b/(pi*d))) - 81*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt
(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 96*(8*(b^2*d*x + b^2*c)*cos(b*x + a)^6
 - 12*(b^2*d*x + b^2*c)*cos(b*x + a)^4 + 2*b^2*d*x + 2*b^2*c - (2*b*d*cos(
b*x + a)^5 - 2*b*d*cos(b*x + a)^3 - 3*b*d*cos(b*x + a))*sin(b*x + a))*sqrt
(d*x + c))/b^3
```

## 3.197.6 Sympy [F]

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx=\int (c+dx)^{\frac{3}{2}}\sin^3(a+bx)\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)**3*cos(a + b*x)**3, x)
```

## 3.197.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.46

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx=\frac{\left(\frac{192\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{6\,((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{1728\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)}{d}-48\sqrt{dx+c}\,b^2\sin\left(\frac{6\,((dx+c)b-bc+\cdots}{d}\right.\cdots}{\cdots}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/36864*(192*(d*x + c)^(3/2)*b^3*cos(6*((d*x + c)*b - b*c + a*d)/d)/d - 17
28*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 48*sqrt(d*x
+ c)*b^2*sin(6*((d*x + c)*b - b*c + a*d)/d) + 1296*sqrt(d*x + c)*b^2*sin(2
*((d*x + c)*b - b*c + a*d)/d) + ((I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^
2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d
*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) +
 81*(-(I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a
*d)/d) + (I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c
- a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + 81*((I - 1)*4^(1/4)*sqrt(2)*
sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(
2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(-2*I*b/d)) + (-(I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*c
os(-6*(b*c - a*d)/d) + (I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/
4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))*d/b^4
```

### 3.197.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.79 (sec) , antiderivative size = 1514, normalized size of antiderivative = 4.31

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/9216*(48*(sqrt(3)*sqrt(pi)*d*erf(-I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)) + sqrt(3)*sqrt(pi)*d*erf(I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)) - 9*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)) - 9*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)))*c^2 + d^2*((sqrt(3)*sqrt(pi)*(48*b^2*c^2 - 8*I*b*c*d - d^2)*d*erf(-I*s
qrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c -
 I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(-4*I*(d*x + c)
^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-6*(-I*(d*x +
 c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (sqrt(3)*sqrt(pi)*(48*b^2*c^2 + 8*I*b
*c*d - d^2)*d*erf(I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^
2) - 6*I*(-4*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - sqrt(d*x +
c)*d^2)*e^(-6*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 - 27*(sqrt(pi)*(
16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b^2) + 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d -...
```

### 3.197.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \cos\left(a+b\,x\right)^3\sin\left(a+b\,x\right)^3\left(c+d\,x\right)^{3/2}\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

# 3.198 $\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx$

3.198.1 Optimal result . . . 1581
3.198.2 Mathematica [C] (verified) . . . 1582
3.198.3 Rubi [A] (verified) . . . 1582
3.198.4 Maple [A] (verified) . . . 1584
3.198.5 Fricas [A] (verification not implemented) . . . 1584
3.198.6 Sympy [F] . . . 1585
3.198.7 Maxima [C] (verification not implemented) . . . 1585
3.198.8 Giac [C] (verification not implemented) . . . 1586
3.198.9 Mupad [F(-1)] . . . 1587

## 3.198.1 Optimal result

Integrand size = 26, antiderivative size = 299

$$\begin{aligned}\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx &= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} \\ &+ \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} \\ &- \frac{\sqrt{d}\sqrt{\frac{\pi}{3}}\cos\left(6a-\frac{6bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}} \\ &+ \frac{3\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{3/2}} \\ &+ \frac{\sqrt{d}\sqrt{\frac{\pi}{3}}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(6a-\frac{6bc}{d}\right)}{384b^{3/2}} \\ &- \frac{3\sqrt{d}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{128b^{3/2}}\end{aligned}$$

output
```
-1/1152*cos(6*a-6*b*c/d)*FresnelC(2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)
/d^(1/2))*d^(1/2)*3^(1/2)*Pi^(1/2)/b^(3/2)+1/1152*FresnelS(2*b^(1/2)*3^(1/
2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)*d^(1/2)*3^(1/2)*Pi^(1/
2)/b^(3/2)+3/128*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)
/Pi^(1/2))*d^(1/2)*Pi^(1/2)/b^(3/2)-3/128*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)
/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-3/64*cos(2*b*
x+2*a)*(d*x+c)^(1/2)/b+1/192*cos(6*b*x+6*a)*(d*x+c)^(1/2)/b
```

### 3.198.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.74 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.86

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{e^{-\frac{6i(bc+ad)}{d}}\sqrt{c+dx}\left(-27e^{4i\left(2a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)-27e^{4ia+\frac{8ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right)+\sqrt{3}\right.}{1152\sqrt{2}b\sqrt{\frac{b^2(c+dx)^2}{d^2}}}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(Sqrt[c + d*x]*(-27*E^((4*I)*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*Gamm
a[3/2, ((-2*I)*b*(c + d*x))/d] - 27*E^((4*I)*a + ((8*I)*b*c)/d)*Sqrt[((-I)
*b*(c + d*x))/d]*Gamma[3/2, ((2*I)*b*(c + d*x))/d] + Sqrt[3]*(E^((12*I)*a)
*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-6*I)*b*(c + d*x))/d] + E^(((12*I)*b
*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((6*I)*b*(c + d*x))/d])))/(11
52*Sqrt[2]*b*E^(((6*I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

### 3.198.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^3(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \; 4906$$

$$\int \left(\frac{3}{32}\sqrt{c+dx}\sin(2a+2bx)-\frac{1}{32}\sqrt{c+dx}\sin(6a+6bx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}}+\frac{3\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{3/2}}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\sin\left(6a-\frac{6bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}}-\frac{3\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{3/2}}-\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b}+\frac{\sqrt{c+dx}\cos(6a+6bx)}{192b}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(-3*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b) + (Sqrt[c + d*x]*Cos[6*a + 6*b*
x])/(192*b) - (Sqrt[d]*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelC[(2*Sqrt[b]
*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(384*b^(3/2)) + (3*Sqrt[d]*Sqrt[Pi]*C
os[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]
)/(128*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/3]*FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[
c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(384*b^(3/2)) - (3*Sqrt[d]*Sqrt[P
i]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c
)/d])/(128*b^(3/2))
```

### 3.198.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.198.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.98

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{3d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{3d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{128b\sqrt{\frac{b}{d}}}+}{}$ |
| default | $\frac{-\frac{3d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{3d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{128b\sqrt{\frac{b}{d}}}+}{}$ |

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-3/128/b*d*(d*x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+3/256/b*d*P
i^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*
(d*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x
+c)^(1/2)/d))+1/384/b*d*(d*x+c)^(1/2)*cos(6*b/d*(d*x+c)+6*(a*d-b*c)/d)-1/4
608/b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(
2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(6*(a*d-b*c)/d)
*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

### 3.198.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.81

$$\int\sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx =$$
$$-\frac{\sqrt{3}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{3}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6(bc-ad)}{d}\right)}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/1152*(sqrt(3)*pi*d*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos(2*s
qrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(3)*pi*d*sqrt(b/(pi*d))*fresnel
_sin(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 27*pi
*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b
/(pi*d))) + 27*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*
d)))*sin(-2*(b*c - a*d)/d) - 48*(4*b*cos(b*x + a)^6 - 6*b*cos(b*x + a)^4 +
 b)*sqrt(d*x + c))/b^2
```

## 3.198.6 Sympy [F]

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \sqrt{c+dx}\sin^3(a+bx)\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x)**3, x)
```

## 3.198.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.47

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{\left(\frac{48\sqrt{dx+c}\,b^2\cos\left(\frac{6((dx+c)b-bc+ad)}{d}\right)}{d} - \frac{432\sqrt{dx+c}\,b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d} - \left(-(i-1)\cdot 36^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{6(bc-}{d}\right.\right.\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/9216*(48*sqrt(d*x + c)*b^2*cos(6*((d*x + c)*b - b*c + a*d)/d)/d - 432*sq
rt(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - (-(I - 1)*36^(1/4)*
sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (I + 1)*36^(1/4
)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x +
 c)*sqrt(6*I*b/d)) - 27*((I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4
)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/
4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 27*(-(I + 1)*
4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)
*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(-2*I*b/d)) - ((I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2
)^(1/4)*cos(-6*(b*c - a*d)/d) + (I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d
^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))*d/b^3
```

### 3.198.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.16 (sec) , antiderivative size = 826, normalized size of antiderivative = 2.76

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/2304*(sqrt(3)*sqrt(pi)*(12*b*c - I*d)*d*erf(-I*sqrt(3)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(3)*sqrt(pi)*(12*b*c + I*d)*d*erf(I*sqrt(
3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(-I*b*c + I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 12*(sqrt(3)*sqrt(pi)*d
*erf(-I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6
*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(3)*sqrt(p
i)*d*erf(I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 9*sqrt(p
i)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*
b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 9*sqrt(pi)*d*erf(I
*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 27*sqrt(pi)*(4*b*c - I*
d)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*
b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 27*sqrt(pi)*(4*b
*c + I*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 54*sqrt
(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 6*sqrt(d*x + c)*d
*e^(-6*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 54*sqrt(d*x + c)*d*e^(-2*(-I
*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^(-6*(-I*(d*x +...
```

### 3.198.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^3\,\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

# 3.199 $\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx$

3.199.1 Optimal result . . . 1588
3.199.2 Mathematica [C] (verified) . . . 1589
3.199.3 Rubi [A] (verified) . . . 1589
3.199.4 Maple [A] (verified) . . . 1591
3.199.5 Fricas [A] (verification not implemented) . . . 1591
3.199.6 Sympy [F] . . . 1592
3.199.7 Maxima [C] (verification not implemented) . . . 1592
3.199.8 Giac [C] (verification not implemented) . . . 1593
3.199.9 Mupad [F(-1)] . . . 1594

## 3.199.1 Optimal result

Integrand size = 26, antiderivative size = 299

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{\sqrt{d}\sqrt{\frac{\pi}{3}}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}} + \frac{3\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{3/2}} + \frac{\sqrt{d}\sqrt{\frac{\pi}{3}}\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(6a-\frac{6bc}{d}\right)}{384b^{3/2}} - \frac{3\sqrt{d}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{128b^{3/2}}$$

output
```
-1/1152*cos(6*a-6*b*c/d)*FresnelC(2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)
/d^(1/2))*d^(1/2)*3^(1/2)*Pi^(1/2)/b^(3/2)+1/1152*FresnelS(2*b^(1/2)*3^(1/
2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)*d^(1/2)*3^(1/2)*Pi^(1/
2)/b^(3/2)+3/128*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)
/Pi^(1/2))*d^(1/2)*Pi^(1/2)/b^(3/2)-3/128*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)
/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-3/64*cos(2*b*
x+2*a)*(d*x+c)^(1/2)/b+1/192*cos(6*b*x+6*a)*(d*x+c)^(1/2)/b
```

### 3.199.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.86

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{e^{-\frac{6i(bc+ad)}{d}}\sqrt{c+dx}\left(-27e^{4i\left(2a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{2ib(c+dx)}{d}\right)-27e^{4ia+\frac{8ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{3}{2},\frac{2ib(c+dx)}{d}\right)+\sqrt{3}\right.}{1152\sqrt{2}b\sqrt{\frac{b^2(c+dx)^2}{d^2}}}$$

input
```
Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(Sqrt[c + d*x]*(-27*E^((4*I)*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*Gamm
a[3/2, ((-2*I)*b*(c + d*x))/d] - 27*E^((4*I)*a + ((8*I)*b*c)/d)*Sqrt[((-I)
*b*(c + d*x))/d]*Gamma[3/2, ((2*I)*b*(c + d*x))/d] + Sqrt[3]*(E^((12*I)*a)
*Sqrt[(I*b*(c + d*x))/d]*Gamma[3/2, ((-6*I)*b*(c + d*x))/d] + E^(((12*I)*b
*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[3/2, ((6*I)*b*(c + d*x))/d])))/(11
52*Sqrt[2]*b*E^(((6*I)*(b*c + a*d))/d)*Sqrt[(b^2*(c + d*x)^2)/d^2])
```

### 3.199.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\sin^3(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{3}{32}\sqrt{c+dx}\sin(2a+2bx)-\frac{1}{32}\sqrt{c+dx}\sin(6a+6bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}}+\frac{3\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{3/2}}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\sin\left(6a-\frac{6bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}}-\frac{3\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{3/2}}-\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b}+\frac{\sqrt{c+dx}\cos(6a+6bx)}{192b}$$

input
```
Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(-3*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b) + (Sqrt[c + d*x]*Cos[6*a + 6*b*
x])/(192*b) - (Sqrt[d]*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelC[(2*Sqrt[b]
*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(384*b^(3/2)) + (3*Sqrt[d]*Sqrt[Pi]*C
os[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]
)/(128*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/3]*FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[
c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(384*b^(3/2)) - (3*Sqrt[d]*Sqrt[P
i]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c
)/d])/(128*b^(3/2))
```

### 3.199.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.199.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.98

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{3d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{3d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{128b\sqrt{\frac{b}{d}}}+\frac{d}{}}{}$ |
| default | $\frac{-\frac{3d\sqrt{dx+c}\,\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{3d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{128b\sqrt{\frac{b}{d}}}+\frac{d}{}}{}$ |

input
```
int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-3/128/b*d*(d*x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+3/256/b*d*P
i^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*
(d*x+c)^(1/2)/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x
+c)^(1/2)/d))+1/384/b*d*(d*x+c)^(1/2)*cos(6*b/d*(d*x+c)+6*(a*d-b*c)/d)-1/4
608/b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(
2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-sin(6*(a*d-b*c)/d)
*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

### 3.199.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.81

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{\sqrt{3}\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{3}\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right)}{}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/1152*(sqrt(3)*pi*d*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos(2*s
qrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(3)*pi*d*sqrt(b/(pi*d))*fresnel
_sin(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 27*pi
*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b
/(pi*d))) + 27*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*
d)))*sin(-2*(b*c - a*d)/d) - 48*(4*b*cos(b*x + a)^6 - 6*b*cos(b*x + a)^4 +
 b)*sqrt(d*x + c))/b^2
```

### 3.199.6 Sympy [F]

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \sqrt{c+dx}\sin^3(a+bx)\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

output
```
Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x)**3, x)
```

### 3.199.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.47

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{\left(\frac{48\sqrt{dx+c}\,b^2\cos\left(\frac{6((dx+c)b-bc+ad)}{d}\right)}{d} - \frac{432\sqrt{dx+c}\,b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d} - \left(-(i-1)\cdot 36^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{6(bc-\cdots}{d}\right.\right.\right.}{\cdots}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/9216*(48*sqrt(d*x + c)*b^2*cos(6*((d*x + c)*b - b*c + a*d)/d)/d - 432*sq
rt(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - (-(I - 1)*36^(1/4)*
sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (I + 1)*36^(1/4
)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x +
 c)*sqrt(6*I*b/d)) - 27*((I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4
)*cos(-2*(b*c - a*d)/d) + (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/
4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 27*(-(I + 1)*
4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)
*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(-2*I*b/d)) - ((I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2
)^(1/4)*cos(-6*(b*c - a*d)/d) + (I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d
^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))*d/b^3
```

### 3.199.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 826, normalized size of antiderivative = 2.76

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
1/2304*(sqrt(3)*sqrt(pi)*(12*b*c - I*d)*d*erf(-I*sqrt(3)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(3)*sqrt(pi)*(12*b*c + I*d)*d*erf(I*sqrt(
3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(-I*b*c + I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 12*(sqrt(3)*sqrt(pi)*d
*erf(-I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6
*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(3)*sqrt(p
i)*d*erf(I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 9*sqrt(p
i)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*
b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 9*sqrt(pi)*d*erf(I
*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 27*sqrt(pi)*(4*b*c - I*
d)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*
b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 27*sqrt(pi)*(4*b
*c + I*d)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 54*sqrt
(d*x + c)*d*e^(-2*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 6*sqrt(d*x + c)*d
*e^(-6*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 54*sqrt(d*x + c)*d*e^(-2*(-I
*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^(-6*(-I*(d*x +...
```

### 3.199.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \cos\left(a+b\,x\right)^3\sin\left(a+b\,x\right)^3\sqrt{c+d\,x}\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

# 3.200 $\int (c+dx)^{3/2} \cos^3(a+bx) \sin^3(a+bx) \, dx$

3.200.1 Optimal result . . . 1595
3.200.2 Mathematica [C] (verified) . . . 1596
3.200.3 Rubi [A] (verified) . . . 1596
3.200.4 Maple [A] (verified) . . . 1598
3.200.5 Fricas [A] (verification not implemented) . . . 1598
3.200.6 Sympy [F] . . . 1599
3.200.7 Maxima [C] (verification not implemented) . . . 1599
3.200.8 Giac [C] (verification not implemented) . . . 1600
3.200.9 Mupad [F(-1)] . . . 1601

## 3.200.1 Optimal result

Integrand size = 26, antiderivative size = 351

$$\int (c+dx)^{3/2} \cos^3(a+bx) \sin^3(a+bx) \, dx = -\frac{3(c+dx)^{3/2} \cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2} \cos(6a+6bx)}{192b} + \frac{d^{3/2} \sqrt{\frac{\pi}{3}} \cos\left(6a-\frac{6bc}{d}\right) \text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}} - \frac{9d^{3/2}\sqrt{\pi} \cos\left(2a-\frac{2bc}{d}\right) \text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{512b^{5/2}} + \frac{d^{3/2} \sqrt{\frac{\pi}{3}} \text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right) \sin\left(6a-\frac{6bc}{d}\right)}{1536b^{5/2}} - \frac{9d^{3/2}\sqrt{\pi} \text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right) \sin\left(2a-\frac{2bc}{d}\right)}{512b^{5/2}} + \frac{9d\sqrt{c+dx} \sin(2a+2bx)}{256b^2} - \frac{d\sqrt{c+dx} \sin(6a+6bx)}{768b^2}$$

output
```
-3/64*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b+1/192*(d*x+c)^(3/2)*cos(6*b*x+6*a)/b+
1/4608*d^(3/2)*cos(6*a-6*b*c/d)*FresnelS(2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(5/2)+1/4608*d^(3/2)*FresnelC(2*b^(1/2
)*3^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)*3^(1/2)*Pi^(1/2
)/b^(5/2)-9/512*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/
d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(5/2)-9/512*d^(3/2)*FresnelC(2*b^(1/2)*(d*x+c
)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)+9/256*d*sin(2*
b*x+2*a)*(d*x+c)^(1/2)/b^2-1/768*d*sin(6*b*x+6*a)*(d*x+c)^(1/2)/b^2
```

### 3.200.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.74

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{ie^{-\frac{6i(bc+ad)}{d}}(c+dx)^{5/2}\left(-81e^{4i\left(2a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{5}{2},-\frac{2ib(c+dx)}{d}\right)+81e^{4ia+\frac{8ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{5}{2},\frac{2ib(c\cdots}{6912\sqrt{2}d\left(\frac{b^2(c+dx)^2}{d^2}\cdots}$$

input
```
Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
((I/6912)*(c + d*x)^(5/2)*(-81*E^((4*I)*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*
x))/d]*Gamma[5/2, ((-2*I)*b*(c + d*x))/d] + 81*E^((4*I)*a + ((8*I)*b*c)/d)
*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[5/2, ((2*I)*b*(c + d*x))/d] + Sqrt[3]*(E
^((12*I)*a)*Sqrt[(I*b*(c + d*x))/d]*Gamma[5/2, ((-6*I)*b*(c + d*x))/d] - E
^(((12*I)*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[5/2, ((6*I)*b*(c + d*x)
)/d])))/(Sqrt[2]*d*E^(((6*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^(3/2)
)
```

### 3.200.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\sin^3(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \text{4906}$$

$$\int \left(\frac{3}{32}(c+dx)^{3/2}\sin(2a+2bx)-\frac{1}{32}(c+dx)^{3/2}\sin(6a+6bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\frac{\pi}{3}}d^{3/2}\sin\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}}-\frac{9\sqrt{\pi}d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{512b^{5/2}}+\frac{\sqrt{\frac{\pi}{3}}d^{3/2}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}}-\frac{9\sqrt{\pi}d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{512b^{5/2}}+\frac{9d\sqrt{c+dx}\sin(2a+2bx)}{256b^2}-\frac{d\sqrt{c+dx}\sin(6a+6bx)}{768b^2}-\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b}+\frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b}$$

input
```
Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(-3*(c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)^(3/2)*Cos[6*a +
6*b*x])/(192*b) + (d^(3/2)*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelS[(2*Sqr
t[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[
Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[
Pi])])/(512*b^(5/2)) + (d^(3/2)*Sqrt[Pi/3]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(1536*b^(5/2)) - (9*d^(3/2)*
Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a -
(2*b*c)/d])/(512*b^(5/2)) + (9*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(256*b^2)
 - (d*Sqrt[c + d*x]*Sin[6*a + 6*b*x])/(768*b^2)
```

### 3.200.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

### 3.200.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.09

| method | result |
|---|---|
| derivativedivides | $-\frac{3d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{9d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\right.}{8b\sqrt{\frac{b}{d}}}\right.}{64b}$ |
| default | $-\frac{3d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{9d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2ad-2cb}{d}\right)\mathrm{FresnelS}\left(\frac{2b\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\right.}{8b\sqrt{\frac{b}{d}}}\right.}{64b}$ |

input
```
int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-3/128/b*d*(d*x+c)^(3/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+9/128/b*d*(
1/4/b*d*(d*x+c)^(1/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b
/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1
/2)/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/
d)))+1/384/b*d*(d*x+c)^(3/2)*cos(6*b/d*(d*x+c)+6*(a*d-b*c)/d)-1/128/b*d*(1
/12/b*d*(d*x+c)^(1/2)*sin(6*b/d*(d*x+c)+6*(a*d-b*c)/d)-1/144/b*d*2^(1/2)*P
i^(1/2)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*
6^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)+sin(6*(a*d-b*c)/d)*FresnelC(2^(1/2)
/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))
```

### 3.200.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.93

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{\sqrt{3}\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+\sqrt{3}\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4608*(sqrt(3)*pi*d^2*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_sin(2*
sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(3)*pi*d^2*sqrt(b/(pi*d))*fres
nel_cos(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 81
*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*s
qrt(b/(pi*d))) - 81*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt
(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 96*(8*(b^2*d*x + b^2*c)*cos(b*x + a)^6
 - 12*(b^2*d*x + b^2*c)*cos(b*x + a)^4 + 2*b^2*d*x + 2*b^2*c - (2*b*d*cos(
b*x + a)^5 - 2*b*d*cos(b*x + a)^3 - 3*b*d*cos(b*x + a))*sin(b*x + a))*sqrt
(d*x + c))/b^3
```

## 3.200.6 Sympy [F]

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int (c+dx)^{\frac{3}{2}}\sin^3(a+bx)\cos^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**(3/2)*sin(a + b*x)**3*cos(a + b*x)**3, x)
```

## 3.200.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.46

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{\left(\frac{192\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{6\,((dx+c)b-bc+ad)}{d}\right)}{d} - \frac{1728\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)}{d} - 48\sqrt{dx+c}b^2\sin\left(\frac{6\,((dx+c)b-bc+}{d}\right.\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/36864*(192*(d*x + c)^(3/2)*b^3*cos(6*((d*x + c)*b - b*c + a*d)/d)/d - 17
28*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 48*sqrt(d*x
+ c)*b^2*sin(6*((d*x + c)*b - b*c + a*d)/d) + 1296*sqrt(d*x + c)*b^2*sin(2
*((d*x + c)*b - b*c + a*d)/d) + ((I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^
2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d
*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) +
 81*(-(I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a
*d)/d) + (I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c
- a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + 81*((I - 1)*4^(1/4)*sqrt(2)*
sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I + 1)*4^(1/4)*sqrt(
2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(-2*I*b/d)) + (-(I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*c
os(-6*(b*c - a*d)/d) + (I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/
4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))*d/b^4
```

### 3.200.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.79 (sec) , antiderivative size = 1514, normalized size of antiderivative = 4.31

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/9216*(48*(sqrt(3)*sqrt(pi)*d*erf(-I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)) + sqrt(3)*sqrt(pi)*d*erf(I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)) - 9*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)) - 9*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)))*c^2 + d^2*((sqrt(3)*sqrt(pi)*(48*b^2*c^2 - 8*I*b*c*d - d^2)*d*erf(-I*s
qrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c -
 I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(-4*I*(d*x + c)
^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-6*(-I*(d*x +
 c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (sqrt(3)*sqrt(pi)*(48*b^2*c^2 + 8*I*b
*c*d - d^2)*d*erf(I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^
2) - 6*I*(-4*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - sqrt(d*x +
c)*d^2)*e^(-6*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 - 27*(sqrt(pi)*(
16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b^2) + 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c*d -...
```

### 3.200.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx=\int \cos\left(a+b\,x\right)^3\sin\left(a+b\,x\right)^3\left(c+d\,x\right)^{3/2}\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(3/2),x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

# 3.201 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$

3.201.1 Optimal result . . . . . . . . . . 1602
3.201.2 Mathematica [C] (verified) . . . . . . . . . . 1603
3.201.3 Rubi [A] (verified) . . . . . . . . . . 1604
3.201.4 Maple [A] (verified) . . . . . . . . . . 1605
3.201.5 Fricas [A] (verification not implemented) . . . . . . . . . . 1606
3.201.6 Sympy [F(-1)] . . . . . . . . . . 1607
3.201.7 Maxima [C] (verification not implemented) . . . . . . . . . . 1607
3.201.8 Giac [C] (verification not implemented) . . . . . . . . . . 1608
3.201.9 Mupad [F(-1)] . . . . . . . . . . 1609

## 3.201.1 Optimal result

Integrand size = 26, antiderivative size = 407

$$\begin{aligned}\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx &= \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3} \\ &- \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d^2\sqrt{c+dx}\cos(6a+6bx)}{9216b^3} \\ &+ \frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b} + \frac{5d^{5/2}\sqrt{\frac{\pi}{3}}\cos\left(6a-\frac{6bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}} \\ &- \frac{45d^{5/2}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2048b^{7/2}} \\ &- \frac{5d^{5/2}\sqrt{\frac{\pi}{3}}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)\sin\left(6a-\frac{6bc}{d}\right)}{18432b^{7/2}} \\ &+ \frac{45d^{5/2}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)\sin\left(2a-\frac{2bc}{d}\right)}{2048b^{7/2}} \\ &+ \frac{15d(c+dx)^{3/2}\sin(2a+2bx)}{256b^2} - \frac{5d(c+dx)^{3/2}\sin(6a+6bx)}{2304b^2}\end{aligned}$$

output
```
-3/64*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b+1/192*(d*x+c)^(5/2)*cos(6*b*x+6*a)/b+
15/256*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2-5/2304*d*(d*x+c)^(3/2)*sin(6*b*x
+6*a)/b^2+5/55296*d^(5/2)*cos(6*a-6*b*c/d)*FresnelC(2*b^(1/2)*3^(1/2)/Pi^(
1/2)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)-5/55296*d^(5/2)*Fresn
elS(2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)*3^(
1/2)*Pi^(1/2)/b^(7/2)-45/2048*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*
(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+45/2048*d^(5/2)*FresnelS(
2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2
)+45/1024*d^2*cos(2*b*x+2*a)*(d*x+c)^(1/2)/b^3-5/9216*d^2*cos(6*b*x+6*a)*(
d*x+c)^(1/2)/b^3
```

### 3.201.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.63

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \frac{e^{-\frac{6i(bc+ad)}{d}}(c+dx)^{5/2}\left(243e^{4i\left(2a+\frac{bc}{d}\right)}\sqrt{\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{2ib(c+dx)}{d}\right)+243e^{4ia+\frac{8ibc}{d}}\sqrt{-\frac{ib(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{2ib(c\cdots}{\cdots}\right.\right.}{41472\sqrt{2}b\left(\frac{b^2(c+dx)^2}{d^2}\cdots\right.}$$

input
```
Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
((c + d*x)^(5/2)*(243*E^((4*I)*(2*a + (b*c)/d))*Sqrt[(I*b*(c + d*x))/d]*Ga
mma[7/2, ((-2*I)*b*(c + d*x))/d] + 243*E^((4*I)*a + ((8*I)*b*c)/d)*Sqrt[((
-I)*b*(c + d*x))/d]*Gamma[7/2, ((2*I)*b*(c + d*x))/d] - Sqrt[3]*(E^((12*I)
*a)*Sqrt[(I*b*(c + d*x))/d]*Gamma[7/2, ((-6*I)*b*(c + d*x))/d] + E^(((12*I
)*b*c)/d)*Sqrt[((-I)*b*(c + d*x))/d]*Gamma[7/2, ((6*I)*b*(c + d*x))/d])))/
(41472*Sqrt[2]*b*E^(((6*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^(3/2))
```

### 3.201.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\sin^3(a+bx)\cos^3(a+bx)\,dx$$

$$\Big\downarrow \ 4906$$

$$\int \left(\frac{3}{32}(c+dx)^{5/2}\sin(2a+2bx)-\frac{1}{32}(c+dx)^{5/2}\sin(6a+6bx)\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{5\sqrt{\frac{\pi}{3}}d^{5/2}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}}-\frac{45\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2048b^{7/2}}-\frac{5\sqrt{\frac{\pi}{3}}d^{5/2}\sin\left(6a-\frac{6bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}}+\frac{45\sqrt{\pi}d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelS}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2048b^{7/2}}+\frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3}-\frac{5d^2\sqrt{c+dx}\cos(6a+6bx)}{9216b^3}+\frac{15d(c+dx)^{3/2}\sin(2a+2bx)}{256b^2}-\frac{5d(c+dx)^{3/2}\sin(6a+6bx)}{2304b^2}-\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b}+\frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b}$$

input
```
Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

output
```
(45*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(1024*b^3) - (3*(c + d*x)^(5/2)*Co
s[2*a + 2*b*x])/(64*b) - (5*d^2*Sqrt[c + d*x]*Cos[6*a + 6*b*x])/(9216*b^3)
 + ((c + d*x)^(5/2)*Cos[6*a + 6*b*x])/(192*b) + (5*d^(5/2)*Sqrt[Pi/3]*Cos[
6*a - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(
18432*b^(7/2)) - (45*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqr
t[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2048*b^(7/2)) - (5*d^(5/2)*Sqrt[
Pi/3]*FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[6*a - (6*
b*c)/d])/(18432*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c
 + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2048*b^(7/2)) + (15*d*
(c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(256*b^2) - (5*d*(c + d*x)^(3/2)*Sin[6*a
 + 6*b*x])/(2304*b^2)
```

### 3.201.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

## 3.201.4 Maple [A] (verified)

Time = 34.41 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.17

| method | result |
|---|---|
| derivativedivides | $-\frac{3d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{15d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}\right.}{}\right.}{}$ |
| default | $-\frac{3d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{64b}+\frac{15d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\cos\left(\frac{2b(dx+c)}{d}+\frac{2ad-2cb}{d}\right)}{4b}\right.}{}\right.}{}$ |

input
```
int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/d*(-3/128/b*d*(d*x+c)^(5/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+15/128/b*d*
(1/4/b*d*(d*x+c)^(3/2)*sin(2*b/d*(d*x+c)+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*
(d*x+c)^(1/2)*cos(2*b/d*(d*x+c)+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2
)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)-s
in(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d))))+1/
384/b*d*(d*x+c)^(5/2)*cos(6*b/d*(d*x+c)+6*(a*d-b*c)/d)-5/384/b*d*(1/12/b*d
*(d*x+c)^(3/2)*sin(6*b/d*(d*x+c)+6*(a*d-b*c)/d)-1/4/b*d*(-1/12/b*d*(d*x+c)
^(1/2)*cos(6*b/d*(d*x+c)+6*(a*d-b*c)/d)+1/144/b*d*2^(1/2)*Pi^(1/2)*6^(1/2)
/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(
1/2)*b*(d*x+c)^(1/2)/d)-sin(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/
2)/(b/d)^(1/2)*b*(d*x+c)^(1/2)/d)))))
```

### 3.201.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.09

$$\int(c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx=\frac{5\sqrt{3}\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-5\sqrt{3}\pi d^3\sqrt{\frac{b}{\pi d}}\mathrm{S}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/55296*(5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos
(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))
*fresnel_sin(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d)
 - 1215*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 1215*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x
+ c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 96*(24*b^3*d^2*x^2 + 2*(48*b^
3*d^2*x^2 + 96*b^3*c*d*x + 48*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^6 + 48*b^3*c
*d*x + 24*b^3*c^2 + 45*b*d^2*cos(b*x + a)^2 - 3*(48*b^3*d^2*x^2 + 96*b^3*c
*d*x + 48*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^4 - 25*b*d^2 - 20*(2*(b^2*d^2*x
+ b^2*c*d)*cos(b*x + a)^5 - 2*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3 - 3*(b^
2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^4
```

### 3.201.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

output
```
Timed out
```

### 3.201.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.38

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx =$$

$$-\frac{\left(960\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{6\,((dx+c)b-bc+ad)}{d}\right)-25920\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-48\left(\frac{48\,(dx+c)^{\frac{5}{2}}b^4}{d}-5\sqrt{\right.\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/442368*(960*(d*x + c)^(3/2)*b^3*sin(6*((d*x + c)*b - b*c + a*d)/d) - 25
920*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 48*(48*(d*x +
 c)^(5/2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*cos(6*((d*x + c)*b - b*c + a*d)/d
) + 1296*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x +
 c)*b - b*c + a*d)/d) - 5*(-(I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d
^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*
(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) -
1215*((I - 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c -
 a*d)/d) + (I + 1)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(
b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - 1215*(-(I + 1)*4^(1/4)*s
qrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (I - 1)*4^(1
/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt
(d*x + c)*sqrt(-2*I*b/d)) - 5*((I + 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^
2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) + (I - 1)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d
^2*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)
))*d/b^5
```

### 3.201.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.67 (sec) , antiderivative size = 2434, normalized size of antiderivative = 5.98

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/110592*(576*(sqrt(3)*sqrt(pi)*d*erf(-I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)) + sqrt(3)*sqrt(pi)*d*erf(I*sqrt(3)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)) - 9*sqrt(pi)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)) - 9*sqrt(pi)*d*erf(I*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^(-2*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)))*c^3 + 36*c*d^2*((sqrt(3)*sqrt(pi)*(48*b^2*c^2 - 8*I*b*c*d - d^2)*d*
erf(-I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-6*
(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(-4*I*(
d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(-6*(-
I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + (sqrt(3)*sqrt(pi)*(48*b^2*c^2
 + 8*I*b*c*d - d^2)*d*erf(I*sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^(-6*(-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
 + 1)*b^2) - 6*I*(-4*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - sqr
t(d*x + c)*d^2)*e^(-6*(I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 - 27*(sq
rt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-I*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(-2*(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b^2) + 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)...
```

### 3.201.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx=\int \cos\left(a+b\,x\right)^3\sin\left(a+b\,x\right)^3\left(c+d\,x\right)^{5/2}\,dx$$

input
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

output
```
int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

# 3.202 $\int x^3 \cos^2(x) \cot^2(x) \, dx$

3.202.1 Optimal result . . . . . 1610
3.202.2 Mathematica [A] (verified) . . . . . 1610
3.202.3 Rubi [A] (verified) . . . . . 1611
3.202.4 Maple [A] (verified) . . . . . 1615
3.202.5 Fricas [B] (verification not implemented) . . . . . 1616
3.202.6 Sympy [F] . . . . . 1616
3.202.7 Maxima [F(-2)] . . . . . 1617
3.202.8 Giac [F] . . . . . 1617
3.202.9 Mupad [F(-1)] . . . . . 1617

## 3.202.1 Optimal result

Integrand size = 12, antiderivative size = 112

$$\int x^3 \cos^2(x) \cot^2(x) \, dx = \frac{3x^2}{8} - ix^3 - \frac{3x^4}{8} + \frac{3\cos^2(x)}{8} - \frac{3}{4}x^2\cos^2(x) - x^3\cot(x) + 3x^2\log\left(1 - e^{2ix}\right) - 3ix\,\text{PolyLog}\left(2, e^{2ix}\right) + \frac{3}{2}\text{PolyLog}\left(3, e^{2ix}\right) + \frac{3}{4}x\cos(x)\sin(x) - \frac{1}{2}x^3\cos(x)\sin(x)$$

output
```
3/8*x^2-I*x^3-3/8*x^4+3/8*cos(x)^2-3/4*x^2*cos(x)^2-x^3*cot(x)+3*x^2*ln(1-
exp(2*I*x))-3*I*x*polylog(2,exp(2*I*x))+3/2*polylog(3,exp(2*I*x))+3/4*x*co
s(x)*sin(x)-1/2*x^3*cos(x)*sin(x)
```

## 3.202.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.93

$$\int x^3 \cos^2(x) \cot^2(x) \, dx = \frac{1}{16}\left(-2i\pi^3 + 16ix^3 - 6x^4 + 3\cos(2x) - 6x^2\cos(2x) - 16x^3\cot(x) + 48x^2\log\left(1 - e^{-2ix}\right) + 48ix\,\text{PolyLog}\left(2, e^{-2ix}\right) + 24\,\text{PolyLog}\left(3, e^{-2ix}\right) + 6x\sin(2x) - 4x^3\sin(2x)\right)$$

input
```
Integrate[x^3*Cos[x]^2*Cot[x]^2,x]
```

output
```
((-2*I)*Pi^3 + (16*I)*x^3 - 6*x^4 + 3*Cos[2*x] - 6*x^2*Cos[2*x] - 16*x^3*C
ot[x] + 48*x^2*Log[1 - E^((-2*I)*x)] + (48*I)*x*PolyLog[2, E^((-2*I)*x)] +
 24*PolyLog[3, E^((-2*I)*x)] + 6*x*Sin[2*x] - 4*x^3*Sin[2*x])/16
```

### 3.202.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.23, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {4908, 3042, 3792, 15, 3042, 3791, 15, 4203, 15, 25, 3042, 25, 4200, 25, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cos^2(x) \cot^2(x) \, dx$$

$$\Big\downarrow \text{ 4908}$$

$$\int x^3 \cot^2(x)dx - \int x^3 \cos^2(x)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x^3 \tan\left(x+\frac{\pi}{2}\right)^2 dx - \int x^3 \sin\left(x+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{ 3792}$$

$$-\frac{\int x^3dx}{2} + \int x^3 \tan\left(x+\frac{\pi}{2}\right)^2 dx + \frac{3}{2}\int x\cos^2(x)dx - \frac{1}{2}x^3\sin(x)\cos(x) - \frac{3}{4}x^2\cos^2(x)$$

$$\Big\downarrow \text{ 15}$$

$$\int x^3 \tan\left(x+\frac{\pi}{2}\right)^2 dx + \frac{3}{2}\int x\cos^2(x)dx - \frac{x^4}{8} - \frac{1}{2}x^3\sin(x)\cos(x) - \frac{3}{4}x^2\cos^2(x)$$

$$\Big\downarrow \text{ 3042}$$

$$\int x^3 \tan\left(x+\frac{\pi}{2}\right)^2 dx + \frac{3}{2}\int x\sin\left(x+\frac{\pi}{2}\right)^2 dx - \frac{x^4}{8} - \frac{1}{2}x^3\sin(x)\cos(x) - \frac{3}{4}x^2\cos^2(x)$$

$$\Big\downarrow \text{ 3791}$$

$$\int x^3 \tan\left(x+\frac{\pi}{2}\right)^2 dx + \frac{3}{2}\left(\frac{\int xdx}{2} + \frac{\cos^2(x)}{4} + \frac{1}{2}x\sin(x)\cos(x)\right) - \frac{x^4}{8} - \frac{1}{2}x^3\sin(x)\cos(x) - \frac{3}{4}x^2\cos^2(x)$$

$$\Big\downarrow \text{ 15}$$

$$\int x^3\tan\left(x+\frac{\pi}{2}\right)^2dx-\frac{x^4}{8}-\frac{1}{2}x^3\sin(x)\cos(x)-\frac{3}{4}x^2\cos^2(x)+\frac{3}{2}\left(\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)\right)$$

$$\Big\downarrow \text{ 4203}$$

$$-\int x^3dx-3\int -x^2\cot(x)dx-\frac{x^4}{8}-x^3\cot(x)-\frac{1}{2}x^3\sin(x)\cos(x)-\frac{3}{4}x^2\cos^2(x)+\frac{3}{2}\left(\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)\right)$$

$$\Big\downarrow \text{ 15}$$

$$-3\int -x^2\cot(x)dx-\frac{3x^4}{8}-x^3\cot(x)-\frac{1}{2}x^3\sin(x)\cos(x)-\frac{3}{4}x^2\cos^2(x)+\frac{3}{2}\left(\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)\right)$$

$$\Big\downarrow \text{ 25}$$

$$3\int x^2\cot(x)dx-\frac{3x^4}{8}-x^3\cot(x)-\frac{1}{2}x^3\sin(x)\cos(x)-\frac{3}{4}x^2\cos^2(x)+\frac{3}{2}\left(\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)\right)$$

$$\Big\downarrow \text{ 3042}$$

$$3\int -x^2\tan\left(x+\frac{\pi}{2}\right)dx-\frac{3x^4}{8}-x^3\cot(x)-\frac{1}{2}x^3\sin(x)\cos(x)-\frac{3}{4}x^2\cos^2(x)+\frac{3}{2}\left(\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)\right)$$

$$\Big\downarrow \text{ 25}$$

$$-3\int x^2\tan\left(x+\frac{\pi}{2}\right)dx-\frac{3x^4}{8}-x^3\cot(x)-\frac{1}{2}x^3\sin(x)\cos(x)-\frac{3}{4}x^2\cos^2(x)+\frac{3}{2}\left(\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)\right)$$

$$\Big\downarrow \text{ 4200}$$

$$-3\left(\frac{ix^3}{3}-2i\int -\frac{e^{2ix}x^2}{1-e^{2ix}}dx\right)-\frac{3x^4}{8}-x^3\cot(x)-\frac{1}{2}x^3\sin(x)\cos(x)-\frac{3}{4}x^2\cos^2(x)+\frac{3}{2}\left(\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)\right)$$

$$\Big\downarrow \text{ 25}$$

$$-3\left(2i\int \frac{e^{2ix}x^2}{1-e^{2ix}}dx+\frac{ix^3}{3}\right)-\frac{3x^4}{8}-x^3\cot(x)-\frac{1}{2}x^3\sin(x)\cos(x)-\frac{3}{4}x^2\cos^2(x)+\frac{3}{2}\left(\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)\right)$$

3.202. $\int x^3\cos^2(x)\cot^2(x)\,dx$

$$\Big\downarrow \text{2620}$$

$$-3\left(2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\int x\log\left(1-e^{2ix}\right)dx\right)+\frac{ix^3}{3}\right)-\frac{3x^4}{8}-x^3\cot(x)-\frac{1}{2}x^3\sin(x)\cos(x)-\frac{3}{4}x^2\cos^2(x)+\frac{3}{2}\left(\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)\right)$$

$$\Big\downarrow \text{3011}$$

$$-3\left(2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{2}i\int \text{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)+\frac{ix^3}{3}\right)-\frac{3x^4}{8}-x^3\cot(x)-\frac{1}{2}x^3\sin(x)\cos(x)-\frac{3}{4}x^2\cos^2(x)+\frac{3}{2}\left(\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)\right)$$

$$\Big\downarrow \text{2720}$$

$$-3\left(2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)+\frac{ix^3}{3}\right)-\frac{3x^4}{8}-x^3\cot(x)-\frac{1}{2}x^3\sin(x)\cos(x)-\frac{3}{4}x^2\cos^2(x)+\frac{3}{2}\left(\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)\right)$$

$$\Big\downarrow \text{7143}$$

$$-3\left(2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\text{PolyLog}\left(3,e^{2ix}\right)\right)\right)+\frac{ix^3}{3}\right)-\frac{3x^4}{8}-x^3\cot(x)-\frac{1}{2}x^3\sin(x)\cos(x)-\frac{3}{4}x^2\cos^2(x)+\frac{3}{2}\left(\frac{x^2}{4}+\frac{\cos^2(x)}{4}+\frac{1}{2}x\sin(x)\cos(x)\right)$$

input
```
Int[x^3*Cos[x]^2*Cot[x]^2,x]
```

output
```
(-3*x^4)/8 - (3*x^2*Cos[x]^2)/4 - x^3*Cot[x] - 3*((I/3)*x^3 + (2*I)*((I/2)
*x^2*Log[1 - E^((2*I)*x)] - I*((I/2)*x*PolyLog[2, E^((2*I)*x)] - PolyLog[3
, E^((2*I)*x)]/4))) - (x^3*Cos[x]*Sin[x])/2 + (3*(x^2/4 + Cos[x]^2/4 + (x*
Cos[x]*Sin[x])/2))/2
```

### 3.202.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

## 3.202. $\int x^3 \cos^2(x) \cot^2(x) \, dx$

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.202.4 Maple [A] (verified)

Time = 3.78 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.34

| method | result |
|---|---|
| risch | $-\frac{3x^4}{8}+\frac{i(4x^3+6ix^2-6x-3i)\,\mathrm{e}^{2ix}}{32}-\frac{i(4x^3-6ix^2-6x+3i)\,\mathrm{e}^{-2ix}}{32}-\frac{2ix^3}{\mathrm{e}^{2ix}-1}-2ix^3+3x^2\ln\left(\mathrm{e}^{ix}+1\right)-6ix\,\mathrm{polylo}$ |

input
```
int(x^3*cos(x)^2*cot(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/8*x^4+1/32*I*(6*I*x^2+4*x^3-3*I-6*x)*exp(2*I*x)-1/32*I*(-6*I*x^2+4*x^3+
3*I-6*x)*exp(-2*I*x)-2*I*x^3/(exp(2*I*x)-1)-2*I*x^3+3*x^2*ln(exp(I*x)+1)-6
*I*x*polylog(2,-exp(I*x))+6*polylog(3,-exp(I*x))+3*x^2*ln(1-exp(I*x))-6*I*
x*polylog(2,exp(I*x))+6*polylog(3,exp(I*x))
```

### 3.202.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(84) = 168.

Time = 0.28 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.18

$$\int x^3\cos^2(x)\cot^2(x)\,dx = \frac{4\,(2\,x^3-3\,x)\cos(x)^3+24\,x^2\log(\cos(x)+i\,\sin(x)+1)\sin(x)+24\,x^2\log(\cos(x)-i\,\sin(x)+1)\sin(x}{}$$

input
```
integrate(x^3*cos(x)^2*cot(x)^2,x, algorithm="fricas")
```

output
```
1/16*(4*(2*x^3 - 3*x)*cos(x)^3 + 24*x^2*log(cos(x) + I*sin(x) + 1)*sin(x)
+ 24*x^2*log(cos(x) - I*sin(x) + 1)*sin(x) + 24*x^2*log(-cos(x) + I*sin(x)
 + 1)*sin(x) + 24*x^2*log(-cos(x) - I*sin(x) + 1)*sin(x) - 48*I*x*dilog(co
s(x) + I*sin(x))*sin(x) + 48*I*x*dilog(cos(x) - I*sin(x))*sin(x) + 48*I*x*
dilog(-cos(x) + I*sin(x))*sin(x) - 48*I*x*dilog(-cos(x) - I*sin(x))*sin(x)
 - 12*(2*x^3 - x)*cos(x) - 3*(2*x^4 + 2*(2*x^2 - 1)*cos(x)^2 - 2*x^2 + 1)*
sin(x) + 48*polylog(3, cos(x) + I*sin(x))*sin(x) + 48*polylog(3, cos(x) -
I*sin(x))*sin(x) + 48*polylog(3, -cos(x) + I*sin(x))*sin(x) + 48*polylog(3
, -cos(x) - I*sin(x))*sin(x))/sin(x)
```

### 3.202.6 Sympy [F]

$$\int x^3\cos^2(x)\cot^2(x)\,dx = \int x^3\cos^2(x)\cot^2(x)\,dx$$

input
```
integrate(x**3*cos(x)**2*cot(x)**2,x)
```

output
```
Integral(x**3*cos(x)**2*cot(x)**2, x)
```

### 3.202.7 Maxima [F(-2)]

Exception generated.

$$\int x^3 \cos^2(x) \cot^2(x)\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*cos(x)^2*cot(x)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

### 3.202.8 Giac [F]

$$\int x^3 \cos^2(x) \cot^2(x)\,dx = \int x^3 \cos{(x)}^2 \cot{(x)}^2 \,dx$$

input
```
integrate(x^3*cos(x)^2*cot(x)^2,x, algorithm="giac")
```

output
```
integrate(x^3*cos(x)^2*cot(x)^2, x)
```

### 3.202.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \cos^2(x) \cot^2(x)\,dx = \int x^3 \cos{(x)}^2 \cot(x)^2\,dx$$

input
```
int(x^3*cos(x)^2*cot(x)^2,x)
```

output
```
int(x^3*cos(x)^2*cot(x)^2, x)
```

# 3.203 $\int x^2 \cos^2(x) \cot^2(x) \, dx$

3.203.1 Optimal result . . . . . 1618
3.203.2 Mathematica [A] (verified) . . . . . 1618
3.203.3 Rubi [A] (verified) . . . . . 1619
3.203.4 Maple [A] (verified) . . . . . 1623
3.203.5 Fricas [B] (verification not implemented) . . . . . 1623
3.203.6 Sympy [F] . . . . . 1624
3.203.7 Maxima [F(-2)] . . . . . 1624
3.203.8 Giac [F] . . . . . 1624
3.203.9 Mupad [F(-1)] . . . . . 1625

## 3.203.1 Optimal result

Integrand size = 12, antiderivative size = 83

$$\int x^2 \cos^2(x) \cot^2(x) \, dx = \frac{x}{4} - ix^2 - \frac{x^3}{2} - \frac{1}{2}x\cos^2(x) - x^2\cot(x) + 2x\log\left(1 - e^{2ix}\right) - i\,\text{PolyLog}\left(2, e^{2ix}\right) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}x^2\cos(x)\sin(x)$$

output
```
1/4*x-I*x^2-1/2*x^3-1/2*x*cos(x)^2-x^2*cot(x)+2*x*ln(1-exp(2*I*x))-I*polyl
og(2,exp(2*I*x))+1/4*cos(x)*sin(x)-1/2*x^2*cos(x)*sin(x)
```

## 3.203.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87

$$\int x^2 \cos^2(x) \cot^2(x) \, dx = \frac{1}{8}\left(-8ix^2 - 4x^3 - 2x\cos(2x) - 8x^2\cot(x) + 16x\log\left(1 - e^{2ix}\right) - 8i\,\text{PolyLog}\left(2, e^{2ix}\right) + \sin(2x) - 2x^2\sin(2x)\right)$$

input
```
Integrate[x^2*Cos[x]^2*Cot[x]^2,x]
```

output
```
((-8*I)*x^2 - 4*x^3 - 2*x*Cos[2*x] - 8*x^2*Cot[x] + 16*x*Log[1 - E^((2*I)*
x)] - (8*I)*PolyLog[2, E^((2*I)*x)] + Sin[2*x] - 2*x^2*Sin[2*x])/8
```

### 3.203.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.23, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.417$, Rules used = {4908, 3042, 3792, 15, 3042, 3115, 24, 4203, 15, 25, 3042, 25, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cos^2(x) \cot^2(x) \, dx$$

$$\Big\downarrow \text{4908}$$

$$\int x^2 \cot^2(x)dx - \int x^2 \cos^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2 \tan\left(x + \frac{\pi}{2}\right)^2 dx - \int x^2 \sin\left(x + \frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{3792}$$

$$-\frac{\int x^2 dx}{2} + \int x^2 \tan\left(x + \frac{\pi}{2}\right)^2 dx + \frac{1}{2}\int \cos^2(x)dx - \frac{1}{2}x^2 \sin(x)\cos(x) - \frac{1}{2}x\cos^2(x)$$

$$\Big\downarrow \text{15}$$

$$\int x^2 \tan\left(x + \frac{\pi}{2}\right)^2 dx + \frac{1}{2}\int \cos^2(x)dx - \frac{x^3}{6} - \frac{1}{2}x^2 \sin(x)\cos(x) - \frac{1}{2}x\cos^2(x)$$

$$\Big\downarrow \text{3042}$$

$$\int x^2 \tan\left(x + \frac{\pi}{2}\right)^2 dx + \frac{1}{2}\int \sin\left(x + \frac{\pi}{2}\right)^2 dx - \frac{x^3}{6} - \frac{1}{2}x^2 \sin(x)\cos(x) - \frac{1}{2}x\cos^2(x)$$

$$\Big\downarrow \text{3115}$$

$$\int x^2 \tan\left(x + \frac{\pi}{2}\right)^2 dx + \frac{1}{2}\left(\frac{\int 1dx}{2} + \frac{1}{2}\sin(x)\cos(x)\right) - \frac{x^3}{6} - \frac{1}{2}x^2 \sin(x)\cos(x) - \frac{1}{2}x\cos^2(x)$$

$$\Big\downarrow \text{24}$$

$$\int x^2 \tan\left(x + \frac{\pi}{2}\right)^2 dx - \frac{x^3}{6} - \frac{1}{2}x^2 \sin(x)\cos(x) - \frac{1}{2}x\cos^2(x) + \frac{1}{2}\left(\frac{x}{2} + \frac{1}{2}\sin(x)\cos(x)\right)$$

$$\Big\downarrow \text{4203}$$

$$-\int x^2dx-2\int -x\cot(x)dx-\frac{x^3}{6}-x^2\cot(x)-\frac{1}{2}x^2\sin(x)\cos(x)-\frac{1}{2}x\cos^2(x)+\frac{1}{2}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)$$

$$\Big\downarrow \; 15$$

$$-2\int -x\cot(x)dx-\frac{x^3}{2}-x^2\cot(x)-\frac{1}{2}x^2\sin(x)\cos(x)-\frac{1}{2}x\cos^2(x)+\frac{1}{2}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)$$

$$\Big\downarrow \; 25$$

$$2\int x\cot(x)dx-\frac{x^3}{2}-x^2\cot(x)-\frac{1}{2}x^2\sin(x)\cos(x)-\frac{1}{2}x\cos^2(x)+\frac{1}{2}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)$$

$$\Big\downarrow \; 3042$$

$$2\int -x\tan\left(x+\frac{\pi}{2}\right)dx-\frac{x^3}{2}-x^2\cot(x)-\frac{1}{2}x^2\sin(x)\cos(x)-\frac{1}{2}x\cos^2(x)+\frac{1}{2}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)$$

$$\Big\downarrow \; 25$$

$$-2\int x\tan\left(x+\frac{\pi}{2}\right)dx-\frac{x^3}{2}-x^2\cot(x)-\frac{1}{2}x^2\sin(x)\cos(x)-\frac{1}{2}x\cos^2(x)+\frac{1}{2}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)$$

$$\Big\downarrow \; 4200$$

$$-2\left(\frac{ix^2}{2}-2i\int -\frac{e^{2ix}x}{1-e^{2ix}}dx\right)-\frac{x^3}{2}-x^2\cot(x)-\frac{1}{2}x^2\sin(x)\cos(x)-\frac{1}{2}x\cos^2(x)+\frac{1}{2}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)$$

$$\Big\downarrow \; 25$$

$$-2\left(2i\int \frac{e^{2ix}x}{1-e^{2ix}}dx+\frac{ix^2}{2}\right)-\frac{x^3}{2}-x^2\cot(x)-\frac{1}{2}x^2\sin(x)\cos(x)-\frac{1}{2}x\cos^2(x)+\frac{1}{2}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)$$

$$\Big\downarrow \; 2620$$

$$-2\left(2i\left(\frac{1}{2}ix\log\left(1-e^{2ix}\right)-\frac{1}{2}i\int \log\left(1-e^{2ix}\right)dx\right)+\frac{ix^2}{2}\right)-\frac{x^3}{2}-x^2\cot(x)-\frac{1}{2}x^2\sin(x)\cos(x)-\frac{1}{2}x\cos^2(x)+\frac{1}{2}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)$$

$$\Big\downarrow \; 2715$$

$$-2\left(2i\left(\frac{1}{2}ix\log\left(1-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\log\left(1-e^{2ix}\right)de^{2ix}\right)+\frac{ix^2}{2}\right)-\frac{x^3}{2}-x^2\cot(x)-\frac{1}{2}x^2\sin(x)\cos(x)-\frac{1}{2}x\cos^2(x)+\frac{1}{2}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)$$

---

3.203. $\int x^2\cos^2(x)\cot^2(x)\,dx$

$$\Big\downarrow \text{2838}$$

$$-2\left(2i\left(\frac{1}{4}\text{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)-\frac{x^3}{2}-x^2\cot(x)-\frac{1}{2}x^2\sin(x)\cos(x)-\frac{1}{2}x\cos^2(x)+\frac{1}{2}\left(\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)\right)$$

input
```
Int[x^2*Cos[x]^2*Cot[x]^2,x]
```

output
```
-1/2*x^3 - (x*Cos[x]^2)/2 - x^2*Cot[x] - 2*((I/2)*x^2 + (2*I)*((I/2)*x*Log
[1 - E^((2*I)*x)] + PolyLog[2, E^((2*I)*x)]/4)) - (x^2*Cos[x]*Sin[x])/2 +
(x/2 + (Cos[x]*Sin[x])/2)/2
```

### 3.203.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

## 3.203. $\int x^2 \cos^2(x) \cot^2(x) \, dx$

### 3.203.4 Maple [A] (verified)

Time = 2.60 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.35

| method | result |
|---|---|
| risch | $-\frac{x^3}{2}+\frac{i(2x^2+2ix-1)\mathrm{e}^{2ix}}{16}-\frac{i(2x^2-2ix-1)\mathrm{e}^{-2ix}}{16}-\frac{2ix^2}{\mathrm{e}^{2ix}-1}+2x\ln\left(\mathrm{e}^{ix}+1\right)+2x\ln\left(1-\mathrm{e}^{ix}\right)-2ix^2-2i\,\mathrm{p}$ |

input
```
int(x^2*cos(x)^2*cot(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^3+1/16*I*(2*I*x+2*x^2-1)*exp(2*I*x)-1/16*I*(-2*I*x+2*x^2-1)*exp(-2*
I*x)-2*I*x^2/(exp(2*I*x)-1)+2*x*ln(exp(I*x)+1)+2*x*ln(1-exp(I*x))-2*I*x^2-
2*I*polylog(2,-exp(I*x))-2*I*polylog(2,exp(I*x))
```

### 3.203.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(62) = 124.

Time = 0.26 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.95

$$\int x^2\cos^2(x)\cot^2(x)\,dx$$
$$=\frac{(2\,x^2-1)\cos(x)^3+4\,x\log(\cos(x)+i\,\sin(x)+1)\sin(x)+4\,x\log(\cos(x)-i\,\sin(x)+1)\sin(x)+4\,x\,\mathrm{l}}{}$$

input
```
integrate(x^2*cos(x)^2*cot(x)^2,x, algorithm="fricas")
```

output
```
1/4*((2*x^2 - 1)*cos(x)^3 + 4*x*log(cos(x) + I*sin(x) + 1)*sin(x) + 4*x*lo
g(cos(x) - I*sin(x) + 1)*sin(x) + 4*x*log(-cos(x) + I*sin(x) + 1)*sin(x) +
 4*x*log(-cos(x) - I*sin(x) + 1)*sin(x) - (6*x^2 - 1)*cos(x) - (2*x^3 + 2*
x*cos(x)^2 - x)*sin(x) - 4*I*dilog(cos(x) + I*sin(x))*sin(x) + 4*I*dilog(c
os(x) - I*sin(x))*sin(x) + 4*I*dilog(-cos(x) + I*sin(x))*sin(x) - 4*I*dilo
g(-cos(x) - I*sin(x))*sin(x))/sin(x)
```

### 3.203.6 Sympy [F]

$$\int x^2 \cos^2(x) \cot^2(x)\, dx = \int x^2 \cos^2{(x)} \cot^2{(x)}\, dx$$

input
```
integrate(x**2*cos(x)**2*cot(x)**2,x)
```

output
```
Integral(x**2*cos(x)**2*cot(x)**2, x)
```

### 3.203.7 Maxima [F(-2)]

Exception generated.

$$\int x^2 \cos^2(x) \cot^2(x)\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*cos(x)^2*cot(x)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

### 3.203.8 Giac [F]

$$\int x^2 \cos^2(x) \cot^2(x)\, dx = \int x^2 \cos{(x)}^2 \cot{(x)}^2\, dx$$

input
```
integrate(x^2*cos(x)^2*cot(x)^2,x, algorithm="giac")
```

output
```
integrate(x^2*cos(x)^2*cot(x)^2, x)
```

### 3.203.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cos^2(x) \cot^2(x) \, dx = \int x^2 \cos(x)^2 \cot(x)^2 \, dx$$

input
```
int(x^2*cos(x)^2*cot(x)^2,x)
```

output
```
int(x^2*cos(x)^2*cot(x)^2, x)
```

# 3.204 $\int x\cos^2(x)\cot^2(x)\,dx$

3.204.1 Optimal result . . . 1626
3.204.2 Mathematica [A] (verified) . . . 1626
3.204.3 Rubi [A] (verified) . . . 1627
3.204.4 Maple [C] (verified) . . . 1629
3.204.5 Fricas [A] (verification not implemented) . . . 1629
3.204.6 Sympy [F] . . . 1630
3.204.7 Maxima [F(-2)] . . . 1630
3.204.8 Giac [B] (verification not implemented) . . . 1630
3.204.9 Mupad [B] (verification not implemented) . . . 1631

## 3.204.1 Optimal result

Integrand size = 10, antiderivative size = 33

$$\int x\cos^2(x)\cot^2(x)\,dx=-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)+\log(\sin(x))-\frac{1}{2}x\cos(x)\sin(x)$$

output
```
-3/4*x^2-1/4*cos(x)^2-x*cot(x)+ln(sin(x))-1/2*x*cos(x)*sin(x)
```

## 3.204.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x\cos^2(x)\cot^2(x)\,dx=-\frac{3x^2}{4}-\frac{1}{8}\cos(2x)-x\cot(x)+\log(\sin(x))-\frac{1}{4}x\sin(2x)$$

input
```
Integrate[x*Cos[x]^2*Cot[x]^2,x]
```

output
```
(-3*x^2)/4 - Cos[2*x]/8 - x*Cot[x] + Log[Sin[x]] - (x*Sin[2*x])/4
```

### 3.204.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {4908, 3042, 3791, 15, 4203, 15, 25, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos^2(x)\cot^2(x)\,dx$$

$$\Big\downarrow \text{4908}$$

$$\int x\cot^2(x)dx-\int x\cos^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\tan\left(x+\frac{\pi}{2}\right)^2dx-\int x\sin\left(x+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3791}$$

$$-\frac{\int xdx}{2}+\int x\tan\left(x+\frac{\pi}{2}\right)^2dx-\frac{1}{4}\cos^2(x)-\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \text{15}$$

$$\int x\tan\left(x+\frac{\pi}{2}\right)^2dx-\frac{x^2}{4}-\frac{\cos^2(x)}{4}-\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \text{4203}$$

$$-\int xdx-\int -\cot(x)dx-\frac{x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)-\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \text{15}$$

$$-\int -\cot(x)dx-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)-\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \text{25}$$

$$\int \cot(x)dx-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)-\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \text{3042}$$

$$\int -\tan\left(x+\frac{\pi}{2}\right)dx-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)-\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \text{25}$$

$$-\int \tan\left(x+\frac{\pi}{2}\right)dx-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)-\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \ 3956$$

$$-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)+\log(\sin(x))-\frac{1}{2}x\sin(x)\cos(x)$$

input
```
Int[x*Cos[x]^2*Cot[x]^2,x]
```

output
```
(-3*x^2)/4 - Cos[x]^2/4 - x*Cot[x] + Log[Sin[x]] - (x*Cos[x]*Sin[x])/2
```

### 3.204.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

### 3.204.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.53 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.82

| method | result | size |
|---|---|---|
| risch | $-\frac{3x^2}{4}+\frac{i(2x+i)\mathrm{e}^{2ix}}{16}-\frac{i(-i+2x)\mathrm{e}^{-2ix}}{16}-2ix-\frac{2ix}{\mathrm{e}^{2ix}-1}+\ln\left(\mathrm{e}^{2ix}-1\right)$ | 60 |

input
```
int(x*cos(x)^2*cot(x)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/4*x^2+1/16*I*(2*x+I)*exp(2*I*x)-1/16*I*(-I+2*x)*exp(-2*I*x)-2*I*x-2*I*x
/(exp(2*I*x)-1)+ln(exp(2*I*x)-1)
```

### 3.204.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36

$$\int x\cos^2(x)\cot^2(x)\,dx = \frac{4\,x\cos(x)^3-12\,x\cos(x)-\left(6\,x^2+2\,\cos(x)^2-1\right)\sin(x)+8\,\log\left(\frac{1}{2}\,\sin(x)\right)\sin(x)}{8\,\sin(x)}$$

input
```
integrate(x*cos(x)^2*cot(x)^2,x, algorithm="fricas")
```

output
```
1/8*(4*x*cos(x)^3 - 12*x*cos(x) - (6*x^2 + 2*cos(x)^2 - 1)*sin(x) + 8*log(
1/2*sin(x))*sin(x))/sin(x)
```

### 3.204.6 Sympy [F]

$$\int x\cos^2(x)\cot^2(x)\,dx = \int x\cos^2(x)\cot^2(x)\,dx$$

input
```
integrate(x*cos(x)**2*cot(x)**2,x)
```

output
```
Integral(x*cos(x)**2*cot(x)**2, x)
```

### 3.204.7 Maxima [F(-2)]

Exception generated.

$$\int x\cos^2(x)\cot^2(x)\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*cos(x)^2*cot(x)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

### 3.204.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(27) = 54.

Time = 0.30 (sec) , antiderivative size = 206, normalized size of antiderivative = 6.24

$$\int x\cos^2(x)\cot^2(x)\,dx =$$

$$-\frac{6\,x^2\tan\left(\frac{1}{2}\,x\right)^5 - 4\,x\tan\left(\frac{1}{2}\,x\right)^6 - 4\log\left(\frac{16\tan\left(\frac{1}{2}\,x\right)^2}{\tan\left(\frac{1}{2}\,x\right)^4+2\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^5 + 12\,x^2\tan\left(\frac{1}{2}\,x\right)^3 - 12\,x\tan}{}$$

input
```
integrate(x*cos(x)^2*cot(x)^2,x, algorithm="giac")
```

output
```
-1/8*(6*x^2*tan(1/2*x)^5 - 4*x*tan(1/2*x)^6 - 4*log(16*tan(1/2*x)^2/(tan(1
/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^5 + 12*x^2*tan(1/2*x)^3 - 12*x*t
an(1/2*x)^4 + tan(1/2*x)^5 - 8*log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1
/2*x)^2 + 1))*tan(1/2*x)^3 + 6*x^2*tan(1/2*x) + 12*x*tan(1/2*x)^2 - 6*tan(
1/2*x)^3 - 4*log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(
1/2*x) + 4*x + tan(1/2*x))/(tan(1/2*x)^5 + 2*tan(1/2*x)^3 + tan(1/2*x))
```

## 3.204.9 Mupad [B] (verification not implemented)

Time = 24.45 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int x\cos^2(x)\cot^2(x)\,dx = \ln\left(\mathrm{e}^{x\,2\mathrm{i}}-1\right)-\mathrm{e}^{-x\,2\mathrm{i}}\left(\frac{1}{16}+\frac{x\,1\mathrm{i}}{8}\right) + \mathrm{e}^{x\,2\mathrm{i}}\left(-\frac{1}{16}+\frac{x\,1\mathrm{i}}{8}\right)-\frac{3\,x^2}{4}-x\,2\mathrm{i}-\frac{x\,2\mathrm{i}}{\mathrm{e}^{x\,2\mathrm{i}}-1}$$

input
```
int(x*cos(x)^2*cot(x)^2,x)
```

output
```
log(exp(x*2i) - 1) - x*2i - exp(-x*2i)*((x*1i)/8 + 1/16) + exp(x*2i)*((x*1
i)/8 - 1/16) - (x*2i)/(exp(x*2i) - 1) - (3*x^2)/4
```

# 3.205 $\int x^3 \cos^2(x) \cot^3(x)\,dx$

3.205.1 Optimal result . . . . . . . . . . 1632
3.205.2 Mathematica [A] (verified) . . . . . . . . . . 1632
3.205.3 Rubi [F] . . . . . . . . . . 1633
3.205.4 Maple [A] (verified) . . . . . . . . . . 1640
3.205.5 Fricas [B] (verification not implemented) . . . . . . . . . . 1640
3.205.6 Sympy [F] . . . . . . . . . . 1641
3.205.7 Maxima [B] (verification not implemented) . . . . . . . . . . 1641
3.205.8 Giac [F] . . . . . . . . . . 1642
3.205.9 Mupad [F(-1)] . . . . . . . . . . 1643

## 3.205.1 Optimal result

Integrand size = 12, antiderivative size = 180

$$\int x^3 \cos^2(x) \cot^3(x)\,dx = \frac{3x}{8}-\frac{3ix^2}{2}-\frac{3x^3}{4}+\frac{ix^4}{2}-\frac{3}{2}x^2\cot(x)-\frac{1}{2}x^3\cot^2(x)+3x\log\left(1-e^{2ix}\right) - 2x^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\operatorname{PolyLog}\left(2,e^{2ix}\right)+3ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right) - 3x\operatorname{PolyLog}\left(3,e^{2ix}\right)-\frac{3}{2}i\operatorname{PolyLog}\left(4,e^{2ix}\right)-\frac{3}{8}\cos(x)\sin(x) + \frac{3}{4}x^2\cos(x)\sin(x)-\frac{3}{4}x\sin^2(x)+\frac{1}{2}x^3\sin^2(x)$$

output
```
3/8*x-3/2*I*polylog(2,exp(2*I*x))-3/4*x^3-3/2*I*x^2-3/2*x^2*cot(x)-1/2*x^3
*cot(x)^2+3*x*ln(1-exp(2*I*x))-2*x^3*ln(1-exp(2*I*x))-3/2*I*polylog(4,exp(
2*I*x))+3*I*x^2*polylog(2,exp(2*I*x))-3*x*polylog(3,exp(2*I*x))+1/2*I*x^4-
3/8*cos(x)*sin(x)+3/4*x^2*cos(x)*sin(x)-3/4*x*sin(x)^2+1/2*x^3*sin(x)^2
```

## 3.205.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.88

$$\int x^3 \cos^2(x) \cot^3(x)\,dx = \frac{1}{32}\left(i\pi^4-48ix^2-16ix^4+12x\cos(2x)-8x^3\cos(2x)-48x^2\cot(x) - 16x^3\csc^2(x)-64x^3\log\left(1-e^{-2ix}\right)+96x\log\left(1-e^{2ix}\right) - 96ix^2\operatorname{PolyLog}\left(2,e^{-2ix}\right)-48i\operatorname{PolyLog}\left(2,e^{2ix}\right) - 96x\operatorname{PolyLog}\left(3,e^{-2ix}\right)+48i\operatorname{PolyLog}\left(4,e^{-2ix}\right)-6\sin(2x) + 12x^2\sin(2x)\right)$$

input
```
Integrate[x^3*Cos[x]^2*Cot[x]^3,x]
```

output
```
(I*Pi^4 - (48*I)*x^2 - (16*I)*x^4 + 12*x*Cos[2*x] - 8*x^3*Cos[2*x] - 48*x^
2*Cot[x] - 16*x^3*Csc[x]^2 - 64*x^3*Log[1 - E^((-2*I)*x)] + 96*x*Log[1 - E
^((2*I)*x)] - (96*I)*x^2*PolyLog[2, E^((-2*I)*x)] - (48*I)*PolyLog[2, E^((
2*I)*x)] - 96*x*PolyLog[3, E^((-2*I)*x)] + (48*I)*PolyLog[4, E^((-2*I)*x)]
 - 6*Sin[2*x] + 12*x^2*Sin[2*x])/32
```

### 3.205.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \cos^2(x) \cot^3(x) \, dx$$

$$\Big\downarrow \text{4908}$$

$$\int x^3 \cot^3(x)dx - \int x^3 \cos^2(x) \cot(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x^3 \tan\left(x+\frac{\pi}{2}\right)^3 dx - \int x^3 \cos^2(x) \cot(x)dx$$

$$\Big\downarrow \text{25}$$

$$-\int x^3 \tan\left(x+\frac{\pi}{2}\right)^3 dx - \int x^3 \cos^2(x) \cot(x)dx$$

$$\Big\downarrow \text{4203}$$

$$\int -x^3 \cot(x)dx - \int x^3 \cos^2(x) \cot(x)dx + \frac{3}{2}\int x^2 \cot^2(x)dx - \frac{1}{2}x^3 \cot^2(x)$$

$$\Big\downarrow \text{25}$$

$$-\int x^3 \cot(x)dx - \int x^3 \cos^2(x) \cot(x)dx + \frac{3}{2}\int x^2 \cot^2(x)dx - \frac{1}{2}x^3 \cot^2(x)$$

$$\Big\downarrow \text{3042}$$

$$-\int -x^3 \tan\left(x+\frac{\pi}{2}\right) dx - \int x^3 \cos^2(x) \cot(x)dx + \frac{3}{2}\int x^2 \tan\left(x+\frac{\pi}{2}\right)^2 dx - \frac{1}{2}x^3 \cot^2(x)$$

$$\Big\downarrow \text{25}$$

$$\int x^3 \tan\left(x+\frac{\pi}{2}\right) dx - \int x^3 \cos^2(x) \cot(x)dx + \frac{3}{2}\int x^2 \tan\left(x+\frac{\pi}{2}\right)^2 dx - \frac{1}{2}x^3 \cot^2(x)$$

$$\Big\downarrow \text{4200}$$

$$-2i\int -\frac{e^{2ix}x^3}{1-e^{2ix}}dx-\int x^3\cos^2(x)\cot(x)dx+\frac{3}{2}\int x^2\tan\left(x+\frac{\pi}{2}\right)^2dx+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\Big\downarrow \text{25}$$

$$2i\int \frac{e^{2ix}x^3}{1-e^{2ix}}dx-\int x^3\cos^2(x)\cot(x)dx+\frac{3}{2}\int x^2\tan\left(x+\frac{\pi}{2}\right)^2dx+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\Big\downarrow \text{2620}$$

$$-\int x^3\cos^2(x)\cot(x)dx+\frac{3}{2}\int x^2\tan\left(x+\frac{\pi}{2}\right)^2dx+ 2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\int x^2\log\left(1-e^{2ix}\right)dx\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\Big\downarrow \text{3011}$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)- \int x^3\cos^2(x)\cot(x)dx+\frac{3}{2}\int x^2\tan\left(x+\frac{\pi}{2}\right)^2dx+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\Big\downarrow \text{4203}$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)- \int x^3\cos^2(x)\cot(x)dx+\frac{3}{2}\left(-\int x^2dx-2\int -x\cot(x)dx+x^2(-\cot(x))\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\Big\downarrow \text{15}$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)- \int x^3\cos^2(x)\cot(x)dx+\frac{3}{2}\left(-2\int -x\cot(x)dx-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\Big\downarrow \text{25}$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)- \int x^3\cos^2(x)\cot(x)dx+\frac{3}{2}\left(2\int x\cot(x)dx-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\Big\downarrow \text{3042}$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)- \int x^3\cos^2(x)\cot(x)dx+\frac{3}{2}\left(2\int -x\tan\left(x+\frac{\pi}{2}\right)dx-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

---

3.205. $\int x^3\cos^2(x)\cot^3(x)\,dx$

$$\Big\downarrow \text{25}$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)-\int x^3\cos^2(x)\cot(x)dx+\frac{3}{2}\left(-2\int x\tan\left(x+\frac{\pi}{2}\right)dx-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\Big\downarrow \text{4200}$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)-\int x^3\cos^2(x)\cot(x)dx+\frac{3}{2}\left(-2\left(\frac{ix^2}{2}-2i\int -\frac{e^{2ix}x}{1-e^{2ix}}dx\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\Big\downarrow \text{25}$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)-\int x^3\cos^2(x)\cot(x)dx+\frac{3}{2}\left(-2\left(2i\int \frac{e^{2ix}x}{1-e^{2ix}}dx+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\Big\downarrow \text{2620}$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)-\int x^3\cos^2(x)\cot(x)dx+\frac{3}{2}\left(-2\left(2i\left(\frac{1}{2}ix\log\left(1-e^{2ix}\right)-\frac{1}{2}i\int\log\left(1-e^{2ix}\right)dx\right)+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\Big\downarrow \text{2715}$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)-\int x^3\cos^2(x)\cot(x)dx+\frac{3}{2}\left(-2\left(2i\left(\frac{1}{2}ix\log\left(1-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\log\left(1-e^{2ix}\right)de^{2ix}\right)+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\Big\downarrow \text{2838}$$

3.205. $\int x^3\cos^2(x)\cot^3(x)\,dx$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)-\int x^3\cos^2(x)\cot(x)dx+\frac{3}{2}\left(-2\left(2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\Big\downarrow \text{4908}$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)-\int x^3\cot(x)dx+\int x^3\cos(x)\sin(x)dx+\frac{3}{2}\left(-2\left(2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\Big\downarrow \text{3042}$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)-\int -x^3\tan\left(x+\frac{\pi}{2}\right)dx+\int x^3\cos(x)\sin(x)dx+\frac{3}{2}\left(-2\left(2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\Big\downarrow \text{25}$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)+\int x^3\tan\left(x+\frac{\pi}{2}\right)dx+\int x^3\cos(x)\sin(x)dx+\frac{3}{2}\left(-2\left(2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}-\frac{1}{2}x^3\cot^2(x)$$

$$\Big\downarrow \text{3924}$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)+\int x^3\tan\left(x+\frac{\pi}{2}\right)dx-\frac{3}{2}\int x^2\sin^2(x)dx+\frac{3}{2}\left(-2\left(2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}+\frac{1}{2}x^3\sin^2(x)-\frac{1}{2}x^3\cot^2(x)$$

3.205. $\int x^3\cos^2(x)\cot^3(x)\,dx$

$$\Big\downarrow \text{3042}$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\,\text{PolyLog}\left(2,e^{2ix}\right)-i\int x\,\text{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)+\int x^3\tan\left(x+\frac{\pi}{2}\right)dx-\frac{3}{2}\int x^2\sin(x)^2dx+\frac{3}{2}\left(-2\left(2i\left(\frac{1}{4}\,\text{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}+\frac{1}{2}x^3\sin^2(x)-\frac{1}{2}x^3\cot^2(x)$$

$$\Big\downarrow \text{3792}$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\,\text{PolyLog}\left(2,e^{2ix}\right)-i\int x\,\text{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)+\int x^3\tan\left(x+\frac{\pi}{2}\right)dx-\frac{3}{2}\left(\frac{\int x^2dx}{2}-\frac{1}{2}\int\sin^2(x)dx-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{1}{2}x\sin^2(x)\right)+\frac{3}{2}\left(-2\left(2i\left(\frac{1}{4}\,\text{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}+\frac{1}{2}x^3\sin^2(x)-\frac{1}{2}x^3\cot^2(x)$$

$$\Big\downarrow \text{15}$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\,\text{PolyLog}\left(2,e^{2ix}\right)-i\int x\,\text{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)+\int x^3\tan\left(x+\frac{\pi}{2}\right)dx-\frac{3}{2}\left(-\frac{1}{2}\int\sin^2(x)dx+\frac{x^3}{6}-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{1}{2}x\sin^2(x)\right)+\frac{3}{2}\left(-2\left(2i\left(\frac{1}{4}\,\text{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}+\frac{1}{2}x^3\sin^2(x)-\frac{1}{2}x^3\cot^2(x)$$

$$\Big\downarrow \text{3042}$$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\,\text{PolyLog}\left(2,e^{2ix}\right)-i\int x\,\text{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)+\int x^3\tan\left(x+\frac{\pi}{2}\right)dx-\frac{3}{2}\left(-\frac{1}{2}\int\sin(x)^2dx+\frac{x^3}{6}-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{1}{2}x\sin^2(x)\right)+\frac{3}{2}\left(-2\left(2i\left(\frac{1}{4}\,\text{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}+\frac{1}{2}x^3\sin^2(x)-\frac{1}{2}x^3\cot^2(x)$$

$$\Big\downarrow \text{3115}$$

3.205. $\int x^3\cos^2(x)\cot^3(x)\,dx$

$$2i\left(\frac{1}{2}ix^3\log\left(1-e^{2ix}\right)-\frac{3}{2}i\left(\frac{1}{2}ix^2\operatorname{PolyLog}\left(2,e^{2ix}\right)-i\int x\operatorname{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)+ \int x^3\tan\left(x+\frac{\pi}{2}\right)dx-\frac{3}{2}\left(\frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x)-\frac{\int 1dx}{2}\right)+\frac{x^3}{6}-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{1}{2}x\sin^2(x)\right)+ \frac{3}{2}\left(-2\left(2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}\right)-\frac{x^3}{3}-x^2\cot(x)\right)+\frac{ix^4}{4}+ \frac{1}{2}x^3\sin^2(x)-\frac{1}{2}x^3\cot^2(x)$$

input
```
Int[x^3*Cos[x]^2*Cot[x]^3,x]
```

output
```
$Aborted
```

### 3.205.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

## 3.205. $\int x^3 \cos^2(x) \cot^3(x) \, dx$

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

### 3.205.4 Maple [A] (verified)

Time = 4.58 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.33

| method | result |
|---|---|
| risch | $-3i \operatorname{polylog}\left(2, -\mathrm{e}^{ix}\right) - \frac{\left(4x^3+6ix^2-6x-3i\right)\mathrm{e}^{2ix}}{32} - \frac{\left(4x^3-6ix^2-6x+3i\right)\mathrm{e}^{-2ix}}{32} + \frac{x^2\left(2\,\mathrm{e}^{2ix}x-3i\mathrm{e}^{2ix}+3i\right)}{\left(\mathrm{e}^{2ix}-1\right)^2} - 2x^3 \ln\left(\mathrm{e}\right.$ |

input
```
int(x^3*cos(x)^2*cot(x)^3,x,method=_RETURNVERBOSE)
```

output
```
-3*I*polylog(2,-exp(I*x))-1/32*(6*I*x^2+4*x^3-3*I-6*x)*exp(2*I*x)-1/32*(-6
*I*x^2+4*x^3+3*I-6*x)*exp(-2*I*x)+x^2*(2*exp(2*I*x)*x-3*I*exp(2*I*x)+3*I)/
(exp(2*I*x)-1)^2-2*x^3*ln(exp(I*x)+1)-2*x^3*ln(1-exp(I*x))-12*I*polylog(4,
-exp(I*x))+3*x*ln(exp(I*x)+1)+3*x*ln(1-exp(I*x))-12*x*polylog(3,-exp(I*x))
-12*x*polylog(3,exp(I*x))+6*I*polylog(2,-exp(I*x))*x^2-3*I*x^2+1/2*I*x^4+6
*I*polylog(2,exp(I*x))*x^2-12*I*polylog(4,exp(I*x))-3*I*polylog(2,exp(I*x)
)
```

### 3.205.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 508 vs. 2(126) = 252.

Time = 0.28 (sec) , antiderivative size = 508, normalized size of antiderivative = 2.82

$$\int x^3 \cos^2(x) \cot^3(x)\,dx =$$

$$-\frac{2\left(2\,x^3-3\,x\right)\cos\left(x\right)^4-2\,x^3-3\left(2\,x^3-3\,x\right)\cos\left(x\right)^2+12\left(\left(-2i\,x^2+i\right)\cos\left(x\right)^2+2i\,x^2-i\right)\mathrm{Li}_2(\cos\left(x\right)}{}$$

input
```
integrate(x^3*cos(x)^2*cot(x)^3,x, algorithm="fricas")
```

output
```
-1/8*(2*(2*x^3 - 3*x)*cos(x)^4 - 2*x^3 - 3*(2*x^3 - 3*x)*cos(x)^2 + 12*((-
2*I*x^2 + I)*cos(x)^2 + 2*I*x^2 - I)*dilog(cos(x) + I*sin(x)) + 12*((2*I*x
^2 - I)*cos(x)^2 - 2*I*x^2 + I)*dilog(cos(x) - I*sin(x)) + 12*((2*I*x^2 -
I)*cos(x)^2 - 2*I*x^2 + I)*dilog(-cos(x) + I*sin(x)) + 12*((-2*I*x^2 + I)*
cos(x)^2 + 2*I*x^2 - I)*dilog(-cos(x) - I*sin(x)) - 4*(2*x^3 - (2*x^3 - 3*
x)*cos(x)^2 - 3*x)*log(cos(x) + I*sin(x) + 1) - 4*(2*x^3 - (2*x^3 - 3*x)*c
os(x)^2 - 3*x)*log(cos(x) - I*sin(x) + 1) - 4*(2*x^3 - (2*x^3 - 3*x)*cos(x
)^2 - 3*x)*log(-cos(x) + I*sin(x) + 1) - 4*(2*x^3 - (2*x^3 - 3*x)*cos(x)^2
 - 3*x)*log(-cos(x) - I*sin(x) + 1) + 48*(I*cos(x)^2 - I)*polylog(4, cos(x
) + I*sin(x)) + 48*(-I*cos(x)^2 + I)*polylog(4, cos(x) - I*sin(x)) + 48*(-
I*cos(x)^2 + I)*polylog(4, -cos(x) + I*sin(x)) + 48*(I*cos(x)^2 - I)*polyl
og(4, -cos(x) - I*sin(x)) + 48*(x*cos(x)^2 - x)*polylog(3, cos(x) + I*sin(
x)) + 48*(x*cos(x)^2 - x)*polylog(3, cos(x) - I*sin(x)) + 48*(x*cos(x)^2 -
 x)*polylog(3, -cos(x) + I*sin(x)) + 48*(x*cos(x)^2 - x)*polylog(3, -cos(x
) - I*sin(x)) - 3*((2*x^2 - 1)*cos(x)^3 + (2*x^2 + 1)*cos(x))*sin(x) - 3*x
)/(cos(x)^2 - 1)
```

### 3.205.6 Sympy [F]

$$\int x^3 \cos^2(x) \cot^3(x) \, dx = \int x^3 \cos^2(x) \cot^3(x) \, dx$$

input
```
integrate(x**3*cos(x)**2*cot(x)**3,x)
```

output
```
Integral(x**3*cos(x)**2*cot(x)**3, x)
```

### 3.205.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3740 vs. 2(126) = 252.

Time = 0.64 (sec) , antiderivative size = 3740, normalized size of antiderivative = 20.78

$$\int x^3 \cos^2(x) \cot^3(x) \, dx = \text{Too large to display}$$

input
```
integrate(x^3*cos(x)^2*cot(x)^3,x, algorithm="maxima")
```

output
```
-1/32*(4*x^3 + (4*x^3 + 6*I*x^2 - 6*x - 3*I)*cos(6*x)^2 + 4*(8*I*x^4 + 4*x
^3 - 42*I*x^2 - 6*x - 3*I)*cos(4*x)^2 + 4*(8*I*x^4 - 14*x^3 - 24*I*x^2 - 3
*x)*cos(2*x)^2 - (4*x^3 + 6*I*x^2 - 6*x - 3*I)*sin(6*x)^2 + 4*(-8*I*x^4 -
4*x^3 + 42*I*x^2 + 6*x + 3*I)*sin(4*x)^2 + 4*(-8*I*x^4 + 14*x^3 + 24*I*x^2
 + 3*x)*sin(2*x)^2 - 6*I*x^2 + 32*(2*(-2*I*x^3 + 3*I*x)*cos(4*x)^2 + 2*(-2
*I*x^3 + 3*I*x)*cos(2*x)^2 + 2*(2*I*x^3 - 3*I*x)*sin(4*x)^2 + 2*(2*I*x^3 -
 3*I*x)*sin(2*x)^2 + (2*I*x^3 + (2*I*x^3 - 3*I*x)*cos(4*x) + 2*(-2*I*x^3 +
 3*I*x)*cos(2*x) - (2*x^3 - 3*x)*sin(4*x) + 2*(2*x^3 - 3*x)*sin(2*x) - 3*I
*x)*cos(6*x) + (-4*I*x^3 + 5*(2*I*x^3 - 3*I*x)*cos(2*x) - 5*(2*x^3 - 3*x)*
sin(2*x) + 6*I*x)*cos(4*x) + (2*I*x^3 - 3*I*x)*cos(2*x) - (2*x^3 + (2*x^3
- 3*x)*cos(4*x) - 2*(2*x^3 - 3*x)*cos(2*x) - (-2*I*x^3 + 3*I*x)*sin(4*x) -
 2*(2*I*x^3 - 3*I*x)*sin(2*x) - 3*x)*sin(6*x) + (4*x^3 + 4*(2*x^3 - 3*x)*c
os(4*x) - 5*(2*x^3 - 3*x)*cos(2*x) + 5*(-2*I*x^3 + 3*I*x)*sin(2*x) - 6*x)*
sin(4*x) - (2*x^3 - 4*(2*x^3 - 3*x)*cos(2*x) - 3*x)*sin(2*x))*arctan2(sin(
x), cos(x) + 1) + 32*(2*(2*I*x^3 - 3*I*x)*cos(4*x)^2 + 2*(2*I*x^3 - 3*I*x)
*cos(2*x)^2 + 2*(-2*I*x^3 + 3*I*x)*sin(4*x)^2 + 2*(-2*I*x^3 + 3*I*x)*sin(2
*x)^2 + (-2*I*x^3 + (-2*I*x^3 + 3*I*x)*cos(4*x) + 2*(2*I*x^3 - 3*I*x)*cos(
2*x) + (2*x^3 - 3*x)*sin(4*x) - 2*(2*x^3 - 3*x)*sin(2*x) + 3*I*x)*cos(6*x)
 + (4*I*x^3 + 5*(-2*I*x^3 + 3*I*x)*cos(2*x) + 5*(2*x^3 - 3*x)*sin(2*x) - 6
*I*x)*cos(4*x) + (-2*I*x^3 + 3*I*x)*cos(2*x) + (2*x^3 + (2*x^3 - 3*x)*c...
```

### 3.205.8 Giac [F]

$$\int x^3 \cos^2(x) \cot^3(x)\,dx = \int x^3 \cos(x)^2 \cot(x)^3\,dx$$

input
```
integrate(x^3*cos(x)^2*cot(x)^3,x, algorithm="giac")
```

output
```
integrate(x^3*cos(x)^2*cot(x)^3, x)
```

### 3.205.9 Mupad [F(-1)]

Timed out.

$$\int x^3\cos^2(x)\cot^3(x)\,dx = \int x^3\cos(x)^2\cot(x)^3\,dx$$

input
```
int(x^3*cos(x)^2*cot(x)^3,x)
```

output
```
int(x^3*cos(x)^2*cot(x)^3, x)
```

# 3.206 $\int x^2 \cos^2(x) \cot^3(x) \, dx$

3.206.1 Optimal result . . . . . 1644
3.206.2 Mathematica [A] (verified) . . . . . 1644
3.206.3 Rubi [F] . . . . . 1645
3.206.4 Maple [A] (verified) . . . . . 1651
3.206.5 Fricas [B] (verification not implemented) . . . . . 1651
3.206.6 Sympy [F] . . . . . 1652
3.206.7 Maxima [B] (verification not implemented) . . . . . 1652
3.206.8 Giac [F] . . . . . 1653
3.206.9 Mupad [F(-1)] . . . . . 1654

## 3.206.1 Optimal result

Integrand size = 12, antiderivative size = 106

$$\int x^2 \cos^2(x) \cot^3(x) \, dx = -\frac{3x^2}{4} + \frac{2ix^3}{3} - x\cot(x) - \frac{1}{2}x^2\cot^2(x) - 2x^2\log\left(1-e^{2ix}\right) + \log(\sin(x)) + 2ix\,\text{PolyLog}\left(2,e^{2ix}\right) - \text{PolyLog}\left(3,e^{2ix}\right) + \frac{1}{2}x\cos(x)\sin(x) - \frac{\sin^2(x)}{4} + \frac{1}{2}x^2\sin^2(x)$$

output
```
-3/4*x^2+2/3*I*x^3-x*cot(x)-1/2*x^2*cot(x)^2-2*x^2*ln(1-exp(2*I*x))+ln(sin
(x))+2*I*x*polylog(2,exp(2*I*x))-polylog(3,exp(2*I*x))+1/2*x*cos(x)*sin(x)
-1/4*sin(x)^2+1/2*x^2*sin(x)^2
```

## 3.206.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.05

$$\int x^2 \cos^2(x) \cot^3(x) \, dx = \frac{i\pi^3}{12} - \frac{2ix^3}{3} + \frac{1}{8}\cos(2x) - \frac{1}{4}x^2\cos(2x) - x\cot(x) - \frac{1}{2}x^2\csc^2(x) - 2x^2\log\left(1-e^{-2ix}\right) + \log(\cos(x)) + \log(\tan(x)) - 2ix\,\text{PolyLog}\left(2,e^{-2ix}\right) - \text{PolyLog}\left(3,e^{-2ix}\right) + \frac{1}{4}x\sin(2x)$$

input
```
Integrate[x^2*Cos[x]^2*Cot[x]^3,x]
```

output
```
(I/12)*Pi^3 - ((2*I)/3)*x^3 + Cos[2*x]/8 - (x^2*Cos[2*x])/4 - x*Cot[x] - (
x^2*Csc[x]^2)/2 - 2*x^2*Log[1 - E^((-2*I)*x)] + Log[Cos[x]] + Log[Tan[x]]
- (2*I)*x*PolyLog[2, E^((-2*I)*x)] - PolyLog[3, E^((-2*I)*x)] + (x*Sin[2*x
])/4
```

## 3.206.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \cos^2(x) \cot^3(x) \, dx$$

$$\Big\downarrow \; 4908$$

$$\int x^2 \cot^3(x)dx - \int x^2 \cos^2(x) \cot(x)dx$$

$$\Big\downarrow \; 3042$$

$$\int -x^2 \tan\left(x+\frac{\pi}{2}\right)^3 dx - \int x^2 \cos^2(x) \cot(x)dx$$

$$\Big\downarrow \; 25$$

$$-\int x^2 \tan\left(x+\frac{\pi}{2}\right)^3 dx - \int x^2 \cos^2(x) \cot(x)dx$$

$$\Big\downarrow \; 4203$$

$$\int -x^2 \cot(x)dx - \int x^2 \cos^2(x) \cot(x)dx + \int x \cot^2(x)dx - \frac{1}{2}x^2 \cot^2(x)$$

$$\Big\downarrow \; 25$$

$$-\int x^2 \cot(x)dx - \int x^2 \cos^2(x) \cot(x)dx + \int x \cot^2(x)dx - \frac{1}{2}x^2 \cot^2(x)$$

$$\Big\downarrow \; 3042$$

$$-\int -x^2 \tan\left(x+\frac{\pi}{2}\right) dx - \int x^2 \cos^2(x) \cot(x)dx + \int x \tan\left(x+\frac{\pi}{2}\right)^2 dx - \frac{1}{2}x^2 \cot^2(x)$$

$$\Big\downarrow \; 25$$

$$\int x^2 \tan\left(x+\frac{\pi}{2}\right) dx - \int x^2 \cos^2(x) \cot(x)dx + \int x \tan\left(x+\frac{\pi}{2}\right)^2 dx - \frac{1}{2}x^2 \cot^2(x)$$

$$\Big\downarrow \; 4200$$

$$-2i \int -\frac{e^{2ix}x^2}{1-e^{2ix}}dx - \int x^2 \cos^2(x) \cot(x)dx + \int x \tan\left(x+\frac{\pi}{2}\right)^2 dx + \frac{ix^3}{3} - \frac{1}{2}x^2 \cot^2(x)$$

$$\Big\downarrow \textcolor{blue}{25}$$

$$2i\int \frac{e^{2ix}x^2}{1-e^{2ix}}dx - \int x^2\cos^2(x)\cot(x)dx + \int x\tan\left(x+\frac{\pi}{2}\right)^2dx + \frac{ix^3}{3} - \frac{1}{2}x^2\cot^2(x)$$

$$\Big\downarrow \textcolor{blue}{2620}$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right) - i\int x\log\left(1-e^{2ix}\right)dx\right) - \int x^2\cos^2(x)\cot(x)dx + \int x\tan\left(x+\frac{\pi}{2}\right)^2dx + \frac{ix^3}{3} - \frac{1}{2}x^2\cot^2(x)$$

$$\Big\downarrow \textcolor{blue}{3011}$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right) - i\left(\frac{1}{2}ix\,\mathrm{PolyLog}\left(2,e^{2ix}\right) - \frac{1}{2}i\int \mathrm{PolyLog}\left(2,e^{2ix}\right)dx\right)\right) - \int x^2\cos^2(x)\cot(x)dx + \int x\tan\left(x+\frac{\pi}{2}\right)^2dx + \frac{ix^3}{3} - \frac{1}{2}x^2\cot^2(x)$$

$$\Big\downarrow \textcolor{blue}{2720}$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right) - i\left(\frac{1}{2}ix\,\mathrm{PolyLog}\left(2,e^{2ix}\right) - \frac{1}{4}\int e^{-2ix}\,\mathrm{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right) - \int x^2\cos^2(x)\cot(x)dx + \int x\tan\left(x+\frac{\pi}{2}\right)^2dx + \frac{ix^3}{3} - \frac{1}{2}x^2\cot^2(x)$$

$$\Big\downarrow \textcolor{blue}{4203}$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right) - i\left(\frac{1}{2}ix\,\mathrm{PolyLog}\left(2,e^{2ix}\right) - \frac{1}{4}\int e^{-2ix}\,\mathrm{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right) - \int x^2\cos^2(x)\cot(x)dx - \int xdx - \int -\cot(x)dx + \frac{ix^3}{3} - \frac{1}{2}x^2\cot^2(x) - x\cot(x)$$

$$\Big\downarrow \textcolor{blue}{15}$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right) - i\left(\frac{1}{2}ix\,\mathrm{PolyLog}\left(2,e^{2ix}\right) - \frac{1}{4}\int e^{-2ix}\,\mathrm{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right) - \int x^2\cos^2(x)\cot(x)dx - \int -\cot(x)dx + \frac{ix^3}{3} - \frac{x^2}{2} - \frac{1}{2}x^2\cot^2(x) - x\cot(x)$$

$$\Big\downarrow \textcolor{blue}{25}$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right) - i\left(\frac{1}{2}ix\,\mathrm{PolyLog}\left(2,e^{2ix}\right) - \frac{1}{4}\int e^{-2ix}\,\mathrm{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right) - \int x^2\cos^2(x)\cot(x)dx + \int \cot(x)dx + \frac{ix^3}{3} - \frac{x^2}{2} - \frac{1}{2}x^2\cot^2(x) - x\cot(x)$$

$$\Big\downarrow \textcolor{blue}{3042}$$

3.206. $\int x^2\cos^2(x)\cot^3(x)\,dx$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)-\int x^2\cos^2(x)\cot(x)dx+\int -\tan\left(x+\frac{\pi}{2}\right)dx+\frac{ix^3}{3}-\frac{x^2}{2}-\frac{1}{2}x^2\cot^2(x)-x\cot(x)$$

$\Big\downarrow$ 25

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)-\int x^2\cos^2(x)\cot(x)dx-\int \tan\left(x+\frac{\pi}{2}\right)dx+\frac{ix^3}{3}-\frac{x^2}{2}-\frac{1}{2}x^2\cot^2(x)-x\cot(x)$$

$\Big\downarrow$ 3956

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)-\int x^2\cos^2(x)\cot(x)dx+\frac{ix^3}{3}-\frac{x^2}{2}-\frac{1}{2}x^2\cot^2(x)-x\cot(x)+\log(\sin(x))$$

$\Big\downarrow$ 4908

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)-\int x^2\cot(x)dx+\int x^2\cos(x)\sin(x)dx+\frac{ix^3}{3}-\frac{x^2}{2}-\frac{1}{2}x^2\cot^2(x)-x\cot(x)+\log(\sin(x))$$

$\Big\downarrow$ 3042

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)-\int -x^2\tan\left(x+\frac{\pi}{2}\right)dx+\int x^2\cos(x)\sin(x)dx+\frac{ix^3}{3}-\frac{x^2}{2}-\frac{1}{2}x^2\cot^2(x)-x\cot(x)+\log(\sin(x))$$

$\Big\downarrow$ 25

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)+\int x^2\tan\left(x+\frac{\pi}{2}\right)dx+\int x^2\cos(x)\sin(x)dx+\frac{ix^3}{3}-\frac{x^2}{2}-\frac{1}{2}x^2\cot^2(x)-x\cot(x)+\log(\sin(x))$$

$\Big\downarrow$ 3924

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)+\int x^2\tan\left(x+\frac{\pi}{2}\right)dx-\int x\sin^2(x)dx+\frac{ix^3}{3}-\frac{x^2}{2}+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-x\cot(x)+\log(\sin(x))$$

$\Big\downarrow$ 3042

---

3.206. $\int x^2\cos^2(x)\cot^3(x)\,dx$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\operatorname{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\operatorname{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)+\int x^2\tan\left(x+\frac{\pi}{2}\right)dx-\int x\sin(x)^2dx+\frac{ix^3}{3}-\frac{x^2}{2}+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-x\cot(x)+\log(\sin(x))$$

$$\Big\downarrow \; 3791$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\operatorname{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\operatorname{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)+\int x^2\tan\left(x+\frac{\pi}{2}\right)dx-\frac{\int xdx}{2}+\frac{ix^3}{3}-\frac{x^2}{2}+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-\frac{\sin^2(x)}{4}-x\cot(x)+\log(\sin(x))+\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \; 15$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\operatorname{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\operatorname{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)+\int x^2\tan\left(x+\frac{\pi}{2}\right)dx+\frac{ix^3}{3}-\frac{3x^2}{4}+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-\frac{\sin^2(x)}{4}-x\cot(x)+\log(\sin(x))+\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \; 4200$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\operatorname{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\operatorname{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)-2i\int-\frac{e^{2ix}x^2}{1-e^{2ix}}dx+\frac{2ix^3}{3}-\frac{3x^2}{4}+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-\frac{\sin^2(x)}{4}-x\cot(x)+\log(\sin(x))+\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \; 25$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\operatorname{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\operatorname{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)+2i\int\frac{e^{2ix}x^2}{1-e^{2ix}}dx+\frac{2ix^3}{3}-\frac{3x^2}{4}+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-\frac{\sin^2(x)}{4}-x\cot(x)+\log(\sin(x))+\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \; 2620$$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\operatorname{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\operatorname{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)+2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\int x\log\left(1-e^{2ix}\right)dx\right)+\frac{2ix^3}{3}-\frac{3x^2}{4}+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-\frac{\sin^2(x)}{4}-x\cot(x)+\log(\sin(x))+\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \; 3011$$

3.206. $\int x^2\cos^2(x)\cot^3(x)\,dx$

$$2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{2}i\int \text{PolyLog}\left(2,e^{2ix}\right)dx\right)\right)+2i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)+\frac{2ix^3}{3}-\frac{3x^2}{4}+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-\frac{\sin^2(x)}{4}-x\cot(x)+\log(\sin(x))+\frac{1}{2}x\sin(x)\cos(x)$$

$$\Big\downarrow \; 2720$$

$$4i\left(\frac{1}{2}ix^2\log\left(1-e^{2ix}\right)-i\left(\frac{1}{2}ix\,\text{PolyLog}\left(2,e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\,\text{PolyLog}\left(2,e^{2ix}\right)de^{2ix}\right)\right)+\frac{2ix^3}{3}-\frac{3x^2}{4}+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-\frac{\sin^2(x)}{4}-x\cot(x)+\log(\sin(x))+\frac{1}{2}x\sin(x)\cos(x)$$

input
```
Int[x^2*Cos[x]^2*Cot[x]^3,x]
```

output
```
$Aborted
```

### 3.206.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

## 3.206. $\int x^2 \cos^2(x) \cot^3(x)\,dx$

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

## 3.206.4 Maple [A] (verified)

Time = 4.30 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.60

| method | result |
|---|---|
| risch | $\frac{2ix^3}{3}-\frac{(2x^2+2ix-1)\mathrm{e}^{2ix}}{16}-\frac{(2x^2-2ix-1)\mathrm{e}^{-2ix}}{16}+\frac{2x(\mathrm{e}^{2ix}x-i\mathrm{e}^{2ix}+i)}{(\mathrm{e}^{2ix}-1)^2}-2\ln(\mathrm{e}^{ix})+\ln(\mathrm{e}^{ix}+1)+\ln(\mathrm{e}^{ix}-1)$ - |

input
```
int(x^2*cos(x)^2*cot(x)^3,x,method=_RETURNVERBOSE)
```

output
```
2/3*I*x^3-1/16*(2*I*x+2*x^2-1)*exp(2*I*x)-1/16*(-2*I*x+2*x^2-1)*exp(-2*I*x
)+2*x*(exp(2*I*x)*x-I*exp(2*I*x)+I)/(exp(2*I*x)-1)^2-2*ln(exp(I*x))+ln(exp
(I*x)+1)+ln(exp(I*x)-1)-2*x^2*ln(exp(I*x)+1)+4*I*x*polylog(2,-exp(I*x))-4*
polylog(3,-exp(I*x))-2*x^2*ln(1-exp(I*x))+4*I*x*polylog(2,exp(I*x))-4*poly
log(3,exp(I*x))
```

## 3.206.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 370 vs. 2(80) = 160.

Time = 0.30 (sec) , antiderivative size = 370, normalized size of antiderivative = 3.49

$$\int x^2\cos^2(x)\cot^3(x)\,dx =$$

$$-\frac{2\,(2\,x^2-1)\cos(x)^4-3\,(2\,x^2-1)\cos(x)^2-2\,x^2+16\left(-i\,x\cos(x)^2+i\,x\right)\mathrm{Li}_2(\cos(x)+i\,\sin(x))+16}{}$$

input
```
integrate(x^2*cos(x)^2*cot(x)^3,x, algorithm="fricas")
```

output
```
-1/8*(2*(2*x^2 - 1)*cos(x)^4 - 3*(2*x^2 - 1)*cos(x)^2 - 2*x^2 + 16*(-I*x*c
os(x)^2 + I*x)*dilog(cos(x) + I*sin(x)) + 16*(I*x*cos(x)^2 - I*x)*dilog(co
s(x) - I*sin(x)) + 16*(I*x*cos(x)^2 - I*x)*dilog(-cos(x) + I*sin(x)) + 16*
(-I*x*cos(x)^2 + I*x)*dilog(-cos(x) - I*sin(x)) + 4*((2*x^2 - 1)*cos(x)^2
- 2*x^2 + 1)*log(cos(x) + I*sin(x) + 1) + 4*((2*x^2 - 1)*cos(x)^2 - 2*x^2
+ 1)*log(cos(x) - I*sin(x) + 1) - 4*(cos(x)^2 - 1)*log(-1/2*cos(x) + 1/2*I
*sin(x) + 1/2) - 4*(cos(x)^2 - 1)*log(-1/2*cos(x) - 1/2*I*sin(x) + 1/2) +
8*(x^2*cos(x)^2 - x^2)*log(-cos(x) + I*sin(x) + 1) + 8*(x^2*cos(x)^2 - x^2
)*log(-cos(x) - I*sin(x) + 1) + 16*(cos(x)^2 - 1)*polylog(3, cos(x) + I*si
n(x)) + 16*(cos(x)^2 - 1)*polylog(3, cos(x) - I*sin(x)) + 16*(cos(x)^2 - 1
)*polylog(3, -cos(x) + I*sin(x)) + 16*(cos(x)^2 - 1)*polylog(3, -cos(x) -
I*sin(x)) - 4*(x*cos(x)^3 + x*cos(x))*sin(x) - 1)/(cos(x)^2 - 1)
```

### 3.206.6 Sympy [F]

$$\int x^2 \cos^2(x) \cot^3(x) \, dx = \int x^2 \cos^2(x) \cot^3(x) \, dx$$

input
```
integrate(x**2*cos(x)**2*cot(x)**3,x)
```

output
```
Integral(x**2*cos(x)**2*cot(x)**3, x)
```

### 3.206.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2842 vs. $2(80) = 160$.

Time = 0.50 (sec) , antiderivative size = 2842, normalized size of antiderivative = 26.81

$$\int x^2 \cos^2(x) \cot^3(x) \, dx = \text{Too large to display}$$

input
```
integrate(x^2*cos(x)^2*cot(x)^3,x, algorithm="maxima")
```

output
```
-1/48*(3*(2*x^2 + 2*I*x - 1)*cos(6*x)^2 + 4*(16*I*x^3 + 6*x^2 - 42*I*x - 3
)*cos(4*x)^2 + 2*(32*I*x^3 - 42*x^2 - 48*I*x - 3)*cos(2*x)^2 - 3*(2*x^2 +
2*I*x - 1)*sin(6*x)^2 + 4*(-16*I*x^3 - 6*x^2 + 42*I*x + 3)*sin(4*x)^2 + 2*
(-32*I*x^3 + 42*x^2 + 48*I*x + 3)*sin(2*x)^2 + 6*x^2 + 48*(2*(-2*I*x^2 + I
)*cos(4*x)^2 + 2*(-2*I*x^2 + I)*cos(2*x)^2 + 2*(2*I*x^2 - I)*sin(4*x)^2 +
2*(2*I*x^2 - I)*sin(2*x)^2 + (2*I*x^2 + (2*I*x^2 - I)*cos(4*x) + 2*(-2*I*x
^2 + I)*cos(2*x) - (2*x^2 - 1)*sin(4*x) + 2*(2*x^2 - 1)*sin(2*x) - I)*cos(
6*x) + (-4*I*x^2 + 5*(2*I*x^2 - I)*cos(2*x) - 5*(2*x^2 - 1)*sin(2*x) + 2*I
)*cos(4*x) + (2*I*x^2 - I)*cos(2*x) - (2*x^2 + (2*x^2 - 1)*cos(4*x) - 2*(2
*x^2 - 1)*cos(2*x) - (-2*I*x^2 + I)*sin(4*x) - 2*(2*I*x^2 - I)*sin(2*x) -
1)*sin(6*x) + (4*x^2 + 4*(2*x^2 - 1)*cos(4*x) - 5*(2*x^2 - 1)*cos(2*x) + 5
*(-2*I*x^2 + I)*sin(2*x) - 2)*sin(4*x) - (2*x^2 - 4*(2*x^2 - 1)*cos(2*x) -
 1)*sin(2*x))*arctan2(sin(x), cos(x) + 1) + 48*((-I*cos(4*x) + 2*I*cos(2*x
) + sin(4*x) - 2*sin(2*x) - I)*cos(6*x) + (-5*I*cos(2*x) + 5*sin(2*x) + 2*
I)*cos(4*x) + 2*I*cos(4*x)^2 + 2*I*cos(2*x)^2 + (cos(4*x) - 2*cos(2*x) + I
*sin(4*x) - 2*I*sin(2*x) + 1)*sin(6*x) - (4*cos(4*x) - 5*cos(2*x) - 5*I*si
n(2*x) + 2)*sin(4*x) - 2*I*sin(4*x)^2 - (4*cos(2*x) - 1)*sin(2*x) - 2*I*si
n(2*x)^2 - I*cos(2*x))*arctan2(sin(x), cos(x) - 1) + 96*(2*I*x^2*cos(4*x)^
2 + 2*I*x^2*cos(2*x)^2 - 2*I*x^2*sin(4*x)^2 - 2*I*x^2*sin(2*x)^2 - I*x^2*c
os(2*x) + (-I*x^2*cos(4*x) + 2*I*x^2*cos(2*x) + x^2*sin(4*x) - 2*x^2*si...
```

### 3.206.8 Giac [F]

$$\int x^2\cos^2(x)\cot^3(x)\,dx = \int x^2\cos(x)^2\cot(x)^3\,dx$$

input
```
integrate(x^2*cos(x)^2*cot(x)^3,x, algorithm="giac")
```

output
```
integrate(x^2*cos(x)^2*cot(x)^3, x)
```

### 3.206.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \cos^2(x) \cot^3(x) \, dx = \int x^2 \cos(x)^2 \cot(x)^3 \, dx$$

input
```
int(x^2*cos(x)^2*cot(x)^3,x)
```

output
```
int(x^2*cos(x)^2*cot(x)^3, x)
```

# 3.207 $\int x\cos^2(x)\cot^3(x)\,dx$


3.207.1 Optimal result . . . 1655
3.207.2 Mathematica [A] (verified) . . . 1655
3.207.3 Rubi [A] (verified) . . . 1656
3.207.4 Maple [A] (verified) . . . 1661
3.207.5 Fricas [B] (verification not implemented) . . . 1661
3.207.6 Sympy [F] . . . 1662
3.207.7 Maxima [B] (verification not implemented) . . . 1662
3.207.8 Giac [F] . . . 1663
3.207.9 Mupad [F(-1)] . . . 1663


## 3.207.1 Optimal result

Integrand size = 10, antiderivative size = 73

$$\int x\cos^2(x)\cot^3(x)\,dx=-\frac{3x}{4}+ix^2-\frac{\cot(x)}{2}-\frac{1}{2}x\cot^2(x)-2x\log\left(1-e^{2ix}\right)+i\,\text{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{4}\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)$$

output
```
-3/4*x+I*x^2-1/2*cot(x)-1/2*x*cot(x)^2-2*x*ln(1-exp(2*I*x))+I*polylog(2,ex
p(2*I*x))+1/4*cos(x)*sin(x)+1/2*x*sin(x)^2
```

## 3.207.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

$$\int x\cos^2(x)\cot^3(x)\,dx=\frac{1}{8}\left(8ix^2-2x\cos(2x)-4\cot(x)-4x\csc^2(x)-16x\log\left(1-e^{2ix}\right)+8i\,\text{PolyLog}\left(2,e^{2ix}\right)+\sin(2x)\right)$$

input
```
Integrate[x*Cos[x]^2*Cot[x]^3,x]
```

output
```
((8*I)*x^2 - 2*x*Cos[2*x] - 4*Cot[x] - 4*x*Csc[x]^2 - 16*x*Log[1 - E^((2*I
)*x)] + (8*I)*PolyLog[2, E^((2*I)*x)] + Sin[2*x])/8
```

### 3.207.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2.600, Rules used = {4908, 3042, 25, 4203, 25, 3042, 25, 3954, 24, 4200, 25, 2620, 2715, 2838, 4908, 3042, 25, 3924, 3042, 3115, 24, 4200, 25, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos^2(x)\cot^3(x)\,dx$$

$$\Big\downarrow \text{4908}$$

$$\int x\cot^3(x)dx-\int x\cos^2(x)\cot(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\int -x\tan\left(x+\frac{\pi}{2}\right)^3dx-\int x\cos^2(x)\cot(x)dx$$

$$\Big\downarrow \text{25}$$

$$-\int x\tan\left(x+\frac{\pi}{2}\right)^3dx-\int x\cos^2(x)\cot(x)dx$$

$$\Big\downarrow \text{4203}$$

$$\frac{1}{2}\int\cot^2(x)dx+\int -x\cot(x)dx-\int x\cos^2(x)\cot(x)dx-\frac{1}{2}x\cot^2(x)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int\cot^2(x)dx-\int x\cot(x)dx-\int x\cos^2(x)\cot(x)dx-\frac{1}{2}x\cot^2(x)$$

$$\Big\downarrow \text{3042}$$

$$-\int -x\tan\left(x+\frac{\pi}{2}\right)dx+\frac{1}{2}\int\tan\left(x+\frac{\pi}{2}\right)^2dx-\int x\cos^2(x)\cot(x)dx-\frac{1}{2}x\cot^2(x)$$

$$\Big\downarrow \text{25}$$

$$\int x\tan\left(x+\frac{\pi}{2}\right)dx+\frac{1}{2}\int\tan\left(x+\frac{\pi}{2}\right)^2dx-\int x\cos^2(x)\cot(x)dx-\frac{1}{2}x\cot^2(x)$$

$$\Big\downarrow \text{3954}$$

$$\int x\tan\left(x+\frac{\pi}{2}\right)dx+\frac{1}{2}\left(-\int 1dx-\cot(x)\right)-\int x\cos^2(x)\cot(x)dx-\frac{1}{2}x\cot^2(x)$$

$$\Big\downarrow \text{24}$$

$$\int x\tan\left(x+\frac{\pi}{2}\right)dx-\int x\cos^2(x)\cot(x)dx-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))$$

$$\Big\downarrow \text{4200}$$

$$-2i\int -\frac{e^{2ix}x}{1-e^{2ix}}dx-\int x\cos^2(x)\cot(x)dx+\frac{ix^2}{2}-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))$$

$$\Big\downarrow \text{25}$$

$$2i\int \frac{e^{2ix}x}{1-e^{2ix}}dx-\int x\cos^2(x)\cot(x)dx+\frac{ix^2}{2}-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))$$

$$\Big\downarrow \text{2620}$$

$$2i\left(\frac{1}{2}ix\log\left(1-e^{2ix}\right)-\frac{1}{2}i\int \log\left(1-e^{2ix}\right)dx\right)-\int x\cos^2(x)\cot(x)dx+\frac{ix^2}{2}-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))$$

$$\Big\downarrow \text{2715}$$

$$2i\left(\frac{1}{2}ix\log\left(1-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\log\left(1-e^{2ix}\right)de^{2ix}\right)-\int x\cos^2(x)\cot(x)dx+\frac{ix^2}{2}-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))$$

$$\Big\downarrow \text{2838}$$

$$-\int x\cos^2(x)\cot(x)dx+2i\left(\frac{1}{4}\text{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))$$

$$\Big\downarrow \text{4908}$$

$$-\int x\cot(x)dx+\int x\cos(x)\sin(x)dx+2i\left(\frac{1}{4}\text{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))$$

$$\Big\downarrow \text{3042}$$

$$-\int -x\tan\left(x+\frac{\pi}{2}\right)dx+\int x\cos(x)\sin(x)dx+2i\left(\frac{1}{4}\text{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))$$

$$\Big\downarrow \text{25}$$

$$\int x\tan\left(x+\frac{\pi}{2}\right)dx+\int x\cos(x)\sin(x)dx+2i\left(\frac{1}{4}\text{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))$$

---

3.207. $\int x\cos^2(x)\cot^3(x)\,dx$

$$\Big\downarrow \; 3924$$

$$-\frac{1}{2}\int \sin^2(x)dx+\int x\tan\left(x+\frac{\pi}{2}\right)dx+2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))$$

$$\Big\downarrow \; 3042$$

$$-\frac{1}{2}\int \sin(x)^2dx+\int x\tan\left(x+\frac{\pi}{2}\right)dx+2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))$$

$$\Big\downarrow \; 3115$$

$$\int x\tan\left(x+\frac{\pi}{2}\right)dx+\frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x)-\frac{\int 1dx}{2}\right)+2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))$$

$$\Big\downarrow \; 24$$

$$\int x\tan\left(x+\frac{\pi}{2}\right)dx+2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+\frac{ix^2}{2}+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))+\frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x)-\frac{x}{2}\right)$$

$$\Big\downarrow \; 4200$$

$$-2i\int -\frac{e^{2ix}x}{1-e^{2ix}}dx+2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+ix^2+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))+\frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x)-\frac{x}{2}\right)$$

$$\Big\downarrow \; 25$$

$$2i\int \frac{e^{2ix}x}{1-e^{2ix}}dx+2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+ix^2+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))+\frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x)-\frac{x}{2}\right)$$

$$\Big\downarrow \; 2620$$

$$2i\left(\frac{1}{2}ix\log\left(1-e^{2ix}\right)-\frac{1}{2}i\int \log\left(1-e^{2ix}\right)dx\right)+2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+ix^2+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))+\frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x)-\frac{x}{2}\right)$$

$$\Big\downarrow \; 2715$$

3.207. $\int x\cos^2(x)\cot^3(x)\,dx$

$$2i\left(\frac{1}{2}ix\log\left(1-e^{2ix}\right)-\frac{1}{4}\int e^{-2ix}\log\left(1-e^{2ix}\right)de^{2ix}\right)+$$
$$2i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+ix^2+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))+$$
$$\frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x)-\frac{x}{2}\right)$$

$$\Big\downarrow \text{2838}$$

$$4i\left(\frac{1}{4}\operatorname{PolyLog}\left(2,e^{2ix}\right)+\frac{1}{2}ix\log\left(1-e^{2ix}\right)\right)+ix^2+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)+\frac{1}{2}(-x-\cot(x))+$$
$$\frac{1}{2}\left(\frac{1}{2}\sin(x)\cos(x)-\frac{x}{2}\right)$$

input
```
Int[x*Cos[x]^2*Cot[x]^3,x]
```

output
```
I*x^2 + (-x - Cot[x])/2 - (x*Cot[x]^2)/2 + (4*I)*((I/2)*x*Log[1 - E^((2*I)
*x)] + PolyLog[2, E^((2*I)*x)]/4) + (x*Sin[x]^2)/2 + (-1/2*x + (Cos[x]*Sin
[x])/2)/2
```

### 3.207.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4200
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^
m*E^(2*I*k*Pi)*(E^(2*I*(e + f*x))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x)))), x]
, x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4908
```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

## 3.207. $\int x \cos^2(x) \cot^3(x) \, dx$

### 3.207.4 Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.49

| method | result |
|---|---|
| risch | $ix^2 - \frac{(2x+i)e^{2ix}}{16} - \frac{(-i+2x)e^{-2ix}}{16} + \frac{2e^{2ix}x - ie^{2ix}+i}{(e^{2ix}-1)^2} - 2x\ln(e^{ix}+1) - 2x\ln(1-e^{ix}) + 2i\,\mathrm{polylog}(2, -$ |

input
```
int(x*cos(x)^2*cot(x)^3,x,method=_RETURNVERBOSE)
```

output
```
I*x^2-1/16*(2*x+I)*exp(2*I*x)-1/16*(-I+2*x)*exp(-2*I*x)+(2*exp(2*I*x)*x-I*
exp(2*I*x)+I)/(exp(2*I*x)-1)^2-2*x*ln(exp(I*x)+1)-2*x*ln(1-exp(I*x))+2*I*p
olylog(2,-exp(I*x))+2*I*polylog(2,exp(I*x))
```

### 3.207.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(52) = 104.

Time = 0.27 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.78

$$\int x\cos^2(x)\cot^3(x)\,dx =$$
$$-\frac{2\,x\cos(x)^4 - 3\,x\cos(x)^2 + 4\left(-i\,\cos(x)^2 + i\right)\mathrm{Li}_2(\cos(x) + i\,\sin(x)) + 4\left(i\,\cos(x)^2 - i\right)\mathrm{Li}_2(\cos(x) -}{}$$

input
```
integrate(x*cos(x)^2*cot(x)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*x*cos(x)^4 - 3*x*cos(x)^2 + 4*(-I*cos(x)^2 + I)*dilog(cos(x) + I*s
in(x)) + 4*(I*cos(x)^2 - I)*dilog(cos(x) - I*sin(x)) + 4*(I*cos(x)^2 - I)*
dilog(-cos(x) + I*sin(x)) + 4*(-I*cos(x)^2 + I)*dilog(-cos(x) - I*sin(x))
 + 4*(x*cos(x)^2 - x)*log(cos(x) + I*sin(x) + 1) + 4*(x*cos(x)^2 - x)*log(c
os(x) - I*sin(x) + 1) + 4*(x*cos(x)^2 - x)*log(-cos(x) + I*sin(x) + 1) + 4
*(x*cos(x)^2 - x)*log(-cos(x) - I*sin(x) + 1) - (cos(x)^3 + cos(x))*sin(x)
 - x)/(cos(x)^2 - 1)
```

### 3.207.6 Sympy [F]

$$\int x\cos^2(x)\cot^3(x)\,dx = \int x\cos^2(x)\cot^3(x)\,dx$$

input
```
integrate(x*cos(x)**2*cot(x)**3,x)
```

output
```
Integral(x*cos(x)**2*cot(x)**3, x)
```

### 3.207.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1718 vs. $2(52) = 104$.

Time = 0.36 (sec) , antiderivative size = 1718, normalized size of antiderivative = 23.53

$$\int x\cos^2(x)\cot^3(x)\,dx = \text{Too large to display}$$

input
```
integrate(x*cos(x)^2*cot(x)^3,x, algorithm="maxima")
```

output
```
-1/16*((2*x + I)*cos(6*x)^2 + 4*(8*I*x^2 + 2*x + I)*cos(4*x)^2 + 4*(8*I*x^
2 - 7*x + 4*I)*cos(2*x)^2 - (2*x + I)*sin(6*x)^2 + 4*(-8*I*x^2 - 2*x - I)*
sin(4*x)^2 + 4*(-8*I*x^2 + 7*x - 4*I)*sin(2*x)^2 + 32*(-2*I*x*cos(4*x)^2 -
 2*I*x*cos(2*x)^2 + 2*I*x*sin(4*x)^2 + 2*I*x*sin(2*x)^2 + (I*x*cos(4*x) -
2*I*x*cos(2*x) - x*sin(4*x) + 2*x*sin(2*x) + I*x)*cos(6*x) + (5*I*x*cos(2*
x) - 5*x*sin(2*x) - 2*I*x)*cos(4*x) + I*x*cos(2*x) - (x*cos(4*x) - 2*x*cos
(2*x) + I*x*sin(4*x) - 2*I*x*sin(2*x) + x)*sin(6*x) + (4*x*cos(4*x) - 5*x*
cos(2*x) - 5*I*x*sin(2*x) + 2*x)*sin(4*x) + (4*x*cos(2*x) - x)*sin(2*x))*a
rctan2(sin(x), cos(x) + 1) + 32*(2*I*x*cos(4*x)^2 + 2*I*x*cos(2*x)^2 - 2*I
*x*sin(4*x)^2 - 2*I*x*sin(2*x)^2 + (-I*x*cos(4*x) + 2*I*x*cos(2*x) + x*sin
(4*x) - 2*x*sin(2*x) - I*x)*cos(6*x) + (-5*I*x*cos(2*x) + 5*x*sin(2*x) + 2
*I*x)*cos(4*x) - I*x*cos(2*x) + (x*cos(4*x) - 2*x*cos(2*x) + I*x*sin(4*x)
- 2*I*x*sin(2*x) + x)*sin(6*x) - (4*x*cos(4*x) - 5*x*cos(2*x) - 5*I*x*sin(
2*x) + 2*x)*sin(4*x) - (4*x*cos(2*x) - x)*sin(2*x))*arctan2(sin(x), -cos(x
) + 1) - (16*I*x^2 - 4*(-4*I*x^2 - 2*x - I)*cos(4*x) + (-32*I*x^2 + 26*x -
 17*I)*cos(2*x) - 4*(4*x^2 - 2*I*x + 1)*sin(4*x) + (32*x^2 + 26*I*x + 17)*
sin(2*x) + 4*x + 14*I)*cos(6*x) - (-32*I*x^2 - 2*(-40*I*x^2 + 26*x - 17*I)
*cos(2*x) - 2*(40*x^2 + 26*I*x + 17)*sin(2*x) - 10*x - 27*I)*cos(4*x) + 4*
(-4*I*x^2 - 2*x - 3*I)*cos(2*x) + 32*((-I*cos(4*x) + 2*I*cos(2*x) + sin(4*
x) - 2*sin(2*x) - I)*cos(6*x) + (-5*I*cos(2*x) + 5*sin(2*x) + 2*I)*cos(...
```

### 3.207.8 Giac [F]

$$\int x\cos^2(x)\cot^3(x)\,dx = \int x\cos(x)^2\cot(x)^3\,dx$$

input
```
integrate(x*cos(x)^2*cot(x)^3,x, algorithm="giac")
```

output
```
integrate(x*cos(x)^2*cot(x)^3, x)
```

### 3.207.9 Mupad [F(-1)]

Timed out.

$$\int x\cos^2(x)\cot^3(x)\,dx = \int x\cos(x)^2\cot(x)^3\,dx$$

input
```
int(x*cos(x)^2*cot(x)^3,x)
```

output
```
int(x*cos(x)^2*cot(x)^3, x)
```

# 3.208 $\int (c+dx)^m \tan(a+bx)\,dx$

3.208.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1664
3.208.2 Mathematica [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . 1664
3.208.3 Rubi [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1665
3.208.4 Maple [N/A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 1666
3.208.5 Fricas [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1666
3.208.6 Sympy [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1666
3.208.7 Maxima [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1667
3.208.8 Giac [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1667
3.208.9 Mupad [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1667

## 3.208.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (c+dx)^m \tan(a+bx)\,dx = \text{Int}((c+dx)^m \tan(a+bx),x)$$

output
```
Unintegrable((d*x+c)^m*tan(b*x+a),x)
```

## 3.208.2 Mathematica [N/A]

Not integrable

Time = 3.72 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \tan(a+bx)\,dx = \int (c+dx)^m \tan(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Tan[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Tan[a + b*x], x]
```

### 3.208.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)(c+dx)^m dx$$

$$\Big\downarrow \text{4222}$$

$$\int \tan(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Tan[a + b*x],x]
```

output
```
$Aborted
```

#### 3.208.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

### 3.208.4 Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int (dx+c)^m \sec (xb+a) \sin (xb+a) \, dx$$

input
```
int((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x)
```

output
```
int((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x)
```

### 3.208.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int (c+dx)^m \tan(a+bx) \, dx = \int (dx+c)^m \sec (bx+a) \sin (bx+a) \, dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*sec(b*x + a)*sin(b*x + a), x)
```

### 3.208.6 Sympy [N/A]

Not integrable

Time = 30.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int (c+dx)^m \tan(a+bx) \, dx = \int (c+dx)^m \sin (a+bx) \sec (a+bx) \, dx$$

input
```
integrate((d*x+c)**m*sec(b*x+a)*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**m*sin(a + b*x)*sec(a + b*x), x)
```

### 3.208.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int (c+dx)^m \tan(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\sin(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a), x)
```

### 3.208.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int (c+dx)^m \tan(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\sin(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a), x)
```

### 3.208.9 Mupad [N/A]

Not integrable

Time = 24.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int (c+dx)^m \tan(a+bx)\,dx = \int \frac{\sin(a+b\,x)\,(c+d\,x)^m}{\cos(a+b\,x)}\,dx$$

input
```
int((sin(a + b*x)*(c + d*x)^m)/cos(a + b*x),x)
```

output
```
int((sin(a + b*x)*(c + d*x)^m)/cos(a + b*x), x)
```

# 3.209 $\int (c+dx)^4 \tan(a+bx)\,dx$

3.209.1 Optimal result . . . 1668
3.209.2 Mathematica [A] (verified) . . . 1669
3.209.3 Rubi [A] (verified) . . . 1669
3.209.4 Maple [B] (verified) . . . 1673
3.209.5 Fricas [B] (verification not implemented) . . . 1673
3.209.6 Sympy [F] . . . 1674
3.209.7 Maxima [B] (verification not implemented) . . . 1675
3.209.8 Giac [F] . . . 1676
3.209.9 Mupad [F(-1)] . . . 1676

## 3.209.1 Optimal result

Integrand size = 14, antiderivative size = 158

$$\int (c+dx)^4 \tan(a+bx)\,dx = \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4 \log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} - \frac{3d^2(c+dx)^2 \operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3} - \frac{3id^3(c+dx) \operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4} + \frac{3d^4 \operatorname{PolyLog}\left(5,-e^{2i(a+bx)}\right)}{2b^5}$$

output
```
1/5*I*(d*x+c)^5/d-(d*x+c)^4*ln(1+exp(2*I*(b*x+a)))/b+2*I*d*(d*x+c)^3*polyl
og(2,-exp(2*I*(b*x+a)))/b^2-3*d^2*(d*x+c)^2*polylog(3,-exp(2*I*(b*x+a)))/b
^3-3*I*d^3*(d*x+c)*polylog(4,-exp(2*I*(b*x+a)))/b^4+3/2*d^4*polylog(5,-exp
(2*I*(b*x+a)))/b^5
```

## 3.209.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99

$$\int (c+dx)^4\tan(a+bx)\,dx$$
$$=\frac{i(c+dx)^5}{5d}-\frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{2id(c+dx)^3\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}$$
$$-\frac{3d^2\left(2b^2(c+dx)^2\,\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)+d\left(2ib(c+dx)\,\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)-d\,\text{PolyLog}\left(5,-e^{2i(}\right.\right.\right.}{2b^5}$$

input
```
Integrate[(c + d*x)^4*Tan[a + b*x],x]
```

output
```
((I/5)*(c + d*x)^5)/d - ((c + d*x)^4*Log[1 + E^((2*I)*(a + b*x))])/b + ((2
*I)*d*(c + d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(2*b^2*(c
 + d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x))] + d*((2*I)*b*(c + d*x)*PolyLog[
4, -E^((2*I)*(a + b*x))] - d*PolyLog[5, -E^((2*I)*(a + b*x))])))/(2*b^5)
```

## 3.209.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.23, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 4202, 2620, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4\tan(a+bx)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int (c+dx)^4\tan(a+bx)dx$$
$$\Big\downarrow \text{ 4202}$$
$$\frac{i(c+dx)^5}{5d}-2i\int \frac{e^{2i(a+bx)}(c+dx)^4}{1+e^{2i(a+bx)}}dx$$
$$\Big\downarrow \text{ 2620}$$
$$\frac{i(c+dx)^5}{5d}-2i\left(\frac{2id\int (c+dx)^3\log\left(1+e^{2i(a+bx)}\right)dx}{b}-\frac{i(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

$\Big\downarrow$ 3011

$$2i\left(\frac{i(c+dx)^5}{5d}-\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}(2,-e^{2i(a+bx)})}{2b}-\frac{3id\int(c+dx)^2\operatorname{PolyLog}(2,-e^{2i(a+bx)})dx}{2b}\right)}{b}-\frac{i(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

$\Big\downarrow$ 7163

$$2i\left(\frac{i(c+dx)^5}{5d}-\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}(2,-e^{2i(a+bx)})}{2b}-\frac{3id\left(\frac{id\int(c+dx)\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)dx}{b}-\frac{i(c+dx)^2\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{2b}\right)}{b}-\frac{i(c+dx)^4 \log}{}\right.$$

$\Big\downarrow$ 7163

$$2i\left(\frac{i(c+dx)^5}{5d}-\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}(2,-e^{2i(a+bx)})}{2b}-\frac{3id\left(\frac{id\left(\frac{id\int\operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{2b}\right)}{b}-\frac{i(c+dx)^2\operatorname{PolyLog}\left(3,-e^{2i}\right.}{2b}\right.}{2b}\right.}{b}\right.$$

$\Big\downarrow$ 2720

3.209. $\int(c+dx)^4\tan(a+bx)\,dx$

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}(2,-e^{2i(a+bx)})}{2b}-\frac{3id\left(\frac{id\left(\frac{d\int e^{-2i(a+bx)}\operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{2b}\right)}{b}-\frac{i(c+\cdots}{\cdots}\right.}{2b}\right.}{b}\right.\frac{i(c+dx)^5}{5d}-$$

$\downarrow$ 7143

$$2i\left(\frac{2id\left(\frac{i(c+dx)^3\operatorname{PolyLog}(2,-e^{2i(a+bx)})}{2b}-\frac{3id\left(\frac{id\left(\frac{d\operatorname{PolyLog}\left(5,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{2b}\right)}{b}-\frac{i(c+dx)^2\operatorname{PolyLog}\left(3,-e^{2i(a+b\cdots}\right.}{2b}\right.}{2b}\right.}{b}\right.\frac{i(c+dx)^5}{5d}-$$

input
```
Int[(c + d*x)^4*Tan[a + b*x],x]
```

output
```
((I/5)*(c + d*x)^5)/d - (2*I)*(((-1/2*I)*(c + d*x)^4*Log[1 + E^((2*I)*(a +
 b*x))])/b + ((2*I)*d*(((I/2)*(c + d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))]
)/b - (((3*I)/2)*d*(((-1/2*I)*(c + d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x))]
)/b + (I*d*(((-1/2*I)*(c + d*x)*PolyLog[4, -E^((2*I)*(a + b*x))])/b + (d*P
olyLog[5, -E^((2*I)*(a + b*x))])/(4*b^2)))/b))/b))/b)
```

3.209. $\int (c+dx)^4 \tan(a+bx)\,dx$

### 3.209.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

### 3.209.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 624 vs. 2(141) = 282.

Time = 1.24 (sec) , antiderivative size = 625, normalized size of antiderivative = 3.96

| method | result |
|---|---|
| risch | $\frac{4id\,c^3a^2}{b^2}+\frac{2id\,c^3\,\mathrm{polylog}\left(2,-\mathrm{e}^{2i(xb+a)}\right)}{b^2}-\frac{8id^2c^2a^3}{b^3}+2id\,c^3x^2-\frac{6d^3c\,\mathrm{polylog}\left(3,-\mathrm{e}^{2i(xb+a)}\right)x}{b^3}-\frac{8c\,d^3a^3\ln\left(\mathrm{e}^{i(xb+a)}\right)}{b^4}+$ |

input
```
int((d*x+c)^4*sec(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
6*I/b^2*d^2*c^2*polylog(2,-exp(2*I*(b*x+a)))*x+8*I/b*d*c^3*x*a+8*I/b^3*d^3
*c*a^3*x+6*I/b^2*d^3*c*polylog(2,-exp(2*I*(b*x+a)))*x^2-12*I/b^2*d^2*c^2*a
^2*x-I*c^4*x-1/5*I/d*c^5+I*d^3*c*x^4-1/b*c^4*ln(exp(2*I*(b*x+a))+1)+2/b*c^
4*ln(exp(I*(b*x+a)))+1/5*I*d^4*x^5+3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^
5-1/b*d^4*ln(exp(2*I*(b*x+a))+1)*x^4+2/b^5*d^4*a^4*ln(exp(I*(b*x+a)))+4*I/
b^2*d*c^3*a^2+2*I/b^2*d*c^3*polylog(2,-exp(2*I*(b*x+a)))-8*I/b^3*d^2*c^2*a
^3-2*I/b^4*d^4*a^4*x-3*I/b^4*d^4*polylog(4,-exp(2*I*(b*x+a)))*x+12/b^3*c^2
*d^2*a^2*ln(exp(I*(b*x+a)))-8/b^2*c^3*d*a*ln(exp(I*(b*x+a)))-6/b*d^2*c^2*l
n(exp(2*I*(b*x+a))+1)*x^2-4/b*d^3*c*ln(exp(2*I*(b*x+a))+1)*x^3-4/b*d*c^3*l
n(exp(2*I*(b*x+a))+1)*x+2*I/b^2*d^4*polylog(2,-exp(2*I*(b*x+a)))*x^3+6*I/b
^4*d^3*c*a^4-3*I/b^4*d^3*c*polylog(4,-exp(2*I*(b*x+a)))-3/b^3*d^2*c^2*poly
log(3,-exp(2*I*(b*x+a)))-3/b^3*d^4*polylog(3,-exp(2*I*(b*x+a)))*x^2-8/5*I/
b^5*d^4*a^5+2*I*d^2*c^2*x^3+2*I*d*c^3*x^2-6/b^3*d^3*c*polylog(3,-exp(2*I*(
b*x+a)))*x-8/b^4*c*d^3*a^3*ln(exp(I*(b*x+a)))
```

### 3.209.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1410 vs. 2(137) = 274.

Time = 0.31 (sec) , antiderivative size = 1410, normalized size of antiderivative = 8.92

$$\int (c+dx)^4\tan(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(24*d^4*polylog(5, I*cos(b*x + a) + sin(b*x + a)) + 24*d^4*polylog(5,
I*cos(b*x + a) - sin(b*x + a)) + 24*d^4*polylog(5, -I*cos(b*x + a) + sin(b
*x + a)) + 24*d^4*polylog(5, -I*cos(b*x + a) - sin(b*x + a)) - 4*(I*b^3*d^
4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(I*cos(b
*x + a) + sin(b*x + a)) - 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*
c^2*d^2*x - I*b^3*c^3*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 4*(-I*b^3*
d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*d^2*x - I*b^3*c^3*d)*dilog(-I*co
s(b*x + a) + sin(b*x + a)) - 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^
3*c^2*d^2*x + I*b^3*c^3*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^4*c^
4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(cos(b
*x + a) + I*sin(b*x + a) + I) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d
^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b^
4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*
c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(I*cos(b*x + a) +
sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4
*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4
)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3
+ 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 +
4*a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^4*d^
4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c...
```

### 3.209.6 Sympy [F]

$$\int (c+dx)^4\tan(a+bx)\,dx=\int(c+dx)^4\sin(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**4*sec(b*x+a)*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**4*sin(a + b*x)*sec(a + b*x), x)
```

### 3.209.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 803 vs. $2(137) = 274$.

Time = 0.41 (sec) , antiderivative size = 803, normalized size of antiderivative = 5.08

$$\int (c+dx)^4 \tan(a+bx)\,dx =$$

$$-\frac{15\,c^4\log\left(-\sin\left(bx+a\right)^2+1\right)-\frac{60\,ac^3d\log\left(-\sin(bx+a)^2+1\right)}{b}+\frac{90\,a^2c^2d^2\log\left(-\sin(bx+a)^2+1\right)}{b^2}-\frac{60\,a^3cd^3\log\left(-\sin(bx+\right.}{b^3}}{}$$

input
```
integrate((d*x+c)^4*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/30*(15*c^4*log(-sin(b*x + a)^2 + 1) - 60*a*c^3*d*log(-sin(b*x + a)^2 +
1)/b + 90*a^2*c^2*d^2*log(-sin(b*x + a)^2 + 1)/b^2 - 60*a^3*c*d^3*log(-sin
(b*x + a)^2 + 1)/b^3 + 15*a^4*d^4*log(-sin(b*x + a)^2 + 1)/b^4 + 2*(-3*I*(
b*x + a)^5*d^4 - 15*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^4 - 45*d^4*polylog(5,
-e^(2*I*b*x + 2*I*a)) - 30*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*
x + a)^3 - 30*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 - I*a^3*d
^4)*(b*x + a)^2 - 10*(-3*I*(b*x + a)^4*d^4 + 8*(-I*b*c*d^3 + I*a*d^4)*(b*x
 + a)^3 + 9*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x + a)^2 + 6*(
-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 - 3*I*a^2*b*c*d^3 + I*a^3*d^4)*(b*x + a))
*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 30*(I*b^3*c^3*d - 3*I*a
*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 + 2*I*(b*x + a)^3*d^4 - I*a^3*d^4 + 4*(I*b*
c*d^3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^
4)*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*a)) + 5*(3*(b*x + a)^4*d^4 + 8*(b*c*
d^3 - a*d^4)*(b*x + a)^3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x +
a)^2 + 6*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a)
)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) -
30*(-2*I*b*c*d^3 - 3*I*(b*x + a)*d^4 + 2*I*a*d^4)*polylog(4, -e^(2*I*b*x +
 2*I*a)) + 15*(3*b^2*c^2*d^2 - 6*a*b*c*d^3 + 6*(b*x + a)^2*d^4 + 3*a^2*d^4
 + 8*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, -e^(2*I*b*x + 2*I*a)))/b^4)/b
```

### 3.209.8 Giac [F]

$$\int (c+dx)^4 \tan(a+bx)\,dx = \int (dx+c)^4 \sec(bx+a)\sin(bx+a)\,dx$$

input
```
integrate((d*x+c)^4*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*sec(b*x + a)*sin(b*x + a), x)
```

### 3.209.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \tan(a+bx)\,dx = \int \frac{\sin(a+b\,x)\,(c+d\,x)^4}{\cos(a+b\,x)}\,dx$$

input
```
int((sin(a + b*x)*(c + d*x)^4)/cos(a + b*x),x)
```

output
```
int((sin(a + b*x)*(c + d*x)^4)/cos(a + b*x), x)
```

# 3.210 $\int (c+dx)^3 \tan(a+bx)\,dx$

3.210.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1677
3.210.2 Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1677
3.210.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1678
3.210.4 Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1681
3.210.5 Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1681
3.210.6 Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1682
3.210.7 Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1683
3.210.8 Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1683
3.210.9 Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1684

## 3.210.1 Optimal result

Integrand size = 14, antiderivative size = 132

$$\int (c+dx)^3 \tan(a+bx)\,dx = \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3 \log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} - \frac{3d^2(c+dx) \operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} - \frac{3id^3 \operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4}$$

output
```
1/4*I*(d*x+c)^4/d-(d*x+c)^3*ln(1+exp(2*I*(b*x+a)))/b+3/2*I*d*(d*x+c)^2*pol
ylog(2,-exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*x+c)*polylog(3,-exp(2*I*(b*x+a)))
/b^3-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4
```

## 3.210.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95

$$\int (c+dx)^3 \tan(a+bx)\,dx = \frac{1}{4}i\left(\frac{(c+dx)^4}{d} + \frac{4i(c+dx)^3 \log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3d\left(2b^2(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right) + d\left(2ib(c+dx) \operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right) - d \operatorname{PolyLog}\left(4,-e^{2i(a\right.\right.\right.}{b^4}$$

input
```
Integrate[(c + d*x)^3*Tan[a + b*x],x]
```

output
```
(I/4)*((c + d*x)^4/d + ((4*I)*(c + d*x)^3*Log[1 + E^((2*I)*(a + b*x))])/b
+ (3*d*(2*b^2*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))] + d*((2*I)*b*(c
 + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))] - d*PolyLog[4, -E^((2*I)*(a + b*x
))])))/b^4)
```

### 3.210.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4202, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \tan(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3 \tan(a+bx)dx$$

$$\Big\downarrow \text{4202}$$

$$\frac{i(c+dx)^4}{4d}-2i\int \frac{e^{2i(a+bx)}(c+dx)^3}{1+e^{2i(a+bx)}}dx$$

$$\Big\downarrow \text{2620}$$

$$\frac{i(c+dx)^4}{4d}-2i\left(\frac{3id\int (c+dx)^2\log\left(1+e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{i(c+dx)^4}{4d}-2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int (c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

$$\Big\downarrow \text{7163}$$

$$2i\left(\frac{i(c+dx)^4}{4d}-\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\left(\frac{id\int\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1-\right.}{2b}\right.$$

$$\Big\downarrow \; 2720$$

$$2i\left(\frac{i(c+dx)^4}{4d}-\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\left(\frac{d\int e^{-2i(a+bx)}\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i(c}{}\right.$$

$$\Big\downarrow \; 7143$$

$$2i\left(\frac{i(c+dx)^4}{4d}-\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\left(\frac{d\operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^2\right.}{2b}\right.$$

input
```
Int[(c + d*x)^3*Tan[a + b*x],x]
```

output
```
((I/4)*(c + d*x)^4)/d - (2*I)*(((-1/2*I)*(c + d*x)^3*Log[1 + E^((2*I)*(a +
 b*x))])/b + (((3*I)/2)*d*(((I/2)*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*
x))])/b - (I*d*(((-1/2*I)*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/b +
(d*PolyLog[4, -E^((2*I)*(a + b*x))])/(4*b^2)))/b))/b)
```

3.210. $\int (c+dx)^3\tan(a+bx)\,dx$

### 3.210.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

### 3.210.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(113) = 226.

Time = 1.22 (sec) , antiderivative size = 432, normalized size of antiderivative = 3.27

| method | result |
|---|---|
| risch | $\frac{6id\,c^2xa}{b}+\frac{3ic\,d^2\,\mathrm{polylog}\left(2,-\mathrm{e}^{2i(xb+a)}\right)x}{b^2}-\frac{6ic\,d^2a^2x}{b^2}-ic^3x-\frac{ic^4}{4d}-\frac{c^3\ln\left(\mathrm{e}^{2i(xb+a)}+1\right)}{b}+\frac{2c^3\ln\left(\mathrm{e}^{i(xb+a)}\right)}{b}+\frac{6c\,d^2a^2}{}$ |

input
```
int((d*x+c)^3*sec(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
6*I/b*d*c^2*x*a+3*I/b^2*c*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-6*I/b^2*c*d^2
*a^2*x-I*c^3*x-1/4*I/d*c^4+3/2*I*d*c^2*x^2-1/b*c^3*ln(exp(2*I*(b*x+a))+1)+
1/4*I*d^3*x^4+3/2*I/b^4*d^3*a^4-3/2/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))
-1/b*d^3*ln(exp(2*I*(b*x+a))+1)*x^3-3/2/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)
))*x+I*d^2*c*x^3-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+6/b^3*c*d^2*a^
2*ln(exp(I*(b*x+a)))-6/b^2*c^2*d*a*ln(exp(I*(b*x+a)))+2/b*c^3*ln(exp(I*(b*
x+a)))-3/b*d*c^2*ln(exp(2*I*(b*x+a))+1)*x-3/b*c*d^2*ln(exp(2*I*(b*x+a))+1)
*x^2+3*I/b^2*d*c^2*a^2+3/2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2-4*I/
b^3*c*d^2*a^3+3/2*I/b^2*d*c^2*polylog(2,-exp(2*I*(b*x+a)))+2*I/b^3*d^3*a^3
*x-2/b^4*d^3*a^3*ln(exp(I*(b*x+a)))
```

### 3.210.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 974 vs. 2(109) = 218.

Time = 0.29 (sec) , antiderivative size = 974, normalized size of antiderivative = 7.38

$$\int (c+dx)^3\tan(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(6*I*d^3*polylog(4, I*cos(b*x + a) + sin(b*x + a)) - 6*I*d^3*polylog(4
, I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) + si
n(b*x + a)) + 6*I*d^3*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - 3*(I*b^
2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(I*cos(b*x + a) + sin(b*x
+ a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(I*cos(b*x
 + a) - sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)
*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*
x + I*b^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^3*c^3 - 3*a*b^
2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*sin(b*x + a) + I)
- (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I
*sin(b*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1
) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^3*d^3*x^3
+ 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^
3)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^
2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*
x + a) - sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 6*...
```

### 3.210.6 Sympy [F]

$$\int (c+dx)^3 \tan(a+bx)\,dx = \int (c+dx)^3 \sin(a+bx) \sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*sec(b*x+a)*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**3*sin(a + b*x)*sec(a + b*x), x)
```

### 3.210.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(109) = 218$.

Time = 0.38 (sec) , antiderivative size = 497, normalized size of antiderivative = 3.77

$$\int (c+dx)^3 \tan(a+bx)\,dx = -\frac{6\,c^3\log\left(-\sin\left(bx+a\right)^2+1\right) - \frac{18\,ac^2d\log\left(-\sin(bx+a)^2+1\right)}{b} + \frac{18\,a^2cd^2\log\left(-\sin(bx+a)^2+1\right)}{b^2} - \frac{6\,a^3d^3\log\left(-\sin(bx+a)^2+\right.}{b^3}}{}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/12*(6*c^3*log(-sin(b*x + a)^2 + 1) - 18*a*c^2*d*log(-sin(b*x + a)^2 + 1
)/b + 18*a^2*c*d^2*log(-sin(b*x + a)^2 + 1)/b^2 - 6*a^3*d^3*log(-sin(b*x +
 a)^2 + 1)/b^3 + (-3*I*(b*x + a)^4*d^3 - 12*(I*b*c*d^2 - I*a*d^3)*(b*x + a
)^3 + 12*I*d^3*polylog(4, -e^(2*I*b*x + 2*I*a)) - 18*(I*b^2*c^2*d - 2*I*a*
b*c*d^2 + I*a^2*d^3)*(b*x + a)^2 - 4*(-4*I*(b*x + a)^3*d^3 + 9*(-I*b*c*d^2
 + I*a*d^3)*(b*x + a)^2 + 9*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*(b*
x + a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 6*(3*I*b^2*c^2*d
 - 6*I*a*b*c*d^2 + 4*I*(b*x + a)^2*d^3 + 3*I*a^2*d^3 + 6*(I*b*c*d^2 - I*a*
d^3)*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*a)) + 2*(4*(b*x + a)^3*d^3 + 9*(b*
c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x +
a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)
+ 6*(3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*polylog(3, -e^(2*I*b*x + 2*I*a
)))/b^3)/b
```

### 3.210.8 Giac [F]

$$\int (c+dx)^3 \tan(a+bx)\,dx = \int (dx+c)^3 \sec\left(bx+a\right)\sin\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*sec(b*x + a)*sin(b*x + a), x)
```

### 3.210.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \tan(a+bx)\,dx = \int \frac{\sin(a+bx)\,(c+dx)^3}{\cos(a+bx)}\,dx$$

input
```
int((sin(a + b*x)*(c + d*x)^3)/cos(a + b*x),x)
```

output
```
int((sin(a + b*x)*(c + d*x)^3)/cos(a + b*x), x)
```

# 3.211 $\int (c+dx)^2 \tan(a+bx)\,dx$

3.211.1 Optimal result . . . 1685
3.211.2 Mathematica [A] (verified) . . . 1685
3.211.3 Rubi [A] (verified) . . . 1686
3.211.4 Maple [B] (verified) . . . 1688
3.211.5 Fricas [B] (verification not implemented) . . . 1688
3.211.6 Sympy [F] . . . 1689
3.211.7 Maxima [B] (verification not implemented) . . . 1689
3.211.8 Giac [F] . . . 1690
3.211.9 Mupad [F(-1)] . . . 1690

## 3.211.1 Optimal result

Integrand size = 14, antiderivative size = 96

$$\int (c+dx)^2 \tan(a+bx)\,dx = \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2 \log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} - \frac{d^2 \operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}$$

output
```
1/3*I*(d*x+c)^3/d-(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b+I*d*(d*x+c)*polylog(2
,-exp(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3
```

## 3.211.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.04

$$\int (c+dx)^2 \tan(a+bx)\,dx = \frac{2ib^2(c+dx)^2\left(b(c+dx)+3id\log\left(1+e^{2i(a+bx)}\right)\right)+6ibd^2(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)-3d^3\operatorname{PolyLog}}{6b^3d}$$

input
```
Integrate[(c + d*x)^2*Tan[a + b*x],x]
```

output
```
((2*I)*b^2*(c + d*x)^2*(b*(c + d*x) + (3*I)*d*Log[1 + E^((2*I)*(a + b*x))]
) + (6*I)*b*d^2*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))] - 3*d^3*PolyLog
[3, -E^((2*I)*(a + b*x))])/(6*b^3*d)
```

### 3.211.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \tan(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (c+dx)^2 \tan(a+bx)dx$$

$$\Big\downarrow \; 4202$$

$$\frac{i(c+dx)^3}{3d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}dx$$

$$\Big\downarrow \; 2620$$

$$\frac{i(c+dx)^3}{3d} - 2i\left(\frac{id\int (c+dx)\log\left(1+e^{2i(a+bx)}\right)dx}{b} - \frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

$$\Big\downarrow \; 3011$$

$$\frac{i(c+dx)^3}{3d} - 2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{2i(a+bx)})}{2b} - \frac{id\int \operatorname{PolyLog}(2,-e^{2i(a+bx)})dx}{2b}\right)}{b} - \frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

$$\Big\downarrow \; 2720$$

$$\frac{i(c+dx)^3}{3d} - 2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{2i(a+bx)})}{2b} - \frac{d\int e^{-2i(a+bx)}\operatorname{PolyLog}(2,-e^{2i(a+bx)})de^{2i(a+bx)}}{4b^2}\right)}{b} - \frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{i(c+dx)^3}{3d} - 2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{2i(a+bx)})}{2b} - \frac{d\operatorname{PolyLog}(3,-e^{2i(a+bx)})}{4b^2}\right)}{b} - \frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

input
```
Int[(c + d*x)^2*Tan[a + b*x],x]
```

output
```
((I/3)*(c + d*x)^3)/d - (2*I)*(((-1/2*I)*(c + d*x)^2*Log[1 + E^((2*I)*(a +
 b*x))])/b + (I*d*(((I/2)*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b -
(d*PolyLog[3, -E^((2*I)*(a + b*x))])/(4*b^2)))/b)
```

### 3.211.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.211.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(84) = 168$.

Time = 1.19 (sec) , antiderivative size = 266, normalized size of antiderivative = 2.77

| method | result |
|---|---|
| risch | $\frac{4icdxa}{b} - ic^2x + \frac{id^2x^3}{3} - \frac{2id^2a^2x}{b^2} + \frac{2icd\,a^2}{b^2} - \frac{4id^2a^3}{3b^3} - \frac{4cda\ln(\mathrm{e}^{i(xb+a)})}{b^2} - \frac{2cd\ln(\mathrm{e}^{2i(xb+a)}+1)x}{b} + \frac{id^2\operatorname{polylog}(2,-\mathrm{e}}{b^2}$ |

input
```
int((d*x+c)^2*sec(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
4*I/b*c*d*x*a-I*c^2*x+1/3*I*d^2*x^3-2*I/b^2*d^2*a^2*x+2*I/b^2*c*d*a^2-4/3*
I/b^3*d^2*a^3-4/b^2*c*d*a*ln(exp(I*(b*x+a)))-2/b*c*d*ln(exp(2*I*(b*x+a))+1
)*x+I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-1/b*d^2*ln(exp(2*I*(b*x+a))+1
)*x^2-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3-1/3*I/d*c^3+I*d*c*x^2+I/b^2
*c*d*polylog(2,-exp(2*I*(b*x+a)))+2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))-1/b*c^2
*ln(exp(2*I*(b*x+a))+1)+2/b*c^2*ln(exp(I*(b*x+a)))
```

### 3.211.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(81) = 162$.

Time = 0.30 (sec) , antiderivative size = 594, normalized size of antiderivative = 6.19

$$\int (c+dx)^2\tan(a+bx)\,dx = -\frac{2\,d^2\mathrm{polylog}(3, i\,\cos{(bx+a)}+\sin{(bx+a)}) + 2\,d^2\mathrm{polylog}(3, i\,\cos{(bx+a)}-\sin{(bx+a)}) + 2\,d^2\mathrm{polylo}}{}$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 2*d^2*polylog(3, I
*cos(b*x + a) - sin(b*x + a)) + 2*d^2*polylog(3, -I*cos(b*x + a) + sin(b*x
 + a)) + 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 2*(I*b*d^2*x +
 I*b*c*d)*dilog(I*cos(b*x + a) + sin(b*x + a)) + 2*(-I*b*d^2*x - I*b*c*d)*
dilog(I*cos(b*x + a) - sin(b*x + a)) + 2*(-I*b*d^2*x - I*b*c*d)*dilog(-I*c
os(b*x + a) + sin(b*x + a)) + 2*(I*b*d^2*x + I*b*c*d)*dilog(-I*cos(b*x + a
) - sin(b*x + a)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) + I*s
in(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) - I*si
n(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*
cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d
- a^2*d^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c
*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2
*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*
x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*
x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*
x + a) + I))/b^3
```

### 3.211.6 Sympy [F]

$$\int (c+dx)^2 \tan(a+bx)\,dx = \int (c+dx)^2 \sin(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*sec(b*x+a)*sin(b*x+a),x)
```

output
```
Integral((c + d*x)**2*sin(a + b*x)*sec(a + b*x), x)
```

### 3.211.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(81) = 162$.

Time = 0.39 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.96

$$\int (c+dx)^2 \tan(a+bx)\,dx =$$

$$-\frac{3\,c^2\log\left(-\sin\left(bx+a\right)^2+1\right) - \frac{6\,acd\log\left(-\sin(bx+a)^2+1\right)}{b} + \frac{3\,a^2d^2\log\left(-\sin(bx+a)^2+1\right)}{b^2} + \frac{-2i\,(bx+a)^3d^2-6\,(i\,bcd-i\,ad^2)}{\cdots}}{\cdots}$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/6*(3*c^2*log(-sin(b*x + a)^2 + 1) - 6*a*c*d*log(-sin(b*x + a)^2 + 1)/b
+ 3*a^2*d^2*log(-sin(b*x + a)^2 + 1)/b^2 + (-2*I*(b*x + a)^3*d^2 - 6*(I*b*
c*d - I*a*d^2)*(b*x + a)^2 + 3*d^2*polylog(3, -e^(2*I*b*x + 2*I*a)) - 6*(-
I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*arctan2(sin(2*b*x +
2*a), cos(2*b*x + 2*a) + 1) - 6*(I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*dilo
g(-e^(2*I*b*x + 2*I*a)) + 3*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a)
)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1))/b
^2)/b
```

### 3.211.8 Giac [F]

$$\int (c+dx)^2 \tan(a+bx)\,dx = \int (dx+c)^2 \sec(bx+a)\sin(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*sec(b*x + a)*sin(b*x + a), x)
```

### 3.211.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \tan(a+bx)\,dx = \int \frac{\sin(a+b\,x)\,(c+d\,x)^2}{\cos(a+b\,x)}\,dx$$

input
```
int((sin(a + b*x)*(c + d*x)^2)/cos(a + b*x),x)
```

output
```
int((sin(a + b*x)*(c + d*x)^2)/cos(a + b*x), x)
```

# 3.212 $\int (c+dx)\tan(a+bx)\,dx$

3.212.1 Optimal result . . . . . 1691
3.212.2 Mathematica [A] (verified) . . . . . 1691
3.212.3 Rubi [A] (verified) . . . . . 1692
3.212.4 Maple [B] (verified) . . . . . 1693
3.212.5 Fricas [B] (verification not implemented) . . . . . 1694
3.212.6 Sympy [F] . . . . . 1694
3.212.7 Maxima [B] (verification not implemented) . . . . . 1695
3.212.8 Giac [F] . . . . . 1695
3.212.9 Mupad [B] (verification not implemented) . . . . . 1695

## 3.212.1 Optimal result

Integrand size = 12, antiderivative size = 66

$$\int (c+dx)\tan(a+bx)\,dx=\frac{i(c+dx)^2}{2d}-\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}$$

output
```
1/2*I*(d*x+c)^2/d-(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b+1/2*I*d*polylog(2,-exp(
2*I*(b*x+a)))/b^2
```

## 3.212.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int (c+dx)\tan(a+bx)\,dx=\frac{1}{2}idx^2-\frac{dx\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{c\log(\cos(a+bx))}{b}+\frac{id\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}$$

input
```
Integrate[(c + d*x)*Tan[a + b*x],x]
```

output
```
(I/2)*d*x^2 - (d*x*Log[1 + E^((2*I)*(a + b*x))])/b - (c*Log[Cos[a + b*x]])
/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2
```

### 3.212.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\tan(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (c+dx)\tan(a+bx)dx$$

$$\Big\downarrow \; 4202$$

$$\frac{i(c+dx)^2}{2d}-2i\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}dx$$

$$\Big\downarrow \; 2620$$

$$\frac{i(c+dx)^2}{2d}-2i\left(\frac{id\int \log\left(1+e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

$$\Big\downarrow \; 2715$$

$$\frac{i(c+dx)^2}{2d}-2i\left(\frac{d\int e^{-2i(a+bx)}\log\left(1+e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

$$\Big\downarrow \; 2838$$

$$\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)$$

input
```
Int[(c + d*x)*Tan[a + b*x],x]
```

output
```
((I/2)*(c + d*x)^2)/d - (2*I)*(((-1/2*I)*(c + d*x)*Log[1 + E^((2*I)*(a + b
*x))])/b - (d*PolyLog[2, -E^((2*I)*(a + b*x))])/(4*b^2))
```

### 3.212.3.1 Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

## 3.212.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(56) = 112$.

Time = 1.03 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.86

| method | result |
|---|---|
| risch | $\frac{i d\,x^{2}}{2}-i c x-\frac{c \ln\left({\mathrm e}^{2 i \left(x b +a \right)}+1\right)}{b}+\frac{2 c \ln\left({\mathrm e}^{i \left(x b +a \right)}\right)}{b}+\frac{2 i d x a}{b}+\frac{i d\,a^{2}}{b^{2}}-\frac{d \ln\left({\mathrm e}^{2 i \left(x b +a \right)}+1\right) x}{b}+\frac{i d \,\operatorname{polylog}\left(2,-{\mathrm e}^{2 i \left(x b +a \right)}\right)}{2 b^{2}}-$ |

input
```
int((d*x+c)*sec(b*x+a)*sin(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*d*x^2-I*c*x-1/b*c*ln(exp(2*I*(b*x+a))+1)+2/b*c*ln(exp(I*(b*x+a)))+2*
I/b*d*x*a+I/b^2*d*a^2-1/b*d*ln(exp(2*I*(b*x+a))+1)*x+1/2*I*d*polylog(2,-ex
p(2*I*(b*x+a)))/b^2-2/b^2*d*a*ln(exp(I*(b*x+a)))
```

### 3.212.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(53) = 106.

Time = 0.26 (sec) , antiderivative size = 310, normalized size of antiderivative = 4.70

$$\int (c+dx)\tan(a+bx)\,dx = \frac{-i\,d\mathrm{Li}_2(i\,\cos{(bx+a)}+\sin{(bx+a)})+i\,d\mathrm{Li}_2(i\,\cos{(bx+a)}-\sin{(bx+a)})+i\,d\mathrm{Li}_2(-i\,\cos{(bx+a)}+\mathrm{si}}{}$$

input
```
integrate((d*x+c)*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(-I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*dilog(I*cos(b*x + a)
- sin(b*x + a)) + I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d*dilog(-I
*cos(b*x + a) - sin(b*x + a)) - (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x +
 a) + I) - (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b*d*x + a
*d)*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*x + a*d)*log(I*cos(b*x +
 a) - sin(b*x + a) + 1) - (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x + a)
 + 1) - (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b*c - a*d
)*log(-cos(b*x + a) + I*sin(b*x + a) + I) - (b*c - a*d)*log(-cos(b*x + a)
- I*sin(b*x + a) + I))/b^2
```

### 3.212.6 Sympy [F]

$$\int (c+dx)\tan(a+bx)\,dx = \int (c+dx)\sin{(a+bx)}\sec{(a+bx)}\,dx$$

input
```
integrate((d*x+c)*sec(b*x+a)*sin(b*x+a),x)
```

output
```
Integral((c + d*x)*sin(a + b*x)*sec(a + b*x), x)
```

### 3.212.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 115 vs. 2(53) = 106.

Time = 0.37 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.74

$$\int (c+dx)\tan(a+bx)\,dx = -\frac{-i\,b^2dx^2 - 2i\,b^2cx - 2\left(-i\,bdx - i\,bc\right)\arctan\left(\sin\left(2\,bx+2\,a\right),\cos\left(2\,bx+2\,a\right)+1\right) - i\,d\mathrm{Li}_2\left(-e^{(2i\,bx+2i\,a}\right.}{2\,b^2}$$

input
```
integrate((d*x+c)*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(-I*b^2*d*x^2 - 2*I*b^2*c*x - 2*(-I*b*d*x - I*b*c)*arctan2(sin(2*b*x
+ 2*a), cos(2*b*x + 2*a) + 1) - I*d*dilog(-e^(2*I*b*x + 2*I*a)) + (b*d*x +
 b*c)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1
))/b^2
```

### 3.212.8 Giac [F]

$$\int (c+dx)\tan(a+bx)\,dx = \int (dx+c)\sec\left(bx+a\right)\sin\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*sec(b*x + a)*sin(b*x + a), x)
```

### 3.212.9 Mupad [B] (verification not implemented)

Time = 25.46 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.24

$$\int (c+dx)\tan(a+bx)\,dx = \frac{c\,\ln\left(\tan(a+b\,x)^2+1\right)}{2\,b} - \frac{d\left(\pi\,\ln\left(\cos\left(b\,x\right)\right) + \mathrm{polylog}\left(2,-\mathrm{e}^{-a\,2\mathrm{i}}\,\mathrm{e}^{-b\,x\,2\mathrm{i}}\right)\,1\mathrm{i} - \pi\,\ln\left(\mathrm{e}^{-a\,2\mathrm{i}}\,\mathrm{e}^{-b\,x\,2\mathrm{i}}+1\right) + 2\,a\,\ln\left(\mathrm{e}^{-a\,2\mathrm{i}}\,\mathrm{e}^{-b\,x\,2\mathrm{i}}+1\right) - \right.}{2\,b^2}$$

input
```
int((sin(a + b*x)*(c + d*x))/cos(a + b*x),x)
```

output
```
(c*log(tan(a + b*x)^2 + 1))/(2*b) - (d*(polylog(2, -exp(-a*2i)*exp(-b*x*2i
))*1i - pi*log(exp(b*x*2i) + 1) - pi*log(exp(-a*2i)*exp(-b*x*2i) + 1) + 2*
a*log(exp(-a*2i)*exp(-b*x*2i) + 1) + pi*log(cos(b*x)) + b^2*x^2*1i - log(c
os(a + b*x))*(2*a - pi) + 2*b*x*log(exp(-a*2i)*exp(-b*x*2i) + 1) + a*b*x*2
i))/(2*b^2)
```

## 3.212. $\int (c+dx)\tan(a+bx)\,dx$

# 3.213 $\int \frac{\tan(a+bx)}{c+dx} dx$

3.213.1 Optimal result . . . . . 1697
3.213.2 Mathematica [N/A] . . . . . 1697
3.213.3 Rubi [N/A] . . . . . 1698
3.213.4 Maple [N/A] (verified) . . . . . 1699
3.213.5 Fricas [N/A] . . . . . 1699
3.213.6 Sympy [N/A] . . . . . 1699
3.213.7 Maxima [N/A] . . . . . 1700
3.213.8 Giac [N/A] . . . . . 1700
3.213.9 Mupad [N/A] . . . . . 1700

## 3.213.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\tan(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\tan(a+bx)}{c+dx},x\right)$$

output
```
Unintegrable(tan(b*x+a)/(d*x+c),x)
```

## 3.213.2 Mathematica [N/A]

Not integrable

Time = 5.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tan(a+bx)}{c+dx}\,dx = \int \frac{\tan(a+bx)}{c+dx}\,dx$$

input
```
Integrate[Tan[a + b*x]/(c + d*x),x]
```

output
```
Integrate[Tan[a + b*x]/(c + d*x), x]
```

### 3.213.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(a+bx)}{c+dx} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(a+bx)}{c+dx}dx$$
$$\Big\downarrow \text{4222}$$
$$\int \frac{\tan(a+bx)}{c+dx}dx$$

input
```
Int[Tan[a + b*x]/(c + d*x),x]
```

output
```
$Aborted
```

#### 3.213.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

### 3.213.4 Maple [N/A] (verified)

Not integrable

Time = 0.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{\sec (xb+a)\sin (xb+a)}{dx+c}dx$$

input
```
int(sec(b*x+a)*sin(b*x+a)/(d*x+c),x)
```

output
```
int(sec(b*x+a)*sin(b*x+a)/(d*x+c),x)
```

### 3.213.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec (bx+a)\sin (bx+a)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)*sin(b*x + a)/(d*x + c), x)
```

### 3.213.6 Sympy [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \frac{\tan(a+bx)}{c+dx}\,dx = \int \frac{\sin (a+bx)\sec (a+bx)}{c+dx}\,dx$$

input
```
integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)*sec(a + b*x)/(c + d*x), x)
```

### 3.213.7 Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)\sin(bx+a)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)*sin(b*x + a)/(d*x + c), x)
```

### 3.213.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)\sin(bx+a)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)*sin(b*x + a)/(d*x + c), x)
```

### 3.213.9 Mupad [N/A]

Not integrable

Time = 25.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{\tan(a+bx)}{c+dx}\,dx = \int \frac{\sin(a+b\,x)}{\cos(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(sin(a + b*x)/(cos(a + b*x)*(c + d*x)),x)
```

output
```
int(sin(a + b*x)/(cos(a + b*x)*(c + d*x)), x)
```

# 3.214 $\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx$

3.214.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1701
3.214.2 Mathematica [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1701
3.214.3 Rubi [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1702
3.214.4 Maple [N/A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1703
3.214.5 Fricas [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1703
3.214.6 Sympy [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1703
3.214.7 Maxima [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1704
3.214.8 Giac [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1704
3.214.9 Mupad [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1704

## 3.214.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\tan(a+bx)}{(c+dx)^2},x\right)$$

output
```
Unintegrable(tan(b*x+a)/(d*x+c)^2,x)
```

## 3.214.2 Mathematica [N/A]

Not integrable

Time = 3.48 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[Tan[a + b*x]/(c + d*x)^2,x]
```

output
```
Integrate[Tan[a + b*x]/(c + d*x)^2, x]
```

## 3.214.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(a+bx)}{(c+dx)^2} dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{\tan(a+bx)}{(c+dx)^2} dx$$
$$\Big\downarrow \ 4222$$
$$\int \frac{\tan(a+bx)}{(c+dx)^2} dx$$

input
```
Int[Tan[a + b*x]/(c + d*x)^2,x]
```

output
```
$Aborted
```

### 3.214.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

### 3.214.4 Maple [N/A] (verified)

Not integrable

Time = 0.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{\sec(xb+a)\sin(xb+a)}{(dx+c)^2}dx$$

input
```
int(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x)
```

output
```
int(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x)
```

### 3.214.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.36

$$\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec(bx+a)\sin(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)*sin(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.214.6 Sympy [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)*sec(a + b*x)/(c + d*x)**2, x)
```

### 3.214.7 Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec(bx+a)\sin(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(sec(b*x + a)*sin(b*x + a)/(d*x + c)^2, x)
```

### 3.214.8 Giac [N/A]

Not integrable

Time = 2.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec(bx+a)\sin(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)*sin(b*x + a)/(d*x + c)^2, x)
```

### 3.214.9 Mupad [N/A]

Not integrable

Time = 26.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(a+b\,x)}{\cos(a+b\,x)\,(c+d\,x)^2}\,dx$$

input
```
int(sin(a + b*x)/(cos(a + b*x)*(c + d*x)^2),x)
```

output
```
int(sin(a + b*x)/(cos(a + b*x)*(c + d*x)^2), x)
```

# 3.215 $\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx$

3.215.1 Optimal result . . . . . 1705
3.215.2 Mathematica [N/A] . . . . . 1705
3.215.3 Rubi [N/A] . . . . . 1706
3.215.4 Maple [N/A] (verified) . . . . . 1708
3.215.5 Fricas [N/A] . . . . . 1708
3.215.6 Sympy [N/A] . . . . . 1708
3.215.7 Maxima [N/A] . . . . . 1709
3.215.8 Giac [N/A] . . . . . 1709
3.215.9 Mupad [N/A] . . . . . 1709

## 3.215.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx = \frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b} - \frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)}{2b} + \text{Int}((c+dx)^m\sec(a+bx),x)$$

output
```
1/2*I*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c
)/d)^m)-1/2*I*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*
(d*x+c)/d)^m)+Unintegrable((d*x+c)^m*sec(b*x+a),x)
```

## 3.215.2 Mathematica [N/A]

Not integrable

Time = 11.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx = \int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x], x]
```

### 3.215.3 Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4907, 3042, 3788, 26, 2612, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\tan(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{4907}$$

$$\int (c+dx)^m\sec(a+bx)dx-\int (c+dx)^m\cos(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m\csc\left(a+bx+\frac{\pi}{2}\right)dx-\int (c+dx)^m\sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3788}$$

$$-\frac{1}{2}i\int -ie^{-i(a+bx)}(c+dx)^m dx+\frac{1}{2}i\int ie^{i(a+bx)}(c+dx)^m dx+\int (c+dx)^m\csc\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{26}$$

$$-\frac{1}{2}\int e^{-i(a+bx)}(c+dx)^m dx-\frac{1}{2}\int e^{i(a+bx)}(c+dx)^m dx+\int (c+dx)^m\csc\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{2612}$$

$$\int (c+dx)^m\csc\left(a+bx+\frac{\pi}{2}\right)dx+\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}-\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}$$

$$\Big\downarrow \text{4680}$$

$$\int (c+dx)^m\sec(a+bx)dx+\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}-\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}$$

input
```
Int[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x],x]
```

output
```
$Aborted
```

### 3.215.3.1 Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

### 3.215.4 Maple [N/A] (verified)

Not integrable

Time = 0.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (dx+c)^m \sec(xb+a)\sin(xb+a)^2\,dx$$

input
```
int((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x)
```

### 3.215.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\sin(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*(d*x + c)^m*sec(b*x + a), x)
```

### 3.215.6 Sympy [N/A]

Not integrable

Time = 121.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx = \int (c+dx)^m \sin^2(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*sec(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*sin(a + b*x)**2*sec(a + b*x), x)
```

### 3.215.7 Maxima [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\sin(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a)^2, x)
```

### 3.215.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\sin(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a)^2, x)
```

### 3.215.9 Mupad [N/A]

Not integrable

Time = 25.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx = \int \frac{\sin(a+b\,x)^2\,(c+d\,x)^m}{\cos(a+b\,x)}\,dx$$

input
```
int((sin(a + b*x)^2*(c + d*x)^m)/cos(a + b*x),x)
```

output
```
int((sin(a + b*x)^2*(c + d*x)^m)/cos(a + b*x), x)
```

# 3.216 $\int (c+dx)^3 \sin(a+bx) \tan(a+bx) \, dx$


3.216.1 Optimal result . . . 1710
3.216.2 Mathematica [B] (verified) . . . 1711
3.216.3 Rubi [A] (verified) . . . 1712
3.216.4 Maple [B] (verified) . . . 1717
3.216.5 Fricas [B] (verification not implemented) . . . 1718
3.216.6 Sympy [F] . . . 1718
3.216.7 Maxima [B] (verification not implemented) . . . 1719
3.216.8 Giac [F] . . . 1719
3.216.9 Mupad [F(-1)] . . . 1720


## 3.216.1 Optimal result

Integrand size = 20, antiderivative size = 275

$$\begin{aligned}\int (c+dx)^3 \sin(a+bx) \tan(a+bx) \, dx = & -\frac{2i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b} \\ & + \frac{6d^3 \cos(a+bx)}{b^4} - \frac{3d(c+dx)^2 \cos(a+bx)}{b^2} \\ & + \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} \\ & - \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} \\ & - \frac{6d^2(c+dx) \operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} \\ & + \frac{6d^2(c+dx) \operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3} \\ & - \frac{6id^3 \operatorname{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^4} \\ & + \frac{6id^3 \operatorname{PolyLog}\left(4,ie^{i(a+bx)}\right)}{b^4} \\ & + \frac{6d^2(c+dx) \sin(a+bx)}{b^3} - \frac{(c+dx)^3 \sin(a+bx)}{b}\end{aligned}$$

output
```
-2*I*(d*x+c)^3*arctan(exp(I*(b*x+a)))/b+6*d^3*cos(b*x+a)/b^4-3*d*(d*x+c)^2
*cos(b*x+a)/b^2+3*I*d*(d*x+c)^2*polylog(2,-I*exp(I*(b*x+a)))/b^2-3*I*d*(d*
x+c)^2*polylog(2,I*exp(I*(b*x+a)))/b^2-6*d^2*(d*x+c)*polylog(3,-I*exp(I*(b
*x+a)))/b^3+6*d^2*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))/b^3-6*I*d^3*polylog(
4,-I*exp(I*(b*x+a)))/b^4+6*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4+6*d^2*(d*
x+c)*sin(b*x+a)/b^3-(d*x+c)^3*sin(b*x+a)/b
```

### 3.216.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 557 vs. 2(275) = 550.

Time = 1.57 (sec) , antiderivative size = 557, normalized size of antiderivative = 2.03

$$\int (c+dx)^3 \sin(a+bx)\tan(a+bx)\,dx = -\frac{2ib^3c^3\arctan\left(e^{i(a+bx)}\right)+3b^2c^2d\cos(a+bx)-6d^3\cos(a+bx)+6b^2cd^2x\cos(a+bx)+3b^2d^3x^2\cos(a+\cdots}{\cdots}$$

input
```
Integrate[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x],x]
```

output
```
-(((2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^2*c^2*d*Cos[a + b*x] - 6*d^
3*Cos[a + b*x] + 6*b^2*c*d^2*x*Cos[a + b*x] + 3*b^2*d^3*x^2*Cos[a + b*x] -
 3*b^3*c^2*d*x*Log[1 - I*E^(I*(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 - I*E^(I
*(a + b*x))] - b^3*d^3*x^3*Log[1 - I*E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[
1 + I*E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + b^3*
d^3*x^3*Log[1 + I*E^(I*(a + b*x))] - (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-
I)*E^(I*(a + b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))
] + 6*b*c*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, (-I)
*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, I*E^(I*(a + b*x))] - 6*b*d^3*x*Po
lyLog[3, I*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] -
 (6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))] + b^3*c^3*Sin[a + b*x] - 6*b*c*d^
2*Sin[a + b*x] + 3*b^3*c^2*d*x*Sin[a + b*x] - 6*b*d^3*x*Sin[a + b*x] + 3*b
^3*c*d^2*x^2*Sin[a + b*x] + b^3*d^3*x^3*Sin[a + b*x])/b^4)
```

### 3.216.3 Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.09, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {4907, 3042, 3777, 25, 3042, 3777, 3042, 3777, 25, 3042, 3118, 4669, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\sin(a+bx)\tan(a+bx)\,dx$$

$$\Big\downarrow \text{4907}$$

$$\int (c+dx)^3\sec(a+bx)dx-\int (c+dx)^3\cos(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3\csc\left(a+bx+\frac{\pi}{2}\right)dx-\int (c+dx)^3\sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$-\frac{3d\int -(c+dx)^2\sin(a+bx)dx}{b}+\int (c+dx)^3\csc\left(a+bx+\frac{\pi}{2}\right)dx-\frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{3d\int (c+dx)^2\sin(a+bx)dx}{b}+\int (c+dx)^3\csc\left(a+bx+\frac{\pi}{2}\right)dx-\frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d\int (c+dx)^2\sin(a+bx)dx}{b}+\int (c+dx)^3\csc\left(a+bx+\frac{\pi}{2}\right)dx-\frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{3d\left(\frac{2d\int (c+dx)\cos(a+bx)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}+\int (c+dx)^3\csc\left(a+bx+\frac{\pi}{2}\right)dx-\frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3\csc\left(a+bx+\frac{\pi}{2}\right)dx+\frac{3d\left(\frac{2d\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right)dx+\frac{3d\left(\frac{2d\left(\frac{d\int -\sin(a+bx)dx}{b}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right)dx+\frac{3d\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int \sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right)dx+\frac{3d\left(\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int \sin(a+bx)dx}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\Big\downarrow \text{3118}$$

$$\int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right)dx+\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{3d\int (c+dx)^2\log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{3d\int (c+dx)^2\log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\Big\downarrow \text{3011}$$

3.216. $\int (c+dx)^3\sin(a+bx)\tan(a+bx)\,dx$

$$\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{2id\int(c+dx)\operatorname{PolyLog}(2,-ie^{i(a+bx)})dx}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}(2,ie^{i(a+bx)})}{b}-\frac{2id\int(c+dx)\operatorname{PolyLog}(2,ie^{i(a+bx)})dx}{b}\right)}{b}-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\Big\downarrow \ 7163$$

$$\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{2id\left(\frac{id\int\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}(2,ie^{i(a+bx)})}{b}-\frac{2id\left(\frac{id\int\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\Big\downarrow \ 2720$$

$$\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}(2,ie^{i(a+bx)})}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\sin(a+bx)}{b}$$

$$\Big\downarrow \ 7143$$

3.216. $\int (c+dx)^3 \sin(a+bx)\tan(a+bx)\,dx$

$$-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\text{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\text{PolyLog}\left(4,ie^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}+\frac{3d\left(\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^3\sin(a+bx)}{b}$$

input
```
Int[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x],x]
```

output
```
((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b + (3*d*((I*(c + d*x)^2*Poly
Log[2, (-I)*E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(c + d*x)*PolyLog[3, (-I
)*E^(I*(a + b*x))])/b + (d*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^2))/b))/b -
 (3*d*((I*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(
c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b + (d*PolyLog[4, I*E^(I*(a + b*x)
)])/b^2))/b))/b - ((c + d*x)^3*Sin[a + b*x])/b + (3*d*(-(((c + d*x)^2*Cos[
a + b*x])/b) + (2*d*((d*Cos[a + b*x])/b^2 + ((c + d*x)*Sin[a + b*x])/b))/b
))/b
```

### 3.216.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.216. $\int (c+dx)^3 \sin(a+bx) \tan(a+bx) \, dx$

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

### 3.216.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 900 vs. $2(250) = 500$.

Time = 2.51 (sec) , antiderivative size = 901, normalized size of antiderivative = 3.28

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 901 |

input
```
int((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
6/b^3*d^2*c*polylog(3,I*exp(I*(b*x+a)))-1/b^4*a^3*d^3*ln(1+I*exp(I*(b*x+a)
))+6/b^3*d^3*polylog(3,I*exp(I*(b*x+a)))*x-6/b^3*d^2*c*polylog(3,-I*exp(I*
(b*x+a)))-1/b*d^3*ln(1+I*exp(I*(b*x+a)))*x^3+1/b*d^3*ln(1-I*exp(I*(b*x+a))
)*x^3+1/b^4*a^3*d^3*ln(1-I*exp(I*(b*x+a)))-6/b^3*d^3*polylog(3,-I*exp(I*(b
*x+a)))*x-2*I/b*c^3*arctan(exp(I*(b*x+a)))-6*I*d^3*polylog(4,-I*exp(I*(b*x
+a)))/b^4+6*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4-6*I/b^3*c*d^2*a^2*arctan
(exp(I*(b*x+a)))+6*I/b^2*c^2*d*a*arctan(exp(I*(b*x+a)))-6*I/b^2*d^2*c*poly
log(2,I*exp(I*(b*x+a)))*x+6*I/b^2*d^2*c*polylog(2,-I*exp(I*(b*x+a)))*x-1/2
*I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*
d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))
+1/2*I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-
6*b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+
a))+3*I/b^2*c^2*d*polylog(2,-I*exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,I*e
xp(I*(b*x+a)))-3/b*c^2*d*ln(1+I*exp(I*(b*x+a)))*x-3/b^2*c^2*d*ln(1+I*exp(I
*(b*x+a)))*a+3/b*c^2*d*ln(1-I*exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-I*exp(I*(
b*x+a)))*a+3/b^3*a^2*c*d^2*ln(1+I*exp(I*(b*x+a)))-3/b^3*a^2*c*d^2*ln(1-I*e
xp(I*(b*x+a)))+3/b*d^2*c*ln(1-I*exp(I*(b*x+a)))*x^2-3/b*d^2*c*ln(1+I*exp(I
*(b*x+a)))*x^2+2*I/b^4*d^3*a^3*arctan(exp(I*(b*x+a)))-3*I/b^2*d^3*polylog(
2,I*exp(I*(b*x+a)))*x^2+3*I/b^2*d^3*polylog(2,-I*exp(I*(b*x+a)))*x^2
```

### 3.216.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1075 vs. $2(237) = 474$.

Time = 0.31 (sec) , antiderivative size = 1075, normalized size of antiderivative = 3.91

$$\int (c+dx)^3 \sin(a+bx) \tan(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(6*I*d^3*polylog(4, I*cos(b*x + a) + sin(b*x + a)) + 6*I*d^3*polylog(4
, I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) + si
n(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - 6*(b^2*
d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a) - 3*(I*b^2*d^3*x
^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(I*cos(b*x + a) + sin(b*x + a)) -
 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(I*cos(b*x + a) -
sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-
I*cos(b*x + a) + sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b
^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (b^3*c^3 - 3*a*b^2*c^2*d
 + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*sin(b*x + a) + I) - (b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I*sin(b*
x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b^
3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^
2 + a^3*d^3)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3
*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(
-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b
^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a)
- sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*
log(-cos(b*x + a) + I*sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*...
```

### 3.216.6 Sympy [F]

$$\int (c+dx)^3 \sin(a+bx) \tan(a+bx) \, dx = \int (c+dx)^3 \sin^2(a+bx) \sec(a+bx) \, dx$$

input
```
integrate((d*x+c)**3*sec(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**3*sin(a + b*x)**2*sec(a + b*x), x)
```

### 3.216.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 934 vs. $2(237) = 474$.

Time = 0.52 (sec) , antiderivative size = 934, normalized size of antiderivative = 3.40

$$\int (c+dx)^3 \sin(a+bx) \tan(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(c^3*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a))
- 3*a*c^2*d*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a
))/b + 3*a^2*c*d^2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(
b*x + a))/b^2 - a^3*d^3*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2
*sin(b*x + a))/b^3 + (12*I*d^3*polylog(4, I*e^(I*b*x + I*a)) - 12*I*d^3*po
lylog(4, -I*e^(I*b*x + I*a)) - 2*(I*(b*x + a)^3*d^3 + 3*(I*b*c*d^2 - I*a*d
^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2*d^3)*(b*x + a))*a
rctan2(cos(b*x + a), sin(b*x + a) + 1) - 2*(I*(b*x + a)^3*d^3 + 3*(I*b*c*d
^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2*d^3)*(b
*x + a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 6*(b^2*c^2*d - 2*a*b*c
*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*co
s(b*x + a) - 6*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^
3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*dilog(I*e^(I*b*x + I*a)) - 6*(-I*b^
2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 +
I*a*d^3)*(b*x + a))*dilog(-I*e^(I*b*x + I*a)) + ((b*x + a)^3*d^3 + 3*(b*c*
d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a)
)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - ((b*x + a)^3
*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1)
+ 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, I*e^(I*b*x + I*a)) - ...
```

### 3.216.8 Giac [F]

$$\int (c+dx)^3 \sin(a+bx) \tan(a+bx)\,dx = \int (dx+c)^3 \sec(bx+a) \sin(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*sec(b*x + a)*sin(b*x + a)^2, x)
```

### 3.216.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3\sin(a+bx)\tan(a+bx)\,dx=\int \frac{\sin\left(a+b\,x\right)^2\,\left(c+d\,x\right)^3}{\cos\left(a+b\,x\right)}\,dx$$

input
```
int((sin(a + b*x)^2*(c + d*x)^3)/cos(a + b*x),x)
```

output
```
int((sin(a + b*x)^2*(c + d*x)^3)/cos(a + b*x), x)
```

# 3.217 $\int (c+dx)^2 \sin(a+bx)\tan(a+bx)\,dx$


3.217.1 Optimal result . . . 1721
3.217.2 Mathematica [A] (verified) . . . 1722
3.217.3 Rubi [A] (verified) . . . 1722
3.217.4 Maple [B] (verified) . . . 1726
3.217.5 Fricas [B] (verification not implemented) . . . 1726
3.217.6 Sympy [F] . . . 1727
3.217.7 Maxima [B] (verification not implemented) . . . 1727
3.217.8 Giac [F] . . . 1728
3.217.9 Mupad [F(-1)] . . . 1728


## 3.217.1 Optimal result

Integrand size = 20, antiderivative size = 186

$$\int (c+dx)^2 \sin(a+bx)\tan(a+bx)\,dx = -\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+dx)\cos(a+bx)}{b^2} + \frac{2id(c+dx)\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} - \frac{2id(c+dx)\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} - \frac{2d^2\,\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} + \frac{2d^2\,\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3} + \frac{2d^2\sin(a+bx)}{b^3} - \frac{(c+dx)^2\sin(a+bx)}{b}$$

output
```
-2*I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b-2*d*(d*x+c)*cos(b*x+a)/b^2+2*I*d*(
d*x+c)*polylog(2,-I*exp(I*(b*x+a)))/b^2-2*I*d*(d*x+c)*polylog(2,I*exp(I*(b
*x+a)))/b^2-2*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,I*exp(I
*(b*x+a)))/b^3+2*d^2*sin(b*x+a)/b^3-(d*x+c)^2*sin(b*x+a)/b
```

### 3.217.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.69

$$\int (c+dx)^2 \sin(a+bx) \tan(a+bx)\,dx =$$
$$-\frac{2ib^2c^2 \arctan\left(e^{i(a+bx)}\right) + 2bcd\cos(a+bx) + 2bd^2x\cos(a+bx) - 2b^2cdx\log\left(1-ie^{i(a+bx)}\right) - b^2d^2x^2\log}{}$$

input
```
Integrate[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x],x]
```

output
```
-(((2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b*c*d*Cos[a + b*x] + 2*b*d^2*
x*Cos[a + b*x] - 2*b^2*c*d*x*Log[1 - I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[
1 - I*E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x))] + b^2*d^2*
x^2*Log[1 + I*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*
(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))] + 2*d^2*Po
lyLog[3, (-I)*E^(I*(a + b*x))] - 2*d^2*PolyLog[3, I*E^(I*(a + b*x))] + b^2
*c^2*Sin[a + b*x] - 2*d^2*Sin[a + b*x] + 2*b^2*c*d*x*Sin[a + b*x] + b^2*d^
2*x^2*Sin[a + b*x])/b^3)
```

### 3.217.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4907, 3042, 3777, 25, 3042, 3777, 3042, 3117, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin(a+bx)\tan(a+bx)\,dx$$
$$\Big\downarrow \text{ 4907}$$
$$\int (c+dx)^2 \sec(a+bx)dx - \int (c+dx)^2 \cos(a+bx)dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int (c+dx)^2 \csc\left(a+bx+\frac{\pi}{2}\right)dx - \int (c+dx)^2 \sin\left(a+bx+\frac{\pi}{2}\right)dx$$
$$\Big\downarrow \text{ 3777}$$

$$-\frac{2d\int -((c+dx)\sin(a+bx))dx}{b}+\int (c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)dx-\frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{2d\int (c+dx)\sin(a+bx)dx}{b}+\int (c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)dx-\frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d\int (c+dx)\sin(a+bx)dx}{b}+\int (c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)dx-\frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{2d\left(\frac{d\int \cos(a+bx)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}+\int (c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)dx-\frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)dx+\frac{2d\left(\frac{d\int \sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\Big\downarrow \text{3117}$$

$$\int (c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)dx+\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{2d\int (c+dx)\log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{2d\int (c+dx)\log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\Big\downarrow \text{3011}$$

$$\frac{2d\left(\frac{i(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{id\int \text{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{id\int \text{PolyLog}\left(2,ie^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2720}$$

---

3.217. $\int (c+dx)^2\sin(a+bx)\tan(a+bx)\,dx$

$$\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\sin(a+bx)}{b}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{d\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{d\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^2}\right)}{b}+\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}-\frac{(c+dx)^2\sin(a+bx)}{b}$$

input
```
Int[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x],x]
```

output
```
((-2*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (2*d*((I*(c + d*x)*PolyLo
g[2, (-I)*E^(I*(a + b*x))])/b - (d*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^2))
/b - (2*d*((I*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b - (d*PolyLog[3, I
*E^(I*(a + b*x))])/b^2))/b - ((c + d*x)^2*Sin[a + b*x])/b + (2*d*(-(((c +
d*x)*Cos[a + b*x])/b) + (d*Sin[a + b*x])/b^2))/b
```

### 3.217.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.217. $\int (c+dx)^2 \sin(a+bx) \tan(a+bx) \, dx$

### 3.217.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(169) = 338.

Time = 2.19 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.75

| method | result |
|---|---|
| risch | $-\frac{2ic^2 \arctan(e^{i(xb+a)})}{b} - \frac{i(x^2d^2b^2+2b^2cdx-2ib\,d^2x+b^2c^2-2ibcd-2d^2)e^{-i(xb+a)}}{2b^3} + \frac{2cd\ln(1-ie^{i(xb+a)})a}{b^2} - \frac{a^2d^2\ln(1-ie^{i(x}}{b^3}$ |

input
```
int((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*I/b*c^2*arctan(exp(I*(b*x+a)))-1/2*I*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2-2
*I*b*d^2*x-2*d^2-2*I*b*c*d)/b^3*exp(-I*(b*x+a))+2/b^2*c*d*ln(1-I*exp(I*(b*
x+a)))*a-1/b^3*a^2*d^2*ln(1-I*exp(I*(b*x+a)))+1/b^3*a^2*d^2*ln(1+I*exp(I*(
b*x+a)))+2*d^2*polylog(3,I*exp(I*(b*x+a)))/b^3-2/b^2*c*d*ln(1+I*exp(I*(b*x
+a)))*a+1/b*d^2*ln(1-I*exp(I*(b*x+a)))*x^2+1/2*I*(x^2*d^2*b^2+2*b^2*c*d*x+
b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp(I*(b*x+a))+2*I/b^2*d^2*polylo
g(2,-I*exp(I*(b*x+a)))*x-2*I/b^3*d^2*a^2*arctan(exp(I*(b*x+a)))-1/b*d^2*ln
(1+I*exp(I*(b*x+a)))*x^2-2*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+2*I/b^2*c*
d*polylog(2,-I*exp(I*(b*x+a)))+4*I/b^2*c*d*a*arctan(exp(I*(b*x+a)))-2*I/b^
2*c*d*polylog(2,I*exp(I*(b*x+a)))-2*I/b^2*d^2*polylog(2,I*exp(I*(b*x+a)))*
x-2/b*c*d*ln(1+I*exp(I*(b*x+a)))*x+2/b*c*d*ln(1-I*exp(I*(b*x+a)))*x
```

### 3.217.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 656 vs. 2(160) = 320.

Time = 0.29 (sec) , antiderivative size = 656, normalized size of antiderivative = 3.53

$$\int (c+dx)^2 \sin(a+bx)\tan(a+bx)\,dx =$$

$$-\frac{2\,d^2\mathrm{polylog}(3, i\,\cos{(bx+a)} + \sin{(bx+a)}) - 2\,d^2\mathrm{polylog}(3, i\,\cos{(bx+a)} - \sin{(bx+a)}) + 2\,d^2\mathrm{polylo}}{}$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*polylog(3, I
*cos(b*x + a) - sin(b*x + a)) + 2*d^2*polylog(3, -I*cos(b*x + a) + sin(b*x
 + a)) - 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 4*(b*d^2*x + b
*c*d)*cos(b*x + a) + 2*(I*b*d^2*x + I*b*c*d)*dilog(I*cos(b*x + a) + sin(b*
x + a)) + 2*(I*b*d^2*x + I*b*c*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) + 2
*(-I*b*d^2*x - I*b*c*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + 2*(-I*b*d^
2*x - I*b*c*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^2*c^2 - 2*a*b*c*
d + a^2*d^2)*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*
c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2
*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) - sin(b*x
 + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(
b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^
2*d^2)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^
2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 2*(b^2*d^2*x^2 + 2*b^2*c*
d*x + b^2*c^2 - 2*d^2)*sin(b*x + a))/b^3
```

### 3.217.6 Sympy [F]

$$\int (c+dx)^2 \sin(a+bx)\tan(a+bx)\,dx = \int (c+dx)^2 \sin^2(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*sec(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*sin(a + b*x)**2*sec(a + b*x), x)
```

### 3.217.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(160) = 320$.

Time = 0.39 (sec) , antiderivative size = 516, normalized size of antiderivative = 2.77

$$\int (c+dx)^2 \sin(a+bx)\tan(a+bx)\,dx$$

$$= \frac{c^2(\log(\sin(bx+a)+1) - \log(\sin(bx+a)-1) - 2\sin(bx+a)) - \frac{2acd(\log(\sin(bx+a)+1)-\log(\sin(bx+a)-1)-2\sin}{b}}{}$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(c^2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a))
- 2*a*c*d*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a))
/b + a^2*d^2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x +
a))/b^2 + (4*d^2*polylog(3, I*e^(I*b*x + I*a)) - 4*d^2*polylog(3, -I*e^(I*
b*x + I*a)) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*arct
an2(cos(b*x + a), sin(b*x + a) + 1) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d -
I*a*d^2)*(b*x + a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 4*(b*c*d +
(b*x + a)*d^2 - a*d^2)*cos(b*x + a) - 4*(I*b*c*d + I*(b*x + a)*d^2 - I*a*d
^2)*dilog(I*e^(I*b*x + I*a)) - 4*(-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*di
log(-I*e^(I*b*x + I*a)) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*
log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - ((b*x + a)^2*d
^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*
sin(b*x + a) + 1) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d
^2)*sin(b*x + a))/b^2)/b
```

### 3.217.8 Giac [F]

$$\int (c+dx)^2 \sin(a+bx)\tan(a+bx)\,dx = \int (dx+c)^2 \sec(bx+a)\sin(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*sec(b*x + a)*sin(b*x + a)^2, x)
```

### 3.217.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \sin(a+bx)\tan(a+bx)\,dx = \int \frac{\sin(a+b\,x)^2\,(c+d\,x)^2}{\cos(a+b\,x)}\,dx$$

input
```
int((sin(a + b*x)^2*(c + d*x)^2)/cos(a + b*x),x)
```

output
```
int((sin(a + b*x)^2*(c + d*x)^2)/cos(a + b*x), x)
```

# 3.218 $\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx$

3.218.1 Optimal result . . . . . 1729
3.218.2 Mathematica [B] (verified) . . . . . 1729
3.218.3 Rubi [A] (verified) . . . . . 1730
3.218.4 Maple [A] (verified) . . . . . 1732
3.218.5 Fricas [B] (verification not implemented) . . . . . 1733
3.218.6 Sympy [F] . . . . . 1733
3.218.7 Maxima [F] . . . . . 1734
3.218.8 Giac [F] . . . . . 1734
3.218.9 Mupad [F(-1)] . . . . . 1734

## 3.218.1 Optimal result

Integrand size = 18, antiderivative size = 103

$$\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx = -\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)}{b^2} + \frac{id\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} - \frac{id\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)\sin(a+bx)}{b}$$

output
```
-2*I*(d*x+c)*arctan(exp(I*(b*x+a)))/b-d*cos(b*x+a)/b^2+I*d*polylog(2,-I*ex
p(I*(b*x+a)))/b^2-I*d*polylog(2,I*exp(I*(b*x+a)))/b^2-(d*x+c)*sin(b*x+a)/b
```

## 3.218.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 213 vs. 2(103) = 206.

Time = 0.50 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.07

$$\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx = \frac{c\,\text{arctanh}(\sin(a+bx))}{b} + \frac{d\left(\left(-a+\frac{\pi}{2}-bx\right)\left(\log\left(1-e^{i\left(-a+\frac{\pi}{2}-bx\right)}\right)-\log\left(1+e^{i\left(-a+\frac{\pi}{2}-bx\right)}\right)\right)-\left(-a+\frac{\pi}{2}\right)\log\left(\tan\left(\frac{1}{2}\left(-a+\frac{\pi}{2}-\right.\right.\right.}{b^2} - \frac{d\cos(bx)(\cos(a)+bx\sin(a))}{b^2} - \frac{d(bx\cos(a)-\sin(a))\sin(bx)}{b^2} - \frac{c\sin(a+bx)}{b}$$

input
```
Integrate[(c + d*x)*Sin[a + b*x]*Tan[a + b*x],x]
```

output
```
(c*ArcTanh[Sin[a + b*x]])/b + (d*((-a + Pi/2 - b*x)*(Log[1 - E^(I*(-a + Pi
/2 - b*x))] - Log[1 + E^(I*(-a + Pi/2 - b*x))]) - (-a + Pi/2)*Log[Tan[(-a
+ Pi/2 - b*x)/2]] + I*(PolyLog[2, -E^(I*(-a + Pi/2 - b*x))] - PolyLog[2, E
^(I*(-a + Pi/2 - b*x))])))/b^2 - (d*Cos[b*x]*(Cos[a] + b*x*Sin[a]))/b^2 -
(d*(b*x*Cos[a] - Sin[a])*Sin[b*x])/b^2 - (c*Sin[a + b*x])/b
```

### 3.218.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4907, 3042, 3777, 25, 3042, 3118, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx$$

$$\Big\downarrow \text{4907}$$

$$\int (c+dx)\sec(a+bx)dx - \int (c+dx)\cos(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)dx - \int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)dx - \frac{d\int -\sin(a+bx)dx}{b} - \frac{(c+dx)\sin(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)dx + \frac{d\int \sin(a+bx)dx}{b} - \frac{(c+dx)\sin(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)dx + \frac{d\int \sin(a+bx)dx}{b} - \frac{(c+dx)\sin(a+bx)}{b}$$

$$\Big\downarrow \text{3118}$$

$$\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)dx-\frac{d\cos(a+bx)}{b^2}-\frac{(c+dx)\sin(a+bx)}{b}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{d\int \log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{d\int \log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{d\cos(a+bx)}{b^2}-\frac{(c+dx)\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2715}$$

$$\frac{id\int e^{-i(a+bx)}\log\left(1-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{id\int e^{-i(a+bx)}\log\left(1+ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{d\cos(a+bx)}{b^2}-\frac{(c+dx)\sin(a+bx)}{b}$$

$$\Big\downarrow \text{2838}$$

$$-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}-\frac{d\cos(a+bx)}{b^2}-\frac{(c+dx)\sin(a+bx)}{b}$$

input
```
Int[(c + d*x)*Sin[a + b*x]*Tan[a + b*x],x]
```

output
```
((-2*I)*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b - (d*Cos[a + b*x])/b^2 + (I*d
*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))
])/b^2 - ((c + d*x)*Sin[a + b*x])/b
```

### 3.218.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.218. $\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

### 3.218.4 Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.75

| method | result |
|---|---|
| default | $\frac{\frac{da\sin(xb+a)}{b}-c\sin(xb+a)-\frac{d(\cos(xb+a)+(xb+a)\sin(xb+a))}{b}}{b}+\frac{-\frac{da\ln(\sec(xb+a)+\tan(xb+a))}{b}+c\ln(\sec(xb+a)+\tan(xb+a))+\frac{d\left(-(xb+a)\right.}{}}{}$ |
| risch | $\frac{i(dxb+cb+id)\mathrm{e}^{i(xb+a)}}{2b^2}-\frac{i(dxb+cb-id)\mathrm{e}^{-i(xb+a)}}{2b^2}-\frac{2ic\arctan\left(\mathrm{e}^{i(xb+a)}\right)}{b}-\frac{d\ln\left(1+i\mathrm{e}^{i(xb+a)}\right)x}{b}-\frac{d\ln\left(1+i\mathrm{e}^{i(xb+a)}\right)a}{b^2}+\frac{d\ln}{}$ |

input
```
int((d*x+c)*sec(b*x+a)*sin(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/b*d*a*sin(b*x+a)-c*sin(b*x+a)-1/b*d*(cos(b*x+a)+(b*x+a)*sin(b*x+a))
)+1/b*(-1/b*d*a*ln(sec(b*x+a)+tan(b*x+a))+c*ln(sec(b*x+a)+tan(b*x+a))+1/b*
d*(-(b*x+a)*ln(1+I*exp(I*(b*x+a)))+(b*x+a)*ln(1-I*exp(I*(b*x+a)))+I*dilog(
1+I*exp(I*(b*x+a)))-I*dilog(1-I*exp(I*(b*x+a)))))
```

### 3.218.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(88) = 176.

Time = 0.28 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.21

$$\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx =$$
$$-\frac{2\,d\cos\left(bx+a\right)+i\,d\mathrm{Li}_2(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right))+i\,d\mathrm{Li}_2(i\,\cos\left(bx+a\right)-\sin\left(bx+a\right))-i\,d\mathrm{Li}_2(-$$

input
```
integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*d*cos(b*x + a) + I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*di
log(I*cos(b*x + a) - sin(b*x + a)) - I*d*dilog(-I*cos(b*x + a) + sin(b*x +
 a)) - I*d*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b*c - a*d)*log(cos(b*x
 + a) + I*sin(b*x + a) + I) + (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a
) + I) - (b*d*x + a*d)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x + a
*d)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b*d*x + a*d)*log(-I*cos(b*x
+ a) + sin(b*x + a) + 1) + (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a
) + 1) - (b*c - a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*c - a*d)
*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 2*(b*d*x + b*c)*sin(b*x + a))/b
^2
```

### 3.218.6 Sympy [F]

$$\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx = \int (c+dx)\sin^2\left(a+bx\right)\sec\left(a+bx\right)dx$$

input
```
integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*sin(a + b*x)**2*sec(a + b*x), x)
```

### 3.218.7 Maxima [F]

$$\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx = \int (dx+c)\sec(bx+a)\sin(bx+a)^2\,dx$$

input
```
integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(4*b^2*d*integrate((x*cos(2*b*x + 2*a)*cos(b*x + a) + x*sin(2*b*x + 2*
a)*sin(b*x + a) + x*cos(b*x + a))/(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
 + 2*cos(2*b*x + 2*a) + 1), x) + b*c*log(cos(b*x + a)^2 + sin(b*x + a)^2 +
 2*sin(b*x + a) + 1) - b*c*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x
 + a) + 1) - 2*d*cos(b*x + a) - 2*(b*d*x + b*c)*sin(b*x + a))/b^2
```

### 3.218.8 Giac [F]

$$\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx = \int (dx+c)\sec(bx+a)\sin(bx+a)^2\,dx$$

input
```
integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)*sec(b*x + a)*sin(b*x + a)^2, x)
```

### 3.218.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx = \int \frac{\sin(a+b\,x)^2\,(c+d\,x)}{\cos(a+b\,x)}\,dx$$

input
```
int((sin(a + b*x)^2*(c + d*x))/cos(a + b*x),x)
```

output
```
int((sin(a + b*x)^2*(c + d*x))/cos(a + b*x), x)
```

# 3.219 $\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx$

3.219.1 Optimal result . . . . . 1735
3.219.2 Mathematica [N/A] . . . . . 1735
3.219.3 Rubi [N/A] . . . . . 1736
3.219.4 Maple [N/A] (verified) . . . . . 1738
3.219.5 Fricas [N/A] . . . . . 1738
3.219.6 Sympy [N/A] . . . . . 1738
3.219.7 Maxima [N/A] . . . . . 1739
3.219.8 Giac [N/A] . . . . . 1739
3.219.9 Mupad [N/A] . . . . . 1739

## 3.219.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx = -\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d} + \frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d} + \text{Int}\left(\frac{\sec(a+bx)}{c+dx},x\right)$$

output
```
-Ci(b*c/d+b*x)*cos(a-b*c/d)/d+Si(b*c/d+b*x)*sin(a-b*c/d)/d+Unintegrable(se
c(b*x+a)/(d*x+c),x)
```

## 3.219.2 Mathematica [N/A]

Not integrable

Time = 3.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x),x]
```

output
```
Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x), x]
```

### 3.219.3 Rubi [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4907, 3042, 3784, 3042, 3780, 3783, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}dx$$
$$\Big\downarrow \text{4907}$$
$$\int \frac{\sec(a+bx)}{c+dx}dx-\int \frac{\cos(a+bx)}{c+dx}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx-\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx$$
$$\Big\downarrow \text{3784}$$
$$\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx-\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx$$
$$\Big\downarrow \text{3042}$$
$$\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx-\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx$$
$$\Big\downarrow \text{3780}$$
$$\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx-\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$
$$\Big\downarrow \text{3783}$$
$$\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx-\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$
$$\Big\downarrow \text{4680}$$
$$\int \frac{\sec(a+bx)}{c+dx}dx-\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

input
```
Int[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x),x]
```

output
```
$Aborted
```

### 3.219.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

## 3.219. $\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx$

### 3.219.4 Maple [N/A] (verified)

Not integrable

Time = 0.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sec(xb+a)\sin(xb+a)^2}{dx+c}dx$$

input
```
int(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x)
```

output
```
int(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x)
```

### 3.219.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)\sin(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sec(b*x + a)/(d*x + c), x)
```

### 3.219.6 Sympy [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sin^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

input
```
integrate(sec(b*x+a)*sin(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)**2*sec(a + b*x)/(c + d*x), x)
```

### 3.219.7 Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 210, normalized size of antiderivative = 10.50

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec\left(bx+a\right)\sin\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
1/2*((exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x
 + I*b*c)/d))*cos(-(b*c - a*d)/d) + 4*d*integrate((cos(2*b*x + 2*a)*cos(b*
x + a) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))/((d*x + c)*cos(2*b*
x + 2*a)^2 + (d*x + c)*sin(2*b*x + 2*a)^2 + d*x + 2*(d*x + c)*cos(2*b*x +
2*a) + c), x) + (-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) + I*exp_integra
l_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))/d
```

### 3.219.8 Giac [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec\left(bx+a\right)\sin\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)*sin(b*x + a)^2/(d*x + c), x)
```

### 3.219.9 Mupad [N/A]

Not integrable

Time = 24.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{{\sin\left(a+b\,x\right)}^2}{\cos\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

input
```
int(sin(a + b*x)^2/(cos(a + b*x)*(c + d*x)),x)
```

output
```
int(sin(a + b*x)^2/(cos(a + b*x)*(c + d*x)), x)
```

3.219. $\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx$

# 3.220 $\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$

3.220.1 Optimal result . . . . . 1741
3.220.2 Mathematica [N/A] . . . . . 1741
3.220.3 Rubi [N/A] . . . . . 1742
3.220.4 Maple [N/A] (verified) . . . . . 1744
3.220.5 Fricas [N/A] . . . . . 1745
3.220.6 Sympy [N/A] . . . . . 1745
3.220.7 Maxima [N/A] . . . . . 1745
3.220.8 Giac [N/A] . . . . . 1746
3.220.9 Mupad [N/A] . . . . . 1746

## 3.220.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \frac{\cos(a+bx)}{d(c+dx)} + \frac{b\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^2} + \frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2} + \text{Int}\left(\frac{\sec(a+bx)}{(c+dx)^2},x\right)$$

output
```
cos(b*x+a)/d/(d*x+c)+b*cos(a-b*c/d)*Si(b*c/d+b*x)/d^2+b*Ci(b*c/d+b*x)*sin(
a-b*c/d)/d^2+Unintegrable(sec(b*x+a)/(d*x+c)^2,x)
```

## 3.220.2 Mathematica [N/A]

Not integrable

Time = 7.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

### 3.220.3 Rubi [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4907, 3042, 3778, 25, 3042, 3784, 3042, 3780, 3783, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{4907}$$

$$\int \frac{\sec(a+bx)}{(c+dx)^2}dx-\int \frac{\cos(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx-\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3778}$$

$$-\frac{b\int -\frac{\sin(a+bx)}{c+dx}dx}{d}+\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx+\frac{\cos(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{25}$$

$$\frac{b\int \frac{\sin(a+bx)}{c+dx}dx}{d}+\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx+\frac{\cos(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\int \frac{\sin(a+bx)}{c+dx}dx}{d}+\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx+\frac{\cos(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3784}$$

$$\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx+\frac{b\left(\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}+\frac{\cos(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx+\frac{b\left(\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx+\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}+\frac{\cos(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3780}$$

$$\frac{b\left(\sin\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}+\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx+\frac{\cos(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3783}$$

$$\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx+\frac{b\left(\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}+\frac{\cos(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{4680}$$

$$\int \frac{\sec(a+bx)}{(c+dx)^2}dx+\frac{b\left(\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}+\frac{\cos(a+bx)}{d(c+dx)}$$

input
```
Int[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

### 3.220.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

### 3.220.4 Maple [N/A] (verified)

Not integrable

Time = 0.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sec(xb+a)\sin(xb+a)^2}{(dx+c)^2}dx$$

input
```
int(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x)
```

### 3.220.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2} \, dx = \int \frac{\sec(bx+a)\sin(bx+a)^2}{(dx+c)^2} \, dx$$

input
```
integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.220.6 Sympy [N/A]

Not integrable

Time = 2.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2} \, dx = \int \frac{\sin^2(a+bx)\sec(a+bx)}{(c+dx)^2} \, dx$$

input
```
integrate(sec(b*x+a)*sin(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)**2*sec(a + b*x)/(c + d*x)**2, x)
```

### 3.220.7 Maxima [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 270, normalized size of antiderivative = 13.50

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2} \, dx = \int \frac{\sec(bx+a)\sin(bx+a)^2}{(dx+c)^2} \, dx$$

input
```
integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*((exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x
 + I*b*c)/d))*cos(-(b*c - a*d)/d) + 4*(d^2*x + c*d)*integrate((cos(2*b*x +
 2*a)*cos(b*x + a) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))/(d^2*x^
2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(2*b*x + 2*a)^2 + (d^2*x^2 + 2*
c*d*x + c^2)*sin(2*b*x + 2*a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(2*
b*x + 2*a)), x) + (-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) + I*exp_integ
ral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))/(d^2*x + c*d)
```

### 3.220.8 Giac [N/A]

Not integrable

Time = 3.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx=\int \frac{\sec\left(bx+a\right)\sin\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)*sin(b*x + a)^2/(d*x + c)^2, x)
```

### 3.220.9 Mupad [N/A]

Not integrable

Time = 24.92 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx=\int \frac{\sin\left(a+b\,x\right)^2}{\cos\left(a+b\,x\right)\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(sin(a + b*x)^2/(cos(a + b*x)*(c + d*x)^2),x)
```

output
```
int(sin(a + b*x)^2/(cos(a + b*x)*(c + d*x)^2), x)
```

# 3.221 $\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx$

3.221.1 Optimal result . . . 1747
3.221.2 Mathematica [N/A] . . . 1747
3.221.3 Rubi [N/A] . . . 1748
3.221.4 Maple [N/A] (verified) . . . 1750
3.221.5 Fricas [N/A] . . . 1751
3.221.6 Sympy [F(-2)] . . . 1751
3.221.7 Maxima [N/A] . . . 1751
3.221.8 Giac [N/A] . . . 1752
3.221.9 Mupad [N/A] . . . 1752

## 3.221.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx = \frac{2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} + \frac{2^{-3-m}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2ib(c+dx)}{d}\right)}{b} + \mathrm{Int}((c+dx)^m\tan(a+bx),x)$$

output
```
2^(-3-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*b
*(d*x+c)/d)^m)+2^(-3-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a-
b*c/d))/((I*b*(d*x+c)/d)^m)+Unintegrable((d*x+c)^m*tan(b*x+a),x)
```

## 3.221.2 Mathematica [N/A]

Not integrable

Time = 13.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx = \int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Sin[a + b*x]^2*Tan[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Sin[a + b*x]^2*Tan[a + b*x], x]
```

### 3.221.3 Rubi [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4907, 3042, 4222, 4906, 27, 3042, 3789, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin^2(a+bx)\tan(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{4907}$$

$$\int (c+dx)^m\tan(a+bx)dx-\int (c+dx)^m\cos(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m\tan(a+bx)dx-\int (c+dx)^m\cos(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{4222}$$

$$\int (c+dx)^m\tan(a+bx)dx-\int (c+dx)^m\cos(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{4906}$$

$$\int (c+dx)^m\tan(a+bx)dx-\int \frac{1}{2}(c+dx)^m\sin(2a+2bx)dx$$

$$\Big\downarrow \text{27}$$

$$\int (c+dx)^m\tan(a+bx)dx-\frac{1}{2}\int (c+dx)^m\sin(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m\tan(a+bx)dx-\frac{1}{2}\int (c+dx)^m\sin(2a+2bx)dx$$

$$\Big\downarrow \text{3789}$$

$$\int (c+dx)^m \tan(a+bx)dx + \frac{1}{2}\left(\frac{1}{2}i\int e^{2i(a+bx)}(c+dx)^m dx - \frac{1}{2}i\int e^{-2i(a+bx)}(c+dx)^m dx\right)$$

$$\Big\downarrow \; 2612$$

$$\int (c+dx)^m \tan(a+bx)dx +$$

$$\frac{1}{2}\left(\frac{2^{-m-2}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b} + \frac{2^{-m-2}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma}{b}\right.$$

input
```
Int[(c + d*x)^m*Sin[a + b*x]^2*Tan[a + b*x],x]
```

output
```
$Aborted
```

### 3.221.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

### 3.221.4 Maple [N/A] (verified)

Not integrable

Time = 0.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \sec (xb+a) \sin (xb+a)^3 \, dx$$

input
```
int((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x)
```

output
```
int((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x)
```

### 3.221.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\sin(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*(d*x + c)^m*sec(b*x + a)*sin(b*x + a), x)
```

### 3.221.6 Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d*x+c)**m*sec(b*x+a)*sin(b*x+a)**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.221.7 Maxima [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\sin(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a)^3, x)
```

### 3.221.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\sin(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a)^3, x)
```

### 3.221.9 Mupad [N/A]

Not integrable

Time = 25.79 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx = \int \frac{\sin(a+b\,x)^3\,(c+d\,x)^m}{\cos(a+b\,x)}\,dx$$

input
```
int((sin(a + b*x)^3*(c + d*x)^m)/cos(a + b*x),x)
```

output
```
int((sin(a + b*x)^3*(c + d*x)^m)/cos(a + b*x), x)
```

# 3.222 $\int (c+dx)^3 \sin^2(a+bx) \tan(a+bx) \, dx$

3.222.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1753
3.222.2 Mathematica [B] (warning: unable to verify) . . . . . . . . . . . . . . . . . . . 1754
3.222.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1754
3.222.4 Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1760
3.222.5 Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1761
3.222.6 Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1762
3.222.7 Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 1763
3.222.8 Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1763
3.222.9 Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1764

## 3.222.1 Optimal result

Integrand size = 22, antiderivative size = 251

$$\begin{aligned}\int (c+dx)^3 \sin^2(a+bx) \tan(a+bx) \, dx = & -\frac{3d^3x}{8b^3}+\frac{(c+dx)^3}{4b}+\frac{i(c+dx)^4}{4d} \\ & -\frac{(c+dx)^3 \log\left(1+e^{2i(a+bx)}\right)}{b} \\ & +\frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} \\ & -\frac{3d^2(c+dx) \operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} \\ & -\frac{3id^3 \operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4} \\ & +\frac{3d^3 \cos(a+bx) \sin(a+bx)}{8b^4} \\ & -\frac{3d(c+dx)^2 \cos(a+bx) \sin(a+bx)}{4b^2} \\ & +\frac{3d^2(c+dx) \sin^2(a+bx)}{4b^3} \\ & -\frac{(c+dx)^3 \sin^2(a+bx)}{2b}\end{aligned}$$

output
```
-3/8*d^3*x/b^3+1/4*(d*x+c)^3/b+1/4*I*(d*x+c)^4/d-(d*x+c)^3*ln(1+exp(2*I*(b
*x+a)))/b+3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*x+
c)*polylog(3,-exp(2*I*(b*x+a)))/b^3-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))
/b^4+3/8*d^3*cos(b*x+a)*sin(b*x+a)/b^4-3/4*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+
a)/b^2+3/4*d^2*(d*x+c)*sin(b*x+a)^2/b^3-1/2*(d*x+c)^3*sin(b*x+a)^2/b
```

### 3.222.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1731 vs. 2(251) = 502.

Time = 6.53 (sec) , antiderivative size = 1731, normalized size of antiderivative = 6.90

$$\int (c+dx)^3 \sin^2(a+bx) \tan(a+bx)\,dx = \text{Too large to display}$$

input
```
Integrate[(c + d*x)^3*Sin[a + b*x]^2*Tan[a + b*x],x]
```

output
```
((-1/4*I)*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2
*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x)
)] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b
^3*E^(I*a)) - ((I/8)*d^3*E^(I*a)*((2*b^4*x^4)/E^((2*I)*a) - (4*I)*b^3*(1 +
 E^((-2*I)*a))*x^3*Log[1 + E^((-2*I)*(a + b*x))] + 6*b^2*(1 + E^((-2*I)*a)
)*x^2*PolyLog[2, -E^((-2*I)*(a + b*x))] - (6*I)*b*(1 + E^((-2*I)*a))*x*Pol
yLog[3, -E^((-2*I)*(a + b*x))] - 3*(1 + E^((-2*I)*a))*PolyLog[4, -E^((-2*I
)*(a + b*x))])*Sec[a])/b^4 - (c^3*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin
[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (3*c^2*d*Csc[a]*(
(b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) -
 Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b
*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x -
 ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[
1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + Sec[
a]*(Cos[2*a + 2*b*x]/(64*b^4) - ((I/64)*Sin[2*a + 2*b*x])/b^4)*(8*b^3*c^3*
Cos[a] - (12*I)*b^2*c^2*d*Cos[a] - 12*b*c*d^2*Cos[a] + (6*I)*d^3*Cos[a] +
24*b^3*c^2*d*x*Cos[a] - (24*I)*b^2*c*d^2*x*Cos[a] - 12*b*d^3*x*Cos[a] + 24
*b^3*c*d^2*x^2*Cos[a] - (12*I)*b^2*d^3*x^2*Cos[a] + 8*b^3*d^3*x^3*Cos[a] +
 (32*I)*b^4*c^3*x*Cos[a + 2*b*x] + (48*I)*b^4*c^2*d*x^2*Cos[a + 2*b*x] + (
32*I)*b^4*c*d^2*x^3*Cos[a + 2*b*x] + (8*I)*b^4*d^3*x^4*Cos[a + 2*b*x] -...
```

### 3.222.3 Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.12, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {4907, 3042, 4202, 2620, 3011, 4904, 3042, 3792, 17, 3042, 3115, 24, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin^2(a+bx)\tan(a+bx)\,dx$$

$$\Big\downarrow \text{ 4907}$$

$$\int (c+dx)^3\tan(a+bx)dx-\int (c+dx)^3\cos(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^3\tan(a+bx)dx-\int (c+dx)^3\cos(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{ 4202}$$

$$-2i\int \frac{e^{2i(a+bx)}(c+dx)^3}{1+e^{2i(a+bx)}}dx-\int (c+dx)^3\cos(a+bx)\sin(a+bx)dx+\frac{i(c+dx)^4}{4d}$$

$$\Big\downarrow \text{ 2620}$$

$$-2i\left(\frac{3id\int (c+dx)^2\log\left(1+e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\int (c+dx)^3\cos(a+bx)\sin(a+bx)dx+\frac{i(c+dx)^4}{4d}$$

$$\Big\downarrow \text{ 3011}$$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int (c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\int (c+dx)^3\cos(a+bx)\sin(a+bx)dx+\frac{i(c+dx)^4}{4d}$$

$$\Big\downarrow \text{ 4904}$$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int (c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)+\frac{3d\int (c+dx)^2\sin^2(a+bx)dx}{2b}-\frac{(c+dx)^3\sin^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$$\Big\downarrow \text{ 3042}$$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int (c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)+\frac{3d\int (c+dx)^2\sin(a+bx)^2dx}{2b}-\frac{(c+dx)^3\sin^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

---

3.222. $\int (c+dx)^3\sin^2(a+bx)\tan(a+bx)\,dx$

$$\Big\downarrow \text{3792}$$

$$\frac{3d\left(-\frac{d^2\int \sin^2(a+bx)dx}{2b^2}+\frac{1}{2}\int (c+dx)^2dx+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b}-2i\left(\frac{3id\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int (c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{(c+dx)^3\sin^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$$\Big\downarrow \text{17}$$

$$\frac{3d\left(-\frac{d^2\int \sin^2(a+bx)dx}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}-2i\left(\frac{3id\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int (c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{(c+dx)^3\sin^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d\left(-\frac{d^2\int \sin(a+bx)^2dx}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}-2i\left(\frac{3id\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int (c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{(c+dx)^3\sin^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{3d\left(-\frac{d^2\left(\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}+\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}-2i\left(\frac{3id\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int (c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{(c+dx)^3\sin^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$$\Big\downarrow \text{24}$$

3.222. $\int (c+dx)^3\sin^2(a+bx)\tan(a+bx)\,dx$

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)+\frac{3d\left(\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}-\frac{(c+dx)^3\sin^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$\Big\downarrow$ 7163

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\left(\frac{id\int\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i(c+dx)^3\log(1}{2b}\right.$$
$$\frac{3d\left(\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}-\frac{(c+dx)^3\sin^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$\Big\downarrow$ 2720

$$-2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\left(\frac{d\int e^{-2i(a+bx)}\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i(}{}\right.$$
$$\frac{3d\left(\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}-\frac{(c+dx)^3\sin^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

$\Big\downarrow$ 7143

3.222. $\int(c+dx)^3\sin^2(a+bx)\tan(a+bx)\,dx$

$$\frac{3d\left(\frac{d(c+dx)\sin^2(a+bx)}{2b^2}-\frac{d^2\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b^2}-\frac{(c+dx)^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{2b}-$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}(2,-e^{2i(a+bx)})}{2b}-\frac{id\left(\frac{d\operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^2\right.}{2b}\right.$$

$$\frac{(c+dx)^3\sin^2(a+bx)}{2b}+\frac{i(c+dx)^4}{4d}$$

input
```
Int[(c + d*x)^3*Sin[a + b*x]^2*Tan[a + b*x],x]
```

output
```
((I/4)*(c + d*x)^4)/d - (2*I)*(((-1/2*I)*(c + d*x)^3*Log[1 + E^((2*I)*(a +
 b*x))])/b + (((3*I)/2)*d*(((I/2)*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*
x))])/b - (I*d*(((-1/2*I)*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/b +
(d*PolyLog[4, -E^((2*I)*(a + b*x))])/(4*b^2)))/b))/b) - ((c + d*x)^3*Sin[a
 + b*x]^2)/(2*b) + (3*d*((c + d*x)^3/(6*d) - ((c + d*x)^2*Cos[a + b*x]*Sin
[a + b*x])/(2*b) + (d*(c + d*x)*Sin[a + b*x]^2)/(2*b^2) - (d^2*(x/2 - (Cos
[a + b*x]*Sin[a + b*x])/(2*b)))/(2*b^2)))/(2*b)
```

### 3.222.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

## 3.222. $\int (c+dx)^3 \sin^2(a+bx) \tan(a+bx) \, dx$

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

### 3.222.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(220) = 440$.

Time = 3.11 (sec) , antiderivative size = 650, normalized size of antiderivative = 2.59

| method | result |
|---|---|
| risch | $\frac{6id\,c^2xa}{b}+\frac{3ic\,d^2\operatorname{polylog}(2,-e^{2i(xb+a)})x}{b^2}-\frac{6ic\,d^2a^2x}{b^2}-ic^3x-\frac{ic^4}{4d}-\frac{c^3\ln(e^{2i(xb+a)}+1)}{b}+\frac{2c^3\ln(e^{i(xb+a)})}{b}+\frac{6c\,d^2a^2}{}$ |

input
```
int((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+3/2*I*d*c^2*x^2-1/b*c^3*ln(exp
(2*I*(b*x+a))+1)-3/b*d*c^2*ln(exp(2*I*(b*x+a))+1)*x-3/b*c*d^2*ln(exp(2*I*(
b*x+a))+1)*x^2+2*I/b^3*d^3*a^3*x+3/2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a))
)*x^2+3/2*I/b^2*d*c^2*polylog(2,-exp(2*I*(b*x+a)))+3*I/b^2*d*c^2*a^2-4*I/b
^3*c*d^2*a^3+1/4*I*d^3*x^4+6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a)))-6/b^2*c^2*d*
a*ln(exp(I*(b*x+a)))-1/b*d^3*ln(exp(2*I*(b*x+a))+1)*x^3-3/2/b^3*d^3*polylo
g(3,-exp(2*I*(b*x+a)))*x-3/2/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))+3/2*I/
b^4*d^3*a^4-I*c^3*x-1/4*I/d*c^4+2/b*c^3*ln(exp(I*(b*x+a)))-2/b^4*d^3*a^3*l
n(exp(I*(b*x+a)))-6*I/b^2*c*d^2*a^2*x+6*I/b*d*c^2*x*a+3*I/b^2*c*d^2*polylo
g(2,-exp(2*I*(b*x+a)))*x+1/32*(4*d^3*x^3*b^3-6*I*b^2*d^3*x^2+12*b^3*c*d^2*
x^2-12*I*b^2*c*d^2*x+12*b^3*c^2*d*x-6*I*b^2*c^2*d+4*b^3*c^3-6*b*d^3*x+3*I*
d^3-6*c*d^2*b)/b^4*exp(-2*I*(b*x+a))+1/32*(4*d^3*x^3*b^3+6*I*b^2*d^3*x^2+1
2*b^3*c*d^2*x^2+12*I*b^2*c*d^2*x+12*b^3*c^2*d*x+6*I*b^2*c^2*d+4*b^3*c^3-6*
b*d^3*x-3*I*d^3-6*c*d^2*b)/b^4*exp(2*I*(b*x+a))+I*d^2*c*x^3
```

### 3.222.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1134 vs. $2(216) = 432$.

Time = 0.32 (sec) , antiderivative size = 1134, normalized size of antiderivative = 4.52

$$\int (c+dx)^3 \sin^2(a+bx)\tan(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 24*I*d^3*polylog(4, I*cos(b*x + a)
 + sin(b*x + a)) + 24*I*d^3*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 24
*I*d^3*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) - 24*I*d^3*polylog(4, -I
*cos(b*x + a) - sin(b*x + a)) - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3
*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2*b^2*d^
3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 3*(
2*b^3*c^2*d - b*d^3)*x + 12*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d
)*dilog(I*cos(b*x + a) + sin(b*x + a)) + 12*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^
2*x - I*b^2*c^2*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) + 12*(-I*b^2*d^3*x
^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-I*cos(b*x + a) + sin(b*x + a))
+ 12*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(-I*cos(b*x + a)
 - sin(b*x + a)) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*l
og(cos(b*x + a) + I*sin(b*x + a) + I) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2
*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I*sin(b*x + a) + I) + 4*(b^3*d^3*x^
3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*
d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2
*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(
b*x + a) - sin(b*x + a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^
2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) + sin
(b*x + a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a...
```

### 3.222.6 Sympy [F]

$$\int (c+dx)^3 \sin^2(a+bx)\tan(a+bx)\,dx = \int (c+dx)^3 \sin^3{(a+bx)}\sec{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**3*sec(b*x+a)*sin(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**3*sin(a + b*x)**3*sec(a + b*x), x)
```

### 3.222.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(216) = 432$.

Time = 0.44 (sec) , antiderivative size = 692, normalized size of antiderivative = 2.76

$$\int (c + dx)^3 \sin^2(a + bx) \tan(a + bx)\,dx =$$

$$-\frac{24\left(\sin\left(bx+a\right)^2+\log\left(\sin\left(bx+a\right)^2-1\right)\right)c^3-\frac{72\left(\sin(bx+a)^2+\log\left(\sin(bx+a)^2-1\right)\right)ac^2d}{b}+\frac{72\left(\sin(bx+a)^2+\log\left(\sin(b\right.\right.}{b^2}}{}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/48*(24*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1))*c^3 - 72*(sin(b*x + a
)^2 + log(sin(b*x + a)^2 - 1))*a*c^2*d/b + 72*(sin(b*x + a)^2 + log(sin(b*
x + a)^2 - 1))*a^2*c*d^2/b^2 - 24*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1
))*a^3*d^3/b^3 + (-12*I*(b*x + a)^4*d^3 - 48*(I*b*c*d^2 - I*a*d^3)*(b*x +
a)^3 + 48*I*d^3*polylog(4, -e^(2*I*b*x + 2*I*a)) - 72*(I*b^2*c^2*d - 2*I*a
*b*c*d^2 + I*a^2*d^3)*(b*x + a)^2 - 16*(-4*I*(b*x + a)^3*d^3 + 9*(-I*b*c*d
^2 + I*a*d^3)*(b*x + a)^2 + 9*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*(
b*x + a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 6*(2*(b*x + a)
^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*
c^2*d - 4*a*b*c*d^2 + (2*a^2 - 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 24*(3
*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 4*I*(b*x + a)^2*d^3 + 3*I*a^2*d^3 + 6*(I*b*
c*d^2 - I*a*d^3)*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*a)) + 8*(4*(b*x + a)^3
*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3)*(b*x + a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
+ 2*a) + 1) + 24*(3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*polylog(3, -e^(2*
I*b*x + 2*I*a)) + 9*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^2*d^3 + (2*a^
2 - 1)*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))/b^3)/b
```

### 3.222.8 Giac [F]

$$\int (c + dx)^3 \sin^2(a + bx) \tan(a + bx)\,dx = \int (dx + c)^3 \sec(bx + a) \sin(bx + a)^3\,dx$$

input
```
integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*sec(b*x + a)*sin(b*x + a)^3, x)
```

### 3.222.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \sin^2(a+bx) \tan(a+bx) \, dx = \int \frac{\sin(a+b\,x)^3\,(c+d\,x)^3}{\cos(a+b\,x)} \, dx$$

input
```
int((sin(a + b*x)^3*(c + d*x)^3)/cos(a + b*x),x)
```

output
```
int((sin(a + b*x)^3*(c + d*x)^3)/cos(a + b*x), x)
```

# 3.223 $\int (c+dx)^2 \sin^2(a+bx)\tan(a+bx)\,dx$

3.223.1 Optimal result . . . . . 1765
3.223.2 Mathematica [B] (verified) . . . . . 1766
3.223.3 Rubi [A] (verified) . . . . . 1767
3.223.4 Maple [B] (verified) . . . . . 1770
3.223.5 Fricas [B] (verification not implemented) . . . . . 1771
3.223.6 Sympy [F] . . . . . 1772
3.223.7 Maxima [B] (verification not implemented) . . . . . 1772
3.223.8 Giac [F] . . . . . 1773
3.223.9 Mupad [F(-1)] . . . . . 1773

## 3.223.1 Optimal result

Integrand size = 22, antiderivative size = 184

$$\int (c+dx)^2\sin^2(a+bx)\tan(a+bx)\,dx = \frac{cdx}{2b}+\frac{d^2x^2}{4b}+\frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} - \frac{d^2\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} - \frac{d(c+dx)\cos(a+bx)\sin(a+bx)}{2b^2} + \frac{d^2\sin^2(a+bx)}{4b^3} - \frac{(c+dx)^2\sin^2(a+bx)}{2b}$$

output
```
1/2*c*d*x/b+1/4*d^2*x^2/b+1/3*I*(d*x+c)^3/d-(d*x+c)^2*ln(1+exp(2*I*(b*x+a)
))/b+I*d*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,-exp(2
*I*(b*x+a)))/b^3-1/2*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2+1/4*d^2*sin(b*x+a
)^2/b^3-1/2*(d*x+c)^2*sin(b*x+a)^2/b
```

### 3.223.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 518 vs. 2(184) = 368.

Time = 6.58 (sec) , antiderivative size = 518, normalized size of antiderivative = 2.82

$$\int (c+dx)^2\sin^2(a+bx)\tan(a+bx)\,dx = -\frac{id^2e^{-ia}\left(2b^2x^2\left(2bx-3i(1+e^{2ia})\log\left(1+e^{-2i(a+bx)}\right)\right)+6b(1+e^{2ia})\,x\,\mathrm{PolyLog}\left(2,-e^{-2i(a+bx)}\right)-3i(1+\dots\right.}{12b^3}$$
$$-\frac{c^2\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))}{b\,(\cos^2(a)+\sin^2(a))}$$
$$-\frac{cd\csc(a)\left(b^2e^{-i\arctan(\cot(a))}x^2-\frac{\cot(a)(ibx(-\pi-2\arctan(\cot(a)))-\pi\log(1+e^{-2ibx})-2(bx-\arctan(\cot(a)))\log(1-e^{2i(bx-\arctan(\dots}{\dots}\right.}{b^2\sqrt{\csc^2(a)\,(\cos^2(a)+\dots}}$$
$$+\frac{\cos(2bx)\left(2b^2c^2\cos(2a)-d^2\cos(2a)+4b^2cdx\cos(2a)+2b^2d^2x^2\cos(2a)-2bcd\sin(2a)-2bd^2x\sin(2a)\dots\right.}{8b^3}$$
$$-\frac{(2bcd\cos(2a)+2bd^2x\cos(2a)+2b^2c^2\sin(2a)-d^2\sin(2a)+4b^2cdx\sin(2a)+2b^2d^2x^2\sin(2a))\sin(2bx)\dots}{8b^3}$$
$$+\frac{1}{3}x\left(3c^2+3cdx+d^2x^2\right)\tan(a)$$

input
```
Integrate[(c + d*x)^2*Sin[a + b*x]^2*Tan[a + b*x],x]
```

output
```
((-1/12*I)*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*
I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))
] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^
3*E^(I*a)) - (c^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] +
b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (c*d*Csc[a]*((b^2*x^2)/E^(I*ArcTa
n[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I
)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]])
)] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] +
I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a
])/(b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Cos[2*b*x]*(2*b^2*c^2*Cos
[2*a] - d^2*Cos[2*a] + 4*b^2*c*d*x*Cos[2*a] + 2*b^2*d^2*x^2*Cos[2*a] - 2*b
*c*d*Sin[2*a] - 2*b*d^2*x*Sin[2*a]))/(8*b^3) - ((2*b*c*d*Cos[2*a] + 2*b*d^
2*x*Cos[2*a] + 2*b^2*c^2*Sin[2*a] - d^2*Sin[2*a] + 4*b^2*c*d*x*Sin[2*a] +
2*b^2*d^2*x^2*Sin[2*a])*Sin[2*b*x])/(8*b^3) + (x*(3*c^2 + 3*c*d*x + d^2*x^
2)*Tan[a])/3
```

### 3.223.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4907, 3042, 4202, 2620, 3011, 2720, 4904, 3042, 3791, 17, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin^2(a+bx)\tan(a+bx)\,dx$$

$$\Big\downarrow \text{4907}$$

$$\int (c+dx)^2\tan(a+bx)dx-\int (c+dx)^2\cos(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2\tan(a+bx)dx-\int (c+dx)^2\cos(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{4202}$$

$$-2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}dx-\int (c+dx)^2\cos(a+bx)\sin(a+bx)dx+\frac{i(c+dx)^3}{3d}$$

$$\Big\downarrow \text{2620}$$

$$-2i\left(\frac{id\int (c+dx)\log\left(1+e^{2i(a+bx)}\right)dx}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\int (c+dx)^2\cos(a+bx)\sin(a+bx)dx+\frac{i(c+dx)^3}{3d}$$

$$\Big\downarrow \text{3011}$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\text{PolyLog}(2,-e^{2i(a+bx)})}{2b}-\frac{id\int \text{PolyLog}(2,-e^{2i(a+bx)})dx}{2b}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\int (c+dx)^2\cos(a+bx)\sin(a+bx)dx+\frac{i(c+dx)^3}{3d}$$

$$\Big\downarrow \text{2720}$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\text{PolyLog}(2,-e^{2i(a+bx)})}{2b}-\frac{d\int e^{-2i(a+bx)}\text{PolyLog}(2,-e^{2i(a+bx)})de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\int (c+dx)^2\cos(a+bx)\sin(a+bx)dx+\frac{i(c+dx)^3}{3d}$$

$$\Big\downarrow \text{4904}$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{2i(a+bx)})}{2b}-\frac{d\int e^{-2i(a+bx)}\operatorname{PolyLog}(2,-e^{2i(a+bx)})de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)+\frac{d\int(c+dx)\sin^2(a+bx)dx}{b}-\frac{(c+dx)^2\sin^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{2i(a+bx)})}{2b}-\frac{d\int e^{-2i(a+bx)}\operatorname{PolyLog}(2,-e^{2i(a+bx)})de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)+\frac{d\int(c+dx)\sin(a+bx)^2dx}{b}-\frac{(c+dx)^2\sin^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

$$\Big\downarrow \text{3791}$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{2i(a+bx)})}{2b}-\frac{d\int e^{-2i(a+bx)}\operatorname{PolyLog}(2,-e^{2i(a+bx)})de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)+\frac{d\left(\frac{1}{2}\int(c+dx)dx+\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}\right)}{b}-\frac{(c+dx)^2\sin^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

$$\Big\downarrow \text{17}$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{2i(a+bx)})}{2b}-\frac{d\int e^{-2i(a+bx)}\operatorname{PolyLog}(2,-e^{2i(a+bx)})de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)+\frac{d\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{b}-\frac{(c+dx)^2\sin^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

$$\Big\downarrow \text{7143}$$

$$-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{2i(a+bx)})}{2b}-\frac{d\operatorname{PolyLog}(3,-e^{2i(a+bx)})}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)+\frac{d\left(\frac{d\sin^2(a+bx)}{4b^2}-\frac{(c+dx)\sin(a+bx)\cos(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{b}-\frac{(c+dx)^2\sin^2(a+bx)}{2b}+\frac{i(c+dx)^3}{3d}$$

input
```
Int[(c + d*x)^2*Sin[a + b*x]^2*Tan[a + b*x],x]
```

## 3.223. $\int (c+dx)^2\sin^2(a+bx)\tan(a+bx)\,dx$

output
```
((I/3)*(c + d*x)^3)/d - (2*I)*(((-1/2*I)*(c + d*x)^2*Log[1 + E^((2*I)*(a +
 b*x))])/b + (I*d*(((I/2)*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b -
(d*PolyLog[3, -E^((2*I)*(a + b*x))])/(4*b^2)))/b) - ((c + d*x)^2*Sin[a + b
*x]^2)/(2*b) + (d*((c + d*x)^2/(4*d) - ((c + d*x)*Cos[a + b*x]*Sin[a + b*x
])/(2*b) + (d*Sin[a + b*x]^2)/(4*b^2)))/b
```

### 3.223.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.223. $\int (c+dx)^2 \sin^2(a+bx) \tan(a+bx) \, dx$

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.223.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(162) = 324$.

Time = 2.63 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.11

| method | result |
|---|---|
| risch | $\frac{id^2 \operatorname{polylog}\left(2,-e^{2i(xb+a)}\right)x}{b^2} - ic^2x + \frac{id^2x^3}{3} - \frac{2id^2a^2x}{b^2} + \frac{\left(2x^2d^2b^2+4b^2cdx+2ib\,d^2x+2b^2c^2+2ibcd-d^2\right)e^{2i(xb+a)}}{16b^3} + \frac{(2x^2d^2}{}$ |

input
```
int((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-I*c^2*x+1/3*I*d^2*x^3-2*I/b^2*d^2
*a^2*x+1/16*(2*x^2*d^2*b^2+2*I*b*d^2*x+4*b^2*c*d*x+2*I*b*c*d+2*b^2*c^2-d^2
)/b^3*exp(2*I*(b*x+a))+1/16*(2*x^2*d^2*b^2-2*I*b*d^2*x+4*b^2*c*d*x-2*I*b*c
*d+2*b^2*c^2-d^2)/b^3*exp(-2*I*(b*x+a))+2*I/b^2*c*d*a^2+I/b^2*c*d*polylog(
2,-exp(2*I*(b*x+a)))-1/3*I/d*c^3+I*d*c*x^2+2/b^3*d^2*a^2*ln(exp(I*(b*x+a))
)-4/3*I/b^3*d^2*a^3-2/b*c*d*ln(exp(2*I*(b*x+a))+1)*x+4*I/b*c*d*x*a-1/b*d^2
*ln(exp(2*I*(b*x+a))+1)*x^2-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3-1/b*c
^2*ln(exp(2*I*(b*x+a))+1)+2/b*c^2*ln(exp(I*(b*x+a)))-4/b^2*c*d*a*ln(exp(I*
(b*x+a)))
```

### 3.223.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 688 vs. $2(159) = 318$.

Time = 0.29 (sec) , antiderivative size = 688, normalized size of antiderivative = 3.74

$$\int (c+dx)^2 \sin^2(a+bx)\tan(a+bx)\,dx =$$
$$-\frac{b^2d^2x^2+2\,b^2cdx-(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2)\cos{(bx+a)}^2+4\,d^2\text{polylog}(3,i\,\cos{(bx+a)}+\sin$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2
 - d^2)*cos(b*x + a)^2 + 4*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) +
 4*d^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 4*d^2*polylog(3, -I*cos
(b*x + a) + sin(b*x + a)) + 4*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a
)) + 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) + 4*(I*b*d^2*x + I*b*c*
d)*dilog(I*cos(b*x + a) + sin(b*x + a)) + 4*(-I*b*d^2*x - I*b*c*d)*dilog(I
*cos(b*x + a) - sin(b*x + a)) + 4*(-I*b*d^2*x - I*b*c*d)*dilog(-I*cos(b*x
+ a) + sin(b*x + a)) + 4*(I*b*d^2*x + I*b*c*d)*dilog(-I*cos(b*x + a) - sin
(b*x + a)) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) + I*sin(b*
x + a) + I) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) - I*sin(b
*x + a) + I) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*c
os(b*x + a) + sin(b*x + a) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d
 - a^2*d^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + 2*(b^2*d^2*x^2 + 2*b^
2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 2
*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - s
in(b*x + a) + 1) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I
*sin(b*x + a) + I) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) -
 I*sin(b*x + a) + I))/b^3
```

### 3.223.6 Sympy [F]

$$\int (c+dx)^2 \sin^2(a+bx) \tan(a+bx)\, dx = \int (c+dx)^2 \sin^3(a+bx) \sec(a+bx)\, dx$$

input
```
integrate((d*x+c)**2*sec(b*x+a)*sin(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**2*sin(a + b*x)**3*sec(a + b*x), x)
```

### 3.223.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(159) = 318$.

Time = 0.43 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.08

$$\int (c+dx)^2 \sin^2(a+bx) \tan(a+bx)\, dx =$$

$$-\frac{12\left(\sin(bx+a)^2 + \log\left(\sin(bx+a)^2 - 1\right)\right)c^2 - \frac{24\left(\sin(bx+a)^2 + \log\left(\sin(bx+a)^2 - 1\right)\right)acd}{b} + \frac{12\left(\sin(bx+a)^2 + \log\left(\sin(bx\right.\right.}{b^2}}{}$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/24*(12*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1))*c^2 - 24*(sin(b*x + a
)^2 + log(sin(b*x + a)^2 - 1))*a*c*d/b + 12*(sin(b*x + a)^2 + log(sin(b*x
+ a)^2 - 1))*a^2*d^2/b^2 + (-8*I*(b*x + a)^3*d^2 - 24*(I*b*c*d - I*a*d^2)*
(b*x + a)^2 + 12*d^2*polylog(3, -e^(2*I*b*x + 2*I*a)) - 24*(-I*(b*x + a)^2
*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*arctan2(sin(2*b*x + 2*a), cos(2*b
*x + 2*a) + 1) - 3*(2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) - d^2)
*cos(2*b*x + 2*a) - 24*(I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*dilog(-e^(2*I
*b*x + 2*I*a)) + 12*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(co
s(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 6*(b*c*d
 + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))/b^2)/b
```

### 3.223.8 Giac [F]

$$\int (c+dx)^2 \sin^2(a+bx) \tan(a+bx)\, dx = \int (dx+c)^2 \sec(bx+a) \sin(bx+a)^3\, dx$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*sec(b*x + a)*sin(b*x + a)^3, x)
```

### 3.223.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \sin^2(a+bx) \tan(a+bx)\, dx = \int \frac{\sin(a+b\,x)^3\,(c+d\,x)^2}{\cos(a+b\,x)}\, dx$$

input
```
int((sin(a + b*x)^3*(c + d*x)^2)/cos(a + b*x),x)
```

output
```
int((sin(a + b*x)^3*(c + d*x)^2)/cos(a + b*x), x)
```

# 3.224 $\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx$

3.224.1 Optimal result . . . 1774
3.224.2 Mathematica [A] (verified) . . . 1774
3.224.3 Rubi [A] (verified) . . . 1775
3.224.4 Maple [A] (verified) . . . 1778
3.224.5 Fricas [B] (verification not implemented) . . . 1778
3.224.6 Sympy [F] . . . 1779
3.224.7 Maxima [A] (verification not implemented) . . . 1779
3.224.8 Giac [F] . . . 1780
3.224.9 Mupad [F(-1)] . . . 1780

## 3.224.1 Optimal result

Integrand size = 20, antiderivative size = 115

$$\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx = \frac{dx}{4b}+\frac{i(c+dx)^2}{2d}-\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{d\cos(a+bx)\sin(a+bx)}{4b^2}-\frac{(c+dx)\sin^2(a+bx)}{2b}$$

output
```
1/4*d*x/b+1/2*I*(d*x+c)^2/d-(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b+1/2*I*d*polyl
og(2,-exp(2*I*(b*x+a)))/b^2-1/4*d*cos(b*x+a)*sin(b*x+a)/b^2-1/2*(d*x+c)*si
n(b*x+a)^2/b
```

## 3.224.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.17

$$\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx = \frac{dx\cos(2(a+bx))}{4b}+\frac{ad\log(\cos(a+bx))}{b^2}-\frac{c\left(-\frac{1}{2}\cos^2(a+bx)+\log(\cos(a+bx))\right)}{b}+\frac{d\left(\frac{1}{2}i(a+bx)^2-(a+bx)\log\left(1+e^{2i(a+bx)}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)\right)}{b^2}-\frac{d\sin(2(a+bx))}{8b^2}$$

input
```
Integrate[(c + d*x)*Sin[a + b*x]^2*Tan[a + b*x],x]
```

output
```
(d*x*Cos[2*(a + b*x)])/(4*b) + (a*d*Log[Cos[a + b*x]])/b^2 - (c*(-1/2*Cos[
a + b*x]^2 + Log[Cos[a + b*x]]))/b + (d*((I/2)*(a + b*x)^2 - (a + b*x)*Log
[1 + E^((2*I)*(a + b*x))] + (I/2)*PolyLog[2, -E^((2*I)*(a + b*x))]))/b^2 -
 (d*Sin[2*(a + b*x)])/(8*b^2)
```

### 3.224.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4907, 3042, 4202, 2620, 2715, 2838, 4904, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx$$

$$\Big\downarrow \text{ 4907}$$

$$\int (c+dx)\tan(a+bx)dx-\int (c+dx)\cos(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)\tan(a+bx)dx-\int (c+dx)\cos(a+bx)\sin(a+bx)dx$$

$$\Big\downarrow \text{ 4202}$$

$$-2i\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}dx-\int (c+dx)\cos(a+bx)\sin(a+bx)dx+\frac{i(c+dx)^2}{2d}$$

$$\Big\downarrow \text{ 2620}$$

$$-2i\left(\frac{id\int \log\left(1+e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\int (c+dx)\cos(a+bx)\sin(a+bx)dx+\frac{i(c+dx)^2}{2d}$$

$$\Big\downarrow \text{ 2715}$$

$$-2i\left(\frac{d\int e^{-2i(a+bx)}\log\left(1+e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\int (c+dx)\cos(a+bx)\sin(a+bx)dx+\frac{i(c+dx)^2}{2d}$$

$$\Big\downarrow \text{2838}$$

$$-\int (c+dx)\cos(a+bx)\sin(a+bx)dx - 2i\left(-\frac{d\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)+\frac{i(c+dx)^2}{2d}$$

$$\Big\downarrow \text{4904}$$

$$\frac{d\int \sin^2(a+bx)dx}{2b}-2i\left(-\frac{d\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{(c+dx)\sin^2(a+bx)}{2b}+\frac{i(c+dx)^2}{2d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d\int \sin(a+bx)^2dx}{2b}-2i\left(-\frac{d\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{(c+dx)\sin^2(a+bx)}{2b}+\frac{i(c+dx)^2}{2d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{d\left(\frac{\int 1dx}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b}-2i\left(-\frac{d\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{(c+dx)\sin^2(a+bx)}{2b}+\frac{i(c+dx)^2}{2d}$$

$$\Big\downarrow \text{24}$$

$$-2i\left(-\frac{d\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{(c+dx)\sin^2(a+bx)}{2b}+\frac{d\left(\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}\right)}{2b}+\frac{i(c+dx)^2}{2d}$$

input
```
Int[(c + d*x)*Sin[a + b*x]^2*Tan[a + b*x],x]
```

output
```
((I/2)*(c + d*x)^2)/d - (2*I)*((((-1/2*I)*(c + d*x)*Log[1 + E^((2*I)*(a + b
*x))])/b - (d*PolyLog[2, -E^((2*I)*(a + b*x))])/(4*b^2)) - ((c + d*x)*Sin[
a + b*x]^2)/(2*b) + (d*(x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)))/(2*b)
```

## 3.224. $\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx$

### 3.224.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4904
```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sin[a + b*x]^(n + 1)/(b*(n + 1)))
, x] - Simp[d*(m/(b*(n + 1)))   Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

### 3.224.4 Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.56

| method | result |
|---|---|
| risch | $\frac{idx^2}{2} - icx + \frac{(2dxb+2cb+id)e^{2i(xb+a)}}{16b^2} + \frac{(2dxb+2cb-id)e^{-2i(xb+a)}}{16b^2} - \frac{c\ln(e^{2i(xb+a)}+1)}{b} + \frac{2c\ln(e^{i(xb+a)})}{b} + \frac{2idxa}{b} + i$ |

input
```
int((d*x+c)*sec(b*x+a)*sin(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*d*x^2-I*c*x+1/16*(2*d*x*b+I*d+2*c*b)/b^2*exp(2*I*(b*x+a))+1/16*(2*d*
x*b-I*d+2*c*b)/b^2*exp(-2*I*(b*x+a))-1/b*c*ln(exp(2*I*(b*x+a))+1)+2/b*c*ln
(exp(I*(b*x+a)))+2*I/b*d*x*a+I/b^2*d*a^2-1/b*d*ln(exp(2*I*(b*x+a))+1)*x+1/
2*I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2-2/b^2*d*a*ln(exp(I*(b*x+a)))
```

### 3.224.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(96) = 192.

Time = 0.28 (sec) , antiderivative size = 346, normalized size of antiderivative = 3.01

$$\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx = -\frac{bdx - 2\,(bdx+bc)\cos{(bx+a)}^2 + d\cos{(bx+a)}\sin{(bx+a)} + 2i\,d\mathrm{Li}_2(i\,\cos{(bx+a)} + \sin{(bx+a)}) -}{}$$

input
```
integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/4*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*x + a)
 + 2*I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) - 2*I*d*dilog(I*cos(b*x + a)
 - sin(b*x + a)) - 2*I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) + 2*I*d*dil
og(-I*cos(b*x + a) - sin(b*x + a)) + 2*(b*c - a*d)*log(cos(b*x + a) + I*si
n(b*x + a) + I) + 2*(b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) + 2
*(b*d*x + a*d)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(b*d*x + a*d)*lo
g(I*cos(b*x + a) - sin(b*x + a) + 1) + 2*(b*d*x + a*d)*log(-I*cos(b*x + a)
 + sin(b*x + a) + 1) + 2*(b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a)
+ 1) + 2*(b*c - a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + 2*(b*c - a*
d)*log(-cos(b*x + a) - I*sin(b*x + a) + I))/b^2
```

### 3.224.6 Sympy [F]

$$\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx=\int (c+dx)\sin^3(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)**3,x)
```

output
```
Integral((c + d*x)*sin(a + b*x)**3*sec(a + b*x), x)
```

### 3.224.7 Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.27

$$\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx =$$
$$-\frac{-4i\,b^2dx^2-8i\,b^2cx-8\left(-i\,bdx-i\,bc\right)\arctan\left(\sin\left(2\,bx+2\,a\right),\cos\left(2\,bx+2\,a\right)+1\right)-2\left(bdx+bc\right)\cos\left(\right.}{}$$

input
```
integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(-4*I*b^2*d*x^2 - 8*I*b^2*c*x - 8*(-I*b*d*x - I*b*c)*arctan2(sin(2*b*
x + 2*a), cos(2*b*x + 2*a) + 1) - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - 4*I*d
*dilog(-e^(2*I*b*x + 2*I*a)) + 4*(b*d*x + b*c)*log(cos(2*b*x + 2*a)^2 + si
n(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + d*sin(2*b*x + 2*a))/b^2
```

### 3.224.8 Giac [F]

$$\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx=\int (dx+c)\sec(bx+a)\sin(bx+a)^3\,dx$$

input
```
integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)*sec(b*x + a)*sin(b*x + a)^3, x)
```

### 3.224.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx=\int \frac{\sin(a+b\,x)^3\,(c+d\,x)}{\cos(a+b\,x)}\,dx$$

input
```
int((sin(a + b*x)^3*(c + d*x))/cos(a + b*x),x)
```

output
```
int((sin(a + b*x)^3*(c + d*x))/cos(a + b*x), x)
```

# 3.225 $\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx$

3.225.1 Optimal result . . . 1781
3.225.2 Mathematica [N/A] . . . 1781
3.225.3 Rubi [N/A] . . . 1782
3.225.4 Maple [N/A] (verified) . . . 1784
3.225.5 Fricas [N/A] . . . 1785
3.225.6 Sympy [F(-2)] . . . 1785
3.225.7 Maxima [N/A] . . . 1785
3.225.8 Giac [N/A] . . . 1786
3.225.9 Mupad [N/A] . . . 1786

## 3.225.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx = -\frac{\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d} - \frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d} + \text{Int}\left(\frac{\tan(a+bx)}{c+dx},x\right)$$

output
```
-1/2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d-1/2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*
b*c/d)/d+Unintegrable(tan(b*x+a)/(d*x+c),x)
```

## 3.225.2 Mathematica [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x),x]
```

output
```
Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]
```

### 3.225.3 Rubi [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4907, 3042, 4222, 4906, 27, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{ 4907}$$

$$\int \frac{\tan(a+bx)}{c+dx}dx-\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(a+bx)}{c+dx}dx-\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{ 4222}$$

$$\int \frac{\tan(a+bx)}{c+dx}dx-\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{ 4906}$$

$$\int \frac{\tan(a+bx)}{c+dx}dx-\int \frac{\sin(2a+2bx)}{2(c+dx)}dx$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{\tan(a+bx)}{c+dx}dx-\frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(a+bx)}{c+dx}dx-\frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}dx$$

$$\Big\downarrow \text{ 3784}$$

$$\int \frac{\tan(a+bx)}{c+dx}dx+\frac{1}{2}\left(-\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx-\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(a+bx)}{c+dx}dx+\frac{1}{2}\left(-\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx-\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)$$

$\big\downarrow$ 3780

$$\frac{1}{2}\left(-\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx-\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)+\int \frac{\tan(a+bx)}{c+dx}dx$$

$\big\downarrow$ 3783

$$\int \frac{\tan(a+bx)}{c+dx}dx+\frac{1}{2}\left(-\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}-\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)$$

input
```
Int[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x),x]
```

output
```
$Aborted
```

### 3.225.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

### 3.225.4 Maple [N/A] (verified)

Not integrable

Time = 1.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sec (xb+a) \sin (xb+a)^3}{dx+c} dx$$

input
```
int(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x)
```

output
```
int(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x)
```

### 3.225.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)\sin(bx+a)^3}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sec(b*x + a)*sin(b*x + a)/(d*x + c), x)
```

### 3.225.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(sec(b*x+a)*sin(b*x+a)**3/(d*x+c),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.225.7 Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 182, normalized size of antiderivative = 8.27

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)\sin(bx+a)^3}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output
```
1/4*((-I*exp_integral_e(1, 2*(-I*b*d*x - I*b*c)/d) + I*exp_integral_e(1, -
2*(-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 8*d*integrate(sin(2*b*x +
 2*a)/((d*x + c)*cos(2*b*x + 2*a)^2 + (d*x + c)*sin(2*b*x + 2*a)^2 + d*x +
 2*(d*x + c)*cos(2*b*x + 2*a) + c), x) + (exp_integral_e(1, 2*(-I*b*d*x -
I*b*c)/d) + exp_integral_e(1, -2*(-I*b*d*x - I*b*c)/d))*sin(-2*(b*c - a*d)
/d))/d
```

### 3.225.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)\sin(bx+a)^3}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)*sin(b*x + a)^3/(d*x + c), x)
```

### 3.225.9 Mupad [N/A]

Not integrable

Time = 25.53 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sin(a+b\,x)^3}{\cos(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(sin(a + b*x)^3/(cos(a + b*x)*(c + d*x)),x)
```

output
```
int(sin(a + b*x)^3/(cos(a + b*x)*(c + d*x)), x)
```

# 3.226 $\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$


3.226.1 Optimal result . . . 1787
3.226.2 Mathematica [N/A] . . . 1787
3.226.3 Rubi [N/A] . . . 1788
3.226.4 Maple [N/A] (verified) . . . 1791
3.226.5 Fricas [N/A] . . . 1791
3.226.6 Sympy [F(-2)] . . . 1792
3.226.7 Maxima [N/A] . . . 1792
3.226.8 Giac [N/A] . . . 1792
3.226.9 Mupad [N/A] . . . 1793


## 3.226.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = -\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{\sin(2a+2bx)}{2d(c+dx)} + \frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\text{Int}\left(\frac{\tan(a+bx)}{(c+dx)^2},x\right)$$

output
```
-b*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^2+b*Si(2*b*c/d+2*b*x)*sin(2*a-2*b*
c/d)/d^2+1/2*sin(2*b*x+2*a)/d/(d*x+c)+Unintegrable(tan(b*x+a)/(d*x+c)^2,x)
```

## 3.226.2 Mathematica [N/A]

Not integrable

Time = 2.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]
```

### 3.226.3 Rubi [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4907, 3042, 4222, 4906, 27, 3042, 3778, 3042, 3784, 3042, 3780, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{4907}$$

$$\int \frac{\tan(a+bx)}{(c+dx)^2}dx-\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(a+bx)}{(c+dx)^2}dx-\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{4222}$$

$$\int \frac{\tan(a+bx)}{(c+dx)^2}dx-\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{4906}$$

$$\int \frac{\tan(a+bx)}{(c+dx)^2}dx-\int \frac{\sin(2a+2bx)}{2(c+dx)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\tan(a+bx)}{(c+dx)^2}dx-\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(a+bx)}{(c+dx)^2}dx-\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3778}$$

$$\int \frac{\tan(a+bx)}{(c+dx)^2}dx+\frac{1}{2}\left(\frac{\sin(2a+2bx)}{d(c+dx)}-\frac{2b\int \frac{\cos(2a+2bx)}{c+dx}dx}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{2}\left(\frac{\sin(2a+2bx)}{d(c+dx)}-\frac{2b\int\frac{\sin\left(2a+2bx+\frac{\pi}{2}\right)}{c+dx}dx}{d}\right)+\int\frac{\tan(a+bx)}{(c+dx)^2}dx$$

$\downarrow$ 3784

$$\int\frac{\tan(a+bx)}{(c+dx)^2}dx+\frac{1}{2}\left(\frac{\sin(2a+2bx)}{d(c+dx)}-\frac{2b\left(\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx-\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}\right)$$

$\downarrow$ 3042

$$\int\frac{\tan(a+bx)}{(c+dx)^2}dx+\frac{1}{2}\left(\frac{\sin(2a+2bx)}{d(c+dx)}-\frac{2b\left(\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx-\sin\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}\right)$$

$\downarrow$ 3780

$$\frac{1}{2}\left(\frac{\sin(2a+2bx)}{d(c+dx)}-\frac{2b\left(\cos\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2bc}{d}+2bx+\frac{\pi}{2}\right)}{c+dx}dx-\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}\right)+\int\frac{\tan(a+bx)}{(c+dx)^2}dx$$

$\downarrow$ 3783

$$\int\frac{\tan(a+bx)}{(c+dx)^2}dx+\frac{1}{2}\left(\frac{\sin(2a+2bx)}{d(c+dx)}-\frac{2b\left(\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}-\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}\right)$$

input
```
Int[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

3.226. $\int\frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$

### 3.226.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

## 3.226. $\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

### 3.226.4 Maple [N/A] (verified)

Not integrable

Time = 1.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sec (xb+a) \sin (xb+a)^3}{(dx+c)^2} dx$$

input
```
int(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x)
```

### 3.226.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2} dx = \int \frac{\sec (bx+a) \sin (bx+a)^3}{(dx+c)^2} dx$$

input
```
integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(-(cos(b*x + a)^2 - 1)*sec(b*x + a)*sin(b*x + a)/(d^2*x^2 + 2*c*d*
x + c^2), x)
```

### 3.226.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(sec(b*x+a)*sin(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.226.7 Maxima [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 242, normalized size of antiderivative = 11.00

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec(bx+a)\sin(bx+a)^3}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/4*((-I*exp_integral_e(2, 2*(-I*b*d*x - I*b*c)/d) + I*exp_integral_e(2, -
2*(-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 8*(d^2*x + c*d)*integrate
(sin(2*b*x + 2*a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(2*b*x
 + 2*a)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(2*b*x + 2*a)^2 + c^2 + 2*(d^2*x^
2 + 2*c*d*x + c^2)*cos(2*b*x + 2*a)), x) + (exp_integral_e(2, 2*(-I*b*d*x
- I*b*c)/d) + exp_integral_e(2, -2*(-I*b*d*x - I*b*c)/d))*sin(-2*(b*c - a*
d)/d))/(d^2*x + c*d)
```

### 3.226.8 Giac [N/A]

Not integrable

Time = 2.80 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec(bx+a)\sin(bx+a)^3}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)*sin(b*x + a)^3/(d*x + c)^2, x)
```

### 3.226.9 Mupad [N/A]

Not integrable

Time = 25.57 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin\left(a+b\,x\right)^3}{\cos\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(sin(a + b*x)^3/(cos(a + b*x)*(c + d*x)^2),x)
```

output
```
int(sin(a + b*x)^3/(cos(a + b*x)*(c + d*x)^2), x)
```

# 3.227 $\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx$

3.227.1 Optimal result . . . . . . . . . . 1794
3.227.2 Mathematica [N/A] . . . . . . . . . . 1794
3.227.3 Rubi [N/A] . . . . . . . . . . 1795
3.227.4 Maple [N/A] (verified) . . . . . . . . . . 1795
3.227.5 Fricas [N/A] . . . . . . . . . . 1796
3.227.6 Sympy [N/A] . . . . . . . . . . 1796
3.227.7 Maxima [N/A] . . . . . . . . . . 1796
3.227.8 Giac [N/A] . . . . . . . . . . 1797
3.227.9 Mupad [N/A] . . . . . . . . . . 1797

## 3.227.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx = \text{Int}((c+dx)^m \csc(a+bx)\sec(a+bx),x)$$

output
```
CannotIntegrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x)
```

## 3.227.2 Mathematica [N/A]

Not integrable

Time = 10.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx = \int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x], x]
```

## 3.227.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\sec(a+bx)(c+dx)^m\,dx$$

$$\big\downarrow \text{7299}$$

$$\int \csc(a+bx)\sec(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x],x]
```

output
```
$Aborted
```

### 3.227.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.227.4 Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \csc(xb+a)\sec(xb+a)\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x)
```

output
```
int((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x)
```

### 3.227.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx=\int (dx+c)^m \csc{(bx+a)}\sec{(bx+a)}\;dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*csc(b*x + a)*sec(b*x + a), x)
```

### 3.227.6 Sympy [N/A]

Not integrable

Time = 33.99 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx=\int (c+dx)^m \csc{(a+bx)}\sec{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**m*csc(b*x+a)*sec(b*x+a),x)
```

output
```
Integral((c + d*x)**m*csc(a + b*x)*sec(a + b*x), x)
```

### 3.227.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx=\int (dx+c)^m \csc{(bx+a)}\sec{(bx+a)}\;dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a), x)
```

### 3.227.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)\sec(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a), x)
```

### 3.227.9 Mupad [N/A]

Not integrable

Time = 25.96 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\cos(a+b\,x)\,\sin(a+b\,x)}\,dx$$

input
```
int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)),x)
```

output
```
int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)), x)
```

# 3.228 $\int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx$


3.228.1 Optimal result . . . . . 1798
3.228.2 Mathematica [B] (verified) . . . . . 1799
3.228.3 Rubi [A] (verified) . . . . . 1800
3.228.4 Maple [B] (verified) . . . . . 1803
3.228.5 Fricas [F(-2)] . . . . . 1804
3.228.6 Sympy [F] . . . . . 1805
3.228.7 Maxima [B] (verification not implemented) . . . . . 1805
3.228.8 Giac [F] . . . . . 1806
3.228.9 Mupad [F(-1)] . . . . . 1806


## 3.228.1 Optimal result

Integrand size = 20, antiderivative size = 247

$$\begin{aligned}\int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx = & -\frac{2(c+dx)^4\operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b} \\ & +\frac{2id(c+dx)^3\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} \\ & -\frac{2id(c+dx)^3\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2} \\ & -\frac{3d^2(c+dx)^2\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3} \\ & +\frac{3d^2(c+dx)^2\operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3} \\ & -\frac{3id^3(c+dx)\operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4} \\ & +\frac{3id^3(c+dx)\operatorname{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4} \\ & +\frac{3d^4\operatorname{PolyLog}\left(5,-e^{2i(a+bx)}\right)}{2b^5} \\ & -\frac{3d^4\operatorname{PolyLog}\left(5,e^{2i(a+bx)}\right)}{2b^5}\end{aligned}$$

output
```
-2*(d*x+c)^4*arctanh(exp(2*I*(b*x+a)))/b+2*I*d*(d*x+c)^3*polylog(2,-exp(2*
I*(b*x+a)))/b^2-2*I*d*(d*x+c)^3*polylog(2,exp(2*I*(b*x+a)))/b^2-3*d^2*(d*x
+c)^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+3*d^2*(d*x+c)^2*polylog(3,exp(2*I*(
b*x+a)))/b^3-3*I*d^3*(d*x+c)*polylog(4,-exp(2*I*(b*x+a)))/b^4+3*I*d^3*(d*x
+c)*polylog(4,exp(2*I*(b*x+a)))/b^4+3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b
^5-3/2*d^4*polylog(5,exp(2*I*(b*x+a)))/b^5
```

### 3.228.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 578 vs. 2(247) = 494.

Time = 1.23 (sec) , antiderivative size = 578, normalized size of antiderivative = 2.34

$$\int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx$$
$$= \frac{-4b^4c^4\text{arctanh}\left(e^{2i(a+bx)}\right)+8b^4c^3dx\log\left(1-e^{2i(a+bx)}\right)+12b^4c^2d^2x^2\log\left(1-e^{2i(a+bx)}\right)+8b^4cd^3x^3\log\left(1-\right.}{}$$

input
```
Integrate[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x],x]
```

output
```
(-4*b^4*c^4*ArcTanh[E^((2*I)*(a + b*x))] + 8*b^4*c^3*d*x*Log[1 - E^((2*I)*
(a + b*x))] + 12*b^4*c^2*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] + 8*b^4*c*d^
3*x^3*Log[1 - E^((2*I)*(a + b*x))] + 2*b^4*d^4*x^4*Log[1 - E^((2*I)*(a + b
*x))] - 8*b^4*c^3*d*x*Log[1 + E^((2*I)*(a + b*x))] - 12*b^4*c^2*d^2*x^2*Lo
g[1 + E^((2*I)*(a + b*x))] - 8*b^4*c*d^3*x^3*Log[1 + E^((2*I)*(a + b*x))]
- 2*b^4*d^4*x^4*Log[1 + E^((2*I)*(a + b*x))] + (4*I)*b^3*d*(c + d*x)^3*Pol
yLog[2, -E^((2*I)*(a + b*x))] - (4*I)*b^3*d*(c + d*x)^3*PolyLog[2, E^((2*I
)*(a + b*x))] - 6*b^2*c^2*d^2*PolyLog[3, -E^((2*I)*(a + b*x))] - 12*b^2*c*
d^3*x*PolyLog[3, -E^((2*I)*(a + b*x))] - 6*b^2*d^4*x^2*PolyLog[3, -E^((2*I
)*(a + b*x))] + 6*b^2*c^2*d^2*PolyLog[3, E^((2*I)*(a + b*x))] + 12*b^2*c*d
^3*x*PolyLog[3, E^((2*I)*(a + b*x))] + 6*b^2*d^4*x^2*PolyLog[3, E^((2*I)*(
a + b*x))] - (6*I)*b*c*d^3*PolyLog[4, -E^((2*I)*(a + b*x))] - (6*I)*b*d^4*
x*PolyLog[4, -E^((2*I)*(a + b*x))] + (6*I)*b*c*d^3*PolyLog[4, E^((2*I)*(a
+ b*x))] + (6*I)*b*d^4*x*PolyLog[4, E^((2*I)*(a + b*x))] + 3*d^4*PolyLog[5
, -E^((2*I)*(a + b*x))] - 3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5)
```

### 3.228.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4919, 3042, 4671, 3011, 7163, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4\csc(a+bx)\sec(a+bx)\,dx$$

$$\Big\downarrow \text{ 4919}$$

$$2\int (c+dx)^4\csc(2a+2bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$2\int (c+dx)^4\csc(2a+2bx)dx$$

$$\Big\downarrow \text{ 4671}$$

$$2\left(-\frac{2d\int (c+dx)^3\log\left(1-e^{2i(a+bx)}\right)dx}{b}+\frac{2d\int (c+dx)^3\log\left(1+e^{2i(a+bx)}\right)dx}{b}-\frac{(c+dx)^4\text{arctanh}\left(e^{2i(a+bx)}\right)}{b}\right)$$

$$\Big\downarrow \text{ 3011}$$

$$2\left(\frac{2d\left(\frac{i(c+dx)^3\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{3id\int (c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{2d\left(\frac{i(c+dx)^3\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b}-\frac{3id\int (c+dx)\right.}{b}\right.$$

$$\Big\downarrow \text{ 7163}$$

$$2\left(\frac{2d\left(\frac{i(c+dx)^3\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{3id\left(\frac{id\int (c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)dx}{b}-\frac{i(c+dx)^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{2b}\right)}{b}-\frac{2d\left(\frac{i(c+dx)^3\text{I}\right.}{}\right.$$

$$\Big\downarrow \text{ 7163}$$

$$2\left(\frac{2d\left(\frac{i(c+dx)^3\operatorname{PolyLog}(2,-e^{2i(a+bx)})}{2b}-\frac{3id\left(\frac{id\left(\frac{id\int \operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{2b}\right)}{b}-\frac{i(c+dx)^2\operatorname{PolyLog}\left(3,-e^{2i(a\cdots}\right.}{2b}\right.}{2b}\right.}{b}\right.$$

$\downarrow$ 2720

$$2\left(\frac{2d\left(\frac{i(c+dx)^3\operatorname{PolyLog}(2,-e^{2i(a+bx)})}{2b}-\frac{3id\left(\frac{id\left(\frac{d\int e^{-2i(a+bx)}\operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{2b}\right)}{b}-\frac{i(c+dx\cdots}{\cdots}\right.}{2b}\right.}{b}\right.$$

$\downarrow$ 7143

$$2\left(-\frac{(c+dx)^4\operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)^3\operatorname{PolyLog}(2,-e^{2i(a+bx)})}{2b}-\frac{3id\left(\frac{id\left(\frac{d\operatorname{PolyLog}\left(5,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(4,-e^{2\cdots}\right)}{2b}\right.}{b}\right.}{2b}\right.}{b}\right.$$

3.228. $\int (c+dx)^4\csc(a+bx)\sec(a+bx)\,dx$

input
```
Int[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x],x]
```

output
```
2*(-(((c + d*x)^4*ArcTanh[E^((2*I)*(a + b*x))])/b) + (2*d*(((I/2)*(c + d*x
)^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d*(((-1/2*I)*(c + d*x
)^2*PolyLog[3, -E^((2*I)*(a + b*x))])/b + (I*d*(((-1/2*I)*(c + d*x)*PolyLo
g[4, -E^((2*I)*(a + b*x))])/b + (d*PolyLog[5, -E^((2*I)*(a + b*x))])/(4*b^
2)))/b))/b))/b - (2*d*(((I/2)*(c + d*x)^3*PolyLog[2, E^((2*I)*(a + b*x))])
/b - (((3*I)/2)*d*(((-1/2*I)*(c + d*x)^2*PolyLog[3, E^((2*I)*(a + b*x))])/
b + (I*d*(((-1/2*I)*(c + d*x)*PolyLog[4, E^((2*I)*(a + b*x))])/b + (d*Poly
Log[5, E^((2*I)*(a + b*x))])/(4*b^2)))/b))/b))/b)
```

### 3.228.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

### 3.228.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1241 vs. $2(221) = 442$.

Time = 1.33 (sec) , antiderivative size = 1242, normalized size of antiderivative = 5.03

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1242 |

input
```
int((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-3*I/b^4*d^4*polylog(4,-exp(2*I*(b*x+a)))*x+24*I/b^4*d^4*polylog(4,exp(I*(
b*x+a)))*x-4*I/b^2*c^3*d*polylog(2,-exp(I*(b*x+a)))+2*I/b^2*c^3*d*polylog(
2,-exp(2*I*(b*x+a)))-4*I/b^2*c^3*d*polylog(2,exp(I*(b*x+a)))+24*I/b^4*c*d^
3*polylog(4,-exp(I*(b*x+a)))-3*I/b^4*c*d^3*polylog(4,-exp(2*I*(b*x+a)))+24
*I/b^4*c*d^3*polylog(4,exp(I*(b*x+a)))+2*I/b^2*d^4*polylog(2,-exp(2*I*(b*x
+a)))*x^3-4*I/b^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3-4*I/b^2*d^4*polylog(2
,exp(I*(b*x+a)))*x^3-12*I/b^2*d^2*c^2*polylog(2,-exp(I*(b*x+a)))*x+6*I/b^2
*d^2*c^2*polylog(2,-exp(2*I*(b*x+a)))*x-12*I/b^2*d^2*c^2*polylog(2,exp(I*(
b*x+a)))*x-12*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2+1/b*c^4*ln(exp(I*(
b*x+a))+1)-1/b*c^4*ln(exp(2*I*(b*x+a))+1)+1/b*c^4*ln(exp(I*(b*x+a))-1)+3/2
*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5+4/b*c*d^3*ln(1-exp(I*(b*x+a)))*x^3-2
4*d^4*polylog(5,-exp(I*(b*x+a)))/b^5-24*d^4*polylog(5,exp(I*(b*x+a)))/b^5-
12*I/b^2*c*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+6*I/b^2*c*d^3*polylog(2,-exp
(2*I*(b*x+a)))*x^2+1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4+1/b*d^4*ln(exp(I*(b*x+
a))+1)*x^4-1/b*d^4*ln(exp(2*I*(b*x+a))+1)*x^4+12/b^3*d^4*polylog(3,exp(I*(
b*x+a)))*x^2-1/b^5*d^4*ln(1-exp(I*(b*x+a)))*a^4+12/b^3*d^4*polylog(3,-exp(
I*(b*x+a)))*x^2-3/b^3*d^4*polylog(3,-exp(2*I*(b*x+a)))*x^2+12/b^3*d^2*c^2*
polylog(3,-exp(I*(b*x+a)))-3/b^3*d^2*c^2*polylog(3,-exp(2*I*(b*x+a)))+12/b
^3*d^2*c^2*polylog(3,exp(I*(b*x+a)))+1/b^5*d^4*a^4*ln(exp(I*(b*x+a))-1)+4/
b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3+4/b*c^3*d*ln(1-exp(I*(b*x+a)))*x-6/b^3...
```

### 3.228.5 Fricas [F(-2)]

Exception generated.

$$\int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   Too
many variables
```

### 3.228.6 Sympy [F]

$$\int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx = \int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**4*csc(b*x+a)*sec(b*x+a),x)
```

output
```
Integral((c + d*x)**4*csc(a + b*x)*sec(a + b*x), x)
```

### 3.228.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1794 vs. $2(215) = 430$.

Time = 0.52 (sec) , antiderivative size = 1794, normalized size of antiderivative = 7.26

$$\int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

output
```
-1/6*(3*c^4*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)) - 12*a*c^3*d*(
log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b + 18*a^2*c^2*d^2*(log(sin
(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^2 - 12*a^3*c*d^3*(log(sin(b*x +
a)^2 - 1) - log(sin(b*x + a)^2))/b^3 + 3*a^4*d^4*(log(sin(b*x + a)^2 - 1)
- log(sin(b*x + a)^2))/b^4 - (18*d^4*polylog(5, -e^(2*I*b*x + 2*I*a)) - 14
4*d^4*polylog(5, -e^(I*b*x + I*a)) - 144*d^4*polylog(5, e^(I*b*x + I*a)) +
 4*(-3*I*(b*x + a)^4*d^4 + 8*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 9*(-I*b^
2*c^2*d^2 + 2*I*a*b*c*d^3 - I*a^2*d^4)*(b*x + a)^2 + 6*(-I*b^3*c^3*d + 3*I
*a*b^2*c^2*d^2 - 3*I*a^2*b*c*d^3 + I*a^3*d^4)*(b*x + a))*arctan2(sin(2*b*x
 + 2*a), cos(2*b*x + 2*a) + 1) + 6*(I*(b*x + a)^4*d^4 + 4*(I*b*c*d^3 - I*a
*d^4)*(b*x + a)^3 + 6*(I*b^2*c^2*d^2 - 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a
)^2 + 4*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 - I*a^3*d^4)*(b
*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 6*(-I*(b*x + a)^4*d^4 +
 4*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 6*(-I*b^2*c^2*d^2 + 2*I*a*b*c*d^3
- I*a^2*d^4)*(b*x + a)^2 + 4*(-I*b^3*c^3*d + 3*I*a*b^2*c^2*d^2 - 3*I*a^2*b
*c*d^3 + I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) +
12*(I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 + 2*I*(b*x + a)^3*d^
4 - I*a^3*d^4 + 4*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2
*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*a)) + 24*(-I*
b^3*c^3*d + 3*I*a*b^2*c^2*d^2 - 3*I*a^2*b*c*d^3 - I*(b*x + a)^3*d^4 + I...
```

### 3.228.8 Giac [F]

$$\int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx = \int (dx+c)^4 \csc(bx+a)\sec(bx+a)\ dx$$

input
```
integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*csc(b*x + a)*sec(b*x + a), x)
```

### 3.228.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx = \int \frac{(c+d\,x)^4}{\cos(a+b\,x)\ \sin(a+b\,x)}\,dx$$

input
```
int((c + d*x)^4/(cos(a + b*x)*sin(a + b*x)),x)
```

output
```
int((c + d*x)^4/(cos(a + b*x)*sin(a + b*x)), x)
```

# 3.229 $\int (c+dx)^3 \csc(a+bx) \sec(a+bx)\, dx$

3.229.1 Optimal result . . . 1807
3.229.2 Mathematica [A] (verified) . . . 1808
3.229.3 Rubi [A] (verified) . . . 1808
3.229.4 Maple [B] (verified) . . . 1811
3.229.5 Fricas [B] (verification not implemented) . . . 1812
3.229.6 Sympy [F] . . . 1813
3.229.7 Maxima [B] (verification not implemented) . . . 1814
3.229.8 Giac [F] . . . 1814
3.229.9 Mupad [F(-1)] . . . 1815

## 3.229.1 Optimal result

Integrand size = 20, antiderivative size = 197

$$\begin{aligned}\int (c+dx)^3 \csc(a+bx) \sec(a+bx)\, dx = &-\frac{2(c+dx)^3 \operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b} \\ &+\frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} \\ &-\frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2} \\ &-\frac{3d^2(c+dx) \operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} \\ &+\frac{3d^2(c+dx) \operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3} \\ &-\frac{3id^3 \operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4} \\ &+\frac{3id^3 \operatorname{PolyLog}\left(4,e^{2i(a+bx)}\right)}{4b^4}\end{aligned}$$

output
```
-2*(d*x+c)^3*arctanh(exp(2*I*(b*x+a)))/b+3/2*I*d*(d*x+c)^2*polylog(2,-exp(
2*I*(b*x+a)))/b^2-3/2*I*d*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2-3/2*d^
2*(d*x+c)*polylog(3,-exp(2*I*(b*x+a)))/b^3+3/2*d^2*(d*x+c)*polylog(3,exp(2
*I*(b*x+a)))/b^3-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+3/4*I*d^3*poly
log(4,exp(2*I*(b*x+a)))/b^4
```

### 3.229.2 Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.78

$$\int (c+dx)^3 \csc(a+bx)\sec(a+bx)\,dx$$
$$= \frac{-8b^3c^3\text{arctanh}\left(e^{2i(a+bx)}\right)+12b^3c^2dx\log\left(1-e^{2i(a+bx)}\right)+12b^3cd^2x^2\log\left(1-e^{2i(a+bx)}\right)+4b^3d^3x^3\log\left(1-\right.}{}$$

input
```
Integrate[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x],x]
```

output
```
(-8*b^3*c^3*ArcTanh[E^((2*I)*(a + b*x))] + 12*b^3*c^2*d*x*Log[1 - E^((2*I)
*(a + b*x))] + 12*b^3*c*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] + 4*b^3*d^3*x
^3*Log[1 - E^((2*I)*(a + b*x))] - 12*b^3*c^2*d*x*Log[1 + E^((2*I)*(a + b*x
))] - 12*b^3*c*d^2*x^2*Log[1 + E^((2*I)*(a + b*x))] - 4*b^3*d^3*x^3*Log[1
+ E^((2*I)*(a + b*x))] + (6*I)*b^2*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a +
 b*x))] - (6*I)*b^2*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))] - 6*b*d^
2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))] + 6*b*c*d^2*PolyLog[3, E^((2*
I)*(a + b*x))] + 6*b*d^3*x*PolyLog[3, E^((2*I)*(a + b*x))] - (3*I)*d^3*Pol
yLog[4, -E^((2*I)*(a + b*x))] + (3*I)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])
/(4*b^4)
```

### 3.229.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {4919, 3042, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \csc(a+bx)\sec(a+bx)\,dx$$
$$\Big\downarrow \; 4919$$
$$2\int (c+dx)^3\csc(2a+2bx)dx$$
$$\Big\downarrow \; 3042$$
$$2\int (c+dx)^3\csc(2a+2bx)dx$$

$$\Big\downarrow \; 4671$$

$$2\left(-\frac{3d\int (c+dx)^2\log\left(1-e^{2i(a+bx)}\right)dx}{2b}+\frac{3d\int (c+dx)^2\log\left(1+e^{2i(a+bx)}\right)dx}{2b}-\frac{(c+dx)^3\text{arctanh}\left(e^{2i(a+bx)}\right)}{b}\right)$$

$$\Big\downarrow \; 3011$$

$$2\left(\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int (c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b}-\frac{id\int (c+dx)\operatorname{Po}\right.}{2b}\right.$$

$$\Big\downarrow \; 7163$$

$$2\left(\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\left(\frac{id\int \operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}(}{2b}\right.}{}\right.$$

$$\Big\downarrow \; 2720$$

$$2\left(\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\left(\frac{d\int e^{-2i(a+bx)}\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{3d\left(\right.}{}\right.$$

$$\Big\downarrow \; 7143$$

$$2\left(-\frac{(c+dx)^3\text{arctanh}\left(e^{2i(a+bx)}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\left(\frac{d\operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{2i(a+b}\right.}{2b}\right.}{b}\right.}{2b}\right.$$

input
```
Int[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x],x]
```

3.229. $\int (c+dx)^3\csc(a+bx)\sec(a+bx)\,dx$

output
```
2*(-(((c + d*x)^3*ArcTanh[E^((2*I)*(a + b*x))])/b) + (3*d*(((I/2)*(c + d*x
)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b - (I*d*(((-1/2*I)*(c + d*x)*PolyLo
g[3, -E^((2*I)*(a + b*x))])/b + (d*PolyLog[4, -E^((2*I)*(a + b*x))])/(4*b^
2)))/b))/(2*b) - (3*d*(((I/2)*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])
/b - (I*d*(((-1/2*I)*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])/b + (d*Pol
yLog[4, E^((2*I)*(a + b*x))])/(4*b^2)))/b))/(2*b))
```

### 3.229.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

### 3.229.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 815 vs. 2(167) = 334.

Time = 1.30 (sec) , antiderivative size = 816, normalized size of antiderivative = 4.14

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 816 |

input
```
int((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4-6*I/b^2*c*d^2*polylog(2,exp(I*
(b*x+a)))*x-6*I/b^2*c*d^2*polylog(2,-exp(I*(b*x+a)))*x-1/b*c^3*ln(exp(2*I*
(b*x+a))+1)-3/b*d*c^2*ln(exp(2*I*(b*x+a))+1)*x-3/b*c*d^2*ln(exp(2*I*(b*x+a
))+1)*x^2+3/2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2+3/2*I/b^2*d*c^2*p
olylog(2,-exp(2*I*(b*x+a)))+3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)-3/b^2*c^2
*d*a*ln(exp(I*(b*x+a))-1)+3/b^2*d*c^2*ln(1-exp(I*(b*x+a)))*a-3/b^3*c*d^2*l
n(1-exp(I*(b*x+a)))*a^2-1/b*d^3*ln(exp(2*I*(b*x+a))+1)*x^3-3/2/b^3*d^3*pol
ylog(3,-exp(2*I*(b*x+a)))*x-3/2/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))+1/b
*c^3*ln(exp(I*(b*x+a))+1)+1/b*c^3*ln(exp(I*(b*x+a))-1)+3/b*d*c^2*ln(1-exp(
I*(b*x+a)))*x+3/b*d*c^2*ln(exp(I*(b*x+a))+1)*x+3/b*c*d^2*ln(1-exp(I*(b*x+a
)))*x^2+3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2-3*I/b^2*d^3*polylog(2,-exp(I*(b
*x+a)))*x^2-3*I/b^2*d*c^2*polylog(2,exp(I*(b*x+a)))-3*I/b^2*d*c^2*polylog(
2,-exp(I*(b*x+a)))-3*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2-1/b^4*d^3*a^3
*ln(exp(I*(b*x+a))-1)+1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+6/b^3*d^3*polylog
(3,exp(I*(b*x+a)))*x+6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+1/b*d^3*ln(1-e
xp(I*(b*x+a)))*x^3+1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3+6/b^3*c*d^2*polylog(3,
exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+6*I/b^4*d^3*polylog
(4,-exp(I*(b*x+a)))+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+3*I/b^2*c*d^2*po
lylog(2,-exp(2*I*(b*x+a)))*x
```

### 3.229.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1786 vs. $2(161) = 322$.

Time = 0.38 (sec) , antiderivative size = 1786, normalized size of antiderivative = 9.07

$$\int (c+dx)^3 \csc(a+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(6*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*polylog(4
, cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*polylog(4, I*cos(b*x + a) + sin
(b*x + a)) - 6*I*d^3*polylog(4, I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*p
olylog(4, -I*cos(b*x + a) + sin(b*x + a)) + 6*I*d^3*polylog(4, -I*cos(b*x
+ a) - sin(b*x + a)) - 6*I*d^3*polylog(4, -cos(b*x + a) + I*sin(b*x + a))
+ 6*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) - 3*(I*b^2*d^3*x^2 +
2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(cos(b*x + a) + I*sin(b*x + a)) - 3*(-
I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(cos(b*x + a) - I*sin(
b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(I*cos(
b*x + a) + sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2
*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d
^2*x - I*b^2*c^2*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 3*(I*b^2*d^3*x
^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x + a))
- 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-cos(b*x + a) +
 I*sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog
(-cos(b*x + a) - I*sin(b*x + a)) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*
c^2*d*x + b^3*c^3)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*sin(b*x + a) +
I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x
 + a) - I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 ...
```

### 3.229.6 Sympy [F]

$$\int (c+dx)^3 \csc(a+bx)\sec(a+bx)\,dx = \int (c+dx)^3 \csc(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*csc(b*x+a)*sec(b*x+a),x)
```

output
```
Integral((c + d*x)**3*csc(a + b*x)*sec(a + b*x), x)
```

### 3.229.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1078 vs. $2(161) = 322$.

Time = 0.51 (sec) , antiderivative size = 1078, normalized size of antiderivative = 5.47

$$\int (c+dx)^3 \csc(a+bx) \sec(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

output
```
-1/6*(3*c^3*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)) - 9*a*c^2*d*(l
og(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b + 9*a^2*c*d^2*(log(sin(b*x
 + a)^2 - 1) - log(sin(b*x + a)^2))/b^2 - 3*a^3*d^3*(log(sin(b*x + a)^2 -
1) - log(sin(b*x + a)^2))/b^3 + (6*I*d^3*polylog(4, -e^(2*I*b*x + 2*I*a))
- 36*I*d^3*polylog(4, -e^(I*b*x + I*a)) - 36*I*d^3*polylog(4, e^(I*b*x + I
*a)) - 2*(-4*I*(b*x + a)^3*d^3 + 9*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 9*
(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*(b*x + a))*arctan2(sin(2*b*x +
2*a), cos(2*b*x + 2*a) + 1) - 6*(I*(b*x + a)^3*d^3 + 3*(I*b*c*d^2 - I*a*d^
3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2*d^3)*(b*x + a))*ar
ctan2(sin(b*x + a), cos(b*x + a) + 1) - 6*(-I*(b*x + a)^3*d^3 + 3*(-I*b*c*
d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*a^2*d^3)*
(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 3*(3*I*b^2*c^2*d - 6
*I*a*b*c*d^2 + 4*I*(b*x + a)^2*d^3 + 3*I*a^2*d^3 + 6*(I*b*c*d^2 - I*a*d^3)
*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*a)) - 18*(-I*b^2*c^2*d + 2*I*a*b*c*d^2
 - I*(b*x + a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*dil
og(-e^(I*b*x + I*a)) - 18*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^
3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*dilog(e^(I*b*x + I*a))
 + (4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2
*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)
^2 + 2*cos(2*b*x + 2*a) + 1) - 3*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)...
```

### 3.229.8 Giac [F]

$$\int (c+dx)^3 \csc(a+bx) \sec(a+bx)\,dx = \int (dx+c)^3 \csc(bx+a) \sec(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*csc(b*x + a)*sec(b*x + a), x)
```

### 3.229.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc(a+bx)\sec(a+bx)\,dx = \int \frac{(c+d\,x)^3}{\cos(a+b\,x)\,\sin(a+b\,x)}\,dx$$

input
```
int((c + d*x)^3/(cos(a + b*x)*sin(a + b*x)),x)
```

output
```
int((c + d*x)^3/(cos(a + b*x)*sin(a + b*x)), x)
```

# 3.230 $\int (c+dx)^2 \csc(a+bx)\sec(a+bx)\,dx$

3.230.1 Optimal result . . . 1816
3.230.2 Mathematica [A] (verified) . . . 1817
3.230.3 Rubi [A] (verified) . . . 1817
3.230.4 Maple [B] (verified) . . . 1819
3.230.5 Fricas [B] (verification not implemented) . . . 1820
3.230.6 Sympy [F] . . . 1821
3.230.7 Maxima [B] (verification not implemented) . . . 1822
3.230.8 Giac [F] . . . 1822
3.230.9 Mupad [F(-1)] . . . 1823

## 3.230.1 Optimal result

Integrand size = 20, antiderivative size = 127

$$\int (c+dx)^2 \csc(a+bx)\sec(a+bx)\,dx = -\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} - \frac{id(c+dx)\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2} - \frac{d^2\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} + \frac{d^2\operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}$$

output
```
-2*(d*x+c)^2*arctanh(exp(2*I*(b*x+a)))/b+I*d*(d*x+c)*polylog(2,-exp(2*I*(b
*x+a)))/b^2-I*d*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,
-exp(2*I*(b*x+a)))/b^3+1/2*d^2*polylog(3,exp(2*I*(b*x+a)))/b^3
```

### 3.230.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.68

$$\int (c+dx)^2\csc(a+bx)\sec(a+bx)\,dx$$
$$= \frac{-4b^2c^2\text{arctanh}\left(e^{2i(a+bx)}\right)+4b^2cdx\log\left(1-e^{2i(a+bx)}\right)+2b^2d^2x^2\log\left(1-e^{2i(a+bx)}\right)-4b^2cdx\log\left(1+e^{2i(a+bx)}\right)\dots}{}$$

input
```
Integrate[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x],x]
```

output
```
(-4*b^2*c^2*ArcTanh[E^((2*I)*(a + b*x))] + 4*b^2*c*d*x*Log[1 - E^((2*I)*(a
 + b*x))] + 2*b^2*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] - 4*b^2*c*d*x*Log[1
 + E^((2*I)*(a + b*x))] - 2*b^2*d^2*x^2*Log[1 + E^((2*I)*(a + b*x))] + (2*
I)*b*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))] - (2*I)*b*d*(c + d*x)*Po
lyLog[2, E^((2*I)*(a + b*x))] - d^2*PolyLog[3, -E^((2*I)*(a + b*x))] + d^2
*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3)
```

### 3.230.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4919, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\csc(a+bx)\sec(a+bx)\,dx$$

$$\Big\downarrow \text{4919}$$

$$2\int (c+dx)^2\csc(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$2\int (c+dx)^2\csc(2a+2bx)dx$$

$$\Big\downarrow \text{4671}$$

$$2\left(-\frac{d\int (c+dx)\log\left(1-e^{2i(a+bx)}\right)dx}{b}+\frac{d\int (c+dx)\log\left(1+e^{2i(a+bx)}\right)dx}{b}-\frac{(c+dx)^2\text{arctanh}\left(e^{2i(a+bx)}\right)}{b}\right)$$

$$\downarrow \text{3011}$$

$$2\left(\frac{d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b}-\frac{id\int \operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)dx}{2b}\right)}{b}\right.$$

$$\downarrow \text{2720}$$

$$2\left(\frac{d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\int e^{-2i(a+bx)}\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b}-\frac{d\int e^{-2i}\dots}{}\right)}{b}\right.$$

$$\downarrow \text{7143}$$

$$2\left(-\frac{(c+dx)^2\operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b}+\frac{d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{4b^2}\right)}{b}-\frac{d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,e^{2i(\dots}\right)}{2b}\right.}{}\right.$$

input
```
Int[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x],x]
```

output
```
2*(-(((c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/b) + (d*(((I/2)*(c + d*x)*
PolyLog[2, -E^((2*I)*(a + b*x))])/b - (d*PolyLog[3, -E^((2*I)*(a + b*x))])
/(4*b^2)))/b - (d*(((I/2)*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b - (
d*PolyLog[3, E^((2*I)*(a + b*x))])/(4*b^2)))/b)
```

### 3.230.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.230.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(111) = 222$.

Time = 1.21 (sec) , antiderivative size = 469, normalized size of antiderivative = 3.69

| method | result |
|---|---|
| risch | $-\frac{2id^2 \operatorname{polylog}\left(2,e^{i(xb+a)}\right)x}{b^2}-\frac{2id^2 \operatorname{polylog}\left(2,-e^{i(xb+a)}\right)x}{b^2}-\frac{2idc \operatorname{polylog}\left(2,e^{i(xb+a)}\right)}{b^2}+\frac{d^2 \ln\left(e^{i(xb+a)}+1\right)x^2}{b}+\frac{2d^2 \operatorname{polylog}(}{}$ |

input
```
int((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x-2*I/b^2*d^2*polylog(2,-exp(I*(b*x
+a)))*x-2*I/b^2*d*c*polylog(2,exp(I*(b*x+a)))+1/b*d^2*ln(exp(I*(b*x+a))+1)
*x^2+2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3-1/b*d^2*ln(exp(2*I*(b*x+a))+1)*x
^2-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+1/b*d^2*ln(1-exp(I*(b*x+a)))*x
^2+2*d^2*polylog(3,exp(I*(b*x+a)))/b^3+2/b*d*c*ln(exp(I*(b*x+a))+1)*x-2/b*
c*d*ln(exp(2*I*(b*x+a))+1)*x+2/b*d*c*ln(1-exp(I*(b*x+a)))*x-2/b^2*c*d*a*ln
(exp(I*(b*x+a))-1)+1/b*c^2*ln(exp(I*(b*x+a))+1)-1/b*c^2*ln(exp(2*I*(b*x+a)
)+1)+1/b*c^2*ln(exp(I*(b*x+a))-1)+2/b^2*d*c*ln(1-exp(I*(b*x+a)))*a+I/b^2*d
^2*polylog(2,-exp(2*I*(b*x+a)))*x+I/b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))-2
*I/b^2*d*c*polylog(2,-exp(I*(b*x+a)))-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+1
/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)
```

### 3.230.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1098 vs. $2(107) = 214$.

Time = 0.32 (sec) , antiderivative size = 1098, normalized size of antiderivative = 8.65

$$\int (c+dx)^2\csc(a+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylog(3, co
s(b*x + a) - I*sin(b*x + a)) - 2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x +
 a)) - 2*d^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 2*d^2*polylog(3,
-I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b
*x + a)) + 2*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylo
g(3, -cos(b*x + a) - I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*dilog(cos(b
*x + a) + I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*dilog(cos(b*x + a) -
I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*dilog(I*cos(b*x + a) + sin(b*x +
 a)) - 2*(-I*b*d^2*x - I*b*c*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 2*(
-I*b*d^2*x - I*b*c*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 2*(I*b*d^2*x
 + I*b*c*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*
d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*dilog(-
cos(b*x + a) - I*sin(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log
(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(
cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*
log(cos(b*x + a) - I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*
b*c*d - a^2*d^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2
*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) -
 (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) ...
```

### 3.230.6 Sympy [F]

$$\int (c+dx)^2 \csc(a+bx) \sec(a+bx)\,dx = \int (c+dx)^2 \csc(a+bx) \sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*csc(b*x+a)*sec(b*x+a),x)
```

output
```
Integral((c + d*x)**2*csc(a + b*x)*sec(a + b*x), x)
```

### 3.230.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 599 vs. $2(107) = 214$.

Time = 0.42 (sec) , antiderivative size = 599, normalized size of antiderivative = 4.72

$$\int (c+dx)^2 \csc(a+bx)\sec(a+bx)\,dx =$$

$$-\frac{c^2\left(\log\left(\sin\left(bx+a\right)^2-1\right)-\log\left(\sin\left(bx+a\right)^2\right)\right)-\frac{2\,acd\left(\log\left(\sin(bx+a)^2-1\right)-\log\left(\sin(bx+a)^2\right)\right)}{b}+\frac{a^2d^2\left(\log\left(\sin(bx+a\right.\right.}{}}{}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(c^2*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)) - 2*a*c*d*(log(s
in(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b + a^2*d^2*(log(sin(b*x + a)^2
- 1) - log(sin(b*x + a)^2))/b^2 + (d^2*polylog(3, -e^(2*I*b*x + 2*I*a)) -
4*d^2*polylog(3, -e^(I*b*x + I*a)) - 4*d^2*polylog(3, e^(I*b*x + I*a)) - 2
*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*arctan2(sin(2*b*x
 + 2*a), cos(2*b*x + 2*a) + 1) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d
^2)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*(-I*(b*x + a)^2
*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x +
a) + 1) - 2*(I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*dilog(-e^(2*I*b*x + 2*I*
a)) - 4*(-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*dilog(-e^(I*b*x + I*a)) - 4
*(-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*dilog(e^(I*b*x + I*a)) + ((b*x + a
)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x
+ 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*
(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - ((b
*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x
+ a)^2 - 2*cos(b*x + a) + 1))/b^2)/b
```

### 3.230.8 Giac [F]

$$\int (c+dx)^2 \csc(a+bx)\sec(a+bx)\,dx = \int (dx+c)^2 \csc(bx+a)\sec(bx+a)\;dx$$

input
```
integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csc(b*x + a)*sec(b*x + a), x)
```

### 3.230.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc(a+bx) \sec(a+bx)\, dx = \int \frac{(c+d\,x)^2}{\cos(a+b\,x)\,\sin(a+b\,x)}\, dx$$

input
```
int((c + d*x)^2/(cos(a + b*x)*sin(a + b*x)),x)
```

output
```
int((c + d*x)^2/(cos(a + b*x)*sin(a + b*x)), x)
```

# 3.231 $\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx$

3.231.1 Optimal result . . . 1824
3.231.2 Mathematica [A] (verified) . . . 1824
3.231.3 Rubi [A] (verified) . . . 1825
3.231.4 Maple [B] (verified) . . . 1826
3.231.5 Fricas [B] (verification not implemented) . . . 1827
3.231.6 Sympy [F] . . . 1828
3.231.7 Maxima [B] (verification not implemented) . . . 1828
3.231.8 Giac [F] . . . 1829
3.231.9 Mupad [F(-1)] . . . 1829

## 3.231.1 Optimal result

Integrand size = 18, antiderivative size = 71

$$\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx = -\frac{2(c+dx)\text{arctanh}\left(e^{2i(a+bx)}\right)}{b} + \frac{id\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} - \frac{id\,\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}$$

output
```
-2*(d*x+c)*arctanh(exp(2*I*(b*x+a)))/b+1/2*I*d*polylog(2,-exp(2*I*(b*x+a))
)/b^2-1/2*I*d*polylog(2,exp(2*I*(b*x+a)))/b^2
```

## 3.231.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.99

$$\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx = -\frac{c\log(\cos(a+bx))}{b} + \frac{c\log(\sin(a+bx))}{b} + \frac{d\left((2a+2bx)\left(\log\left(1-e^{i(2a+2bx)}\right)-\log\left(1+e^{i(2a+2bx)}\right)\right)-2a\log\left(\tan\left(\frac{1}{2}(2a+2bx)\right)\right)+i(\text{PolyLog}(2,-\right.}{2b^2}$$

input
```
Integrate[(c + d*x)*Csc[a + b*x]*Sec[a + b*x],x]
```

output
```
-((c*Log[Cos[a + b*x]])/b) + (c*Log[Sin[a + b*x]])/b + (d*((2*a + 2*b*x)*(
Log[1 - E^(I*(2*a + 2*b*x))] - Log[1 + E^(I*(2*a + 2*b*x))]) - 2*a*Log[Tan
[(2*a + 2*b*x)/2]] + I*(PolyLog[2, -E^(I*(2*a + 2*b*x))] - PolyLog[2, E^(I
*(2*a + 2*b*x))])))/(2*b^2)
```

### 3.231.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4919, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx$$

$$\Big\downarrow \text{4919}$$

$$2\int (c+dx)\csc(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$2\int (c+dx)\csc(2a+2bx)dx$$

$$\Big\downarrow \text{4671}$$

$$2\left(-\frac{d\int \log\left(1-e^{2i(a+bx)}\right)dx}{2b}+\frac{d\int \log\left(1+e^{2i(a+bx)}\right)dx}{2b}-\frac{(c+dx)\text{arctanh}\left(e^{2i(a+bx)}\right)}{b}\right)$$

$$\Big\downarrow \text{2715}$$

$$2\left(\frac{id\int e^{-2i(a+bx)}\log\left(1-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{id\int e^{-2i(a+bx)}\log\left(1+e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{(c+dx)\text{arctanh}(e}{b}\right.$$

$$\Big\downarrow \text{2838}$$

$$2\left(-\frac{(c+dx)\text{arctanh}\left(e^{2i(a+bx)}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{id\,\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{4b^2}\right)$$

input
```
Int[(c + d*x)*Csc[a + b*x]*Sec[a + b*x],x]
```

output
```
2*(-(((c + d*x)*ArcTanh[E^((2*I)*(a + b*x))])/b) + ((I/4)*d*PolyLog[2, -E^
((2*I)*(a + b*x))])/b^2 - ((I/4)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2)
```

#### 3.231.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

### 3.231.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 207 vs. $2(59) = 118$.

Time = 1.01 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.93

| method | result |
|---|---|
| risch | $\frac{c\ln(\mathrm{e}^{i(xb+a)}+1)}{b} - \frac{c\ln(\mathrm{e}^{2i(xb+a)}+1)}{b} + \frac{c\ln(\mathrm{e}^{i(xb+a)}-1)}{b} + \frac{d\ln(\mathrm{e}^{i(xb+a)}+1)x}{b} - \frac{id\,\mathrm{polylog}(2,-\mathrm{e}^{i(xb+a)})}{b^2} - \frac{d\ln(\mathrm{e}^{2i(xb+a)}-}{b}$ |

input
```
int((d*x+c)*csc(b*x+a)*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*c*ln(exp(I*(b*x+a))+1)-1/b*c*ln(exp(2*I*(b*x+a))+1)+1/b*c*ln(exp(I*(b*
x+a))-1)+1/b*d*ln(exp(I*(b*x+a))+1)*x-I*d*polylog(2,-exp(I*(b*x+a)))/b^2-1
/b*d*ln(exp(2*I*(b*x+a))+1)*x+1/2*I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2+1/b
*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-exp(I*(b*x+a)))*a-I*d*polylog(2,exp
(I*(b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(b*x+a))-1)
```

### 3.231.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(55) = 110$.

Time = 0.32 (sec) , antiderivative size = 554, normalized size of antiderivative = 7.80

$$\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx$$
$$= \frac{-i\,d\mathrm{Li}_2(\cos{(bx+a)}+i\,\sin{(bx+a)})+i\,d\mathrm{Li}_2(\cos{(bx+a)}-i\,\sin{(bx+a)})-i\,d\mathrm{Li}_2(i\,\cos{(bx+a)}+\sin}{}$$

input
```
integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(-I*d*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d*dilog(cos(b*x + a) -
I*sin(b*x + a)) - I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*dilog(I*c
os(b*x + a) - sin(b*x + a)) + I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) -
I*d*dilog(-I*cos(b*x + a) - sin(b*x + a)) + I*d*dilog(-cos(b*x + a) + I*si
n(b*x + a)) - I*d*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (b*d*x + b*c)*lo
g(cos(b*x + a) + I*sin(b*x + a) + 1) - (b*c - a*d)*log(cos(b*x + a) + I*si
n(b*x + a) + I) + (b*d*x + b*c)*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (
b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b*d*x + a*d)*log(I*co
s(b*x + a) + sin(b*x + a) + 1) - (b*d*x + a*d)*log(I*cos(b*x + a) - sin(b*
x + a) + 1) - (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d
*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a*d)*log(-1/2*c
os(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b*c - a*d)*log(-1/2*cos(b*x + a
) - 1/2*I*sin(b*x + a) + 1/2) + (b*d*x + a*d)*log(-cos(b*x + a) + I*sin(b*
x + a) + 1) - (b*c - a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*d*x
 + a*d)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - (b*c - a*d)*log(-cos(b*x
 + a) - I*sin(b*x + a) + I))/b^2
```

### 3.231.6 Sympy [F]

$$\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx=\int (c+dx)\csc{(a+bx)}\sec{(a+bx)}\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x)
```

output
```
Integral((c + d*x)*csc(a + b*x)*sec(a + b*x), x)
```

### 3.231.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(55)=110$.

Time = 0.41 (sec) , antiderivative size = 269, normalized size of antiderivative = 3.79

$$\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx=$$
$$-\frac{2i\,bdx\arctan{(\sin{(bx+a)},-\cos{(bx+a)}+1)}-2i\,bc\arctan{(\sin{(bx+a)},\cos{(bx+a)}-1)}-2\,(-i\,bc}{}$$

input
```
integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(2*I*b*d*x*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*I*b*c*arctan2
(sin(b*x + a), cos(b*x + a) - 1) - 2*(-I*b*d*x - I*b*c)*arctan2(sin(2*b*x
+ 2*a), cos(2*b*x + 2*a) + 1) - 2*(I*b*d*x + I*b*c)*arctan2(sin(b*x + a),
cos(b*x + a) + 1) - I*d*dilog(-e^(2*I*b*x + 2*I*a)) + 2*I*d*dilog(-e^(I*b*
x + I*a)) + 2*I*d*dilog(e^(I*b*x + I*a)) + (b*d*x + b*c)*log(cos(2*b*x + 2
*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - (b*d*x + b*c)*log(c
os(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (b*d*x + b*c)*log(c
os(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1))/b^2
```

### 3.231.8 Giac [F]

$$\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx=\int (dx+c)\csc(bx+a)\sec(bx+a)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*csc(b*x + a)*sec(b*x + a), x)
```

### 3.231.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx=\int \frac{c+d\,x}{\cos(a+b\,x)\,\sin(a+b\,x)}\,dx$$

input
```
int((c + d*x)/(cos(a + b*x)*sin(a + b*x)),x)
```

output
```
int((c + d*x)/(cos(a + b*x)*sin(a + b*x)), x)
```

# 3.232 $\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx$


3.232.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1830
3.232.2 Mathematica [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1830
3.232.3 Rubi [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1831
3.232.4 Maple [N/A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 1832
3.232.5 Fricas [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1832
3.232.6 Sympy [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1832
3.232.7 Maxima [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1833
3.232.8 Giac [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1833
3.232.9 Mupad [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1834


## 3.232.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx = 2\mathrm{Int}\left(\frac{\csc(2a+2bx)}{c+dx},x\right)$$

output
```
2*Unintegrable(csc(2*b*x+2*a)/(d*x+c),x)
```

## 3.232.2 Mathematica [N/A]

Not integrable

Time = 5.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x),x]
```

output
```
Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x), x]
```

### 3.232.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4919, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}dx$$
$$\Big\downarrow \text{4919}$$
$$2\int \frac{\csc(2a+2bx)}{c+dx}dx$$
$$\Big\downarrow \text{3042}$$
$$2\int \frac{\csc(2a+2bx)}{c+dx}dx$$
$$\Big\downarrow \text{4680}$$
$$2\int \frac{\csc(2a+2bx)}{c+dx}dx$$

input
```
Int[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x),x]
```

output
```
$Aborted
```

#### 3.232.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

### 3.232.4 Maple [N/A] (verified)

Not integrable

Time = 0.72 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\csc \left(xb+a\right) \sec \left(xb+a\right)}{dx+c}dx$$

input
```
int(csc(b*x+a)*sec(b*x+a)/(d*x+c),x)
```

output
```
int(csc(b*x+a)*sec(b*x+a)/(d*x+c),x)
```

### 3.232.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx=\int \frac{\csc \left(bx+a\right) \sec \left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)*sec(b*x + a)/(d*x + c), x)
```

### 3.232.6 Sympy [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx=\int \frac{\csc \left(a+bx\right) \sec \left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x)
```

output
```
Integral(csc(a + b*x)*sec(a + b*x)/(c + d*x), x)
```

## 3.232.7 Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c), x)
```

## 3.232.8 Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c), x)
```

### 3.232.9 Mupad [N/A]

Not integrable

Time = 25.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/(cos(a + b*x)*sin(a + b*x)*(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)*sin(a + b*x)*(c + d*x)), x)
```

# 3.233 $\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$

3.233.1 Optimal result . . . 1835
3.233.2 Mathematica [N/A] . . . 1835
3.233.3 Rubi [N/A] . . . 1836
3.233.4 Maple [N/A] (verified) . . . 1837
3.233.5 Fricas [N/A] . . . 1837
3.233.6 Sympy [N/A] . . . 1837
3.233.7 Maxima [N/A] . . . 1838
3.233.8 Giac [N/A] . . . 1838
3.233.9 Mupad [N/A] . . . 1839

## 3.233.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = 2\mathrm{Int}\left(\frac{\csc(2a+2bx)}{(c+dx)^2},x\right)$$

output
```
2*Unintegrable(csc(2*b*x+2*a)/(d*x+c)^2,x)
```

## 3.233.2 Mathematica [N/A]

Not integrable

Time = 6.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x)^2, x]
```

### 3.233.3 Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4919, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{4919}$$

$$2\int \frac{\csc(2a+2bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{\csc(2a+2bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{4680}$$

$$2\int \frac{\csc(2a+2bx)}{(c+dx)^2}dx$$

input
```
Int[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

#### 3.233.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

### 3.233.4 Maple [N/A] (verified)

Not integrable

Time = 0.73 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\csc (xb+a) \sec (xb+a)}{(dx+c)^2} dx$$

input
```
int(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x)
```

### 3.233.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc (bx+a) \sec (bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.233.6 Sympy [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc (a+bx) \sec (a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)*sec(a + b*x)/(c + d*x)**2, x)
```

### 3.233.7 Maxima [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c)^2, x)
```

### 3.233.8 Giac [N/A]

Not integrable

Time = 4.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c)^2, x)
```

### 3.233.9 Mupad [N/A]

Not integrable

Time = 24.84 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx=\int \frac{1}{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(1/(cos(a + b*x)*sin(a + b*x)*(c + d*x)^2),x)
```

output
```
int(1/(cos(a + b*x)*sin(a + b*x)*(c + d*x)^2), x)
```

# 3.234 $\int (c+dx)^m \csc^2(a+bx) \sec(a+bx) \, dx$

3.234.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1840
3.234.2 Mathematica [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . 1840
3.234.3 Rubi [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1841
3.234.4 Maple [N/A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 1841
3.234.5 Fricas [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1842
3.234.6 Sympy [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1842
3.234.7 Maxima [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1842
3.234.8 Giac [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1843
3.234.9 Mupad [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1843

## 3.234.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (c+dx)^m \csc^2(a+bx) \sec(a+bx) \, dx = \operatorname{Int}\left((c+dx)^m \csc^2(a+bx) \sec(a+bx), x\right)$$

output
```
CannotIntegrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x)
```

## 3.234.2 Mathematica [N/A]

Not integrable

Time = 23.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc^2(a+bx) \sec(a+bx) \, dx = \int (c+dx)^m \csc^2(a+bx) \sec(a+bx) \, dx$$

input
```
Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x], x]
```

### 3.234.3 Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\sec(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \ 7299$$

$$\int \csc^2(a+bx)\sec(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x],x]
```

output
```
$Aborted
```

#### 3.234.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

### 3.234.4 Maple [N/A] (verified)

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \csc(xb+a)^2 \sec(xb+a)\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x)
```

output
```
int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x)
```

### 3.234.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a), x)
```

### 3.234.6 Sympy [N/A]

Not integrable

Time = 118.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx = \int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*csc(b*x+a)**2*sec(b*x+a),x)
```

output
```
Integral((c + d*x)**m*csc(a + b*x)**2*sec(a + b*x), x)
```

### 3.234.7 Maxima [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a), x)
```

### 3.234.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc^2(a+bx) \sec(a+bx) \, dx = \int (dx+c)^m \csc(bx+a)^2 \sec(bx+a) \, dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a), x)
```

### 3.234.9 Mupad [N/A]

Not integrable

Time = 25.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (c+dx)^m \csc^2(a+bx) \sec(a+bx) \, dx = \int \frac{(c+d\,x)^m}{\cos(a+b\,x)\,\sin(a+b\,x)^2} \, dx$$

input
```
int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)^2),x)
```

output
```
int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)^2), x)
```

# 3.235 $\int (c+dx)^3 \csc^2(a+bx) \sec(a+bx) \, dx$

3.235.1 Optimal result . . . 1844
3.235.2 Mathematica [B] (verified) . . . 1845
3.235.3 Rubi [A] (verified) . . . 1846
3.235.4 Maple [B] (verified) . . . 1848
3.235.5 Fricas [B] (verification not implemented) . . . 1849
3.235.6 Sympy [F] . . . 1849
3.235.7 Maxima [B] (verification not implemented) . . . 1850
3.235.8 Giac [F] . . . 1850
3.235.9 Mupad [F(-1)] . . . 1851

## 3.235.1 Optimal result

Integrand size = 22, antiderivative size = 350

$$\begin{aligned}
&\int (c+dx)^3 \csc^2(a+bx) \sec(a+bx) \, dx \\
&= -\frac{2i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2 \text{arctanh}\left(e^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{(c+dx)^3 \csc(a+bx)}{b} + \frac{6id^2(c+dx) \operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3} \\
&\quad + \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} - \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} \\
&\quad - \frac{6id^2(c+dx) \operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3} - \frac{6d^3 \operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^4} \\
&\quad - \frac{6d^2(c+dx) \operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} + \frac{6d^2(c+dx) \operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3} \\
&\quad + \frac{6d^3 \operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^4} - \frac{6id^3 \operatorname{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^4} + \frac{6id^3 \operatorname{PolyLog}\left(4,ie^{i(a+bx)}\right)}{b^4}
\end{aligned}$$

output
```
-2*I*(d*x+c)^3*arctan(exp(I*(b*x+a)))/b-6*d*(d*x+c)^2*arctanh(exp(I*(b*x+a
)))/b^2-(d*x+c)^3*csc(b*x+a)/b+6*I*d^2*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/
b^3+3*I*d*(d*x+c)^2*polylog(2,-I*exp(I*(b*x+a)))/b^2-3*I*d*(d*x+c)^2*polyl
og(2,I*exp(I*(b*x+a)))/b^2-6*I*d^2*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^3-6
*d^3*polylog(3,-exp(I*(b*x+a)))/b^4-6*d^2*(d*x+c)*polylog(3,-I*exp(I*(b*x+
a)))/b^3+6*d^2*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))/b^3+6*d^3*polylog(3,exp
(I*(b*x+a)))/b^4-6*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4+6*I*d^3*polylog(
4,I*exp(I*(b*x+a)))/b^4
```

### 3.235.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 739 vs. 2(350) = 700.

Time = 6.39 (sec) , antiderivative size = 739, normalized size of antiderivative = 2.11

$$\int (c+dx)^3 \csc^2(a+bx)\sec(a+bx)\,dx = -\frac{(c+dx)^3\csc(a)}{b}$$
$$+\frac{3d\Big((c+dx)^2\log\left(1-e^{i(a+bx)}\right)-(c+dx)^2\log\left(1+e^{i(a+bx)}\right)+\frac{2id(b(c+dx)\operatorname{PolyLog}(2,-e^{i(a+bx)})+id\operatorname{PolyLog}(3,-e^{i}}{b^2}}{b^2}$$
$$+\frac{-2ib^3c^3\arctan\left(e^{i(a+bx)}\right)+3b^3c^2dx\log\left(1-ie^{i(a+bx)}\right)+3b^3cd^2x^2\log\left(1-ie^{i(a+bx)}\right)+b^3d^3x^3\log\left(1-ie\right.}{}$$
$$+\frac{\sec\left(\frac{a}{2}\right)\sec\left(\frac{a}{2}+\frac{bx}{2}\right)\left(-c^3\sin\left(\frac{bx}{2}\right)-3c^2dx\sin\left(\frac{bx}{2}\right)-3cd^2x^2\sin\left(\frac{bx}{2}\right)-d^3x^3\sin\left(\frac{bx}{2}\right)\right)}{2b}$$
$$+\frac{\csc\left(\frac{a}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)\left(c^3\sin\left(\frac{bx}{2}\right)+3c^2dx\sin\left(\frac{bx}{2}\right)+3cd^2x^2\sin\left(\frac{bx}{2}\right)+d^3x^3\sin\left(\frac{bx}{2}\right)\right)}{2b}$$

input
```
Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x],x]
```

output
```
-(((c + d*x)^3*Csc[a])/b) + (3*d*((c + d*x)^2*Log[1 - E^(I*(a + b*x))] - (
c + d*x)^2*Log[1 + E^(I*(a + b*x))] + ((2*I)*d*(b*(c + d*x)*PolyLog[2, -E^
(I*(a + b*x))] + I*d*PolyLog[3, -E^(I*(a + b*x))]))/b^2 + (2*d*((-I)*b*(c
+ d*x)*PolyLog[2, E^(I*(a + b*x))] + d*PolyLog[3, E^(I*(a + b*x))]))/b^2))
/b^2 + ((-2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 - I*E
^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] + b^3*d^3*x^3
*Log[1 - I*E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] - 3
*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + I*E^(I*(a
+ b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))] - (3*I
)*b^2*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, (
-I)*E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*c*
d^2*PolyLog[3, I*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x))
] - (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, I*E^
(I*(a + b*x))])/b^4 + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c^3*Sin[(b*x)/2]) -
3*c^2*d*x*Sin[(b*x)/2] - 3*c*d^2*x^2*Sin[(b*x)/2] - d^3*x^3*Sin[(b*x)/2]))
/(2*b) + (Csc[a/2]*Csc[a/2 + (b*x)/2]*(c^3*Sin[(b*x)/2] + 3*c^2*d*x*Sin[(b
*x)/2] + 3*c*d^2*x^2*Sin[(b*x)/2] + d^3*x^3*Sin[(b*x)/2]))/(2*b)
```

### 3.235.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4920, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \csc^2(a+bx)\sec(a+bx)\,dx$$

$$\Big\downarrow \ 4920$$

$$-3d\int (c+dx)^2\left(\frac{\text{arctanh}(\sin(a+bx))}{b}-\frac{\csc(a+bx)}{b}\right)dx+\frac{(c+dx)^3\text{arctanh}(\sin(a+bx))}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

$$\Big\downarrow \ 7292$$

$$-3d\int \frac{(c+dx)^2(\text{arctanh}(\sin(a+bx))-\csc(a+bx))}{b}dx+\frac{(c+dx)^3\text{arctanh}(\sin(a+bx))}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

$$\Big\downarrow \ 27$$

$$-\frac{3d\int (c+dx)^2(\text{arctanh}(\sin(a+bx))-\csc(a+bx))dx}{b}+\frac{(c+dx)^3\text{arctanh}(\sin(a+bx))}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

$$\Big\downarrow \ 7293$$

$$-\frac{3d\int \left((c+dx)^2\text{arctanh}(\sin(a+bx))-(c+dx)^2\csc(a+bx)\right)dx}{b}+\frac{(c+dx)^3\text{arctanh}(\sin(a+bx))}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

$$\Big\downarrow \ 2009$$

$$-\frac{3d\left(\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{3d}+\frac{2(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{(c+dx)^3\text{arctanh}(\sin(a+bx))}{3d}+\frac{2d^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}-\frac{2d^2\text{Po}}{}\right.}{}$$

$$\frac{(c+dx)^3\text{arctanh}(\sin(a+bx))}{b}-\frac{(c+dx)^3\csc(a+bx)}{b}$$

input

```
Int[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x],x]
```

output
```
((c + d*x)^3*ArcTanh[Sin[a + b*x]])/b - ((c + d*x)^3*Csc[a + b*x])/b - (3*
d*((((2*I)/3)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/d + (2*(c + d*x)^2*ArcT
anh[E^(I*(a + b*x))])/b + ((c + d*x)^3*ArcTanh[Sin[a + b*x]])/(3*d) - ((2*
I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (I*(c + d*x)^2*PolyLog[
2, (-I)*E^(I*(a + b*x))])/b + (I*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))]
)/b + ((2*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 + (2*d^2*PolyLog
[3, -E^(I*(a + b*x))])/b^3 + (2*d*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x)
)])/b^2 - (2*d*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^2 - (2*d^2*PolyL
og[3, E^(I*(a + b*x))])/b^3 + ((2*I)*d^2*PolyLog[4, (-I)*E^(I*(a + b*x))])
/b^3 - ((2*I)*d^2*PolyLog[4, I*E^(I*(a + b*x))])/b^3))/b
```

### 3.235.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### 3.235.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1157 vs. 2(313) = 626.

Time = 1.92 (sec) , antiderivative size = 1158, normalized size of antiderivative = 3.31

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1158 |

input
```
int((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-6*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4+6*I*d^3*polylog(4,I*exp(I*(b*x+a
)))/b^4+6*I/b^2*a*c^2*d*arctan(exp(I*(b*x+a)))+6*I/b^2*d^2*c*polylog(2,-I*
exp(I*(b*x+a)))*x-6*I/b^2*d^2*c*polylog(2,I*exp(I*(b*x+a)))*x-6*I/b^3*a^2*
c*d^2*arctan(exp(I*(b*x+a)))-3/b^2*d^3*ln(exp(I*(b*x+a))+1)*x^2+3/b^2*d^3*
ln(1-exp(I*(b*x+a)))*x^2-3/b^2*c^2*d*ln(exp(I*(b*x+a))+1)+3/b^2*c^2*d*ln(e
xp(I*(b*x+a))-1)-1/b*d^3*ln(1+I*exp(I*(b*x+a)))*x^3-6/b^3*d^2*c*polylog(3,
-I*exp(I*(b*x+a)))+1/b*d^3*ln(1-I*exp(I*(b*x+a)))*x^3+3/b^4*d^3*ln(1-exp(I
*(b*x+a)))*a^2+6/b^3*d^2*c*polylog(3,I*exp(I*(b*x+a)))-1/b^4*a^3*d^3*ln(1+
I*exp(I*(b*x+a)))+1/b^4*a^3*d^3*ln(1-I*exp(I*(b*x+a)))-6/b^3*d^3*polylog(3
,-I*exp(I*(b*x+a)))*x+3/b^4*d^3*a^2*ln(exp(I*(b*x+a))-1)+6/b^3*d^3*polylog
(3,I*exp(I*(b*x+a)))*x-2*I/b*c^3*arctan(exp(I*(b*x+a)))-6*d^3*polylog(3,-e
xp(I*(b*x+a)))/b^4+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4+6/b^3*d^3*ln(1-exp(
I*(b*x+a)))*a*x-3/b*c^2*d*ln(1+I*exp(I*(b*x+a)))*x-3/b^3*a^2*c*d^2*ln(1-I*
exp(I*(b*x+a)))+3/b^2*c^2*d*ln(1-I*exp(I*(b*x+a)))*a-3/b^2*c^2*d*ln(1+I*ex
p(I*(b*x+a)))*a-2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*exp(I*(b*x+a))/b/(
exp(2*I*(b*x+a))-1)-6/b^2*c*d^2*ln(exp(I*(b*x+a))+1)*x+3/b*c^2*d*ln(1-I*ex
p(I*(b*x+a)))*x-6*I/b^4*d^3*polylog(2,exp(I*(b*x+a)))*a-6*I/b^4*a*d^3*dilo
g(exp(I*(b*x+a))+1)+6*I/b^3*d^3*polylog(2,-exp(I*(b*x+a)))*x+6*I/b^4*d^3*p
olylog(2,-exp(I*(b*x+a)))*a-6*I/b^3*d^3*polylog(2,exp(I*(b*x+a)))*x-3*I/b^
2*c^2*d*polylog(2,I*exp(I*(b*x+a)))+2*I/b^4*a^3*d^3*arctan(exp(I*(b*x+a...
```

### 3.235.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1753 vs. 2(296) = 592.

Time = 0.36 (sec) , antiderivative size = 1753, normalized size of antiderivative = 5.01

$$\int (c+dx)^3 \csc^2(a+bx) \sec(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 6*I*d^
3*polylog(4, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 6*I*d^3*polylog
(4, I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) + 6*I*d^3*polylog(4, -I*co
s(b*x + a) + sin(b*x + a))*sin(b*x + a) + 6*I*d^3*polylog(4, -I*cos(b*x +
a) - sin(b*x + a))*sin(b*x + a) - 6*d^3*polylog(3, cos(b*x + a) + I*sin(b*
x + a))*sin(b*x + a) - 6*d^3*polylog(3, cos(b*x + a) - I*sin(b*x + a))*sin
(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a)
+ 6*d^3*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*(I*b*d
^3*x + I*b*c*d^2)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*(-
I*b*d^3*x - I*b*c*d^2)*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) +
 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(I*cos(b*x + a) +
sin(b*x + a))*sin(b*x + a) + 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^
2*d)*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) + 3*(-I*b^2*d^3*x^2
 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-I*cos(b*x + a) + sin(b*x + a))*si
n(b*x + a) + 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-I*c
os(b*x + a) - sin(b*x + a))*sin(b*x + a) + 6*(I*b*d^3*x + I*b*c*d^2)*dilog
(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*(-I*b*d^3*x - I*b*c*d^2)
*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b
^2*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a
) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a...
```

### 3.235.6 Sympy [F]

$$\int (c+dx)^3 \csc^2(a+bx) \sec(a+bx) \, dx = \int (c+dx)^3 \csc^2(a+bx) \sec(a+bx) \, dx$$

input
```
integrate((d*x+c)**3*csc(b*x+a)**2*sec(b*x+a),x)
```

output
```
Integral((c + d*x)**3*csc(a + b*x)**2*sec(a + b*x), x)
```

### 3.235.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3257 vs. $2(296) = 592$.

Time = 0.99 (sec) , antiderivative size = 3257, normalized size of antiderivative = 9.31

$$\int (c+dx)^3 \csc^2(a+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(c^3*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))
 - 3*a*c^2*d*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) -
1))/b + 3*a^2*c*d^2*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x
+ a) - 1))/b^2 - a^3*d^3*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin
(b*x + a) - 1))/b^3 - 2*(2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a
)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a) - ((b*x + a)^3*d^3 +
 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(
b*x + a))*cos(2*b*x + 2*a) - (I*(b*x + a)^3*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*
(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*a^2*d^3)*(b*x + a))*sin(2
*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + 2*((b*x + a)^3*d^3
+ 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*
(b*x + a) - ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^
2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (I*(b*x + a)^3*
d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2
 + I*a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x
+ a) + 1) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*
c*d^2 - a*d^3)*(b*x + a) - (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^
2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (I*b^2*c^2*d - 2
*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*
x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 6*(...
```

### 3.235.8 Giac [F]

$$\int (c+dx)^3 \csc^2(a+bx)\sec(a+bx)\,dx = \int (dx+c)^3 \csc(bx+a)^2 \sec(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*csc(b*x + a)^2*sec(b*x + a), x)
```

### 3.235.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^2(a+bx)\sec(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)^3/(cos(a + b*x)*sin(a + b*x)^2),x)
```

output
```
\text{Hanged}
```

# 3.236 $\int (c+dx)^2 \csc^2(a+bx) \sec(a+bx) \, dx$


3.236.1 Optimal result . . . . . . . . . . 1852
3.236.2 Mathematica [B] (verified) . . . . . . . . . . 1853
3.236.3 Rubi [A] (verified) . . . . . . . . . . 1854
3.236.4 Maple [B] (verified) . . . . . . . . . . 1855
3.236.5 Fricas [B] (verification not implemented) . . . . . . . . . . 1856
3.236.6 Sympy [F] . . . . . . . . . . 1857
3.236.7 Maxima [B] (verification not implemented) . . . . . . . . . . 1858
3.236.8 Giac [F] . . . . . . . . . . 1858
3.236.9 Mupad [F(-1)] . . . . . . . . . . 1859


## 3.236.1 Optimal result

Integrand size = 22, antiderivative size = 226

$$\begin{aligned}\int (c+dx)^2 \csc^2(a+bx) \sec(a+bx) \, dx &= -\frac{2i(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b} - \frac{4d(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2 \csc(a+bx)}{b} \\ &+ \frac{2id^2 \text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3} + \frac{2id(c+dx) \text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} \\ &- \frac{2id(c+dx) \text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} - \frac{2id^2 \text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3} \\ &- \frac{2d^2 \text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} + \frac{2d^2 \text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3}\end{aligned}$$

output
```
-2*I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b-4*d*(d*x+c)*arctanh(exp(I*(b*x+a))
)/b^2-(d*x+c)^2*csc(b*x+a)/b+2*I*d^2*polylog(2,-exp(I*(b*x+a)))/b^3+2*I*d*
(d*x+c)*polylog(2,-I*exp(I*(b*x+a)))/b^2-2*I*d*(d*x+c)*polylog(2,I*exp(I*(
b*x+a)))/b^2-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^3-2*d^2*polylog(3,-I*exp(
I*(b*x+a)))/b^3+2*d^2*polylog(3,I*exp(I*(b*x+a)))/b^3
```

### 3.236.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 593 vs. 2(226) = 452.

Time = 6.28 (sec) , antiderivative size = 593, normalized size of antiderivative = 2.62

$$\int (c+dx)^2 \csc^2(a+bx)\sec(a+bx)\,dx = -\frac{(c+dx)^2\csc(a)}{b} + \frac{-2ib^2c^2\arctan\left(e^{i(a+bx)}\right)+2b^2cdx\log\left(1-ie^{i(a+bx)}\right)+b^2d^2x^2\log\left(1-ie^{i(a+bx)}\right)-2b^2cdx\log\left(1+ie^{i(a-}\right.}{} + \frac{4icd\arctan\left(\frac{i\cos(a)-i\sin(a)\tan\left(\frac{bx}{2}\right)}{\sqrt{\cos^2(a)+\sin^2(a)}}\right)}{b^2\sqrt{\cos^2(a)+\sin^2(a)}} + \frac{\sec\left(\frac{a}{2}\right)\sec\left(\frac{a}{2}+\frac{bx}{2}\right)\left(-c^2\sin\left(\frac{bx}{2}\right)-2cdx\sin\left(\frac{bx}{2}\right)-d^2x^2\sin\left(\frac{bx}{2}\right)\right)}{2b} + \frac{\csc\left(\frac{a}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)\left(c^2\sin\left(\frac{bx}{2}\right)+2cdx\sin\left(\frac{bx}{2}\right)+d^2x^2\sin\left(\frac{bx}{2}\right)\right)}{2b} + \frac{2d^2\left(-\frac{2\arctan(\tan(a))\operatorname{arctanh}\left(\frac{-\cos(a)+\sin(a)\tan\left(\frac{bx}{2}\right)}{\sqrt{\cos^2(a)+\sin^2(a)}}\right)}{\sqrt{\cos^2(a)+\sin^2(a)}}+\frac{((bx+\arctan(\tan(a)))(\log(1-e^{i(bx+\arctan(\tan(a)))})-\log(1+e^{i(bx+\arctan}}{}\right.}{b^3}$$

input
```
Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x],x]
```

output
```
-(((c + d*x)^2*Csc[a])/b) + ((-2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b^
2*c*d*x*Log[1 - I*E^(I*(a + b*x))] + b^2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))
] - 2*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 + I*E^(I*(a
 + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] - (2*I)*b
*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a
 + b*x))] + 2*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + ((4*I)*c*d*ArcTan[(
I*Cos[a] - I*Sin[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]])/(b^2*Sqrt[Co
s[a]^2 + Sin[a]^2]) + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c^2*Sin[(b*x)/2]) -
2*c*d*x*Sin[(b*x)/2] - d^2*x^2*Sin[(b*x)/2]))/(2*b) + (Csc[a/2]*Csc[a/2 +
(b*x)/2]*(c^2*Sin[(b*x)/2] + 2*c*d*x*Sin[(b*x)/2] + d^2*x^2*Sin[(b*x)/2]))
/(2*b) + (2*d^2*((-2*ArcTan[Tan[a]]*ArcTanh[(-Cos[a] + Sin[a]*Tan[(b*x)/2]
)/Sqrt[Cos[a]^2 + Sin[a]^2]])/Sqrt[Cos[a]^2 + Sin[a]^2] + (((b*x + ArcTan[
Tan[a]])*(Log[1 - E^(I*(b*x + ArcTan[Tan[a]]))] - Log[1 + E^(I*(b*x + ArcT
an[Tan[a]]))]) + I*(PolyLog[2, -E^(I*(b*x + ArcTan[Tan[a]]))] - PolyLog[2,
 E^(I*(b*x + ArcTan[Tan[a]]))]))*Sec[a])/Sqrt[1 + Tan[a]^2]))/b^3
```

### 3.236.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4920, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\csc^2(a+bx)\sec(a+bx)\,dx$$

$$\Big\downarrow \text{4920}$$

$$-2d\int (c+dx)\left(\frac{\text{arctanh}(\sin(a+bx))}{b}-\frac{\csc(a+bx)}{b}\right)dx+\frac{(c+dx)^2\text{arctanh}(\sin(a+bx))}{b}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

$$\Big\downarrow \text{7292}$$

$$-2d\int \frac{(c+dx)(\text{arctanh}(\sin(a+bx))-\csc(a+bx))}{b}dx+\frac{(c+dx)^2\text{arctanh}(\sin(a+bx))}{b}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2d\int (c+dx)(\text{arctanh}(\sin(a+bx))-\csc(a+bx))dx}{b}+\frac{(c+dx)^2\text{arctanh}(\sin(a+bx))}{b}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{2d\int ((c+dx)\text{arctanh}(\sin(a+bx))-(c+dx)\csc(a+bx))dx}{b}+\frac{(c+dx)^2\text{arctanh}(\sin(a+bx))}{b}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2d\left(\frac{i(c+dx)^2\arctan(e^{i(a+bx)})}{d}+\frac{2(c+dx)\text{arctanh}(e^{i(a+bx)})}{b}+\frac{(c+dx)^2\text{arctanh}(\sin(a+bx))}{2d}-\frac{id\,\text{PolyLog}(2,-e^{i(a+bx)})}{b^2}+\frac{id\,\text{PolyLo}}{}\right.}{b}$$

$$\frac{(c+dx)^2\text{arctanh}(\sin(a+bx))}{b}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

input
```
Int[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x],x]
```

output
```
((c + d*x)^2*ArcTanh[Sin[a + b*x]])/b - ((c + d*x)^2*Csc[a + b*x])/b - (2*
d*((I*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/d + (2*(c + d*x)*ArcTanh[E^(I*(
a + b*x))])/b + ((c + d*x)^2*ArcTanh[Sin[a + b*x]])/(2*d) - (I*d*PolyLog[2
, -E^(I*(a + b*x))])/b^2 - (I*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/
b + (I*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b + (I*d*PolyLog[2, E^(I*(
a + b*x))])/b^2 + (d*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^2 - (d*PolyLog[3,
 I*E^(I*(a + b*x))])/b^2))/b
```

### 3.236.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## 3.236.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 555 vs. $2(201) = 402$.

Time = 1.61 (sec) , antiderivative size = 556, normalized size of antiderivative = 2.46

| method | result |
|---|---|
| risch | $\frac{2id^2\,\text{polylog}(2,-ie^{i(xb+a)})x}{b^2}-\frac{2cd\ln(1+ie^{i(xb+a)})a}{b^2}+\frac{2cd\ln(1-ie^{i(xb+a)})x}{b}-\frac{2d^2a\ln(e^{i(xb+a)}-1)}{b^3}-\frac{2d^2\ln(e^{i(xb+a)}+1)x}{b^2}$ |

input
```
int((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*I/b^2*d^2*polylog(2,-I*exp(I*(b*x+a)))*x-2/b^2*c*d*ln(1+I*exp(I*(b*x+a))
)*a+2/b*c*d*ln(1-I*exp(I*(b*x+a)))*x-2*d^2/b^3*a*ln(exp(I*(b*x+a))-1)-2*d^
2/b^2*ln(exp(I*(b*x+a))+1)*x+2/b^2*c*d*ln(1-I*exp(I*(b*x+a)))*a+2*I/b^2*c*
d*polylog(2,-I*exp(I*(b*x+a)))+2*d/b^2*c*ln(exp(I*(b*x+a))-1)-2*I/b*c^2*ar
ctan(exp(I*(b*x+a)))+1/b*d^2*ln(1-I*exp(I*(b*x+a)))*x^2-2*d^2*polylog(3,-I
*exp(I*(b*x+a)))/b^3-1/b*d^2*ln(1+I*exp(I*(b*x+a)))*x^2-2*I/b^2*c*d*polylo
g(2,I*exp(I*(b*x+a)))+1/b^3*a^2*d^2*ln(1+I*exp(I*(b*x+a)))-2/b*c*d*ln(1+I*
exp(I*(b*x+a)))*x-1/b^3*a^2*d^2*ln(1-I*exp(I*(b*x+a)))-2*d/b^2*c*ln(exp(I*
(b*x+a))+1)+2*d^2*polylog(3,I*exp(I*(b*x+a)))/b^3+2*I/b^3*d^2*dilog(exp(I*
(b*x+a))+1)-2*I/b^3*d^2*a^2*arctan(exp(I*(b*x+a)))+4*I/b^2*c*d*a*arctan(ex
p(I*(b*x+a)))-2*I*(d^2*x^2+2*c*d*x+c^2)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))
-1)+2*I/b^3*d^2*dilog(exp(I*(b*x+a)))-2*I/b^2*d^2*polylog(2,I*exp(I*(b*x+a
)))*x
```

### 3.236.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1067 vs. $2(188) = 376$.

Time = 0.32 (sec) , antiderivative size = 1067, normalized size of antiderivative = 4.72

$$\int (c+dx)^2\csc^2(a+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 2*I*d^2*dilog(cos(b*x + a)
 + I*sin(b*x + a))*sin(b*x + a) - 2*I*d^2*dilog(cos(b*x + a) - I*sin(b*x +
 a))*sin(b*x + a) + 2*I*d^2*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x
+ a) - 2*I*d^2*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 2*d^2*
polylog(3, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 2*d^2*polylog(3,
I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) + 2*d^2*polylog(3, -I*cos(b*x
+ a) + sin(b*x + a))*sin(b*x + a) - 2*d^2*polylog(3, -I*cos(b*x + a) - sin
(b*x + a))*sin(b*x + a) + 2*(I*b*d^2*x + I*b*c*d)*dilog(I*cos(b*x + a) + s
in(b*x + a))*sin(b*x + a) + 2*(I*b*d^2*x + I*b*c*d)*dilog(I*cos(b*x + a) -
 sin(b*x + a))*sin(b*x + a) + 2*(-I*b*d^2*x - I*b*c*d)*dilog(-I*cos(b*x +
a) + sin(b*x + a))*sin(b*x + a) + 2*(-I*b*d^2*x - I*b*c*d)*dilog(-I*cos(b*
x + a) - sin(b*x + a))*sin(b*x + a) + 2*(b*d^2*x + b*c*d)*log(cos(b*x + a)
 + I*sin(b*x + a) + 1)*sin(b*x + a) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(
cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + 2*(b*d^2*x + b*c*d)*log(
cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) + (b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) - (b^2*d^2*x^2
 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) + sin(b*x + a) +
1)*sin(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*
cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) - (b^2*d^2*x^2 + 2*b^2*c*d*x
 + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x...
```

### 3.236.6 Sympy [F]

$$\int (c+dx)^2 \csc^2(a+bx) \sec(a+bx)\, dx = \int (c+dx)^2 \csc^2(a+bx) \sec(a+bx)\, dx$$

input
```
integrate((d*x+c)**2*csc(b*x+a)**2*sec(b*x+a),x)
```

output
```
Integral((c + d*x)**2*csc(a + b*x)**2*sec(a + b*x), x)
```

### 3.236.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1638 vs. 2(188) = 376.

Time = 0.50 (sec) , antiderivative size = 1638, normalized size of antiderivative = 7.25

$$\int (c+dx)^2 \csc^2(a+bx) \sec(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(c^2*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))
 - 2*a*c*d*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1)
)/b + a^2*d^2*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) -
 1))/b^2 - 2*(2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - ((b*x + a
)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (I*(b*x + a)^2*d
^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x +
a), sin(b*x + a) + 1) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) -
 ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (I*(b*
x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(
cos(b*x + a), -sin(b*x + a) + 1) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2 - (b*c
*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (I*b*c*d + I*(b*x + a)*d^2
- I*a*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 4*(
b*c*d - a*d^2 - (b*c*d - a*d^2)*cos(2*b*x + 2*a) + (-I*b*c*d + I*a*d^2)*si
n(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) - 4*((b*x + a)*d^2
*cos(2*b*x + 2*a) + I*(b*x + a)*d^2*sin(2*b*x + 2*a) - (b*x + a)*d^2)*arct
an2(sin(b*x + a), -cos(b*x + a) + 1) - 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d
^2)*(b*x + a))*cos(b*x + a) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2 - (b*c*d +
(b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (I*b*c*d + I*(b*x + a)*d^2 - I*a
*d^2)*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) - 4*(b*c*d + (b*x + a)*d^
2 - a*d^2 - (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (-I*b*c*...
```

### 3.236.8 Giac [F]

$$\int (c+dx)^2 \csc^2(a+bx) \sec(a+bx)\, dx = \int (dx+c)^2 \csc(bx+a)^2 \sec(bx+a)\, dx$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csc(b*x + a)^2*sec(b*x + a), x)
```

### 3.236.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc^2(a+bx) \sec(a+bx)\, dx = \text{Hanged}$$

input
```
int((c + d*x)^2/(cos(a + b*x)*sin(a + b*x)^2),x)
```

output
```
\text{Hanged}
```

# 3.237 $\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx$

3.237.1 Optimal result . . . . . . . . . . 1860
3.237.2 Mathematica [C] (warning: unable to verify) . . . . . . . . . . 1861
3.237.3 Rubi [A] (verified) . . . . . . . . . . 1862
3.237.4 Maple [A] (verified) . . . . . . . . . . 1863
3.237.5 Fricas [B] (verification not implemented) . . . . . . . . . . 1863
3.237.6 Sympy [F] . . . . . . . . . . 1864
3.237.7 Maxima [F] . . . . . . . . . . 1864
3.237.8 Giac [F] . . . . . . . . . . 1865
3.237.9 Mupad [F(-1)] . . . . . . . . . . 1865

## 3.237.1 Optimal result

Integrand size = 20, antiderivative size = 131

$$\begin{aligned}\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx = &-\frac{2idx\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{d\text{arctanh}(\cos(a+bx))}{b^2}\\ &-\frac{dx\text{arctanh}(\sin(a+bx))}{b}\\ &+\frac{(c+dx)\text{arctanh}(\sin(a+bx))}{b}\\ &-\frac{(c+dx)\csc(a+bx)}{b}+\frac{id\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}\\ &-\frac{id\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}\end{aligned}$$

output
```
-2*I*d*x*arctan(exp(I*(b*x+a)))/b-d*arctanh(cos(b*x+a))/b^2-d*x*arctanh(si
n(b*x+a))/b+(d*x+c)*arctanh(sin(b*x+a))/b-(d*x+c)*csc(b*x+a)/b+I*d*polylog
(2,-I*exp(I*(b*x+a)))/b^2-I*d*polylog(2,I*exp(I*(b*x+a)))/b^2
```

### 3.237.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.83 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.95

$$\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx$$
$$= \frac{d\left(a\cos\left(\frac{1}{2}(a+bx)\right)-(a+bx)\cos\left(\frac{1}{2}(a+bx)\right)\right)\csc\left(\frac{1}{2}(a+bx)\right)}{2b^2}$$
$$-\frac{c\csc(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\sin^2(a+bx)\right)}{b}$$
$$-\frac{d\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b^2}+\frac{d\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b^2}$$
$$-\frac{dx\left(a\log\left(1-\tan\left(\frac{1}{2}(a+bx)\right)\right)-a\log\left(1+\tan\left(\frac{1}{2}(a+bx)\right)\right)-i\left(\log\left(1+i\tan\left(\frac{1}{2}(a+bx)\right)\right)\log\left(\left(\frac{1}{2}-\right.\right.\right.\right.}{}$$
$$+\frac{d\sec\left(\frac{1}{2}(a+bx)\right)\left(a\sin\left(\frac{1}{2}(a+bx)\right)-(a+bx)\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b^2}$$

input
```
Integrate[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x],x]
```

output
```
(d*(a*Cos[(a + b*x)/2] - (a + b*x)*Cos[(a + b*x)/2])*Csc[(a + b*x)/2])/(2*
b^2) - (c*Csc[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[a + b*x]^2])/b
- (d*Log[Cos[(a + b*x)/2]])/b^2 + (d*Log[Sin[(a + b*x)/2]])/b^2 - (d*x*(a*
Log[1 - Tan[(a + b*x)/2]] - a*Log[1 + Tan[(a + b*x)/2]] - I*(Log[1 + I*Tan
[(a + b*x)/2]]*Log[(1/2 - I/2)*(1 + Tan[(a + b*x)/2])] + PolyLog[2, ((1 +
I) - (1 - I)*Tan[(a + b*x)/2])/2]) + I*(Log[1 - I*Tan[(a + b*x)/2]]*Log[(1
/2 + I/2)*(1 + Tan[(a + b*x)/2])] + PolyLog[2, (-1/2 - I/2)*(I + Tan[(a +
b*x)/2])]) - I*(Log[1 - I*Tan[(a + b*x)/2]]*Log[(-1/2 + I/2)*(-1 + Tan[(a
+ b*x)/2])] + PolyLog[2, ((1 + I) + (1 - I)*Tan[(a + b*x)/2])/2]) + I*(Log
[1 + I*Tan[(a + b*x)/2]]*Log[(-1/2 - I/2)*(-1 + Tan[(a + b*x)/2])] + PolyL
og[2, ((1 - I) + (1 + I)*Tan[(a + b*x)/2])/2])))/(b*(a - I*Log[1 - I*Tan[(
a + b*x)/2]] + I*Log[1 + I*Tan[(a + b*x)/2]])) + (d*Sec[(a + b*x)/2]*(a*Si
n[(a + b*x)/2] - (a + b*x)*Sin[(a + b*x)/2]))/(2*b^2)
```

### 3.237.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4920, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx$$

$$\Big\downarrow \ 4920$$

$$-d\int \left(\frac{\text{arctanh}(\sin(a+bx))}{b}-\frac{\csc(a+bx)}{b}\right)dx+\frac{(c+dx)\text{arctanh}(\sin(a+bx))}{b}-\frac{(c+dx)\csc(a+bx)}{b}$$

$$\Big\downarrow \ 2009$$

$$-d\left(\frac{2ix\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{\text{arctanh}(\cos(a+bx))}{b^2}+\frac{x\text{arctanh}(\sin(a+bx))}{b}-\frac{i\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}+\frac{i\,\text{PolyL}}{}\right.$$
$$\frac{(c+dx)\text{arctanh}(\sin(a+bx))}{b}-\frac{(c+dx)\csc(a+bx)}{b}$$

input
```
Int[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x],x]
```

output
```
((c + d*x)*ArcTanh[Sin[a + b*x]])/b - ((c + d*x)*Csc[a + b*x])/b - d*(((2*
I)*x*ArcTan[E^(I*(a + b*x))])/b + ArcTanh[Cos[a + b*x]]/b^2 + (x*ArcTanh[S
in[a + b*x]])/b - (I*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 + (I*PolyLog[2,
 I*E^(I*(a + b*x))])/b^2)
```

#### 3.237.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

### 3.237.4 Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.79

| method | result |
|---|---|
| risch | $-\frac{2i(dx+c)e^{i(xb+a)}}{b(e^{2i(xb+a)}-1)}-\frac{d\ln(1+ie^{i(xb+a)})x}{b}+\frac{d\ln(1-ie^{i(xb+a)})x}{b}-\frac{2ic\arctan(e^{i(xb+a)})}{b}+\frac{id\operatorname{dilog}(1+ie^{i(xb+a)})}{b^2}-\frac{id\operatorname{dilog}($ |

input
```
int((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-2*I*(d*x+c)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)-1/b*d*ln(1+I*exp(I*(b*x
+a)))*x+1/b*d*ln(1-I*exp(I*(b*x+a)))*x-2*I/b*c*arctan(exp(I*(b*x+a)))+I/b^
2*d*dilog(1+I*exp(I*(b*x+a)))-I/b^2*d*dilog(1-I*exp(I*(b*x+a)))+2*I/b^2*a*
d*arctan(exp(I*(b*x+a)))-d/b^2*ln(exp(I*(b*x+a))+1)+d/b^2*ln(exp(I*(b*x+a)
)-1)-1/b^2*d*ln(1+I*exp(I*(b*x+a)))*a+1/b^2*d*ln(1-I*exp(I*(b*x+a)))*a
```

### 3.237.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(113) = 226.

Time = 0.29 (sec) , antiderivative size = 434, normalized size of antiderivative = 3.31

$$\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx =$$
$$-\frac{2\,bdx + i\,d\mathrm{Li}_2(i\,\cos(bx+a)+\sin(bx+a))\sin(bx+a) + i\,d\mathrm{Li}_2(i\,\cos(bx+a)-\sin(bx+a))\sin(bx+$$

input
```
integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*b*d*x + I*d*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + I*
d*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - I*d*dilog(-I*cos(b*x
 + a) + sin(b*x + a))*sin(b*x + a) - I*d*dilog(-I*cos(b*x + a) - sin(b*x +
 a))*sin(b*x + a) - (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x + a) + I)*sin
(b*x + a) + (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a
) + d*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - (b*d*x + a*d)*log(I*cos(b
*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + (b*d*x + a*d)*log(I*cos(b*x + a
) - sin(b*x + a) + 1)*sin(b*x + a) - (b*d*x + a*d)*log(-I*cos(b*x + a) + s
in(b*x + a) + 1)*sin(b*x + a) + (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*
x + a) + 1)*sin(b*x + a) - d*log(-1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - (
b*c - a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + (b*c - a
*d)*log(-cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + 2*b*c)/(b^2*sin
(b*x + a))
```

### 3.237.6 Sympy [F]

$$\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx = \int (c+dx)\csc^2{(a+bx)}\sec{(a+bx)}\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)**2*sec(b*x+a),x)
```

output
```
Integral((c + d*x)*csc(a + b*x)**2*sec(a + b*x), x)
```

### 3.237.7 Maxima [F]

$$\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx = \int (dx+c)\csc(bx+a)^2\sec(bx+a)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(4*(b*d*x + b*c)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*d*x + b*c)*cos(
2*b*x + 2*a)*sin(b*x + a) - 4*(b^2*d*cos(2*b*x + 2*a)^2 + b^2*d*sin(2*b*x
+ 2*a)^2 - 2*b^2*d*cos(2*b*x + 2*a) + b^2*d)*integrate((x*cos(2*b*x + 2*a)
*cos(b*x + a) + x*sin(2*b*x + 2*a)*sin(b*x + a) + x*cos(b*x + a))/(cos(2*b
*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1), x) - (b*c*cos(
2*b*x + 2*a)^2 + b*c*sin(2*b*x + 2*a)^2 - 2*b*c*cos(2*b*x + 2*a) + b*c)*lo
g(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) + (b*c*cos(2*b*x +
 2*a)^2 + b*c*sin(2*b*x + 2*a)^2 - 2*b*c*cos(2*b*x + 2*a) + b*c)*log(cos(b
*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + (d*cos(2*b*x + 2*a)^2 +
 d*sin(2*b*x + 2*a)^2 - 2*d*cos(2*b*x + 2*a) + d)*log(cos(b*x)^2 + 2*cos(b
*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) - (d*co
s(2*b*x + 2*a)^2 + d*sin(2*b*x + 2*a)^2 - 2*d*cos(2*b*x + 2*a) + d)*log(co
s(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) +
 sin(a)^2) + 4*(b*d*x + b*c)*sin(b*x + a))/(b^2*cos(2*b*x + 2*a)^2 + b^2*s
in(2*b*x + 2*a)^2 - 2*b^2*cos(2*b*x + 2*a) + b^2)
```

### 3.237.8 Giac [F]

$$\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx = \int (dx+c)\csc(bx+a)^2\sec(bx+a)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*csc(b*x + a)^2*sec(b*x + a), x)
```

### 3.237.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)/(cos(a + b*x)*sin(a + b*x)^2),x)
```

output
```
\text{Hanged}
```

# 3.238 $\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$


3.238.1 Optimal result . . . . . . . . . . 1866
3.238.2 Mathematica [N/A] . . . . . . . . . . 1866
3.238.3 Rubi [N/A] . . . . . . . . . . 1867
3.238.4 Maple [N/A] (verified) . . . . . . . . . . 1867
3.238.5 Fricas [N/A] . . . . . . . . . . 1868
3.238.6 Sympy [N/A] . . . . . . . . . . 1868
3.238.7 Maxima [N/A] . . . . . . . . . . 1868
3.238.8 Giac [N/A] . . . . . . . . . . 1869
3.238.9 Mupad [N/A] . . . . . . . . . . 1869


## 3.238.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\csc^2(a+bx)\sec(a+bx)}{c+dx},x\right)$$

output
```
CannotIntegrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x)
```

## 3.238.2 Mathematica [N/A]

Not integrable

Time = 22.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x),x]
```

output
```
Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x), x]
```

## 3.238.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}dx$$

input
```
Int[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x),x]
```

output
```
$Aborted
```

### 3.238.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.238.4 Maple [N/A] (verified)

Not integrable

Time = 0.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\csc(xb+a)^2\sec(xb+a)}{dx+c}dx$$

input
```
int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x)
```

output
```
int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x)
```

### 3.238.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc\left(bx+a\right)^2\sec\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^2*sec(b*x + a)/(d*x + c), x)
```

### 3.238.6 Sympy [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc^2\left(a+bx\right)\sec\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(csc(b*x+a)**2*sec(b*x+a)/(d*x+c),x)
```

output
```
Integral(csc(a + b*x)**2*sec(a + b*x)/(c + d*x), x)
```

### 3.238.7 Maxima [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 677, normalized size of antiderivative = 30.77

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc\left(bx+a\right)^2\sec\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
(2*(b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*b*x + 2
*a)^2 + b*c - 2*(b*d*x + b*c)*cos(2*b*x + 2*a))*integrate((cos(2*b*x + 2*a
)*cos(b*x + a) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))/((d*x + c)*
cos(2*b*x + 2*a)^2 + (d*x + c)*sin(2*b*x + 2*a)^2 + d*x + 2*(d*x + c)*cos(
2*b*x + 2*a) + c), x) - (b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2
*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^2*x + b*c*d)*cos(2*b
*x + 2*a))*integrate(sin(b*x + a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*
x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*
sin(b*x + a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)), x) - (b*
d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c*d)*s
in(2*b*x + 2*a)^2 - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a))*integrate(sin(b*
x + a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*co
s(b*x + a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 - 2*(b*d^2*x
^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)), x) - 2*cos(b*x + a)*sin(2*b*x + 2*a
) + 2*cos(2*b*x + 2*a)*sin(b*x + a) - 2*sin(b*x + a))/(b*d*x + (b*d*x + b*
c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*b*x + 2*a)^2 + b*c - 2*(b*d*x
+ b*c)*cos(2*b*x + 2*a))
```

## 3.238.8 Giac [N/A]

Not integrable

Time = 5.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc\left(bx+a\right)^2\sec\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sec(b*x + a)/(d*x + c), x)
```

## 3.238.9 Mupad [N/A]

Not integrable

Time = 24.82 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)), x)
```

3.238. $\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$

# 3.239 $\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2} dx$


3.239.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1871
3.239.2 Mathematica [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1871
3.239.3 Rubi [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1872
3.239.4 Maple [N/A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1872
3.239.5 Fricas [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1873
3.239.6 Sympy [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1873
3.239.7 Maxima [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1873
3.239.8 Giac [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1874
3.239.9 Mupad [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1875


## 3.239.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2} dx = \text{Int}\left(\frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}, x\right)$$

output
```
CannotIntegrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x)
```

## 3.239.2 Mathematica [N/A]

Not integrable

Time = 21.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2} dx = \int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2} dx$$

input
```
Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2, x]
```

## 3.239.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}dx$$

input
```
Int[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

### 3.239.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.239.4 Maple [N/A] (verified)

Not integrable

Time = 0.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\csc\left(xb+a\right)^{2}\sec\left(xb+a\right)}{\left(dx+c\right)^{2}}dx$$

input
```
int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x)
```

### 3.239.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)^2\sec(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^2*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.239.6 Sympy [N/A]

Not integrable

Time = 2.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(csc(b*x+a)**2*sec(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)**2*sec(a + b*x)/(c + d*x)**2, x)
```

### 3.239.7 Maxima [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 1008, normalized size of antiderivative = 45.82

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)^2\sec(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
2*((b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*
b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*a)^2 - 2*(b*d
^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a))*integrate((cos(2*b*x + 2*a)*
cos(b*x + a) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))/(d^2*x^2 + 2*
c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(2*b*x + 2*a)^2 + (d^2*x^2 + 2*c*d*x
+ c^2)*sin(2*b*x + 2*a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(2*b*x +
2*a)), x) - (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d + (b*d^3*x^2 + 2*b*c*d^2*x
+ b*c^2*d)*cos(2*b*x + 2*a)^2 + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*sin(2*
b*x + 2*a)^2 - 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a))*int
egrate(sin(b*x + a)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*
d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)^2 + (b*d^3*x^3
 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(b*x + a)^2 + 2*(b*d^3*x^3 + 3*
b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)), x) - (b*d^3*x^2 + 2*b*c*
d^2*x + b*c^2*d + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a)^2 +
 (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^3*x^2 + 2
*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^3*x^3
+ 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c
^2*d*x + b*c^3)*cos(b*x + a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x
+ b*c^3)*sin(b*x + a)^2 - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c
^3)*cos(b*x + a)), x) - cos(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a...
```

### 3.239.8 Giac [N/A]

Not integrable

Time = 4.25 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)^2\sec(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

### 3.239.9 Mupad [N/A]

Not integrable

Time = 25.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^2),x)
```

output
```
int(1/(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^2), x)
```

# 3.240 $\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx$

3.240.1 Optimal result . . . 1876
3.240.2 Mathematica [N/A] . . . 1876
3.240.3 Rubi [N/A] . . . 1877
3.240.4 Maple [N/A] (verified) . . . 1877
3.240.5 Fricas [N/A] . . . 1878
3.240.6 Sympy [F(-1)] . . . 1878
3.240.7 Maxima [N/A] . . . 1878
3.240.8 Giac [N/A] . . . 1879
3.240.9 Mupad [N/A] . . . 1879

## 3.240.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx = \text{Int}\left((c+dx)^m \csc^3(a+bx)\sec(a+bx),x\right)$$

output
```
CannotIntegrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x)
```

## 3.240.2 Mathematica [N/A]

Not integrable

Time = 26.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx = \int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x], x]
```

## 3.240.3 Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\sec(a+bx)(c+dx)^m\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \csc^3(a+bx)\sec(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x],x]
```

output
```
$Aborted
```

### 3.240.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.240.4 Maple [N/A] (verified)

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \csc(xb+a)^3 \sec(xb+a)\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x)
```

output
```
int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x)
```

### 3.240.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a), x)
```

### 3.240.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*csc(b*x+a)**3*sec(b*x+a),x)
```

output
```
Timed out
```

### 3.240.7 Maxima [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a), x)
```

### 3.240.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a), x)
```

### 3.240.9 Mupad [N/A]

Not integrable

Time = 24.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\cos(a+b\,x)\,\sin(a+b\,x)^3}\,dx$$

input
```
int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)^3),x)
```

output
```
int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)^3), x)
```

# 3.241 $\int (c+dx)^3 \csc^3(a+bx) \sec(a+bx) \, dx$

3.241.1 Optimal result . . . . . 1880
3.241.2 Mathematica [B] (warning: unable to verify) . . . . . 1881
3.241.3 Rubi [A] (verified) . . . . . 1882
3.241.4 Maple [B] (verified) . . . . . 1884
3.241.5 Fricas [B] (verification not implemented) . . . . . 1885
3.241.6 Sympy [F] . . . . . 1885
3.241.7 Maxima [B] (verification not implemented) . . . . . 1886
3.241.8 Giac [F] . . . . . 1886
3.241.9 Mupad [F(-1)] . . . . . 1887

## 3.241.1 Optimal result

Integrand size = 22, antiderivative size = 325

$$\begin{aligned}\int (c+dx)^3 \csc^3(a+bx) \sec(a+bx) \, dx = & -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} \\ & - \frac{2(c+dx)^3 \operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b} \\ & - \frac{3d(c+dx)^2 \cot(a+bx)}{2b^2} \\ & - \frac{(c+dx)^3 \cot^2(a+bx)}{2b} \\ & + \frac{3d^2(c+dx) \log\left(1-e^{2i(a+bx)}\right)}{b^3} \\ & + \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} \\ & - \frac{3id^3 \operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^4} \\ & - \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2} \\ & - \frac{3d^2(c+dx) \operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} \\ & + \frac{3d^2(c+dx) \operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3} \\ & - \frac{3id^3 \operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4} \\ & + \frac{3id^3 \operatorname{PolyLog}\left(4,e^{2i(a+bx)}\right)}{4b^4}\end{aligned}$$

output
```
-3/2*I*d*(d*x+c)^2/b^2-1/2*(d*x+c)^3/b-2*(d*x+c)^3*arctanh(exp(2*I*(b*x+a)
))/b-3/2*d*(d*x+c)^2*cot(b*x+a)/b^2-1/2*(d*x+c)^3*cot(b*x+a)^2/b+3*d^2*(d*
x+c)*ln(1-exp(2*I*(b*x+a)))/b^3+3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+
a)))/b^2-3/2*I*d^3*polylog(2,exp(2*I*(b*x+a)))/b^4-3/2*I*d*(d*x+c)^2*polyl
og(2,exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*x+c)*polylog(3,-exp(2*I*(b*x+a)))/b^
3+3/2*d^2*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/b^3-3/4*I*d^3*polylog(4,-exp
(2*I*(b*x+a)))/b^4+3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^4
```

### 3.241.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1526 vs. $2(325) = 650$.

Time = 7.00 (sec) , antiderivative size = 1526, normalized size of antiderivative = 4.70

$$\int (c+dx)^3 \csc^3(a+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input
```
Integrate[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x],x]
```

output
```
-1/2*((c + d*x)^3*Csc[a + b*x]^2)/b - (c*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E
^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))]
+ (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - 6*b*(1 -
E^((-2*I)*a))*x*PolyLog[2, -E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x
*PolyLog[2, E^((-I)*(a + b*x))] + (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, -E^(
(-I)*(a + b*x))] + (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, E^((-I)*(a + b*x))]
))/(2*b^3) - (d^3*E^(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a) + (2*I)*b^3*(1 - E
^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b^3*(1 - E^((-2*I)*a)
)*x^3*Log[1 + E^((-I)*(a + b*x))] - 6*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[2
, -E^((-I)*(a + b*x))] - 6*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[2, E^((-I)*(
a + b*x))] + (12*I)*b*(1 - E^((-2*I)*a))*x*PolyLog[3, -E^((-I)*(a + b*x))]
 + (12*I)*b*(1 - E^((-2*I)*a))*x*PolyLog[3, E^((-I)*(a + b*x))] + 12*(1 -
E^((-2*I)*a))*PolyLog[4, -E^((-I)*(a + b*x))] + 12*(1 - E^((-2*I)*a))*Poly
Log[4, E^((-I)*(a + b*x))]))/(4*b^4) + (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2
 + d^3*x^3)*Csc[a]*Sec[a])/4 - ((I/4)*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 +
 E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*Pol
yLog[2, -E^((-2*I)*(a + b*x))] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-
2*I)*(a + b*x))])*Sec[a])/(b^3*E^(I*a)) - ((I/8)*d^3*E^(I*a)*((2*b^4*x^4)/
E^((2*I)*a) - (4*I)*b^3*(1 + E^((-2*I)*a))*x^3*Log[1 + E^((-2*I)*(a + b*x)
)] + 6*b^2*(1 + E^((-2*I)*a))*x^2*PolyLog[2, -E^((-2*I)*(a + b*x))] - (...
```

### 3.241.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4920, 27, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\csc^3(a+bx)\sec(a+bx)\,dx$$

$$\Big\downarrow \text{4920}$$

$$-3d\int -\frac{1}{2}(c+dx)^2\left(\frac{\cot^2(a+bx)}{b}-\frac{2\log(\tan(a+bx))}{b}\right)dx-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{(c+dx)^3\log(\tan(a+bx))}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{2}d\int (c+dx)^2\left(\frac{\cot^2(a+bx)}{b}-\frac{2\log(\tan(a+bx))}{b}\right)dx-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{(c+dx)^3\log(\tan(a+bx))}{b}$$

$$\Big\downarrow \text{7292}$$

$$\frac{3}{2}d\int \frac{(c+dx)^2\left(\cot^2(a+bx)-2\log(\tan(a+bx))\right)}{b}dx-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{(c+dx)^3\log(\tan(a+bx))}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{3d\int (c+dx)^2\left(\cot^2(a+bx)-2\log(\tan(a+bx))\right)dx}{2b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{(c+dx)^3\log(\tan(a+bx))}{b}$$

$$\Big\downarrow \text{7293}$$

$$\frac{3d\int \left((c+dx)^2\cot^2(a+bx)-2(c+dx)^2\log(\tan(a+bx))\right)dx}{2b}-\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{(c+dx)^3\log(\tan(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$3d\left(-\frac{4(c+dx)^3\text{arctanh}\left(e^{2i(a+bx)}\right)}{3d}-\frac{id^2\,\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}-\frac{id^2\,\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{2b^3}+\frac{id^2\,\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{2b^3}-\frac{d(c+dx)\,\text{Po}}{}\right.$$

$$\frac{(c+dx)^3\cot^2(a+bx)}{2b}+\frac{(c+dx)^3\log(\tan(a+bx))}{b}$$

input
```
Int[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x],x]
```

output
```
-1/2*((c + d*x)^3*Cot[a + b*x]^2)/b + ((c + d*x)^3*Log[Tan[a + b*x]])/b +
(3*d*(((-I)*(c + d*x)^2)/b - (c + d*x)^3/(3*d) - (4*(c + d*x)^3*ArcTanh[E^
((2*I)*(a + b*x))])/(3*d) - ((c + d*x)^2*Cot[a + b*x])/b + (2*d*(c + d*x)*
Log[1 - E^((2*I)*(a + b*x))])/b^2 - (2*(c + d*x)^3*Log[Tan[a + b*x]])/(3*d
) + (I*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b - (I*d^2*PolyLog[2,
 E^((2*I)*(a + b*x))])/b^3 - (I*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))
])/b - (d*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/b^2 + (d*(c + d*x)*P
olyLog[3, E^((2*I)*(a + b*x))])/b^2 - ((I/2)*d^2*PolyLog[4, -E^((2*I)*(a +
 b*x))])/b^3 + ((I/2)*d^2*PolyLog[4, E^((2*I)*(a + b*x))])/b^3))/(2*b)
```

### 3.241.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### 3.241.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1222 vs. $2(279) = 558$.

Time = 1.83 (sec) , antiderivative size = 1223, normalized size of antiderivative = 3.76

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1223 |

input
```
int((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4+(2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d
^3*x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*exp(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I
*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-3*I*c^2*d*exp(2*I*(b*x+a))+3*I*d^3*
x^2+2*b*c^3*exp(2*I*(b*x+a))+6*I*c*d^2*x+3*I*c^2*d)/b^2/(exp(2*I*(b*x+a))-
1)^2-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4-3/b*c^2*d*ln(exp(2*I*(b*x+
a))+1)*x-3/b*c*d^2*ln(exp(2*I*(b*x+a))+1)*x^2+3/2*I/b^2*d^3*polylog(2,-exp
(2*I*(b*x+a)))*x^2+3/2*I/b^2*c^2*d*polylog(2,-exp(2*I*(b*x+a)))-6*I/b^3*d^
3*x*a-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x-6*I/b^2*c*d^2*polylog(2,-e
xp(I*(b*x+a)))*x+3*I/b^2*c*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-3/2/b^3*c*d^
2*polylog(3,-exp(2*I*(b*x+a)))+6/b^4*a*d^3*ln(exp(I*(b*x+a)))-3/b^4*a*d^3*
ln(exp(I*(b*x+a))-1)+3/b^4*d^3*ln(1-exp(I*(b*x+a)))*a-1/b*d^3*ln(exp(2*I*(
b*x+a))+1)*x^3+3/b^3*d^3*ln(exp(I*(b*x+a))+1)*x+3/b^3*d^3*ln(1-exp(I*(b*x+
a)))*x-3/2/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x-6/b^3*c*d^2*ln(exp(I*(b*
x+a)))+3/b^3*c*d^2*ln(exp(I*(b*x+a))-1)+3/b^3*c*d^2*ln(exp(I*(b*x+a))+1)+3
/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)-3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)+3/b
^2*d*c^2*ln(1-exp(I*(b*x+a)))*a-3/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2+1/b*c
^3*ln(exp(I*(b*x+a))+1)+1/b*c^3*ln(exp(I*(b*x+a))-1)+3/b*d*c^2*ln(1-exp(I*
(b*x+a)))*x+3/b*d*c^2*ln(exp(I*(b*x+a))+1)*x+3/b*c*d^2*ln(1-exp(I*(b*x+a))
)*x^2+3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2-3*I/b^2*d^3*polylog(2,-exp(I*(b*x
+a)))*x^2-3*I/b^2*d*c^2*polylog(2,exp(I*(b*x+a)))-3*I/b^2*d*c^2*polylog...
```

### 3.241.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3475 vs. 2(270) = 540.

Time = 0.42 (sec) , antiderivative size = 3475, normalized size of antiderivative = 10.69

$$\int (c+dx)^3\csc^3(a+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 + 3*(b^2*d^3*
x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x + a) - 3*(-I*b^2*d^3
*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d - I*d^3 + (I*b^2*d^3*x^2 + 2*I*b^2*c*
d^2*x + I*b^2*c^2*d + I*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) + I*sin(b*
x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d + I*d^3 + (-I*b
^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d - I*d^3)*cos(b*x + a)^2)*dilog(
cos(b*x + a) - I*sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b
^2*c^2*d + (I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*cos(b*x + a)^2)
*dilog(I*cos(b*x + a) + sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x
 + I*b^2*c^2*d + (-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*cos(b*x
+ a)^2)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*
c*d^2*x + I*b^2*c^2*d + (-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*c
os(b*x + a)^2)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 -
 2*I*b^2*c*d^2*x - I*b^2*c^2*d + (I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*
c^2*d)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*(I*b^2*d^
3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d + I*d^3 + (-I*b^2*d^3*x^2 - 2*I*b^2*
c*d^2*x - I*b^2*c^2*d - I*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin
(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d - I*d^3 + (
I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d + I*d^3)*cos(b*x + a)^2)*dil
og(-cos(b*x + a) - I*sin(b*x + a)) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b...
```

### 3.241.6 Sympy [F]

$$\int (c+dx)^3\csc^3(a+bx)\sec(a+bx)\,dx = \int (c+dx)^3\csc^3(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*csc(b*x+a)**3*sec(b*x+a),x)
```

output
```
Integral((c + d*x)**3*csc(a + b*x)**3*sec(a + b*x), x)
```

### 3.241.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5165 vs. $2(270) = 540$.

Time = 1.62 (sec) , antiderivative size = 5165, normalized size of antiderivative = 15.89

$$\int (c+dx)^3 \csc^3(a+bx) \sec(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(c^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2
)) - 3*a*c^2*d*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x +
 a)^2))/b + 3*a^2*c*d^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(
sin(b*x + a)^2))/b^2 - a^3*d^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1)
 - log(sin(b*x + a)^2))/b^3 - 2*(18*b^2*c^2*d - 36*a*b*c*d^2 + 18*a^2*d^3
- 2*(4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d -
2*a*b*c*d^2 + a^2*d^3)*(b*x + a) + (4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3
)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(4*b*x
 + 4*a) - 2*(4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*
c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-4*I*(b*x +
a)^3*d^3 + 9*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 9*(-I*b^2*c^2*d + 2*I*a*
b*c*d^2 - I*a^2*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*(4*I*(b*x + a)^3*d^3
+ 9*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 9*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + I
*a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x
 + 2*a) + 1) + 6*((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d
^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a) +
((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 +
 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) -
 2*((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^
2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(2*b*x + ...
```

### 3.241.8 Giac [F]

$$\int (c+dx)^3 \csc^3(a+bx) \sec(a+bx)\,dx = \int (dx+c)^3 \csc(bx+a)^3 \sec(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*csc(b*x + a)^3*sec(b*x + a), x)
```

### 3.241.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^3(a+bx) \sec(a+bx) \, dx = \text{Hanged}$$

input
```
int((c + d*x)^3/(cos(a + b*x)*sin(a + b*x)^3),x)
```

output
```
\text{Hanged}
```

# 3.242 $\int (c+dx)^2 \csc^3(a+bx)\sec(a+bx)\,dx$

3.242.1 Optimal result . . . 1888
3.242.2 Mathematica [B] (warning: unable to verify) . . . 1889
3.242.3 Rubi [A] (verified) . . . 1890
3.242.4 Maple [B] (verified) . . . 1892
3.242.5 Fricas [B] (verification not implemented) . . . 1893
3.242.6 Sympy [F] . . . 1894
3.242.7 Maxima [B] (verification not implemented) . . . 1895
3.242.8 Giac [F] . . . 1895
3.242.9 Mupad [F(-1)] . . . 1896

## 3.242.1 Optimal result

Integrand size = 22, antiderivative size = 201

$$\int (c+dx)^2 \csc^3(a+bx)\sec(a+bx)\,dx = -\frac{cdx}{b}-\frac{d^2x^2}{2b}-\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{d^2\log(\sin(a+bx))}{b^3} + \frac{id(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} - \frac{id(c+dx)\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2} - \frac{d^2\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} + \frac{d^2\operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}$$

output
```
-c*d*x/b-1/2*d^2*x^2/b-2*(d*x+c)^2*arctanh(exp(2*I*(b*x+a)))/b-d*(d*x+c)*c
ot(b*x+a)/b^2-1/2*(d*x+c)^2*cot(b*x+a)^2/b+d^2*ln(sin(b*x+a))/b^3+I*d*(d*x
+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2-I*d*(d*x+c)*polylog(2,exp(2*I*(b*x+a)
))/b^2-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+1/2*d^2*polylog(3,exp(2*I*
(b*x+a)))/b^3
```

### 3.242.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 880 vs. 2(201) = 402.

Time = 6.92 (sec) , antiderivative size = 880, normalized size of antiderivative = 4.38

$$\int (c+dx)^2\csc^3(a+bx)\sec(a+bx)\,dx = -\frac{(c+dx)^2\csc^2(a+bx)}{2b}$$
$$-\frac{d^2e^{ia}\csc(a)\left(2b^3e^{-2ia}x^3+3ib^2(1-e^{-2ia})\,x^2\log\left(1-e^{-i(a+bx)}\right)+3ib^2(1-e^{-2ia})\,x^2\log\left(1+e^{-i(a+bx)}\right)-\cdots\right.}{\cdots}$$
$$+\frac{1}{3}x\left(3c^2+3cdx+d^2x^2\right)\csc(a)\sec(a)$$
$$-\frac{id^2e^{-ia}\left(2b^2x^2\left(2bx-3i(1+e^{2ia})\log\left(1+e^{-2i(a+bx)}\right)\right)+6b(1+e^{2ia})\,x\,\mathrm{PolyLog}\left(2,-e^{-2i(a+bx)}\right)-3i(1+\cdots\right.}{12b^3}$$
$$-\frac{c^2\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))}{b\left(\cos^2(a)+\sin^2(a)\right)}$$
$$+\frac{c^2\csc(a)(-bx\cos(a)+\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a))}{b\left(\cos^2(a)+\sin^2(a)\right)}$$
$$+\frac{d^2\csc(a)(-bx\cos(a)+\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a))}{b^3\left(\cos^2(a)+\sin^2(a)\right)}$$
$$-\frac{cd\csc(a)\left(b^2e^{-i\arctan(\cot(a))}x^2-\frac{\cot(a)(ibx(-\pi-2\arctan(\cot(a)))-\pi\log(1+e^{-2ibx})-2(bx-\arctan(\cot(a)))\log(1-e^{2i(bx-\arctan(\cdots}{\cdots}\right.}{b^2\sqrt{\csc^2(a)\left(\cos^2(a)+\cdots\right.}}$$
$$+\frac{\csc(a)\csc(a+bx)\left(cd\sin(bx)+d^2x\sin(bx)\right)}{b^2}$$
$$-\frac{cd\csc(a)\sec(a)\left(b^2e^{i\arctan(\tan(a))}x^2+\frac{(ibx(-\pi+2\arctan(\tan(a)))-\pi\log(1+e^{-2ibx})-2(bx+\arctan(\tan(a)))\log(1-e^{2i(bx+\arctan\cdots}{\cdots}\right.}{b^2\sqrt{\sec^2(a)\left(\cos^2(a)+\cdots\right.}}$$

input
```
Integrate[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x],x]
```

output
```
-1/2*((c + d*x)^2*Csc[a + b*x]^2)/b - (d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^(
(2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] +
(3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - 6*b*(1 - E^
((-2*I)*a))*x*PolyLog[2, -E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*P
olyLog[2, E^((-I)*(a + b*x))] + (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, -E^((-
I)*(a + b*x))] + (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, E^((-I)*(a + b*x))]))
/(6*b^3) + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Csc[a]*Sec[a])/3 - ((I/12)*d^2*(
2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))])
+ 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (3*I)*(1 + E
^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(I*a)) - (c^
2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*
(Cos[a]^2 + Sin[a]^2)) + (c^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a]
+ Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (d^2*Csc[a]*(-(b*x
*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos[a]^2
+ Sin[a]^2)) - (c*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*
x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[
Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*
ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x
 - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*Sqrt[Csc[a]^2*(Cos
[a]^2 + Sin[a]^2)]) + (Csc[a]*Csc[a + b*x]*(c*d*Sin[b*x] + d^2*x*Sin[b*...
```

### 3.242.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4920, 27, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \csc^3(a+bx) \sec(a+bx) \, dx$$

$$\Big\downarrow \text{4920}$$

$$-2d\int -\frac{1}{2}(c+dx)\left(\frac{\cot^2(a+bx)}{b}-\frac{2\log(\tan(a+bx))}{b}\right)dx-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{(c+dx)^2\log(\tan(a+bx))}{b}$$

$$\Big\downarrow \text{27}$$

$$d\int (c+dx)\left(\frac{\cot^2(a+bx)}{b}-\frac{2\log(\tan(a+bx))}{b}\right)dx-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{(c+dx)^2\log(\tan(a+bx))}{b}$$

$\Big\downarrow$ 7292

$$d\int \frac{(c+dx)\left(\cot^2(a+bx)-2\log(\tan(a+bx))\right)}{b}dx-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{(c+dx)^2\log(\tan(a+bx))}{b}$$

$\Big\downarrow$ 27

$$\frac{d\int (c+dx)\left(\cot^2(a+bx)-2\log(\tan(a+bx))\right)dx}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{(c+dx)^2\log(\tan(a+bx))}{b}$$

$\Big\downarrow$ 7293

$$\frac{d\int \left((c+dx)\cot^2(a+bx)-2(c+dx)\log(\tan(a+bx))\right)dx}{b}-\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{(c+dx)^2\log(\tan(a+bx))}{b}$$

$\Big\downarrow$ 2009

$$\frac{d\left(-\frac{2(c+dx)^2\text{arctanh}\left(e^{2i(a+bx)}\right)}{d}-\frac{d\,\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^2}+\frac{d\,\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^2}+\frac{d\log(\sin(a+bx))}{b^2}+\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b}\right.}{b}$$
$$\frac{(c+dx)^2\cot^2(a+bx)}{2b}+\frac{(c+dx)^2\log(\tan(a+bx))}{b}$$

input
```
Int[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x],x]
```

output
```
-1/2*((c + d*x)^2*Cot[a + b*x]^2)/b + ((c + d*x)^2*Log[Tan[a + b*x]])/b +
(d*(-1/2*(c + d*x)^2/d - (2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/d -
((c + d*x)*Cot[a + b*x])/b + (d*Log[Sin[a + b*x]])/b^2 - ((c + d*x)^2*Log[
Tan[a + b*x]])/d + (I*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b - (I*(
c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b - (d*PolyLog[3, -E^((2*I)*(a +
 b*x))])/(2*b^2) + (d*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^2)))/b
```

3.242. $\int (c+dx)^2\csc^3(a+bx)\sec(a+bx)\,dx$

### 3.242.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## 3.242.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(181) = 362$.

Time = 1.50 (sec) , antiderivative size = 632, normalized size of antiderivative = 3.14

| method | result |
|---|---|
| risch | $-\frac{2cd\ln\left(\mathrm{e}^{2i(xb+a)}+1\right)x}{b}-\frac{2id^{2}\operatorname{polylog}\left(2,\mathrm{e}^{i(xb+a)}\right)x}{b^{2}}-\frac{2idc\operatorname{polylog}\left(2,\mathrm{e}^{i(xb+a)}\right)}{b^{2}}+\frac{2dc\ln\left(1-\mathrm{e}^{i(xb+a)}\right)x}{b}+\frac{2dc\ln\left(1-\mathrm{e}^{i(xb+a)}\right)}{b^{2}}$ |

input
```
int((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b*d^2*ln(exp(2*I*(b*x+a))+1)*x^2+1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+I/b
^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a))
)*x-2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+1/b*c^2*ln(exp(I*(b*x+a))+1)+
1/b*c^2*ln(exp(I*(b*x+a))-1)-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+1/b*d^2*ln
(1-exp(I*(b*x+a)))*x^2+1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+2*(b*d^2*x^2*exp(2
*I*(b*x+a))+2*b*c*d*x*exp(2*I*(b*x+a))+b*c^2*exp(2*I*(b*x+a))-I*d^2*x*exp(
2*I*(b*x+a))-I*c*d*exp(2*I*(b*x+a))+I*d^2*x+I*d*c)/b^2/(exp(2*I*(b*x+a))-1
)^2-1/b*c^2*ln(exp(2*I*(b*x+a))+1)+2/b*d*c*ln(exp(I*(b*x+a))+1)*x+2/b^2*d*
c*ln(1-exp(I*(b*x+a)))*a-2/b^2*c*d*a*ln(exp(I*(b*x+a))-1)+2/b*d*c*ln(1-exp
(I*(b*x+a)))*x-2*I/b^2*d*c*polylog(2,exp(I*(b*x+a)))-2*I/b^2*d*c*polylog(2
,-exp(I*(b*x+a)))-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3-2/b*c*d*ln(exp(
2*I*(b*x+a))+1)*x+2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp
(I*(b*x+a)))/b^3-2/b^3*d^2*ln(exp(I*(b*x+a)))+1/b^3*d^2*ln(exp(I*(b*x+a))-
1)+1/b^3*d^2*ln(exp(I*(b*x+a))+1)+I/b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))
```

### 3.242.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1995 vs. 2(177) = 354.

Time = 0.36 (sec) , antiderivative size = 1995, normalized size of antiderivative = 9.93

$$\int (c+dx)^2\csc^3(a+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a
)*sin(b*x + a) - 2*(-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(b*x +
 a)^2)*dilog(cos(b*x + a) + I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d + (-I
*b*d^2*x - I*b*c*d)*cos(b*x + a)^2)*dilog(cos(b*x + a) - I*sin(b*x + a)) -
 2*(-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2)*dilog(I*c
os(b*x + a) + sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b*c
*d)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 2*(I*b*d^2*x +
I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) + s
in(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(b*x + a
)^2)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d + (-I*
b*d^2*x - I*b*c*d)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a)) -
 2*(-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2)*dilog(-co
s(b*x + a) - I*sin(b*x + a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - (b^2
*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + d^2)*cos(b*x + a)^2 + d^2)*log(cos(b*x
+ a) + I*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*c^2 - 2
*a*b*c*d + a^2*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a) + I)
 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2 + d^2)*cos(b*x + a)^2 + d^2)*log(cos(b*x + a) - I*sin(b*x + a) + 1) +
 (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(b*x
+ a)^2)*log(cos(b*x + a) - I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c...
```

### 3.242.6 Sympy [F]

$$\int (c+dx)^2 \csc^3(a+bx) \sec(a+bx) \, dx = \int (c+dx)^2 \csc^3{(a+bx)} \sec{(a+bx)} \, dx$$

input
```
integrate((d*x+c)**2*csc(b*x+a)**3*sec(b*x+a),x)
```

output
```
Integral((c + d*x)**2*csc(a + b*x)**3*sec(a + b*x), x)
```

### 3.242.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2528 vs. 2(177) = 354.

Time = 0.61 (sec) , antiderivative size = 2528, normalized size of antiderivative = 12.58

$$\int (c+dx)^2 \csc^3(a+bx) \sec(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(c^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2
)) - 2*a*c*d*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a
)^2))/b + a^2*d^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*
x + a)^2))/b^2 + 2*(4*(b*x + a)*d^2*cos(4*b*x + 4*a) + 4*I*(b*x + a)*d^2*s
in(4*b*x + 4*a) - 4*b*c*d + 4*a*d^2 + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^
2)*(b*x + a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x +
 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a)
 - (-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a
) - 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*sin(2*b*x + 2*
a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 2*((b*x + a)^2*d^2 +
 2*(b*c*d - a*d^2)*(b*x + a) + d^2 + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*
(b*x + a) + d^2)*cos(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)
*(b*x + a) + d^2)*cos(2*b*x + 2*a) + (I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a
*d^2)*(b*x + a) + I*d^2)*sin(4*b*x + 4*a) + 2*(-I*(b*x + a)^2*d^2 + 2*(-I*
b*c*d + I*a*d^2)*(b*x + a) - I*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a)
, cos(b*x + a) + 1) - 2*(d^2*cos(4*b*x + 4*a) - 2*d^2*cos(2*b*x + 2*a) + I
*d^2*sin(4*b*x + 4*a) - 2*I*d^2*sin(2*b*x + 2*a) + d^2)*arctan2(sin(b*x +
a), cos(b*x + a) - 1) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) +
 ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a) - 2*((b*
x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*x...
```

### 3.242.8 Giac [F]

$$\int (c+dx)^2 \csc^3(a+bx) \sec(a+bx)\,dx = \int (dx+c)^2 \csc(bx+a)^3 \sec(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csc(b*x + a)^3*sec(b*x + a), x)
```

### 3.242.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc^3(a+bx) \sec(a+bx)\, dx = \text{Hanged}$$

input
```
int((c + d*x)^2/(cos(a + b*x)*sin(a + b*x)^3),x)
```

output
```
\text{Hanged}
```

# 3.243 $\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx$

3.243.1 Optimal result . . . . . 1897
3.243.2 Mathematica [A] (verified) . . . . . 1898
3.243.3 Rubi [A] (verified) . . . . . 1898
3.243.4 Maple [B] (verified) . . . . . 1899
3.243.5 Fricas [B] (verification not implemented) . . . . . 1900
3.243.6 Sympy [F] . . . . . 1901
3.243.7 Maxima [B] (verification not implemented) . . . . . 1901
3.243.8 Giac [F] . . . . . 1902
3.243.9 Mupad [F(-1)] . . . . . 1902

## 3.243.1 Optimal result

Integrand size = 20, antiderivative size = 141

$$\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx = -\frac{dx}{2b}-\frac{2dx\,\text{arctanh}\left(e^{2i(a+bx)}\right)}{b} - \frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+bx)}{2b} - \frac{dx\log(\tan(a+bx))}{b}+\frac{(c+dx)\log(\tan(a+bx))}{b} + \frac{id\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} - \frac{id\,\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}$$

output
```
-1/2*d*x/b-2*d*x*arctanh(exp(2*I*(b*x+a)))/b-1/2*d*cot(b*x+a)/b^2-1/2*(d*x
+c)*cot(b*x+a)^2/b-d*x*ln(tan(b*x+a))/b+(d*x+c)*ln(tan(b*x+a))/b+1/2*I*d*p
olylog(2,-exp(2*I*(b*x+a)))/b^2-1/2*I*d*polylog(2,exp(2*I*(b*x+a)))/b^2
```

### 3.243.2 Mathematica [A] (verified)

Time = 1.71 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.59

$$\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx$$
$$= -\frac{d\cot(a+bx)}{2b^2}-\frac{c\csc^2(a+bx)}{2b}-\frac{dx\csc^2(a+bx)}{2b}-\frac{c\log(\cos(a+bx))}{b}$$
$$+\frac{ad\log(\cos(a+bx))}{b^2}+\frac{c\log(\sin(a+bx))}{b}-\frac{ad(\log(\cos(a+bx))+\log(\tan(a+bx)))}{b^2}$$
$$+\frac{d\left(\frac{1}{2}i(a+bx)^2-(a+bx)\log\left(1+e^{2i(a+bx)}\right)+\frac{1}{2}i\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)\right)}{b^2}$$
$$+\frac{d\left((a+bx)\log\left(1-e^{2i(a+bx)}\right)-\frac{1}{2}i\left((a+bx)^2+\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)\right)\right)}{b^2}$$

input
```
Integrate[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x],x]
```

output
```
-1/2*(d*Cot[a + b*x])/b^2 - (c*Csc[a + b*x]^2)/(2*b) - (d*x*Csc[a + b*x]^2
)/(2*b) - (c*Log[Cos[a + b*x]])/b + (a*d*Log[Cos[a + b*x]])/b^2 + (c*Log[S
in[a + b*x]])/b - (a*d*(Log[Cos[a + b*x]] + Log[Tan[a + b*x]]))/b^2 + (d*(
(I/2)*(a + b*x)^2 - (a + b*x)*Log[1 + E^((2*I)*(a + b*x))] + (I/2)*PolyLog
[2, -E^((2*I)*(a + b*x))]))/b^2 + (d*((a + b*x)*Log[1 - E^((2*I)*(a + b*x)
)] - (I/2)*((a + b*x)^2 + PolyLog[2, E^((2*I)*(a + b*x))])))/b^2
```

### 3.243.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4920, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx$$
$$\Big\downarrow \text{4920}$$
$$-d\int\left(\frac{\log(\tan(a+bx))}{b}-\frac{\cot^2(a+bx)}{2b}\right)dx-\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{(c+dx)\log(\tan(a+bx))}{b}$$
$$\Big\downarrow \text{2009}$$

$$-d\left(\frac{2x\text{arctanh}\left(e^{2i(a+bx)}\right)}{b}-\frac{i\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}+\frac{i\,\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}+\frac{\cot(a+bx)}{2b^2}+\frac{x\log(\tan(a+b\right.$$
$$\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{(c+dx)\log(\tan(a+bx))}{b}$$

input
```
Int[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x],x]
```

output
```
-1/2*((c + d*x)*Cot[a + b*x]^2)/b + ((c + d*x)*Log[Tan[a + b*x]])/b - d*(x
/(2*b) + (2*x*ArcTanh[E^((2*I)*(a + b*x))])/b + Cot[a + b*x]/(2*b^2) + (x*
Log[Tan[a + b*x]])/b - ((I/2)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 + ((I/
2)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2)
```

### 3.243.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

## 3.243.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(123) = 246$.

Time = 1.06 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.91

| method | result |
|---|---|
| risch | $\frac{2bdx\,e^{2i(xb+a)}-id\,e^{2i(xb+a)}+2bc\,e^{2i(xb+a)}+id}{b^2\left(e^{2i(xb+a)}-1\right)^2}+\frac{c\ln\left(e^{i(xb+a)}+1\right)}{b}-\frac{c\ln\left(e^{2i(xb+a)}+1\right)}{b}+\frac{c\ln\left(e^{i(xb+a)}-1\right)}{b}+\frac{d\ln\left(e^{i(xb+a)}+\right.}{b}$ |

input
```
int((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(2*b*d*x*exp(2*I*(b*x+a))-I*d*exp(2*I*(b*x+a))+2*b*c*exp(2*I*(b*x+a))+I*d)
/b^2/(exp(2*I*(b*x+a))-1)^2+1/b*c*ln(exp(I*(b*x+a))+1)-1/b*c*ln(exp(2*I*(b
*x+a))+1)+1/b*c*ln(exp(I*(b*x+a))-1)+1/b*d*ln(exp(I*(b*x+a))+1)*x-I*d*poly
log(2,-exp(I*(b*x+a)))/b^2-1/b*d*ln(exp(2*I*(b*x+a))+1)*x+1/2*I*d*polylog(
2,-exp(2*I*(b*x+a)))/b^2+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-exp(I*(
b*x+a)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(b*x+a))-1
)
```

### 3.243.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 942 vs. 2(119) = 238.

Time = 0.31 (sec) , antiderivative size = 942, normalized size of antiderivative = 6.68

$$\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b*d*x + d*cos(b*x + a)*sin(b*x + a) + b*c + (-I*d*cos(b*x + a)^2 + I*
d)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog
(cos(b*x + a) - I*sin(b*x + a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(I*cos(
b*x + a) + sin(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(I*cos(b*x + a)
 - sin(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(-I*cos(b*x + a) + sin(
b*x + a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(-I*cos(b*x + a) - sin(b*x +
a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (
-I*d*cos(b*x + a)^2 + I*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b*d*x
- (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a) + I*sin(b*x + a) +
1) - ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(cos(b*x + a) + I*sin(b*x
 + a) + I) - (b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a)
 - I*sin(b*x + a) + 1) - ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(cos(
b*x + a) - I*sin(b*x + a) + I) + (b*d*x - (b*d*x + a*d)*cos(b*x + a)^2 + a
*d)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x - (b*d*x + a*d)*cos(b*
x + a)^2 + a*d)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b*d*x - (b*d*x +
 a*d)*cos(b*x + a)^2 + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d
*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d)*log(-I*cos(b*x + a) - sin(b*x + a
) + 1) + ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(-1/2*cos(b*x + a) +
1/2*I*sin(b*x + a) + 1/2) + ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(-
1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - (b*d*x - (b*d*x + a*d)*c...
```

### 3.243.6 Sympy [F]

$$\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx = \int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)**3*sec(b*x+a),x)
```

output
```
Integral((c + d*x)*csc(a + b*x)**3*sec(a + b*x), x)
```

### 3.243.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1028 vs. 2(119) = 238.

Time = 0.46 (sec) , antiderivative size = 1028, normalized size of antiderivative = 7.29

$$\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
```

output
```
-(2*(b*d*x + b*c + (b*d*x + b*c)*cos(4*b*x + 4*a) - 2*(b*d*x + b*c)*cos(2*
b*x + 2*a) - (-I*b*d*x - I*b*c)*sin(4*b*x + 4*a) - 2*(I*b*d*x + I*b*c)*sin
(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 2*(b*d*x
+ b*c + (b*d*x + b*c)*cos(4*b*x + 4*a) - 2*(b*d*x + b*c)*cos(2*b*x + 2*a)
+ (I*b*d*x + I*b*c)*sin(4*b*x + 4*a) + 2*(-I*b*d*x - I*b*c)*sin(2*b*x + 2*
a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*(b*c*cos(4*b*x + 4*a) - 2*
b*c*cos(2*b*x + 2*a) + I*b*c*sin(4*b*x + 4*a) - 2*I*b*c*sin(2*b*x + 2*a) +
 b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 2*(b*d*x*cos(4*b*x + 4*a)
- 2*b*d*x*cos(2*b*x + 2*a) + I*b*d*x*sin(4*b*x + 4*a) - 2*I*b*d*x*sin(2*b*
x + 2*a) + b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*(-2*I*b*d*x
 - 2*I*b*c - d)*cos(2*b*x + 2*a) - (d*cos(4*b*x + 4*a) - 2*d*cos(2*b*x + 2
*a) + I*d*sin(4*b*x + 4*a) - 2*I*d*sin(2*b*x + 2*a) + d)*dilog(-e^(2*I*b*x
 + 2*I*a)) + 2*(d*cos(4*b*x + 4*a) - 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x
+ 4*a) - 2*I*d*sin(2*b*x + 2*a) + d)*dilog(-e^(I*b*x + I*a)) + 2*(d*cos(4*
b*x + 4*a) - 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) - 2*I*d*sin(2*b*x
 + 2*a) + d)*dilog(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*
c)*cos(4*b*x + 4*a) - 2*(-I*b*d*x - I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c
)*sin(4*b*x + 4*a) - 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a
)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (I*b*d*x + I*b*c + (I
*b*d*x + I*b*c)*cos(4*b*x + 4*a) - 2*(I*b*d*x + I*b*c)*cos(2*b*x + 2*a)...
```

### 3.243.8 Giac [F]

$$\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx = \int (dx+c)\csc(bx+a)^3\sec(bx+a)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*csc(b*x + a)^3*sec(b*x + a), x)
```

### 3.243.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)/(cos(a + b*x)*sin(a + b*x)^3),x)
```

output
```
\text{Hanged}
```

# 3.244 $\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$

3.244.1 Optimal result . . . . . 1903
3.244.2 Mathematica [N/A] . . . . . 1903
3.244.3 Rubi [N/A] . . . . . 1904
3.244.4 Maple [N/A] (verified) . . . . . 1904
3.244.5 Fricas [N/A] . . . . . 1905
3.244.6 Sympy [N/A] . . . . . 1905
3.244.7 Maxima [N/A] . . . . . 1905
3.244.8 Giac [N/A] . . . . . 1906
3.244.9 Mupad [N/A] . . . . . 1907

## 3.244.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\csc^3(a+bx)\sec(a+bx)}{c+dx},x\right)$$

output
```
CannotIntegrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x)
```

## 3.244.2 Mathematica [N/A]

Not integrable

Time = 19.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x),x]
```

output
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x), x]
```

### 3.244.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \ 7299$$

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}dx$$

input
```
Int[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x),x]
```

output
```
$Aborted
```

#### 3.244.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

### 3.244.4 Maple [N/A] (verified)

Not integrable

Time = 0.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\csc\left(xb+a\right)^{3}\sec\left(xb+a\right)}{dx+c}dx$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x)
```

output
```
int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x)
```

### 3.244.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx=\int \frac{\csc\left(bx+a\right)^3\sec\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^3*sec(b*x + a)/(d*x + c), x)
```

### 3.244.6 Sympy [N/A]

Not integrable

Time = 2.59 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx=\int \frac{\csc^3\left(a+bx\right)\sec\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(csc(b*x+a)**3*sec(b*x+a)/(d*x+c),x)
```

output
```
Integral(csc(a + b*x)**3*sec(a + b*x)/(c + d*x), x)
```

### 3.244.7 Maxima [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 2040, normalized size of antiderivative = 92.73

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx=\int \frac{\csc\left(bx+a\right)^3\sec\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
-(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2
- (2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
 - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x
^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*
sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate((b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2 + d^2)*sin(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*
x + b^2*c^3)*cos(b*x + a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3)*sin(b*x + a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^
2*d*x + b^2*c^3)*cos(b*x + a)), x) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
+ (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*s
in(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d...
```

### 3.244.8 Giac [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3*sec(b*x + a)/(d*x + c), x)
```

### 3.244.9 Mupad [N/A]

Not integrable

Time = 24.86 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx=\int \frac{1}{\cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^3\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)), x)
```

# 3.245 $\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$

3.245.1 Optimal result . . . 1908
3.245.2 Mathematica [N/A] . . . 1908
3.245.3 Rubi [N/A] . . . 1909
3.245.4 Maple [N/A] (verified) . . . 1909
3.245.5 Fricas [N/A] . . . 1910
3.245.6 Sympy [N/A] . . . 1910
3.245.7 Maxima [N/A] . . . 1910
3.245.8 Giac [F(-1)] . . . 1911
3.245.9 Mupad [N/A] . . . 1912

## 3.245.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx=\operatorname{Int}\left(\frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2},x\right)$$

output
```
CannotIntegrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x)
```

## 3.245.2 Mathematica [N/A]

Not integrable

Time = 18.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx=\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2, x]
```

## 3.245.3 Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}dx$$

input
```
Int[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

### 3.245.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.245.4 Maple [N/A] (verified)

Not integrable

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\csc\left(xb+a\right)^{3}\sec\left(xb+a\right)}{\left(dx+c\right)^{2}}dx$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x)
```

### 3.245.5 Fricas [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^3*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.245.6 Sympy [N/A]

Not integrable

Time = 4.94 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(csc(b*x+a)**3*sec(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)**3*sec(a + b*x)/(c + d*x)**2, x)
```

### 3.245.7 Maxima [N/A]

Not integrable

Time = 9.09 (sec) , antiderivative size = 2702, normalized size of antiderivative = 122.82

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2
- 2*((b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
 - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b
^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^
2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*
c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 +
 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x
^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3
*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*
a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x
 + 2*a))*integrate((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 3*d^2)*sin(b*x +
 a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b
^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*
x + b^2*c^4)*cos(b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d
^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(b*x + a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*
c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x)
 - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b...
```

### 3.245.8 Giac [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

## 3.245.9 Mupad [N/A]

Not integrable

Time = 24.83 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{1}{\cos(a+b\,x)\,\sin(a+b\,x)^3\,(c+d\,x)^2}\,dx$$

input
```
int(1/(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^2),x)
```

output
```
int(1/(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^2), x)
```

# 3.246 $\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx$

3.246.1 Optimal result . . . . . . . . . . 1913
3.246.2 Mathematica [N/A] . . . . . . . . . . 1913
3.246.3 Rubi [N/A] . . . . . . . . . . 1914
3.246.4 Maple [N/A] (verified) . . . . . . . . . . 1914
3.246.5 Fricas [N/A] . . . . . . . . . . 1915
3.246.6 Sympy [N/A] . . . . . . . . . . 1915
3.246.7 Maxima [N/A] . . . . . . . . . . 1915
3.246.8 Giac [N/A] . . . . . . . . . . 1916
3.246.9 Mupad [N/A] . . . . . . . . . . 1916

## 3.246.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx = \mathrm{Int}((c+dx)^m \sec(a+bx)\tan(a+bx),x)$$

output
```
CannotIntegrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x)
```

## 3.246.2 Mathematica [N/A]

Not integrable

Time = 3.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx = \int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x], x]
```

## 3.246.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(a+bx)\sec(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \tan(a+bx)\sec(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x],x]
```

output
```
$Aborted
```

### 3.246.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.246.4 Maple [N/A] (verified)

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \sec(xb+a)\tan(xb+a)\,dx$$

input
```
int((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x)
```

output
```
int((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x)
```

### 3.246.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\tan(bx+a)\;dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*sec(b*x + a)*tan(b*x + a), x)
```

### 3.246.6 Sympy [N/A]

Not integrable

Time = 3.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx = \int (c+dx)^m \tan(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*sec(b*x+a)*tan(b*x+a),x)
```

output
```
Integral((c + d*x)**m*tan(a + b*x)*sec(a + b*x), x)
```

### 3.246.7 Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\tan(bx+a)\;dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a), x)
```

### 3.246.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\tan(bx+a)\;dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a), x)
```

### 3.246.9 Mupad [N/A]

Not integrable

Time = 23.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,dx = \int \frac{\tan(a+b\,x)\,\left(c+d\,x\right)^m}{\cos\left(a+b\,x\right)}\,dx$$

input
```
int((tan(a + b*x)*(c + d*x)^m)/cos(a + b*x),x)
```

output
```
int((tan(a + b*x)*(c + d*x)^m)/cos(a + b*x), x)
```

# 3.247 $\int (c+dx)^4 \sec(a+bx)\tan(a+bx)\,dx$


3.247.1 Optimal result . . . . . 1917
3.247.2 Mathematica [A] (verified) . . . . . 1918
3.247.3 Rubi [A] (verified) . . . . . 1918
3.247.4 Maple [B] (verified) . . . . . 1921
3.247.5 Fricas [B] (verification not implemented) . . . . . 1922
3.247.6 Sympy [F] . . . . . 1923
3.247.7 Maxima [B] (verification not implemented) . . . . . 1924
3.247.8 Giac [F] . . . . . 1924
3.247.9 Mupad [F(-1)] . . . . . 1925


## 3.247.1 Optimal result

Integrand size = 20, antiderivative size = 227

$$\int (c+dx)^4 \sec(a+bx)\tan(a+bx)\,dx = \frac{8id(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b^2} - \frac{12id^2(c+dx)^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} + \frac{12id^2(c+dx)^2\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} + \frac{24d^3(c+dx)\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4} - \frac{24d^3(c+dx)\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^4} + \frac{24id^4\operatorname{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^5} - \frac{24id^4\operatorname{PolyLog}\left(4,ie^{i(a+bx)}\right)}{b^5} + \frac{(c+dx)^4\sec(a+bx)}{b}$$

output
```
8*I*d*(d*x+c)^3*arctan(exp(I*(b*x+a)))/b^2-12*I*d^2*(d*x+c)^2*polylog(2,-I
*exp(I*(b*x+a)))/b^3+12*I*d^2*(d*x+c)^2*polylog(2,I*exp(I*(b*x+a)))/b^3+24
*d^3*(d*x+c)*polylog(3,-I*exp(I*(b*x+a)))/b^4-24*d^3*(d*x+c)*polylog(3,I*e
xp(I*(b*x+a)))/b^4+24*I*d^4*polylog(4,-I*exp(I*(b*x+a)))/b^5-24*I*d^4*poly
log(4,I*exp(I*(b*x+a)))/b^5+(d*x+c)^4*sec(b*x+a)/b
```

### 3.247.2 Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.89

$$\int (c+dx)^4 \sec(a+bx)\tan(a+bx)\,dx =$$
$$-\frac{4d\left(-2ib^3c^3\arctan\left(e^{i(a+bx)}\right)+3b^3c^2dx\log\left(1-ie^{i(a+bx)}\right)+3b^3cd^2x^2\log\left(1-ie^{i(a+bx)}\right)+b^3d^3x^3\log\left(1\right.\right.}{}$$
$$+\frac{(c+dx)^4\sec(a+bx)}{b}$$

input
```
Integrate[(c + d*x)^4*Sec[a + b*x]*Tan[a + b*x],x]
```

output
```
(-4*d*((-2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 - I*E^
(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] + b^3*d^3*x^3*
Log[1 - I*E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] - 3*
b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + I*E^(I*(a +
 b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))] - (3*I)
*b^2*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, (-
I)*E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*c*d
^2*PolyLog[3, I*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x))]
 - (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, I*E^(
I*(a + b*x))]))/b^5 + ((c + d*x)^4*Sec[a + b*x])/b
```

### 3.247.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {4909, 3042, 4669, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4\tan(a+bx)\sec(a+bx)\,dx$$
$$\Big\downarrow \text{4909}$$
$$\frac{(c+dx)^4\sec(a+bx)}{b}-\frac{4d\int (c+dx)^3\sec(a+bx)dx}{b}$$
$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^4\sec(a+bx)}{b}-\frac{4d\int(c+dx)^3\csc\left(a+bx+\frac{\pi}{2}\right)dx}{b}$$

$\big\downarrow$ 4669

$$\frac{(c+dx)^4\sec(a+bx)}{b}-\frac{4d\left(-\frac{3d\int(c+dx)^2\log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{3d\int(c+dx)^2\log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}\right)}{b}$$

$\big\downarrow$ 3011

$$\frac{(c+dx)^4\sec(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{2id\int(c+dx)\operatorname{PolyLog}\left(2,ie^{i(a}\right.}{b}\right.}{b}\right.}{b}$$

$\big\downarrow$ 7163

$$\frac{(c+dx)^4\sec(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{id\int\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)dx}{b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,ie^{i(a+}\right.}{b}\right.}{}\right.}{b}$$

$\big\downarrow$ 2720

$$\frac{(c+dx)^4\sec(a+bx)}{b}-\frac{4d\left(\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2}{}\right.}{}\right.}{b}$$

$\big\downarrow$ 7143

3.247. $\int(c+dx)^4\sec(a+bx)\tan(a+bx)\,dx$

$$\frac{(c+dx)^4\sec(a+bx)}{b} - \frac{4d\left(-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\left(\frac{d\operatorname{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\right.}{b}$$

input
```
Int[(c + d*x)^4*Sec[a + b*x]*Tan[a + b*x],x]
```

output
```
(-4*d*(((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b + (3*d*((I*(c + d*x)
^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(c + d*x)*PolyLog
[3, (-I)*E^(I*(a + b*x))])/b + (d*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^2))/
b))/b - (3*d*((I*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b - ((2*I)*d*(
((-I)*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b + (d*PolyLog[4, I*E^(I*(a
 + b*x))])/b^2))/b))/b))/b + ((c + d*x)^4*Sec[a + b*x])/b
```

### 3.247.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

### 3.247.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 766 vs. $2(202) = 404$.

Time = 4.67 (sec) , antiderivative size = 767, normalized size of antiderivative = 3.38

| method | result |
|---|---|
| risch | $\frac{12id^2c^2\operatorname{polylog}(2,ie^{i(xb+a)})}{b^3}-\frac{12id^2c^2\operatorname{polylog}(2,-ie^{i(xb+a)})}{b^3}+\frac{8id\,c^3\arctan(e^{i(xb+a)})}{b^2}-\frac{8id^4a^3\arctan(e^{i(xb+a)})}{b^5}-\frac{12id^4}{}$ |

input
```
int((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
12*I/b^3*d^2*c^2*polylog(2,I*exp(I*(b*x+a)))-12*I/b^3*d^2*c^2*polylog(2,-I
*exp(I*(b*x+a)))+8*I/b^2*d*c^3*arctan(exp(I*(b*x+a)))-8*I/b^5*d^4*a^3*arct
an(exp(I*(b*x+a)))-12*I/b^3*d^4*polylog(2,-I*exp(I*(b*x+a)))*x^2+12*I/b^3*
d^4*polylog(2,I*exp(I*(b*x+a)))*x^2-24*I*d^4*polylog(4,I*exp(I*(b*x+a)))/b
^5+24/b^4*d^3*c*polylog(3,-I*exp(I*(b*x+a)))+4/b^5*d^4*a^3*ln(1+I*exp(I*(b
*x+a)))-24/b^4*d^4*polylog(3,I*exp(I*(b*x+a)))*x-4/b^5*d^4*a^3*ln(1-I*exp(
I*(b*x+a)))+24/b^4*d^4*polylog(3,-I*exp(I*(b*x+a)))*x-24/b^4*d^3*c*polylog
(3,I*exp(I*(b*x+a)))+4/b^2*d^4*ln(1+I*exp(I*(b*x+a)))*x^3-4/b^2*d^4*ln(1-I
*exp(I*(b*x+a)))*x^3+24*I/b^4*d^3*c*a^2*arctan(exp(I*(b*x+a)))-24*I/b^3*d^
2*c^2*a*arctan(exp(I*(b*x+a)))-24*I/b^3*d^3*c*polylog(2,-I*exp(I*(b*x+a)))
*x+24*I/b^3*d^3*c*polylog(2,I*exp(I*(b*x+a)))*x-12/b^3*d^2*c^2*ln(1-I*exp(
I*(b*x+a)))*a+12/b^2*d^3*c*ln(1+I*exp(I*(b*x+a)))*x^2-12/b^2*d^3*c*ln(1-I*
exp(I*(b*x+a)))*x^2+12/b^2*d^2*c^2*ln(1+I*exp(I*(b*x+a)))*x+12/b^3*d^2*c^2
*ln(1+I*exp(I*(b*x+a)))*a-12/b^4*d^3*a^2*c*ln(1+I*exp(I*(b*x+a)))+12/b^4*d
^3*a^2*c*ln(1-I*exp(I*(b*x+a)))-12/b^2*d^2*c^2*ln(1-I*exp(I*(b*x+a)))*x+24
*I*d^4*polylog(4,-I*exp(I*(b*x+a)))/b^5+2*exp(I*(b*x+a))*(d^4*x^4+4*c*d^3*
x^3+6*c^2*d^2*x^2+4*c^3*d*x+c^4)/b/(exp(2*I*(b*x+a))+1)
```

### 3.247.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1190 vs. $2(189) = 378$.

Time = 0.31 (sec) , antiderivative size = 1190, normalized size of antiderivative = 5.24

$$\int (c+dx)^4 \sec(a+bx)\tan(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

output
```
(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c
^4 - 12*I*d^4*cos(b*x + a)*polylog(4, I*cos(b*x + a) + sin(b*x + a)) - 12*
I*d^4*cos(b*x + a)*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 12*I*d^4*co
s(b*x + a)*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) + 12*I*d^4*cos(b*x +
 a)*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - 6*(-I*b^2*d^4*x^2 - 2*I*b
^2*c*d^3*x - I*b^2*c^2*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x +
a)) - 6*(-I*b^2*d^4*x^2 - 2*I*b^2*c*d^3*x - I*b^2*c^2*d^2)*cos(b*x + a)*di
log(I*cos(b*x + a) - sin(b*x + a)) - 6*(I*b^2*d^4*x^2 + 2*I*b^2*c*d^3*x +
I*b^2*c^2*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 6*(I*b
^2*d^4*x^2 + 2*I*b^2*c*d^3*x + I*b^2*c^2*d^2)*cos(b*x + a)*dilog(-I*cos(b*
x + a) - sin(b*x + a)) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 -
a^3*d^4)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I) + 2*(b^3*c^3*
d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(cos(b*x +
a) - I*sin(b*x + a) + I) - 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^
2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(I*cos(b*
x + a) + sin(b*x + a) + 1) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*
d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(I*cos(
b*x + a) - sin(b*x + a) + 1) - 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^
2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(-I*c
os(b*x + a) + sin(b*x + a) + 1) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*...
```

### 3.247.6 Sympy [F]

$$\int (c+dx)^4 \sec(a+bx)\tan(a+bx)\,dx = \int (c+dx)^4 \tan{(a+bx)} \sec{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**4*sec(b*x+a)*tan(b*x+a),x)
```

output
```
Integral((c + d*x)**4*tan(a + b*x)*sec(a + b*x), x)
```

### 3.247.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2944 vs. 2(189) = 378.

Time = 0.59 (sec) , antiderivative size = 2944, normalized size of antiderivative = 12.97

$$\int (c+dx)^4 \sec(a+bx)\tan(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

output
```
(2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x + 2*
a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2*b
*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2
+ 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2
*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1)
)*c^3*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1
)*b) - 6*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*
x + 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + s
in(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x +
 a)^2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2
*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a
) + 1))*a*c^2*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
+ 2*a) + 1)*b^2) + 6*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x +
 a)*sin(2*b*x + 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x
+ 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2
 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x
+ 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2
*sin(b*x + a) + 1))*a^2*c*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 +
2*cos(2*b*x + 2*a) + 1)*b^3) - 2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a
) + 4*(b*x + a)*sin(2*b*x + 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a...
```

### 3.247.8 Giac [F]

$$\int (c+dx)^4 \sec(a+bx)\tan(a+bx)\,dx = \int (dx+c)^4 \sec(bx+a)\tan(bx+a)\,dx$$

input
```
integrate((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*sec(b*x + a)*tan(b*x + a), x)
```

### 3.247.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \sec(a+bx)\tan(a+bx)\,dx = \int \frac{\tan(a+b\,x)\,\left(c+d\,x\right)^4}{\cos\left(a+b\,x\right)}\,dx$$

input
```
int((tan(a + b*x)*(c + d*x)^4)/cos(a + b*x),x)
```

output
```
int((tan(a + b*x)*(c + d*x)^4)/cos(a + b*x), x)
```

# 3.248 $\int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx$

3.248.1 Optimal result . . . 1926
3.248.2 Mathematica [A] (verified) . . . 1927
3.248.3 Rubi [A] (verified) . . . 1927
3.248.4 Maple [B] (verified) . . . 1930
3.248.5 Fricas [B] (verification not implemented) . . . 1930
3.248.6 Sympy [F] . . . 1931
3.248.7 Maxima [B] (verification not implemented) . . . 1932
3.248.8 Giac [F] . . . 1932
3.248.9 Mupad [F(-1)] . . . 1933

## 3.248.1 Optimal result

Integrand size = 20, antiderivative size = 159

$$\int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx = \frac{6id(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b^2} - \frac{6id^2(c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} + \frac{6id^2(c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} + \frac{6d^3\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4} - \frac{6d^3\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^4} + \frac{(c+dx)^3\sec(a+bx)}{b}$$

output
```
6*I*d*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b^2-6*I*d^2*(d*x+c)*polylog(2,-I*ex
p(I*(b*x+a)))/b^3+6*I*d^2*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^3+6*d^3*po
lylog(3,-I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4+(d*x+
c)^3*sec(b*x+a)/b
```

### 3.248.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.61

$$\int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx =$$
$$-\frac{3d\left(-2ib^2c^2\arctan\left(e^{i(a+bx)}\right)+2b^2cdx\log\left(1-ie^{i(a+bx)}\right)+b^2d^2x^2\log\left(1-ie^{i(a+bx)}\right)-2b^2cdx\log\left(1+i\right.\right.}{}$$
$$+\frac{(c+dx)^3\sec(a+bx)}{b}$$

input
```
Integrate[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x],x]
```

output
```
(-3*d*((-2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 - I*E^(I
*(a + b*x))] + b^2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - 2*b^2*c*d*x*Log[1
+ I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + (2*I)*b*d*
(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2
, I*E^(I*(a + b*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 2*d^2*Poly
Log[3, I*E^(I*(a + b*x))]))/b^4 + ((c + d*x)^3*Sec[a + b*x])/b
```

### 3.248.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4909, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \tan(a+bx)\sec(a+bx)\,dx$$
$$\Big\downarrow \text{4909}$$
$$\frac{(c+dx)^3\sec(a+bx)}{b}-\frac{3d\int (c+dx)^2\sec(a+bx)dx}{b}$$
$$\Big\downarrow \text{3042}$$
$$\frac{(c+dx)^3\sec(a+bx)}{b}-\frac{3d\int (c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)dx}{b}$$
$$\Big\downarrow \text{4669}$$

$$\frac{(c+dx)^3\sec(a+bx)}{b}-\frac{3d\left(-\frac{2d\int(c+dx)\log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{2d\int(c+dx)\log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 3011

$$\frac{(c+dx)^3\sec(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{id\int\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{id\int\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2i}{}\right.}{b}$$

$\Big\downarrow$ 2720

$$\frac{(c+dx)^3\sec(a+bx)}{b}-\frac{3d\left(\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyL}}{}\right.}{b}\right.}{b}$$

$\Big\downarrow$ 7143

$$\frac{(c+dx)^3\sec(a+bx)}{b}-\frac{3d\left(-\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{d\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{d\operatorname{Poly}}{}\right.}{b}\right.}{b}$$

input
```
Int[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x],x]
```

output
```
(-3*d*(((-2*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (2*d*((I*(c + d*x)
*PolyLog[2, (-I)*E^(I*(a + b*x))])/b - (d*PolyLog[3, (-I)*E^(I*(a + b*x))]
)/b^2))/b - (2*d*((I*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b - (d*PolyL
og[3, I*E^(I*(a + b*x))])/b^2))/b))/b + ((c + d*x)^3*Sec[a + b*x])/b
```

## 3.248. $\int(c+dx)^3\sec(a+bx)\tan(a+bx)\,dx$

### 3.248.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.248. $\int (c+dx)^3 \sec(a+bx) \tan(a+bx)\,dx$

### 3.248.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 462 vs. 2(142) = 284.

Time = 2.24 (sec) , antiderivative size = 463, normalized size of antiderivative = 2.91

| method | result |
|---|---|
| risch | $\frac{2\,\mathrm{e}^{i(xb+a)}\left(d^3x^3+3c\,d^2x^2+3c^2dx+c^3\right)}{b\left(\mathrm{e}^{2i(xb+a)}+1\right)}+\frac{6id\,c^2\arctan\left(\mathrm{e}^{i(xb+a)}\right)}{b^2}+\frac{3d^3\ln\left(1+i\mathrm{e}^{i(xb+a)}\right)x^2}{b^2}+\frac{6d^2c\ln\left(1+i\mathrm{e}^{i(xb+a)}\right)x}{b^2}+\frac{6ic\,d^2\,\mathrm{p}}{}$ |

input
```
int((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*exp(I*(b*x+a))*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(exp(2*I*(b*x+a))+1
)+6*I/b^2*d*c^2*arctan(exp(I*(b*x+a)))+3/b^2*d^3*ln(1+I*exp(I*(b*x+a)))*x^
2+6/b^2*d^2*c*ln(1+I*exp(I*(b*x+a)))*x+6*I*c*d^2*polylog(2,I*exp(I*(b*x+a)
))/b^3-12*I/b^3*d^2*c*a*arctan(exp(I*(b*x+a)))+6*I/b^4*d^3*a^2*arctan(exp(
I*(b*x+a)))+6*I*d^3*x*polylog(2,I*exp(I*(b*x+a)))/b^3+3/b^4*d^3*a^2*ln(1-I
*exp(I*(b*x+a)))+6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4-6/b^3*d^2*c*ln(1-I
*exp(I*(b*x+a)))*a-3/b^2*d^3*ln(1-I*exp(I*(b*x+a)))*x^2-6*d^3*polylog(3,I*
exp(I*(b*x+a)))/b^4-3/b^4*d^3*a^2*ln(1+I*exp(I*(b*x+a)))-6*I*c*d^2*polylog
(2,-I*exp(I*(b*x+a)))/b^3-6*I*d^3*x*polylog(2,-I*exp(I*(b*x+a)))/b^3+6/b^3
*d^2*c*ln(1+I*exp(I*(b*x+a)))*a-6/b^2*d^2*c*ln(1-I*exp(I*(b*x+a)))*x
```

### 3.248.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 783 vs. 2(133) = 266.

Time = 0.29 (sec) , antiderivative size = 783, normalized size of antiderivative = 4.92

$$\int (c+dx)^3\sec(a+bx)\tan(a+bx)\,dx$$
$$=\frac{2\,b^3d^3x^3+6\,b^3cd^2x^2+6\,b^3c^2dx+2\,b^3c^3+6\,d^3\cos\left(bx+a\right)\mathrm{polylog}(3,i\,\cos\left(bx+a\right)+\sin\left(bx+a\right))-6\,d^3}{}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*c
os(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)
*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3,
 -I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*
x + a) - sin(b*x + a)) - 6*(-I*b*d^3*x - I*b*c*d^2)*cos(b*x + a)*dilog(I*c
os(b*x + a) + sin(b*x + a)) - 6*(-I*b*d^3*x - I*b*c*d^2)*cos(b*x + a)*dilo
g(I*cos(b*x + a) - sin(b*x + a)) - 6*(I*b*d^3*x + I*b*c*d^2)*cos(b*x + a)*
dilog(-I*cos(b*x + a) + sin(b*x + a)) - 6*(I*b*d^3*x + I*b*c*d^2)*cos(b*x
+ a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*(b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I) + 3*(b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a)
+ I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a
)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x
+ 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) +
1) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*
log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x +
 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) +
1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a)
+ I*sin(b*x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)
*log(-cos(b*x + a) - I*sin(b*x + a) + I))/(b^4*cos(b*x + a))
```

### 3.248.6 Sympy [F]

$$\int (c+dx)^3\sec(a+bx)\tan(a+bx)\,dx=\int (c+dx)^3\tan{(a+bx)}\sec{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**3*sec(b*x+a)*tan(b*x+a),x)
```

output
```
Integral((c + d*x)**3*tan(a + b*x)*sec(a + b*x), x)
```

### 3.248.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1770 vs. 2(133) = 266.

Time = 0.44 (sec) , antiderivative size = 1770, normalized size of antiderivative = 11.13

$$\int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(3*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x
+ 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin
(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a
)^2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*c
os(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a)
+ 1))*c^2*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)
 + 1)*b) - 6*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(
2*b*x + 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2
 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b
*x + a)^2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x
 + a) + 1))*a*c*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*
x + 2*a) + 1)*b^2) + 3*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x
 + a)*sin(2*b*x + 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*
x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)
^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*
x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 -
 2*sin(b*x + a) + 1))*a^2*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 +
2*cos(2*b*x + 2*a) + 1)*b^3) + 2*c^3/cos(b*x + a) - 6*a*c^2*d/(b*cos(b*x +
 a)) + 6*a^2*c*d^2/(b^2*cos(b*x + a)) - 2*a^3*d^3/(b^3*cos(b*x + a)) + ...
```

### 3.248.8 Giac [F]

$$\int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx = \int (dx+c)^3 \sec(bx+a)\tan(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*sec(b*x + a)*tan(b*x + a), x)
```

### 3.248.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx = \int \frac{\tan(a+b\,x)\,\left(c+d\,x\right)^3}{\cos\left(a+b\,x\right)}\,dx$$

input
```
int((tan(a + b*x)*(c + d*x)^3)/cos(a + b*x),x)
```

output
```
int((tan(a + b*x)*(c + d*x)^3)/cos(a + b*x), x)
```

# 3.249 $\int (c+dx)^2 \sec(a+bx) \tan(a+bx) \, dx$

3.249.1 Optimal result . . . 1934
3.249.2 Mathematica [A] (verified) . . . 1934
3.249.3 Rubi [A] (verified) . . . 1935
3.249.4 Maple [B] (verified) . . . 1937
3.249.5 Fricas [B] (verification not implemented) . . . 1937
3.249.6 Sympy [F] . . . 1938
3.249.7 Maxima [F] . . . 1938
3.249.8 Giac [F] . . . 1939
3.249.9 Mupad [F(-1)] . . . 1939

## 3.249.1 Optimal result

Integrand size = 20, antiderivative size = 97

$$\int (c+dx)^2 \sec(a+bx) \tan(a+bx) \, dx = \frac{4id(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b^2} - \frac{2id^2 \operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} + \frac{2id^2 \operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^2\sec(a+bx)}{b}$$

output
```
4*I*d*(d*x+c)*arctan(exp(I*(b*x+a)))/b^2-2*I*d^2*polylog(2,-I*exp(I*(b*x+a
)))/b^3+2*I*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3+(d*x+c)^2*sec(b*x+a)/b
```

## 3.249.2 Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.79

$$\int (c+dx)^2 \sec(a+bx)\tan(a+bx)\,dx$$
$$= \frac{-4bcd\operatorname{arctanh}\left(\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)\right) - 4d^2\arctan(\cot(a))\operatorname{arctanh}\left(\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)\right) + \frac{2d^2\csc(a}{}}{}$$

input
```
Integrate[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x],x]
```

output
```
(-4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - 4*d^2*ArcTan[Cot[a]]*Arc
Tanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + (2*d^2*Csc[a]*((b*x - ArcTan[Cot[a]])
*(Log[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcTan[Cot[a
]]))]) + I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - I*PolyLog[2, E^(I*(
b*x - ArcTan[Cot[a]]))]))/Sqrt[Csc[a]^2] + b^2*(c + d*x)^2*Sec[a + b*x])/b
^3
```

### 3.249.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4909, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \tan(a+bx)\sec(a+bx)\,dx$$

$$\Big\downarrow \; 4909$$

$$\frac{(c+dx)^2\sec(a+bx)}{b}-\frac{2d\int (c+dx)\sec(a+bx)dx}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{(c+dx)^2\sec(a+bx)}{b}-\frac{2d\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \; 4669$$

$$\frac{(c+dx)^2\sec(a+bx)}{b}-\frac{2d\left(-\frac{d\int \log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{d\int \log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 2715$$

$$\frac{(c+dx)^2\sec(a+bx)}{b}-\frac{2d\left(\frac{id\int e^{-i(a+bx)}\log\left(1-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{id\int e^{-i(a+bx)}\log\left(1+ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 2838$$

$$\frac{(c+dx)^2\sec(a+bx)}{b}-\frac{2d\left(-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}\right)}{b}$$

input
```
Int[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x],x]
```

output
```
(-2*d*(((-2*I)*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b + (I*d*PolyLog[2, (-I)
*E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2))/b + ((c
 + d*x)^2*Sec[a + b*x])/b
```

### 3.249.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

### 3.249.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 226 vs. 2(86) = 172.

Time = 1.23 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.34

| method | result |
|---|---|
| risch | $\frac{2\,\mathrm{e}^{i(xb+a)}\left(x^{2} d^{2}+2 c d x+c^{2}\right)}{b\left(\mathrm{e}^{2 i(xb+a)}+1\right)}+\frac{4 i d c \arctan\left(\mathrm{e}^{i(xb+a)}\right)}{b^{2}}+\frac{2 d^{2} \ln\left(1+i \mathrm{e}^{i(xb+a)}\right) x}{b^{2}}+\frac{2 d^{2} \ln\left(1+i \mathrm{e}^{i(xb+a)}\right) a}{b^{3}}-\frac{2 d^{2} \ln(1$ |
| derivativedivides | $\frac{\frac{a^{2} d^{2}}{b^{2} \cos(xb+a)}-\frac{2 a c d}{b \cos(xb+a)}-\frac{2 a\, d^{2}\left(\frac{xb+a}{\cos(xb+a)}-\ln(\sec(xb+a)+\tan(xb+a))\right)}{b^{2}}+\frac{c^{2}}{\cos(xb+a)}+\frac{2 c d\left(\frac{xb+a}{\cos(xb+a)}-\ln(\sec(xb+a)+\tan(xb+\right.}{b}}{b}$ |
| default | $\frac{\frac{a^{2} d^{2}}{b^{2} \cos(xb+a)}-\frac{2 a c d}{b \cos(xb+a)}-\frac{2 a\, d^{2}\left(\frac{xb+a}{\cos(xb+a)}-\ln(\sec(xb+a)+\tan(xb+a))\right)}{b^{2}}+\frac{c^{2}}{\cos(xb+a)}+\frac{2 c d\left(\frac{xb+a}{\cos(xb+a)}-\ln(\sec(xb+a)+\tan(xb+\right.}{b}}{b}$ |

input
```
int((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*exp(I*(b*x+a))*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*x+a))+1)+4*I/b^2*d*c*
arctan(exp(I*(b*x+a)))+2/b^2*d^2*ln(1+I*exp(I*(b*x+a)))*x+2/b^3*d^2*ln(1+I
*exp(I*(b*x+a)))*a-2/b^2*d^2*ln(1-I*exp(I*(b*x+a)))*x-2/b^3*d^2*ln(1-I*exp
(I*(b*x+a)))*a-2*I/b^3*d^2*dilog(1+I*exp(I*(b*x+a)))+2*I/b^3*d^2*dilog(1-I
*exp(I*(b*x+a)))-4*I/b^3*d^2*a*arctan(exp(I*(b*x+a)))
```

### 3.249.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 446 vs. 2(79) = 158.

Time = 0.29 (sec) , antiderivative size = 446, normalized size of antiderivative = 4.60

$$\int (c+dx)^2\sec(a+bx)\tan(a+bx)\,dx$$
$$=\frac{b^2d^2x^2+2\,b^2cdx+b^2c^2+i\,d^2\cos\left(bx+a\right)\mathrm{Li}_2(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right))+i\,d^2\cos\left(bx+a\right)\mathrm{Li}_2(i\,\cos\left(b\right.}{}$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

output
```
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*cos(b*x + a)*dilog(I*cos(b*x
+ a) + sin(b*x + a)) + I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x +
 a)) - I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d^2*co
s(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b*c*d - a*d^2)*cos(b*x
 + a)*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a
)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b*d^2*x + a*d^2)*cos(b*x + a)*
log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*lo
g(I*cos(b*x + a) - sin(b*x + a) + 1) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(
-I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-
I*cos(b*x + a) - sin(b*x + a) + 1) - (b*c*d - a*d^2)*cos(b*x + a)*log(-cos
(b*x + a) + I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*
x + a) - I*sin(b*x + a) + I))/(b^3*cos(b*x + a))
```

### 3.249.6 Sympy [F]

$$\int (c+dx)^2 \sec(a+bx)\tan(a+bx)\,dx = \int (c+dx)^2 \tan{(a+bx)}\sec{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**2*sec(b*x+a)*tan(b*x+a),x)
```

output
```
Integral((c + d*x)**2*tan(a + b*x)*sec(a + b*x), x)
```

### 3.249.7 Maxima [F]

$$\int (c+dx)^2 \sec(a+bx)\tan(a+bx)\,dx = \int (dx+c)^2 \sec{(bx+a)}\tan{(bx+a)}\;dx$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

output
```
(2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)*cos(b*x + a) + 2*(b*d^
2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*a)*sin(b*x + a) + 2*(b*d^2*x^2 +
2*b*c*d*x + b*c^2)*cos(b*x + a) - 4*(b^2*d^2*cos(2*b*x + 2*a)^2 + b^2*d^2*
sin(2*b*x + 2*a)^2 + 2*b^2*d^2*cos(2*b*x + 2*a) + b^2*d^2)*integrate((x*co
s(2*b*x + 2*a)*cos(b*x + a) + x*sin(2*b*x + 2*a)*sin(b*x + a) + x*cos(b*x
+ a))/(b*cos(2*b*x + 2*a)^2 + b*sin(2*b*x + 2*a)^2 + 2*b*cos(2*b*x + 2*a)
+ b), x) - (c*d*cos(2*b*x + 2*a)^2 + c*d*sin(2*b*x + 2*a)^2 + 2*c*d*cos(2*
b*x + 2*a) + c*d)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1
) + (c*d*cos(2*b*x + 2*a)^2 + c*d*sin(2*b*x + 2*a)^2 + 2*c*d*cos(2*b*x + 2
*a) + c*d)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1))/(b^2
*cos(2*b*x + 2*a)^2 + b^2*sin(2*b*x + 2*a)^2 + 2*b^2*cos(2*b*x + 2*a) + b^
2)
```

### 3.249.8 Giac [F]

$$\int (c+dx)^2\sec(a+bx)\tan(a+bx)\,dx=\int (dx+c)^2\sec(bx+a)\tan(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*sec(b*x + a)*tan(b*x + a), x)
```

### 3.249.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2\sec(a+bx)\tan(a+bx)\,dx=\int \frac{\tan(a+b\,x)\,\left(c+d\,x\right)^2}{\cos\left(a+b\,x\right)}\,dx$$

input
```
int((tan(a + b*x)*(c + d*x)^2)/cos(a + b*x),x)
```

output
```
int((tan(a + b*x)*(c + d*x)^2)/cos(a + b*x), x)
```

# 3.250 $\int (c+dx)\sec(a+bx)\tan(a+bx)\,dx$


3.250.1 Optimal result . . . 1940
3.250.2 Mathematica [B] (verified) . . . 1940
3.250.3 Rubi [A] (verified) . . . 1941
3.250.4 Maple [B] (verified) . . . 1942
3.250.5 Fricas [B] (verification not implemented) . . . 1942
3.250.6 Sympy [F] . . . 1943
3.250.7 Maxima [B] (verification not implemented) . . . 1943
3.250.8 Giac [B] (verification not implemented) . . . 1944
3.250.9 Mupad [B] (verification not implemented) . . . 1944


## 3.250.1 Optimal result

Integrand size = 18, antiderivative size = 29

$$\int (c+dx)\sec(a+bx)\tan(a+bx)\,dx=-\frac{d\,\mathrm{arctanh}(\sin(a+bx))}{b^2}+\frac{(c+dx)\sec(a+bx)}{b}$$

output
```
-d*arctanh(sin(b*x+a))/b^2+(d*x+c)*sec(b*x+a)/b
```

## 3.250.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 93 vs. 2(29) = 58.

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.21

$$\int (c+dx)\sec(a+bx)\tan(a+bx)\,dx=\frac{d\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}-\frac{d\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)+\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}+\frac{c\sec(a+bx)}{b}+\frac{dx\sec(a+bx)}{b}$$

input
```
Integrate[(c + d*x)*Sec[a + b*x]*Tan[a + b*x],x]
```

output
```
(d*Log[Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2]])/b^2 - (d*Log[Cos[a/2 + (b
*x)/2] + Sin[a/2 + (b*x)/2]])/b^2 + (c*Sec[a + b*x])/b + (d*x*Sec[a + b*x]
)/b
```

## 3.250.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4909, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\tan(a+bx)\sec(a+bx)\,dx$$

$$\Big\downarrow \text{ 4909}$$

$$\frac{(c+dx)\sec(a+bx)}{b}-\frac{d\int \sec(a+bx)dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(c+dx)\sec(a+bx)}{b}-\frac{d\int \csc\left(a+bx+\frac{\pi}{2}\right)dx}{b}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{(c+dx)\sec(a+bx)}{b}-\frac{d\text{arctanh}(\sin(a+bx))}{b^2}$$

input
```
Int[(c + d*x)*Sec[a + b*x]*Tan[a + b*x],x]
```

output
```
-((d*ArcTanh[Sin[a + b*x]])/b^2) + ((c + d*x)*Sec[a + b*x])/b
```

### 3.250.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

### 3.250.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(29) = 58$.

Time = 0.62 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.31

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\frac{da}{b\cos(xb+a)}+\frac{c}{\cos(xb+a)}+\frac{d\left(\frac{xb+a}{\cos(xb+a)}-\ln(\sec(xb+a)+\tan(xb+a))\right)}{b}}{b}$ | 67 |
| default | $\frac{-\frac{da}{b\cos(xb+a)}+\frac{c}{\cos(xb+a)}+\frac{d\left(\frac{xb+a}{\cos(xb+a)}-\ln(\sec(xb+a)+\tan(xb+a))\right)}{b}}{b}$ | 67 |
| risch | $\frac{2\,\mathrm{e}^{i(xb+a)}(dx+c)}{b\left(\mathrm{e}^{2i(xb+a)}+1\right)}-\frac{d\ln\left(i+\mathrm{e}^{i(xb+a)}\right)}{b^2}+\frac{d\ln\left(\mathrm{e}^{i(xb+a)}-i\right)}{b^2}$ | 71 |

input
```
int((d*x+c)*sec(b*x+a)*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/b*d*a/cos(b*x+a)+1/cos(b*x+a)*c+1/b*d*((b*x+a)/cos(b*x+a)-ln(sec(b
*x+a)+tan(b*x+a))))
```

### 3.250.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(29) = 58$.

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.07

$$\int (c+dx)\sec(a+bx)\tan(a+bx)\,dx$$
$$=\frac{2\,bdx-d\cos(bx+a)\log(\sin(bx+a)+1)+d\cos(bx+a)\log(-\sin(bx+a)+1)+2\,bc}{2\,b^2\cos(bx+a)}$$

input
```
integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(2*b*d*x - d*cos(b*x + a)*log(sin(b*x + a) + 1) + d*cos(b*x + a)*log(-
sin(b*x + a) + 1) + 2*b*c)/(b^2*cos(b*x + a))
```

### 3.250.6 Sympy [F]

$$\int (c+dx)\sec(a+bx)\tan(a+bx)\,dx = \int (c+dx)\tan(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x)
```

output
```
Integral((c + d*x)*tan(a + b*x)*sec(a + b*x), x)
```

### 3.250.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(29) = 58$.

Time = 0.33 (sec) , antiderivative size = 259, normalized size of antiderivative = 8.93

$$\int (c+dx)\sec(a+bx)\tan(a+bx)\,dx$$
$$= \frac{\frac{\left(4\,(bx+a)\cos(2\,bx+2\,a)\cos(bx+a)+4\,(bx+a)\sin(2\,bx+2\,a)\sin(bx+a)+4\,(bx+a)\cos(bx+a)-\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\,a)+\cdots\right.\right.}{\left(\cos(2\,bx+2\,a)^2+\sin(\cdots\right.}}{\cdots}$$

input
```
integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

output
```
1/2*((4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x +
2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^
2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos
(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) +
1))*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*
b) + 2*c/cos(b*x + a) - 2*a*d/(b*cos(b*x + a)))/b
```

### 3.250.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1267 vs. $2(29) = 58$.

Time = 0.61 (sec) , antiderivative size = 1267, normalized size of antiderivative = 43.69

$$\int (c+dx)\sec(a+bx)\tan(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

output
```
1/2*(2*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b*c*tan(1/2*b*x)^2*tan(1/2*a)
^2 + d*log(2*(tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2*tan(1/2*a) +
2*tan(1/2*b*x)*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*
x) - 2*tan(1/2*a) + 1)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan
(1/2*a)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a)^2 - d*log(2*(tan(1/2*b*x)^2*tan(
1/2*a)^2 - 2*tan(1/2*b*x)^2*tan(1/2*a) - 2*tan(1/2*b*x)*tan(1/2*a)^2 + tan
(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*b
*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2*ta
n(1/2*a)^2 + 2*b*d*x*tan(1/2*b*x)^2 + 2*b*d*x*tan(1/2*a)^2 + 2*b*c*tan(1/2
*b*x)^2 - d*log(2*(tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2*tan(1/2*
a) + 2*tan(1/2*b*x)*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1
/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2
+ tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2 + d*log(2*(tan(1/2*b*x)^2*tan(1/2*a)^2
 - 2*tan(1/2*b*x)^2*tan(1/2*a) - 2*tan(1/2*b*x)*tan(1/2*a)^2 + tan(1/2*b*x
)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*b*x)^2*ta
n(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2 - 4*d*log(
2*(tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2*tan(1/2*a) + 2*tan(1/2*b
*x)*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(
1/2*a) + 1)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 +
 1))*tan(1/2*b*x)*tan(1/2*a) + 4*d*log(2*(tan(1/2*b*x)^2*tan(1/2*a)^2 -...
```

### 3.250.9 Mupad [B] (verification not implemented)

Time = 27.60 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.69

$$\int (c+dx)\sec(a+bx)\tan(a+bx)\,dx = \frac{d\,\ln\left(\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}-\mathrm{i}\right)}{b^2} - \frac{d\,\ln\left(\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}+1\mathrm{i}\right)}{b^2} + \frac{2\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(c+d\,x\right)}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}$$

input
```
int((tan(a + b*x)*(c + d*x))/cos(a + b*x),x)
```

output
```
(d*log(exp(a*1i + b*x*1i) - 1i))/b^2 - (d*log(exp(a*1i + b*x*1i) + 1i))/b^
2 + (2*exp(a*1i + b*x*1i)*(c + d*x))/(b*(exp(a*2i + b*x*2i) + 1))
```

3.250. $\int (c + dx) \sec(a + bx) \tan(a + bx) \, dx$

# 3.251 $\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx$

3.251.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1946
3.251.2 Mathematica [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1946
3.251.3 Rubi [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1947
3.251.4 Maple [N/A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1947
3.251.5 Fricas [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1948
3.251.6 Sympy [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1948
3.251.7 Maxima [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1948
3.251.8 Giac [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1949
3.251.9 Mupad [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1949

## 3.251.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\sec(a+bx)\tan(a+bx)}{c+dx},x\right)$$

output
```
CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

## 3.251.2 Mathematica [N/A]

Not integrable

Time = 12.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x),x]
```

output
```
Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x), x]
```

## 3.251.3 Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(a+bx)\sec(a+bx)}{c+dx}\,dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{\tan(a+bx)\sec(a+bx)}{c+dx}dx$$

input
```
Int[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x),x]
```

output
```
$Aborted
```

### 3.251.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.251.4 Maple [N/A] (verified)

Not integrable

Time = 0.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sec(xb+a)\tan(xb+a)}{dx+c}dx$$

input
```
int(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

output
```
int(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

### 3.251.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)\tan(bx+a)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)*tan(b*x + a)/(d*x + c), x)
```

### 3.251.6 Sympy [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\tan(a+bx)\sec(a+bx)}{c+dx}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

output
```
Integral(tan(a + b*x)*sec(a + b*x)/(c + d*x), x)
```

### 3.251.7 Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 349, normalized size of antiderivative = 17.45

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)\tan(bx+a)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
2*(cos(2*b*x + 2*a)*cos(b*x + a) + (b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*co
s(2*b*x + 2*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x + b*c
*d)*cos(2*b*x + 2*a))*integrate((cos(2*b*x + 2*a)*cos(b*x + a) + sin(2*b*x
 + 2*a)*sin(b*x + a) + cos(b*x + a))/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d
^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x +
b*c^2)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x +
2*a)), x) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))/(b*d*x + (b*d*x
+ b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*b*x + 2*a)^2 + b*c + 2*(b*
d*x + b*c)*cos(2*b*x + 2*a))
```

## 3.251.8 Giac [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec\left(bx+a\right)\tan\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)*tan(b*x + a)/(d*x + c), x)
```

## 3.251.9 Mupad [N/A]

Not integrable

Time = 25.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\tan(a+b\,x)}{\cos\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

input
```
int(tan(a + b*x)/(cos(a + b*x)*(c + d*x)),x)
```

output
```
int(tan(a + b*x)/(cos(a + b*x)*(c + d*x)), x)
```

# 3.252 $\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$


3.252.1 Optimal result . . . . . . . . . . 1950
3.252.2 Mathematica [N/A] . . . . . . . . . . 1950
3.252.3 Rubi [N/A] . . . . . . . . . . 1951
3.252.4 Maple [N/A] (verified) . . . . . . . . . . 1951
3.252.5 Fricas [N/A] . . . . . . . . . . 1952
3.252.6 Sympy [N/A] . . . . . . . . . . 1952
3.252.7 Maxima [N/A] . . . . . . . . . . 1952
3.252.8 Giac [N/A] . . . . . . . . . . 1953
3.252.9 Mupad [N/A] . . . . . . . . . . 1953


## 3.252.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2},x\right)$$

output
```
CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)
```

## 3.252.2 Mathematica [N/A]

Not integrable

Time = 19.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

### 3.252.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(a+bx)\sec(a+bx)}{(c+dx)^2}dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{\tan(a+bx)\sec(a+bx)}{(c+dx)^2}dx$$

input
```
Int[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

#### 3.252.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

### 3.252.4 Maple [N/A] (verified)

Not integrable

Time = 0.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sec(xb+a)\tan(xb+a)}{(dx+c)^2}dx$$

input
```
int(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)
```

output
```
int(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)
```

### 3.252.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec{(bx+a)}\tan{(bx+a)}}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)*tan(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.252.6 Sympy [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan{(a+bx)}\sec{(a+bx)}}{(c+dx)^2}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(tan(a + b*x)*sec(a + b*x)/(c + d*x)**2, x)
```

### 3.252.7 Maxima [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 494, normalized size of antiderivative = 24.70

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec{(bx+a)}\tan{(bx+a)}}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
2*(cos(2*b*x + 2*a)*cos(b*x + a) + 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d +
(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a)^2 + (b*d^3*x^2 + 2*b*
c*d^2*x + b*c^2*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2
*d)*cos(2*b*x + 2*a))*integrate((cos(2*b*x + 2*a)*cos(b*x + a) + sin(2*b*x
 + 2*a)*sin(b*x + a) + cos(b*x + a))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*
d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(2*b*x
+ 2*a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(2*b*x + 2
*a)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(2*b*x + 2*
a)), x) + sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))/(b*d^2*x^2 + 2*b*c
*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2
*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x +
b*c^2)*cos(2*b*x + 2*a))
```

### 3.252.8 Giac [N/A]

Not integrable

Time = 9.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec\left(bx+a\right)\tan\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)*tan(b*x + a)/(d*x + c)^2, x)
```

### 3.252.9 Mupad [N/A]

Not integrable

Time = 25.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(a+b\,x)}{\cos\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(tan(a + b*x)/(cos(a + b*x)*(c + d*x)^2),x)
```

output
```
int(tan(a + b*x)/(cos(a + b*x)*(c + d*x)^2), x)
```

# 3.253 $\int (c+dx)^m \tan^2(a+bx)\,dx$

3.253.1 Optimal result . . . . . 1954
3.253.2 Mathematica [N/A] . . . . . 1954
3.253.3 Rubi [N/A] . . . . . 1955
3.253.4 Maple [N/A] (verified) . . . . . 1956
3.253.5 Fricas [N/A] . . . . . 1956
3.253.6 Sympy [N/A] . . . . . 1956
3.253.7 Maxima [N/A] . . . . . 1957
3.253.8 Giac [N/A] . . . . . 1957
3.253.9 Mupad [N/A] . . . . . 1957

## 3.253.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (c+dx)^m \tan^2(a+bx)\,dx = \mathrm{Int}\big((c+dx)^m \tan^2(a+bx),x\big)$$

output
```
Unintegrable((d*x+c)^m*tan(b*x+a)^2,x)
```

## 3.253.2 Mathematica [N/A]

Not integrable

Time = 3.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \tan^2(a+bx)\,dx = \int (c+dx)^m \tan^2(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Tan[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^m*Tan[a + b*x]^2, x]
```

### 3.253.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(a+bx)^2(c+dx)^m dx$$

$$\Big\downarrow \text{4222}$$

$$\int \tan^2(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Tan[a + b*x]^2,x]
```

output
```
$Aborted
```

#### 3.253.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

### 3.253.4 Maple [N/A] (verified)

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \tan(xb+a)^2 \, dx$$

input
```
int((d*x+c)^m*tan(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*tan(b*x+a)^2,x)
```

### 3.253.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \tan^2(a+bx) \, dx = \int (dx+c)^m \tan(bx+a)^2 \, dx$$

input
```
integrate((d*x+c)^m*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*tan(b*x + a)^2, x)
```

### 3.253.6 Sympy [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (c+dx)^m \tan^2(a+bx) \, dx = \int (c+dx)^m \tan^2(a+bx) \, dx$$

input
```
integrate((d*x+c)**m*tan(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*tan(a + b*x)**2, x)
```

### 3.253.7 Maxima [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \tan^2(a+bx)\,dx = \int (dx+c)^m \tan(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*tan(b*x + a)^2, x)
```

### 3.253.8 Giac [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \tan^2(a+bx)\,dx = \int (dx+c)^m \tan(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*tan(b*x + a)^2, x)
```

### 3.253.9 Mupad [N/A]

Not integrable

Time = 24.71 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \tan^2(a+bx)\,dx = \int \tan(a+b\,x)^2\,(c+d\,x)^m\,dx$$

input
```
int(tan(a + b*x)^2*(c + d*x)^m,x)
```

output
```
int(tan(a + b*x)^2*(c + d*x)^m, x)
```

# 3.254 $\int (c+dx)^3 \tan^2(a+bx)\,dx$


3.254.1 Optimal result . . . . . 1958
3.254.2 Mathematica [B] (verified) . . . . . 1959
3.254.3 Rubi [A] (verified) . . . . . 1960
3.254.4 Maple [B] (verified) . . . . . 1963
3.254.5 Fricas [B] (verification not implemented) . . . . . 1963
3.254.6 Sympy [F] . . . . . 1964
3.254.7 Maxima [B] (verification not implemented) . . . . . 1964
3.254.8 Giac [F] . . . . . 1965
3.254.9 Mupad [F(-1)] . . . . . 1966


## 3.254.1 Optimal result

Integrand size = 16, antiderivative size = 128

$$\int (c+dx)^3 \tan^2(a+bx)\,dx = -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2 \log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3} + \frac{3d^3\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^4} + \frac{(c+dx)^3\tan(a+bx)}{b}$$

output

```
-I*(d*x+c)^3/b-1/4*(d*x+c)^4/d+3*d*(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b^2-3*
I*d^2*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^3+3/2*d^3*polylog(3,-exp(2*I*
(b*x+a)))/b^4+(d*x+c)^3*tan(b*x+a)/b
```

### 3.254.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 424 vs. 2(128) = 256.

Time = 6.54 (sec) , antiderivative size = 424, normalized size of antiderivative = 3.31

$$\int (c+dx)^3\tan^2(a+bx)\,dx = -\frac{1}{4}x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)$$
$$+\frac{id^3e^{-ia}\left(2b^2x^2\left(2bx-3i(1+e^{2ia})\log\left(1+e^{-2i(a+bx)}\right)\right)+6b(1+e^{2ia})\,x\,\mathrm{PolyLog}\left(2,-e^{-2i(a+bx)}\right)-3i(1+\right.}{4b^4}$$
$$+\frac{3c^2d\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))}{b^2\left(\cos^2(a)+\sin^2(a)\right)}$$
$$+\frac{3cd^2\csc(a)\left(b^2e^{-i\arctan(\cot(a))}x^2-\frac{\cot(a)(ibx(-\pi-2\arctan(\cot(a)))-\pi\log(1+e^{-2ibx})-2(bx-\arctan(\cot(a)))\log(1-e^{2i(bx-\mathrm{arcta}}}{}\right.}{b^3\sqrt{\csc^2(a)\left(\cos^2(a)+\right.}}$$
$$+\frac{\sec(a)\sec(a+bx)\left(c^3\sin(bx)+3c^2dx\sin(bx)+3cd^2x^2\sin(bx)+d^3x^3\sin(bx)\right)}{b}$$

input
```
Integrate[(c + d*x)^3*Tan[a + b*x]^2,x]
```

output
```
-1/4*(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)) + ((I/4)*d^3*(2*b^2*x
^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(
1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (3*I)*(1 + E^((2*I)
*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^4*E^(I*a)) + (3*c^2*d*S
ec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^2*(
Cos[a]^2 + Sin[a]^2)) + (3*c*d^2*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) -
(Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(
b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[C
os[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2,
E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^3*Sqrt[
Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Sec[a]*Sec[a + b*x]*(c^3*Sin[b*x] + 3*
c^2*d*x*Sin[b*x] + 3*c*d^2*x^2*Sin[b*x] + d^3*x^3*Sin[b*x]))/b
```

### 3.254.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {3042, 4203, 17, 3042, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \tan^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^3 \tan(a+bx)^2dx$$

$$\Big\downarrow \text{ 4203}$$

$$-\frac{3d\int (c+dx)^2\tan(a+bx)dx}{b}-\int (c+dx)^3dx+\frac{(c+dx)^3\tan(a+bx)}{b}$$

$$\Big\downarrow \text{ 17}$$

$$-\frac{3d\int (c+dx)^2\tan(a+bx)dx}{b}+\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{(c+dx)^4}{4d}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3d\int (c+dx)^2\tan(a+bx)dx}{b}+\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{(c+dx)^4}{4d}$$

$$\Big\downarrow \text{ 4202}$$

$$-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}dx\right)}{b}+\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{(c+dx)^4}{4d}$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\int (c+dx)\log\left(1+e^{2i(a+bx)}\right)dx}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}+\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{(c+dx)^4}{4d}$$

$$\Big\downarrow \text{ 3011}$$

$$-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}+\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{(c+dx)^4}{4d}$$

$$\Big\downarrow \; 2720$$

$$-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\int e^{-2i(a+bx)}\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}+\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{(c+dx)^4}{4d}$$

$$\Big\downarrow \; 7143$$

$$-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}+\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{(c+dx)^4}{4d}$$

input
```
Int[(c + d*x)^3*Tan[a + b*x]^2,x]
```

output
```
-1/4*(c + d*x)^4/d - (3*d*(((I/3)*(c + d*x)^3)/d - (2*I)*(((-1/2*I)*(c + d
*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b + (I*d*(((I/2)*(c + d*x)*PolyLog[2,
-E^((2*I)*(a + b*x))])/b - (d*PolyLog[3, -E^((2*I)*(a + b*x))])/(4*b^2)))/
b)))/b + ((c + d*x)^3*Tan[a + b*x])/b
```

## 3.254. $\int (c+dx)^3 \tan^2(a+bx)\,dx$

### 3.254.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

## 3.254. $\int (c+dx)^3 \tan^2(a+bx)\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.254.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(116) = 232.

Time = 1.70 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.78

| method | result |
|---|---|
| risch | $-\frac{d^3x^4}{4}-d^2c\,x^3-\frac{3d\,c^2x^2}{2}-c^3x-\frac{c^4}{4d}-\frac{3id^3\operatorname{polylog}\left(2,-\mathrm{e}^{2i(xb+a)}\right)x}{b^3}+\frac{3d^3\ln\left(\mathrm{e}^{2i(xb+a)}+1\right)x^2}{b^2}+\frac{3d^3\operatorname{polylog}\left(3,-\mathrm{e}^{2i\ldots}\right.}{2b^4}$ |

input
```
int((d*x+c)^3*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*d^3*x^4-d^2*c*x^3-3/2*d*c^2*x^2-c^3*x-1/4/d*c^4-3*I*d^3/b^3*polylog(2
,-exp(2*I*(b*x+a)))*x+3*d^3/b^2*ln(exp(2*I*(b*x+a))+1)*x^2+3/2*d^3*polylog
(3,-exp(2*I*(b*x+a)))/b^4+6*d^2/b^2*c*ln(exp(2*I*(b*x+a))+1)*x-6*I*d^2/b*c
*x^2+6*I*d^3/b^3*a^2*x+2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(exp(2*I*
(b*x+a))+1)+12*d^2/b^3*c*a*ln(exp(I*(b*x+a)))-3*I*d^2/b^3*c*polylog(2,-exp
(2*I*(b*x+a)))-6*I*d^2/b^3*c*a^2-6*d^3/b^4*a^2*ln(exp(I*(b*x+a)))+3*d/b^2*
c^2*ln(exp(2*I*(b*x+a))+1)-6*d/b^2*c^2*ln(exp(I*(b*x+a)))-12*I*d^2/b^2*c*x
*a-2*I*d^3/b*x^3+4*I*d^3/b^4*a^3
```

### 3.254.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 373 vs. 2(113) = 226.

Time = 0.26 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.91

$$\int (c+dx)^3\tan^2(a+bx)\,dx =$$

$$-\frac{b^4d^3x^4+4\,b^4cd^2x^3+6\,b^4c^2dx^2+4\,b^4c^3x-3\,d^3\operatorname{polylog}\left(3,\frac{\tan(bx+a)^2+2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)-3\,d^3\operatorname{polylog}\left(3,\frac{\tan\ldots}{\ldots}\right.}{\ldots}$$

input
```
integrate((d*x+c)^3*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 6*b^4*c^2*d*x^2 + 4*b^4*c^3*x - 3*d^
3*polylog(3, (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1))
 - 3*d^3*polylog(3, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^
2 + 1)) + 6*(-I*b*d^3*x - I*b*c*d^2)*dilog(2*(I*tan(b*x + a) - 1)/(tan(b*x
 + a)^2 + 1) + 1) + 6*(I*b*d^3*x + I*b*c*d^2)*dilog(2*(-I*tan(b*x + a) - 1
)/(tan(b*x + a)^2 + 1) + 1) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*
log(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) - 6*(b^2*d^3*x^2 + 2*b^2
*c*d^2*x + b^2*c^2*d)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) -
 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*tan(b*x + a))
/b^4
```

### 3.254.6 Sympy [F]

$$\int (c+dx)^3\tan^2(a+bx)\,dx = \int (c+dx)^3\tan^2{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**3*tan(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**3*tan(a + b*x)**2, x)
```

### 3.254.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1363 vs. 2(113) = 226.

Time = 0.40 (sec) , antiderivative size = 1363, normalized size of antiderivative = 10.65

$$\int (c+dx)^3\tan^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(b*x + a - tan(b*x + a))*c^3 - 6*(b*x + a - tan(b*x + a))*a*c^2*d/
b + 6*(b*x + a - tan(b*x + a))*a^2*c*d^2/b^2 - 2*(b*x + a - tan(b*x + a))*
a^3*d^3/b^3 + 3*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x +
2*a)^2 + 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^
2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 +
sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a
))*c^2*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) +
1)*b) - 6*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2
 + 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + si
n(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*
b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*a*c
*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b
^2) + 3*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 +
 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*
x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*a^2*d
^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b^3
) - 2*(I*(b*x + a)^4*d^3 - 4*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^3 + 12*((b*x
 + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) + ((b*x + a)^2*d^3 + 2*(b*c*d^
2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^3 + 2*(-I*b*...
```

### 3.254.8 Giac [F]

$$\int (c+dx)^3 \tan^2(a+bx)\,dx = \int (dx+c)^3 \tan(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^3*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*tan(b*x + a)^2, x)
```

### 3.254.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \tan^2(a+bx)\,dx = \int \tan(a+b\,x)^2\,(c+d\,x)^3\,dx$$

input
```
int(tan(a + b*x)^2*(c + d*x)^3,x)
```

output
```
int(tan(a + b*x)^2*(c + d*x)^3, x)
```

# 3.255 $\int (c+dx)^2 \tan^2(a+bx)\,dx$

3.255.1 Optimal result . . . . . 1967
3.255.2 Mathematica [B] (verified) . . . . . 1967
3.255.3 Rubi [A] (verified) . . . . . 1968
3.255.4 Maple [B] (verified) . . . . . 1970
3.255.5 Fricas [B] (verification not implemented) . . . . . 1971
3.255.6 Sympy [F] . . . . . 1971
3.255.7 Maxima [B] (verification not implemented) . . . . . 1972
3.255.8 Giac [F] . . . . . 1972
3.255.9 Mupad [F(-1)] . . . . . 1973

## 3.255.1 Optimal result

Integrand size = 16, antiderivative size = 96

$$\int (c+dx)^2 \tan^2(a+bx)\,dx = -\frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d} + \frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{id^2\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)^2\tan(a+bx)}{b}$$

output
```
-I*(d*x+c)^2/b-1/3*(d*x+c)^3/d+2*d*(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b^2-I*d^
2*polylog(2,-exp(2*I*(b*x+a)))/b^3+(d*x+c)^2*tan(b*x+a)/b
```

## 3.255.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 276 vs. 2(96) = 192.

Time = 6.40 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.88

$$\int (c+dx)^2 \tan^2(a+bx)\,dx = -\frac{1}{3}x\left(3c^2+3cdx+d^2x^2\right) + \frac{2cd\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))}{b^2\left(\cos^2(a)+\sin^2(a)\right)} + \frac{d^2\csc(a)\left(b^2e^{-i\arctan(\cot(a))}x^2 - \frac{\cot(a)\left(ibx(-\pi-2\arctan(\cot(a)))-\pi\log\left(1+e^{-2ibx}\right)-2(bx-\arctan(\cot(a)))\log\left(1-e^{2i(bx-\arctan(c\ldots}\right.\right.}{\sqrt{\ldots}}\right.}{b^3\sqrt{\csc^2(a)\left(\cos^2(a)+s\ldots\right.}} + \frac{\sec(a)\sec(a+bx)\left(c^2\sin(bx)+2cdx\sin(bx)+d^2x^2\sin(bx)\right)}{b}$$

input
```
Integrate[(c + d*x)^2*Tan[a + b*x]^2,x]
```

output
```
-1/3*(x*(3*c^2 + 3*c*d*x + d^2*x^2)) + (2*c*d*Sec[a]*(Cos[a]*Log[Cos[a]*Co
s[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^2*(Cos[a]^2 + Sin[a]^2)) + (d^
2*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[
Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E
^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log
[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]])
)]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^3*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]
) + (Sec[a]*Sec[a + b*x]*(c^2*Sin[b*x] + 2*c*d*x*Sin[b*x] + d^2*x^2*Sin[b*
x]))/b
```

### 3.255.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4203, 17, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \tan^2(a+bx) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2 \tan(a+bx)^2dx$$

$$\Big\downarrow \text{4203}$$

$$-\frac{2d\int (c+dx)\tan(a+bx)dx}{b}-\int (c+dx)^2dx+\frac{(c+dx)^2\tan(a+bx)}{b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{2d\int (c+dx)\tan(a+bx)dx}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2d\int (c+dx)\tan(a+bx)dx}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

$$\Big\downarrow \text{4202}$$

$$-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}dx\right)}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

$\big\downarrow$ 2620

$$-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{id\int \log\left(1+e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

$\big\downarrow$ 2715

$$-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{d\int e^{-2i(a+bx)}\log\left(1+e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

$\big\downarrow$ 2838

$$-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}$$

input
```
Int[(c + d*x)^2*Tan[a + b*x]^2,x]
```

output
```
-1/3*(c + d*x)^3/d - (2*d*(((I/2)*(c + d*x)^2)/d - (2*I)*(((-1/2*I)*(c + d
*x)*Log[1 + E^((2*I)*(a + b*x))])/b - (d*PolyLog[2, -E^((2*I)*(a + b*x))])
/(4*b^2))))/b + ((c + d*x)^2*Tan[a + b*x])/b
```

### 3.255.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

### 3.255.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(88) = 176.

Time = 1.58 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.07

| method | result |
|---|---|
| risch | $-\frac{d^2x^3}{3} - dc\,x^2 - c^2x - \frac{c^3}{3d} + \frac{2i\left(x^2d^2+2cdx+c^2\right)}{b\left(\mathrm{e}^{2i(xb+a)}+1\right)} + \frac{2dc\ln\left(\mathrm{e}^{2i(xb+a)}+1\right)}{b^2} - \frac{4dc\ln\left(\mathrm{e}^{i(xb+a)}\right)}{b^2} - \frac{2id^2x^2}{b} - \frac{4id^2xa}{b^2} -$ |

input
```
int((d*x+c)^2*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*d^2*x^3-d*c*x^2-c^2*x-1/3/d*c^3+2*I*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*
(b*x+a))+1)+2*d/b^2*c*ln(exp(2*I*(b*x+a))+1)-4*d/b^2*c*ln(exp(I*(b*x+a)))-
2*I*d^2/b*x^2-4*I*d^2/b^2*x*a-2*I*d^2/b^3*a^2+2*d^2/b^2*ln(exp(2*I*(b*x+a)
)+1)*x-I*d^2*polylog(2,-exp(2*I*(b*x+a)))/b^3+4*d^2/b^3*a*ln(exp(I*(b*x+a)
))
```

### 3.255.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(85) = 170.

Time = 0.24 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.19

$$\int (c+dx)^2\tan^2(a+bx)\,dx = -\frac{2\,b^3d^2x^3+6\,b^3cdx^2+6\,b^3c^2x-3i\,d^2\mathrm{Li}_2\left(\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)+3i\,d^2\mathrm{Li}_2\left(\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)-6\,(bd^2x}{}$$

input
```
integrate((d*x+c)^2*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/6*(2*b^3*d^2*x^3 + 6*b^3*c*d*x^2 + 6*b^3*c^2*x - 3*I*d^2*dilog(2*(I*tan
(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) + 3*I*d^2*dilog(2*(-I*tan(b*x + a
) - 1)/(tan(b*x + a)^2 + 1) + 1) - 6*(b*d^2*x + b*c*d)*log(-2*(I*tan(b*x +
 a) - 1)/(tan(b*x + a)^2 + 1)) - 6*(b*d^2*x + b*c*d)*log(-2*(-I*tan(b*x +
a) - 1)/(tan(b*x + a)^2 + 1)) - 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*ta
n(b*x + a))/b^3
```

### 3.255.6 Sympy [F]

$$\int (c+dx)^2\tan^2(a+bx)\,dx = \int (c+dx)^2\tan^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*tan(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*tan(a + b*x)**2, x)
```

### 3.255.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(85) = 170.

Time = 0.39 (sec) , antiderivative size = 418, normalized size of antiderivative = 4.35

$$\int (c+dx)^2\tan^2(a+bx)\,dx$$
$$=\frac{i\,b^3d^2x^3+3i\,b^3cdx^2+3i\,b^3c^2x+6\,b^2c^2+6\,(bd^2x+bcd+(bd^2x+bcd)\cos(2\,bx+2\,a)-(-i\,bd^2x-i\,bcd)}{}$$

input
```
integrate((d*x+c)^2*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
(I*b^3*d^2*x^3 + 3*I*b^3*c*d*x^2 + 3*I*b^3*c^2*x + 6*b^2*c^2 + 6*(b*d^2*x
+ b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a) - (-I*b*d^2*x - I*b*c*d)*sin(
2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + (I*b^3*d^2
*x^3 - 3*(-I*b^3*c*d + 2*b^2*d^2)*x^2 - 3*(-I*b^3*c^2 + 4*b^2*c*d)*x)*cos(
2*b*x + 2*a) - 3*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) + d^2)*dil
og(-e^(2*I*b*x + 2*I*a)) - 3*(I*b*d^2*x + I*b*c*d + (I*b*d^2*x + I*b*c*d)*
cos(2*b*x + 2*a) - (b*d^2*x + b*c*d)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a
)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - (b^3*d^2*x^3 + 3*(b^3
*c*d + 2*I*b^2*d^2)*x^2 + 3*(b^3*c^2 + 4*I*b^2*c*d)*x)*sin(2*b*x + 2*a))/(
-3*I*b^3*cos(2*b*x + 2*a) + 3*b^3*sin(2*b*x + 2*a) - 3*I*b^3)
```

### 3.255.8 Giac [F]

$$\int (c+dx)^2\tan^2(a+bx)\,dx=\int (dx+c)^2\tan(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^2*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*tan(b*x + a)^2, x)
```

### 3.255.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \tan^2(a+bx)\,dx = \int \tan(a+b\,x)^2\,(c+d\,x)^2\,dx$$

input
```
int(tan(a + b*x)^2*(c + d*x)^2,x)
```

output
```
int(tan(a + b*x)^2*(c + d*x)^2, x)
```

# 3.256 $\int (c+dx)\tan^2(a+bx)\,dx$

3.256.1 Optimal result . . . . . 1974
3.256.2 Mathematica [A] (verified) . . . . . 1974
3.256.3 Rubi [A] (verified) . . . . . 1975
3.256.4 Maple [A] (verified) . . . . . 1976
3.256.5 Fricas [A] (verification not implemented) . . . . . 1977
3.256.6 Sympy [A] (verification not implemented) . . . . . 1977
3.256.7 Maxima [B] (verification not implemented) . . . . . 1978
3.256.8 Giac [B] (verification not implemented) . . . . . 1978
3.256.9 Mupad [B] (verification not implemented) . . . . . 1979

## 3.256.1 Optimal result

Integrand size = 14, antiderivative size = 40

$$\int (c+dx)\tan^2(a+bx)\,dx=-cx-\frac{dx^2}{2}+\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}$$

output
```
-c*x-1/2*d*x^2+d*ln(cos(b*x+a))/b^2+(d*x+c)*tan(b*x+a)/b
```

## 3.256.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.90

$$\int (c+dx)\tan^2(a+bx)\,dx=-\frac{c\arctan(\tan(a+bx))}{b}+\frac{d\log(\cos(a+bx))}{b^2}-\frac{dx\sec(a)(bx\cos(a)-2\sin(a))}{2b}+\frac{dx\sec(a)\sec(a+bx)\sin(bx)}{b}+\frac{c\tan(a+bx)}{b}$$

input
```
Integrate[(c + d*x)*Tan[a + b*x]^2,x]
```

output
```
-((c*ArcTan[Tan[a + b*x]])/b) + (d*Log[Cos[a + b*x]])/b^2 - (d*x*Sec[a]*(b
*x*Cos[a] - 2*Sin[a]))/(2*b) + (d*x*Sec[a]*Sec[a + b*x]*Sin[b*x])/b + (c*T
an[a + b*x])/b
```

### 3.256.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4203, 17, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\tan^2(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (c+dx)\tan(a+bx)^2dx$$

$$\Big\downarrow \; 4203$$

$$-\frac{d\int \tan(a+bx)dx}{b}-\int (c+dx)dx+\frac{(c+dx)\tan(a+bx)}{b}$$

$$\Big\downarrow \; 17$$

$$-\frac{d\int \tan(a+bx)dx}{b}+\frac{(c+dx)\tan(a+bx)}{b}-\frac{(c+dx)^2}{2d}$$

$$\Big\downarrow \; 3042$$

$$-\frac{d\int \tan(a+bx)dx}{b}+\frac{(c+dx)\tan(a+bx)}{b}-\frac{(c+dx)^2}{2d}$$

$$\Big\downarrow \; 3956$$

$$\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}-\frac{(c+dx)^2}{2d}$$

input
```
Int[(c + d*x)*Tan[a + b*x]^2,x]
```

output
```
-1/2*(c + d*x)^2/d + (d*Log[Cos[a + b*x]])/b^2 + ((c + d*x)*Tan[a + b*x])/
b
```

### 3.256.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

## 3.256.4 Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.30

| method | result | size |
|---|---|---|
| norman | $\frac{c\tan(xb+a)}{b}+\frac{dx\tan(xb+a)}{b}-cx-\frac{d\,x^{2}}{2}-\frac{d\ln\left(1+\tan(xb+a)^{2}\right)}{2b^{2}}$ | 52 |
| parallelrisch | $-\frac{d\,x^{2}b^{2}-2d\tan(xb+a)xb+2cx\,b^{2}-2c\tan(xb+a)b+d\ln\left(1+\tan(xb+a)^{2}\right)}{2b^{2}}$ | 56 |
| default | $-\frac{d\,x^{2}}{2}-cx+\frac{-\frac{da\tan(xb+a)}{b}+c\tan(xb+a)+\frac{d((xb+a)\tan(xb+a)+\ln(\cos(xb+a)))}{b}}{b}$ | 63 |
| risch | $-\frac{d\,x^{2}}{2}-cx-\frac{2idx}{b}-\frac{2ida}{b^{2}}+\frac{2i(dx+c)}{b\left(e^{2i(xb+a)}+1\right)}+\frac{d\ln\left(e^{2i(xb+a)}+1\right)}{b^{2}}$ | 69 |

input
```
int((d*x+c)*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
c/b*tan(b*x+a)+d*x/b*tan(b*x+a)-c*x-1/2*d*x^2-1/2*d/b^2*ln(1+tan(b*x+a)^2)
```

### 3.256.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.32

$$\int (c+dx)\tan^2(a+bx)\,dx = -\frac{b^2dx^2 + 2\,b^2cx - d\log\left(\frac{1}{\tan(bx+a)^2+1}\right) - 2\,(bdx+bc)\tan(bx+a)}{2\,b^2}$$

input
```
integrate((d*x+c)*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(b^2*d*x^2 + 2*b^2*c*x - d*log(1/(tan(b*x + a)^2 + 1)) - 2*(b*d*x + b
*c)*tan(b*x + a))/b^2
```

### 3.256.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.62

$$\int (c+dx)\tan^2(a+bx)\,dx = \begin{cases} -cx - \frac{dx^2}{2} + \frac{c\tan(a+bx)}{b} + \frac{dx\tan(a+bx)}{b} - \frac{d\log(\tan^2(a+bx)+1)}{2b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\tan^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*tan(b*x+a)**2,x)
```

output
```
Piecewise((-c*x - d*x**2/2 + c*tan(a + b*x)/b + d*x*tan(a + b*x)/b - d*log
(tan(a + b*x)**2 + 1)/(2*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*tan(a)**2, Tr
ue))
```

### 3.256.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(38) = 76$.

Time = 0.31 (sec) , antiderivative size = 237, normalized size of antiderivative = 5.92

$$\int (c+dx)\tan^2(a+bx)\,dx =$$

$$-\frac{2\,(bx+a-\tan(bx+a))c-\frac{2\,(bx+a-\tan(bx+a))ad}{b}+\frac{\left((bx+a)^2\cos(2\,bx+2\,a)^2+(bx+a)^2\sin(2\,bx+2\,a)^2+2\,(bx+a)^2\cos(2\,bx+2}{}}{}$$

input
```
integrate((d*x+c)*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(b*x + a - tan(b*x + a))*c - 2*(b*x + a - tan(b*x + a))*a*d/b + ((
b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 + 2*(b*x +
a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*
a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
 + 2*cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*d/((cos(2*b*x +
 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b))/b
```

### 3.256.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(38) = 76$.

Time = 0.45 (sec) , antiderivative size = 203, normalized size of antiderivative = 5.08

$$\int (c+dx)\tan^2(a+bx)\,dx =$$

$$-\frac{b^2dx^2\tan(bx)\tan(a)+2\,b^2cx\tan(bx)\tan(a)-b^2dx^2-2\,b^2cx+2\,bdx\tan(bx)+2\,bdx\tan(a)-d\log}{2\,(b}$$

input
```
integrate((d*x+c)*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*(b^2*d*x^2*tan(b*x)*tan(a) + 2*b^2*c*x*tan(b*x)*tan(a) - b^2*d*x^2 -
2*b^2*c*x + 2*b*d*x*tan(b*x) + 2*b*d*x*tan(a) - d*log(4*(tan(b*x)^2*tan(a)
^2 - 2*tan(b*x)*tan(a) + 1)/(tan(b*x)^2*tan(a)^2 + tan(b*x)^2 + tan(a)^2 +
 1))*tan(b*x)*tan(a) + 2*b*c*tan(b*x) + 2*b*c*tan(a) + d*log(4*(tan(b*x)^2
*tan(a)^2 - 2*tan(b*x)*tan(a) + 1)/(tan(b*x)^2*tan(a)^2 + tan(b*x)^2 + tan
(a)^2 + 1)))/(b^2*tan(b*x)*tan(a) - b^2)
```

### 3.256.9 Mupad [B] (verification not implemented)

Time = 25.92 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.30

$$\int (c+dx)\tan^2(a+bx)\,dx = -c\,x-\frac{d\,x^2}{2}-\frac{\frac{d\,\ln\left(\tan(a+b\,x)^2+1\right)}{2}-b\,(c\tan(a+b\,x)+d\,x\tan(a+b\,x))}{b^2}$$

input
```
int(tan(a + b*x)^2*(c + d*x),x)
```

output
```
- c*x - (d*x^2)/2 - ((d*log(tan(a + b*x)^2 + 1))/2 - b*(c*tan(a + b*x) + d
*x*tan(a + b*x)))/b^2
```

# 3.257 $\int \frac{\tan^2(a+bx)}{c+dx} \, dx$

3.257.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1980
3.257.2 Mathematica [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . 1980
3.257.3 Rubi [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1981
3.257.4 Maple [N/A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 1982
3.257.5 Fricas [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1982
3.257.6 Sympy [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1982
3.257.7 Maxima [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1983
3.257.8 Giac [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1983
3.257.9 Mupad [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1984

## 3.257.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\tan^2(a+bx)}{c+dx} \, dx = \operatorname{Int}\left(\frac{\tan^2(a+bx)}{c+dx}, x\right)$$

output
```
Unintegrable(tan(b*x+a)^2/(d*x+c),x)
```

## 3.257.2 Mathematica [N/A]

Not integrable

Time = 5.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^2(a+bx)}{c+dx} \, dx = \int \frac{\tan^2(a+bx)}{c+dx} \, dx$$

input
```
Integrate[Tan[a + b*x]^2/(c + d*x),x]
```

output
```
Integrate[Tan[a + b*x]^2/(c + d*x), x]
```

### 3.257.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(a+bx)}{c+dx} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(a+bx)^2}{c+dx}dx$$
$$\Big\downarrow \text{4222}$$
$$\int \frac{\tan^2(a+bx)}{c+dx}dx$$

input
```
Int[Tan[a + b*x]^2/(c + d*x),x]
```

output
```
$Aborted
```

#### 3.257.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

### 3.257.4 Maple [N/A] (verified)

Not integrable

Time = 0.67 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\tan (xb+a)^2}{dx+c}dx$$

input
```
int(tan(b*x+a)^2/(d*x+c),x)
```

output
```
int(tan(b*x+a)^2/(d*x+c),x)
```

### 3.257.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\tan (bx+a)^2}{dx+c}\,dx$$

input
```
integrate(tan(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(tan(b*x + a)^2/(d*x + c), x)
```

### 3.257.6 Sympy [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\tan^2(a+bx)}{c+dx}\,dx$$

input
```
integrate(tan(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(tan(a + b*x)**2/(c + d*x), x)
```

### 3.257.7 Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 388, normalized size of antiderivative = 24.25

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\tan(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(tan(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
(2*(b*d^3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a)^2 + (b*d^3*x
+ b*c*d^2)*sin(2*b*x + 2*a)^2 + 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*in
tegrate(sin(2*b*x + 2*a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b
*c*d*x + b*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2
*b*x + 2*a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)), x) -
(b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*b*x + 2*a)
^2 + b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a))*log(d*x + c) + 2*d*sin(2*b*x
+ 2*a))/(b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x
 + b*c*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a))
```

### 3.257.8 Giac [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\tan(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(tan(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(tan(b*x + a)^2/(d*x + c), x)
```

### 3.257.9 Mupad [N/A]

Not integrable

Time = 25.44 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx=\int \frac{\tan(a+b\,x)^2}{c+d\,x}\,dx$$

input
```
int(tan(a + b*x)^2/(c + d*x),x)
```

output
```
int(tan(a + b*x)^2/(c + d*x), x)
```

# 3.258 $\int \frac{\tan^2(a+bx)}{(c+dx)^2} \, dx$

3.258.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1985
3.258.2 Mathematica [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1985
3.258.3 Rubi [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1986
3.258.4 Maple [N/A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1987
3.258.5 Fricas [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1987
3.258.6 Sympy [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1987
3.258.7 Maxima [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1988
3.258.8 Giac [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1988
3.258.9 Mupad [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1989

## 3.258.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2} \, dx = \mathrm{Int}\left(\frac{\tan^2(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Unintegrable(tan(b*x+a)^2/(d*x+c)^2,x)
```

## 3.258.2 Mathematica [N/A]

Not integrable

Time = 6.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2} \, dx = \int \frac{\tan^2(a+bx)}{(c+dx)^2} \, dx$$

input
```
Integrate[Tan[a + b*x]^2/(c + d*x)^2,x]
```

output
```
Integrate[Tan[a + b*x]^2/(c + d*x)^2, x]
```

### 3.258.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(a+bx)^2}{(c+dx)^2}dx$$
$$\Big\downarrow \text{4222}$$
$$\int \frac{\tan^2(a+bx)}{(c+dx)^2}dx$$

input
```
Int[Tan[a + b*x]^2/(c + d*x)^2,x]
```

output
```
$Aborted
```

#### 3.258.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

### 3.258.4 Maple [N/A] (verified)

Not integrable

Time = 0.67 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\tan (xb+a)^2}{(dx+c)^2}dx$$

input
```
int(tan(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(tan(b*x+a)^2/(d*x+c)^2,x)
```

### 3.258.5 Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2} \, dx = \int \frac{\tan (bx+a)^2}{(dx+c)^2} \, dx$$

input
```
integrate(tan(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(tan(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.258.6 Sympy [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2} \, dx = \int \frac{\tan^2 (a+bx)}{(c+dx)^2} \, dx$$

input
```
integrate(tan(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(tan(a + b*x)**2/(c + d*x)**2, x)
```

### 3.258.7 Maxima [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 523, normalized size of antiderivative = 32.69

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(tan(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
(b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*b*x + 2*a)
^2 + b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + 4*(b*d^4*x^2 + 2*b*c*d^3*x +
 b*c^2*d^2 + (b*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2)*cos(2*b*x + 2*a)^2 + (b
*d^4*x^2 + 2*b*c*d^3*x + b*c^2*d^2)*sin(2*b*x + 2*a)^2 + 2*(b*d^4*x^2 + 2*
b*c*d^3*x + b*c^2*d^2)*cos(2*b*x + 2*a))*integrate(sin(2*b*x + 2*a)/(b*d^3
*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 +
3*b*c^2*d*x + b*c^3)*cos(2*b*x + 2*a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b
*c^2*d*x + b*c^3)*sin(2*b*x + 2*a)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*
c^2*d*x + b*c^3)*cos(2*b*x + 2*a)), x) + 2*d*sin(2*b*x + 2*a))/(b*d^3*x^2
+ 2*b*c*d^2*x + b*c^2*d + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x +
2*a)^2 + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^3
*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a))
```

### 3.258.8 Giac [N/A]

Not integrable

Time = 7.92 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(tan(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(tan(b*x + a)^2/(d*x + c)^2, x)
```

### 3.258.9 Mupad [N/A]

Not integrable

Time = 25.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(a+b\,x)^2}{(c+d\,x)^2}\,dx$$

input
```
int(tan(a + b*x)^2/(c + d*x)^2,x)
```

output
```
int(tan(a + b*x)^2/(c + d*x)^2, x)
```

# 3.259 $\int (c+dx)^m \sin(a+bx) \tan^2(a+bx)\,dx$

3.259.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1990
3.259.2 Mathematica [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1990
3.259.3 Rubi [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1991
3.259.4 Maple [N/A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 1992
3.259.5 Fricas [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1993
3.259.6 Sympy [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1993
3.259.7 Maxima [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1993
3.259.8 Giac [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1994
3.259.9 Mupad [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 1994

## 3.259.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (c+dx)^m \sin(a+bx) \tan^2(a+bx)\,dx = \frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b} + \frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)}{2b} + \mathrm{Int}((c+dx)^m\sec(a+bx)\tan(a+bx),x)$$

output
```
CannotIntegrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x)+1/2*exp(I*(a-b*c/d))*(d
*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)+1/2*(d*x+c)^m*GAM
MA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

## 3.259.2 Mathematica [N/A]

Not integrable

Time = 29.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sin(a+bx) \tan^2(a+bx)\,dx = \int (c+dx)^m \sin(a+bx) \tan^2(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x]^2, x]
```

### 3.259.3 Rubi [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4907, 3042, 3789, 2612, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(a+bx)\tan^2(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \; 4907$$

$$\int (c+dx)^m\sec(a+bx)\tan(a+bx)dx-\int (c+dx)^m\sin(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$\int (c+dx)^m\sec(a+bx)\tan(a+bx)dx-\int (c+dx)^m\sin(a+bx)dx$$

$$\Big\downarrow \; 3789$$

$$-\frac{1}{2}i\int e^{-i(a+bx)}(c+dx)^mdx+\frac{1}{2}i\int e^{i(a+bx)}(c+dx)^mdx+\int (c+dx)^m\sec(a+bx)\tan(a+bx)dx$$

$$\Big\downarrow \; 2612$$

$$\int (c+dx)^m\sec(a+bx)\tan(a+bx)dx+\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}+\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}$$

$$\Big\downarrow \; 7299$$

$$\int (c+dx)^m\sec(a+bx)\tan(a+bx)dx+\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}+\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}$$

input
```
Int[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
$Aborted
```

### 3.259.3.1 Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.259.4 Maple [N/A] (verified)

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \sin(xb+a)\tan(xb+a)^2 \,dx$$

input
```
int((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x)
```

### 3.259.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx = \int (dx+c)^m \sin(bx+a)\tan(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*sin(b*x + a)*tan(b*x + a)^2, x)
```

### 3.259.6 Sympy [N/A]

Not integrable

Time = 32.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx = \int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*sin(b*x+a)*tan(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*sin(a + b*x)*tan(a + b*x)**2, x)
```

### 3.259.7 Maxima [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx = \int (dx+c)^m \sin(bx+a)\tan(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*sin(b*x + a)*tan(b*x + a)^2, x)
```

### 3.259.8 Giac [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx = \int (dx+c)^m \sin(bx+a)\tan(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sin(b*x + a)*tan(b*x + a)^2, x)
```

### 3.259.9 Mupad [N/A]

Not integrable

Time = 26.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx = \int \sin(a+b\,x)\,\tan(a+b\,x)^2\,(c+d\,x)^m\,dx$$

input
```
int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^m,x)
```

output
```
int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^m, x)
```

# 3.260 $\int (c+dx)^3 \sin(a+bx)\tan^2(a+bx)\,dx$


3.260.1 Optimal result . . . 1995
3.260.2 Mathematica [B] (verified) . . . 1996
3.260.3 Rubi [A] (verified) . . . 1996
3.260.4 Maple [B] (verified) . . . 2001
3.260.5 Fricas [B] (verification not implemented) . . . 2002
3.260.6 Sympy [F] . . . 2002
3.260.7 Maxima [B] (verification not implemented) . . . 2003
3.260.8 Giac [F] . . . 2003
3.260.9 Mupad [F(-1)] . . . 2004


## 3.260.1 Optimal result

Integrand size = 22, antiderivative size = 228

$$\int (c+dx)^3\sin(a+bx)\tan^2(a+bx)\,dx=\frac{6id(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b^2}-\frac{6d^2(c+dx)\cos(a+bx)}{b^3}+\frac{(c+dx)^3\cos(a+bx)}{b}-\frac{6id^2(c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}+\frac{6d^3\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4}-\frac{6d^3\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^4}+\frac{(c+dx)^3\sec(a+bx)}{b}+\frac{6d^3\sin(a+bx)}{b^4}-\frac{3d(c+dx)^2\sin(a+bx)}{b^2}$$

output
```
6*I*d*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b^2-6*d^2*(d*x+c)*cos(b*x+a)/b^3+(d
*x+c)^3*cos(b*x+a)/b-6*I*d^2*(d*x+c)*polylog(2,-I*exp(I*(b*x+a)))/b^3+6*I*
d^2*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^3+6*d^3*polylog(3,-I*exp(I*(b*x+
a)))/b^4-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4+(d*x+c)^3*sec(b*x+a)/b+6*d^
3*sin(b*x+a)/b^4-3*d*(d*x+c)^2*sin(b*x+a)/b^2
```

### 3.260.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 532 vs. 2(228) = 456.

Time = 1.68 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.33

$$\int (c+dx)^3\sin(a+bx)\tan^2(a+bx)\,dx$$
$$= \frac{\sec(a+bx)\left(3b^3c^3-6bcd^2+9b^3c^2dx-6bd^3x+9b^3cd^2x^2+3b^3d^3x^3+12ib^2c^2d\arctan\left(e^{i(a+bx)}\right)\cos(a+b$$

input
```
Integrate[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
(Sec[a + b*x]*(3*b^3*c^3 - 6*b*c*d^2 + 9*b^3*c^2*d*x - 6*b*d^3*x + 9*b^3*c
*d^2*x^2 + 3*b^3*d^3*x^3 + (12*I)*b^2*c^2*d*ArcTan[E^(I*(a + b*x))]*Cos[a
+ b*x] + b^3*c^3*Cos[2*(a + b*x)] - 6*b*c*d^2*Cos[2*(a + b*x)] + 3*b^3*c^2
*d*x*Cos[2*(a + b*x)] - 6*b*d^3*x*Cos[2*(a + b*x)] + 3*b^3*c*d^2*x^2*Cos[2
*(a + b*x)] + b^3*d^3*x^3*Cos[2*(a + b*x)] - 12*b^2*c*d^2*x*Cos[a + b*x]*L
og[1 - I*E^(I*(a + b*x))] - 6*b^2*d^3*x^2*Cos[a + b*x]*Log[1 - I*E^(I*(a +
 b*x))] + 12*b^2*c*d^2*x*Cos[a + b*x]*Log[1 + I*E^(I*(a + b*x))] + 6*b^2*d
^3*x^2*Cos[a + b*x]*Log[1 + I*E^(I*(a + b*x))] - (12*I)*b*d^2*(c + d*x)*Co
s[a + b*x]*PolyLog[2, (-I)*E^(I*(a + b*x))] + (12*I)*b*d^2*(c + d*x)*Cos[a
 + b*x]*PolyLog[2, I*E^(I*(a + b*x))] + 12*d^3*Cos[a + b*x]*PolyLog[3, (-I
)*E^(I*(a + b*x))] - 12*d^3*Cos[a + b*x]*PolyLog[3, I*E^(I*(a + b*x))] - 3
*b^2*c^2*d*Sin[2*(a + b*x)] + 6*d^3*Sin[2*(a + b*x)] - 6*b^2*c*d^2*x*Sin[2
*(a + b*x)] - 3*b^2*d^3*x^2*Sin[2*(a + b*x)]))/(2*b^4)
```

### 3.260.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.07, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {4907, 3042, 3777, 3042, 3777, 25, 3042, 3777, 3042, 3117, 4909, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\sin(a+bx)\tan^2(a+bx)\,dx$$
$$\Big\downarrow \text{ 4907}$$

$$\int (c+dx)^3 \sec(a+bx)\tan(a+bx)dx - \int (c+dx)^3 \sin(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$\int (c+dx)^3 \sec(a+bx)\tan(a+bx)dx - \int (c+dx)^3 \sin(a+bx)dx$$

$$\Big\downarrow \; 3777$$

$$-\frac{3d\int (c+dx)^2\cos(a+bx)dx}{b} + \int (c+dx)^3\sec(a+bx)\tan(a+bx)dx + \frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3d\int (c+dx)^2\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b} + \int (c+dx)^3\sec(a+bx)\tan(a+bx)dx + \frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\Big\downarrow \; 3777$$

$$-\frac{3d\left(\frac{2d\int -((c+dx)\sin(a+bx))dx}{b} + \frac{(c+dx)^2\sin(a+bx)}{b}\right)}{b} + \int (c+dx)^3\sec(a+bx)\tan(a+bx)dx + \frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\Big\downarrow \; 25$$

$$-\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b} - \frac{2d\int (c+dx)\sin(a+bx)dx}{b}\right)}{b} + \int (c+dx)^3\sec(a+bx)\tan(a+bx)dx + \frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\Big\downarrow \; 3042$$

$$-\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b} - \frac{2d\int (c+dx)\sin(a+bx)dx}{b}\right)}{b} + \int (c+dx)^3\sec(a+bx)\tan(a+bx)dx + \frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\Big\downarrow \; 3777$$

$$-\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b} - \frac{2d\left(\frac{d\int \cos(a+bx)dx}{b} - \frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b} + \int (c+dx)^3\sec(a+bx)\tan(a+bx)dx + \frac{(c+dx)^3\cos(a+bx)}{b}$$

$$\Big\downarrow \; 3042$$

---

3.260. $\int (c+dx)^3\sin(a+bx)\tan^2(a+bx)\,dx$

$$-\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\int\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}+\int(c+dx)^3\sec(a+bx)\tan(a+bx)dx+\frac{(c+dx)^3\cos(a+bx)}{b}$$

$\big\downarrow$ 3117

$$\int(c+dx)^3\sec(a+bx)\tan(a+bx)dx-\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^3\cos(a+bx)}{b}$$

$\big\downarrow$ 4909

$$-\frac{3d\int(c+dx)^2\sec(a+bx)dx}{b}-\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^3\cos(a+bx)}{b}+\frac{(c+dx)^3\sec(a+bx)}{b}$$

$\big\downarrow$ 3042

$$-\frac{3d\int(c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^3\cos(a+bx)}{b}+\frac{(c+dx)^3\sec(a+bx)}{b}$$

$\big\downarrow$ 4669

$$-\frac{3d\left(-\frac{2d\int(c+dx)\log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{2d\int(c+dx)\log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^3\cos(a+bx)}{b}+\frac{(c+dx)^3\sec(a+bx)}{b}$$

$\big\downarrow$ 3011

3.260. $\int(c+dx)^3\sin(a+bx)\tan^2(a+bx)\,dx$

$$-\frac{3d\left(\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{id\int \mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{id\int \mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)dx}{b}\right)}{b}-\right.}{b}$$

$$\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^3\cos(a+bx)}{b}+\frac{(c+dx)^3\sec(a+bx)}{b}$$

$\downarrow$ 2720

$$-\frac{3d\left(\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\mathrm{Pol}}{}\right.}{b}\right.}{b}$$

$$\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^3\cos(a+bx)}{b}+\frac{(c+dx)^3\sec(a+bx)}{b}$$

$\downarrow$ 7143

$$-\frac{3d\left(-\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{d\,\mathrm{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{d\,\mathrm{P}}{}\right.}{b}\right.}{b}$$

$$\frac{3d\left(\frac{(c+dx)^2\sin(a+bx)}{b}-\frac{2d\left(\frac{d\sin(a+bx)}{b^2}-\frac{(c+dx)\cos(a+bx)}{b}\right)}{b}\right)}{b}+\frac{(c+dx)^3\cos(a+bx)}{b}+\frac{(c+dx)^3\sec(a+bx)}{b}$$

input
```
Int[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
((c + d*x)^3*Cos[a + b*x])/b - (3*d*(((-2*I)*(c + d*x)^2*ArcTan[E^(I*(a +
b*x))])/b + (2*d*((I*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b - (d*Po
lyLog[3, (-I)*E^(I*(a + b*x))])/b^2))/b - (2*d*((I*(c + d*x)*PolyLog[2, I*
E^(I*(a + b*x))])/b - (d*PolyLog[3, I*E^(I*(a + b*x))])/b^2))/b))/b + ((c
+ d*x)^3*Sec[a + b*x])/b - (3*d*(((c + d*x)^2*Sin[a + b*x])/b - (2*d*(-(((
c + d*x)*Cos[a + b*x])/b) + (d*Sin[a + b*x])/b^2))/b))/b
```

3.260. $\int (c+dx)^3\sin(a+bx)\tan^2(a+bx)\,dx$

### 3.260.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.260.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(211) = 422$.

Time = 3.57 (sec) , antiderivative size = 677, normalized size of antiderivative = 2.97

| method | result |
|---|---|
| risch | $\frac{\left(d^3x^3b^3+3b^3c\,d^2x^2+3ib^2d^3x^2+3b^3c^2dx+6ib^2c\,d^2x+b^3c^3+3ib^2c^2d-6b\,d^3x-6c\,d^2b-6id^3\right)\mathrm{e}^{i(xb+a)}}{2b^4}+\frac{\left(d^3x^3b^3+3b^3c\,d^2x^2-3ib^2d^3x\right.}{}$ |

input
```
int((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b
*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))
+1/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*
b*d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a
))+2*exp(I*(b*x+a))*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(exp(2*I*(b*x+a)
)+1)+6/b^2*d^2*c*ln(1+I*exp(I*(b*x+a)))*x-6*d^3*polylog(3,I*exp(I*(b*x+a))
)/b^4+6*I/b^4*d^3*a^2*arctan(exp(I*(b*x+a)))+6*I*c*d^2*polylog(2,I*exp(I*(
b*x+a)))/b^3-6*I*c*d^2*polylog(2,-I*exp(I*(b*x+a)))/b^3-6*I*d^3*x*polylog(
2,-I*exp(I*(b*x+a)))/b^3+6*I*d^3*x*polylog(2,I*exp(I*(b*x+a)))/b^3-12*I/b^
3*d^2*c*a*arctan(exp(I*(b*x+a)))+3/b^2*d^3*ln(1+I*exp(I*(b*x+a)))*x^2+3/b^
4*d^3*a^2*ln(1-I*exp(I*(b*x+a)))+6*I/b^2*d*c^2*arctan(exp(I*(b*x+a)))+6*d^
3*polylog(3,-I*exp(I*(b*x+a)))/b^4-6/b^2*d^2*c*ln(1-I*exp(I*(b*x+a)))*x+6/
b^3*d^2*c*ln(1+I*exp(I*(b*x+a)))*a-3/b^4*d^3*a^2*ln(1+I*exp(I*(b*x+a)))-3/
b^2*d^3*ln(1-I*exp(I*(b*x+a)))*x^2-6/b^3*d^2*c*ln(1-I*exp(I*(b*x+a)))*a
```

### 3.260.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 896 vs. $2(202) = 404$.

Time = 0.31 (sec) , antiderivative size = 896, normalized size of antiderivative = 3.93

$$\int (c+dx)^3 \sin(a+bx) \tan^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*c
os(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)
*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3,
 -I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*
x + a) - sin(b*x + a)) + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*
c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 - 6*(-I*b*d^3*x - I*b*c*
d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)) - 6*(-I*b*d^3*x - I
*b*c*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 6*(I*b*d^3*x
 + I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 6*(I*b*
d^3*x + I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*
(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(
b*x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos
(b*x + a) - I*sin(b*x + a) + I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c
*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b
^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos
(b*x + a) - sin(b*x + a) + 1) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d
^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^
2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos
(b*x + a) - sin(b*x + a) + 1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(
b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + 3*(b^2*c^2*d - 2*a*b...
```

### 3.260.6 Sympy [F]

$$\int (c+dx)^3 \sin(a+bx) \tan^2(a+bx)\, dx = \int (c+dx)^3 \sin(a+bx) \tan^2(a+bx)\, dx$$

input
```
integrate((d*x+c)**3*sin(b*x+a)*tan(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**3*sin(a + b*x)*tan(a + b*x)**2, x)
```

### 3.260.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 11010 vs. 2(202) = 404.

Time = 1.31 (sec) , antiderivative size = 11010, normalized size of antiderivative = 48.29

$$\int (c+dx)^3 \sin(a+bx) \tan^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(2*c^3*(1/cos(b*x + a) + cos(b*x + a)) - 6*a*c^2*d*(1/cos(b*x + a) + c
os(b*x + a))/b + 6*a^2*c*d^2*(1/cos(b*x + a) + cos(b*x + a))/b^2 - 2*a^3*d
^3*(1/cos(b*x + a) + cos(b*x + a))/b^3 + 3*((b*x + (b*x + a)*cos(2*b*x + 2
*a) + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)^
3 + ((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a)^3
 + 6*(b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + 2*(4*(b*x + a)*cos(2*b*x + 2*
a)*cos(b*x + a) + 4*(b*x + a)*cos(b*x + a) + (3*(b*x + a)*sin(b*x + a) + c
os(b*x + a))*sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^2 + ((b*x + a)*cos(b*x + a
) - sin(b*x + a))*cos(2*b*x + 2*a)^2 + (8*(b*x + a)*sin(2*b*x + 2*a)*sin(b
*x + a) + (b*x + (b*x + a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a))*cos(3*
b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a)
+ 6*(b*x + a)*cos(b*x + a) - 2*sin(b*x + a))*sin(3*b*x + 3*a)^2 + ((b*x +
a)*cos(b*x + a) - sin(b*x + a))*sin(2*b*x + 2*a)^2 + ((b*x + a)*cos(2*b*x
+ 2*a)^2 + 13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(2*b*x + 2*a)^2 + (b
*x + a)*sin(b*x + a)^2 + b*x + (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*si
n(b*x + a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) + (12*(b*x + a)*cos(b*x + a)*
sin(b*x + a) + cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a) + a)*cos(
3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a)^3 + 3*(b*x + a)*cos(b*x + a)*si
n(b*x + a)^2 + (b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a) + (
b*x + a)*cos(b*x + a) - ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*c...
```

### 3.260.8 Giac [F]

$$\int (c+dx)^3 \sin(a+bx) \tan^2(a+bx)\,dx = \int (dx+c)^3 \sin(bx+a) \tan(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*sin(b*x + a)*tan(b*x + a)^2, x)
```

### 3.260.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \sin(a+bx)\tan^2(a+bx)\,dx=\int \sin\left(a+b\,x\right)\,\tan(a+b\,x)^2\,{\left(c+d\,x\right)}^3\,dx$$

input
```
int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^3,x)
```

output
```
int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^3, x)
```

# 3.261 $\int (c+dx)^2 \sin(a+bx) \tan^2(a+bx) \, dx$


3.261.1 Optimal result . . . 2005
3.261.2 Mathematica [B] (warning: unable to verify) . . . 2006
3.261.3 Rubi [A] (verified) . . . 2006
3.261.4 Maple [B] (verified) . . . 2009
3.261.5 Fricas [B] (verification not implemented) . . . 2010
3.261.6 Sympy [F] . . . 2011
3.261.7 Maxima [F] . . . 2011
3.261.8 Giac [F] . . . 2012
3.261.9 Mupad [F(-1)] . . . 2012


## 3.261.1 Optimal result

Integrand size = 22, antiderivative size = 145

$$\int (c+dx)^2 \sin(a+bx) \tan^2(a+bx) \, dx = \frac{4id(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b^2} - \frac{2d^2\cos(a+bx)}{b^3} + \frac{(c+dx)^2\cos(a+bx)}{b} - \frac{2id^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} + \frac{2id^2\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^2\sec(a+bx)}{b} - \frac{2d(c+dx)\sin(a+bx)}{b^2}$$

output
```
4*I*d*(d*x+c)*arctan(exp(I*(b*x+a)))/b^2-2*d^2*cos(b*x+a)/b^3+(d*x+c)^2*co
s(b*x+a)/b-2*I*d^2*polylog(2,-I*exp(I*(b*x+a)))/b^3+2*I*d^2*polylog(2,I*ex
p(I*(b*x+a)))/b^3+(d*x+c)^2*sec(b*x+a)/b-2*d*(d*x+c)*sin(b*x+a)/b^2
```

### 3.261.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 362 vs. $2(145) = 290$.

Time = 3.48 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.50

$$\int (c+dx)^2 \sin(a+bx)\tan^2(a+bx)\,dx$$
$$= \frac{-4bcd\text{arctanh}\left(\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)\right) - 4d^2\arctan(\cot(a))\text{arctanh}\left(\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)\right) + \frac{2d^2\csc(a}{}}{}$$

input
```
Integrate[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
(-4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - 4*d^2*ArcTan[Cot[a]]*Arc
Tanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + (2*d^2*Csc[a]*((b*x - ArcTan[Cot[a]])
*(Log[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcTan[Cot[a
]]))]) + I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - I*PolyLog[2, E^(I*(
b*x - ArcTan[Cot[a]]))]))/Sqrt[Csc[a]^2] + b^2*(c + d*x)^2*Sec[a] + Cos[b*
x]*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] - 2*b*d*(c + d*x)*Sin[a]) - (2*b*d*(
c + d*x)*Cos[a] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x] + (b^2*(c +
d*x)^2*Sin[(b*x)/2])/((Cos[a/2] - Sin[a/2])*(Cos[(a + b*x)/2] - Sin[(a + b
*x)/2])) - (b^2*(c + d*x)^2*Sin[(b*x)/2])/((Cos[a/2] + Sin[a/2])*(Cos[(a +
 b*x)/2] + Sin[(a + b*x)/2])))/b^3
```

### 3.261.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {4907, 3042, 3777, 3042, 3777, 25, 3042, 3118, 4909, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin(a+bx)\tan^2(a+bx)\,dx$$
$$\Big\downarrow \text{4907}$$
$$\int (c+dx)^2 \sec(a+bx)\tan(a+bx)dx - \int (c+dx)^2 \sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2\sec(a+bx)\tan(a+bx)dx-\int (c+dx)^2\sin(a+bx)dx$$

$$\Big\downarrow \text{3777}$$

$$-\frac{2d\int (c+dx)\cos(a+bx)dx}{b}+\int (c+dx)^2\sec(a+bx)\tan(a+bx)dx+\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2d\int (c+dx)\sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}+\int (c+dx)^2\sec(a+bx)\tan(a+bx)dx+\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3777}$$

$$-\frac{2d\left(\frac{d\int -\sin(a+bx)dx}{b}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}+\int (c+dx)^2\sec(a+bx)\tan(a+bx)dx+\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$-\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int \sin(a+bx)dx}{b}\right)}{b}+\int (c+dx)^2\sec(a+bx)\tan(a+bx)dx+\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2d\left(\frac{(c+dx)\sin(a+bx)}{b}-\frac{d\int \sin(a+bx)dx}{b}\right)}{b}+\int (c+dx)^2\sec(a+bx)\tan(a+bx)dx+\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3118}$$

$$\int (c+dx)^2\sec(a+bx)\tan(a+bx)dx-\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}+\frac{(c+dx)^2\cos(a+bx)}{b}$$

$$\Big\downarrow \text{4909}$$

$$-\frac{2d\int (c+dx)\sec(a+bx)dx}{b}-\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}+\frac{(c+dx)^2\cos(a+bx)}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2d\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}+\frac{(c+dx)^2\cos(a+bx)}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

---

3.261. $\int (c+dx)^2\sin(a+bx)\tan^2(a+bx)\,dx$

$$\Big\downarrow \; 4669$$

$$-\frac{2d\left(-\frac{d\int \log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{d\int \log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}+\frac{(c+dx)^2\cos(a+bx)}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

$$\Big\downarrow \; 2715$$

$$-\frac{2d\left(\frac{id\int e^{-i(a+bx)}\log\left(1-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{id\int e^{-i(a+bx)}\log\left(1+ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}\right)}{b}-\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}+\frac{(c+dx)^2\cos(a+bx)}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

$$\Big\downarrow \; 2838$$

$$-\frac{2d\left(-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}\right)}{b}-\frac{2d\left(\frac{d\cos(a+bx)}{b^2}+\frac{(c+dx)\sin(a+bx)}{b}\right)}{b}+\frac{(c+dx)^2\cos(a+bx)}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

input
```
Int[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
((c + d*x)^2*Cos[a + b*x])/b - (2*d*(((-2*I)*(c + d*x)*ArcTan[E^(I*(a + b*
x))])/b + (I*d*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, I*E
^(I*(a + b*x))])/b^2))/b + ((c + d*x)^2*Sec[a + b*x])/b - (2*d*((d*Cos[a +
 b*x])/b^2 + ((c + d*x)*Sin[a + b*x])/b))/b
```

### 3.261.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.261. $\int (c+dx)^2\sin(a+bx)\tan^2(a+bx)\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

### 3.261.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(134) = 268$.

Time = 2.27 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.38

| method | result |
|---|---|
| risch | $\frac{\left(x^2 d^2 b^2+2 b^2 c d x+2 i b\, d^2 x+b^2 c^2+2 i b c d-2 d^2\right) \mathrm{e}^{i(x b+a)}}{2 b^3}+\frac{\left(x^2 d^2 b^2+2 b^2 c d x-2 i b\, d^2 x+b^2 c^2-2 i b c d-2 d^2\right) \mathrm{e}^{-i(x b+a)}}{2 b^3}+\frac{2\, \mathrm{e}^{i(x b+a)}\left(x^2\right.}{b\left(\mathrm{e}^{2 i(x b\right.}}$ |

input
```
int((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp(
I*(b*x+a))+1/2*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*
d)/b^3*exp(-I*(b*x+a))+2*exp(I*(b*x+a))*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(
b*x+a))+1)+4*I/b^2*d*c*arctan(exp(I*(b*x+a)))+2/b^2*d^2*ln(1+I*exp(I*(b*x+
a)))*x+2/b^3*d^2*ln(1+I*exp(I*(b*x+a)))*a-2/b^2*d^2*ln(1-I*exp(I*(b*x+a)))
*x-2/b^3*d^2*ln(1-I*exp(I*(b*x+a)))*a-2*I/b^3*d^2*dilog(1+I*exp(I*(b*x+a))
)+2*I/b^3*d^2*dilog(1-I*exp(I*(b*x+a)))-4*I/b^3*d^2*a*arctan(exp(I*(b*x+a)
))
```

### 3.261.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 511 vs. 2(127) = 254.

Time = 0.28 (sec) , antiderivative size = 511, normalized size of antiderivative = 3.52

$$\int (c+dx)^2 \sin(a+bx)\tan^2(a+bx)\,dx = \frac{b^2d^2x^2 + 2\,b^2cdx + b^2c^2 + i\,d^2\cos\left(bx+a\right)\mathrm{Li}_2(i\,\cos\left(bx+a\right) + \sin\left(bx+a\right)) + i\,d^2\cos\left(bx+a\right)\mathrm{Li}_2(i\,\cos\left(b\right.}{}$$

input
```
integrate((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*cos(b*x + a)*dilog(I*cos(b*x
+ a) + sin(b*x + a)) + I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x +
 a)) - I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d^2*co
s(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*
d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - (b*c*d - a*d^2)*cos(b*x + a)*log(c
os(b*x + a) + I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b
*x + a) - I*sin(b*x + a) + I) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b
*x + a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x
 + a) - sin(b*x + a) + 1) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x
+ a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x +
 a) - sin(b*x + a) + 1) - (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) +
 I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*
sin(b*x + a) + I) - 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3*co
s(b*x + a))
```

### 3.261.6 Sympy [F]

$$\int (c+dx)^2\sin(a+bx)\tan^2(a+bx)\,dx=\int (c+dx)^2\sin{(a+bx)}\tan^2{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**2*sin(b*x+a)*tan(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*sin(a + b*x)*tan(a + b*x)**2, x)
```

### 3.261.7 Maxima [F]

$$\int (c+dx)^2\sin(a+bx)\tan^2(a+bx)\,dx=\int (dx+c)^2\sin{(bx+a)}\tan{(bx+a)}^2\,dx$$

input
```
integrate((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(2*((3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*cos(2*b*x + 3*a)
*cos(b*x + 2*a) + (3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*sin(2*
b*x + 3*a)*sin(b*x + 2*a) + (3*b^2*d^2*x^2*cos(a) + 6*b^2*c*d*x*cos(a) + 3
*b^2*c^2*cos(a) - 2*d^2*cos(a))*cos(b*x + 2*a) + (3*b^2*d^2*x^2*sin(a) + 6
*b^2*c*d*x*sin(a) + 3*b^2*c^2*sin(a) - 2*d^2*sin(a))*sin(b*x + 2*a))*cos(3
*b*x + 3*a)^2 + ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a
) - 2*(b*d^2*x + b*c*d)*sin(b*x + a))*cos(2*b*x + 3*a)^2 + 2*((3*b^2*d^2*x
^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*cos(2*b*x + 3*a)*cos(b*x + 2*a) + (3
*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*sin(2*b*x + 3*a)*sin(b*x +
 2*a) + (3*b^2*d^2*x^2*cos(a) + 6*b^2*c*d*x*cos(a) + 3*b^2*c^2*cos(a) - 2*
d^2*cos(a))*cos(b*x + 2*a) + (3*b^2*d^2*x^2*sin(a) + 6*b^2*c*d*x*sin(a) +
3*b^2*c^2*sin(a) - 2*d^2*sin(a))*sin(b*x + 2*a))*sin(3*b*x + 3*a)^2 + ((b^
2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a) - 2*(b*d^2*x + b*c
*d)*sin(b*x + a))*sin(2*b*x + 3*a)^2 + ((b^2*d^2*x^2*cos(a) + b^2*c^2*cos(
a) + 2*b*c*d*sin(a) - 2*d^2*cos(a) + 2*(b^2*c*d*cos(a) + b*d^2*sin(a))*x +
 (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(2*b*x + 3*a) + 2*(b*d^2
*x + b*c*d)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a)^2 + (b^2*d^2*x^2*cos(a) + b
^2*c^2*cos(a) + 2*b*c*d*sin(a) - 2*d^2*cos(a) + 2*(b^2*c*d*cos(a) + b*d^2*
sin(a))*x)*cos(b*x + a)^2 + (b^2*d^2*x^2*cos(a) + b^2*c^2*cos(a) + 2*b*c*d
*sin(a) - 2*d^2*cos(a) + 2*(b^2*c*d*cos(a) + b*d^2*sin(a))*x + (b^2*d^2...
```

### 3.261.8 Giac [F]

$$\int (c+dx)^2 \sin(a+bx) \tan^2(a+bx)\, dx = \int (dx+c)^2 \sin(bx+a) \tan(bx+a)^2 \, dx$$

input
```
integrate((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*sin(b*x + a)*tan(b*x + a)^2, x)
```

### 3.261.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \sin(a+bx) \tan^2(a+bx)\, dx = \int \sin(a+b\,x)\ \tan(a+b\,x)^2\, (c+d\,x)^2\, dx$$

input
```
int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^2,x)
```

output
```
int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^2, x)
```

# 3.262 $\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx$

3.262.1 Optimal result . . . . . 2013
3.262.2 Mathematica [A] (verified) . . . . . 2013
3.262.3 Rubi [A] (verified) . . . . . 2014
3.262.4 Maple [C] (verified) . . . . . 2016
3.262.5 Fricas [A] (verification not implemented) . . . . . 2016
3.262.6 Sympy [F] . . . . . 2017
3.262.7 Maxima [B] (verification not implemented) . . . . . 2017
3.262.8 Giac [B] (verification not implemented) . . . . . 2018
3.262.9 Mupad [B] (verification not implemented) . . . . . 2019

## 3.262.1 Optimal result

Integrand size = 20, antiderivative size = 56

$$\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx=-\frac{d\operatorname{arctanh}(\sin(a+bx))}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}+\frac{(c+dx)\sec(a+bx)}{b}-\frac{d\sin(a+bx)}{b^2}$$

output
```
-d*arctanh(sin(b*x+a))/b^2+(d*x+c)*cos(b*x+a)/b+(d*x+c)*sec(b*x+a)/b-d*sin
(b*x+a)/b^2
```

## 3.262.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.91

$$\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx$$
$$=\frac{\sec(a+bx)\left(3bc+3bdx+b(c+dx)\cos(2(a+bx))+2d\cos(a+bx)\left(\log\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+b\right.\right.\right.\right.}{2b^2}$$

input
```
Integrate[(c + d*x)*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
(Sec[a + b*x]*(3*b*c + 3*b*d*x + b*(c + d*x)*Cos[2*(a + b*x)] + 2*d*Cos[a
+ b*x]*(Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]] - Log[Cos[(a + b*x)/2] +
Sin[(a + b*x)/2]]) - d*Sin[2*(a + b*x)]))/(2*b^2)
```

### 3.262.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4907, 3042, 3777, 3042, 3117, 4909, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx$$

$$\Big\downarrow \text{4907}$$

$$\int (c+dx)\sec(a+bx)\tan(a+bx)dx-\int (c+dx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\sec(a+bx)\tan(a+bx)dx-\int (c+dx)\sin(a+bx)dx$$

$$\Big\downarrow \text{3777}$$

$$\int (c+dx)\sec(a+bx)\tan(a+bx)dx-\frac{d\int \cos(a+bx)dx}{b}+\frac{(c+dx)\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\sec(a+bx)\tan(a+bx)dx-\frac{d\int \sin\left(a+bx+\frac{\pi}{2}\right)dx}{b}+\frac{(c+dx)\cos(a+bx)}{b}$$

$$\Big\downarrow \text{3117}$$

$$\int (c+dx)\sec(a+bx)\tan(a+bx)dx-\frac{d\sin(a+bx)}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}$$

$$\Big\downarrow \text{4909}$$

$$-\frac{d\int \sec(a+bx)dx}{b}-\frac{d\sin(a+bx)}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}+\frac{(c+dx)\sec(a+bx)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{d\int \csc\left(a+bx+\frac{\pi}{2}\right)dx}{b}-\frac{d\sin(a+bx)}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}+\frac{(c+dx)\sec(a+bx)}{b}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{d\,\text{arctanh}(\sin(a+bx))}{b^2}-\frac{d\sin(a+bx)}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}+\frac{(c+dx)\sec(a+bx)}{b}$$

input
```
Int[(c + d*x)*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
-((d*ArcTanh[Sin[a + b*x]])/b^2) + ((c + d*x)*Cos[a + b*x])/b + ((c + d*x)
*Sec[a + b*x])/b - (d*Sin[a + b*x])/b^2
```

### 3.262.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

### 3.262.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.20

| method | result | size |
|---|---|---|
| risch | $\frac{(dxb+cb+id)e^{i(xb+a)}}{2b^2}+\frac{(dxb+cb-id)e^{-i(xb+a)}}{2b^2}+\frac{2\,e^{i(xb+a)}(dx+c)}{b\left(e^{2i(xb+a)}+1\right)}+\frac{d\ln\left(e^{i(xb+a)}-i\right)}{b^2}-\frac{d\ln\left(i+e^{i(xb+a)}\right)}{b^2}$ | 123 |

input
```
int((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(d*x*b+c*b+I*d)/b^2*exp(I*(b*x+a))+1/2*(d*x*b+c*b-I*d)/b^2*exp(-I*(b*x
+a))+2*exp(I*(b*x+a))*(d*x+c)/b/(exp(2*I*(b*x+a))+1)+d/b^2*ln(exp(I*(b*x+a
))-I)-d/b^2*ln(I+exp(I*(b*x+a)))
```

### 3.262.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.66

$$\int(c+dx)\sin(a+bx)\tan^2(a+bx)\,dx$$
$$=\frac{2\,bdx+2\,(bdx+bc)\cos{(bx+a)}^2-d\cos{(bx+a)}\log{(\sin{(bx+a)}+1)}+d\cos{(bx+a)}\log{(-\sin{(bx+a}}{2\,b^2\cos{(bx+a)}}$$

input
```
integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(2*b*d*x + 2*(b*d*x + b*c)*cos(b*x + a)^2 - d*cos(b*x + a)*log(sin(b*x
 + a) + 1) + d*cos(b*x + a)*log(-sin(b*x + a) + 1) - 2*d*cos(b*x + a)*sin(
b*x + a) + 2*b*c)/(b^2*cos(b*x + a))
```

### 3.262.6 Sympy [F]

$$\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx=\int (c+dx)\sin{(a+bx)}\tan^2{(a+bx)}\,dx$$

input
```
integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*sin(a + b*x)*tan(a + b*x)**2, x)
```

### 3.262.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2123 vs. $2(56)=112$.

Time = 0.32 (sec) , antiderivative size = 2123, normalized size of antiderivative = 37.91

$$\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(2*c*(1/cos(b*x + a) + cos(b*x + a)) - 2*a*d*(1/cos(b*x + a) + cos(b*x
 + a))/b + ((b*x + (b*x + a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a))*cos(
3*b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)^3 + ((b*x + a)*sin(2*b*x + 2*a)
- cos(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)*sin(
b*x + a)^2 + 2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*co
s(b*x + a) + (3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a))*c
os(3*b*x + 3*a)^2 + ((b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*
a)^2 + (8*(b*x + a)*sin(2*b*x + 2*a)*sin(b*x + a) + (b*x + (b*x + a)*cos(2
*b*x + 2*a) + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(
b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a) + 6*(b*x + a)*cos(b*x + a) - 2*s
in(b*x + a))*sin(3*b*x + 3*a)^2 + ((b*x + a)*cos(b*x + a) - sin(b*x + a))*
sin(2*b*x + 2*a)^2 + ((b*x + a)*cos(2*b*x + 2*a)^2 + 13*(b*x + a)*cos(b*x
+ a)^2 + (b*x + a)*sin(2*b*x + 2*a)^2 + (b*x + a)*sin(b*x + a)^2 + b*x + (
13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2 + 2*b*x + 2*a)*cos(
2*b*x + 2*a) + (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x + a)^2 +
sin(b*x + a)^2)*sin(2*b*x + 2*a) + a)*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*co
s(b*x + a)^3 + 3*(b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + (b*x + a)*cos(b*x
 + a) - sin(b*x + a))*cos(2*b*x + 2*a) + (b*x + a)*cos(b*x + a) - ((cos(2*
b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*
a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*...
```

### 3.262.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2330 vs. $2(56) = 112$.

Time = 1.11 (sec) , antiderivative size = 2330, normalized size of antiderivative = 41.61

$$\int (c + dx) \sin(a + bx) \tan^2(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*(4*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b*c*tan(1/2*b*x)^4*tan(1/2*a)
^4 + d*log(2*(tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2*tan(1/2*a) +
2*tan(1/2*b*x)*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*
x) - 2*tan(1/2*a) + 1)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan
(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a)^4 - d*log(2*(tan(1/2*b*x)^2*tan(
1/2*a)^2 - 2*tan(1/2*b*x)^2*tan(1/2*a) - 2*tan(1/2*b*x)*tan(1/2*a)^2 + tan
(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*b
*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*ta
n(1/2*a)^4 - 16*b*d*x*tan(1/2*b*x)^3*tan(1/2*a)^3 - 16*b*c*tan(1/2*b*x)^3*
tan(1/2*a)^3 - 4*d*log(2*(tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2*t
an(1/2*a) + 2*tan(1/2*b*x)*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 -
2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*
b*x)^2 + tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a)^3 + 4*d*log(2*(tan(1
/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^2*tan(1/2*a) - 2*tan(1/2*b*x)*tan(
1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) +
 1)/(tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + tan(1/2*a)^2 + 1))*tan
(1/2*b*x)^3*tan(1/2*a)^3 + 4*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + 4*d*tan(1/2*b
*x)^3*tan(1/2*a)^4 + 4*b*d*x*tan(1/2*b*x)^4 + 16*b*d*x*tan(1/2*b*x)^3*tan(
1/2*a) + 48*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + 16*b*d*x*tan(1/2*b*x)*tan(
1/2*a)^3 + 4*b*d*x*tan(1/2*a)^4 + 4*b*c*tan(1/2*b*x)^4 - d*log(2*(tan(1...
```

### 3.262.9 Mupad [B] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.70

$$\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx = e^{a\,1i+b\,x\,1i}\left(\frac{b\,c+d\,1i}{2\,b^2}+\frac{d\,x}{2\,b}\right) - e^{-a\,1i-b\,x\,1i}\left(\frac{-b\,c+d\,1i}{2\,b^2}-\frac{d\,x}{2\,b}\right) + \frac{d\,\ln\left(e^{a\,1i+b\,x\,1i}-i\right)}{b^2} - \frac{d\,\ln\left(e^{a\,1i+b\,x\,1i}+1i\right)}{b^2} + \frac{e^{a\,1i+b\,x\,1i}\,(c+d\,x)\,2i}{b\,\left(e^{a\,2i+b\,x\,2i}\,1i+1i\right)}$$

input
```
int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x),x)
```

output
```
exp(a*1i + b*x*1i)*((d*1i + b*c)/(2*b^2) + (d*x)/(2*b)) - exp(- a*1i - b*x
*1i)*((d*1i - b*c)/(2*b^2) - (d*x)/(2*b)) + (d*log(exp(a*1i + b*x*1i) - 1i
))/b^2 - (d*log(exp(a*1i + b*x*1i) + 1i))/b^2 + (exp(a*1i + b*x*1i)*(c + d
*x)*2i)/(b*(exp(a*2i + b*x*2i)*1i + 1i))
```

# 3.263 $\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$

3.263.1 Optimal result . . . 2020
3.263.2 Mathematica [N/A] . . . 2020
3.263.3 Rubi [N/A] . . . 2021
3.263.4 Maple [N/A] (verified) . . . 2023
3.263.5 Fricas [N/A] . . . 2023
3.263.6 Sympy [N/A] . . . 2023
3.263.7 Maxima [N/A] . . . 2024
3.263.8 Giac [N/A] . . . 2024
3.263.9 Mupad [N/A] . . . 2025

## 3.263.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx = -\frac{\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d} - \frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d} + \text{Int}\left(\frac{\sec(a+bx)\tan(a+bx)}{c+dx},x\right)$$

output
```
CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)-cos(a-b*c/d)*Si(b*c/d+b*x
)/d-Ci(b*c/d+b*x)*sin(a-b*c/d)/d
```

## 3.263.2 Mathematica [N/A]

Not integrable

Time = 4.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x),x]
```

output
```
Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x), x]
```

### 3.263.3 Rubi [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4907, 3042, 3784, 3042, 3780, 3783, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \; 4907$$

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}dx-\int \frac{\sin(a+bx)}{c+dx}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}dx-\int \frac{\sin(a+bx)}{c+dx}dx$$

$$\Big\downarrow \; 3784$$

$$-\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx-\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}dx$$

$$\Big\downarrow \; 3042$$

$$-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx-\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}dx$$

$$\Big\downarrow \; 3780$$

$$-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx+\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}dx-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

$$\Big\downarrow \; 3783$$

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}dx-\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}dx-\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

input
```
Int[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x),x]
```

output
```
$Aborted
```

### 3.263.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

### 3.263.4 Maple [N/A] (verified)

Not integrable

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin(xb+a)\tan(xb+a)^2}{dx+c}dx$$

input
```
int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
```

output
```
int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
```

### 3.263.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\sin(bx+a)\tan(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(sin(b*x + a)*tan(b*x + a)^2/(d*x + c), x)
```

### 3.263.6 Sympy [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(sin(a + b*x)*tan(a + b*x)**2/(c + d*x), x)
```

### 3.263.7 Maxima [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 1268, normalized size of antiderivative = 57.64

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\sin(bx+a)\tan(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
1/2*(b*c*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(1, -
(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*c*(exp_integral_e(1, (I*b*d*
x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/
d) + (b*c*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(1,
-(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*c*(exp_integral_e(1, (I*b*d
*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)
/d) + (b*d*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(1,
 -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*d*(exp_integral_e(1, (I*b*
d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d
)/d))*x)*cos(2*b*x + 2*a)^2 + (b*c*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/
d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*c*
(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*
b*c)/d))*sin(-(b*c - a*d)/d) + (b*d*(I*exp_integral_e(1, (I*b*d*x + I*b*c)
/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*d
*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I
*b*c)/d))*sin(-(b*c - a*d)/d))*x)*sin(2*b*x + 2*a)^2 + 4*d*sin(2*b*x + 2*a
)*sin(b*x + a) + (b*d*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_in
tegral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*d*(exp_integral
_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(
-(b*c - a*d)/d))*x - 2*(b*c*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) ...
```

### 3.263.8 Giac [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\sin(bx+a)\tan(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)*tan(b*x + a)^2/(d*x + c), x)
```

### 3.263.9 Mupad [N/A]

Not integrable

Time = 27.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\sin\left(a+b\,x\right)\,\tan\left(a+b\,x\right)^2}{c+d\,x}\,dx$$

input
```
int((sin(a + b*x)*tan(a + b*x)^2)/(c + d*x),x)
```

output
```
int((sin(a + b*x)*tan(a + b*x)^2)/(c + d*x), x)
```

# 3.264 $\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$

3.264.1 Optimal result . . . . . 2026
3.264.2 Mathematica [N/A] . . . . . 2026
3.264.3 Rubi [N/A] . . . . . 2027
3.264.4 Maple [N/A] (verified) . . . . . 2029
3.264.5 Fricas [N/A] . . . . . 2029
3.264.6 Sympy [N/A] . . . . . 2030
3.264.7 Maxima [N/A] . . . . . 2030
3.264.8 Giac [N/A] . . . . . 2031
3.264.9 Mupad [N/A] . . . . . 2032

## 3.264.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = -\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2} + \frac{\sin(a+bx)}{d(c+dx)} + \frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2} + \text{Int}\left(\frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2},x\right)$$

output
```
CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)-b*Ci(b*c/d+b*x)*cos(a-b
*c/d)/d^2+b*Si(b*c/d+b*x)*sin(a-b*c/d)/d^2+sin(b*x+a)/d/(d*x+c)
```

## 3.264.2 Mathematica [N/A]

Not integrable

Time = 4.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2, x]
```

### 3.264.3 Rubi [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4907, 3042, 3778, 3042, 3784, 3042, 3780, 3783, 7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{4907}$$

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}dx-\int \frac{\sin(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}dx-\int \frac{\sin(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3778}$$

$$-\frac{b\int \frac{\cos(a+bx)}{c+dx}dx}{d}+\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}dx+\frac{\sin(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\int \frac{\sin\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx}{d}+\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}dx+\frac{\sin(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{b\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}dx-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}+\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}dx+\frac{\sin(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx-\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}+\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}dx+\frac{\sin(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \; 3780$$

$$-\frac{b\left(\cos\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{bc}{d}+bx+\frac{\pi}{2}\right)}{c+dx}dx-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}+\int\frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}dx+\frac{\sin(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \; 3783$$

$$\int\frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}dx-\frac{b\left(\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}+\frac{\sin(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \; 7299$$

$$\int\frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}dx-\frac{b\left(\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}+\frac{\sin(a+bx)}{d(c+dx)}$$

input
```
Int[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
$Aborted
```

### 3.264.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4907
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

### 3.264.4 Maple [N/A] (verified)

Not integrable

Time = 0.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin (xb+a) \tan (xb+a)^{2}}{(dx+c)^{2}}dx$$

input
```
int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
```

### 3.264.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin (bx+a) \tan (bx+a)^{2}}{(dx+c)^{2}}\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(sin(b*x + a)*tan(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.264.6 Sympy [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin{\left(a+bx \right)}\tan^{2}{\left(a+bx \right)}}{\left(c+dx\right)^{2}}\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(sin(a + b*x)*tan(a + b*x)**2/(c + d*x)**2, x)
```

### 3.264.7 Maxima [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 1412, normalized size of antiderivative = 64.18

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin\left(bx+a\right)\tan\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*(b*c*(I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(2, -
(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*c*(exp_integral_e(2, (I*b*d*
x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/
d) + (b*c*(I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(2,
-(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*c*(exp_integral_e(2, (I*b*d
*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)
/d) + (b*d*(I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(2,
 -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*d*(exp_integral_e(2, (I*b*
d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d
)/d))*x)*cos(2*b*x + 2*a)^2 + (b*c*(I*exp_integral_e(2, (I*b*d*x + I*b*c)/
d) - I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*c*
(exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*
b*c)/d))*sin(-(b*c - a*d)/d) + (b*d*(I*exp_integral_e(2, (I*b*d*x + I*b*c)
/d) - I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*d
*(exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I
*b*c)/d))*sin(-(b*c - a*d)/d))*x)*sin(2*b*x + 2*a)^2 + 4*d*sin(2*b*x + 2*a
)*sin(b*x + a) + (b*d*(I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) - I*exp_in
tegral_e(2, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*d*(exp_integral
_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(
-(b*c - a*d)/d))*x - 2*(b*c*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) ...
```

### 3.264.8 Giac [N/A]

Not integrable

Time = 18.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin(bx+a)\tan(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sin(b*x + a)*tan(b*x + a)^2/(d*x + c)^2, x)
```

### 3.264.9 Mupad [N/A]

Not integrable

Time = 28.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sin\left(a+b\,x\right)\,\tan(a+b\,x)^2}{\left(c+d\,x\right)^2}\,dx$$

input
```
int((sin(a + b*x)*tan(a + b*x)^2)/(c + d*x)^2,x)
```

output
```
int((sin(a + b*x)*tan(a + b*x)^2)/(c + d*x)^2, x)
```

# 3.265 $\int (c+dx)^m \csc(a+bx) \sec^2(a+bx) \, dx$

3.265.1 Optimal result . . . . . . . . . . 2033
3.265.2 Mathematica [N/A] . . . . . . . . . . 2033
3.265.3 Rubi [N/A] . . . . . . . . . . 2034
3.265.4 Maple [N/A] (verified) . . . . . . . . . . 2034
3.265.5 Fricas [N/A] . . . . . . . . . . 2035
3.265.6 Sympy [N/A] . . . . . . . . . . 2035
3.265.7 Maxima [N/A] . . . . . . . . . . 2035
3.265.8 Giac [N/A] . . . . . . . . . . 2036
3.265.9 Mupad [N/A] . . . . . . . . . . 2036

## 3.265.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (c+dx)^m \csc(a+bx) \sec^2(a+bx) \, dx = \text{Int}\left((c+dx)^m \csc(a+bx) \sec^2(a+bx), x\right)$$

output
```
CannotIntegrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x)
```

## 3.265.2 Mathematica [N/A]

Not integrable

Time = 24.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc(a+bx) \sec^2(a+bx) \, dx = \int (c+dx)^m \csc(a+bx) \sec^2(a+bx) \, dx$$

input
```
Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2, x]
```

## 3.265.3 Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\sec^2(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{ 7299}$$

$$\int \csc(a+bx)\sec^2(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

output
```
$Aborted
```

### 3.265.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.265.4 Maple [N/A] (verified)

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \csc(xb+a)\sec(xb+a)^2\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x)
```

### 3.265.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)\sec(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^2, x)
```

### 3.265.6 Sympy [N/A]

Not integrable

Time = 118.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx = \int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*csc(b*x+a)*sec(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*csc(a + b*x)*sec(a + b*x)**2, x)
```

### 3.265.7 Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)\sec(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^2, x)
```

### 3.265.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)\sec(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^2, x)
```

### 3.265.9 Mupad [N/A]

Not integrable

Time = 26.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\cos(a+b\,x)^2\,\sin(a+b\,x)}\,dx$$

input
```
int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)),x)
```

output
```
int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)), x)
```

# 3.266 $\int (c+dx)^4 \csc(a+bx) \sec^2(a+bx) \, dx$

3.266.1 Optimal result . . . . . 2038
3.266.2 Mathematica [A] (verified) . . . . . 2039
3.266.3 Rubi [A] (verified) . . . . . 2040
3.266.4 Maple [B] (verified) . . . . . 2042
3.266.5 Fricas [F(-2)] . . . . . 2043
3.266.6 Sympy [F] . . . . . 2043
3.266.7 Maxima [B] (verification not implemented) . . . . . 2043
3.266.8 Giac [F] . . . . . 2044
3.266.9 Mupad [F(-1)] . . . . . 2045

### 3.266.1 Optimal result

Integrand size = 22, antiderivative size = 469

$$\int (c+dx)^4 \csc(a+bx)\sec^2(a+bx)\,dx = \frac{8id(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^4\text{arctanh}\left(e^{i(a+bx)}\right)}{b} + \frac{4id(c+dx)^3\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} - \frac{12id^2(c+dx)^2\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} + \frac{12id^2(c+dx)^2\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} - \frac{4id(c+dx)^3\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} - \frac{12d^2(c+dx)^2\,\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} + \frac{24d^3(c+dx)\,\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4} - \frac{24d^3(c+dx)\,\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^4} + \frac{12d^2(c+dx)^2\,\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3} - \frac{24id^3(c+dx)\,\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4} + \frac{24id^4\,\text{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^5} - \frac{24id^4\,\text{PolyLog}\left(4,ie^{i(a+bx)}\right)}{b^5} + \frac{24id^3(c+dx)\,\text{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^4} + \frac{24d^4\,\text{PolyLog}\left(5,-e^{i(a+bx)}\right)}{b^5} - \frac{24d^4\,\text{PolyLog}\left(5,e^{i(a+bx)}\right)}{b^5} + \frac{(c+dx)^4\sec(a+bx)}{b}$$

output
```
-12*I*d^2*(d*x+c)^2*polylog(2,-I*exp(I*(b*x+a)))/b^3-2*(d*x+c)^4*arctanh(e
xp(I*(b*x+a)))/b+12*I*d^2*(d*x+c)^2*polylog(2,I*exp(I*(b*x+a)))/b^3+8*I*d*
(d*x+c)^3*arctan(exp(I*(b*x+a)))/b^2+4*I*d*(d*x+c)^3*polylog(2,-exp(I*(b*x
+a)))/b^2+24*I*d^4*polylog(4,-I*exp(I*(b*x+a)))/b^5-12*d^2*(d*x+c)^2*polyl
og(3,-exp(I*(b*x+a)))/b^3+24*d^3*(d*x+c)*polylog(3,-I*exp(I*(b*x+a)))/b^4-
24*d^3*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))/b^4+12*d^2*(d*x+c)^2*polylog(3,
exp(I*(b*x+a)))/b^3-24*I*d^3*(d*x+c)*polylog(4,-exp(I*(b*x+a)))/b^4-4*I*d*
(d*x+c)^3*polylog(2,exp(I*(b*x+a)))/b^2+24*I*d^3*(d*x+c)*polylog(4,exp(I*(
b*x+a)))/b^4-24*I*d^4*polylog(4,I*exp(I*(b*x+a)))/b^5+24*d^4*polylog(5,-ex
p(I*(b*x+a)))/b^5-24*d^4*polylog(5,exp(I*(b*x+a)))/b^5+(d*x+c)^4*sec(b*x+a
)/b
```

### 3.266.2 Mathematica [A] (verified)

Time = 3.67 (sec) , antiderivative size = 694, normalized size of antiderivative = 1.48

$$\int (c+dx)^4 \csc(a+bx)\sec^2(a+bx)\,dx$$
$$= \frac{b^4(c+dx)^4\log\left(1-e^{i(a+bx)}\right)-b^4(c+dx)^4\log\left(1+e^{i(a+bx)}\right)-4d\left(-2ib^3c^3\arctan\left(e^{i(a+bx)}\right)+3b^3c^2dx\log\right.}{}$$

input
```
Integrate[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

output
```
(b^4*(c + d*x)^4*Log[1 - E^(I*(a + b*x))] - b^4*(c + d*x)^4*Log[1 + E^(I*(
a + b*x))] - 4*d*((-2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^3*c^2*d*x*L
og[1 - I*E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] + b
^3*d^3*x^3*Log[1 - I*E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a +
b*x))] - 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 +
I*E^(I*(a + b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x
))] - (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] - 6*b*c*d^2*Po
lyLog[3, (-I)*E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))
] + 6*b*c*d^2*PolyLog[3, I*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, I*E^(I*
(a + b*x))] - (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] + (6*I)*d^3*PolyL
og[4, I*E^(I*(a + b*x))]) + (4*I)*d*(b^3*(c + d*x)^3*PolyLog[2, -E^(I*(a +
 b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[3, -E^(I*(a + b*x))] - 6*d^2*(b*
(c + d*x)*PolyLog[4, -E^(I*(a + b*x))] + I*d*PolyLog[5, -E^(I*(a + b*x))])
) - (4*I)*d*(b^3*(c + d*x)^3*PolyLog[2, E^(I*(a + b*x))] + (3*I)*b^2*d*(c
+ d*x)^2*PolyLog[3, E^(I*(a + b*x))] - 6*d^2*(b*(c + d*x)*PolyLog[4, E^(I*
(a + b*x))] + I*d*PolyLog[5, E^(I*(a + b*x))])) + b^4*(c + d*x)^4*Sec[a +
b*x])/b^5
```

### 3.266.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4920, 25, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \csc(a+bx)\sec^2(a+bx)\,dx$$

$$\Big\downarrow \text{4920}$$

$$-4d\int -(c+dx)^3\left(\frac{\text{arctanh}(\cos(a+bx))}{b}-\frac{\sec(a+bx)}{b}\right)dx-\frac{(c+dx)^4\text{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^4\sec(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$4d\int (c+dx)^3\left(\frac{\text{arctanh}(\cos(a+bx))}{b}-\frac{\sec(a+bx)}{b}\right)dx-\frac{(c+dx)^4\text{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^4\sec(a+bx)}{b}$$

$$\Big\downarrow \text{7292}$$

$$4d\int \frac{(c+dx)^3(\text{arctanh}(\cos(a+bx))-\sec(a+bx))}{b}dx-\frac{(c+dx)^4\text{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^4\sec(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{4d\int (c+dx)^3(\text{arctanh}(\cos(a+bx))-\sec(a+bx))dx}{b}-\frac{(c+dx)^4\text{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^4\sec(a+bx)}{b}$$

$$\Big\downarrow \text{7293}$$

$$\frac{4d\int \left((c+dx)^3\text{arctanh}(\cos(a+bx))-(c+dx)^3\sec(a+bx)\right)dx}{b}-\frac{(c+dx)^4\text{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^4\sec(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4d\left(\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{(c+dx)^4\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{2d}+\frac{(c+dx)^4\operatorname{arctanh}(\cos(a+bx))}{4d}+\frac{6id^3\operatorname{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^4}-\frac{6id^3\operatorname{Po}}{}\right.}{}$$

$$\frac{(c+dx)^4\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^4\sec(a+bx)}{b}$$

input
```
Int[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

output
```
-(((c + d*x)^4*ArcTanh[Cos[a + b*x]])/b) + (4*d*(((2*I)*(c + d*x)^3*ArcTan
[E^(I*(a + b*x))])/b - ((c + d*x)^4*ArcTanh[E^(I*(a + b*x))])/(2*d) + ((c
+ d*x)^4*ArcTanh[Cos[a + b*x]])/(4*d) + (I*(c + d*x)^3*PolyLog[2, -E^(I*(a
 + b*x))])/b - ((3*I)*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2
+ ((3*I)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (I*(c + d*x)^3
*PolyLog[2, E^(I*(a + b*x))])/b - (3*d*(c + d*x)^2*PolyLog[3, -E^(I*(a + b
*x))])/b^2 + (6*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 - (6*d
^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + (3*d*(c + d*x)^2*PolyLog
[3, E^(I*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)*PolyLog[4, -E^(I*(a + b*x
))])/b^3 + ((6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^4 - ((6*I)*d^3*P
olyLog[4, I*E^(I*(a + b*x))])/b^4 + ((6*I)*d^2*(c + d*x)*PolyLog[4, E^(I*(
a + b*x))])/b^3 + (6*d^3*PolyLog[5, -E^(I*(a + b*x))])/b^4 - (6*d^3*PolyLo
g[5, E^(I*(a + b*x))])/b^4))/b + ((c + d*x)^4*Sec[a + b*x])/b
```

### 3.266.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### 3.266.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1865 vs. 2(422) = 844.

Time = 2.05 (sec) , antiderivative size = 1866, normalized size of antiderivative = 3.98

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1866 |

input
```
int((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^5*a^4*d^4*ln(exp(I*(b*x+a))-1)+12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a))
)-24/b^4*d^4*polylog(3,I*exp(I*(b*x+a)))*x+24/b^4*d^4*polylog(3,-I*exp(I*(
b*x+a)))*x+12/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x^2-12/b^3*d^4*polylog(3,-
exp(I*(b*x+a)))*x^2+4/b^5*a^3*d^4*ln(1+I*exp(I*(b*x+a)))+4/b^2*d^4*ln(1+I*
exp(I*(b*x+a)))*x^3-4/b^2*d^4*ln(1-I*exp(I*(b*x+a)))*x^3-24/b^4*c*d^3*poly
log(3,I*exp(I*(b*x+a)))+24/b^4*c*d^3*polylog(3,-I*exp(I*(b*x+a)))-4/b^5*a^
3*d^4*ln(1-I*exp(I*(b*x+a)))+1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4-1/b^5*d^4*ln
(1-exp(I*(b*x+a)))*a^4-1/b*d^4*ln(exp(I*(b*x+a))+1)*x^4-24*I*d^4*polylog(4
,I*exp(I*(b*x+a)))/b^5+12/b^3*c^2*d^2*ln(1+I*exp(I*(b*x+a)))*a-12/b^3*c^2*
d^2*ln(1-I*exp(I*(b*x+a)))*a-4/b*c^3*d*ln(exp(I*(b*x+a))+1)*x+4/b*c^3*d*ln
(1-exp(I*(b*x+a)))*x-24/b^3*d^3*c*polylog(3,-exp(I*(b*x+a)))*x+12/b^4*c*d^
3*ln(1-I*exp(I*(b*x+a)))*a^2+4/b*d^3*c*ln(1-exp(I*(b*x+a)))*x^3+4/b^4*d^3*
c*ln(1-exp(I*(b*x+a)))*a^3-4/b*d^3*c*ln(exp(I*(b*x+a))+1)*x^3+24/b^3*d^3*c
*polylog(3,exp(I*(b*x+a)))*x-12/b^4*c*d^3*ln(1+I*exp(I*(b*x+a)))*a^2+6/b^3
*a^2*c^2*d^2*ln(exp(I*(b*x+a))-1)+6/b*c^2*d^2*ln(1-exp(I*(b*x+a)))*x^2-4/b
^4*a^3*c*d^3*ln(exp(I*(b*x+a))-1)+24*d^4*polylog(5,-exp(I*(b*x+a)))/b^5-24
*d^4*polylog(5,exp(I*(b*x+a)))/b^5-12/b^3*c^2*d^2*polylog(3,-exp(I*(b*x+a)
))-1/b*c^4*ln(exp(I*(b*x+a))+1)+1/b*c^4*ln(exp(I*(b*x+a))-1)+2*exp(I*(b*x+
a))*(d^4*x^4+4*c*d^3*x^3+6*c^2*d^2*x^2+4*c^3*d*x+c^4)/b/(exp(2*I*(b*x+a))+
1)+24*I*d^4*polylog(4,-I*exp(I*(b*x+a)))/b^5-24*I/b^3*c^2*d^2*a*arctan(...
```

### 3.266.5 Fricas [F(-2)]

Exception generated.

$$\int (c+dx)^4 \csc(a+bx) \sec^2(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   Too
many variables
```

### 3.266.6 Sympy [F]

$$\int (c+dx)^4 \csc(a+bx) \sec^2(a+bx)\,dx = \int (c+dx)^4 \csc(a+bx) \sec^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**4*csc(b*x+a)*sec(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**4*csc(a + b*x)*sec(a + b*x)**2, x)
```

### 3.266.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5709 vs. $2(403) = 806$.

Time = 1.77 (sec) , antiderivative size = 5709, normalized size of antiderivative = 12.17

$$\int (c+dx)^4 \csc(a+bx) \sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(c^4*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
- 4*a*c^3*d*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1
))/b + 6*a^2*c^2*d^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x
 + a) - 1))/b^2 - 4*a^3*c*d^3*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + lo
g(cos(b*x + a) - 1))/b^3 + a^4*d^4*(2/cos(b*x + a) - log(cos(b*x + a) + 1)
 + log(cos(b*x + a) - 1))/b^4 + 2*(8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*
b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*
(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) + (b^3*c^3*d - 3*a*b^2*c^2
*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*
x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x +
2*a) + (I*b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 + I*(b*x + a)^3*
d^4 - I*a^3*d^4 + 3*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 -
 2*I*a*b*c*d^3 + I*a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x +
 a), sin(b*x + a) + 1) + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 +
(b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d
^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) + (b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a
^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 +
 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (I*
b^3*c^3*d - 3*I*a*b^2*c^2*d^2 + 3*I*a^2*b*c*d^3 + I*(b*x + a)^3*d^4 - I*a^
3*d^4 + 3*(I*b*c*d^3 - I*a*d^4)*(b*x + a)^2 + 3*(I*b^2*c^2*d^2 - 2*I*a*...
```

### 3.266.8 Giac [F]

$$\int (c+dx)^4 \csc(a+bx) \sec^2(a+bx) \, dx = \int (dx+c)^4 \csc(bx+a) \sec(bx+a)^2 \, dx$$

input
```
integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*csc(b*x + a)*sec(b*x + a)^2, x)
```

### 3.266.9 Mupad [F(-1)]

Timed out.

$$\int (c + dx)^4 \csc(a + bx) \sec^2(a + bx) \, dx = \text{Hanged}$$

input
```
int((c + d*x)^4/(cos(a + b*x)^2*sin(a + b*x)),x)
```

output
```
\text{Hanged}
```

# 3.267 $\int (c+dx)^3 \csc(a+bx) \sec^2(a+bx)\,dx$

3.267.1 Optimal result . . . . . . . . . . 2047
3.267.2 Mathematica [A] (verified) . . . . . . . . . . 2048
3.267.3 Rubi [A] (verified) . . . . . . . . . . 2048
3.267.4 Maple [B] (verified) . . . . . . . . . . 2051
3.267.5 Fricas [B] (verification not implemented) . . . . . . . . . . 2052
3.267.6 Sympy [F] . . . . . . . . . . 2052
3.267.7 Maxima [B] (verification not implemented) . . . . . . . . . . 2053
3.267.8 Giac [F] . . . . . . . . . . 2053
3.267.9 Mupad [F(-1)] . . . . . . . . . . 2054

## 3.267.1 Optimal result

Integrand size = 22, antiderivative size = 343

$$\int (c+dx)^3 \csc(a+bx)\sec^2(a+bx)\,dx = \frac{6id(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} - \frac{6id^2(c+dx)\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} + \frac{6id^2(c+dx)\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} - \frac{3id(c+dx)^2\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} - \frac{6d^2(c+dx)\,\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} + \frac{6d^3\,\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4} - \frac{6d^3\,\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^4} + \frac{6d^2(c+dx)\,\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3} - \frac{6id^3\,\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4} + \frac{6id^3\,\text{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^4} + \frac{(c+dx)^3\sec(a+bx)}{b}$$

output
```
6*I*d*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b^2-2*(d*x+c)^3*arctanh(exp(I*(b*x+
a)))/b+3*I*d*(d*x+c)^2*polylog(2,-exp(I*(b*x+a)))/b^2-6*I*d^2*(d*x+c)*poly
log(2,-I*exp(I*(b*x+a)))/b^3+6*I*d^2*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b
^3-3*I*d*(d*x+c)^2*polylog(2,exp(I*(b*x+a)))/b^2-6*d^2*(d*x+c)*polylog(3,-
exp(I*(b*x+a)))/b^3+6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3
,I*exp(I*(b*x+a)))/b^4+6*d^2*(d*x+c)*polylog(3,exp(I*(b*x+a)))/b^3-6*I*d^3
*polylog(4,-exp(I*(b*x+a)))/b^4+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+(d*x
+c)^3*sec(b*x+a)/b
```

### 3.267.2 Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.38

$$\int (c+dx)^3\csc(a+bx)\sec^2(a+bx)\,dx$$
$$=\frac{-2b^3(c+dx)^3\text{arctanh}(\cos(a+bx)+i\sin(a+bx))-3d\left(-2ib^2c^2\arctan\left(e^{i(a+bx)}\right)+2b^2cdx\log\left(1-ie^{i(a-}\right.\right.}{}$$

input
```
Integrate[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

output
```
(-2*b^3*(c + d*x)^3*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]] - 3*d*((-2*I)*b
^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 - I*E^(I*(a + b*x))] +
b^2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - 2*b^2*c*d*x*Log[1 + I*E^(I*(a + b
*x))] - b^2*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyL
og[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b
*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 2*d^2*PolyLog[3, I*E^(I*(
a + b*x))]) + (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[a
+ b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] -
 2*d^2*PolyLog[4, -Cos[a + b*x] - I*Sin[a + b*x]]) - (3*I)*d*(b^2*(c + d*x
)^2*PolyLog[2, Cos[a + b*x] + I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLo
g[3, Cos[a + b*x] + I*Sin[a + b*x]] - 2*d^2*PolyLog[4, Cos[a + b*x] + I*Si
n[a + b*x]]) + b^3*(c + d*x)^3*Sec[a + b*x])/b^4
```

### 3.267.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4920, 25, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\csc(a+bx)\sec^2(a+bx)\,dx$$
$$\Big\downarrow \text{4920}$$
$$-3d\int -(c+dx)^2\left(\frac{\text{arctanh}(\cos(a+bx))}{b}-\frac{\sec(a+bx)}{b}\right)dx-\frac{(c+dx)^3\text{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^3\sec(a+bx)}{b}$$

$$\Big\downarrow \text{25}$$

$$3d\int (c+dx)^2\left(\frac{\operatorname{arctanh}(\cos(a+bx))}{b}-\frac{\sec(a+bx)}{b}\right)dx-\frac{(c+dx)^3\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^3\sec(a+bx)}{b}$$

$$\Big\downarrow \text{7292}$$

$$3d\int \frac{(c+dx)^2(\operatorname{arctanh}(\cos(a+bx))-\sec(a+bx))}{b}dx-\frac{(c+dx)^3\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^3\sec(a+bx)}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{3d\int (c+dx)^2(\operatorname{arctanh}(\cos(a+bx))-\sec(a+bx))dx}{b}-\frac{(c+dx)^3\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^3\sec(a+bx)}{b}$$

$$\Big\downarrow \text{7293}$$

$$\frac{3d\int \left((c+dx)^2\operatorname{arctanh}(\cos(a+bx))-(c+dx)^2\sec(a+bx)\right)dx}{b}-\frac{(c+dx)^3\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^3\sec(a+bx)}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3d\left(\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{2(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{3d}+\frac{(c+dx)^3\operatorname{arctanh}(\cos(a+bx))}{3d}+\frac{2d^2\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}-\frac{2d^2\operatorname{Pol}}{}\right.}{}$$

$$\frac{(c+dx)^3\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^3\sec(a+bx)}{b}$$

input
```
Int[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

3.267. $\int (c+dx)^3\csc(a+bx)\sec^2(a+bx)\,dx$

output
```
-(((c + d*x)^3*ArcTanh[Cos[a + b*x]])/b) + (3*d*(((2*I)*(c + d*x)^2*ArcTan
[E^(I*(a + b*x))])/b - (2*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/(3*d) + ((
c + d*x)^3*ArcTanh[Cos[a + b*x]])/(3*d) + (I*(c + d*x)^2*PolyLog[2, -E^(I*
(a + b*x))])/b - ((2*I)*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2
+ ((2*I)*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (I*(c + d*x)^2*P
olyLog[2, E^(I*(a + b*x))])/b - (2*d*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))
])/b^2 + (2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 - (2*d^2*PolyLog[3,
I*E^(I*(a + b*x))])/b^3 + (2*d*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^2
- ((2*I)*d^2*PolyLog[4, -E^(I*(a + b*x))])/b^3 + ((2*I)*d^2*PolyLog[4, E^(
I*(a + b*x))])/b^3))/b + ((c + d*x)^3*Sec[a + b*x])/b
```

### 3.267.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### 3.267.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1151 vs. $2(308) = 616$.

Time = 1.86 (sec) , antiderivative size = 1152, normalized size of antiderivative = 3.36

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1152 |

input
```
int((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
6*I/b^4*a*d^3*dilog(1+I*exp(I*(b*x+a)))-6*I*d^3*polylog(4,-exp(I*(b*x+a)))
/b^4+6/b^2*d^2*c*ln(1+I*exp(I*(b*x+a)))*x-6/b^2*d^2*c*ln(1-I*exp(I*(b*x+a)
))*x+6/b^3*d^2*c*ln(1+I*exp(I*(b*x+a)))*a-6/b^3*d^2*c*ln(1-I*exp(I*(b*x+a)
))*a+6*I/b^2*d*c^2*arctan(exp(I*(b*x+a)))+6*I/b^4*d^3*a^2*arctan(exp(I*(b*
x+a)))+6*I*d^3*x*polylog(2,I*exp(I*(b*x+a)))/b^3-6*I/b^2*c*d^2*polylog(2,e
xp(I*(b*x+a)))*x+6*I/b^2*c*d^2*polylog(2,-exp(I*(b*x+a)))*x+3/b^3*c*d^2*a^
2*ln(exp(I*(b*x+a))-1)-3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)+3/b^2*d*c^2*ln(1
-exp(I*(b*x+a)))*a-3/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2+6*d^3*polylog(3,-I
*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4-1/b*c^3*ln(exp(
I*(b*x+a))+1)+1/b*c^3*ln(exp(I*(b*x+a))-1)+3/b^2*d^3*ln(1+I*exp(I*(b*x+a))
)*x^2+3/b^4*d^3*a^2*ln(1-I*exp(I*(b*x+a)))-3/b^4*d^3*a^2*ln(1+I*exp(I*(b*x
+a)))-3/b^2*d^3*ln(1-I*exp(I*(b*x+a)))*x^2+3/b*d*c^2*ln(1-exp(I*(b*x+a)))*
x-3/b*d*c^2*ln(exp(I*(b*x+a))+1)*x+3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-3/b*
c*d^2*ln(exp(I*(b*x+a))+1)*x^2-3*I/b^2*d*c^2*polylog(2,exp(I*(b*x+a)))-3*I
/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+
1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-6
/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3-1/b
*d^3*ln(exp(I*(b*x+a))+1)*x^3+6/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))-6/b^3*
c*d^2*polylog(3,-exp(I*(b*x+a)))-12*I/b^3*d^2*c*a*arctan(exp(I*(b*x+a)))+6
*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+2*exp(I*(b*x+a))*(d^3*x^3+3*c*d^2*...
```

### 3.267.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1705 vs. $2(293) = 586$.

Time = 0.37 (sec) , antiderivative size = 1705, normalized size of antiderivative = 4.97

$$\int (c+dx)^3 \csc(a+bx) \sec^2(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*I*d^3
*cos(b*x + a)*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*cos(b*x
+ a)*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*cos(b*x + a)*poly
log(4, -cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*cos(b*x + a)*polylog(4, -
cos(b*x + a) - I*sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3, I*cos(b*x +
 a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*polylog(3, I*cos(b*x + a) - sin(b
*x + a)) + 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) -
 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 3*(I*b^2*
d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*cos(b*x + a)*dilog(cos(b*x + a) +
 I*sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*cos(
b*x + a)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 6*(-I*b*d^3*x - I*b*c*d^2)
*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)) - 6*(-I*b*d^3*x - I*b*c
*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 6*(I*b*d^3*x + I
*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 6*(I*b*d^3*
x + I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*(I*b
^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*cos(b*x + a)*dilog(-cos(b*x +
a) + I*sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*
cos(b*x + a)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^3*d^3*x^3 + 3*b^3*
c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin
(b*x + a) + 1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log...
```

### 3.267.6 Sympy [F]

$$\int (c+dx)^3 \csc(a+bx) \sec^2(a+bx) \, dx = \int (c+dx)^3 \csc(a+bx) \sec^2(a+bx) \, dx$$

input
```
integrate((d*x+c)**3*csc(b*x+a)*sec(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**3*csc(a + b*x)*sec(a + b*x)**2, x)
```

### 3.267.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3202 vs. 2(293) = 586.

Time = 0.85 (sec) , antiderivative size = 3202, normalized size of antiderivative = 9.34

$$\int (c+dx)^3 \csc(a+bx) \sec^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(c^3*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
- 3*a*c^2*d*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1
))/b + 3*a^2*c*d^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x +
 a) - 1))/b^2 - a^3*d^3*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(
b*x + a) - 1))/b^3 + 2*(6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2
*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) + (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a
)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (I*b
^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 -
I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) +
 1) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2
- a*d^3)*(b*x + a) + (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3
+ 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (I*b^2*c^2*d - 2*I*a*b
*c*d^2 + I*(b*x + a)^2*d^3 + I*a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a)
)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 2*((b*x + a
)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)*(b*x + a) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3
*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*
x + a)^3*d^3 + 3*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 3*(-I*b^2*c^2*d + 2*
I*a*b*c*d^2 - I*a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a)
, cos(b*x + a) + 1) - 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a...
```

### 3.267.8 Giac [F]

$$\int (c+dx)^3 \csc(a+bx) \sec^2(a+bx)\, dx = \int (dx+c)^3 \csc(bx+a) \sec(bx+a)^2 \, dx$$

input
```
integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*csc(b*x + a)*sec(b*x + a)^2, x)
```

### 3.267.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc(a+bx) \sec^2(a+bx)\, dx = \text{Hanged}$$

input
```
int((c + d*x)^3/(cos(a + b*x)^2*sin(a + b*x)),x)
```

output
```
\text{Hanged}
```

# 3.268 $\int (c+dx)^2 \csc(a+bx) \sec^2(a+bx) \, dx$


3.268.1 Optimal result . . . 2055
3.268.2 Mathematica [A] (verified) . . . 2056
3.268.3 Rubi [A] (verified) . . . 2056
3.268.4 Maple [B] (verified) . . . 2058
3.268.5 Fricas [B] (verification not implemented) . . . 2059
3.268.6 Sympy [F] . . . 2060
3.268.7 Maxima [B] (verification not implemented) . . . 2061
3.268.8 Giac [F] . . . 2061
3.268.9 Mupad [F(-1)] . . . 2062


## 3.268.1 Optimal result

Integrand size = 22, antiderivative size = 219

$$\int (c+dx)^2 \csc(a+bx) \sec^2(a+bx) \, dx = \frac{4id(c+dx) \arctan\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^2 \operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c+dx) \operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} - \frac{2id^2 \operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} + \frac{2id^2 \operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} - \frac{2id(c+dx) \operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} - \frac{2d^2 \operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} + \frac{2d^2 \operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^2 \sec(a+bx)}{b}$$

output
```
4*I*d*(d*x+c)*arctan(exp(I*(b*x+a)))/b^2-2*(d*x+c)^2*arctanh(exp(I*(b*x+a)
))/b+2*I*d*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^2-2*I*d^2*polylog(2,-I*exp
(I*(b*x+a)))/b^3+2*I*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3-2*I*d*(d*x+c)*pol
ylog(2,exp(I*(b*x+a)))/b^2-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*poly
log(3,exp(I*(b*x+a)))/b^3+(d*x+c)^2*sec(b*x+a)/b
```

### 3.268.2 Mathematica [A] (verified)

Time = 2.81 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.45

$$\int (c+dx)^2 \csc(a+bx)\sec^2(a+bx)\,dx$$
$$= \frac{-4bcd\text{arctanh}\left(\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)\right)-4d^2\arctan(\cot(a))\text{arctanh}\left(\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)\right)+b^2(c+}{}$$

input
```
Integrate[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

output
```
(-4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - 4*d^2*ArcTan[Cot[a]]*Arc
Tanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + b^2*(c + d*x)^2*Log[1 - E^(I*(a + b*x
))] - b^2*(c + d*x)^2*Log[1 + E^(I*(a + b*x))] + (2*d^2*Csc[a]*((b*x - Arc
Tan[Cot[a]])*(Log[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x -
ArcTan[Cot[a]]))]) + I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - I*PolyL
og[2, E^(I*(b*x - ArcTan[Cot[a]]))]))/Sqrt[Csc[a]^2] + (2*I)*d*(b*(c + d*x
)*PolyLog[2, -E^(I*(a + b*x))] + I*d*PolyLog[3, -E^(I*(a + b*x))]) + 2*d*(
(-I)*b*(c + d*x)*PolyLog[2, E^(I*(a + b*x))] + d*PolyLog[3, E^(I*(a + b*x)
)]) + b^2*(c + d*x)^2*Sec[a + b*x])/b^3
```

### 3.268.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4920, 25, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \csc(a+bx)\sec^2(a+bx)\,dx$$
$$\Big\downarrow \text{4920}$$
$$-2d\int -\left((c+dx)\left(\frac{\text{arctanh}(\cos(a+bx))}{b}-\frac{\sec(a+bx)}{b}\right)\right)dx-\frac{(c+dx)^2\text{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}$$
$$\Big\downarrow \text{25}$$

$$2d\int (c+dx)\left(\frac{\text{arctanh}(\cos(a+bx))}{b}-\frac{\sec(a+bx)}{b}\right)dx-\frac{(c+dx)^2\text{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

$$\Big\downarrow \; 7292$$

$$2d\int \frac{(c+dx)(\text{arctanh}(\cos(a+bx))-\sec(a+bx))}{b}dx-\frac{(c+dx)^2\text{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{2d\int (c+dx)(\text{arctanh}(\cos(a+bx))-\sec(a+bx))dx}{b}-\frac{(c+dx)^2\text{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

$$\Big\downarrow \; 7293$$

$$\frac{2d\int ((c+dx)\text{arctanh}(\cos(a+bx))-(c+dx)\sec(a+bx))dx}{b}-\frac{(c+dx)^2\text{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

$$\Big\downarrow \; 2009$$

$$\frac{2d\left(\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{d}+\frac{(c+dx)^2\text{arctanh}(\cos(a+bx))}{2d}-\frac{id\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}+\frac{id\,\text{PolyLog}}{}\right.}{b}$$
$$\frac{(c+dx)^2\text{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

input
```
Int[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

output
```
-(((c + d*x)^2*ArcTanh[Cos[a + b*x]])/b) + (2*d*(((2*I)*(c + d*x)*ArcTan[E
^(I*(a + b*x))])/b - ((c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/d + ((c + d*x)
^2*ArcTanh[Cos[a + b*x]])/(2*d) + (I*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))
])/b - (I*d*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 + (I*d*PolyLog[2, I*E^(I
*(a + b*x))])/b^2 - (I*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b - (d*PolyL
og[3, -E^(I*(a + b*x))])/b^2 + (d*PolyLog[3, E^(I*(a + b*x))])/b^2))/b + (
(c + d*x)^2*Sec[a + b*x])/b
```

3.268. $\int (c+dx)^2\csc(a+bx)\sec^2(a+bx)\,dx$

#### 3.268.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### 3.268.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(196) = 392$.

Time = 1.65 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.59

| method | result |
|---|---|
| risch | $-\frac{2id^2 \operatorname{polylog}(2,e^{i(xb+a)})x}{b^2}-\frac{2idc \operatorname{polylog}(2,e^{i(xb+a)})}{b^2}+\frac{2dc\ln(1-e^{i(xb+a)})x}{b}+\frac{2dc\ln(1-e^{i(xb+a)})a}{b^2}+\frac{2d^2 \operatorname{polylog}(3,e^{i(xb}}{b^3}$ |

input
```
int((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*
x+2*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-2*I/b^2*d^2*polylog(2,exp(I*(b*x+
a)))*x+4*I/b^2*d*c*arctan(exp(I*(b*x+a)))-4*I/b^3*d^2*a*arctan(exp(I*(b*x+
a)))-1/b*c^2*ln(exp(I*(b*x+a))+1)+1/b*c^2*ln(exp(I*(b*x+a))-1)-1/b^3*d^2*l
n(1-exp(I*(b*x+a)))*a^2+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b*d^2*ln(exp(I*
(b*x+a))+1)*x^2+2*exp(I*(b*x+a))*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*x+a))
+1)+2*I/b^3*d^2*dilog(1-I*exp(I*(b*x+a)))-2/b*d*c*ln(exp(I*(b*x+a))+1)*x+2
/b^2*d*c*ln(1-exp(I*(b*x+a)))*a-2/b^2*c*d*a*ln(exp(I*(b*x+a))-1)+2/b*d*c*l
n(1-exp(I*(b*x+a)))*x-2*I/b^2*d*c*polylog(2,exp(I*(b*x+a)))+2/b^2*d^2*ln(1
+I*exp(I*(b*x+a)))*x+2/b^3*d^2*ln(1+I*exp(I*(b*x+a)))*a-2/b^2*d^2*ln(1-I*e
xp(I*(b*x+a)))*x-2/b^3*d^2*ln(1-I*exp(I*(b*x+a)))*a-2*I/b^3*d^2*dilog(1+I*
exp(I*(b*x+a)))-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp(I
*(b*x+a)))/b^3
```

### 3.268.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1035 vs. $2(185) = 370$.

Time = 0.32 (sec) , antiderivative size = 1035, normalized size of antiderivative = 4.73

$$\int (c+dx)^2 \csc(a+bx) \sec^2(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 2*I*d^2*cos(b*x + a)*dilog(
I*cos(b*x + a) + sin(b*x + a)) + 2*I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a)
 - sin(b*x + a)) - 2*I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x +
a)) - 2*I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*c
os(b*x + a)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*cos(b*x + a)
*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 2*d^2*cos(b*x + a)*polylog(3,
 -cos(b*x + a) + I*sin(b*x + a)) - 2*d^2*cos(b*x + a)*polylog(3, -cos(b*x
+ a) - I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*cos(b*x + a)*dilog(cos(b*
x + a) + I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*cos(b*x + a)*dilog(cos
(b*x + a) - I*sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*cos(b*x + a)*dilog(-
cos(b*x + a) + I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*cos(b*x + a)*dil
og(-cos(b*x + a) - I*sin(b*x + a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b*c*d - a*d^2)*c
os(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*
c*d*x + b^2*c^2)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + 2*(
b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + I) - 2*(b*
d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(b*
d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - 2*(b*
d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(b
*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + ...
```

### 3.268.6 Sympy [F]

$$\int (c+dx)^2 \csc(a+bx)\sec^2(a+bx)\,dx = \int (c+dx)^2 \csc(a+bx)\sec^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*csc(b*x+a)*sec(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*csc(a + b*x)*sec(a + b*x)**2, x)
```

### 3.268.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1590 vs. 2(185) = 370.

Time = 0.47 (sec) , antiderivative size = 1590, normalized size of antiderivative = 7.26

$$\int (c+dx)^2 \csc(a+bx)\sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(c^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
- 2*a*c*d*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b + a^2*d^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) -
1))/b^2 + 2*(4*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a
*d^2)*cos(2*b*x + 2*a) + (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(2*b*x +
 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + 4*(b*c*d + (b*x + a)*d^2
- a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (I*b*c*d + I*
(b*x + a)*d^2 - I*a*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x
+ a) + 1) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + ((b*x + a)^
2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^
2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x +
a), cos(b*x + a) + 1) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) +
 ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (-I*(b
*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan
2(sin(b*x + a), -cos(b*x + a) + 1) + 4*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d +
 I*a*d^2)*(b*x + a))*cos(b*x + a) + 4*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*
b*x + 2*a) + d^2)*dilog(I*e^(I*b*x + I*a)) - 4*(d^2*cos(2*b*x + 2*a) + I*d
^2*sin(2*b*x + 2*a) + d^2)*dilog(-I*e^(I*b*x + I*a)) + 4*(b*c*d + (b*x + a
)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (I*b*c*
d + I*(b*x + a)*d^2 - I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)...
```

### 3.268.8 Giac [F]

$$\int (c+dx)^2 \csc(a+bx)\sec^2(a+bx)\,dx = \int (dx+c)^2 \csc(bx+a)\sec(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csc(b*x + a)*sec(b*x + a)^2, x)
```

### 3.268.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc(a+bx) \sec^2(a+bx)\, dx = \text{Hanged}$$

input
```
int((c + d*x)^2/(cos(a + b*x)^2*sin(a + b*x)),x)
```

output
```
\text{Hanged}
```

# 3.269 $\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx$

3.269.1 Optimal result . . . 2063
3.269.2 Mathematica [A] (verified) . . . 2064
3.269.3 Rubi [A] (verified) . . . 2064
3.269.4 Maple [A] (verified) . . . 2065
3.269.5 Fricas [B] (verification not implemented) . . . 2066
3.269.6 Sympy [F] . . . 2066
3.269.7 Maxima [B] (verification not implemented) . . . 2067
3.269.8 Giac [F] . . . 2067
3.269.9 Mupad [F(-1)] . . . 2068

## 3.269.1 Optimal result

Integrand size = 20, antiderivative size = 113

$$\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx = -\frac{2dx\text{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{c\text{arctanh}(\cos(a+bx))}{b} - \frac{d\text{arctanh}(\sin(a+bx))}{b^2} + \frac{id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} - \frac{id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} + \frac{c\sec(a+bx)}{b} + \frac{dx\sec(a+bx)}{b}$$

output
```
-2*d*x*arctanh(exp(I*(b*x+a)))/b-c*arctanh(cos(b*x+a))/b-d*arctanh(sin(b*x
+a))/b^2+I*d*polylog(2,-exp(I*(b*x+a)))/b^2-I*d*polylog(2,exp(I*(b*x+a)))/
b^2+c*sec(b*x+a)/b+d*x*sec(b*x+a)/b
```

### 3.269.2 Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.88

$$\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx = -\frac{c\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b} + \frac{d\log\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b^2} + \frac{c\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b} - \frac{d\log\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b^2} - \frac{ad\log\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)}{b^2} + \frac{d\left((a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)+i\left(\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)-\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)\right)\right)}{b^2} + \frac{c\sec(a+bx)}{b} + \frac{dx\sec(a+bx)}{b}$$

input
```
Integrate[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

output
```
-((c*Log[Cos[(a + b*x)/2]])/b) + (d*Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2
]])/b^2 + (c*Log[Sin[(a + b*x)/2]])/b - (d*Log[Cos[(a + b*x)/2] + Sin[(a +
 b*x)/2]])/b^2 - (a*d*Log[Tan[(a + b*x)/2]])/b^2 + (d*((a + b*x)*(Log[1 -
E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + I*(PolyLog[2, -E^(I*(a + b*
x))] - PolyLog[2, E^(I*(a + b*x))])))/b^2 + (c*Sec[a + b*x])/b + (d*x*Sec[
a + b*x])/b
```

### 3.269.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4920, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx$$

$$\Big\downarrow \text{4920}$$

$$-d\int\left(\frac{\sec(a+bx)}{b}-\frac{\operatorname{arctanh}(\cos(a+bx))}{b}\right)dx - \frac{(c+dx)\operatorname{arctanh}(\cos(a+bx))}{b} + \frac{(c+dx)\sec(a+bx)}{b}$$

$$\downarrow \text{2009}$$

$$-d\left(\frac{\operatorname{arctanh}(\sin(a+bx))}{b^2}+\frac{2x\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{x\operatorname{arctanh}(\cos(a+bx))}{b}-\frac{i\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}+\frac{i\operatorname{PolyLo}}{}\right.$$
$$\frac{(c+dx)\operatorname{arctanh}(\cos(a+bx))}{b}+\frac{(c+dx)\sec(a+bx)}{b}$$

input
```
Int[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

output
```
-(((c + d*x)*ArcTanh[Cos[a + b*x]])/b) - d*((2*x*ArcTanh[E^(I*(a + b*x))])
/b - (x*ArcTanh[Cos[a + b*x]])/b + ArcTanh[Sin[a + b*x]]/b^2 - (I*PolyLog[
2, -E^(I*(a + b*x))])/b^2 + (I*PolyLog[2, E^(I*(a + b*x))])/b^2) + ((c + d
*x)*Sec[a + b*x])/b
```

### 3.269.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

## 3.269.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.42

| method | result |
|---|---|
| risch | $\frac{2\,\mathrm{e}^{i(xb+a)}(dx+c)}{b\left(\mathrm{e}^{2i(xb+a)}+1\right)}+\frac{c\ln\left(\mathrm{e}^{i(xb+a)}-1\right)}{b}+\frac{2id\arctan\left(\mathrm{e}^{i(xb+a)}\right)}{b^2}+\frac{id\operatorname{dilog}\left(\mathrm{e}^{i(xb+a)}\right)}{b^2}+\frac{id\operatorname{dilog}\left(\mathrm{e}^{i(xb+a)}+1\right)}{b^2}-\frac{da\ln\left(\mathrm{e}^{i(xb+a)}-\right.}{b^2}$ |

input
```
int((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*exp(I*(b*x+a))*(d*x+c)/b/(exp(2*I*(b*x+a))+1)+1/b*c*ln(exp(I*(b*x+a))-1)
+2*I/b^2*d*arctan(exp(I*(b*x+a)))+I/b^2*d*dilog(exp(I*(b*x+a)))+I/b^2*d*di
log(exp(I*(b*x+a))+1)-1/b^2*d*a*ln(exp(I*(b*x+a))-1)-1/b*d*ln(exp(I*(b*x+a
))+1)*x-1/b*c*ln(exp(I*(b*x+a))+1)
```

### 3.269.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 366 vs. 2(101) = 202.

Time = 0.27 (sec) , antiderivative size = 366, normalized size of antiderivative = 3.24

$$\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx = \frac{2\,bdx - i\,d\cos(bx+a)\,\text{Li}_2(\cos(bx+a)+i\,\sin(bx+a)) + i\,d\cos(bx+a)\,\text{Li}_2(\cos(bx+a)-i\,\sin(bx+a)}{}$$

input
```
integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(2*b*d*x - I*d*cos(b*x + a)*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d
*cos(b*x + a)*dilog(cos(b*x + a) - I*sin(b*x + a)) - I*d*cos(b*x + a)*dilo
g(-cos(b*x + a) + I*sin(b*x + a)) + I*d*cos(b*x + a)*dilog(-cos(b*x + a) -
 I*sin(b*x + a)) - (b*d*x + b*c)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x
 + a) + 1) - (b*d*x + b*c)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a)
+ 1) + (b*c - a*d)*cos(b*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a)
 + 1/2) + (b*c - a*d)*cos(b*x + a)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x +
 a) + 1/2) + (b*d*x + a*d)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a)
 + 1) + (b*d*x + a*d)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)
 - d*cos(b*x + a)*log(sin(b*x + a) + 1) + d*cos(b*x + a)*log(-sin(b*x + a)
 + 1) + 2*b*c)/(b^2*cos(b*x + a))
```

### 3.269.6 Sympy [F]

$$\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx = \int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*csc(a + b*x)*sec(a + b*x)**2, x)
```

### 3.269.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 800 vs. 2(101) = 202.

Time = 0.44 (sec) , antiderivative size = 800, normalized size of antiderivative = 7.08

$$\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(2*(d*cos(2*b*x + 2*a) + I*d*sin(2*b*x + 2*a) + d)*arctan2(2*(cos(b*x + 2
*a)*cos(a) + sin(b*x + 2*a)*sin(a))/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a
)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2),
 (cos(b*x + 2*a)^2 - cos(a)^2 + sin(b*x + 2*a)^2 - sin(a)^2)/(cos(b*x + 2*
a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x +
 2*a)*sin(a) + sin(a)^2)) + 2*(b*d*x + b*c + (b*d*x + b*c)*cos(2*b*x + 2*a
) + (I*b*d*x + I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a)
 + 1) - 2*(b*c*cos(2*b*x + 2*a) + I*b*c*sin(2*b*x + 2*a) + b*c)*arctan2(si
n(b*x + a), cos(b*x + a) - 1) + 2*(b*d*x*cos(2*b*x + 2*a) + I*b*d*x*sin(2*
b*x + 2*a) + b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 4*(I*b*d*x
+ I*b*c)*cos(b*x + a) - 2*(d*cos(2*b*x + 2*a) + I*d*sin(2*b*x + 2*a) + d)*
dilog(-e^(I*b*x + I*a)) + 2*(d*cos(2*b*x + 2*a) + I*d*sin(2*b*x + 2*a) + d
)*dilog(e^(I*b*x + I*a)) - (I*b*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(2*b*x
+ 2*a) - (b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)
^2 + 2*cos(b*x + a) + 1) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(2*b*
x + 2*a) + (b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x +
a)^2 - 2*cos(b*x + a) + 1) - (-I*d*cos(2*b*x + 2*a) + d*sin(2*b*x + 2*a) -
 I*d)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x
 + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)
^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(...
```

### 3.269.8 Giac [F]

$$\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx = \int (dx+c)\csc(bx+a)\sec(bx+a)^2\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)*csc(b*x + a)*sec(b*x + a)^2, x)
```

### 3.269.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)/(cos(a + b*x)^2*sin(a + b*x)),x)
```

output
```
\text{Hanged}
```

# 3.270 $\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx$


3.270.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2069
3.270.2 Mathematica [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . 2069
3.270.3 Rubi [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2070
3.270.4 Maple [N/A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 2070
3.270.5 Fricas [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2071
3.270.6 Sympy [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2071
3.270.7 Maxima [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2071
3.270.8 Giac [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2072
3.270.9 Mupad [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2072


## 3.270.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \operatorname{Int}\left(\frac{\csc(a+bx)\sec^2(a+bx)}{c+dx},x\right)$$

output
```
CannotIntegrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x)
```

## 3.270.2 Mathematica [N/A]

Not integrable

Time = 19.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x),x]
```

output
```
Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x), x]
```

## 3.270.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx} \, dx$$

$$\Big\downarrow \ 7299$$

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}dx$$

input
```
Int[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x),x]
```

output
```
$Aborted
```

### 3.270.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.270.4 Maple [N/A] (verified)

Not integrable

Time = 0.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\csc \left(xb+a\right) \sec \left(xb+a\right)^{2}}{dx+c}dx$$

input
```
int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x)
```

output
```
int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x)
```

### 3.270.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)*sec(b*x + a)^2/(d*x + c), x)
```

### 3.270.6 Sympy [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(csc(a + b*x)*sec(a + b*x)**2/(c + d*x), x)
```

### 3.270.7 Maxima [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 603, normalized size of antiderivative = 27.41

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
(2*cos(2*b*x + 2*a)*cos(b*x + a) + 2*(b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*
cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x + b
*c*d)*cos(2*b*x + 2*a))*integrate((cos(2*b*x + 2*a)*cos(b*x + a) + sin(2*b
*x + 2*a)*sin(b*x + a) + cos(b*x + a))/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b
*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x
+ b*c^2)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x
+ 2*a)), x) + (b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*si
n(2*b*x + 2*a)^2 + b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a))*integrate(sin(b
*x + a)/((d*x + c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*
x + c)*cos(b*x + a) + c), x) + (b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 +
 (b*d*x + b*c)*sin(2*b*x + 2*a)^2 + b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a)
)*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a
)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) + 2*sin(2*b*x + 2*a)*sin(b*x
 + a) + 2*cos(b*x + a))/(b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x
 + b*c)*sin(2*b*x + 2*a)^2 + b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a))
```

### 3.270.8 Giac [N/A]

Not integrable

Time = 4.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)*sec(b*x + a)^2/(d*x + c), x)
```

### 3.270.9 Mupad [N/A]

Not integrable

Time = 25.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{1}{\cos(a+b\,x)^2\,\sin(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(1/(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)), x)
```

3.270. $\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx$

# 3.271 $\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2} dx$

3.271.1 Optimal result . . . 2074
3.271.2 Mathematica [N/A] . . . 2074
3.271.3 Rubi [N/A] . . . 2075
3.271.4 Maple [N/A] (verified) . . . 2075
3.271.5 Fricas [N/A] . . . 2076
3.271.6 Sympy [N/A] . . . 2076
3.271.7 Maxima [N/A] . . . 2076
3.271.8 Giac [N/A] . . . 2077
3.271.9 Mupad [N/A] . . . 2078

## 3.271.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2} dx = \text{Int}\left(\frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}, x\right)$$

output
```
CannotIntegrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x)
```

## 3.271.2 Mathematica [N/A]

Not integrable

Time = 19.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2} dx = \int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2} dx$$

input
```
Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

## 3.271.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}dx$$

input
```
Int[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
$Aborted
```

### 3.271.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.271.4 Maple [N/A] (verified)

Not integrable

Time = 0.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\csc\left(xb+a\right)\sec\left(xb+a\right)^{2}}{\left(dx+c\right)^{2}}dx$$

input
```
int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x)
```

### 3.271.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)*sec(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.271.6 Sympy [N/A]

Not integrable

Time = 1.97 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)*sec(a + b*x)**2/(c + d*x)**2, x)
```

### 3.271.7 Maxima [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 931, normalized size of antiderivative = 42.32

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
(2*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d +
(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(2*b*x + 2*a)^2 + (b*d^3*x^2 + 2*b*
c*d^2*x + b*c^2*d)*sin(2*b*x + 2*a)^2 + 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2
*d)*cos(2*b*x + 2*a))*integrate((cos(2*b*x + 2*a)*cos(b*x + a) + sin(2*b*x
 + 2*a)*sin(b*x + a) + cos(b*x + a))/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*
d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(2*b*x
+ 2*a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(2*b*x + 2
*a)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(2*b*x + 2*
a)), x) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)
*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*a)^2 +
 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a
)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2
 + 2*c*d*x + c^2)*sin(b*x + a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b
*x + a)), x) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b
*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2*a
)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a))*integrate(sin(b*
x + a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^
2*x^2 + 2*c*d*x + c^2)*sin(b*x + a)^2 + c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*
cos(b*x + a)), x) + 2*sin(2*b*x + 2*a)*sin(b*x + a) + 2*cos(b*x + a))/(b*d
^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x ...
```

### 3.271.8 Giac [N/A]

Not integrable

Time = 4.10 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
sage0*x
```

### 3.271.9 Mupad [N/A]

Not integrable

Time = 26.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx=\int \frac{1}{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(1/(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^2),x)
```

output
```
int(1/(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^2), x)
```

# 3.272 $\int (c+dx)^m \csc^2(a+bx) \sec^2(a+bx) \, dx$

3.272.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2079
3.272.2 Mathematica [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . 2079
3.272.3 Rubi [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2080
3.272.4 Maple [N/A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 2080
3.272.5 Fricas [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2081
3.272.6 Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . 2081
3.272.7 Maxima [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . 2081
3.272.8 Giac [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2082
3.272.9 Mupad [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2082

## 3.272.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int (c+dx)^m \csc^2(a+bx) \sec^2(a+bx) \, dx = \operatorname{Int}\left((c+dx)^m \csc^2(a+bx) \sec^2(a+bx), x\right)$$

output
```
CannotIntegrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

## 3.272.2 Mathematica [N/A]

Not integrable

Time = 3.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^2(a+bx) \sec^2(a+bx) \, dx = \int (c+dx)^m \csc^2(a+bx) \sec^2(a+bx) \, dx$$

input
```
Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2, x]
```

### 3.272.3 Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\sec^2(a+bx)(c+dx)^m\,dx$$
$$\Big\downarrow \text{7299}$$
$$\int \csc^2(a+bx)\sec^2(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

output
```
$Aborted
```

#### 3.272.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

### 3.272.4 Maple [N/A] (verified)

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \csc(xb+a)^2 \sec(xb+a)^2\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

### 3.272.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^2(a+bx) \sec^2(a+bx) \, dx = \int (dx+c)^m \csc{(bx+a)}^2 \sec{(bx+a)}^2 \, dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

### 3.272.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m \csc^2(a+bx) \sec^2(a+bx) \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*csc(b*x+a)**2*sec(b*x+a)**2,x)
```

output
```
Timed out
```

### 3.272.7 Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^2(a+bx) \sec^2(a+bx) \, dx = \int (dx+c)^m \csc{(bx+a)}^2 \sec{(bx+a)}^2 \, dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

### 3.272.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

### 3.272.9 Mupad [N/A]

Not integrable

Time = 25.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\cos(a+b\,x)^2\sin(a+b\,x)^2}\,dx$$

input
```
int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)^2),x)
```

output
```
int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)^2), x)
```

# 3.273 $\int (c+dx)^3 \csc^2(a+bx) \sec^2(a+bx) \, dx$


3.273.1 Optimal result . . . 2083
3.273.2 Mathematica [B] (verified) . . . 2083
3.273.3 Rubi [A] (verified) . . . 2084
3.273.4 Maple [B] (verified) . . . 2087
3.273.5 Fricas [B] (verification not implemented) . . . 2088
3.273.6 Sympy [F] . . . 2089
3.273.7 Maxima [B] (verification not implemented) . . . 2090
3.273.8 Giac [F] . . . 2090
3.273.9 Mupad [F(-1)] . . . 2091


## 3.273.1 Optimal result

Integrand size = 24, antiderivative size = 118

$$\int (c+dx)^3 \csc^2(a+bx) \sec^2(a+bx) \, dx = -\frac{2i(c+dx)^3}{b} - \frac{2(c+dx)^3 \cot(2a+2bx)}{b} + \frac{3d(c+dx)^2 \log\left(1-e^{4i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx) \operatorname{PolyLog}\left(2, e^{4i(a+bx)}\right)}{2b^3} + \frac{3d^3 \operatorname{PolyLog}\left(3, e^{4i(a+bx)}\right)}{8b^4}$$

output
```
-2*I*(d*x+c)^3/b-2*(d*x+c)^3*cot(2*b*x+2*a)/b+3*d*(d*x+c)^2*ln(1-exp(4*I*(
b*x+a)))/b^2-3/2*I*d^2*(d*x+c)*polylog(2,exp(4*I*(b*x+a)))/b^3+3/8*d^3*pol
ylog(3,exp(4*I*(b*x+a)))/b^4
```

## 3.273.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 285 vs. 2(118) = 236.

Time = 2.29 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.42

$$\int (c+dx)^3 \csc^2(a+bx) \sec^2(a+bx) \, dx$$
$$= \frac{-\frac{8ib^3(c+dx)^3}{-1+e^{4ia}} + 6b^2d(c+dx)^2 \log\left(1-e^{-i(a+bx)}\right) + 6b^2d(c+dx)^2 \log\left(1+e^{-i(a+bx)}\right) + 6b^2d(c+dx)^2 \log(1 \cdots}{\cdots}$$

input
```
Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

output
```
(((-8*I)*b^3*(c + d*x)^3)/(-1 + E^((4*I)*a)) + 6*b^2*d*(c + d*x)^2*Log[1 -
 E^((-I)*(a + b*x))] + 6*b^2*d*(c + d*x)^2*Log[1 + E^((-I)*(a + b*x))] + 6
*b^2*d*(c + d*x)^2*Log[1 + E^((-2*I)*(a + b*x))] + (12*I)*b*d^2*(c + d*x)*
PolyLog[2, -E^((-I)*(a + b*x))] + (12*I)*b*d^2*(c + d*x)*PolyLog[2, E^((-I
)*(a + b*x))] + (6*I)*b*d^2*(c + d*x)*PolyLog[2, -E^((-2*I)*(a + b*x))] +
12*d^3*PolyLog[3, -E^((-I)*(a + b*x))] + 12*d^3*PolyLog[3, E^((-I)*(a + b*
x))] + 3*d^3*PolyLog[3, -E^((-2*I)*(a + b*x))] + 4*b^3*(c + d*x)^3*Csc[2*a
]*Csc[2*(a + b*x)]*Sin[2*b*x])/(2*b^4)
```

### 3.273.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.36, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {4919, 3042, 4672, 3042, 25, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \csc^2(a+bx) \sec^2(a+bx) \, dx$$

$$\Big\downarrow \text{4919}$$

$$4 \int (c+dx)^3 \csc^2(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$4 \int (c+dx)^3 \csc(2a+2bx)^2dx$$

$$\Big\downarrow \text{4672}$$

$$4\left(\frac{3d\int (c+dx)^2 \cot(2a+2bx)dx}{2b}-\frac{(c+dx)^3 \cot(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$4\left(\frac{3d\int -(c+dx)^2 \tan\left(2a+2bx+\frac{\pi}{2}\right)dx}{2b}-\frac{(c+dx)^3 \cot(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{25}$$

$$4\left(-\frac{3d\int (c+dx)^2\tan\left(\frac{1}{2}(4a+\pi)+2bx\right)dx}{2b}-\frac{(c+dx)^3\cot(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{4202}$$

$$4\left(-\frac{(c+dx)^3\cot(2a+2bx)}{2b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\int\frac{e^{i(4a+4bx+\pi)}(c+dx)^2}{1+e^{i(4a+4bx+\pi)}}dx\right)}{2b}\right)$$

$$\Big\downarrow \text{2620}$$

$$4\left(-\frac{(c+dx)^3\cot(2a+2bx)}{2b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\int(c+dx)\log\left(1+e^{i(4a+4bx+\pi)}\right)dx}{2b}-\frac{i(c+dx)^2\log\left(1+e^{i(4a+4bx+\pi)}\right)}{4b}\right)\right)}{2b}\right)$$

$$\Big\downarrow \text{3011}$$

$$4\left(-\frac{(c+dx)^3\cot(2a+2bx)}{2b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(4a+4bx+\pi)}\right)}{4b}-\frac{id\int\operatorname{PolyLog}\left(2,-e^{i(4a+4bx+\pi)}\right)dx}{4b}\right)}{2b}-\frac{i(\ldots}{}\right.\right.}{2b}\right.$$

$$\Big\downarrow \text{2720}$$

$$4\left(-\frac{(c+dx)^3\cot(2a+2bx)}{2b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(4a+4bx+\pi)}\right)}{4b}-\frac{d\int e^{-i(4a+4bx+\pi)}\operatorname{PolyLog}\left(2,-e^{i(4a+4bx+\ldots}\right.}{16b^2}\right.}{2b}\right.\right.}{2b}\right.$$

$$\Big\downarrow \text{7143}$$

$$4\left(-\frac{(c+dx)^3\cot(2a+2bx)}{2b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{i(4a+4bx+\pi)}\right)}{4b}-\frac{d\operatorname{PolyLog}\left(3,-e^{i(4a+4bx+\pi)}\right)}{16b^2}\right)}{2b}-\frac{i(c+dx\ldots}{}\right.\right.}{2b}\right.$$

3.273. $\int (c+dx)^3\csc^2(a+bx)\sec^2(a+bx)\,dx$

input
```
Int[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

output
```
4*(-1/2*((c + d*x)^3*Cot[2*a + 2*b*x])/b - (3*d*(((I/3)*(c + d*x)^3)/d - (
2*I)*(((-1/4*I)*(c + d*x)^2*Log[1 + E^(I*(4*a + Pi + 4*b*x))])/b + ((I/2)*
d*(((I/4)*(c + d*x)*PolyLog[2, -E^(I*(4*a + Pi + 4*b*x))])/b - (d*PolyLog[
3, -E^(I*(4*a + Pi + 4*b*x))])/(16*b^2)))/b)))/(2*b))
```

### 3.273.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.273.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 686 vs. $2(106) = 212$.

Time = 1.99 (sec) , antiderivative size = 687, normalized size of antiderivative = 5.82

| method | result |
|---|---|
| risch | $\frac{3d\,c^2 \ln(\mathrm{e}^{i(xb+a)}+1)}{b^2} - \frac{3d^3 \ln(1-\mathrm{e}^{i(xb+a)})a^2}{b^4} + \frac{3d^3 \ln(1-\mathrm{e}^{i(xb+a)})x^2}{b^2} + \frac{3d^3 \ln(\mathrm{e}^{i(xb+a)}+1)x^2}{b^2} + \frac{3d^3 a^2 \ln(\mathrm{e}^{i(xb+a)}-1)}{b^4} +$ |

input
```
int((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
3*d/b^2*c^2*ln(exp(I*(b*x+a))+1)+3*d/b^2*c^2*ln(exp(2*I*(b*x+a))+1)-3*d^3/
b^4*ln(1-exp(I*(b*x+a)))*a^2+3*d^3/b^2*ln(1-exp(I*(b*x+a)))*x^2+3*d^3/b^2*
ln(exp(I*(b*x+a))+1)*x^2+3*d^3/b^2*ln(exp(2*I*(b*x+a))+1)*x^2-12*d^3/b^4*a
^2*ln(exp(I*(b*x+a)))+3*d^3/b^4*a^2*ln(exp(I*(b*x+a))-1)-12*d/b^2*c^2*ln(e
xp(I*(b*x+a)))+3*d/b^2*c^2*ln(exp(I*(b*x+a))-1)-4*I*d^3/b*x^3+8*I*d^3/b^4*
a^3-24*I*d^2/b^2*c*x*a+3/2*d^3*polylog(3,-exp(2*I*(b*x+a)))/b^4+6*d^2/b^2*
c*ln(exp(I*(b*x+a))+1)*x+6*d^2/b^2*c*ln(exp(2*I*(b*x+a))+1)*x+6*d^2/b^2*c*
ln(1-exp(I*(b*x+a)))*x+24*d^2/b^3*c*a*ln(exp(I*(b*x+a)))-6*d^2/b^3*c*a*ln(
exp(I*(b*x+a))-1)+6*d^2/b^3*c*ln(1-exp(I*(b*x+a)))*a-3*I*d^2/b^3*c*polylog
(2,-exp(2*I*(b*x+a)))-6*I*d^2/b^3*c*polylog(2,exp(I*(b*x+a)))-12*I*d^2/b*c
*x^2-12*I*d^2/b^3*c*a^2-6*I*d^2/b^3*c*polylog(2,-exp(I*(b*x+a)))+12*I*d^3/
b^3*a^2*x-6*I*d^3/b^3*polylog(2,-exp(I*(b*x+a)))*x-3*I*d^3/b^3*polylog(2,-
exp(2*I*(b*x+a)))*x-6*I*d^3/b^3*polylog(2,exp(I*(b*x+a)))*x-4*I*(d^3*x^3+3
*c*d^2*x^2+3*c^2*d*x+c^3)/b/(exp(2*I*(b*x+a))+1)/(exp(2*I*(b*x+a))-1)+6*d^
3*polylog(3,-exp(I*(b*x+a)))/b^4+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4
```

### 3.273.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1635 vs. $2(103) = 206$.

Time = 0.35 (sec) , antiderivative size = 1635, normalized size of antiderivative = 13.86

$$\int (c+dx)^3 \csc^2(a+bx) \sec^2(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*c
os(b*x + a)*polylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*d^3
*cos(b*x + a)*polylog(3, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*d
^3*cos(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + 6
*d^3*cos(b*x + a)*polylog(3, I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) +
 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a
) + 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x + a) - sin(b*x + a))*sin(b*x
+ a) + 6*d^3*cos(b*x + a)*polylog(3, -cos(b*x + a) + I*sin(b*x + a))*sin(b
*x + a) + 6*d^3*cos(b*x + a)*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*si
n(b*x + a) - 6*(I*b*d^3*x + I*b*c*d^2)*cos(b*x + a)*dilog(cos(b*x + a) + I
*sin(b*x + a))*sin(b*x + a) - 6*(-I*b*d^3*x - I*b*c*d^2)*cos(b*x + a)*dilo
g(cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - 6*(-I*b*d^3*x - I*b*c*d^2)
*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 6*(I*b*d
^3*x + I*b*c*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*
x + a) - 6*(I*b*d^3*x + I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + si
n(b*x + a))*sin(b*x + a) - 6*(-I*b*d^3*x - I*b*c*d^2)*cos(b*x + a)*dilog(-
I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - 6*(-I*b*d^3*x - I*b*c*d^2)*c
os(b*x + a)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 6*(I*b*d^
3*x + I*b*c*d^2)*cos(b*x + a)*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*
x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*log(c...
```

### 3.273.6 Sympy [F]

$$\int (c+dx)^3 \csc^2(a+bx) \sec^2(a+bx) \, dx = \int (c+dx)^3 \csc^2(a+bx) \sec^2(a+bx) \, dx$$

input
```
integrate((d*x+c)**3*csc(b*x+a)**2*sec(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**3*csc(a + b*x)**2*sec(a + b*x)**2, x)
```

### 3.273.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2360 vs. 2(103) = 206.

Time = 0.51 (sec) , antiderivative size = 2360, normalized size of antiderivative = 20.00

$$\int (c+dx)^3 \csc^2(a+bx)\sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*c^3*(1/tan(b*x + a) - tan(b*x + a)) - 6*a*c^2*d*(1/tan(b*x + a) -
tan(b*x + a))/b + 6*a^2*c*d^2*(1/tan(b*x + a) - tan(b*x + a))/b^2 - 2*a^3*
d^3*(1/tan(b*x + a) - tan(b*x + a))/b^3 - 3*((cos(4*b*x + 4*a)^2 + sin(4*b
*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x +
 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)
^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b
*x + a) + 1) + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*
a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 8*(b*x
 + a)*sin(4*b*x + 4*a))*c^2*d/((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 -
2*cos(4*b*x + 4*a) + 1)*b) + 6*((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 -
 2*cos(4*b*x + 4*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*c
os(2*b*x + 2*a) + 1) + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*
b*x + 4*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
+ (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(c
os(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 8*(b*x + a)*sin(4*b
*x + 4*a))*a*c*d^2/((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x
 + 4*a) + 1)*b^2) - 3*((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*
b*x + 4*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
+ 2*a) + 1) + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a
) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos...
```

### 3.273.8 Giac [F]

$$\int (c+dx)^3 \csc^2(a+bx)\sec^2(a+bx)\,dx = \int (dx+c)^3 \csc(bx+a)^2 \sec(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

### 3.273.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^2(a+bx) \sec^2(a+bx) \, dx = \int \frac{(c+d\,x)^3}{\cos{(a+b\,x)}^2 \sin{(a+b\,x)}^2} \, dx$$

input
```
int((c + d*x)^3/(cos(a + b*x)^2*sin(a + b*x)^2),x)
```

output
```
int((c + d*x)^3/(cos(a + b*x)^2*sin(a + b*x)^2), x)
```

# 3.274 $\int (c+dx)^2 \csc^2(a+bx) \sec^2(a+bx) \, dx$

3.274.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2092
3.274.2 Mathematica [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 2092
3.274.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2093
3.274.4 Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2096
3.274.5 Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 2096
3.274.6 Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2097
3.274.7 Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . . . 2098
3.274.8 Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2099
3.274.9 Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2099

## 3.274.1 Optimal result

Integrand size = 24, antiderivative size = 88

$$\int (c+dx)^2 \csc^2(a+bx) \sec^2(a+bx) \, dx = -\frac{2i(c+dx)^2}{b} - \frac{2(c+dx)^2 \cot(2a+2bx)}{b} + \frac{2d(c+dx) \log \left(1-e^{4i(a+bx)}\right)}{b^2} - \frac{id^2 \operatorname{PolyLog}\left(2, e^{4i(a+bx)}\right)}{2b^3}$$

output
```
-2*I*(d*x+c)^2/b-2*(d*x+c)^2*cot(2*b*x+2*a)/b+2*d*(d*x+c)*ln(1-exp(4*I*(b*
x+a)))/b^2-1/2*I*d^2*polylog(2,exp(4*I*(b*x+a)))/b^3
```

## 3.274.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 277 vs. $2(88) = 176$.

Time = 1.88 (sec) , antiderivative size = 277, normalized size of antiderivative = 3.15

$$\int (c+dx)^2 \csc^2(a+bx) \sec^2(a+bx) \, dx$$
$$= \frac{-\frac{ie^{4ia}\left(4b^2e^{-4ia}(c+dx)^2+2ibd\left(1-e^{-4ia}\right)(c+dx)\log\left(1-e^{-i(a+bx)}\right)+2ibd\left(1-e^{-4ia}\right)(c+dx)\log\left(1+e^{-i(a+bx)}\right)+2ibd\left(1-e^{-4ia}\right)(c+dx)\log\left(1+e\right.}{-1+e^{4ia}}}{}$$

input
```
Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

output
```
(((-I)*E^((4*I)*a)*((4*b^2*(c + d*x)^2)/E^((4*I)*a) + (2*I)*b*d*(1 - E^((-
4*I)*a))*(c + d*x)*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b*d*(1 - E^((-4*I)*
a))*(c + d*x)*Log[1 + E^((-I)*(a + b*x))] + (2*I)*b*d*(1 - E^((-4*I)*a))*(
c + d*x)*Log[1 + E^((-2*I)*(a + b*x))] - 2*d^2*(1 - E^((-4*I)*a))*PolyLog[
2, -E^((-I)*(a + b*x))] - 2*d^2*(1 - E^((-4*I)*a))*PolyLog[2, E^((-I)*(a +
 b*x))] - d^2*(1 - E^((-4*I)*a))*PolyLog[2, -E^((-2*I)*(a + b*x))]))/(-1 +
 E^((4*I)*a)) + 2*b^2*(c + d*x)^2*Csc[2*a]*Csc[2*(a + b*x)]*Sin[2*b*x])/b^
3
```

### 3.274.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.28, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4919, 3042, 4672, 3042, 25, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \csc^2(a+bx) \sec^2(a+bx) \, dx$$

$$\Big\downarrow \text{4919}$$

$$4 \int (c+dx)^2 \csc^2(2a+2bx) dx$$

$$\Big\downarrow \text{3042}$$

$$4 \int (c+dx)^2 \csc(2a+2bx)^2 dx$$

$$\Big\downarrow \text{4672}$$

$$4 \left(\frac{d \int (c+dx) \cot(2a+2bx) dx}{b} - \frac{(c+dx)^2 \cot(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$4 \left(\frac{d \int -\left((c+dx) \tan\left(2a+2bx+\frac{\pi}{2}\right)\right) dx}{b} - \frac{(c+dx)^2 \cot(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{25}$$

$$4\left(-\frac{d\int (c+dx)\tan\left(\frac{1}{2}(4a+\pi)+2bx\right)dx}{b}-\frac{(c+dx)^2\cot(2a+2bx)}{2b}\right)$$

$\Big\downarrow$ 4202

$$4\left(-\frac{(c+dx)^2\cot(2a+2bx)}{2b}-\frac{d\left(\frac{i(c+dx)^2}{2d}-2i\int \frac{e^{i(4a+4bx+\pi)}(c+dx)}{1+e^{i(4a+4bx+\pi)}}dx\right)}{b}\right)$$

$\Big\downarrow$ 2620

$$4\left(-\frac{(c+dx)^2\cot(2a+2bx)}{2b}-\frac{d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{id\int \log\left(1+e^{i(4a+4bx+\pi)}\right)dx}{4b}-\frac{i(c+dx)\log\left(1+e^{i(4a+4bx+\pi)}\right)}{4b}\right)\right)}{b}\right)$$

$\Big\downarrow$ 2715

$$4\left(-\frac{(c+dx)^2\cot(2a+2bx)}{2b}-\frac{d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{d\int e^{-i(4a+4bx+\pi)}\log\left(1+e^{i(4a+4bx+\pi)}\right)de^{i(4a+4bx+\pi)}}{16b^2}-\frac{i(c+dx)\log\left(1+e^{i(4a+4bx+\pi)}\right)}{4b}\right)\right)}{b}\right)$$

$\Big\downarrow$ 2838

$$4\left(-\frac{(c+dx)^2\cot(2a+2bx)}{2b}-\frac{d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\,\text{PolyLog}\left(2,-e^{i(4a+4bx+\pi)}\right)}{16b^2}-\frac{i(c+dx)\log\left(1+e^{i(4a+4bx+\pi)}\right)}{4b}\right)\right)}{b}\right)$$

input
```
Int[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

output
```
4*(-1/2*((c + d*x)^2*Cot[2*a + 2*b*x])/b - (d*(((I/2)*(c + d*x)^2)/d - (2*
I)*(((-1/4*I)*(c + d*x)*Log[1 + E^(I*(4*a + Pi + 4*b*x))])/b - (d*PolyLog[
2, -E^(I*(4*a + Pi + 4*b*x))])/(16*b^2))))/b)
```

## 3.274. $\int (c+dx)^2\csc^2(a+bx)\sec^2(a+bx)\,dx$

### 3.274.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

### 3.274.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(80) = 160.

Time = 1.90 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.99

| method | result |
|---|---|
| risch | $-\frac{2id^2 \operatorname{polylog}(2,e^{i(xb+a)})}{b^3}-\frac{8dc\ln(e^{i(xb+a)})}{b^2}+\frac{2dc\ln(e^{i(xb+a)}-1)}{b^2}+\frac{2dc\ln(e^{i(xb+a)}+1)}{b^2}+\frac{2dc\ln(e^{2i(xb+a)}+1)}{b^2}-\frac{4}{b(e^{2i(a}}$ |

input
```
int((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^3-8*d/b^2*c*ln(exp(I*(b*x+a)))+2*d/b^
2*c*ln(exp(I*(b*x+a))-1)+2*d/b^2*c*ln(exp(I*(b*x+a))+1)+2*d/b^2*c*ln(exp(2
*I*(b*x+a))+1)-4*I*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*x+a))+1)/(exp(2*I*(
b*x+a))-1)-4*I*d^2/b^3*a^2-8*I*d^2/b^2*x*a+2*d^2/b^2*ln(exp(I*(b*x+a))+1)*
x-2*I*d^2/b^3*polylog(2,-exp(I*(b*x+a)))+2*d^2/b^2*ln(exp(2*I*(b*x+a))+1)*
x-I*d^2*polylog(2,-exp(2*I*(b*x+a)))/b^3+2*d^2/b^2*ln(1-exp(I*(b*x+a)))*x+
2*d^2/b^3*ln(1-exp(I*(b*x+a)))*a-4*I*d^2/b*x^2+8*d^2/b^3*a*ln(exp(I*(b*x+a
)))-2*d^2/b^3*a*ln(exp(I*(b*x+a))-1)
```

### 3.274.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 950 vs. 2(77) = 154.

Time = 0.32 (sec) , antiderivative size = 950, normalized size of antiderivative = 10.80

$$\int (c+dx)^2 \csc^2(a+bx)\sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
(b^2*d^2*x^2 + 2*b^2*c*d*x - I*d^2*cos(b*x + a)*dilog(cos(b*x + a) + I*sin
(b*x + a))*sin(b*x + a) + I*d^2*cos(b*x + a)*dilog(cos(b*x + a) - I*sin(b*
x + a))*sin(b*x + a) + I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x +
 a))*sin(b*x + a) - I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a)
)*sin(b*x + a) - I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a))*
sin(b*x + a) + I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a))*si
n(b*x + a) + I*d^2*cos(b*x + a)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(
b*x + a) - I*d^2*cos(b*x + a)*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*
x + a) + b^2*c^2 + (b*d^2*x + b*c*d)*cos(b*x + a)*log(cos(b*x + a) + I*sin
(b*x + a) + 1)*sin(b*x + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*x + a
) + I*sin(b*x + a) + I)*sin(b*x + a) + (b*d^2*x + b*c*d)*cos(b*x + a)*log(
cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) + (b*c*d - a*d^2)*cos(b*x
+ a)*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + (b*d^2*x + a*d^
2)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + (b*d
^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x
+ a) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) +
 1)*sin(b*x + a) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - si
n(b*x + a) + 1)*sin(b*x + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(-1/2*cos(b
*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + (b*c*d - a*d^2)*cos(b*x
 + a)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + ...
```

### 3.274.6 Sympy [F]

$$\int (c+dx)^2 \csc^2(a+bx) \sec^2(a+bx)\,dx = \int (c+dx)^2 \csc^2{(a+bx)} \sec^2{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**2*csc(b*x+a)**2*sec(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*csc(a + b*x)**2*sec(a + b*x)**2, x)
```

### 3.274.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(77) = 154$.

Time = 0.41 (sec) , antiderivative size = 772, normalized size of antiderivative = 8.77

$$\int (c+dx)^2 \csc^2(a+bx) \sec^2(a+bx)\, dx =$$
$$-\frac{4\, b^2c^2 + 2\,(bd^2x + bcd - (bd^2x + bcd)\cos(4\, bx + 4\, a) + (-i\, bd^2x - i\, bcd)\sin(4\, bx + 4\, a))\arctan(\sin(2}{}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(4*b^2*c^2 + 2*(b*d^2*x + b*c*d - (b*d^2*x + b*c*d)*cos(4*b*x + 4*a) + (-
I*b*d^2*x - I*b*c*d)*sin(4*b*x + 4*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x
 + 2*a) + 1) + 2*(b*d^2*x + b*c*d - (b*d^2*x + b*c*d)*cos(4*b*x + 4*a) + (
-I*b*d^2*x - I*b*c*d)*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), cos(b*x + a)
 + 1) - 2*(b*c*d*cos(4*b*x + 4*a) + I*b*c*d*sin(4*b*x + 4*a) - b*c*d)*arct
an2(sin(b*x + a), cos(b*x + a) - 1) + 2*(b*d^2*x*cos(4*b*x + 4*a) + I*b*d^
2*x*sin(4*b*x + 4*a) - b*d^2*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) +
 4*(b^2*d^2*x^2 + 2*b^2*c*d*x)*cos(4*b*x + 4*a) + (d^2*cos(4*b*x + 4*a) +
I*d^2*sin(4*b*x + 4*a) - d^2)*dilog(-e^(2*I*b*x + 2*I*a)) + 2*(d^2*cos(4*b
*x + 4*a) + I*d^2*sin(4*b*x + 4*a) - d^2)*dilog(-e^(I*b*x + I*a)) + 2*(d^2
*cos(4*b*x + 4*a) + I*d^2*sin(4*b*x + 4*a) - d^2)*dilog(e^(I*b*x + I*a)) -
 (I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(4*b*x + 4*a) + (b*d^2*x
 + b*c*d)*sin(4*b*x + 4*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 +
2*cos(2*b*x + 2*a) + 1) - (I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*co
s(4*b*x + 4*a) + (b*d^2*x + b*c*d)*sin(4*b*x + 4*a))*log(cos(b*x + a)^2 +
sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (I*b*d^2*x + I*b*c*d + (-I*b*d^2*x
- I*b*c*d)*cos(4*b*x + 4*a) + (b*d^2*x + b*c*d)*sin(4*b*x + 4*a))*log(cos(
b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(I*b^2*d^2*x^2 + 2*I
*b^2*c*d*x)*sin(4*b*x + 4*a))/(-I*b^3*cos(4*b*x + 4*a) + b^3*sin(4*b*x + 4
*a) + I*b^3)
```

### 3.274.8 Giac [F]

$$\int (c+dx)^2 \csc^2(a+bx)\sec^2(a+bx)\,dx = \int (dx+c)^2 \csc(bx+a)^2 \sec(bx+a)^2 \,dx$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

### 3.274.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc^2(a+bx)\sec^2(a+bx)\,dx = \int \frac{(c+d\,x)^2}{\cos(a+b\,x)^2 \sin(a+b\,x)^2}\,dx$$

input
```
int((c + d*x)^2/(cos(a + b*x)^2*sin(a + b*x)^2),x)
```

output
```
int((c + d*x)^2/(cos(a + b*x)^2*sin(a + b*x)^2), x)
```

# 3.275 $\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx$

3.275.1 Optimal result . . . . . . . . . . 2100
3.275.2 Mathematica [A] (verified) . . . . . . . . . . 2100
3.275.3 Rubi [A] (verified) . . . . . . . . . . 2101
3.275.4 Maple [C] (verified) . . . . . . . . . . 2102
3.275.5 Fricas [B] (verification not implemented) . . . . . . . . . . 2103
3.275.6 Sympy [F] . . . . . . . . . . 2103
3.275.7 Maxima [B] (verification not implemented) . . . . . . . . . . 2104
3.275.8 Giac [B] (verification not implemented) . . . . . . . . . . 2104
3.275.9 Mupad [B] (verification not implemented) . . . . . . . . . . 2105

## 3.275.1 Optimal result

Integrand size = 22, antiderivative size = 35

$$\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx = -\frac{2(c+dx)\cot(2a+2bx)}{b}+\frac{d\log(\sin(2a+2bx))}{b^2}$$

output
```
-2*(d*x+c)*cot(2*b*x+2*a)/b+d*ln(sin(2*b*x+2*a))/b^2
```

## 3.275.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx = \frac{-2b(c+dx)\cot(2(a+bx))+d\log(\sin(2(a+bx)))}{b^2}$$

input
```
Integrate[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

output
```
(-2*b*(c + d*x)*Cot[2*(a + b*x)] + d*Log[Sin[2*(a + b*x)]])/b^2
```

### 3.275.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4919, 3042, 4672, 3042, 25, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx$$

$$\Big\downarrow \text{4919}$$

$$4\int (c+dx)\csc^2(2a+2bx)dx$$

$$\Big\downarrow \text{3042}$$

$$4\int (c+dx)\csc(2a+2bx)^2dx$$

$$\Big\downarrow \text{4672}$$

$$4\left(\frac{d\int \cot(2a+2bx)dx}{2b}-\frac{(c+dx)\cot(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{3042}$$

$$4\left(\frac{d\int -\tan\left(2a+2bx+\frac{\pi}{2}\right)dx}{2b}-\frac{(c+dx)\cot(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{25}$$

$$4\left(-\frac{d\int \tan\left(\frac{1}{2}(4a+\pi)+2bx\right)dx}{2b}-\frac{(c+dx)\cot(2a+2bx)}{2b}\right)$$

$$\Big\downarrow \text{3956}$$

$$4\left(\frac{d\log(-\sin(2a+2bx))}{4b^2}-\frac{(c+dx)\cot(2a+2bx)}{2b}\right)$$

input
```
Int[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

output
```
4*(-1/2*((c + d*x)*Cot[2*a + 2*b*x])/b + (d*Log[-Sin[2*a + 2*b*x]])/(4*b^2
))
```

### 3.275.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

## 3.275.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.06

| method | result |
|---|---|
| risch | $-\frac{4idx}{b}-\frac{4ida}{b^{2}}-\frac{4i(dx+c)}{b\left(\mathrm{e}^{2i(xb+a)}+1\right)\left(\mathrm{e}^{2i(xb+a)}-1\right)}+\frac{d\ln\left(\mathrm{e}^{4i(xb+a)}-1\right)}{b^{2}}$ |
| parallelrisch | $\frac{-4\ln\left(\sec\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}\right)d\cos(xb+a)+2\ln\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right)d\cos(xb+a)+2\ln\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)+1\right)d\cos(xb+a)+2\ln\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\right.}{2b^{2}\cos(xb+a)}$ |
| norman | $\frac{\frac{c}{2b}-\frac{3c\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}}{b}+\frac{c\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{4}}{2b}+\frac{dx}{2b}-\frac{3dx\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}}{b}+\frac{dx\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{4}}{2b}}{\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)^{2}-1\right)}+\frac{d\ln\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)\right)}{b^{2}}+\frac{d\ln\left(\tan\left(\frac{a}{2}+\frac{xb}{2}\right)-1\right.}{b^{2}}$ |

input
```
int((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-4*I*d/b*x-4*I*d/b^2*a-4*I*(d*x+c)/b/(exp(2*I*(b*x+a))+1)/(exp(2*I*(b*x+a)
)-1)+d/b^2*ln(exp(4*I*(b*x+a))-1)
```

### 3.275.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(35) = 70$.

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.14

$$\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx = \frac{d\cos(bx+a)\log\left(-\frac{1}{2}\cos(bx+a)\sin(bx+a)\right)\sin(bx+a)+bdx-2\,(bdx+bc)\cos(bx+a)^2+bc}{b^2\cos(bx+a)\sin(bx+a)}$$

input
```
integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
(d*cos(b*x + a)*log(-1/2*cos(b*x + a)*sin(b*x + a))*sin(b*x + a) + b*d*x -
 2*(b*d*x + b*c)*cos(b*x + a)^2 + b*c)/(b^2*cos(b*x + a)*sin(b*x + a))
```

### 3.275.6 Sympy [F]

$$\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx = \int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)**2*sec(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*csc(a + b*x)**2*sec(a + b*x)**2, x)
```

### 3.275.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(35) = 70$.

Time = 0.32 (sec) , antiderivative size = 308, normalized size of antiderivative = 8.80

$$\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx = -\frac{2c\left(\frac{1}{\tan(bx+a)} - \tan(bx+a)\right) - \frac{2ad\left(\frac{1}{\tan(bx+a)}-\tan(bx+a)\right)}{b} - \frac{\left(\left(\cos(4bx+4a)^2+\sin(4bx+4a)^2-2\cos(4bx+4a)+1\right)\log\left(\cos\right.\right.}{}}{}$$

input
```
integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*c*(1/tan(b*x + a) - tan(b*x + a)) - 2*a*d*(1/tan(b*x + a) - tan(b*
x + a))/b - ((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a)
 + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1
) + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log
(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(4*b*x + 4*a)
^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(b*x + a)^2 + sin
(b*x + a)^2 - 2*cos(b*x + a) + 1) - 8*(b*x + a)*sin(4*b*x + 4*a))*d/((cos(
4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*b))/b
```

### 3.275.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10271 vs. $2(35) = 70$.

Time = 2.84 (sec) , antiderivative size = 10271, normalized size of antiderivative = 293.46

$$\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*(b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4*tan(1/2*a)^4 -
 6*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 16*b*d*x*tan(1/2*b*x)^3*tan(1/2*a)^
3 + d*log(64*(tan(1/2*b*x)^8*tan(1/2*a)^6 + 2*tan(1/2*b*x)^7*tan(1/2*a)^7
+ tan(1/2*b*x)^6*tan(1/2*a)^8 - 2*tan(1/2*b*x)^8*tan(1/2*a)^4 - 14*tan(1/2
*b*x)^7*tan(1/2*a)^5 - 24*tan(1/2*b*x)^6*tan(1/2*a)^6 - 14*tan(1/2*b*x)^5*
tan(1/2*a)^7 - 2*tan(1/2*b*x)^4*tan(1/2*a)^8 + tan(1/2*b*x)^8*tan(1/2*a)^2
 + 14*tan(1/2*b*x)^7*tan(1/2*a)^3 + 62*tan(1/2*b*x)^6*tan(1/2*a)^4 + 98*ta
n(1/2*b*x)^5*tan(1/2*a)^5 + 62*tan(1/2*b*x)^4*tan(1/2*a)^6 + 14*tan(1/2*b*
x)^3*tan(1/2*a)^7 + tan(1/2*b*x)^2*tan(1/2*a)^8 - 2*tan(1/2*b*x)^7*tan(1/2
*a) - 24*tan(1/2*b*x)^6*tan(1/2*a)^2 - 98*tan(1/2*b*x)^5*tan(1/2*a)^3 - 15
2*tan(1/2*b*x)^4*tan(1/2*a)^4 - 98*tan(1/2*b*x)^3*tan(1/2*a)^5 - 24*tan(1/
2*b*x)^2*tan(1/2*a)^6 - 2*tan(1/2*b*x)*tan(1/2*a)^7 + tan(1/2*b*x)^6 + 14*
tan(1/2*b*x)^5*tan(1/2*a) + 62*tan(1/2*b*x)^4*tan(1/2*a)^2 + 98*tan(1/2*b*
x)^3*tan(1/2*a)^3 + 62*tan(1/2*b*x)^2*tan(1/2*a)^4 + 14*tan(1/2*b*x)*tan(1
/2*a)^5 + tan(1/2*a)^6 - 2*tan(1/2*b*x)^4 - 14*tan(1/2*b*x)^3*tan(1/2*a) -
 24*tan(1/2*b*x)^2*tan(1/2*a)^2 - 14*tan(1/2*b*x)*tan(1/2*a)^3 - 2*tan(1/2
*a)^4 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/
2*b*x)^8*tan(1/2*a)^8 + 4*tan(1/2*b*x)^8*tan(1/2*a)^6 + 4*tan(1/2*b*x)^6*t
an(1/2*a)^8 + 6*tan(1/2*b*x)^8*tan(1/2*a)^4 + 16*tan(1/2*b*x)^6*tan(1/2*a)
^6 + 6*tan(1/2*b*x)^4*tan(1/2*a)^8 + 4*tan(1/2*b*x)^8*tan(1/2*a)^2 + 24...
```

### 3.275.9 Mupad [B] (verification not implemented)

Time = 27.60 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.57

$$\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx = \frac{d\,\ln\left(\mathrm{e}^{a\,4\mathrm{i}}\,\mathrm{e}^{b\,x\,4\mathrm{i}}-1\right)}{b^2}-\frac{(c+d\,x)\,4\mathrm{i}}{b\,\left(\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}-1\right)}-\frac{d\,x\,4\mathrm{i}}{b}$$

input
```
int((c + d*x)/(cos(a + b*x)^2*sin(a + b*x)^2),x)
```

output
```
(d*log(exp(a*4i)*exp(b*x*4i) - 1))/b^2 - ((c + d*x)*4i)/(b*(exp(a*4i + b*x
*4i) - 1)) - (d*x*4i)/b
```

# 3.276 $\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$

3.276.1 Optimal result . . . . . . . . . . 2106
3.276.2 Mathematica [N/A] . . . . . . . . . . 2106
3.276.3 Rubi [N/A] . . . . . . . . . . 2107
3.276.4 Maple [N/A] (verified) . . . . . . . . . . 2108
3.276.5 Fricas [N/A] . . . . . . . . . . 2108
3.276.6 Sympy [N/A] . . . . . . . . . . 2108
3.276.7 Maxima [N/A] . . . . . . . . . . 2109
3.276.8 Giac [N/A] . . . . . . . . . . 2110
3.276.9 Mupad [N/A] . . . . . . . . . . 2110

## 3.276.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx = 4\mathrm{Int}\left(\frac{\csc^2(2a+2bx)}{c+dx},x\right)$$

output
```
4*Unintegrable(csc(2*b*x+2*a)^2/(d*x+c),x)
```

## 3.276.2 Mathematica [N/A]

Not integrable

Time = 9.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x),x]
```

output
```
Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x), x]
```

### 3.276.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4919, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{4919}$$

$$4\int \frac{\csc^2(2a+2bx)}{c+dx}dx$$

$$\Big\downarrow \text{3042}$$

$$4\int \frac{\csc(2a+2bx)^2}{c+dx}dx$$

$$\Big\downarrow \text{4680}$$

$$4\int \frac{\csc^2(2a+2bx)}{c+dx}dx$$

input
```
Int[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x),x]
```

output
```
$Aborted
```

#### 3.276.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

### 3.276.4 Maple [N/A] (verified)

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc(xb+a)^2 \sec(xb+a)^2}{dx+c}dx$$

input
```
int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x)
```

output
```
int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x)
```

### 3.276.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^2 \sec(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^2*sec(b*x + a)^2/(d*x + c), x)
```

### 3.276.6 Sympy [N/A]

Not integrable

Time = 2.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

input
```
integrate(csc(b*x+a)**2*sec(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(csc(a + b*x)**2*sec(a + b*x)**2/(c + d*x), x)
```

### 3.276.7 Maxima [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 687, normalized size of antiderivative = 28.62

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx=\int \frac{\csc(bx+a)^2\sec(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
(2*(b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(4*b*x + 4*a)^2 + (b*d^2*x + b*
c*d)*sin(4*b*x + 4*a)^2 - 2*(b*d^2*x + b*c*d)*cos(4*b*x + 4*a))*integrate(
sin(2*b*x + 2*a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x +
 b*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x + 2
*a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)), x) + (b*d^2*x
 + b*c*d + (b*d^2*x + b*c*d)*cos(4*b*x + 4*a)^2 + (b*d^2*x + b*c*d)*sin(4*
b*x + 4*a)^2 - 2*(b*d^2*x + b*c*d)*cos(4*b*x + 4*a))*integrate(sin(b*x + a
)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x
 + a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 + 2*(b*d^2*x^2 +
2*b*c*d*x + b*c^2)*cos(b*x + a)), x) - (b*d^2*x + b*c*d + (b*d^2*x + b*c*d
)*cos(4*b*x + 4*a)^2 + (b*d^2*x + b*c*d)*sin(4*b*x + 4*a)^2 - 2*(b*d^2*x +
 b*c*d)*cos(4*b*x + 4*a))*integrate(sin(b*x + a)/(b*d^2*x^2 + 2*b*c*d*x +
b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)^2 + (b*d^2*x^2 + 2*b*
c*d*x + b*c^2)*sin(b*x + a)^2 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x
+ a)), x) - 4*sin(4*b*x + 4*a))/(b*d*x + (b*d*x + b*c)*cos(4*b*x + 4*a)^2
+ (b*d*x + b*c)*sin(4*b*x + 4*a)^2 + b*c - 2*(b*d*x + b*c)*cos(4*b*x + 4*a
))
```

### 3.276.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^2\sec(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sec(b*x + a)^2/(d*x + c), x)
```

### 3.276.9 Mupad [N/A]

Not integrable

Time = 26.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{1}{\cos(a+b\,x)^2\,\sin(a+b\,x)^2\,(c+d\,x)}\,dx$$

input
```
int(1/(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)), x)
```

# 3.277 $\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$

3.277.1 Optimal result . . . . . . . . . . 2111
3.277.2 Mathematica [N/A] . . . . . . . . . . 2111
3.277.3 Rubi [N/A] . . . . . . . . . . 2112
3.277.4 Maple [N/A] (verified) . . . . . . . . . . 2113
3.277.5 Fricas [N/A] . . . . . . . . . . 2113
3.277.6 Sympy [N/A] . . . . . . . . . . 2113
3.277.7 Maxima [N/A] . . . . . . . . . . 2114
3.277.8 Giac [N/A] . . . . . . . . . . 2115
3.277.9 Mupad [N/A] . . . . . . . . . . 2115

## 3.277.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = 4\text{Int}\left(\frac{\csc^2(2a+2bx)}{(c+dx)^2},x\right)$$

output
```
4*Unintegrable(csc(2*b*x+2*a)^2/(d*x+c)^2,x)
```

## 3.277.2 Mathematica [N/A]

Not integrable

Time = 9.97 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

### 3.277.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4919, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{4919}$$

$$4\int \frac{\csc^2(2a+2bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$4\int \frac{\csc(2a+2bx)^2}{(c+dx)^2}dx$$

$$\Big\downarrow \text{4680}$$

$$4\int \frac{\csc^2(2a+2bx)}{(c+dx)^2}dx$$

input
```
Int[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
$Aborted
```

#### 3.277.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

### 3.277.4 Maple [N/A] (verified)

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc \left(xb+a\right)^{2} \sec \left(xb+a\right)^{2}}{\left(dx+c\right)^{2}}dx$$

input
```
int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x)
```

### 3.277.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc \left(bx+a\right)^{2} \sec \left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^2*sec(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.277.6 Sympy [N/A]

Not integrable

Time = 4.80 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^{2}\left(a+bx\right)\sec^{2}\left(a+bx\right)}{\left(c+dx\right)^{2}}\,dx$$

input
```
integrate(csc(b*x+a)**2*sec(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)**2*sec(a + b*x)**2/(c + d*x)**2, x)
```

## 3.277.7 Maxima [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 1024, normalized size of antiderivative = 42.67

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)^2\sec(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
2*(2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2
*d)*cos(4*b*x + 4*a)^2 + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*sin(4*b*x + 4
*a)^2 - 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(4*b*x + 4*a))*integrate(
sin(2*b*x + 2*a)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3
*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(2*b*x + 2*a)^2 + (b*d^3*x^
3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(2*b*x + 2*a)^2 + 2*(b*d^3*x^3
 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(2*b*x + 2*a)), x) + (b*d^3*x^2
 + 2*b*c*d^2*x + b*c^2*d + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(4*b*x +
 4*a)^2 + (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*sin(4*b*x + 4*a)^2 - 2*(b*d^
3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(4*b*x + 4*a))*integrate(sin(b*x + a)/(b
*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^
2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b
*c^2*d*x + b*c^3)*sin(b*x + a)^2 + 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*
d*x + b*c^3)*cos(b*x + a)), x) - (b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d + (b*d
^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*cos(4*b*x + 4*a)^2 + (b*d^3*x^2 + 2*b*c*d^
2*x + b*c^2*d)*sin(4*b*x + 4*a)^2 - 2*(b*d^3*x^2 + 2*b*c*d^2*x + b*c^2*d)*
cos(4*b*x + 4*a))*integrate(sin(b*x + a)/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*
c^2*d*x + b*c^3 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*
x + a)^2 + (b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*sin(b*x + a)^
2 - 2*(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3)*cos(b*x + a)), ...
```

### 3.277.8 Giac [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)^2\sec\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sec(b*x + a)^2/(d*x + c)^2, x)
```

### 3.277.9 Mupad [N/A]

Not integrable

Time = 26.68 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^2\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^2),x)
```

output
```
int(1/(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^2), x)
```

# 3.278 $\int (c+dx)^m \csc^3(a+bx) \sec^2(a+bx)\,dx$

3.278.1 Optimal result . . . . . 2116
3.278.2 Mathematica [N/A] . . . . . 2116
3.278.3 Rubi [N/A] . . . . . 2117
3.278.4 Maple [N/A] (verified) . . . . . 2117
3.278.5 Fricas [N/A] . . . . . 2118
3.278.6 Sympy [F(-1)] . . . . . 2118
3.278.7 Maxima [N/A] . . . . . 2118
3.278.8 Giac [N/A] . . . . . 2119
3.278.9 Mupad [N/A] . . . . . 2119

## 3.278.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int (c+dx)^m \csc^3(a+bx) \sec^2(a+bx)\,dx = \operatorname{Int}\left((c+dx)^m \csc^3(a+bx) \sec^2(a+bx), x\right)$$

output
```
CannotIntegrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

## 3.278.2 Mathematica [N/A]

Not integrable

Time = 27.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^3(a+bx) \sec^2(a+bx)\,dx = \int (c+dx)^m \csc^3(a+bx) \sec^2(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]
```

## 3.278.3 Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\sec^2(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \csc^3(a+bx)\sec^2(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
$Aborted
```

### 3.278.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.278.4 Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \csc(xb+a)^3 \sec(xb+a)^2\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

### 3.278.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

### 3.278.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

output
```
Timed out
```

### 3.278.7 Maxima [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

### 3.278.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^3(a+bx) \sec^2(a+bx) \, dx = \int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

### 3.278.9 Mupad [N/A]

Not integrable

Time = 26.97 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^3(a+bx) \sec^2(a+bx) \, dx = \int \frac{(c+d\,x)^m}{\cos(a+b\,x)^2 \sin(a+b\,x)^3} \, dx$$

input
```
int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

output
```
int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

# 3.279 $\int (c+dx)^3 \csc^3(a+bx)\sec^2(a+bx)\,dx$

3.279.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2121
3.279.2 Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 2122
3.279.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2123
3.279.4 Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2126
3.279.5 Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 2127
3.279.6 Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2127
3.279.7 Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . . . 2128
3.279.8 Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2129
3.279.9 Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2129

### 3.279.1 Optimal result

Integrand size = 24, antiderivative size = 601

$$\begin{aligned}\int (c+dx)^3 \csc^3(a+bx)\sec^2(a+bx)\,dx &= \frac{12icd^2x\arctan\left(e^{i(a+bx)}\right)}{b^2} \\
&+ \frac{6id^3x^2\arctan\left(e^{i(a+bx)}\right)}{b^2} \\
&- \frac{6d^3x\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b^3} \\
&- \frac{3(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} \\
&- \frac{3cd^2\operatorname{arctanh}(\cos(a+bx))}{b^3} \\
&- \frac{3c^2d\operatorname{arctanh}(\sin(a+bx))}{b^2} - \frac{3c^2d\csc(a+bx)}{2b^2} \\
&- \frac{3cd^2x\csc(a+bx)}{b^2} - \frac{3d^3x^2\csc(a+bx)}{2b^2} \\
&+ \frac{3id^3\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^4} \\
&+ \frac{9id(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2} \\
&- \frac{6icd^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} \\
&- \frac{6id^3x\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} \\
&+ \frac{6icd^2\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} \\
&+ \frac{6id^3x\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} \\
&- \frac{3id^3\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^4} \\
&- \frac{9id(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2} \\
&- \frac{9d^2(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} \\
&+ \frac{6d^3\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4} \\
&- \frac{6d^3\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^4} \\
&+ \frac{9d^2(c+dx)\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3} \\
&- \frac{9id^3\operatorname{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4} \\
&+ \frac{9id^3\operatorname{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^4} \\
&+ \frac{3(c+dx)^3\sec(a+bx)}{2b}\end{aligned}$$

output
```
6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b
^4-3*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b+3/2*(d*x+c)^3*sec(b*x+a)/b-6*d^3*
x*arctanh(exp(I*(b*x+a)))/b^3-3*c*d^2*arctanh(cos(b*x+a))/b^3-3*c^2*d*arct
anh(sin(b*x+a))/b^2-3/2*c^2*d*csc(b*x+a)/b^2-3/2*d^3*x^2*csc(b*x+a)/b^2-1/
2*(d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)/b-9*I*d^3*polylog(4,-exp(I*(b*x+a)))/b
^4+6*I*d^3*x^2*arctan(exp(I*(b*x+a)))/b^2+6*I*c*d^2*polylog(2,I*exp(I*(b*x
+a)))/b^3+6*I*d^3*x*polylog(2,I*exp(I*(b*x+a)))/b^3+9*I*d^3*polylog(4,exp(
I*(b*x+a)))/b^4-9*d^2*(d*x+c)*polylog(3,-exp(I*(b*x+a)))/b^3+9*d^2*(d*x+c)
*polylog(3,exp(I*(b*x+a)))/b^3-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4+12*I*
c*d^2*x*arctan(exp(I*(b*x+a)))/b^2+9/2*I*d*(d*x+c)^2*polylog(2,-exp(I*(b*x
+a)))/b^2+3*I*d^3*polylog(2,-exp(I*(b*x+a)))/b^4-3*c*d^2*x*csc(b*x+a)/b^2-
6*I*c*d^2*polylog(2,-I*exp(I*(b*x+a)))/b^3-6*I*d^3*x*polylog(2,-I*exp(I*(b
*x+a)))/b^3-9/2*I*d*(d*x+c)^2*polylog(2,exp(I*(b*x+a)))/b^2
```

### 3.279.2 Mathematica [A] (verified)

Time = 8.19 (sec) , antiderivative size = 907, normalized size of antiderivative = 1.51

$$\int (c+dx)^3 \csc^3(a+bx)\sec^2(a+bx)\,dx =$$
$$-\frac{3d\left(-2ib^2c^2\arctan\left(e^{i(a+bx)}\right)+2b^2cdx\log\left(1-ie^{i(a+bx)}\right)+b^2d^2x^2\log\left(1-ie^{i(a+bx)}\right)-2b^2cdx\log\left(1+i\right.\right.}{}$$
$$+\frac{3\left(b^3c^3\log\left(1-e^{i(a+bx)}\right)+2bcd^2\log\left(1-e^{i(a+bx)}\right)+3b^3c^2dx\log\left(1-e^{i(a+bx)}\right)+2bd^3x\log\left(1-e^{i(a+bx)}\right)\right.}{}$$
$$-\frac{\csc^2(a+bx)\sec(a+bx)\left(-bc^3-3bc^2dx-3bcd^2x^2-bd^3x^3+3bc^3\cos(2a+2bx)+9bc^2dx\cos(2a+2bx\right.}{}$$

input
```
Integrate[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
(-3*d*((-2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 - I*E^(I
*(a + b*x))] + b^2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - 2*b^2*c*d*x*Log[1
+ I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + (2*I)*b*d*
(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2
, I*E^(I*(a + b*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 2*d^2*Poly
Log[3, I*E^(I*(a + b*x))]))/b^4 + (3*(b^3*c^3*Log[1 - E^(I*(a + b*x))] + 2
*b*c*d^2*Log[1 - E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 - E^(I*(a + b*x))]
 + 2*b*d^3*x*Log[1 - E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - E^(I*(a +
b*x))] + b^3*d^3*x^3*Log[1 - E^(I*(a + b*x))] - b^3*c^3*Log[1 + E^(I*(a +
b*x))] - 2*b*c*d^2*Log[1 + E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + E^(I*(
a + b*x))] - 2*b*d^3*x*Log[1 + E^(I*(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 +
E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + E^(I*(a + b*x))] + I*d*(2*d^2 + 3*b
^2*(c + d*x)^2)*PolyLog[2, -E^(I*(a + b*x))] - I*d*(2*d^2 + 3*b^2*(c + d*x
)^2)*PolyLog[2, E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, -E^(I*(a + b*x))]
- 6*b*d^3*x*PolyLog[3, -E^(I*(a + b*x))] + 6*b*c*d^2*PolyLog[3, E^(I*(a +
b*x))] + 6*b*d^3*x*PolyLog[3, E^(I*(a + b*x))] - (6*I)*d^3*PolyLog[4, -E^(
I*(a + b*x))] + (6*I)*d^3*PolyLog[4, E^(I*(a + b*x))]))/(2*b^4) - (Csc[a +
 b*x]^2*Sec[a + b*x]*(-(b*c^3) - 3*b*c^2*d*x - 3*b*c*d^2*x^2 - b*d^3*x^3 +
 3*b*c^3*Cos[2*a + 2*b*x] + 9*b*c^2*d*x*Cos[2*a + 2*b*x] + 9*b*c*d^2*x^2*C
os[2*a + 2*b*x] + 3*b*d^3*x^3*Cos[2*a + 2*b*x] + 3*c^2*d*Sin[2*a + 2*b*...
```

### 3.279.3 Rubi [A] (verified)

Time = 2.28 (sec) , antiderivative size = 623, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4920, 27, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \csc^3(a+bx)\sec^2(a+bx)\,dx$$

$$\Big\downarrow \text{4920}$$

$$-3d\int -\frac{1}{2}(c+dx)^2\left(\frac{\sec(a+bx)\csc^2(a+bx)}{b}+\frac{3\text{arctanh}(\cos(a+bx))}{b}-\frac{3\sec(a+bx)}{b}\right)dx-\frac{3(c+dx)^3\text{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)^3\sec(a+bx)}{2b}-\frac{(c+dx)^3\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{2}d\int (c+dx)^2\left(\frac{\sec(a+bx)\csc^2(a+bx)}{b}+\frac{3\text{arctanh}(\cos(a+bx))}{b}-\frac{3\sec(a+bx)}{b}\right)dx-\frac{3(c+dx)^3\text{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)^3\sec(a+bx)}{2b}-\frac{(c+dx)^3\csc^2(a+bx)\sec(a+bx)}{2b}$$

$\big\downarrow$ 7292

$$\frac{3}{2}d\int \frac{(c+dx)^2\left(\sec(a+bx)\csc^2(a+bx)+3\text{arctanh}(\cos(a+bx))-3\sec(a+bx)\right)}{b}dx-\frac{3(c+dx)^3\text{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)^3\sec(a+bx)}{2b}-\frac{(c+dx)^3\csc^2(a+bx)\sec(a+bx)}{2b}$$

$\big\downarrow$ 27

$$\frac{3d\int (c+dx)^2\left(\sec(a+bx)\csc^2(a+bx)+3\text{arctanh}(\cos(a+bx))-3\sec(a+bx)\right)dx}{2b}-\frac{3(c+dx)^3\text{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)^3\sec(a+bx)}{2b}-\frac{(c+dx)^3\csc^2(a+bx)\sec(a+bx)}{2b}$$

$\big\downarrow$ 7293

$$\frac{3d\int \left(3\text{arctanh}(\cos(a+bx))(c+dx)^2+\left(\csc^2(a+bx)-3\right)\sec(a+bx)(c+dx)^2\right)dx}{2b}-\frac{3(c+dx)^3\text{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)^3\sec(a+bx)}{2b}-\frac{(c+dx)^3\csc^2(a+bx)\sec(a+bx)}{2b}$$

$\big\downarrow$ 2009

$$\frac{3d\left(\frac{8icdx\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{4id^2x^2\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{2cd\text{arctanh}(\cos(a+bx))}{b^2}-\frac{4d^2x\text{arctanh}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2c^2\text{arctanh}(\sin(a+bx)}{b}\right.}{}$$

$$\frac{3(c+dx)^3\text{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)^3\sec(a+bx)}{2b}-\frac{(c+dx)^3\csc^2(a+bx)\sec(a+bx)}{2b}$$

input
```
Int[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

3.279. $\int (c+dx)^3\csc^3(a+bx)\sec^2(a+bx)\,dx$

output
```
(-3*(c + d*x)^3*ArcTanh[Cos[a + b*x]])/(2*b) + (3*d*(((8*I)*c*d*x*ArcTan[E
^(I*(a + b*x))])/b + ((4*I)*d^2*x^2*ArcTan[E^(I*(a + b*x))])/b - (4*d^2*x*
ArcTanh[E^(I*(a + b*x))])/b^2 - (2*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/d
 - (2*c*d*ArcTanh[Cos[a + b*x]])/b^2 + ((c + d*x)^3*ArcTanh[Cos[a + b*x]])
/d - (2*c^2*ArcTanh[Sin[a + b*x]])/b - (c^2*Csc[a + b*x])/b - (2*c*d*x*Csc
[a + b*x])/b - (d^2*x^2*Csc[a + b*x])/b + ((2*I)*d^2*PolyLog[2, -E^(I*(a +
 b*x))])/b^3 + ((3*I)*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b - ((4*I)
*c*d*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((4*I)*d^2*x*PolyLog[2, (-I)*
E^(I*(a + b*x))])/b^2 + ((4*I)*c*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2 + ((
4*I)*d^2*x*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - ((2*I)*d^2*PolyLog[2, E^(I
*(a + b*x))])/b^3 - ((3*I)*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b - (6
*d*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^2 + (4*d^2*PolyLog[3, (-I)*E^
(I*(a + b*x))])/b^3 - (4*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + (6*d*(c
+ d*x)*PolyLog[3, E^(I*(a + b*x))])/b^2 - ((6*I)*d^2*PolyLog[4, -E^(I*(a +
 b*x))])/b^3 + ((6*I)*d^2*PolyLog[4, E^(I*(a + b*x))])/b^3))/(2*b) + (3*(c
 + d*x)^3*Sec[a + b*x])/(2*b) - ((c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x])/
(2*b)
```

### 3.279.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### 3.279.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1612 vs. 2(535) = 1070.

Time = 2.44 (sec) , antiderivative size = 1613, normalized size of antiderivative = 2.68

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1613 |

input
```
int((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
6*I/b^4*a*d^3*dilog(1+I*exp(I*(b*x+a)))-9*I/b^2*c*d^2*polylog(2,exp(I*(b*x
+a)))*x+9*I/b^2*c*d^2*polylog(2,-exp(I*(b*x+a)))*x+3/b^3*d^3*ln(1-exp(I*(b
*x+a)))*x-3/b^3*d^3*ln(exp(I*(b*x+a))+1)*x-3/b^3*c*d^2*ln(exp(I*(b*x+a))+1
)+3/b^3*c*d^2*ln(exp(I*(b*x+a))-1)-3/b^4*a*d^3*ln(exp(I*(b*x+a))-1)+3/b^4*
d^3*ln(1-exp(I*(b*x+a)))*a-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4-9*I*d^3*p
olylog(4,-exp(I*(b*x+a)))/b^4+6/b^2*d^2*c*ln(1+I*exp(I*(b*x+a)))*x-6/b^2*d
^2*c*ln(1-I*exp(I*(b*x+a)))*x+6/b^3*d^2*c*ln(1+I*exp(I*(b*x+a)))*a-6/b^3*d
^2*c*ln(1-I*exp(I*(b*x+a)))*a+6*I/b^2*d*c^2*arctan(exp(I*(b*x+a)))+6*I/b^4
*d^3*a^2*arctan(exp(I*(b*x+a)))+6*I*d^3*x*polylog(2,I*exp(I*(b*x+a)))/b^3+
9*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+9/2/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-
1)-9/2/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)+9/2/b^2*d*c^2*ln(1-exp(I*(b*x+a)))
*a-9/2/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2+6*d^3*polylog(3,-I*exp(I*(b*x+a)
))/b^4-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4-3/2/b*c^3*ln(exp(I*(b*x+a))+1
)+3/2/b*c^3*ln(exp(I*(b*x+a))-1)+3/b^2*d^3*ln(1+I*exp(I*(b*x+a)))*x^2+3/b^
4*d^3*a^2*ln(1-I*exp(I*(b*x+a)))-3/b^4*d^3*a^2*ln(1+I*exp(I*(b*x+a)))-3/b^
2*d^3*ln(1-I*exp(I*(b*x+a)))*x^2+9/2/b*d*c^2*ln(1-exp(I*(b*x+a)))*x-9/2/b*
d*c^2*ln(exp(I*(b*x+a))+1)*x+9/2/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-9/2/b*c*
d^2*ln(exp(I*(b*x+a))+1)*x^2-3/2/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+3/2/b^4*
d^3*ln(1-exp(I*(b*x+a)))*a^3+9/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-9/b^3*d
^3*polylog(3,-exp(I*(b*x+a)))*x+3/2/b*d^3*ln(1-exp(I*(b*x+a)))*x^3-3/2/...
```

### 3.279.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3173 vs. 2(509) = 1018.

Time = 0.43 (sec) , antiderivative size = 3173, normalized size of antiderivative = 5.28

$$\int (c+dx)^3 \csc^3(a+bx) \sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x + 4*b^3*c^3 - 6*(b
^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a)^2 - 6
*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x + a) + 3*(
(3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a)
^3 + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 2*I*d^3)*cos(b*
x + a))*dilog(cos(b*x + a) + I*sin(b*x + a)) + 3*((-3*I*b^2*d^3*x^2 - 6*I*
b^2*c*d^2*x - 3*I*b^2*c^2*d - 2*I*d^3)*cos(b*x + a)^3 + (3*I*b^2*d^3*x^2 +
 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a))*dilog(cos(b*x +
a) - I*sin(b*x + a)) + 12*((-I*b*d^3*x - I*b*c*d^2)*cos(b*x + a)^3 + (I*b*
d^3*x + I*b*c*d^2)*cos(b*x + a))*dilog(I*cos(b*x + a) + sin(b*x + a)) + 12
*((-I*b*d^3*x - I*b*c*d^2)*cos(b*x + a)^3 + (I*b*d^3*x + I*b*c*d^2)*cos(b*
x + a))*dilog(I*cos(b*x + a) - sin(b*x + a)) + 12*((I*b*d^3*x + I*b*c*d^2)
*cos(b*x + a)^3 + (-I*b*d^3*x - I*b*c*d^2)*cos(b*x + a))*dilog(-I*cos(b*x
+ a) + sin(b*x + a)) + 12*((I*b*d^3*x + I*b*c*d^2)*cos(b*x + a)^3 + (-I*b*
d^3*x - I*b*c*d^2)*cos(b*x + a))*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 3
*((3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 2*I*d^3)*cos(b*x +
a)^3 + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 2*I*d^3)*cos(
b*x + a))*dilog(-cos(b*x + a) + I*sin(b*x + a)) + 3*((-3*I*b^2*d^3*x^2 - 6
*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 2*I*d^3)*cos(b*x + a)^3 + (3*I*b^2*d^3*x^
2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a))*dilog(-cos...
```

### 3.279.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^3(a+bx) \sec^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**3*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

output
```
Timed out
```

### 3.279.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8032 vs. 2(509) = 1018.

Time = 3.62 (sec) , antiderivative size = 8032, normalized size of antiderivative = 13.36

$$\int (c+dx)^3 \csc^3(a+bx) \sec^2(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/4*(c^3*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log
(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1)) - 3*a*c^2*d*(2*(3*cos(b*x +
a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(cos(b*x + a) + 1) + 3*lo
g(cos(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x +
a)^3 - cos(b*x + a)) - 3*log(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1))/
b^2 - a^3*d^3*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) -
3*log(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1))/b^3 + 4*(12*(b^2*c^2*d
- 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a)
+ (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^
3)*(b*x + a))*cos(6*b*x + 6*a) - (b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^
3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - (b^2*c^2*d
 - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a)
)*cos(2*b*x + 2*a) + (I*b^2*c^2*d - 2*I*a*b*c*d^2 + I*(b*x + a)^2*d^3 + I*
a^2*d^3 + 2*(I*b*c*d^2 - I*a*d^3)*(b*x + a))*sin(6*b*x + 6*a) + (-I*b^2*c^
2*d + 2*I*a*b*c*d^2 - I*(b*x + a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a*
d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-I*b^2*c^2*d + 2*I*a*b*c*d^2 - I*(b*x
+ a)^2*d^3 - I*a^2*d^3 + 2*(-I*b*c*d^2 + I*a*d^3)*(b*x + a))*sin(2*b*x + 2
*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + 12*(b^2*c^2*d - 2*a*b*c*d^2
 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a) + (b^2*c^2*d
- 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + ...
```

### 3.279.8 Giac [F]

$$\int (c+dx)^3 \csc^3(a+bx)\sec^2(a+bx)\,dx = \int (dx+c)^3 \csc(bx+a)^3 \sec(bx+a)^2 \,dx$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

### 3.279.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^3(a+bx)\sec^2(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)^3/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

output
```
\text{Hanged}
```

# 3.280 $\int (c+dx)^2 \csc^3(a+bx)\sec^2(a+bx)\,dx$


3.280.1 Optimal result . . . 2130
3.280.2 Mathematica [B] (warning: unable to verify) . . . 2131
3.280.3 Rubi [A] (verified) . . . 2132
3.280.4 Maple [B] (verified) . . . 2134
3.280.5 Fricas [B] (verification not implemented) . . . 2135
3.280.6 Sympy [F] . . . 2136
3.280.7 Maxima [B] (verification not implemented) . . . 2137
3.280.8 Giac [F] . . . 2137
3.280.9 Mupad [F(-1)] . . . 2138


## 3.280.1 Optimal result

Integrand size = 24, antiderivative size = 305

$$\begin{aligned}
&\int (c+dx)^2 \csc^3(a+bx)\sec^2(a+bx)\,dx \\
&= \frac{4id^2x\arctan\left(e^{i(a+bx)}\right)}{b^2}-\frac{3(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{d^2\text{arctanh}(\cos(a+bx))}{b^3} \\
&\quad-\frac{2cd\text{arctanh}(\sin(a+bx))}{b^2}-\frac{cd\csc(a+bx)}{b^2}-\frac{d^2x\csc(a+bx)}{b^2} \\
&\quad+\frac{3id(c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{2id^2\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} \\
&\quad+\frac{2id^2\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-\frac{3id(c+dx)\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} \\
&\quad-\frac{3d^2\,\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{3d^2\,\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3} \\
&\quad+\frac{3(c+dx)^2\sec(a+bx)}{2b}-\frac{(c+dx)^2\csc^2(a+bx)\sec(a+bx)}{2b}
\end{aligned}$$

output
```
4*I*d^2*x*arctan(exp(I*(b*x+a)))/b^2-3*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b
-d^2*arctanh(cos(b*x+a))/b^3-2*c*d*arctanh(sin(b*x+a))/b^2-c*d*csc(b*x+a)/
b^2-d^2*x*csc(b*x+a)/b^2+3*I*d*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^2-2*I*
d^2*polylog(2,-I*exp(I*(b*x+a)))/b^3+2*I*d^2*polylog(2,I*exp(I*(b*x+a)))/b
^3-3*I*d*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^2-3*d^2*polylog(3,-exp(I*(b*x
+a)))/b^3+3*d^2*polylog(3,exp(I*(b*x+a)))/b^3+3/2*(d*x+c)^2*sec(b*x+a)/b-1
/2*(d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)/b
```

### 3.280.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 889 vs. $2(305) = 610$.

Time = 7.93 (sec) , antiderivative size = 889, normalized size of antiderivative = 2.91

$$\int (c+dx)^2\csc^3(a+bx)\sec^2(a+bx)\,dx = \frac{(-c^2-2cdx-d^2x^2)\csc^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}$$
$$+\frac{3b^2c^2\log\left(1-e^{i(a+bx)}\right)+2d^2\log\left(1-e^{i(a+bx)}\right)+6b^2cdx\log\left(1-e^{i(a+bx)}\right)+3b^2d^2x^2\log\left(1-e^{i(a+bx)}\right)-\cdots}{\cdots}$$
$$+\frac{(c^2+2cdx+d^2x^2)\sec^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}$$
$$+\frac{(c+dx)\csc(a)\sec(a)(-d\cos(a)+bc\sin(a)+bdx\sin(a))}{b^2}$$
$$-\frac{4icd\arctan\left(\frac{-i\sin(a)-i\cos(a)\tan\left(\frac{bx}{2}\right)}{\sqrt{\cos^2(a)+\sin^2(a)}}\right)}{b^2\sqrt{\cos^2(a)+\sin^2(a)}}$$
$$-\frac{2d^2\left(-\frac{\csc(a)((bx-\arctan(\cot(a)))(\log(1-e^{i(bx-\arctan(\cot(a)))})-\log(1+e^{i(bx-\arctan(\cot(a)))}))+i(\text{PolyLog}(2,-e^{i(bx-\arctan(\cot(a)))})-\text{Pol}\cdots}{\sqrt{1+\cot^2(a)}}\cdots\right.}{b^3}$$
$$+\frac{\sec\left(\frac{a}{2}\right)\sec\left(\frac{a}{2}+\frac{bx}{2}\right)\left(-cd\sin\left(\frac{bx}{2}\right)-d^2x\sin\left(\frac{bx}{2}\right)\right)}{2b^2}$$
$$+\frac{\csc\left(\frac{a}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)\left(cd\sin\left(\frac{bx}{2}\right)+d^2x\sin\left(\frac{bx}{2}\right)\right)}{2b^2}$$
$$+\frac{c^2\sin\left(\frac{bx}{2}\right)+2cdx\sin\left(\frac{bx}{2}\right)+d^2x^2\sin\left(\frac{bx}{2}\right)}{b\left(\cos\left(\frac{a}{2}\right)-\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}$$
$$+\frac{-c^2\sin\left(\frac{bx}{2}\right)-2cdx\sin\left(\frac{bx}{2}\right)-d^2x^2\sin\left(\frac{bx}{2}\right)}{b\left(\cos\left(\frac{a}{2}\right)+\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)+\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}$$

input
```
Integrate[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
((-c^2 - 2*c*d*x - d^2*x^2)*Csc[a/2 + (b*x)/2]^2)/(8*b) + (3*b^2*c^2*Log[1
 - E^(I*(a + b*x))] + 2*d^2*Log[1 - E^(I*(a + b*x))] + 6*b^2*c*d*x*Log[1 -
 E^(I*(a + b*x))] + 3*b^2*d^2*x^2*Log[1 - E^(I*(a + b*x))] - 3*b^2*c^2*Log
[1 + E^(I*(a + b*x))] - 2*d^2*Log[1 + E^(I*(a + b*x))] - 6*b^2*c*d*x*Log[1
 + E^(I*(a + b*x))] - 3*b^2*d^2*x^2*Log[1 + E^(I*(a + b*x))] + (6*I)*b*d*(
c + d*x)*PolyLog[2, -E^(I*(a + b*x))] - (6*I)*b*d*(c + d*x)*PolyLog[2, E^(
I*(a + b*x))] - 6*d^2*PolyLog[3, -E^(I*(a + b*x))] + 6*d^2*PolyLog[3, E^(I
*(a + b*x))])/(2*b^3) + ((c^2 + 2*c*d*x + d^2*x^2)*Sec[a/2 + (b*x)/2]^2)/(
8*b) + ((c + d*x)*Csc[a]*Sec[a]*(-(d*Cos[a]) + b*c*Sin[a] + b*d*x*Sin[a]))
/b^2 - ((4*I)*c*d*ArcTan[((-I)*Sin[a] - I*Cos[a]*Tan[(b*x)/2])/Sqrt[Cos[a]
^2 + Sin[a]^2]])/(b^2*Sqrt[Cos[a]^2 + Sin[a]^2]) - (2*d^2*(-((Csc[a]*((b*x
 - ArcTan[Cot[a]])*(Log[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(
b*x - ArcTan[Cot[a]]))]) + I*(PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] -
PolyLog[2, E^(I*(b*x - ArcTan[Cot[a]]))])))/Sqrt[1 + Cot[a]^2]) + (2*ArcTa
n[Cot[a]]*ArcTanh[(Sin[a] + Cos[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]
])/Sqrt[Cos[a]^2 + Sin[a]^2]))/b^3 + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c*d*S
in[(b*x)/2]) - d^2*x*Sin[(b*x)/2]))/(2*b^2) + (Csc[a/2]*Csc[a/2 + (b*x)/2]
*(c*d*Sin[(b*x)/2] + d^2*x*Sin[(b*x)/2]))/(2*b^2) + (c^2*Sin[(b*x)/2] + 2*
c*d*x*Sin[(b*x)/2] + d^2*x^2*Sin[(b*x)/2])/(b*(Cos[a/2] - Sin[a/2])*(Cos[a
/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) + (-(c^2*Sin[(b*x)/2]) - 2*c*d*x*S...
```

### 3.280.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4920, 27, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \csc^3(a+bx) \sec^2(a+bx) \, dx$$

$$\Big\downarrow \text{4920}$$

$$-2d \int -\frac{1}{2}(c+dx)\left(\frac{\sec(a+bx)\csc^2(a+bx)}{b}+\frac{3\text{arctanh}(\cos(a+bx))}{b}-\frac{3\sec(a+bx)}{b}\right)dx-\frac{3(c+dx)^2\text{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)^2\sec(a+bx)}{2b}-\frac{(c+dx)^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{27}$$

$$d\int (c+dx)\left(\frac{\sec(a+bx)\csc^2(a+bx)}{b}+\frac{3\text{arctanh}(\cos(a+bx))}{b}-\frac{3\sec(a+bx)}{b}\right)dx-\frac{3(c+dx)^2\text{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)^2\sec(a+bx)}{2b}-\frac{(c+dx)^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

$\Big\downarrow$ 7292

$$d\int \frac{(c+dx)\left(\sec(a+bx)\csc^2(a+bx)+3\text{arctanh}(\cos(a+bx))-3\sec(a+bx)\right)}{b}dx-\frac{3(c+dx)^2\text{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)^2\sec(a+bx)}{2b}-\frac{(c+dx)^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

$\Big\downarrow$ 27

$$\frac{d\int (c+dx)\left(\sec(a+bx)\csc^2(a+bx)+3\text{arctanh}(\cos(a+bx))-3\sec(a+bx)\right)dx}{b}-\frac{3(c+dx)^2\text{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)^2\sec(a+bx)}{2b}-\frac{(c+dx)^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

$\Big\downarrow$ 7293

$$\frac{d\int \left(3(c+dx)\text{arctanh}(\cos(a+bx))+(c+dx)\left(\csc^2(a+bx)-3\right)\sec(a+bx)\right)dx}{b}-\frac{3(c+dx)^2\text{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)^2\sec(a+bx)}{2b}-\frac{(c+dx)^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

$\Big\downarrow$ 2009

$$\frac{d\left(\frac{4idx\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{d\text{arctanh}(\cos(a+bx))}{b^2}-\frac{3(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{d}+\frac{3(c+dx)^2\text{arctanh}(\cos(a+bx))}{2d}-\frac{2c\text{arctanh}(\sin(\ldots}{b}\right.}{}$$

$$\frac{3(c+dx)^2\text{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)^2\sec(a+bx)}{2b}-\frac{(c+dx)^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

input
```
Int[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
(-3*(c + d*x)^2*ArcTanh[Cos[a + b*x]])/(2*b) + (d*(((4*I)*d*x*ArcTan[E^(I*
(a + b*x))])/b - (3*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/d - (d*ArcTanh[C
os[a + b*x]])/b^2 + (3*(c + d*x)^2*ArcTanh[Cos[a + b*x]])/(2*d) - (2*c*Arc
Tanh[Sin[a + b*x]])/b - (c*Csc[a + b*x])/b - (d*x*Csc[a + b*x])/b + ((3*I)
*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b - ((2*I)*d*PolyLog[2, (-I)*E^(I
*(a + b*x))])/b^2 + ((2*I)*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - ((3*I)*(
c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b - (3*d*PolyLog[3, -E^(I*(a + b*x))
])/b^2 + (3*d*PolyLog[3, E^(I*(a + b*x))])/b^2))/b + (3*(c + d*x)^2*Sec[a
+ b*x])/(2*b) - ((c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```

## 3.280. $\int (c+dx)^2\csc^3(a+bx)\sec^2(a+bx)\,dx$

### 3.280.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## 3.280.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 801 vs. 2(278) = 556.

Time = 2.01 (sec) , antiderivative size = 802, normalized size of antiderivative = 2.63

| method | result |
|---|---|
| risch | $-\frac{3id^2 \operatorname{polylog}(2,e^{i(xb+a)})x}{b^2}-\frac{3idc \operatorname{polylog}(2,e^{i(xb+a)})}{b^2}+\frac{3dc\ln(1-e^{i(xb+a)})x}{b}+\frac{3dc\ln(1-e^{i(xb+a)})a}{b^2}+\frac{3d^2 \operatorname{polylog}(3,e^{i(xb}}{b^3}$ |

input
```
int((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
3/2/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+4*I/b^2*d*c*arctan(exp(I*(b*x+a)))-4*
I/b^3*d^2*a*arctan(exp(I*(b*x+a)))-3/2/b*c^2*ln(exp(I*(b*x+a))+1)+3/2/b*c^
2*ln(exp(I*(b*x+a))-1)-3/2/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+3/2/b*d^2*ln(1
-exp(I*(b*x+a)))*x^2-3/2/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+2*I/b^3*d^2*dilog(
1-I*exp(I*(b*x+a)))-3/b*d*c*ln(exp(I*(b*x+a))+1)*x+3/b^2*d*c*ln(1-exp(I*(b
*x+a)))*a-3/b^2*c*d*a*ln(exp(I*(b*x+a))-1)+3/b*d*c*ln(1-exp(I*(b*x+a)))*x+
2/b^2*d^2*ln(1+I*exp(I*(b*x+a)))*x+2/b^3*d^2*ln(1+I*exp(I*(b*x+a)))*a-2/b^
2*d^2*ln(1-I*exp(I*(b*x+a)))*x-2/b^3*d^2*ln(1-I*exp(I*(b*x+a)))*a-2*I/b^3*
d^2*dilog(1+I*exp(I*(b*x+a)))+1/b^2/(exp(2*I*(b*x+a))-1)^2/(exp(2*I*(b*x+a
))+1)*(3*x^2*d^2*b*exp(5*I*(b*x+a))+6*c*d*x*b*exp(5*I*(b*x+a))+3*c^2*b*exp
(5*I*(b*x+a))-2*x^2*d^2*b*exp(3*I*(b*x+a))-4*c*d*x*b*exp(3*I*(b*x+a))-2*I*
d^2*x*exp(5*I*(b*x+a))-2*c^2*b*exp(3*I*(b*x+a))+3*x^2*d^2*b*exp(I*(b*x+a))
-2*I*c*d*exp(5*I*(b*x+a))+6*c*d*x*b*exp(I*(b*x+a))+3*c^2*b*exp(I*(b*x+a))+
2*I*d^2*x*exp(I*(b*x+a))+2*I*d*c*exp(I*(b*x+a)))-3*d^2*polylog(3,-exp(I*(b
*x+a)))/b^3+3*d^2*polylog(3,exp(I*(b*x+a)))/b^3-1/b^3*d^2*ln(exp(I*(b*x+a)
)+1)+1/b^3*d^2*ln(exp(I*(b*x+a))-1)-3*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))+
3*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))+3*I/b^2*d^2*polylog(2,-exp(I*(b*x+a
)))*x-3*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x
```

### 3.280.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1801 vs. $2(267) = 534$.

Time = 0.34 (sec) , antiderivative size = 1801, normalized size of antiderivative = 5.90

$$\int (c+dx)^2 \csc^3(a+bx) \sec^2(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*b^2*d^2*x^2 + 8*b^2*c*d*x + 4*b^2*c^2 - 6*(b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2)*cos(b*x + a)^2 - 4*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x +
a) + 6*((I*b*d^2*x + I*b*c*d)*cos(b*x + a)^3 + (-I*b*d^2*x - I*b*c*d)*cos(
b*x + a))*dilog(cos(b*x + a) + I*sin(b*x + a)) + 6*((-I*b*d^2*x - I*b*c*d)
*cos(b*x + a)^3 + (I*b*d^2*x + I*b*c*d)*cos(b*x + a))*dilog(cos(b*x + a) -
 I*sin(b*x + a)) + 4*(-I*d^2*cos(b*x + a)^3 + I*d^2*cos(b*x + a))*dilog(I*
cos(b*x + a) + sin(b*x + a)) + 4*(-I*d^2*cos(b*x + a)^3 + I*d^2*cos(b*x +
a))*dilog(I*cos(b*x + a) - sin(b*x + a)) + 4*(I*d^2*cos(b*x + a)^3 - I*d^2
*cos(b*x + a))*dilog(-I*cos(b*x + a) + sin(b*x + a)) + 4*(I*d^2*cos(b*x +
a)^3 - I*d^2*cos(b*x + a))*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 6*((I*b
*d^2*x + I*b*c*d)*cos(b*x + a)^3 + (-I*b*d^2*x - I*b*c*d)*cos(b*x + a))*di
log(-cos(b*x + a) + I*sin(b*x + a)) + 6*((-I*b*d^2*x - I*b*c*d)*cos(b*x +
a)^3 + (I*b*d^2*x + I*b*c*d)*cos(b*x + a))*dilog(-cos(b*x + a) - I*sin(b*x
 + a)) + ((3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*cos(b*x + a)^3
 - (3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*cos(b*x + a))*log(cos
(b*x + a) + I*sin(b*x + a) + 1) + 4*((b*c*d - a*d^2)*cos(b*x + a)^3 - (b*c
*d - a*d^2)*cos(b*x + a))*log(cos(b*x + a) + I*sin(b*x + a) + I) + ((3*b^2
*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*cos(b*x + a)^3 - (3*b^2*d^2*x^
2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*cos(b*x + a))*log(cos(b*x + a) - I*si
n(b*x + a) + 1) - 4*((b*c*d - a*d^2)*cos(b*x + a)^3 - (b*c*d - a*d^2)*c...
```

### 3.280.6 Sympy [F]

$$\int (c+dx)^2 \csc^3(a+bx)\sec^2(a+bx)\,dx = \int (c+dx)^2 \csc^3\left(a+bx\right)\sec^2\left(a+bx\right)dx$$

input
```
integrate((d*x+c)**2*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*csc(a + b*x)**3*sec(a + b*x)**2, x)
```

### 3.280.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3814 vs. 2(267) = 534.

Time = 0.99 (sec) , antiderivative size = 3814, normalized size of antiderivative = 12.50

$$\int (c+dx)^2 \csc^3(a+bx) \sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/4*(c^2*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log
(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1)) - 2*a*c*d*(2*(3*cos(b*x + a)
^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(cos(b*x + a) + 1) + 3*log(
cos(b*x + a) - 1))/b + a^2*d^2*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 -
 cos(b*x + a)) - 3*log(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1))/b^2 +
4*(8*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(
6*b*x + 6*a) - (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - (b*c*d +
 (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (I*b*c*d + I*(b*x + a)*d^2 - I*
a*d^2)*sin(6*b*x + 6*a) + (-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(4*b*x
 + 4*a) + (-I*b*c*d - I*(b*x + a)*d^2 + I*a*d^2)*sin(2*b*x + 2*a))*arctan2
(cos(b*x + a), sin(b*x + a) + 1) + 8*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c
*d + (b*x + a)*d^2 - a*d^2)*cos(6*b*x + 6*a) - (b*c*d + (b*x + a)*d^2 - a*
d^2)*cos(4*b*x + 4*a) - (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) +
 (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(6*b*x + 6*a) + (-I*b*c*d - I*(b
*x + a)*d^2 + I*a*d^2)*sin(4*b*x + 4*a) + (-I*b*c*d - I*(b*x + a)*d^2 + I*
a*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 2*(3*(
b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + (3*(b*x + a)^2*d^2
+ 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(6*b*x + 6*a) - (3*(b*x + a)^2*d
^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(4*b*x + 4*a) - (3*(b*x + a)^
2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(2*b*x + 2*a) - (-3*I*(...
```

### 3.280.8 Giac [F]

$$\int (c+dx)^2 \csc^3(a+bx) \sec^2(a+bx)\,dx = \int (dx+c)^2 \csc{(bx+a)}^3 \sec{(bx+a)}^2 \, dx$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

### 3.280.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc^3(a+bx) \sec^2(a+bx) \, dx = \text{Hanged}$$

input
```
int((c + d*x)^2/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

output
```
\text{Hanged}
```

# 3.281 $\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx$

3.281.1 Optimal result . . . 2139
3.281.2 Mathematica [B] (verified) . . . 2140
3.281.3 Rubi [A] (verified) . . . 2141
3.281.4 Maple [A] (verified) . . . 2142
3.281.5 Fricas [B] (verification not implemented) . . . 2142
3.281.6 Sympy [F] . . . 2143
3.281.7 Maxima [B] (verification not implemented) . . . 2143
3.281.8 Giac [F] . . . 2144
3.281.9 Mupad [F(-1)] . . . 2145

## 3.281.1 Optimal result

Integrand size = 22, antiderivative size = 154

$$\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3dx\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{3c\operatorname{arctanh}(\cos(a+bx))}{2b}-\frac{d\operatorname{arctanh}(\sin(a+bx))}{b^2}-\frac{d\csc(a+bx)}{2b^2}+\frac{3id\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}-\frac{3id\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}+\frac{3(c+dx)\sec(a+bx)}{2b}-\frac{(c+dx)\csc^2(a+bx)\sec(a+bx)}{2b}$$

output
```
-3*d*x*arctanh(exp(I*(b*x+a)))/b-3/2*c*arctanh(cos(b*x+a))/b-d*arctanh(sin
(b*x+a))/b^2-1/2*d*csc(b*x+a)/b^2+3/2*I*d*polylog(2,-exp(I*(b*x+a)))/b^2-3
/2*I*d*polylog(2,exp(I*(b*x+a)))/b^2+3/2*(d*x+c)*sec(b*x+a)/b-1/2*(d*x+c)*
csc(b*x+a)^2*sec(b*x+a)/b
```

### 3.281.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 520 vs. 2(154) = 308.

Time = 6.16 (sec) , antiderivative size = 520, normalized size of antiderivative = 3.38

$$\begin{aligned}
\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx &= \frac{dx}{b}-\frac{d\cot\left(\frac{1}{2}(a+bx)\right)}{4b^2} \\
&-\frac{c\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{dx\csc^2\left(\frac{1}{2}(a+bx)\right)}{8b}-\frac{3c\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{2b} \\
&+\frac{d\log\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b^2}+\frac{3c\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b} \\
&-\frac{d\log\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b^2}-\frac{3ad\log\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)}{2b^2} \\
&+\frac{3d\left((a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)+i\left(\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)-\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)\right)\right)}{2b^2} \\
&+\frac{c\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b}+\frac{dx\sec^2\left(\frac{1}{2}(a+bx)\right)}{8b}+\frac{c\sin\left(\frac{1}{2}(a+bx)\right)}{b\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)} \\
&-\frac{c\sin\left(\frac{1}{2}(a+bx)\right)}{b\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right)}+\frac{d\left(a\sin\left(\frac{1}{2}(a+bx)\right)-(a+bx)\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b^2\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right)} \\
&+\frac{d\left(-a\sin\left(\frac{1}{2}(a+bx)\right)+(a+bx)\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b^2\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)}-\frac{d\tan\left(\frac{1}{2}(a+bx)\right)}{4b^2}
\end{aligned}$$

input
```
Integrate[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
(d*x)/b - (d*Cot[(a + b*x)/2])/(4*b^2) - (c*Csc[(a + b*x)/2]^2)/(8*b) - (d
*x*Csc[(a + b*x)/2]^2)/(8*b) - (3*c*Log[Cos[(a + b*x)/2]])/(2*b) + (d*Log[
Cos[(a + b*x)/2] - Sin[(a + b*x)/2]])/b^2 + (3*c*Log[Sin[(a + b*x)/2]])/(2
*b) - (d*Log[Cos[(a + b*x)/2] + Sin[(a + b*x)/2]])/b^2 - (3*a*d*Log[Tan[(a
 + b*x)/2]])/(2*b^2) + (3*d*((a + b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 +
 E^(I*(a + b*x))]) + I*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a
+ b*x))])))/(2*b^2) + (c*Sec[(a + b*x)/2]^2)/(8*b) + (d*x*Sec[(a + b*x)/2]
^2)/(8*b) + (c*Sin[(a + b*x)/2])/(b*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2]))
 - (c*Sin[(a + b*x)/2])/(b*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])) + (d*(a*
Sin[(a + b*x)/2] - (a + b*x)*Sin[(a + b*x)/2]))/(b^2*(Cos[(a + b*x)/2] + S
in[(a + b*x)/2])) + (d*(-(a*Sin[(a + b*x)/2]) + (a + b*x)*Sin[(a + b*x)/2]
))/(b^2*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])) - (d*Tan[(a + b*x)/2])/(4*b
^2)
```

### 3.281.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4920, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx$$

$$\Big\downarrow \text{4920}$$

$$-d\int \left(-\frac{\sec(a+bx)\csc^2(a+bx)}{2b}-\frac{3\text{arctanh}(\cos(a+bx))}{2b}+\frac{3\sec(a+bx)}{2b}\right)dx-\frac{3(c+dx)\text{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)\sec(a+bx)}{2b}-\frac{(c+dx)\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-d\left(\frac{\text{arctanh}(\sin(a+bx))}{b^2}+\frac{3x\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{3x\text{arctanh}(\cos(a+bx))}{2b}-\frac{3i\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}+\frac{3i\,\text{Pol}\right.$$
$$\frac{3(c+dx)\text{arctanh}(\cos(a+bx))}{2b}+\frac{3(c+dx)\sec(a+bx)}{2b}-\frac{(c+dx)\csc^2(a+bx)\sec(a+bx)}{2b}$$

input
```
Int[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
(-3*(c + d*x)*ArcTanh[Cos[a + b*x]])/(2*b) - d*((3*x*ArcTanh[E^(I*(a + b*x
))])/b - (3*x*ArcTanh[Cos[a + b*x]])/(2*b) + ArcTanh[Sin[a + b*x]]/b^2 + C
sc[a + b*x]/(2*b^2) - (((3*I)/2)*PolyLog[2, -E^(I*(a + b*x))])/b^2 + (((3*
I)/2)*PolyLog[2, E^(I*(a + b*x))])/b^2) + (3*(c + d*x)*Sec[a + b*x])/(2*b)
 - ((c + d*x)*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```

#### 3.281.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

### 3.281.4 Maple [A] (verified)

Time = 1.43 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.73

| method | result |
|---|---|
| risch | $\frac{3dxb\,e^{5i(xb+a)}+3cb\,e^{5i(xb+a)}-2dxb\,e^{3i(xb+a)}-2cb\,e^{3i(xb+a)}-id\,e^{5i(xb+a)}+3dxb\,e^{i(xb+a)}+3cb\,e^{i(xb+a)}+id\,e^{i(xb+a)}}{b^2\left(e^{2i(xb+a)}-1\right)^2\left(e^{2i(xb+a)}+1\right)}-\frac{3d\ln\left(e^{i(xb+}\right.}{2b}$ |

input
```
int((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^2/(exp(2*I*(b*x+a))-1)^2/(exp(2*I*(b*x+a))+1)*(3*d*x*b*exp(5*I*(b*x+a)
)+3*c*b*exp(5*I*(b*x+a))-2*d*x*b*exp(3*I*(b*x+a))-2*c*b*exp(3*I*(b*x+a))-I
*d*exp(5*I*(b*x+a))+3*d*x*b*exp(I*(b*x+a))+3*c*b*exp(I*(b*x+a))+I*d*exp(I*
(b*x+a)))-3/2/b*d*ln(exp(I*(b*x+a))+1)*x-3/2/b*c*ln(exp(I*(b*x+a))+1)+3/2/
b*c*ln(exp(I*(b*x+a))-1)-3/2/b^2*d*a*ln(exp(I*(b*x+a))-1)+2*I/b^2*d*arctan
(exp(I*(b*x+a)))+3/2*I/b^2*d*dilog(exp(I*(b*x+a)))+3/2*I/b^2*d*dilog(exp(I
*(b*x+a))+1)
```

### 3.281.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 621 vs. 2(130) = 260.

Time = 0.28 (sec) , antiderivative size = 621, normalized size of antiderivative = 4.03

$$\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx =$$
$$-\frac{4\,bdx-6\,(bdx+bc)\cos(bx+a)^2-2\,d\cos(bx+a)\sin(bx+a)+4\,bc+3\left(i\,d\cos(bx+a)^3-i\,d\cos(b\right.}{}$$

input
```
integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*b*d*x - 6*(b*d*x + b*c)*cos(b*x + a)^2 - 2*d*cos(b*x + a)*sin(b*x
+ a) + 4*b*c + 3*(I*d*cos(b*x + a)^3 - I*d*cos(b*x + a))*dilog(cos(b*x + a
) + I*sin(b*x + a)) + 3*(-I*d*cos(b*x + a)^3 + I*d*cos(b*x + a))*dilog(cos
(b*x + a) - I*sin(b*x + a)) + 3*(I*d*cos(b*x + a)^3 - I*d*cos(b*x + a))*di
log(-cos(b*x + a) + I*sin(b*x + a)) + 3*(-I*d*cos(b*x + a)^3 + I*d*cos(b*x
 + a))*dilog(-cos(b*x + a) - I*sin(b*x + a)) + 3*((b*d*x + b*c)*cos(b*x +
a)^3 - (b*d*x + b*c)*cos(b*x + a))*log(cos(b*x + a) + I*sin(b*x + a) + 1)
+ 3*((b*d*x + b*c)*cos(b*x + a)^3 - (b*d*x + b*c)*cos(b*x + a))*log(cos(b*
x + a) - I*sin(b*x + a) + 1) - 3*((b*c - a*d)*cos(b*x + a)^3 - (b*c - a*d)
*cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 3*((b*c
 - a*d)*cos(b*x + a)^3 - (b*c - a*d)*cos(b*x + a))*log(-1/2*cos(b*x + a) -
 1/2*I*sin(b*x + a) + 1/2) - 3*((b*d*x + a*d)*cos(b*x + a)^3 - (b*d*x + a*
d)*cos(b*x + a))*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 3*((b*d*x + a*d
)*cos(b*x + a)^3 - (b*d*x + a*d)*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b
*x + a) + 1) + 2*(d*cos(b*x + a)^3 - d*cos(b*x + a))*log(sin(b*x + a) + 1)
 - 2*(d*cos(b*x + a)^3 - d*cos(b*x + a))*log(-sin(b*x + a) + 1))/(b^2*cos(
b*x + a)^3 - b^2*cos(b*x + a))
```

### 3.281.6 Sympy [F]

$$\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx = \int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*csc(a + b*x)**3*sec(a + b*x)**2, x)
```

### 3.281.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1481 vs. 2(130) = 260.

Time = 0.58 (sec) , antiderivative size = 1481, normalized size of antiderivative = 9.62

$$\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
-(4*(d*cos(6*b*x + 6*a) - d*cos(4*b*x + 4*a) - d*cos(2*b*x + 2*a) + I*d*si
n(6*b*x + 6*a) - I*d*sin(4*b*x + 4*a) - I*d*sin(2*b*x + 2*a) + d)*arctan2(
2*(cos(b*x + 2*a)*cos(a) + sin(b*x + 2*a)*sin(a))/(cos(b*x + 2*a)^2 + cos(
a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a
) + sin(a)^2), (cos(b*x + 2*a)^2 - cos(a)^2 + sin(b*x + 2*a)^2 - sin(a)^2)
/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2
 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) + 6*(b*d*x + b*c + (b*d*x + b*c)*c
os(6*b*x + 6*a) - (b*d*x + b*c)*cos(4*b*x + 4*a) - (b*d*x + b*c)*cos(2*b*x
 + 2*a) + (I*b*d*x + I*b*c)*sin(6*b*x + 6*a) + (-I*b*d*x - I*b*c)*sin(4*b*
x + 4*a) + (-I*b*d*x - I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(
b*x + a) + 1) - 6*(b*c*cos(6*b*x + 6*a) - b*c*cos(4*b*x + 4*a) - b*c*cos(2
*b*x + 2*a) + I*b*c*sin(6*b*x + 6*a) - I*b*c*sin(4*b*x + 4*a) - I*b*c*sin(
2*b*x + 2*a) + b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 6*(b*d*x*cos
(6*b*x + 6*a) - b*d*x*cos(4*b*x + 4*a) - b*d*x*cos(2*b*x + 2*a) + I*b*d*x*
sin(6*b*x + 6*a) - I*b*d*x*sin(4*b*x + 4*a) - I*b*d*x*sin(2*b*x + 2*a) + b
*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 4*(3*I*b*d*x + 3*I*b*c +
d)*cos(5*b*x + 5*a) + 8*(-I*b*d*x - I*b*c)*cos(3*b*x + 3*a) + 4*(3*I*b*d*x
 + 3*I*b*c - d)*cos(b*x + a) - 6*(d*cos(6*b*x + 6*a) - d*cos(4*b*x + 4*a)
- d*cos(2*b*x + 2*a) + I*d*sin(6*b*x + 6*a) - I*d*sin(4*b*x + 4*a) - I*d*s
in(2*b*x + 2*a) + d)*dilog(-e^(I*b*x + I*a)) + 6*(d*cos(6*b*x + 6*a) - ...
```

### 3.281.8 Giac [F]

$$\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx = \int (dx+c)\csc(bx+a)^3\sec(bx+a)^2\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

### 3.281.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx=\text{Hanged}$$

input
```
int((c + d*x)/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

output
```
\text{Hanged}
```

# 3.282 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx} dx$

3.282.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2146
3.282.2 Mathematica [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . 2146
3.282.3 Rubi [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2147
3.282.4 Maple [N/A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 2147
3.282.5 Fricas [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2148
3.282.6 Sympy [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2148
3.282.7 Maxima [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2148
3.282.8 Giac [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2149
3.282.9 Mupad [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2150

## 3.282.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx} dx = \operatorname{Int}\left(\frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx},x\right)$$

output
```
CannotIntegrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x)
```

## 3.282.2 Mathematica [N/A]

Not integrable

Time = 23.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx} dx = \int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx} dx$$

input
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x),x]
```

output
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x), x]
```

### 3.282.3 Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx} \, dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}dx$$

input
```
Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x),x]
```

output
```
$Aborted
```

#### 3.282.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

### 3.282.4 Maple [N/A] (verified)

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc\left(xb+a\right)^{3}\sec\left(xb+a\right)^{2}}{dx+c}dx$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x)
```

output
```
int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x)
```

### 3.282.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc{(bx+a)}^3\sec{(bx+a)}^2}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^3*sec(b*x + a)^2/(d*x + c), x)
```

### 3.282.6 Sympy [N/A]

Not integrable

Time = 7.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc^3{(a+bx)}\sec^2{(a+bx)}}{c+dx}\,dx$$

input
```
integrate(csc(b*x+a)**3*sec(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(csc(a + b*x)**3*sec(a + b*x)**2/(c + d*x), x)
```

### 3.282.7 Maxima [N/A]

Not integrable

Time = 3.32 (sec) , antiderivative size = 3697, normalized size of antiderivative = 154.04

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc{(bx+a)}^3\sec{(bx+a)}^2}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
(2*(b*d*x + b*c)*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) + (3*(b*d*x + b*c)*cos(
5*b*x + 5*a) - 2*(b*d*x + b*c)*cos(3*b*x + 3*a) + 3*(b*d*x + b*c)*cos(b*x
+ a) - d*sin(5*b*x + 5*a) + d*sin(b*x + a))*cos(6*b*x + 6*a) + (3*b*d*x +
3*b*c - 3*(b*d*x + b*c)*cos(4*b*x + 4*a) - 3*(b*d*x + b*c)*cos(2*b*x + 2*a
) - d*sin(4*b*x + 4*a) - d*sin(2*b*x + 2*a))*cos(5*b*x + 5*a) + (2*(b*d*x
+ b*c)*cos(3*b*x + 3*a) - 3*(b*d*x + b*c)*cos(b*x + a) - d*sin(b*x + a))*c
os(4*b*x + 4*a) - 2*(b*d*x + b*c - (b*d*x + b*c)*cos(2*b*x + 2*a))*cos(3*b
*x + 3*a) - (3*(b*d*x + b*c)*cos(b*x + a) + d*sin(b*x + a))*cos(2*b*x + 2*
a) + 3*(b*d*x + b*c)*cos(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 +
 (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(6*b*x + 6*a)^2 + (b^2*d^2*x^2 +
 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(6*
b*x + 6*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 +
2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a)
+ (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*
x + 4*a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(6*b
*x + 6*a) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - (b^2*d^2*x^2 + 2*b^2*
c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + ...
```

### 3.282.8 Giac [N/A]

Not integrable

Time = 119.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3*sec(b*x + a)^2/(d*x + c), x)
```

### 3.282.9 Mupad [N/A]

Not integrable

Time = 26.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)), x)
```

# 3.283 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2} dx$

3.283.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2151
3.283.2 Mathematica [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2151
3.283.3 Rubi [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2152
3.283.4 Maple [N/A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2152
3.283.5 Fricas [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2153
3.283.6 Sympy [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2153
3.283.7 Maxima [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2153
3.283.8 Giac [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2154
3.283.9 Mupad [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2155

## 3.283.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2} dx = \text{Int}\left(\frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}, x\right)$$

output
```
CannotIntegrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x)
```

## 3.283.2 Mathematica [N/A]

Not integrable

Time = 29.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2} dx = \int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2} dx$$

input
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

### 3.283.3 Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}dx$$

input
```
Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
$Aborted
```

#### 3.283.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

### 3.283.4 Maple [N/A] (verified)

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc\left(xb+a\right)^{3}\sec\left(xb+a\right)^{2}}{\left(dx+c\right)^{2}}dx$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x)
```

### 3.283.5 Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^3*sec(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.283.6 Sympy [N/A]

Not integrable

Time = 14.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(csc(b*x+a)**3*sec(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)**3*sec(a + b*x)**2/(c + d*x)**2, x)
```

### 3.283.7 Maxima [N/A]

Not integrable

Time = 11.08 (sec) , antiderivative size = 4757, normalized size of antiderivative = 198.21

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
(2*(b*d*x + b*c)*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) + (3*(b*d*x + b*c)*cos(
5*b*x + 5*a) - 2*(b*d*x + b*c)*cos(3*b*x + 3*a) + 3*(b*d*x + b*c)*cos(b*x
+ a) - 2*d*sin(5*b*x + 5*a) + 2*d*sin(b*x + a))*cos(6*b*x + 6*a) + (3*b*d*
x + 3*b*c - 3*(b*d*x + b*c)*cos(4*b*x + 4*a) - 3*(b*d*x + b*c)*cos(2*b*x +
 2*a) - 2*d*sin(4*b*x + 4*a) - 2*d*sin(2*b*x + 2*a))*cos(5*b*x + 5*a) + (2
*(b*d*x + b*c)*cos(3*b*x + 3*a) - 3*(b*d*x + b*c)*cos(b*x + a) - 2*d*sin(b
*x + a))*cos(4*b*x + 4*a) - 2*(b*d*x + b*c - (b*d*x + b*c)*cos(2*b*x + 2*a
))*cos(3*b*x + 3*a) - (3*(b*d*x + b*c)*cos(b*x + a) + 2*d*sin(b*x + a))*co
s(2*b*x + 2*a) + 3*(b*d*x + b*c)*cos(b*x + a) + (b^2*d^3*x^3 + 3*b^2*c*d^2
*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^
2*d*x + b^2*c^3)*cos(6*b*x + 6*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b
^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
+ 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2
*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(6*b*x + 6*a)^2 + (b^2*d^3*x^3 + 3*b^2*
c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 + 2*(b^2*d^3*x^3 +
 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2
*a) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x
+ 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - (b
^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a) -
 (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + ...
```

### 3.283.8 Giac [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

### 3.283.9 Mupad [N/A]

Not integrable

Time = 26.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx=\int \frac{1}{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(1/(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^2),x)
```

output
```
int(1/(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^2), x)
```

# 3.284 $\int x^m \csc^3(a+bx) \sec^2(a+bx) \, dx$

3.284.1 Optimal result . . . 2156
3.284.2 Mathematica [N/A] . . . 2156
3.284.3 Rubi [N/A] . . . 2157
3.284.4 Maple [N/A] (verified) . . . 2157
3.284.5 Fricas [N/A] . . . 2158
3.284.6 Sympy [F(-1)] . . . 2158
3.284.7 Maxima [N/A] . . . 2158
3.284.8 Giac [N/A] . . . 2159
3.284.9 Mupad [N/A] . . . 2159

## 3.284.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x^m \csc^3(a+bx) \sec^2(a+bx) \, dx = \text{Int}\left(x^m \csc^3(a+bx) \sec^2(a+bx), x\right)$$

output
```
CannotIntegrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

## 3.284.2 Mathematica [N/A]

Not integrable

Time = 46.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \csc^3(a+bx) \sec^2(a+bx) \, dx = \int x^m \csc^3(a+bx) \sec^2(a+bx) \, dx$$

input
```
Integrate[x^m*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
Integrate[x^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]
```

## 3.284.3 Rubi [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)dx$$

input
```
Int[x^m*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
$Aborted
```

### 3.284.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.284.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m \csc(xb+a)^3 \sec(xb+a)^2\,dx$$

input
```
int(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

output
```
int(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

### 3.284.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \int x^m \csc(bx+a)^3 \sec(bx+a)^2\,dx$$

input
```
integrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

### 3.284.6 Sympy [F(-1)]

Timed out.

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x**m*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

output
```
Timed out
```

### 3.284.7 Maxima [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \int x^m \csc(bx+a)^3 \sec(bx+a)^2\,dx$$

input
```
integrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(x^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

### 3.284.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \int x^m \csc(bx+a)^3 \sec(bx+a)^2\,dx$$

input
```
integrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

### 3.284.9 Mupad [N/A]

Not integrable

Time = 26.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \int \frac{x^m}{\cos(a+b\,x)^2 \sin(a+b\,x)^3}\,dx$$

input
```
int(x^m/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

output
```
int(x^m/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

# 3.285 $\int x^3 \csc^3(a+bx) \sec^2(a+bx)\,dx$

3.285.1 Optimal result . . . 2160
3.285.2 Mathematica [A] (verified) . . . 2161
3.285.3 Rubi [A] (verified) . . . 2162
3.285.4 Maple [F] . . . 2164
3.285.5 Fricas [B] (verification not implemented) . . . 2164
3.285.6 Sympy [F] . . . 2165
3.285.7 Maxima [B] (verification not implemented) . . . 2166
3.285.8 Giac [F] . . . 2166
3.285.9 Mupad [F(-1)] . . . 2167

## 3.285.1 Optimal result

Integrand size = 20, antiderivative size = 387

$$\begin{aligned}\int x^3 \csc^3(a+bx) \sec^2(a+bx)\,dx = {} & \frac{6ix^2 \arctan\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\text{arctanh}\left(e^{i(a+bx)}\right)}{b^3} \\ & - \frac{3x^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{3x^2\csc(a+bx)}{2b^2} \\ & + \frac{3i\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^4} \\ & + \frac{9ix^2\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2} \\ & - \frac{6ix\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} \\ & + \frac{6ix\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} - \frac{3i\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^4} \\ & - \frac{9ix^2\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2} - \frac{9x\,\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} \\ & + \frac{6\,\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4} - \frac{6\,\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^4} \\ & + \frac{9x\,\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3} - \frac{9i\,\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4} \\ & + \frac{9i\,\text{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^4} + \frac{3x^3\sec(a+bx)}{2b} \\ & - \frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b}\end{aligned}$$

output
```
-9*I*polylog(4,-exp(I*(b*x+a)))/b^4-6*x*arctanh(exp(I*(b*x+a)))/b^3-3*x^3*
arctanh(exp(I*(b*x+a)))/b-3/2*x^2*csc(b*x+a)/b^2-3*I*polylog(2,exp(I*(b*x+
a)))/b^4+9*I*polylog(4,exp(I*(b*x+a)))/b^4+6*I*x*polylog(2,I*exp(I*(b*x+a)
))/b^3+3*I*polylog(2,-exp(I*(b*x+a)))/b^4-9/2*I*x^2*polylog(2,exp(I*(b*x+a
)))/b^2-6*I*x*polylog(2,-I*exp(I*(b*x+a)))/b^3-9*x*polylog(3,-exp(I*(b*x+a
)))/b^3+6*polylog(3,-I*exp(I*(b*x+a)))/b^4-6*polylog(3,I*exp(I*(b*x+a)))/b
^4+9*x*polylog(3,exp(I*(b*x+a)))/b^3+9/2*I*x^2*polylog(2,-exp(I*(b*x+a)))/
b^2+6*I*x^2*arctan(exp(I*(b*x+a)))/b^2+3/2*x^3*sec(b*x+a)/b-1/2*x^3*csc(b*
x+a)^2*sec(b*x+a)/b
```

### 3.285.2 Mathematica [A] (verified)

Time = 7.31 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.74

$$\int x^3 \csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{x^3\csc^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}$$
$$+\frac{6(ib^2x^2\arctan(\cos(a+bx)+i\sin(a+bx))+ibx\,\text{PolyLog}(2,i\cos(a+bx)-\sin(a+bx))-ibx\,\text{PolyLog}}{}$$
$$+\frac{3(2bx\log(1-\cos(a+bx)-i\sin(a+bx))+b^3x^3\log(1-\cos(a+bx)-i\sin(a+bx))-2bx\log(1+\cos}{}$$
$$+\frac{x^3\sec^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}+\frac{x^2\csc(a)\sec(a)(-3\cos(a)+2bx\sin(a))}{2b^2}$$
$$+\frac{3x^2\csc\left(\frac{a}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{bx}{2}\right)}{4b^2}-\frac{3x^2\sec\left(\frac{a}{2}\right)\sec\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{bx}{2}\right)}{4b^2}$$
$$+\frac{x^3\sin\left(\frac{bx}{2}\right)}{b\left(\cos\left(\frac{a}{2}\right)-\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}$$
$$-\frac{x^3\sin\left(\frac{bx}{2}\right)}{b\left(\cos\left(\frac{a}{2}\right)+\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)+\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}$$

input
```
Integrate[x^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
-1/8*(x^3*Csc[a/2 + (b*x)/2]^2)/b + (6*(I*b^2*x^2*ArcTan[Cos[a + b*x] + I*
Sin[a + b*x]] + I*b*x*PolyLog[2, I*Cos[a + b*x] - Sin[a + b*x]] - I*b*x*Po
lyLog[2, (-I)*Cos[a + b*x] + Sin[a + b*x]] - PolyLog[3, I*Cos[a + b*x] - S
in[a + b*x]] + PolyLog[3, (-I)*Cos[a + b*x] + Sin[a + b*x]]))/b^4 + (3*(2*
b*x*Log[1 - Cos[a + b*x] - I*Sin[a + b*x]] + b^3*x^3*Log[1 - Cos[a + b*x]
- I*Sin[a + b*x]] - 2*b*x*Log[1 + Cos[a + b*x] + I*Sin[a + b*x]] - b^3*x^3
*Log[1 + Cos[a + b*x] + I*Sin[a + b*x]] + I*(2 + 3*b^2*x^2)*PolyLog[2, -Co
s[a + b*x] - I*Sin[a + b*x]] - I*(2 + 3*b^2*x^2)*PolyLog[2, Cos[a + b*x] +
 I*Sin[a + b*x]] - 6*b*x*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] + 6*b*
x*PolyLog[3, Cos[a + b*x] + I*Sin[a + b*x]] - (6*I)*PolyLog[4, -Cos[a + b*
x] - I*Sin[a + b*x]] + (6*I)*PolyLog[4, Cos[a + b*x] + I*Sin[a + b*x]]))/(
2*b^4) + (x^3*Sec[a/2 + (b*x)/2]^2)/(8*b) + (x^2*Csc[a]*Sec[a]*(-3*Cos[a]
+ 2*b*x*Sin[a]))/(2*b^2) + (3*x^2*Csc[a/2]*Csc[a/2 + (b*x)/2]*Sin[(b*x)/2]
)/(4*b^2) - (3*x^2*Sec[a/2]*Sec[a/2 + (b*x)/2]*Sin[(b*x)/2])/(4*b^2) + (x^
3*Sin[(b*x)/2])/(b*(Cos[a/2] - Sin[a/2])*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (
b*x)/2])) - (x^3*Sin[(b*x)/2])/(b*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (b*x)/2
] + Sin[a/2 + (b*x)/2]))
```

### 3.285.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4920, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \csc^3(a+bx) \sec^2(a+bx) \, dx$$

$$\Big\downarrow \text{ 4920}$$

$$-3 \int -\frac{1}{2} x^2 \left(\frac{\sec(a+bx)\csc^2(a+bx)}{b}+\frac{3\text{arctanh}(\cos(a+bx))}{b}-\frac{3\sec(a+bx)}{b}\right)dx-\frac{3x^3\text{arctanh}(\cos(a+bx))}{2b}+\frac{3x^3\sec(a+bx)}{2b}-\frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{2}\int x^2 \left(\frac{\sec(a+bx)\csc^2(a+bx)}{b}+\frac{3\text{arctanh}(\cos(a+bx))}{b}-\frac{3\sec(a+bx)}{b}\right)dx-\frac{3x^3\text{arctanh}(\cos(a+bx))}{2b}+\frac{3x^3\sec(a+bx)}{2b}-\frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{ 2010}$$

$$\frac{3}{2}\int \left(\frac{3\text{arctanh}(\cos(a+bx))x^2}{b}+\frac{(\csc^2(a+bx)-3)\sec(a+bx)x^2}{b}\right)dx-\frac{3x^3\text{arctanh}(\cos(a+bx))}{2b}+\frac{3x^3\sec(a+bx)}{2b}-\frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b}$$

$\big\downarrow$ 2009

$$\frac{3}{2}\left(\frac{4ix^2\arctan\left(e^{i(a+bx)}\right)}{b^2}-\frac{4x\text{arctanh}\left(e^{i(a+bx)}\right)}{b^3}-\frac{2x^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{x^3\text{arctanh}(\cos(a+bx))}{b}+\frac{2i\,\text{PolyLog}}{}\right.$$
$$\frac{3x^3\text{arctanh}(\cos(a+bx))}{2b}+\frac{3x^3\sec(a+bx)}{2b}-\frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b}$$

input
```
Int[x^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
(-3*x^3*ArcTanh[Cos[a + b*x]])/(2*b) + (3*(((4*I)*x^2*ArcTan[E^(I*(a + b*x
))])/b^2 - (4*x*ArcTanh[E^(I*(a + b*x))])/b^3 - (2*x^3*ArcTanh[E^(I*(a + b
*x))])/b + (x^3*ArcTanh[Cos[a + b*x]])/b - (x^2*Csc[a + b*x])/b^2 + ((2*I)
*PolyLog[2, -E^(I*(a + b*x))])/b^4 + ((3*I)*x^2*PolyLog[2, -E^(I*(a + b*x)
)])/b^2 - ((4*I)*x*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((4*I)*x*PolyLo
g[2, I*E^(I*(a + b*x))])/b^3 - ((2*I)*PolyLog[2, E^(I*(a + b*x))])/b^4 - (
(3*I)*x^2*PolyLog[2, E^(I*(a + b*x))])/b^2 - (6*x*PolyLog[3, -E^(I*(a + b*
x))])/b^3 + (4*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (4*PolyLog[3, I*E^(
I*(a + b*x))])/b^4 + (6*x*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((6*I)*PolyLo
g[4, -E^(I*(a + b*x))])/b^4 + ((6*I)*PolyLog[4, E^(I*(a + b*x))])/b^4))/2
+ (3*x^3*Sec[a + b*x])/(2*b) - (x^3*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```

### 3.285.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

### 3.285.4 Maple [F]

$$\int x^3 \csc(xb+a)^3 \sec(xb+a)^2 \, dx$$

input
```
int(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

output
```
int(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

### 3.285.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1747 vs. 2(315) = 630.

Time = 0.36 (sec) , antiderivative size = 1747, normalized size of antiderivative = 4.51

$$\int x^3 \csc^3(a+bx) \sec^2(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*(6*b^3*x^3*cos(b*x + a)^2 - 4*b^3*x^3 + 6*b^2*x^2*cos(b*x + a)*sin(b*x
 + a) - 3*((3*I*b^2*x^2 + 2*I)*cos(b*x + a)^3 + (-3*I*b^2*x^2 - 2*I)*cos(b
*x + a))*dilog(cos(b*x + a) + I*sin(b*x + a)) - 3*((-3*I*b^2*x^2 - 2*I)*co
s(b*x + a)^3 + (3*I*b^2*x^2 + 2*I)*cos(b*x + a))*dilog(cos(b*x + a) - I*si
n(b*x + a)) - 12*(-I*b*x*cos(b*x + a)^3 + I*b*x*cos(b*x + a))*dilog(I*cos(
b*x + a) + sin(b*x + a)) - 12*(-I*b*x*cos(b*x + a)^3 + I*b*x*cos(b*x + a))
*dilog(I*cos(b*x + a) - sin(b*x + a)) - 12*(I*b*x*cos(b*x + a)^3 - I*b*x*c
os(b*x + a))*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 12*(I*b*x*cos(b*x + a
)^3 - I*b*x*cos(b*x + a))*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*((3*I*
b^2*x^2 + 2*I)*cos(b*x + a)^3 + (-3*I*b^2*x^2 - 2*I)*cos(b*x + a))*dilog(-
cos(b*x + a) + I*sin(b*x + a)) - 3*((-3*I*b^2*x^2 - 2*I)*cos(b*x + a)^3 +
(3*I*b^2*x^2 + 2*I)*cos(b*x + a))*dilog(-cos(b*x + a) - I*sin(b*x + a)) -
3*((b^3*x^3 + 2*b*x)*cos(b*x + a)^3 - (b^3*x^3 + 2*b*x)*cos(b*x + a))*log(
cos(b*x + a) + I*sin(b*x + a) + 1) - 6*(a^2*cos(b*x + a)^3 - a^2*cos(b*x +
 a))*log(cos(b*x + a) + I*sin(b*x + a) + I) - 3*((b^3*x^3 + 2*b*x)*cos(b*x
 + a)^3 - (b^3*x^3 + 2*b*x)*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a
) + 1) + 6*(a^2*cos(b*x + a)^3 - a^2*cos(b*x + a))*log(cos(b*x + a) - I*si
n(b*x + a) + I) - 6*((b^2*x^2 - a^2)*cos(b*x + a)^3 - (b^2*x^2 - a^2)*cos(
b*x + a))*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 6*((b^2*x^2 - a^2)*cos(
b*x + a)^3 - (b^2*x^2 - a^2)*cos(b*x + a))*log(I*cos(b*x + a) - sin(b*x...
```

### 3.285.6 Sympy [F]

$$\int x^3 \csc^3(a+bx) \sec^2(a+bx)\, dx = \int x^3 \csc^3(a+bx) \sec^2(a+bx)\, dx$$

input
```
integrate(x**3*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

output
```
Integral(x**3*csc(a + b*x)**3*sec(a + b*x)**2, x)
```

### 3.285.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3940 vs. 2(315) = 630.

Time = 0.77 (sec) , antiderivative size = 3940, normalized size of antiderivative = 10.18

$$\int x^3 \csc^3(a+bx)\sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(a^3*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*lo
g(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1)) - 4*(12*((b*x + a)^2 - 2*(b
*x + a)*a + a^2 + ((b*x + a)^2 - 2*(b*x + a)*a + a^2)*cos(6*b*x + 6*a) - (
(b*x + a)^2 - 2*(b*x + a)*a + a^2)*cos(4*b*x + 4*a) - ((b*x + a)^2 - 2*(b*
x + a)*a + a^2)*cos(2*b*x + 2*a) + (I*(b*x + a)^2 - 2*I*(b*x + a)*a + I*a^
2)*sin(6*b*x + 6*a) + (-I*(b*x + a)^2 + 2*I*(b*x + a)*a - I*a^2)*sin(4*b*x
 + 4*a) + (-I*(b*x + a)^2 + 2*I*(b*x + a)*a - I*a^2)*sin(2*b*x + 2*a))*arc
tan2(cos(b*x + a), sin(b*x + a) + 1) + 12*((b*x + a)^2 - 2*(b*x + a)*a + a
^2 + ((b*x + a)^2 - 2*(b*x + a)*a + a^2)*cos(6*b*x + 6*a) - ((b*x + a)^2 -
 2*(b*x + a)*a + a^2)*cos(4*b*x + 4*a) - ((b*x + a)^2 - 2*(b*x + a)*a + a^
2)*cos(2*b*x + 2*a) + (I*(b*x + a)^2 - 2*I*(b*x + a)*a + I*a^2)*sin(6*b*x
+ 6*a) + (-I*(b*x + a)^2 + 2*I*(b*x + a)*a - I*a^2)*sin(4*b*x + 4*a) + (-I
*(b*x + a)^2 + 2*I*(b*x + a)*a - I*a^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x
+ a), -sin(b*x + a) + 1) - 6*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*
(b*x + a) + ((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x + a) - 2*a)*
cos(6*b*x + 6*a) - ((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x + a)
- 2*a)*cos(4*b*x + 4*a) - ((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*
x + a) - 2*a)*cos(2*b*x + 2*a) - (-I*(b*x + a)^3 + 3*I*(b*x + a)^2*a + (-3
*I*a^2 - 2*I)*(b*x + a) + 2*I*a)*sin(6*b*x + 6*a) - (I*(b*x + a)^3 - 3*I*(
b*x + a)^2*a + (3*I*a^2 + 2*I)*(b*x + a) - 2*I*a)*sin(4*b*x + 4*a) - (I...
```

### 3.285.8 Giac [F]

$$\int x^3 \csc^3(a+bx)\sec^2(a+bx)\,dx = \int x^3 \csc(bx+a)^3 \sec(bx+a)^2\,dx$$

input
```
integrate(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^3*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

### 3.285.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \csc^3(a+bx)\sec^2(a+bx)\,dx=\int \frac{x^3}{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3}\,dx$$

input
```
int(x^3/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

output
```
int(x^3/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

# 3.286 $\int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx$

3.286.1 Optimal result . . . 2168
3.286.2 Mathematica [B] (verified) . . . 2169
3.286.3 Rubi [A] (verified) . . . 2170
3.286.4 Maple [B] (verified) . . . 2171
3.286.5 Fricas [B] (verification not implemented) . . . 2172
3.286.6 Sympy [F] . . . 2173
3.286.7 Maxima [B] (verification not implemented) . . . 2174
3.286.8 Giac [F] . . . 2174
3.286.9 Mupad [F(-1)] . . . 2175

## 3.286.1 Optimal result

Integrand size = 20, antiderivative size = 235

$$\int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx = \frac{4ix\arctan\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{\text{arctanh}(\cos(a+bx))}{b^3} - \frac{x\csc(a+bx)}{b^2} + \frac{3ix\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} - \frac{2i\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} + \frac{2i\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} - \frac{3ix\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} - \frac{3\,\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} + \frac{3\,\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3} + \frac{3x^2\sec(a+bx)}{2b} - \frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

output
```
4*I*x*arctan(exp(I*(b*x+a)))/b^2-3*x^2*arctanh(exp(I*(b*x+a)))/b-arctanh(c
os(b*x+a))/b^3-x*csc(b*x+a)/b^2+3*I*x*polylog(2,-exp(I*(b*x+a)))/b^2-2*I*p
olylog(2,-I*exp(I*(b*x+a)))/b^3+2*I*polylog(2,I*exp(I*(b*x+a)))/b^3-3*I*x*
polylog(2,exp(I*(b*x+a)))/b^2-3*polylog(3,-exp(I*(b*x+a)))/b^3+3*polylog(3
,exp(I*(b*x+a)))/b^3+3/2*x^2*sec(b*x+a)/b-1/2*x^2*csc(b*x+a)^2*sec(b*x+a)/
b
```

### 3.286.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 613 vs. 2(235) = 470.

Time = 6.90 (sec) , antiderivative size = 613, normalized size of antiderivative = 2.61

$$\int x^2\csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{x^2\csc^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}$$
$$-\frac{2\left(\left(-a+\frac{\pi}{2}-bx\right)\left(\log\left(1-e^{i\left(-a+\frac{\pi}{2}-bx\right)}\right)-\log\left(1+e^{i\left(-a+\frac{\pi}{2}-bx\right)}\right)\right)-\left(-a+\frac{\pi}{2}\right)\log\left(\tan\left(\frac{1}{2}\left(-a+\frac{\pi}{2}-\right.\right.\right.}{b^3}$$
$$+\frac{2\log(1-\cos(a+bx)-i\sin(a+bx))+3b^2x^2\log(1-\cos(a+bx)-i\sin(a+bx))-2\log(1+\cos(a+}{}$$
$$+\frac{x^2\sec^2\left(\frac{a}{2}+\frac{bx}{2}\right)}{8b}+\frac{x\csc(a)\sec(a)(-\cos(a)+bx\sin(a))}{b^2}+\frac{x\csc\left(\frac{a}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{bx}{2}\right)}{2b^2}$$
$$-\frac{x\sec\left(\frac{a}{2}\right)\sec\left(\frac{a}{2}+\frac{bx}{2}\right)\sin\left(\frac{bx}{2}\right)}{2b^2}+\frac{x^2\sin\left(\frac{bx}{2}\right)}{b\left(\cos\left(\frac{a}{2}\right)-\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}$$
$$-\frac{x^2\sin\left(\frac{bx}{2}\right)}{b\left(\cos\left(\frac{a}{2}\right)+\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)+\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}$$

input
```
Integrate[x^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
-1/8*(x^2*Csc[a/2 + (b*x)/2]^2)/b - (2*((-a + Pi/2 - b*x)*(Log[1 - E^(I*(-
a + Pi/2 - b*x))] - Log[1 + E^(I*(-a + Pi/2 - b*x))]) - (-a + Pi/2)*Log[Ta
n[(-a + Pi/2 - b*x)/2]] + I*(PolyLog[2, -E^(I*(-a + Pi/2 - b*x))] - PolyLo
g[2, E^(I*(-a + Pi/2 - b*x))])))/b^3 + (2*Log[1 - Cos[a + b*x] - I*Sin[a +
 b*x]] + 3*b^2*x^2*Log[1 - Cos[a + b*x] - I*Sin[a + b*x]] - 2*Log[1 + Cos[
a + b*x] + I*Sin[a + b*x]] - 3*b^2*x^2*Log[1 + Cos[a + b*x] + I*Sin[a + b*
x]] + (6*I)*b*x*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]] - (6*I)*b*x*Pol
yLog[2, Cos[a + b*x] + I*Sin[a + b*x]] - 6*PolyLog[3, -Cos[a + b*x] - I*Si
n[a + b*x]] + 6*PolyLog[3, Cos[a + b*x] + I*Sin[a + b*x]])/(2*b^3) + (x^2*
Sec[a/2 + (b*x)/2]^2)/(8*b) + (x*Csc[a]*Sec[a]*(-Cos[a] + b*x*Sin[a]))/b^2
 + (x*Csc[a/2]*Csc[a/2 + (b*x)/2]*Sin[(b*x)/2])/(2*b^2) - (x*Sec[a/2]*Sec[
a/2 + (b*x)/2]*Sin[(b*x)/2])/(2*b^2) + (x^2*Sin[(b*x)/2])/(b*(Cos[a/2] - S
in[a/2])*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) - (x^2*Sin[(b*x)/2])/(
b*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2]))
```

### 3.286.3 Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4920, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx$$

$$\Big\downarrow \text{4920}$$

$$-2\int -\frac{1}{2}x\left(\frac{\sec(a+bx)\csc^2(a+bx)}{b}+\frac{3\text{arctanh}(\cos(a+bx))}{b}-\frac{3\sec(a+bx)}{b}\right)dx-\frac{3x^2\text{arctanh}(\cos(a+bx))}{2b}+\frac{3x^2\sec(a+bx)}{2b}-\frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{27}$$

$$\int x\left(\frac{\sec(a+bx)\csc^2(a+bx)}{b}+\frac{3\text{arctanh}(\cos(a+bx))}{b}-\frac{3\sec(a+bx)}{b}\right)dx-\frac{3x^2\text{arctanh}(\cos(a+bx))}{2b}+\frac{3x^2\sec(a+bx)}{2b}-\frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{2010}$$

$$\int\left(\frac{3x\text{arctanh}(\cos(a+bx))}{b}+\frac{x(\csc^2(a+bx)-3)\sec(a+bx)}{b}\right)dx-\frac{3x^2\text{arctanh}(\cos(a+bx))}{2b}+\frac{3x^2\sec(a+bx)}{2b}-\frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4ix\arctan\left(e^{i(a+bx)}\right)}{b^2}-\frac{\text{arctanh}(\cos(a+bx))}{b^3}-\frac{3x^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{2i\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{2i\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-\frac{3\,\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{3\,\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}+\frac{3ix\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{3ix\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{x\csc(a+bx)}{b^2}+\frac{3x^2\sec(a+bx)}{2b}-\frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

input
```
Int[x^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
((4*I)*x*ArcTan[E^(I*(a + b*x))])/b^2 - (3*x^2*ArcTanh[E^(I*(a + b*x))])/b
 - ArcTanh[Cos[a + b*x]]/b^3 - (x*Csc[a + b*x])/b^2 + ((3*I)*x*PolyLog[2,
-E^(I*(a + b*x))])/b^2 - ((2*I)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((
2*I)*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*x*PolyLog[2, E^(I*(a + b*
x))])/b^2 - (3*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (3*PolyLog[3, E^(I*(a +
 b*x))])/b^3 + (3*x^2*Sec[a + b*x])/(2*b) - (x^2*Csc[a + b*x]^2*Sec[a + b*
x])/(2*b)
```

### 3.286.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

## 3.286.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(208) = 416$.

Time = 1.46 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.83

| method | result |
|---|---|
| risch | $\frac{x\left(3xb\,\mathrm{e}^{5i(xb+a)}-2xb\,\mathrm{e}^{3i(xb+a)}-2i\mathrm{e}^{5i(xb+a)}+3xb\,\mathrm{e}^{i(xb+a)}+2i\mathrm{e}^{i(xb+a)}\right)}{b^{2}\left(\mathrm{e}^{2i(xb+a)}-1\right)^{2}\left(\mathrm{e}^{2i(xb+a)}+1\right)}-\frac{\ln\left(\mathrm{e}^{i(xb+a)}+1\right)}{b^{3}}-\frac{3\,\mathrm{polylog}\left(3,-\mathrm{e}^{i(xb+a)}\right)}{b^{3}}+\frac{3\,\mathrm{polylog}}{}$ |

input
```
int(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
x/b^2/(exp(2*I*(b*x+a))-1)^2/(exp(2*I*(b*x+a))+1)*(3*x*b*exp(5*I*(b*x+a))-
2*x*b*exp(3*I*(b*x+a))-2*I*exp(5*I*(b*x+a))+3*x*b*exp(I*(b*x+a))+2*I*exp(I
*(b*x+a)))-1/b^3*ln(exp(I*(b*x+a))+1)-3*polylog(3,-exp(I*(b*x+a)))/b^3+3*p
olylog(3,exp(I*(b*x+a)))/b^3+1/b^3*ln(exp(I*(b*x+a))-1)-3*I*x*polylog(2,ex
p(I*(b*x+a)))/b^2-4*I/b^3*a*arctan(exp(I*(b*x+a)))+3/2/b^3*a^2*ln(exp(I*(b
*x+a))-1)-3/2/b^3*a^2*ln(1-exp(I*(b*x+a)))-2*I/b^3*dilog(1+I*exp(I*(b*x+a)
))-3/2/b*ln(exp(I*(b*x+a))+1)*x^2+2*I/b^3*dilog(1-I*exp(I*(b*x+a)))+2/b^2*
ln(1+I*exp(I*(b*x+a)))*x+3*I*x*polylog(2,-exp(I*(b*x+a)))/b^2+2/b^3*ln(1+I
*exp(I*(b*x+a)))*a-2/b^3*ln(1-I*exp(I*(b*x+a)))*a-2/b^2*ln(1-I*exp(I*(b*x+
a)))*x+3/2/b*ln(1-exp(I*(b*x+a)))*x^2
```

### 3.286.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1237 vs. 2(197) = 394.

Time = 0.33 (sec) , antiderivative size = 1237, normalized size of antiderivative = 5.26

$$\int x^2 \csc^3(a+bx) \sec^2(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*(6*b^2*x^2*cos(b*x + a)^2 - 4*b^2*x^2 + 4*b*x*cos(b*x + a)*sin(b*x + a
) - 6*(I*b*x*cos(b*x + a)^3 - I*b*x*cos(b*x + a))*dilog(cos(b*x + a) + I*s
in(b*x + a)) - 6*(-I*b*x*cos(b*x + a)^3 + I*b*x*cos(b*x + a))*dilog(cos(b*
x + a) - I*sin(b*x + a)) - 4*(-I*cos(b*x + a)^3 + I*cos(b*x + a))*dilog(I*
cos(b*x + a) + sin(b*x + a)) - 4*(-I*cos(b*x + a)^3 + I*cos(b*x + a))*dilo
g(I*cos(b*x + a) - sin(b*x + a)) - 4*(I*cos(b*x + a)^3 - I*cos(b*x + a))*d
ilog(-I*cos(b*x + a) + sin(b*x + a)) - 4*(I*cos(b*x + a)^3 - I*cos(b*x + a
))*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 6*(I*b*x*cos(b*x + a)^3 - I*b*x
*cos(b*x + a))*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 6*(-I*b*x*cos(b*x +
 a)^3 + I*b*x*cos(b*x + a))*dilog(-cos(b*x + a) - I*sin(b*x + a)) - ((3*b^
2*x^2 + 2)*cos(b*x + a)^3 - (3*b^2*x^2 + 2)*cos(b*x + a))*log(cos(b*x + a)
 + I*sin(b*x + a) + 1) + 4*(a*cos(b*x + a)^3 - a*cos(b*x + a))*log(cos(b*x
 + a) + I*sin(b*x + a) + I) - ((3*b^2*x^2 + 2)*cos(b*x + a)^3 - (3*b^2*x^2
 + 2)*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 4*(a*cos(b*x
+ a)^3 - a*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + I) - 4*((b*x
+ a)*cos(b*x + a)^3 - (b*x + a)*cos(b*x + a))*log(I*cos(b*x + a) + sin(b*x
 + a) + 1) + 4*((b*x + a)*cos(b*x + a)^3 - (b*x + a)*cos(b*x + a))*log(I*c
os(b*x + a) - sin(b*x + a) + 1) - 4*((b*x + a)*cos(b*x + a)^3 - (b*x + a)*
cos(b*x + a))*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 4*((b*x + a)*cos(b
*x + a)^3 - (b*x + a)*cos(b*x + a))*log(-I*cos(b*x + a) - sin(b*x + a) ...
```

### 3.286.6 Sympy [F]

$$\int x^2 \csc^3(a+bx) \sec^2(a+bx) \, dx = \int x^2 \csc^3{(a+bx)} \sec^2{(a+bx)} \, dx$$

input
```
integrate(x**2*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

output
```
Integral(x**2*csc(a + b*x)**3*sec(a + b*x)**2, x)
```

### 3.286.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2205 vs. 2(197) = 394.

Time = 0.53 (sec) , antiderivative size = 2205, normalized size of antiderivative = 9.38

$$\int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/4*(a^2*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log
(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1)) + 4*(8*(b*x*cos(6*b*x + 6*a)
 - b*x*cos(4*b*x + 4*a) - b*x*cos(2*b*x + 2*a) + I*b*x*sin(6*b*x + 6*a) -
I*b*x*sin(4*b*x + 4*a) - I*b*x*sin(2*b*x + 2*a) + b*x)*arctan2(cos(b*x + a
), sin(b*x + a) + 1) + 8*(b*x*cos(6*b*x + 6*a) - b*x*cos(4*b*x + 4*a) - b*
x*cos(2*b*x + 2*a) + I*b*x*sin(6*b*x + 6*a) - I*b*x*sin(4*b*x + 4*a) - I*b
*x*sin(2*b*x + 2*a) + b*x)*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 2*(3
*(b*x + a)^2 - 6*(b*x + a)*a + (3*(b*x + a)^2 - 6*(b*x + a)*a + 2)*cos(6*b
*x + 6*a) - (3*(b*x + a)^2 - 6*(b*x + a)*a + 2)*cos(4*b*x + 4*a) - (3*(b*x
 + a)^2 - 6*(b*x + a)*a + 2)*cos(2*b*x + 2*a) - (-3*I*(b*x + a)^2 + 6*I*(b
*x + a)*a - 2*I)*sin(6*b*x + 6*a) - (3*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 2
*I)*sin(4*b*x + 4*a) - (3*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 2*I)*sin(2*b*x
 + 2*a) + 2)*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 4*(cos(6*b*x + 6*a)
 - cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + I*sin(6*b*x + 6*a) - I*sin(4*b*x
+ 4*a) - I*sin(2*b*x + 2*a) + 1)*arctan2(sin(b*x + a), cos(b*x + a) - 1) -
 6*((b*x + a)^2 - 2*(b*x + a)*a + ((b*x + a)^2 - 2*(b*x + a)*a)*cos(6*b*x
+ 6*a) - ((b*x + a)^2 - 2*(b*x + a)*a)*cos(4*b*x + 4*a) - ((b*x + a)^2 - 2
*(b*x + a)*a)*cos(2*b*x + 2*a) - (-I*(b*x + a)^2 + 2*I*(b*x + a)*a)*sin(6*
b*x + 6*a) - (I*(b*x + a)^2 - 2*I*(b*x + a)*a)*sin(4*b*x + 4*a) - (I*(b*x
+ a)^2 - 2*I*(b*x + a)*a)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(...
```

### 3.286.8 Giac [F]

$$\int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx = \int x^2 \csc(bx+a)^3 \sec(bx+a)^2\,dx$$

input
```
integrate(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^2*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

### 3.286.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \csc^3(a+bx) \sec^2(a+bx)\,dx = \int \frac{x^2}{\cos(a+b\,x)^2 \sin(a+b\,x)^3}\,dx$$

input
```
int(x^2/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

output
```
int(x^2/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

# 3.287 $\int x \csc^3(a+bx) \sec^2(a+bx)\,dx$


3.287.1 Optimal result . . . 2176
3.287.2 Mathematica [B] (verified) . . . 2176
3.287.3 Rubi [A] (verified) . . . 2177
3.287.4 Maple [B] (verified) . . . 2178
3.287.5 Fricas [B] (verification not implemented) . . . 2179
3.287.6 Sympy [F] . . . 2179
3.287.7 Maxima [B] (verification not implemented) . . . 2180
3.287.8 Giac [F] . . . 2180
3.287.9 Mupad [F(-1)] . . . 2181


## 3.287.1 Optimal result

Integrand size = 18, antiderivative size = 126

$$\int x \csc^3(a+bx) \sec^2(a+bx)\,dx = -\frac{3x\text{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{\text{arctanh}(\sin(a+bx))}{b^2} - \frac{\csc(a+bx)}{2b^2} + \frac{3i\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2} - \frac{3i\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2} + \frac{3x\sec(a+bx)}{2b} - \frac{x\csc^2(a+bx)\sec(a+bx)}{2b}$$

output
```
-3*x*arctanh(exp(I*(b*x+a)))/b-arctanh(sin(b*x+a))/b^2-1/2*csc(b*x+a)/b^2+
3/2*I*polylog(2,-exp(I*(b*x+a)))/b^2-3/2*I*polylog(2,exp(I*(b*x+a)))/b^2+3
/2*x*sec(b*x+a)/b-1/2*x*csc(b*x+a)^2*sec(b*x+a)/b
```

## 3.287.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 282 vs. 2(126) = 252.

Time = 3.69 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.24

$$\int x \csc^3(a+bx) \sec^2(a+bx)\,dx$$
$$= \frac{8bx - 2\cot\left(\frac{1}{2}(a+bx)\right) - bx\csc^2\left(\frac{1}{2}(a+bx)\right) + 12(a+bx)\left(\log\left(1-e^{i(a+bx)}\right) - \log\left(1+e^{i(a+bx)}\right)\right) + 8\log}{}$$

input
```
Integrate[x*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
(8*b*x - 2*Cot[(a + b*x)/2] - b*x*Csc[(a + b*x)/2]^2 + 12*(a + b*x)*(Log[1
 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + 8*Log[Cos[(a + b*x)/2] -
 Sin[(a + b*x)/2]] - 8*Log[Cos[(a + b*x)/2] + Sin[(a + b*x)/2]] - 12*a*Log
[Tan[(a + b*x)/2]] + (12*I)*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(
I*(a + b*x))]) + b*x*Sec[(a + b*x)/2]^2 + (8*b*x*Sin[(a + b*x)/2])/(Cos[(a
 + b*x)/2] - Sin[(a + b*x)/2]) - (8*b*x*Sin[(a + b*x)/2])/(Cos[(a + b*x)/2
] + Sin[(a + b*x)/2]) - 2*Tan[(a + b*x)/2])/(8*b^2)
```

### 3.287.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4920, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \csc^3(a+bx)\sec^2(a+bx)\,dx$$

$$\Big\downarrow \text{4920}$$

$$-\int \left(-\frac{\sec(a+bx)\csc^2(a+bx)}{2b}-\frac{3\text{arctanh}(\cos(a+bx))}{2b}+\frac{3\sec(a+bx)}{2b}\right)dx-\frac{3x\text{arctanh}(\cos(a+bx))}{2b}+\frac{3x\sec(a+bx)}{2b}-\frac{x\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arctanh}(\sin(a+bx))}{b^2}-\frac{3x\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{3i\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}-\frac{3i\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}-\frac{\csc(a+bx)}{2b^2}+\frac{3x\sec(a+bx)}{2b}-\frac{x\csc^2(a+bx)\sec(a+bx)}{2b}$$

input
```
Int[x*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

output
```
(-3*x*ArcTanh[E^(I*(a + b*x))])/b - ArcTanh[Sin[a + b*x]]/b^2 - Csc[a + b*
x]/(2*b^2) + (((3*I)/2)*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (((3*I)/2)*Pol
yLog[2, E^(I*(a + b*x))])/b^2 + (3*x*Sec[a + b*x])/(2*b) - (x*Csc[a + b*x]
^2*Sec[a + b*x])/(2*b)
```

#### 3.287.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

## 3.287.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(108) = 216$.

Time = 0.70 (sec) , antiderivative size = 519, normalized size of antiderivative = 4.12

| method | result |
|---|---|
| risch | $2i e^{i(xb+a)}+6xb\,e^{5i(xb+a)}-4xb\,e^{3i(xb+a)}+6xb\,e^{i(xb+a)}-3i e^{4i(xb+a)}\operatorname{dilog}\left(e^{i(xb+a)}+1\right)-4i\arctan\left(e^{i(xb+a)}\right)e^{4i(xb+a)}-3i e^{4i(xb+a)}$ di |

input
```
int(x*csc(b*x+a)^3*sec(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(4*I*arctan(exp(I*(b*x+a)))+3*I*dilog(exp(I*(b*x+a)))+3*I*dilog(exp(I*
(b*x+a))+1)-3*a*ln(exp(I*(b*x+a))-1)-3*I*exp(4*I*(b*x+a))*dilog(exp(I*(b*x
+a))+1)-4*I*arctan(exp(I*(b*x+a)))*exp(4*I*(b*x+a))-3*I*exp(4*I*(b*x+a))*d
ilog(exp(I*(b*x+a)))+3*I*exp(6*I*(b*x+a))*dilog(exp(I*(b*x+a)))+3*I*exp(6*
I*(b*x+a))*dilog(exp(I*(b*x+a))+1)+4*I*arctan(exp(I*(b*x+a)))*exp(6*I*(b*x
+a))+3*ln(exp(I*(b*x+a))-1)*exp(4*I*(b*x+a))*a-3*ln(exp(I*(b*x+a))-1)*exp(
6*I*(b*x+a))*a+3*ln(exp(I*(b*x+a))-1)*exp(2*I*(b*x+a))*a-4*I*arctan(exp(I*
(b*x+a)))*exp(2*I*(b*x+a))-3*I*exp(2*I*(b*x+a))*dilog(exp(I*(b*x+a)))-3*I*
exp(2*I*(b*x+a))*dilog(exp(I*(b*x+a))+1)-3*ln(exp(I*(b*x+a))+1)*b*x-2*I*ex
p(5*I*(b*x+a))+2*I*exp(I*(b*x+a))+3*ln(exp(I*(b*x+a))+1)*exp(2*I*(b*x+a))*
b*x-3*ln(exp(I*(b*x+a))+1)*exp(6*I*(b*x+a))*b*x+3*ln(exp(I*(b*x+a))+1)*exp
(4*I*(b*x+a))*b*x+6*x*b*exp(5*I*(b*x+a))-4*x*b*exp(3*I*(b*x+a))+6*x*b*exp(
I*(b*x+a)))/b^2/(exp(2*I*(b*x+a))-1)^2/(exp(2*I*(b*x+a))+1)
```

### 3.287.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 531 vs. 2(104) = 208.

Time = 0.26 (sec) , antiderivative size = 531, normalized size of antiderivative = 4.21

$$\int x\csc^3(a+bx)\sec^2(a+bx)\,dx$$
$$= \frac{6\,bx\cos{(bx+a)}^2-4\,bx-3\left(i\,\cos{(bx+a)}^3-i\,\cos{(bx+a)}\right)\mathrm{Li}_2(\cos{(bx+a)}+i\,\sin{(bx+a)})-3\left(-i\,\mathrm{c}\right.}{}$$

input
```
integrate(x*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*(6*b*x*cos(b*x + a)^2 - 4*b*x - 3*(I*cos(b*x + a)^3 - I*cos(b*x + a))*
dilog(cos(b*x + a) + I*sin(b*x + a)) - 3*(-I*cos(b*x + a)^3 + I*cos(b*x +
a))*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*(I*cos(b*x + a)^3 - I*cos(b*x
 + a))*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 3*(-I*cos(b*x + a)^3 + I*co
s(b*x + a))*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 3*(b*x*cos(b*x + a)^3
- b*x*cos(b*x + a))*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b*x*cos(b*
x + a)^3 - b*x*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 3*(a
*cos(b*x + a)^3 - a*cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x +
a) + 1/2) - 3*(a*cos(b*x + a)^3 - a*cos(b*x + a))*log(-1/2*cos(b*x + a) -
1/2*I*sin(b*x + a) + 1/2) + 3*((b*x + a)*cos(b*x + a)^3 - (b*x + a)*cos(b*
x + a))*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 3*((b*x + a)*cos(b*x + a
)^3 - (b*x + a)*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 2*
(cos(b*x + a)^3 - cos(b*x + a))*log(sin(b*x + a) + 1) + 2*(cos(b*x + a)^3
- cos(b*x + a))*log(-sin(b*x + a) + 1) + 2*cos(b*x + a)*sin(b*x + a))/(b^2
*cos(b*x + a)^3 - b^2*cos(b*x + a))
```

### 3.287.6 Sympy [F]

$$\int x\csc^3(a+bx)\sec^2(a+bx)\,dx = \int x\csc^3{(a+bx)}\sec^2{(a+bx)}\,dx$$

input
```
integrate(x*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

output
```
Integral(x*csc(a + b*x)**3*sec(a + b*x)**2, x)
```

### 3.287.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1173 vs. $2(104) = 208$.

Time = 0.49 (sec) , antiderivative size = 1173, normalized size of antiderivative = 9.31

$$\int x \csc^3(a+bx) \sec^2(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate(x*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

output
```
(8*I*b*x*cos(3*b*x + 3*a) - 8*b*x*sin(3*b*x + 3*a) - 4*(cos(6*b*x + 6*a) -
 cos(4*b*x + 4*a) - cos(2*b*x + 2*a) + I*sin(6*b*x + 6*a) - I*sin(4*b*x +
4*a) - I*sin(2*b*x + 2*a) + 1)*arctan2(2*(cos(b*x + 2*a)*cos(a) + sin(b*x
+ 2*a)*sin(a))/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + si
n(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2), (cos(b*x + 2*a)^2 -
cos(a)^2 + sin(b*x + 2*a)^2 - sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*c
os(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)
^2)) - 6*(b*x*cos(6*b*x + 6*a) - b*x*cos(4*b*x + 4*a) - b*x*cos(2*b*x + 2*
a) + I*b*x*sin(6*b*x + 6*a) - I*b*x*sin(4*b*x + 4*a) - I*b*x*sin(2*b*x + 2
*a) + b*x)*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 6*(b*x*cos(6*b*x + 6*
a) - b*x*cos(4*b*x + 4*a) - b*x*cos(2*b*x + 2*a) + I*b*x*sin(6*b*x + 6*a)
- I*b*x*sin(4*b*x + 4*a) - I*b*x*sin(2*b*x + 2*a) + b*x)*arctan2(sin(b*x +
 a), -cos(b*x + a) + 1) - 4*(3*I*b*x + 1)*cos(5*b*x + 5*a) - 4*(3*I*b*x -
1)*cos(b*x + a) + 6*(cos(6*b*x + 6*a) - cos(4*b*x + 4*a) - cos(2*b*x + 2*a
) + I*sin(6*b*x + 6*a) - I*sin(4*b*x + 4*a) - I*sin(2*b*x + 2*a) + 1)*dilo
g(-e^(I*b*x + I*a)) - 6*(cos(6*b*x + 6*a) - cos(4*b*x + 4*a) - cos(2*b*x +
 2*a) + I*sin(6*b*x + 6*a) - I*sin(4*b*x + 4*a) - I*sin(2*b*x + 2*a) + 1)*
dilog(e^(I*b*x + I*a)) - 3*(-I*b*x*cos(6*b*x + 6*a) + I*b*x*cos(4*b*x + 4*
a) + I*b*x*cos(2*b*x + 2*a) + b*x*sin(6*b*x + 6*a) - b*x*sin(4*b*x + 4*a)
- b*x*sin(2*b*x + 2*a) - I*b*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2...
```

### 3.287.8 Giac [F]

$$\int x \csc^3(a+bx) \sec^2(a+bx) \, dx = \int x \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

input
```
integrate(x*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

### 3.287.9 Mupad [F(-1)]

Timed out.

$$\int x\csc^3(a+bx)\sec^2(a+bx)\,dx=\int \frac{x}{\cos{(a+b\,x)}^2\,\sin{(a+b\,x)}^3}\,dx$$

input
```
int(x/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

output
```
int(x/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

# 3.288 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x} dx$

3.288.1 Optimal result . . . . . . . . . . 2182
3.288.2 Mathematica [N/A] . . . . . . . . . . 2182
3.288.3 Rubi [N/A] . . . . . . . . . . 2183
3.288.4 Maple [N/A] (verified) . . . . . . . . . . 2183
3.288.5 Fricas [N/A] . . . . . . . . . . 2184
3.288.6 Sympy [N/A] . . . . . . . . . . 2184
3.288.7 Maxima [N/A] . . . . . . . . . . 2184
3.288.8 Giac [N/A] . . . . . . . . . . 2185
3.288.9 Mupad [N/A] . . . . . . . . . . 2186

## 3.288.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x} dx = \text{Int}\left(\frac{\csc^3(a+bx)\sec^2(a+bx)}{x}, x\right)$$

output
```
CannotIntegrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x)
```

## 3.288.2 Mathematica [N/A]

Not integrable

Time = 54.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x} dx = \int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x} dx$$

input
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x,x]
```

output
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x, x]
```

## 3.288.3 Rubi [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}dx$$
$$\Big\downarrow \text{7299}$$
$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}dx$$

input
```
Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x,x]
```

output
```
$Aborted
```

### 3.288.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.288.4 Maple [N/A] (verified)

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\csc(xb+a)^3\sec(xb+a)^2}{x}dx$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)^2/x,x)
```

output
```
int(csc(b*x+a)^3*sec(b*x+a)^2/x,x)
```

### 3.288.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)^2}{x}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^3*sec(b*x + a)^2/x, x)
```

### 3.288.6 Sympy [N/A]

Not integrable

Time = 8.70 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx = \int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx$$

input
```
integrate(csc(b*x+a)**3*sec(b*x+a)**2/x,x)
```

output
```
Integral(csc(a + b*x)**3*sec(a + b*x)**2/x, x)
```

### 3.288.7 Maxima [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 1872, normalized size of antiderivative = 93.60

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)^2}{x}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x, algorithm="maxima")
```

output
```
(2*b*x*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) + 3*b*x*cos(b*x + a) + (3*b*x*cos
(5*b*x + 5*a) - 2*b*x*cos(3*b*x + 3*a) + 3*b*x*cos(b*x + a) - sin(5*b*x +
5*a) + sin(b*x + a))*cos(6*b*x + 6*a) - (3*b*x*cos(4*b*x + 4*a) + 3*b*x*co
s(2*b*x + 2*a) - 3*b*x + sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*cos(5*b*x +
5*a) + (2*b*x*cos(3*b*x + 3*a) - 3*b*x*cos(b*x + a) - sin(b*x + a))*cos(4*
b*x + 4*a) + 2*(b*x*cos(2*b*x + 2*a) - b*x)*cos(3*b*x + 3*a) - (3*b*x*cos(
b*x + a) + sin(b*x + a))*cos(2*b*x + 2*a) + (b^2*x^2*cos(6*b*x + 6*a)^2 +
b^2*x^2*cos(4*b*x + 4*a)^2 + b^2*x^2*cos(2*b*x + 2*a)^2 + b^2*x^2*sin(6*b*
x + 6*a)^2 + b^2*x^2*sin(4*b*x + 4*a)^2 + 2*b^2*x^2*sin(4*b*x + 4*a)*sin(2
*b*x + 2*a) + b^2*x^2*sin(2*b*x + 2*a)^2 - 2*b^2*x^2*cos(2*b*x + 2*a) + b^
2*x^2 - 2*(b^2*x^2*cos(4*b*x + 4*a) + b^2*x^2*cos(2*b*x + 2*a) - b^2*x^2)*
cos(6*b*x + 6*a) + 2*(b^2*x^2*cos(2*b*x + 2*a) - b^2*x^2)*cos(4*b*x + 4*a)
 - 2*(b^2*x^2*sin(4*b*x + 4*a) + b^2*x^2*sin(2*b*x + 2*a))*sin(6*b*x + 6*a
))*integrate(1/2*(3*b^2*x^2 + 2)*sin(b*x + a)/(b^2*x^3*cos(b*x + a)^2 + b^
2*x^3*sin(b*x + a)^2 + 2*b^2*x^3*cos(b*x + a) + b^2*x^3), x) + (b^2*x^2*co
s(6*b*x + 6*a)^2 + b^2*x^2*cos(4*b*x + 4*a)^2 + b^2*x^2*cos(2*b*x + 2*a)^2
 + b^2*x^2*sin(6*b*x + 6*a)^2 + b^2*x^2*sin(4*b*x + 4*a)^2 + 2*b^2*x^2*sin
(4*b*x + 4*a)*sin(2*b*x + 2*a) + b^2*x^2*sin(2*b*x + 2*a)^2 - 2*b^2*x^2*co
s(2*b*x + 2*a) + b^2*x^2 - 2*(b^2*x^2*cos(4*b*x + 4*a) + b^2*x^2*cos(2*b*x
 + 2*a) - b^2*x^2)*cos(6*b*x + 6*a) + 2*(b^2*x^2*cos(2*b*x + 2*a) - b^2...
```

### 3.288.8 Giac [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)^2}{x}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3*sec(b*x + a)^2/x, x)
```

### 3.288.9 Mupad [N/A]

Not integrable

Time = 26.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx = \int \frac{1}{x\cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^3}\,dx$$

input
```
int(1/(x*cos(a + b*x)^2*sin(a + b*x)^3),x)
```

output
```
int(1/(x*cos(a + b*x)^2*sin(a + b*x)^3), x)
```

# 3.289 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$

3.289.1 Optimal result . . . . . . . . . . 2187
3.289.2 Mathematica [N/A] . . . . . . . . . . 2187
3.289.3 Rubi [N/A] . . . . . . . . . . 2188
3.289.4 Maple [N/A] (verified) . . . . . . . . . . 2188
3.289.5 Fricas [N/A] . . . . . . . . . . 2189
3.289.6 Sympy [N/A] . . . . . . . . . . 2189
3.289.7 Maxima [N/A] . . . . . . . . . . 2189
3.289.8 Giac [N/A] . . . . . . . . . . 2190
3.289.9 Mupad [N/A] . . . . . . . . . . 2191

## 3.289.1 Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx = \text{Int}\left(\frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2},x\right)$$

output
```
CannotIntegrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x)
```

## 3.289.2 Mathematica [N/A]

Not integrable

Time = 23.74 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx = \int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$$

input
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2,x]
```

output
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2, x]
```

## 3.289.3 Rubi [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$$

$$\Big\downarrow \; 7299$$

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}dx$$

input
```
Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2,x]
```

output
```
$Aborted
```

### 3.289.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.289.4 Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\csc\left(xb+a\right)^{3}\sec\left(xb+a\right)^{2}}{x^{2}}dx$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x)
```

output
```
int(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x)
```

### 3.289.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx = \int \frac{\csc\left(bx+a\right)^3\sec\left(bx+a\right)^2}{x^2}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^3*sec(b*x + a)^2/x^2, x)
```

### 3.289.6 Sympy [N/A]

Not integrable

Time = 11.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx = \int \frac{\csc^3\left(a+bx\right)\sec^2\left(a+bx\right)}{x^2}\,dx$$

input
```
integrate(csc(b*x+a)**3*sec(b*x+a)**2/x**2,x)
```

output
```
Integral(csc(a + b*x)**3*sec(a + b*x)**2/x**2, x)
```

### 3.289.7 Maxima [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 1884, normalized size of antiderivative = 94.20

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx = \int \frac{\csc\left(bx+a\right)^3\sec\left(bx+a\right)^2}{x^2}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x, algorithm="maxima")
```

output
```
(2*b*x*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) + 3*b*x*cos(b*x + a) + (3*b*x*cos
(5*b*x + 5*a) - 2*b*x*cos(3*b*x + 3*a) + 3*b*x*cos(b*x + a) - 2*sin(5*b*x
+ 5*a) + 2*sin(b*x + a))*cos(6*b*x + 6*a) - (3*b*x*cos(4*b*x + 4*a) + 3*b*
x*cos(2*b*x + 2*a) - 3*b*x + 2*sin(4*b*x + 4*a) + 2*sin(2*b*x + 2*a))*cos(
5*b*x + 5*a) + (2*b*x*cos(3*b*x + 3*a) - 3*b*x*cos(b*x + a) - 2*sin(b*x +
a))*cos(4*b*x + 4*a) + 2*(b*x*cos(2*b*x + 2*a) - b*x)*cos(3*b*x + 3*a) - (
3*b*x*cos(b*x + a) + 2*sin(b*x + a))*cos(2*b*x + 2*a) + (b^2*x^3*cos(6*b*x
 + 6*a)^2 + b^2*x^3*cos(4*b*x + 4*a)^2 + b^2*x^3*cos(2*b*x + 2*a)^2 + b^2*
x^3*sin(6*b*x + 6*a)^2 + b^2*x^3*sin(4*b*x + 4*a)^2 + 2*b^2*x^3*sin(4*b*x
+ 4*a)*sin(2*b*x + 2*a) + b^2*x^3*sin(2*b*x + 2*a)^2 - 2*b^2*x^3*cos(2*b*x
 + 2*a) + b^2*x^3 - 2*(b^2*x^3*cos(4*b*x + 4*a) + b^2*x^3*cos(2*b*x + 2*a)
 - b^2*x^3)*cos(6*b*x + 6*a) + 2*(b^2*x^3*cos(2*b*x + 2*a) - b^2*x^3)*cos(
4*b*x + 4*a) - 2*(b^2*x^3*sin(4*b*x + 4*a) + b^2*x^3*sin(2*b*x + 2*a))*sin
(6*b*x + 6*a))*integrate(3/2*(b^2*x^2 + 2)*sin(b*x + a)/(b^2*x^4*cos(b*x +
 a)^2 + b^2*x^4*sin(b*x + a)^2 + 2*b^2*x^4*cos(b*x + a) + b^2*x^4), x) + (
b^2*x^3*cos(6*b*x + 6*a)^2 + b^2*x^3*cos(4*b*x + 4*a)^2 + b^2*x^3*cos(2*b*
x + 2*a)^2 + b^2*x^3*sin(6*b*x + 6*a)^2 + b^2*x^3*sin(4*b*x + 4*a)^2 + 2*b
^2*x^3*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b^2*x^3*sin(2*b*x + 2*a)^2 - 2*
b^2*x^3*cos(2*b*x + 2*a) + b^2*x^3 - 2*(b^2*x^3*cos(4*b*x + 4*a) + b^2*x^3
*cos(2*b*x + 2*a) - b^2*x^3)*cos(6*b*x + 6*a) + 2*(b^2*x^3*cos(2*b*x + ...
```

### 3.289.8 Giac [N/A]

Not integrable

Time = 1.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)^2}{x^2}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3*sec(b*x + a)^2/x^2, x)
```

### 3.289.9 Mupad [N/A]

Not integrable

Time = 26.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx=\int \frac{1}{x^2\cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^3}\,dx$$

input
```
int(1/(x^2*cos(a + b*x)^2*sin(a + b*x)^3),x)
```

output
```
int(1/(x^2*cos(a + b*x)^2*sin(a + b*x)^3), x)
```

# 3.290 $\int (c+dx)^m \sec^2(a+bx) \tan(a+bx)\,dx$

3.290.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2192
3.290.2 Mathematica [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . 2192
3.290.3 Rubi [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2193
3.290.4 Maple [N/A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 2193
3.290.5 Fricas [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2194
3.290.6 Sympy [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2194
3.290.7 Maxima [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2194
3.290.8 Giac [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2195
3.290.9 Mupad [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2195

## 3.290.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (c+dx)^m \sec^2(a+bx) \tan(a+bx)\,dx = \text{Int}\big((c+dx)^m \sec^2(a+bx) \tan(a+bx),x\big)$$

output
```
CannotIntegrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x)
```

## 3.290.2 Mathematica [N/A]

Not integrable

Time = 6.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sec^2(a+bx) \tan(a+bx)\,dx = \int (c+dx)^m \sec^2(a+bx) \tan(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x], x]
```

## 3.290.3 Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(a+bx)\sec^2(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \tan(a+bx)\sec^2(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x],x]
```

output
```
$Aborted
```

### 3.290.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.290.4 Maple [N/A] (verified)

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \sec(xb+a)^2 \tan(xb+a)\,dx$$

input
```
int((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x)
```

output
```
int((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x)
```

### 3.290.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sec^2(a+bx) \tan(a+bx)\, dx = \int (dx+c)^m \sec{(bx+a)}^2 \tan{(bx+a)}\, dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*sec(b*x + a)^2*tan(b*x + a), x)
```

### 3.290.6 Sympy [N/A]

Not integrable

Time = 9.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \sec^2(a+bx) \tan(a+bx)\, dx = \int (c+dx)^m \tan{(a+bx)} \sec^2{(a+bx)}\, dx$$

input
```
integrate((d*x+c)**m*sec(b*x+a)**2*tan(b*x+a),x)
```

output
```
Integral((c + d*x)**m*tan(a + b*x)*sec(a + b*x)**2, x)
```

### 3.290.7 Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sec^2(a+bx) \tan(a+bx)\, dx = \int (dx+c)^m \sec{(bx+a)}^2 \tan{(bx+a)}\, dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*sec(b*x + a)^2*tan(b*x + a), x)
```

### 3.290.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)^2 \tan(bx+a)\;dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sec(b*x + a)^2*tan(b*x + a), x)
```

### 3.290.9 Mupad [N/A]

Not integrable

Time = 25.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx = \int \frac{\tan(a+b\,x)\;(c+d\,x)^m}{\cos(a+b\,x)^2}\,dx$$

input
```
int((tan(a + b*x)*(c + d*x)^m)/cos(a + b*x)^2,x)
```

output
```
int((tan(a + b*x)*(c + d*x)^m)/cos(a + b*x)^2, x)
```

# 3.291 $\int (c+dx)^4 \sec^2(a+bx) \tan(a+bx) \, dx$


3.291.1 Optimal result . . . 2196
3.291.2 Mathematica [B] (verified) . . . 2197
3.291.3 Rubi [A] (verified) . . . 2198
3.291.4 Maple [B] (verified) . . . 2201
3.291.5 Fricas [B] (verification not implemented) . . . 2202
3.291.6 Sympy [F] . . . 2202
3.291.7 Maxima [B] (verification not implemented) . . . 2203
3.291.8 Giac [F] . . . 2203
3.291.9 Mupad [F(-1)] . . . 2204


## 3.291.1 Optimal result

Integrand size = 22, antiderivative size = 139

$$\begin{aligned}\int (c+dx)^4 \sec^2(a+bx) \tan(a+bx) \, dx = {} & \frac{2id(c+dx)^3}{b^2} - \frac{6d^2(c+dx)^2 \log\left(1+e^{2i(a+bx)}\right)}{b^3} \\ & + \frac{6id^3(c+dx) \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^4} \\ & - \frac{3d^4 \operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^5} \\ & + \frac{(c+dx)^4 \sec^2(a+bx)}{2b} \\ & - \frac{2d(c+dx)^3 \tan(a+bx)}{b^2}\end{aligned}$$

output
```
2*I*d*(d*x+c)^3/b^2-6*d^2*(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b^3+6*I*d^3*(d*
x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^4-3*d^4*polylog(3,-exp(2*I*(b*x+a)))/b
^5+1/2*(d*x+c)^4*sec(b*x+a)^2/b-2*d*(d*x+c)^3*tan(b*x+a)/b^2
```

### 3.291.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 418 vs. $2(139) = 278$.

Time = 6.60 (sec) , antiderivative size = 418, normalized size of antiderivative = 3.01

$$\int (c+dx)^4 \sec^2(a+bx)\tan(a+bx)\,dx = -\frac{id^4e^{-ia}\left(2b^2x^2\left(2bx-3i(1+e^{2ia})\log\left(1+e^{-2i(a+bx)}\right)\right)+6b(1+e^{2ia})\,x\,\mathrm{PolyLog}\left(2,-e^{-2i(a+bx)}\right)-3i(1-\right.}{2b^5}$$
$$+\frac{(c+dx)^4\sec^2(a+bx)}{2b}$$
$$-\frac{6c^2d^2\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))}{b^3\left(\cos^2(a)+\sin^2(a)\right)}$$
$$-\frac{6cd^3\csc(a)\left(b^2e^{-i\arctan(\cot(a))}x^2-\frac{\cot(a)(ibx(-\pi-2\arctan(\cot(a)))-\pi\log(1+e^{-2ibx})-2(bx-\arctan(\cot(a)))\log(1-e^{2i(bx-\mathrm{arcta}}}{}\right.}{b^4\sqrt{\csc^2(a)\left(\cos^2(a)+\right.}}$$
$$-\frac{2\sec(a)\sec(a+bx)\left(c^3d\sin(bx)+3c^2d^2x\sin(bx)+3cd^3x^2\sin(bx)+d^4x^3\sin(bx)\right)}{b^2}$$

input
```
Integrate[(c + d*x)^4*Sec[a + b*x]^2*Tan[a + b*x],x]
```

output
```
((-1/2*I)*d^4*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I
)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))]
 - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^5
*E^(I*a)) + ((c + d*x)^4*Sec[a + b*x]^2)/(2*b) - (6*c^2*d^2*Sec[a]*(Cos[a]
*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 + Si
n[a]^2)) - (6*c*d^3*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*
x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[
Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*
ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x
 - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^4*Sqrt[Csc[a]^2*(Cos
[a]^2 + Sin[a]^2)]) - (2*Sec[a]*Sec[a + b*x]*(c^3*d*Sin[b*x] + 3*c^2*d^2*x
*Sin[b*x] + 3*c*d^3*x^2*Sin[b*x] + d^4*x^3*Sin[b*x]))/b^2
```

### 3.291.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {4909, 3042, 4672, 25, 3042, 4202, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4\tan(a+bx)\sec^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 4909}$$

$$\frac{(c+dx)^4\sec^2(a+bx)}{2b}-\frac{2d\int (c+dx)^3\sec^2(a+bx)dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(c+dx)^4\sec^2(a+bx)}{2b}-\frac{2d\int (c+dx)^3\csc\left(a+bx+\frac{\pi}{2}\right)^2dx}{b}$$

$$\Big\downarrow \text{ 4672}$$

$$\frac{(c+dx)^4\sec^2(a+bx)}{2b}-\frac{2d\left(\frac{3d\int -(c+dx)^2\tan(a+bx)dx}{b}+\frac{(c+dx)^3\tan(a+bx)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{(c+dx)^4\sec^2(a+bx)}{2b}-\frac{2d\left(\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{3d\int (c+dx)^2\tan(a+bx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(c+dx)^4\sec^2(a+bx)}{2b}-\frac{2d\left(\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{3d\int (c+dx)^2\tan(a+bx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 4202}$$

$$\frac{(c+dx)^4\sec^2(a+bx)}{2b}-\frac{2d\left(\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}dx\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{(c+dx)^4\sec^2(a+bx)}{2b}-\frac{2d\left(\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\int (c+dx)\log\left(1+e^{2i(a+bx)}\right)dx}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}\right)}{b}$$

$\Big\downarrow$ 3011

$$\frac{(c+dx)^4\sec^2(a+bx)}{2b}-\frac{2d\left(\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}\right)}{b}$$

$\Big\downarrow$ 2720

$$\frac{(c+dx)^4\sec^2(a+bx)}{2b}-\frac{2d\left(\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\int e^{-2i(a+bx)}\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}\right)}{b}$$

$\Big\downarrow$ 7143

$$\frac{(c+dx)^4\sec^2(a+bx)}{2b}-\frac{2d\left(\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{3d\left(\frac{i(c+dx)^3}{3d}-2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}\right)}{b}$$

input
```
Int[(c + d*x)^4*Sec[a + b*x]^2*Tan[a + b*x],x]
```

## 3.291. $\int (c+dx)^4\sec^2(a+bx)\tan(a+bx)\,dx$

output
```
((c + d*x)^4*Sec[a + b*x]^2)/(2*b) - (2*d*((-3*d*(((I/3)*(c + d*x)^3)/d -
(2*I)*(((-1/2*I)*(c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b + (I*d*(((I/2
)*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b - (d*PolyLog[3, -E^((2*I)*
(a + b*x))])/(4*b^2)))/b)))/b + ((c + d*x)^3*Tan[a + b*x])/b))/b
```

### 3.291.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.291.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(129) = 258$.

Time = 4.89 (sec) , antiderivative size = 489, normalized size of antiderivative = 3.52

| method | result |
|---|---|
| risch | $\frac{2b\,d^4x^4 e^{2i(xb+a)}+8bc\,d^3x^3 e^{2i(xb+a)}+12b\,c^2d^2x^2 e^{2i(xb+a)}+8b\,c^3dx\,e^{2i(xb+a)}-4id^4x^3 e^{2i(xb+a)}+2b\,c^4 e^{2i(xb+a)}-12ic\,d^3x^2 e^{2i(xb+a)}-}{b^2\left(e^{2i(xb+a)}+1\right)^2}$ |

input
```
int((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*(b*d^4*x^4*exp(2*I*(b*x+a))+4*b*c*d^3*x^3*exp(2*I*(b*x+a))+6*b*c^2*d^2*x
^2*exp(2*I*(b*x+a))+4*b*c^3*d*x*exp(2*I*(b*x+a))-2*I*d^4*x^3*exp(2*I*(b*x+
a))+b*c^4*exp(2*I*(b*x+a))-6*I*c*d^3*x^2*exp(2*I*(b*x+a))-6*I*c^2*d^2*x*ex
p(2*I*(b*x+a))-2*I*d^4*x^3-2*I*c^3*d*exp(2*I*(b*x+a))-6*I*c*d^3*x^2-6*I*c^
2*d^2*x-2*I*c^3*d)/b^2/(exp(2*I*(b*x+a))+1)^2-12*I/b^4*d^4*a^2*x+12/b^5*d^
4*a^2*ln(exp(I*(b*x+a)))-6/b^3*d^4*ln(exp(2*I*(b*x+a))+1)*x^2-3*d^4*polylo
g(3,-exp(2*I*(b*x+a)))/b^5+12*I/b^4*d^3*c*a^2+12*I/b^2*d^3*c*x^2+24*I/b^3*
d^3*c*x*a+4*I/b^2*d^4*x^3-6/b^3*d^2*c^2*ln(exp(2*I*(b*x+a))+1)+12/b^3*d^2*
c^2*ln(exp(I*(b*x+a)))-12/b^3*d^3*c*ln(exp(2*I*(b*x+a))+1)*x-24/b^4*d^3*c*
a*ln(exp(I*(b*x+a)))+6*I/b^4*d^4*polylog(2,-exp(2*I*(b*x+a)))*x+6*I/b^4*d^
3*c*polylog(2,-exp(2*I*(b*x+a)))-8*I/b^5*d^4*a^3
```

### 3.291.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 892 vs. $2(126) = 252$.

Time = 0.31 (sec) , antiderivative size = 892, normalized size of antiderivative = 6.42

$$\int (c+dx)^4 \sec^2(a+bx)\tan(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b
^4*c^4 - 12*d^4*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) -
 12*d^4*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 12*d^4*
cos(b*x + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 12*d^4*cos(b*x
 + a)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 12*(I*b*d^4*x + I*b*c
*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) - 12*(-I*b*d^4*x
 - I*b*c*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) - 12*(-I
*b*d^4*x - I*b*c*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a))
 - 12*(I*b*d^4*x + I*b*c*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b
*x + a)) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2*log(cos(
b*x + a) + I*sin(b*x + a) + I) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*c
os(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I) - 6*(b^2*d^4*x^2 + 2*
b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*cos(b*x + a)^2*log(I*cos(b*x + a) + s
in(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)
*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - 6*(b^2*d^4*x^2 +
2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*cos(b*x + a)^2*log(-I*cos(b*x + a)
+ sin(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d
^4)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - 6*(b^2*c^2*d^
2 - 2*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x +
a) + I) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2*log(-c...
```

### 3.291.6 Sympy [F]

$$\int (c+dx)^4 \sec^2(a+bx)\tan(a+bx)\,dx = \int (c+dx)^4 \tan(a+bx)\sec^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**4*sec(b*x+a)**2*tan(b*x+a),x)
```

output
```
Integral((c + d*x)**4*tan(a + b*x)*sec(a + b*x)**2, x)
```

### 3.291.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3446 vs. 2(126) = 252.

Time = 0.49 (sec) , antiderivative size = 3446, normalized size of antiderivative = 24.79

$$\int (c+dx)^4 \sec^2(a+bx)\tan(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(c^4*tan(b*x + a)^2 - 4*a*c^3*d*tan(b*x + a)^2/b + 6*a^2*c^2*d^2*tan(b
*x + a)^2/b^2 - 4*a^3*c*d^3*tan(b*x + a)^2/b^3 + a^4*d^4*tan(b*x + a)^2/b^
4 + 8*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (
2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x
 + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)
- 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*c^3*d/((2*(2*cos(2*b*x + 2*a) +
1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*
x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 +
4*cos(2*b*x + 2*a) + 1)*b) - 24*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x +
 a)*sin(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))
*cos(4*b*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x
+ 2*a) - cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*a*c^2*
d^2/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4
*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x +
2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^2) + 24*(4*(b*x +
a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(
2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a)*cos(2*b*x
+ 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(4*b*x +
 4*a) - sin(2*b*x + 2*a))*a^2*c*d^3/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x
 + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^...
```

### 3.291.8 Giac [F]

$$\int (c+dx)^4 \sec^2(a+bx)\tan(a+bx)\,dx = \int (dx+c)^4 \sec(bx+a)^2 \tan(bx+a)\,dx$$

input
```
integrate((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*sec(b*x + a)^2*tan(b*x + a), x)
```

### 3.291.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \sec^2(a+bx) \tan(a+bx) \, dx = \int \frac{\tan(a+b\,x)\,\left(c+d\,x\right)^4}{\cos\left(a+b\,x\right)^2} \, dx$$

input
```
int((tan(a + b*x)*(c + d*x)^4)/cos(a + b*x)^2,x)
```

output
```
int((tan(a + b*x)*(c + d*x)^4)/cos(a + b*x)^2, x)
```

# 3.292 $\int (c+dx)^3 \sec^2(a+bx) \tan(a+bx) \, dx$

3.292.1 Optimal result . . . . . 2205
3.292.2 Mathematica [B] (verified) . . . . . 2205
3.292.3 Rubi [A] (verified) . . . . . 2206
3.292.4 Maple [B] (verified) . . . . . 2209
3.292.5 Fricas [B] (verification not implemented) . . . . . 2210
3.292.6 Sympy [F] . . . . . 2210
3.292.7 Maxima [B] (verification not implemented) . . . . . 2211
3.292.8 Giac [F] . . . . . 2211
3.292.9 Mupad [F(-1)] . . . . . 2212

## 3.292.1 Optimal result

Integrand size = 22, antiderivative size = 115

$$\int (c+dx)^3 \sec^2(a+bx) \tan(a+bx) \, dx = \frac{3id(c+dx)^2}{2b^2} - \frac{3d^2(c+dx) \log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{3id^3 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^4} + \frac{(c+dx)^3 \sec^2(a+bx)}{2b} - \frac{3d(c+dx)^2 \tan(a+bx)}{2b^2}$$

output
```
3/2*I*d*(d*x+c)^2/b^2-3*d^2*(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b^3+3/2*I*d^3*p
olylog(2,-exp(2*I*(b*x+a)))/b^4+1/2*(d*x+c)^3*sec(b*x+a)^2/b-3/2*d*(d*x+c)
^2*tan(b*x+a)/b^2
```

## 3.292.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 286 vs. 2(115) = 230.

Time = 6.40 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.49

$$\int (c+dx)^3 \sec^2(a+bx)\tan(a+bx)\,dx = \frac{(c+dx)^3\sec^2(a+bx)}{2b}$$
$$-\frac{3cd^2\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))}{b^3\left(\cos^2(a)+\sin^2(a)\right)}$$
$$-\frac{3d^3\csc(a)\left(b^2e^{-i\arctan(\cot(a))}x^2-\frac{\cot(a)\left(ibx(-\pi-2\arctan(\cot(a)))-\pi\log\left(1+e^{-2ibx}\right)-2(bx-\arctan(\cot(a)))\log\left(1-e^{2i(bx-\arctan}\right.\right.}{}\right.}{2b^4\sqrt{\csc^2(a)\left(\cos^2(a)+\right.}}$$
$$-\frac{3\sec(a)\sec(a+bx)\left(c^2d\sin(bx)+2cd^2x\sin(bx)+d^3x^2\sin(bx)\right)}{2b^2}$$

input
```
Integrate[(c + d*x)^3*Sec[a + b*x]^2*Tan[a + b*x],x]
```

output
```
((c + d*x)^3*Sec[a + b*x]^2)/(2*b) - (3*c*d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Co
s[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) - (3*
d^3*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTa
n[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 -
 E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*L
og[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]
]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^4*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]
^2)]) - (3*Sec[a]*Sec[a + b*x]*(c^2*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x] + d^3*
x^2*Sin[b*x]))/(2*b^2)
```

### 3.292.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {4909, 3042, 4672, 25, 3042, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\tan(a+bx)\sec^2(a+bx)\,dx$$
$$\Big\downarrow \text{4909}$$
$$\frac{(c+dx)^3\sec^2(a+bx)}{2b}-\frac{3d\int (c+dx)^2\sec^2(a+bx)dx}{2b}$$
$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^3\sec^2(a+bx)}{2b}-\frac{3d\int(c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)^2dx}{2b}$$

$$\Big\downarrow \text{4672}$$

$$\frac{(c+dx)^3\sec^2(a+bx)}{2b}-\frac{3d\left(\frac{2d\int-((c+dx)\tan(a+bx))dx}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}\right)}{2b}$$

$$\Big\downarrow \text{25}$$

$$\frac{(c+dx)^3\sec^2(a+bx)}{2b}-\frac{3d\left(\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{2d\int(c+dx)\tan(a+bx)dx}{b}\right)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^3\sec^2(a+bx)}{2b}-\frac{3d\left(\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{2d\int(c+dx)\tan(a+bx)dx}{b}\right)}{2b}$$

$$\Big\downarrow \text{4202}$$

$$\frac{(c+dx)^3\sec^2(a+bx)}{2b}-\frac{3d\left(\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\int\frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}dx\right)}{b}\right)}{2b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{(c+dx)^3\sec^2(a+bx)}{2b}-\frac{3d\left(\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{id\int\log\left(1+e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}\right)}{2b}$$

$$\Big\downarrow \text{2715}$$

$$\frac{(c+dx)^3\sec^2(a+bx)}{2b}-\frac{3d\left(\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{d\int e^{-2i(a+bx)}\log\left(1+e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}\right)}{2b}$$

$$\Big\downarrow \text{2838}$$

3.292. $\int(c+dx)^3\sec^2(a+bx)\tan(a+bx)\,dx$

$$\frac{(c+dx)^3\sec^2(a+bx)}{2b}-\frac{3d\left(\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}\right)}{2b}$$

input
```
Int[(c + d*x)^3*Sec[a + b*x]^2*Tan[a + b*x],x]
```

output
```
((c + d*x)^3*Sec[a + b*x]^2)/(2*b) - (3*d*((-2*d*(((I/2)*(c + d*x)^2)/d -
(2*I)*(((-1/2*I)*(c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b - (d*PolyLog[2,
 -E^((2*I)*(a + b*x))])/(4*b^2))))/b + ((c + d*x)^2*Tan[a + b*x])/b))/(2*b
)
```

### 3.292.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

## 3.292.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(101) = 202.

Time = 3.18 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.62

| method | result |
|---|---|
| risch | $\frac{2b\,d^3x^3\mathrm{e}^{2i(xb+a)}-3id^3x^2\mathrm{e}^{2i(xb+a)}+6bc\,d^2x^2\mathrm{e}^{2i(xb+a)}-6ic\,d^2x\,\mathrm{e}^{2i(xb+a)}+6b\,c^2dx\,\mathrm{e}^{2i(xb+a)}-3ic^2d\,\mathrm{e}^{2i(xb+a)}-3id^3x^2+2b\,c^3\mathrm{e}^{2i(xb+a}}{b^2\left(\mathrm{e}^{2i(xb+a)}+1\right)^2}$ |

input
```
int((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*e
xp(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-
3*I*c^2*d*exp(2*I*(b*x+a))-3*I*d^3*x^2+2*b*c^3*exp(2*I*(b*x+a))-6*I*c*d^2*
x-3*I*c^2*d)/b^2/(exp(2*I*(b*x+a))+1)^2-3/b^3*d^2*c*ln(exp(2*I*(b*x+a))+1)
+6/b^3*d^2*c*ln(exp(I*(b*x+a)))+3*I/b^2*d^3*x^2+6*I/b^3*d^3*x*a+3*I/b^4*d^
3*a^2-3/b^3*d^3*ln(exp(2*I*(b*x+a))+1)*x+3/2*I*d^3*polylog(2,-exp(2*I*(b*x
+a)))/b^4-6/b^4*d^3*a*ln(exp(I*(b*x+a)))
```

### 3.292.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 540 vs. 2(98) = 196.

Time = 0.29 (sec) , antiderivative size = 540, normalized size of antiderivative = 4.70

$$\int (c+dx)^3 \sec^2(a+bx)\tan(a+bx)\,dx = \frac{b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3 - 3i\,d^3\cos{(bx+a)}^2\,\mathrm{Li}_2(i\,\cos{(bx+a)} + \sin{(bx+a)}) + 3i\,d^3\cos{(bx}}{}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 3*I*d^3*cos
(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + 3*I*d^3*cos(b*x + a)^2*
dilog(I*cos(b*x + a) - sin(b*x + a)) + 3*I*d^3*cos(b*x + a)^2*dilog(-I*cos
(b*x + a) + sin(b*x + a)) - 3*I*d^3*cos(b*x + a)^2*dilog(-I*cos(b*x + a) -
 sin(b*x + a)) - 3*(b*c*d^2 - a*d^3)*cos(b*x + a)^2*log(cos(b*x + a) + I*s
in(b*x + a) + I) - 3*(b*c*d^2 - a*d^3)*cos(b*x + a)^2*log(cos(b*x + a) - I
*sin(b*x + a) + I) - 3*(b*d^3*x + a*d^3)*cos(b*x + a)^2*log(I*cos(b*x + a)
 + sin(b*x + a) + 1) - 3*(b*d^3*x + a*d^3)*cos(b*x + a)^2*log(I*cos(b*x +
a) - sin(b*x + a) + 1) - 3*(b*d^3*x + a*d^3)*cos(b*x + a)^2*log(-I*cos(b*x
 + a) + sin(b*x + a) + 1) - 3*(b*d^3*x + a*d^3)*cos(b*x + a)^2*log(-I*cos(
b*x + a) - sin(b*x + a) + 1) - 3*(b*c*d^2 - a*d^3)*cos(b*x + a)^2*log(-cos
(b*x + a) + I*sin(b*x + a) + I) - 3*(b*c*d^2 - a*d^3)*cos(b*x + a)^2*log(-
cos(b*x + a) - I*sin(b*x + a) + I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*
c^2*d)*cos(b*x + a)*sin(b*x + a))/(b^4*cos(b*x + a)^2)
```

### 3.292.6 Sympy [F]

$$\int (c+dx)^3 \sec^2(a+bx)\tan(a+bx)\,dx = \int (c+dx)^3\tan{(a+bx)}\sec^2{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**3*sec(b*x+a)**2*tan(b*x+a),x)
```

output
```
Integral((c + d*x)**3*tan(a + b*x)*sec(a + b*x)**2, x)
```

### 3.292.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 668 vs. 2(98) = 196.

Time = 0.44 (sec) , antiderivative size = 668, normalized size of antiderivative = 5.81

$$\int (c+dx)^3 \sec^2(a+bx)\tan(a+bx)\,dx =$$
$$-\frac{6\,b^2c^2d+6\,(bd^3x+bcd^2+(bd^3x+bcd^2)\cos{(4\,bx+4\,a)}+2\,(bd^3x+bcd^2)\cos{(2\,bx+2\,a)}-(-i\,bd^3x-}{}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

output
```
-(6*b^2*c^2*d + 6*(b*d^3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*cos(4*b*x + 4*a
) + 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a) - (-I*b*d^3*x - I*b*c*d^2)*sin(
4*b*x + 4*a) - 2*(-I*b*d^3*x - I*b*c*d^2)*sin(2*b*x + 2*a))*arctan2(sin(2*
b*x + 2*a), cos(2*b*x + 2*a) + 1) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x)*cos(4*
b*x + 4*a) - 2*(-2*I*b^3*d^3*x^3 - 2*I*b^3*c^3 - 3*b^2*c^2*d + 3*(-2*I*b^3
*c*d^2 + b^2*d^3)*x^2 + 6*(-I*b^3*c^2*d + b^2*c*d^2)*x)*cos(2*b*x + 2*a) -
 3*(d^3*cos(4*b*x + 4*a) + 2*d^3*cos(2*b*x + 2*a) + I*d^3*sin(4*b*x + 4*a)
 + 2*I*d^3*sin(2*b*x + 2*a) + d^3)*dilog(-e^(2*I*b*x + 2*I*a)) - 3*(I*b*d^
3*x + I*b*c*d^2 + (I*b*d^3*x + I*b*c*d^2)*cos(4*b*x + 4*a) + 2*(I*b*d^3*x
+ I*b*c*d^2)*cos(2*b*x + 2*a) - (b*d^3*x + b*c*d^2)*sin(4*b*x + 4*a) - 2*(
b*d^3*x + b*c*d^2)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x +
2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - 6*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x)*sin
(4*b*x + 4*a) - 2*(2*b^3*d^3*x^3 + 2*b^3*c^3 - 3*I*b^2*c^2*d + 3*(2*b^3*c*
d^2 + I*b^2*d^3)*x^2 + 6*(b^3*c^2*d + I*b^2*c*d^2)*x)*sin(2*b*x + 2*a))/(-
2*I*b^4*cos(4*b*x + 4*a) - 4*I*b^4*cos(2*b*x + 2*a) + 2*b^4*sin(4*b*x + 4*
a) + 4*b^4*sin(2*b*x + 2*a) - 2*I*b^4)
```

### 3.292.8 Giac [F]

$$\int (c+dx)^3 \sec^2(a+bx)\tan(a+bx)\,dx = \int (dx+c)^3 \sec{(bx+a)}^2 \tan{(bx+a)}\;dx$$

input
```
integrate((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*sec(b*x + a)^2*tan(b*x + a), x)
```

### 3.292.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \sec^2(a+bx)\tan(a+bx)\,dx = \int \frac{\tan(a+b\,x)\ (c+d\,x)^3}{\cos\left(a+b\,x\right)^2}\,dx$$

input
```
int((tan(a + b*x)*(c + d*x)^3)/cos(a + b*x)^2,x)
```

output
```
int((tan(a + b*x)*(c + d*x)^3)/cos(a + b*x)^2, x)
```

# 3.293 $\int (c+dx)^2 \sec^2(a+bx) \tan(a+bx) \, dx$

3.293.1 Optimal result . . . . . . . . . . 2213
3.293.2 Mathematica [A] (verified) . . . . . . . . . . 2213
3.293.3 Rubi [A] (verified) . . . . . . . . . . 2214
3.293.4 Maple [C] (verified) . . . . . . . . . . 2215
3.293.5 Fricas [A] (verification not implemented) . . . . . . . . . . 2216
3.293.6 Sympy [F] . . . . . . . . . . 2216
3.293.7 Maxima [B] (verification not implemented) . . . . . . . . . . 2217
3.293.8 Giac [B] (verification not implemented) . . . . . . . . . . 2217
3.293.9 Mupad [B] (verification not implemented) . . . . . . . . . . 2218

## 3.293.1 Optimal result

Integrand size = 22, antiderivative size = 55

$$\int (c+dx)^2 \sec^2(a+bx) \tan(a+bx) \, dx = -\frac{d^2 \log(\cos(a+bx))}{b^3} + \frac{(c+dx)^2 \sec^2(a+bx)}{2b} - \frac{d(c+dx) \tan(a+bx)}{b^2}$$

output
```
-d^2*ln(cos(b*x+a))/b^3+1/2*(d*x+c)^2*sec(b*x+a)^2/b-d*(d*x+c)*tan(b*x+a)/
b^2
```

## 3.293.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

$$\int (c+dx)^2 \sec^2(a+bx) \tan(a+bx) \, dx = \frac{b^2(c+dx)^2 \sec^2(a+bx) - 2bd(c+dx) \sec(a) \sec(a+bx) \sin(bx) - 2d^2(\log(\cos(a+bx)) + bx \tan(a))}{2b^3}$$

input
```
Integrate[(c + d*x)^2*Sec[a + b*x]^2*Tan[a + b*x],x]
```

output
```
(b^2*(c + d*x)^2*Sec[a + b*x]^2 - 2*b*d*(c + d*x)*Sec[a]*Sec[a + b*x]*Sin[
b*x] - 2*d^2*(Log[Cos[a + b*x]] + b*x*Tan[a]))/(2*b^3)
```

### 3.293.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4909, 3042, 4672, 25, 3042, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \tan(a+bx) \sec^2(a+bx) \, dx$$

$$\Big\downarrow \text{ 4909}$$

$$\frac{(c+dx)^2 \sec^2(a+bx)}{2b}-\frac{d \int (c+dx) \sec^2(a+bx)dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(c+dx)^2 \sec^2(a+bx)}{2b}-\frac{d \int (c+dx) \csc \left(a+bx+\frac{\pi}{2}\right)^2dx}{b}$$

$$\Big\downarrow \text{ 4672}$$

$$\frac{(c+dx)^2 \sec^2(a+bx)}{2b}-\frac{d \left(\frac{d \int -\tan(a+bx)dx}{b}+\frac{(c+dx) \tan(a+bx)}{b}\right)}{b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{(c+dx)^2 \sec^2(a+bx)}{2b}-\frac{d \left(\frac{(c+dx) \tan(a+bx)}{b}-\frac{d \int \tan(a+bx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(c+dx)^2 \sec^2(a+bx)}{2b}-\frac{d \left(\frac{(c+dx) \tan(a+bx)}{b}-\frac{d \int \tan(a+bx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{ 3956}$$

$$\frac{(c+dx)^2 \sec^2(a+bx)}{2b}-\frac{d \left(\frac{d \log(\cos(a+bx))}{b^2}+\frac{(c+dx) \tan(a+bx)}{b}\right)}{b}$$

input
```
Int[(c + d*x)^2*Sec[a + b*x]^2*Tan[a + b*x],x]
```

output
```
((c + d*x)^2*Sec[a + b*x]^2)/(2*b) - (d*((d*Log[Cos[a + b*x]])/b^2 + ((c +
 d*x)*Tan[a + b*x])/b))/b
```

### 3.293.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

## 3.293.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.53 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.71

| method | result |
|---|---|
| risch | $\frac{2id^2x}{b^2}+\frac{2id^2a}{b^3}+\frac{2b\,d^2x^2\mathrm{e}^{2i(xb+a)}+4bcdx\,\mathrm{e}^{2i(xb+a)}+2b\,c^2\mathrm{e}^{2i(xb+a)}-2id^2x\,\mathrm{e}^{2i(xb+a)}-2icd\,\mathrm{e}^{2i(xb+a)}-2id^2x-2idc\ \cdots}{b^2\left(\mathrm{e}^{2i(xb+a)}+1\right)^2}\ \cdots$ |
| derivativedivides | $\frac{\frac{a^2d^2}{2b^2\cos(xb+a)^2}-\frac{acd}{b\cos(xb+a)^2}-\frac{2a\,d^2\left(\frac{xb+a}{2\cos(xb+a)^2}-\frac{\tan(xb+a)}{2}\right)}{b^2}+\frac{c^2}{2\cos(xb+a)^2}+\frac{2cd\left(\frac{xb+a}{2\cos(xb+a)^2}-\frac{\tan(xb+a)}{2}\right)}{b}+\frac{d^2\left(\frac{(x\cdots}{2\cos\cdots}\cdots\right)}{\cdots}}{b}$ |
| default | $\frac{\frac{a^2d^2}{2b^2\cos(xb+a)^2}-\frac{acd}{b\cos(xb+a)^2}-\frac{2a\,d^2\left(\frac{xb+a}{2\cos(xb+a)^2}-\frac{\tan(xb+a)}{2}\right)}{b^2}+\frac{c^2}{2\cos(xb+a)^2}+\frac{2cd\left(\frac{xb+a}{2\cos(xb+a)^2}-\frac{\tan(xb+a)}{2}\right)}{b}+\frac{d^2\left(\frac{(x\cdots}{2\cos\cdots}\cdots\right)}{\cdots}}{b}$ |

input
```
int((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*I*d^2/b^2*x+2*I*d^2/b^3*a+2*(b*d^2*x^2*exp(2*I*(b*x+a))+2*b*c*d*x*exp(2*
I*(b*x+a))+b*c^2*exp(2*I*(b*x+a))-I*d^2*x*exp(2*I*(b*x+a))-I*c*d*exp(2*I*(
b*x+a))-I*d^2*x-I*d*c)/b^2/(exp(2*I*(b*x+a))+1)^2-d^2/b^3*ln(exp(2*I*(b*x+
a))+1)
```

### 3.293.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.56

$$\int (c+dx)^2 \sec^2(a+bx)\tan(a+bx)\,dx = \frac{b^2d^2x^2 + 2\,b^2cdx - 2\,d^2\cos\left(bx+a\right)^2\log\left(-\cos\left(bx+a\right)\right) + b^2c^2 - 2\left(bd^2x+bcd\right)\cos\left(bx+a\right)\sin\left(bx+a\right)}{2\,b^3\cos\left(bx+a\right)^2}$$

input
```
integrate((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x - 2*d^2*cos(b*x + a)^2*log(-cos(b*x + a)) +
 b^2*c^2 - 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3*cos(b*x + a
)^2)
```

### 3.293.6 Sympy [F]

$$\int (c+dx)^2 \sec^2(a+bx)\tan(a+bx)\,dx = \int (c+dx)^2\tan\left(a+bx\right)\sec^2\left(a+bx\right)dx$$

input
```
integrate((d*x+c)**2*sec(b*x+a)**2*tan(b*x+a),x)
```

output
```
Integral((c + d*x)**2*tan(a + b*x)*sec(a + b*x)**2, x)
```

### 3.293.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 988 vs. $2(53) = 106$.

Time = 0.31 (sec) , antiderivative size = 988, normalized size of antiderivative = 17.96

$$\int (c + dx)^2 \sec^2(a + bx) \tan(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(c^2*tan(b*x + a)^2 - 2*a*c*d*tan(b*x + a)^2/b + a^2*d^2*tan(b*x + a)^
2/b^2 + 4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2
 + (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*
(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2
*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*c*d/((2*(2*cos(2*b*x + 2*a)
+ 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*
b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2
+ 4*cos(2*b*x + 2*a) + 1)*b) - 4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x
+ a)*sin(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a)
)*cos(4*b*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x
 + 2*a) - cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*a*d^2
/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*co
s(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a
) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^2) + (8*(b*x + a)^2*c
os(2*b*x + 2*a)^2 + 8*(b*x + a)^2*sin(2*b*x + 2*a)^2 + 4*(b*x + a)^2*cos(2
*b*x + 2*a) + 4*((b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2*a)
)*cos(4*b*x + 4*a) - (2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*
b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4
*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(
cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*(...
```

### 3.293.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4331 vs. $2(53) = 106$.

Time = 1.10 (sec) , antiderivative size = 4331, normalized size of antiderivative = 78.75

$$\int (c + dx)^2 \sec^2(a + bx) \tan(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

output
```
1/2*(b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4*
tan(1/2*a)^4 + 2*b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*d^2*x^2*t
an(1/2*b*x)^2*tan(1/2*a)^4 + b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b^2*c
*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 4*b*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^3 +
 4*b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4 + 4*b*d^2*x*tan(1/2*b*x)^3*tan(1/
2*a)^4 - d^2*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^4 - 2*tan(1/2*b*x)^4*tan(1/2
*a)^2 - 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^2*tan(1/2*a)^4 + ta
n(1/2*b*x)^4 + 8*tan(1/2*b*x)^3*tan(1/2*a) + 20*tan(1/2*b*x)^2*tan(1/2*a)^
2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 2*tan(1/2*b*x)^2 - 8*tan(
1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*b*x)^4*tan(1/2*a)^4 + 2
*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2*tan(1/2*a)^4 + tan(1/2*b*x
)^4 + 4*tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*a)^4 + 2*tan(1/2*b*x)^2 + 2*
tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a)^4 + b^2*d^2*x^2*tan(1/2*b*x)^
4 + 4*b^2*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b^2*c^2*tan(1/2*b*x)^4*t
an(1/2*a)^2 + 4*b*c*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + b^2*d^2*x^2*tan(1/2*a)
^4 + 2*b^2*c^2*tan(1/2*b*x)^2*tan(1/2*a)^4 + 4*b*c*d*tan(1/2*b*x)^3*tan(1/
2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4 - 4*b*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)
+ 8*b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - 24*b*d^2*x*tan(1/2*b*x)^3*tan(
1/2*a)^2 + 2*d^2*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^4 - 2*tan(1/2*b*x)^4*tan
(1/2*a)^2 - 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^2*tan(1/2*a)...
```

### 3.293.9 Mupad [B] (verification not implemented)

Time = 27.52 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.73

$$\int (c+dx)^2\sec^2(a+bx)\tan(a+bx)\,dx = -\frac{\frac{(c+d\,x)^2}{b}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,(c+d\,x)^2}{b}}{2\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1}+\frac{d^2\,x\,2\mathrm{i}}{b^2}+\frac{b\,c^2+2\,b\,c\,d\,x-c\,d\,2\mathrm{i}+b\,d^2\,x^2-d^2\,x\,2\mathrm{i}}{b^2\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}-\frac{d^2\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}+1\right)}{b^3}$$

input
```
int((tan(a + b*x)*(c + d*x)^2)/cos(a + b*x)^2,x)
```

output
```
(d^2*x*2i)/b^2 - ((c + d*x)^2/b - (exp(a*2i + b*x*2i)*(c + d*x)^2)/b)/(2*e
xp(a*2i + b*x*2i) + exp(a*4i + b*x*4i) + 1) + (b*c^2 - c*d*2i - d^2*x*2i +
 b*d^2*x^2 + 2*b*c*d*x)/(b^2*(exp(a*2i + b*x*2i) + 1)) - (d^2*log(exp(a*2i
)*exp(b*x*2i) + 1))/b^3
```

3.293. $\int (c+dx)^2 \sec^2(a+bx) \tan(a+bx)\, dx$

# 3.294 $\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx$

3.294.1 Optimal result . . . 2220
3.294.2 Mathematica [A] (verified) . . . 2220
3.294.3 Rubi [A] (verified) . . . 2221
3.294.4 Maple [A] (verified) . . . 2222
3.294.5 Fricas [A] (verification not implemented) . . . 2222
3.294.6 Sympy [F] . . . 2223
3.294.7 Maxima [B] (verification not implemented) . . . 2223
3.294.8 Giac [B] (verification not implemented) . . . 2224
3.294.9 Mupad [B] (verification not implemented) . . . 2224

## 3.294.1 Optimal result

Integrand size = 20, antiderivative size = 35

$$\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx=\frac{(c+dx)\sec^2(a+bx)}{2b}-\frac{d\tan(a+bx)}{2b^2}$$

output
```
1/2*(d*x+c)*sec(b*x+a)^2/b-1/2*d*tan(b*x+a)/b^2
```

## 3.294.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx=\frac{c\sec^2(a+bx)}{2b}+\frac{dx\sec^2(a+bx)}{2b}-\frac{d\tan(a+bx)}{2b^2}$$

input
```
Integrate[(c + d*x)*Sec[a + b*x]^2*Tan[a + b*x],x]
```

output
```
(c*Sec[a + b*x]^2)/(2*b) + (d*x*Sec[a + b*x]^2)/(2*b) - (d*Tan[a + b*x])/(
2*b^2)
```

### 3.294.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4909, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\tan(a+bx)\sec^2(a+bx)\,dx$$

$$\Big\downarrow \text{4909}$$

$$\frac{(c+dx)\sec^2(a+bx)}{2b}-\frac{d\int \sec^2(a+bx)dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)\sec^2(a+bx)}{2b}-\frac{d\int \csc\left(a+bx+\frac{\pi}{2}\right)^2dx}{2b}$$

$$\Big\downarrow \text{4254}$$

$$\frac{d\int 1d(-\tan(a+bx))}{2b^2}+\frac{(c+dx)\sec^2(a+bx)}{2b}$$

$$\Big\downarrow \text{24}$$

$$\frac{(c+dx)\sec^2(a+bx)}{2b}-\frac{d\tan(a+bx)}{2b^2}$$

input
```
Int[(c + d*x)*Sec[a + b*x]^2*Tan[a + b*x],x]
```

output
```
((c + d*x)*Sec[a + b*x]^2)/(2*b) - (d*Tan[a + b*x])/(2*b^2)
```

#### 3.294.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4909
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[(c + d*x)^m*(Sec[a + b*x]^n/(b*n)), x] -
 Simp[d*(m/(b*n))   Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

### 3.294.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.74

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{-\frac{da}{2b\cos(xb+a)^2}+\frac{c}{2\cos(xb+a)^2}+\frac{d\left(\frac{xb+a}{2\cos(xb+a)^2}-\frac{\tan(xb+a)}{2}\right)}{b}}{b}$ | 61 |
| default | $\frac{-\frac{da}{2b\cos(xb+a)^2}+\frac{c}{2\cos(xb+a)^2}+\frac{d\left(\frac{xb+a}{2\cos(xb+a)^2}-\frac{\tan(xb+a)}{2}\right)}{b}}{b}$ | 61 |
| risch | $\frac{2bdx\,\mathrm{e}^{2i(xb+a)}-id\,\mathrm{e}^{2i(xb+a)}+2bc\,\mathrm{e}^{2i(xb+a)}-id}{b^2\left(\mathrm{e}^{2i(xb+a)}+1\right)^2}$ | 63 |

input
```
int((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2/b*d*a/cos(b*x+a)^2+1/2*c/cos(b*x+a)^2+1/b*d*(1/2*(b*x+a)/cos(b*x
+a)^2-1/2*tan(b*x+a)))
```

### 3.294.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx=\frac{bdx-d\cos(bx+a)\sin(bx+a)+bc}{2\,b^2\cos(bx+a)^2}$$

input
```
integrate((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

output
```
1/2*(b*d*x - d*cos(b*x + a)*sin(b*x + a) + b*c)/(b^2*cos(b*x + a)^2)
```

### 3.294.6 Sympy [F]

$$\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx = \int (c+dx)\tan(a+bx)\sec^2(a+bx)\,dx$$

input
```
integrate((d*x+c)*sec(b*x+a)**2*tan(b*x+a),x)
```

output
```
Integral((c + d*x)*tan(a + b*x)*sec(a + b*x)**2, x)
```

### 3.294.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(31) = 62$.

Time = 0.21 (sec) , antiderivative size = 283, normalized size of antiderivative = 8.09

$$\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx$$

$$= \frac{c\tan(bx+a)^2 - \frac{ad\tan(bx+a)^2}{b} + \frac{2\left(4\,(bx+a)\cos(2\,bx+2\,a)^2+4\,(bx+a)\sin(2\,bx+2\,a)^2+(2\,(bx+a)\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a))\cos(4\,b\cdots\right.}{\left(2\,(2\,\cos(2\,bx+2\,a)+1)\cos(4\,bx+4\,a)+\cos(4\,bx+4\,a)^2+4\,\cos(2\,bx+2\,a)^2+\sin\cdots\right.}}{2\,b}$$

input
```
integrate((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

output
```
1/2*(c*tan(b*x + a)^2 - a*d*tan(b*x + a)^2/b + 2*(4*(b*x + a)*cos(2*b*x +
2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) +
sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*
x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b
*x + 2*a))*d/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4
*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin
(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b))/b
```

### 3.294.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 571 vs. 2(31) = 62.

Time = 0.38 (sec) , antiderivative size = 571, normalized size of antiderivative = 16.31

$$\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx = \frac{bdx\tan\left(\frac{1}{2}bx\right)^4\tan\left(\frac{1}{2}a\right)^4 + bc\tan\left(\frac{1}{2}bx\right)^4\tan\left(\frac{1}{2}a\right)^4 + 2\,bdx\tan\left(\frac{1}{2}bx\right)^4\tan\left(\frac{1}{2}a\right)^2 + 2\,bdx\tan\left(\frac{1}{2}bx\right)^2\tan}{}$$

input
```
integrate((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

output
```
1/2*(b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4*tan(1/2*a)^4 +
 2*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4
 + 2*b*c*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + 2
*b*c*tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + b*d*x
*tan(1/2*b*x)^4 + 4*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*d*x*tan(1/2*a)^4
 + b*c*tan(1/2*b*x)^4 - 2*d*tan(1/2*b*x)^4*tan(1/2*a) + 4*b*c*tan(1/2*b*x)
^2*tan(1/2*a)^2 - 12*d*tan(1/2*b*x)^3*tan(1/2*a)^2 - 12*d*tan(1/2*b*x)^2*t
an(1/2*a)^3 + b*c*tan(1/2*a)^4 - 2*d*tan(1/2*b*x)*tan(1/2*a)^4 + 2*b*d*x*t
an(1/2*b*x)^2 + 2*b*d*x*tan(1/2*a)^2 + 2*b*c*tan(1/2*b*x)^2 + 2*d*tan(1/2*
b*x)^3 + 12*d*tan(1/2*b*x)^2*tan(1/2*a) + 2*b*c*tan(1/2*a)^2 + 12*d*tan(1/
2*b*x)*tan(1/2*a)^2 + 2*d*tan(1/2*a)^3 + b*d*x + b*c - 2*d*tan(1/2*b*x) -
2*d*tan(1/2*a))/(b^2*tan(1/2*b*x)^4*tan(1/2*a)^4 - 2*b^2*tan(1/2*b*x)^4*ta
n(1/2*a)^2 - 8*b^2*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*b^2*tan(1/2*b*x)^2*tan(
1/2*a)^4 + b^2*tan(1/2*b*x)^4 + 8*b^2*tan(1/2*b*x)^3*tan(1/2*a) + 20*b^2*t
an(1/2*b*x)^2*tan(1/2*a)^2 + 8*b^2*tan(1/2*b*x)*tan(1/2*a)^3 + b^2*tan(1/2
*a)^4 - 2*b^2*tan(1/2*b*x)^2 - 8*b^2*tan(1/2*b*x)*tan(1/2*a) - 2*b^2*tan(1
/2*a)^2 + b^2)
```

### 3.294.9 Mupad [B] (verification not implemented)

Time = 26.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.51

$$\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx = -\frac{d\,1\mathrm{i} + \mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,(-b\,(2\,c+2\,d\,x)+d\,1\mathrm{i})}{b^2\,(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1)^2}$$

input
```
int((tan(a + b*x)*(c + d*x))/cos(a + b*x)^2,x)
```

output
```
-(d*1i + exp(a*2i + b*x*2i)*(d*1i - b*(2*c + 2*d*x)))/(b^2*(exp(a*2i + b*x
*2i) + 1)^2)
```

3.294. $\int (c + dx) \sec^2(a + bx) \tan(a + bx) \, dx$

# 3.295 $\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx$

3.295.1 Optimal result . . . . . 2226
3.295.2 Mathematica [N/A] . . . . . 2226
3.295.3 Rubi [N/A] . . . . . 2227
3.295.4 Maple [N/A] (verified) . . . . . 2227
3.295.5 Fricas [N/A] . . . . . 2228
3.295.6 Sympy [N/A] . . . . . 2228
3.295.7 Maxima [N/A] . . . . . 2228
3.295.8 Giac [N/A] . . . . . 2229
3.295.9 Mupad [N/A] . . . . . 2230

## 3.295.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \operatorname{Int}\left(\frac{\sec^2(a+bx)\tan(a+bx)}{c+dx},x\right)$$

output
```
CannotIntegrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x)
```

## 3.295.2 Mathematica [N/A]

Not integrable

Time = 8.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x),x]
```

output
```
Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]
```

## 3.295.3 Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$
$$\Big\downarrow \ 7299$$
$$\int \frac{\tan(a+bx)\sec^2(a+bx)}{c+dx}dx$$

input
```
Int[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x),x]
```

output
```
$Aborted
```

### 3.295.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.295.4 Maple [N/A] (verified)

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sec\left(xb+a\right)^{2}\tan\left(xb+a\right)}{dx+c}dx$$

input
```
int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x)
```

output
```
int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x)
```

### 3.295.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)^2\tan(bx+a)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)^2*tan(b*x + a)/(d*x + c), x)
```

### 3.295.6 Sympy [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\tan(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

input
```
integrate(sec(b*x+a)**2*tan(b*x+a)/(d*x+c),x)
```

output
```
Integral(tan(a + b*x)*sec(a + b*x)**2/(c + d*x), x)
```

### 3.295.7 Maxima [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 1090, normalized size of antiderivative = 49.55

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)^2\tan(bx+a)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
 (2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
+ 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + 2*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*
c^2*d^2 + (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(4*b*x + 4*a)^2 +
 4*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(2*b*x + 2*a)^2 + (b^2*d
^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^4*x^2
+ 2*b^2*c*d^3*x + b^2*c^2*d^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*
d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^4*x^2
 + 2*b^2*c*d^3*x + b^2*c^2*d^2 + 2*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*
d^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^4*x^2 + 2*b^2*c*d^3*x +
 b^2*c^2*d^2)*cos(2*b*x + 2*a))*integrate(sin(2*b*x + 2*a)/(b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^
2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b
^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)), x) + (d*cos(2*b
*x + 2*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a) + d)*sin(4*b*x + 4*a) + d*sin
(2*b*x + 2*a))/(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*
c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x
+ 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin...
```

### 3.295.8 Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)^2\tan(bx+a)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^2*tan(b*x + a)/(d*x + c), x)
```

### 3.295.9 Mupad [N/A]

Not integrable

Time = 24.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\tan(a+b\,x)}{\cos{(a+b\,x)}^2\,(c+d\,x)}\,dx$$

input
```
int(tan(a + b*x)/(cos(a + b*x)^2*(c + d*x)),x)
```

output
```
int(tan(a + b*x)/(cos(a + b*x)^2*(c + d*x)), x)
```

# 3.296 $\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$

3.296.1 Optimal result . . . 2231
3.296.2 Mathematica [N/A] . . . 2231
3.296.3 Rubi [N/A] . . . 2232
3.296.4 Maple [N/A] (verified) . . . 2232
3.296.5 Fricas [N/A] . . . 2233
3.296.6 Sympy [N/A] . . . 2233
3.296.7 Maxima [N/A] . . . 2233
3.296.8 Giac [N/A] . . . 2234
3.296.9 Mupad [N/A] . . . 2235

## 3.296.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx=\text{Int}\left(\frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2},x\right)$$

output
```
CannotIntegrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x)
```

## 3.296.2 Mathematica [N/A]

Not integrable

Time = 12.79 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx=\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]
```

## 3.296.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \frac{\tan(a+bx)\sec^2(a+bx)}{(c+dx)^2}dx$$

input
```
Int[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

### 3.296.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.296.4 Maple [N/A] (verified)

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sec(xb+a)^2 \tan(xb+a)}{(dx+c)^2}dx$$

input
```
int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x)
```

output
```
int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x)
```

### 3.296.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec(bx+a)^2\tan(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)^2*tan(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.296.6 Sympy [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(sec(b*x+a)**2*tan(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(tan(a + b*x)*sec(a + b*x)**2/(c + d*x)**2, x)
```

### 3.296.7 Maxima [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 1396, normalized size of antiderivative = 63.45

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec(bx+a)^2\tan(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
2*(2*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 2*(b*d*x + b*c)*sin(2*b*x + 2*a)^2
 + ((b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
+ (b*d*x + b*c)*cos(2*b*x + 2*a) + 3*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^
2*c^2*d^3*x + b^2*c^3*d^2 + (b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3
*x + b^2*c^3*d^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 +
3*b^2*c^2*d^3*x + b^2*c^3*d^2)*cos(2*b*x + 2*a)^2 + (b^2*d^5*x^3 + 3*b^2*c
*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^5*
x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*sin(4*b*x + 4*a)*si
n(2*b*x + 2*a) + 4*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*
c^3*d^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2
*d^3*x + b^2*c^3*d^2 + 2*(b^2*d^5*x^3 + 3*b^2*c*d^4*x^2 + 3*b^2*c^2*d^3*x
+ b^2*c^3*d^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^5*x^3 + 3*b^2
*c*d^4*x^2 + 3*b^2*c^2*d^3*x + b^2*c^3*d^2)*cos(2*b*x + 2*a))*integrate(si
n(2*b*x + 2*a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*
c^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4
*b^2*c^3*d*x + b^2*c^4)*cos(2*b*x + 2*a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^
3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(2*b*x + 2*a)^2 + 2*(b
^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4
)*cos(2*b*x + 2*a)), x) + (d*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(2*b*x +
2*a) + d)*sin(4*b*x + 4*a) + d*sin(2*b*x + 2*a))/(b^2*d^3*x^3 + 3*b^2*c...
```

## 3.296.8 Giac [N/A]

Not integrable

Time = 6.74 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec(bx+a)^2\tan(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^2*tan(b*x + a)/(d*x + c)^2, x)
```

### 3.296.9 Mupad [N/A]

Not integrable

Time = 26.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(a+b\,x)}{\cos\left(a+b\,x\right)^2\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(tan(a + b*x)/(cos(a + b*x)^2*(c + d*x)^2),x)
```

output
```
int(tan(a + b*x)/(cos(a + b*x)^2*(c + d*x)^2), x)
```

# 3.297 $\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx$

3.297.1 Optimal result . . . . . 2236
3.297.2 Mathematica [N/A] . . . . . 2236
3.297.3 Rubi [N/A] . . . . . 2237
3.297.4 Maple [N/A] (verified) . . . . . 2238
3.297.5 Fricas [N/A] . . . . . 2238
3.297.6 Sympy [N/A] . . . . . 2239
3.297.7 Maxima [N/A] . . . . . 2239
3.297.8 Giac [N/A] . . . . . 2239
3.297.9 Mupad [N/A] . . . . . 2240

## 3.297.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx = -\text{Int}((c+dx)^m \sec(a+bx),x) + \text{Int}\left((c+dx)^m \sec^3(a+bx),x\right)$$

output
```
-Unintegrable((d*x+c)^m*sec(b*x+a),x)+Unintegrable((d*x+c)^m*sec(b*x+a)^3,
x)
```

## 3.297.2 Mathematica [N/A]

Not integrable

Time = 51.87 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx = \int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^2, x]
```

### 3.297.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4913, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(a+bx)\sec(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{4913}$$

$$\int (c+dx)^m\sec^3(a+bx)dx-\int (c+dx)^m\sec(a+bx)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m\csc\left(a+bx+\frac{\pi}{2}\right)^3dx-\int (c+dx)^m\csc\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4680}$$

$$\int (c+dx)^m\sec^3(a+bx)dx-\int (c+dx)^m\sec(a+bx)dx$$

input
```
Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
$Aborted
```

#### 3.297.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4913
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

### 3.297.4 Maple [N/A] (verified)

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \sec(xb+a)\tan(xb+a)^2\,dx$$

input
```
int((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x)
```

### 3.297.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\tan(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*sec(b*x + a)*tan(b*x + a)^2, x)
```

### 3.297.6 Sympy [N/A]

Not integrable

Time = 9.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx = \int (c+dx)^m \tan^2(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*sec(b*x+a)*tan(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*tan(a + b*x)**2*sec(a + b*x), x)
```

### 3.297.7 Maxima [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\tan(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a)^2, x)
```

### 3.297.8 Giac [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx = \int (dx+c)^m \sec(bx+a)\tan(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a)^2, x)
```

### 3.297.9 Mupad [N/A]

Not integrable

Time = 28.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx = \int \frac{\tan(a+b\,x)^2\,(c+d\,x)^m}{\cos(a+b\,x)}\,dx$$

input
```
int((tan(a + b*x)^2*(c + d*x)^m)/cos(a + b*x),x)
```

output
```
int((tan(a + b*x)^2*(c + d*x)^m)/cos(a + b*x), x)
```

# 3.298 $\int (c+dx)^3 \sec(a+bx) \tan^2(a+bx) \, dx$


3.298.1 Optimal result . . . 2241
3.298.2 Mathematica [A] (verified) . . . 2242
3.298.3 Rubi [A] (verified) . . . 2242
3.298.4 Maple [B] (verified) . . . 2249
3.298.5 Fricas [B] (verification not implemented) . . . 2250
3.298.6 Sympy [F] . . . 2250
3.298.7 Maxima [B] (verification not implemented) . . . 2251
3.298.8 Giac [F] . . . 2251
3.298.9 Mupad [F(-1)] . . . 2252


## 3.298.1 Optimal result

Integrand size = 22, antiderivative size = 337

$$\begin{aligned}\int (c+dx)^3 \sec(a+bx) \tan^2(a+bx) \, dx &= -\frac{6id^2(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b^3}+\frac{i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b} \\ &\quad+\frac{3id^3\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^4}-\frac{3id(c+dx)^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2} \\ &\quad-\frac{3id^3\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^4}+\frac{3id(c+dx)^2\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2} \\ &\quad+\frac{3d^2(c+dx)\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}-\frac{3d^2(c+dx)\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3} \\ &\quad+\frac{3id^3\operatorname{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^4}-\frac{3id^3\operatorname{PolyLog}\left(4,ie^{i(a+bx)}\right)}{b^4} \\ &\quad-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\sec(a+bx)\tan(a+bx)}{2b}\end{aligned}$$

output
```
-6*I*d^2*(d*x+c)*arctan(exp(I*(b*x+a)))/b^3+I*(d*x+c)^3*arctan(exp(I*(b*x+
a)))/b+3*I*d^3*polylog(2,-I*exp(I*(b*x+a)))/b^4-3/2*I*d*(d*x+c)^2*polylog(
2,-I*exp(I*(b*x+a)))/b^2-3*I*d^3*polylog(2,I*exp(I*(b*x+a)))/b^4+3/2*I*d*(
d*x+c)^2*polylog(2,I*exp(I*(b*x+a)))/b^2+3*d^2*(d*x+c)*polylog(3,-I*exp(I*
(b*x+a)))/b^3-3*d^2*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))/b^3+3*I*d^3*polylo
g(4,-I*exp(I*(b*x+a)))/b^4-3*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4-3/2*d*(
d*x+c)^2*sec(b*x+a)/b^2+1/2*(d*x+c)^3*sec(b*x+a)*tan(b*x+a)/b
```

### 3.298.2 Mathematica [A] (verified)

Time = 3.75 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.57

$$\int (c+dx)^3\sec(a+bx)\tan^2(a+bx)\,dx$$
$$= \frac{2ib^3c^3\arctan\left(e^{i(a+bx)}\right)-12ibcd^2\arctan\left(e^{i(a+bx)}\right)-3b^3c^2dx\log\left(1-ie^{i(a+bx)}\right)+6bd^3x\log\left(1-ie^{i(a+bx)}\right)}{}$$

input
```
Integrate[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
((2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] - (12*I)*b*c*d^2*ArcTan[E^(I*(a + b
*x))] - 3*b^3*c^2*d*x*Log[1 - I*E^(I*(a + b*x))] + 6*b*d^3*x*Log[1 - I*E^(
I*(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - b^3*d^3*x^3*L
og[1 - I*E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] - 6*b
*d^3*x*Log[1 + I*E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*
x))] + b^3*d^3*x^3*Log[1 + I*E^(I*(a + b*x))] - (3*I)*d*(-2*d^2 + b^2*(c +
 d*x)^2)*PolyLog[2, (-I)*E^(I*(a + b*x))] + (3*I)*d*(-2*d^2 + b^2*(c + d*x
)^2)*PolyLog[2, I*E^(I*(a + b*x))] + 6*b*c*d^2*PolyLog[3, (-I)*E^(I*(a + b
*x))] + 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3,
I*E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x))] + (6*I)*d^3*P
olyLog[4, (-I)*E^(I*(a + b*x))] - (6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))]
+ b^2*(c + d*x)^2*Sec[a + b*x]*(-3*d + b*(c + d*x)*Tan[a + b*x]))/(2*b^4)
```

### 3.298.3 Rubi [A] (verified)

Time = 2.07 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.72, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {4913, 3042, 4669, 3011, 4674, 3042, 4669, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\tan^2(a+bx)\sec(a+bx)\,dx$$
$$\Big\downarrow \text{4913}$$
$$\int (c+dx)^3\sec^3(a+bx)dx-\int (c+dx)^3\sec(a+bx)dx$$
$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right)^3 dx-\int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right) dx$$

$$\Big\downarrow \text{ 4669}$$

$$\frac{3d\int (c+dx)^2 \log\left(1-ie^{i(a+bx)}\right)dx}{b}-\frac{3d\int (c+dx)^2 \log\left(1+ie^{i(a+bx)}\right)dx}{b}+\int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right)^3 dx+\frac{2i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{ 3011}$$

$$-\frac{3d\left(\frac{i(c+dx)^2 \text{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{2id\int (c+dx)\text{PolyLog}(2,-ie^{i(a+bx)})dx}{b}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2 \text{PolyLog}(2,ie^{i(a+bx)})}{b}-\frac{2id\int (c+dx)\text{PolyLog}(2,ie^{i(a+bx)})dx}{b}\right)}{b}+\int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right)^3 dx+\frac{2i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{ 4674}$$

$$\frac{3d^2\int (c+dx)\sec(a+bx)dx}{b^2}-\frac{3d\left(\frac{i(c+dx)^2 \text{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{2id\int (c+dx)\text{PolyLog}(2,-ie^{i(a+bx)})dx}{b}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2 \text{PolyLog}(2,ie^{i(a+bx)})}{b}-\frac{2id\int (c+dx)\text{PolyLog}(2,ie^{i(a+bx)})dx}{b}\right)}{b}+\frac{1}{2}\int (c+dx)^3 \sec(a+bx)dx+\frac{2i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2 \sec(a+bx)}{2b^2}+\frac{(c+dx)^3 \tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3d^2\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)dx}{b^2}-\frac{3d\left(\frac{i(c+dx)^2 \text{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{2id\int (c+dx)\text{PolyLog}(2,-ie^{i(a+bx)})dx}{b}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2 \text{PolyLog}(2,ie^{i(a+bx)})}{b}-\frac{2id\int (c+dx)\text{PolyLog}(2,ie^{i(a+bx)})dx}{b}\right)}{b}+\frac{1}{2}\int (c+dx)^3 \csc\left(a+bx+\frac{\pi}{2}\right)dx+\frac{2i(c+dx)^3 \arctan\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2 \sec(a+bx)}{2b^2}+\frac{(c+dx)^3 \tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{ 4669}$$

3.298. $\int (c+dx)^3 \sec(a+bx)\tan^2(a+bx)\,dx$

$$\frac{3d^2\left(-\frac{d\int \log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{d\int \log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}\right)}{b^2}+\frac{1}{2}\left(-\frac{3d\int (c+dx)^2\log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{3d\int (c+dx)^2\log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}\right)-\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\int (c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{2id\int (c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{2715}$$

$$\frac{3d^2\left(\frac{id\int e^{-i(a+bx)}\log\left(1-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{id\int e^{-i(a+bx)}\log\left(1+ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}\right)}{b^2}+\frac{1}{2}\left(-\frac{3d\int (c+dx)^2\log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{3d\int (c+dx)^2\log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}\right)-\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\int (c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{2id\int (c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{2838}$$

$$\frac{1}{2}\left(-\frac{3d\int (c+dx)^2\log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{3d\int (c+dx)^2\log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}\right)-\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{2id\int (c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{2id\int (c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)dx}{b}\right)}{b}+\frac{3d^2\left(-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}\right)}{b^2}+\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{3011}$$

## 3.298. $\int (c+dx)^3\sec(a+bx)\tan^2(a+bx)\,dx$

$$\frac{1}{2}\left(\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{2id\int(c+dx)\operatorname{PolyLog}(2,-ie^{i(a+bx)})dx}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}(2,ie^{i(a+bx)})}{b}-\frac{2id\int(c+dx)\right.}{b}\right.$$

$$\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{2id\int(c+dx)\operatorname{PolyLog}(2,-ie^{i(a+bx)})dx}{b}\right)}{b}+$$

$$\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}(2,ie^{i(a+bx)})}{b}-\frac{2id\int(c+dx)\operatorname{PolyLog}(2,ie^{i(a+bx)})dx}{b}\right)}{b}+$$

$$\frac{3d^2\left(-\frac{2i(c+dx)\arctan(e^{i(a+bx)})}{b}+\frac{id\operatorname{PolyLog}(2,-ie^{i(a+bx)})}{b^2}-\frac{id\operatorname{PolyLog}(2,ie^{i(a+bx)})}{b^2}\right)}{b^2}+$$

$$\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{ 7163}$$

$$\frac{1}{2}\left(\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{2id\left(\frac{id\int\operatorname{PolyLog}(3,-ie^{i(a+bx)})dx}{b}-\frac{i(c+dx)\operatorname{PolyLog}(3,-ie^{i(a+bx)})}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}}{b}\right.}{}\right.$$

$$\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{2id\left(\frac{id\int\operatorname{PolyLog}(3,-ie^{i(a+bx)})dx}{b}-\frac{i(c+dx)\operatorname{PolyLog}(3,-ie^{i(a+bx)})}{b}\right)}{b}\right)}{b}+$$

$$\frac{3d\left(\frac{i(c+dx)^2 \operatorname{PolyLog}(2,ie^{i(a+bx)})}{b}-\frac{2id\left(\frac{id\int\operatorname{PolyLog}(3,ie^{i(a+bx)})dx}{b}-\frac{i(c+dx)\operatorname{PolyLog}(3,ie^{i(a+bx)})}{b}\right)}{b}\right)}{b}+$$

$$\frac{3d^2\left(-\frac{2i(c+dx)\arctan(e^{i(a+bx)})}{b}+\frac{id\operatorname{PolyLog}(2,-ie^{i(a+bx)})}{b^2}-\frac{id\operatorname{PolyLog}(2,ie^{i(a+bx)})}{b^2}\right)}{b^2}+$$

$$\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{ 2720}$$

---

3.298. $\int (c+dx)^3\sec(a+bx)\tan^2(a+bx)\,dx$

$$\frac{1}{2}\left(\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-\frac{3d\left(\frac{i}{}\right.}{}\right.$$

$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}+$$

$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}(2,ie^{i(a+bx)})}{b}-\frac{2id\left(\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}+$$

$$\frac{3d^2\left(-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{id\operatorname{PolyLog}(2,-ie^{i(a+bx)})}{b^2}-\frac{id\operatorname{PolyLog}(2,ie^{i(a+bx)})}{b^2}\right)}{b^2}+$$

$$\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \; 7143$$

$$\frac{3d^2\left(-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{id\operatorname{PolyLog}(2,-ie^{i(a+bx)})}{b^2}-\frac{id\operatorname{PolyLog}(2,ie^{i(a+bx)})}{b^2}\right)}{b^2}+$$

$$\frac{1}{2}\left(-\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{2id\left(\frac{d\operatorname{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-ie^{i(a\right.}{b}\right.}{b}\right.}{b}\right.$$

$$\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b}-$$

$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{2id\left(\frac{d\operatorname{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}+$$

$$\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}(2,ie^{i(a+bx)})}{b}-\frac{2id\left(\frac{d\operatorname{PolyLog}\left(4,ie^{i(a+bx)}\right)}{b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b}\right)}{b}\right)}{b}-$$

$$\frac{3d(c+dx)^2\sec(a+bx)}{2b^2}+\frac{(c+dx)^3\tan(a+bx)\sec(a+bx)}{2b}$$

3.298. $\int (c+dx)^3\sec(a+bx)\tan^2(a+bx)\,dx$

input
```
Int[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
((2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b + (3*d^2*(((-2*I)*(c + d*x)*
ArcTan[E^(I*(a + b*x))])/b + (I*d*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 -
(I*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2))/b^2 - (3*d*((I*(c + d*x)^2*PolyL
og[2, (-I)*E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(c + d*x)*PolyLog[3, (-I)
*E^(I*(a + b*x))])/b + (d*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^2))/b))/b +
(3*d*((I*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(c
 + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b + (d*PolyLog[4, I*E^(I*(a + b*x))
])/b^2))/b))/b + (((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b + (3*d*((
I*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b - ((2*I)*d*(((-I)*(c + d
*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b + (d*PolyLog[4, (-I)*E^(I*(a + b*x
))])/b^2))/b))/b - (3*d*((I*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b -
 ((2*I)*d*(((-I)*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b + (d*PolyLog[4
, I*E^(I*(a + b*x))])/b^2))/b))/b)/2 - (3*d*(c + d*x)^2*Sec[a + b*x])/(2*b
^2) + ((c + d*x)^3*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

### 3.298.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4913
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

## 3.298. $\int (c+dx)^3 \sec(a+bx) \tan^2(a+bx)\,dx$

### 3.298.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1126 vs. 2(293) = 586.

Time = 2.01 (sec) , antiderivative size = 1127, normalized size of antiderivative = 3.34

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1127 |

input
```
int((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-3*I*d^3*polylog(2,I*exp(I*(b*x+a)))/b^4-3*I*d^3*polylog(4,I*exp(I*(b*x+a)
))/b^4-3/2/b*c*d^2*ln(1-I*exp(I*(b*x+a)))*x^2-3/2/b^3*a^2*c*d^2*ln(1+I*exp
(I*(b*x+a)))-3/2/b*c^2*d*ln(1-I*exp(I*(b*x+a)))*x-3/2/b^2*c^2*d*ln(1-I*exp
(I*(b*x+a)))*a+3*I/b^3*c*d^2*a^2*arctan(exp(I*(b*x+a)))+3*I*d^3*polylog(2,
-I*exp(I*(b*x+a)))/b^4+3*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4+3*I/b^2*c*
d^2*polylog(2,I*exp(I*(b*x+a)))*x-3*I/b^2*c*d^2*polylog(2,-I*exp(I*(b*x+a)
))*x-3*I/b^2*c^2*d*a*arctan(exp(I*(b*x+a)))+3/b^3*d^3*polylog(3,-I*exp(I*(
b*x+a)))*x+1/2/b*d^3*ln(1+I*exp(I*(b*x+a)))*x^3-1/2/b*d^3*ln(1-I*exp(I*(b*
x+a)))*x^3-3/b^3*d^3*ln(1+I*exp(I*(b*x+a)))*x+3/b^4*d^3*ln(1-I*exp(I*(b*x+
a)))*a+3/b^3*d^3*ln(1-I*exp(I*(b*x+a)))*x-3/b^4*d^3*ln(1+I*exp(I*(b*x+a)))
*a-3/b^3*d^3*polylog(3,I*exp(I*(b*x+a)))*x+3/b^3*c*d^2*polylog(3,-I*exp(I*
(b*x+a)))-1/2/b^4*a^3*d^3*ln(1-I*exp(I*(b*x+a)))-3/b^3*c*d^2*polylog(3,I*e
xp(I*(b*x+a)))+I/b*c^3*arctan(exp(I*(b*x+a)))+3/2/b*c*d^2*ln(1+I*exp(I*(b*
x+a)))*x^2+3/2/b^3*a^2*c*d^2*ln(1-I*exp(I*(b*x+a)))+3/2/b*c^2*d*ln(1+I*exp
(I*(b*x+a)))*x+3/2/b^2*c^2*d*ln(1+I*exp(I*(b*x+a)))*a-I/b^4*d^3*a^3*arctan
(exp(I*(b*x+a)))+6*I/b^4*d^3*a*arctan(exp(I*(b*x+a)))+3/2*I/b^2*c^2*d*poly
log(2,I*exp(I*(b*x+a)))-6*I/b^3*c*d^2*arctan(exp(I*(b*x+a)))+3/2*I/b^2*d^3
*polylog(2,I*exp(I*(b*x+a)))*x^2-3/2*I/b^2*d^3*polylog(2,-I*exp(I*(b*x+a))
)*x^2-3/2*I/b^2*c^2*d*polylog(2,-I*exp(I*(b*x+a)))-I/b^2/(exp(2*I*(b*x+a))
+1)^2*(d^3*x^3*b*exp(3*I*(b*x+a))+3*c*d^2*x^2*b*exp(3*I*(b*x+a))+3*c^2*...
```

### 3.298.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1315 vs. 2(273) = 546.

Time = 0.33 (sec) , antiderivative size = 1315, normalized size of antiderivative = 3.90

$$\int (c+dx)^3 \sec(a+bx)\tan^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*(-6*I*d^3*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) + sin(b*x + a)) - 6
*I*d^3*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 6*I*d^3*
cos(b*x + a)^2*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) + 6*I*d^3*cos(b*
x + a)^2*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 -
2*I*b^2*c*d^2*x - I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x +
a) + sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d + 2
*I*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) - 3*(I*b^2*d^3
*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d - 2*I*d^3)*cos(b*x + a)^2*dilog(-I*co
s(b*x + a) + sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^
2*d - 2*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^3
*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b*x + a)
^2*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(
a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b*x + a)^2*log(cos(b*x + a) - I*si
n(b*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2*log(I
*cos(b*x + a) + sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b
^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*co
s(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3
*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*
d - 2*b*d^3)*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) ...
```

### 3.298.6 Sympy [F]

$$\int (c+dx)^3 \sec(a+bx)\tan^2(a+bx)\,dx = \int (c+dx)^3 \tan^2(a+bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*sec(b*x+a)*tan(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**3*tan(a + b*x)**2*sec(a + b*x), x)
```

### 3.298.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3831 vs. 2(273) = 546.

Time = 1.37 (sec) , antiderivative size = 3831, normalized size of antiderivative = 11.37

$$\int (c+dx)^3 \sec(a+bx)\tan^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(c^3*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - l
og(sin(b*x + a) - 1)) - 3*a*c^2*d*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + l
og(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*sin(b*x +
 a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/
b^2 - a^3*d^3*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1)
 - log(sin(b*x + a) - 1))/b^3 - 4*(2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^
3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 -
2)*d^3)*(b*x + a) + ((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 -
a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a)
)*cos(4*b*x + 4*a) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2
 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x +
 a))*cos(2*b*x + 2*a) + (I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + 3*(
I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2*I*a*b*c*d^2 + (I*a^2
 - 2*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 2*(I*(b*x + a)^3*d^3 - 6*I*b*c*
d^2 + 6*I*a*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 3*(I*b^2*c^2*d - 2
*I*a*b*c*d^2 + (I*a^2 - 2*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos
(b*x + a), sin(b*x + a) + 1) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 +
3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d
^3)*(b*x + a) + ((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^
3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))...
```

### 3.298.8 Giac [F]

$$\int (c+dx)^3 \sec(a+bx)\tan^2(a+bx)\,dx = \int (dx+c)^3 \sec(bx+a)\tan(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*sec(b*x + a)*tan(b*x + a)^2, x)
```

### 3.298.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3\sec(a+bx)\tan^2(a+bx)\,dx=\text{Hanged}$$

input
```
int((tan(a + b*x)^2*(c + d*x)^3)/cos(a + b*x),x)
```

output
```
\text{Hanged}
```

# 3.299 $\int (c+dx)^2 \sec(a+bx) \tan^2(a+bx)\,dx$

3.299.1 Optimal result . . . 2253
3.299.2 Mathematica [B] (verified) . . . 2254
3.299.3 Rubi [A] (verified) . . . 2255
3.299.4 Maple [B] (verified) . . . 2259
3.299.5 Fricas [B] (verification not implemented) . . . 2260
3.299.6 Sympy [F] . . . 2261
3.299.7 Maxima [B] (verification not implemented) . . . 2262
3.299.8 Giac [F] . . . 2262
3.299.9 Mupad [F(-1)] . . . 2263

## 3.299.1 Optimal result

Integrand size = 22, antiderivative size = 193

$$\begin{aligned}\int (c+dx)^2 \sec(a+bx) \tan^2(a+bx)\,dx &= \frac{i(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b} \\ &+ \frac{d^2 \operatorname{arctanh}(\sin(a+bx))}{b^3} \\ &- \frac{id(c+dx) \operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} \\ &+ \frac{id(c+dx) \operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} \\ &+ \frac{d^2 \operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} \\ &- \frac{d^2 \operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3} \\ &- \frac{d(c+dx)\sec(a+bx)}{b^2} \\ &+ \frac{(c+dx)^2 \sec(a+bx)\tan(a+bx)}{2b}\end{aligned}$$

output

```
I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b+d^2*arctanh(sin(b*x+a))/b^3-I*d*(d*x+
c)*polylog(2,-I*exp(I*(b*x+a)))/b^2+I*d*(d*x+c)*polylog(2,I*exp(I*(b*x+a))
)/b^2+d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3-d^2*polylog(3,I*exp(I*(b*x+a)))
/b^3-d*(d*x+c)*sec(b*x+a)/b^2+1/2*(d*x+c)^2*sec(b*x+a)*tan(b*x+a)/b
```

### 3.299.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 526 vs. $2(193) = 386$.

Time = 7.70 (sec) , antiderivative size = 526, normalized size of antiderivative = 2.73

$$\int (c+dx)^2 \sec(a+bx)\tan^2(a+bx)\,dx$$

$$= \frac{ibc^2\arctan\left(e^{i(a+bx)}\right) - \frac{2id^2\arctan\left(e^{i(a+bx)}\right)}{b} - bcdx\log\left(1-ie^{i(a+bx)}\right) - \frac{1}{2}bd^2x^2\log\left(1-ie^{i(a+bx)}\right) + bcdx\log\left(\cdots\right.}{\cdots}$$

$$- \frac{d(c+dx)\sec(a)}{b^2} + \frac{c^2+2cdx+d^2x^2}{4b\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)^2}$$

$$+ \frac{-cd\sin\left(\frac{bx}{2}\right)-d^2x\sin\left(\frac{bx}{2}\right)}{b^2\left(\cos\left(\frac{a}{2}\right)-\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)} + \frac{-c^2-2cdx-d^2x^2}{4b\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)+\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)^2}$$

$$+ \frac{cd\sin\left(\frac{bx}{2}\right)+d^2x\sin\left(\frac{bx}{2}\right)}{b^2\left(\cos\left(\frac{a}{2}\right)+\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)+\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}$$

input
```
Integrate[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
(I*b*c^2*ArcTan[E^(I*(a + b*x))] - ((2*I)*d^2*ArcTan[E^(I*(a + b*x))])/b -
 b*c*d*x*Log[1 - I*E^(I*(a + b*x))] - (b*d^2*x^2*Log[1 - I*E^(I*(a + b*x))
])/2 + b*c*d*x*Log[1 + I*E^(I*(a + b*x))] + (b*d^2*x^2*Log[1 + I*E^(I*(a +
 b*x))])/2 - I*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] + I*d*(c + d*x
)*PolyLog[2, I*E^(I*(a + b*x))] + (d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])/b
 - (d^2*PolyLog[3, I*E^(I*(a + b*x))])/b)/b^2 - (d*(c + d*x)*Sec[a])/b^2 +
 (c^2 + 2*c*d*x + d^2*x^2)/(4*b*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])^
2) + (-(c*d*Sin[(b*x)/2]) - d^2*x*Sin[(b*x)/2])/(b^2*(Cos[a/2] - Sin[a/2])
*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) + (-c^2 - 2*c*d*x - d^2*x^2)/(
4*b*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2])^2) + (c*d*Sin[(b*x)/2] + d^2
*x*Sin[(b*x)/2])/(b^2*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (b*x)/2] + Sin[a/2
+ (b*x)/2]))
```

### 3.299.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.82, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {4913, 3042, 4669, 3011, 2720, 4674, 3042, 4257, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \tan^2(a+bx)\sec(a+bx)\,dx$$

$$\Big\downarrow \text{ 4913}$$

$$\int (c+dx)^2\sec^3(a+bx)dx-\int (c+dx)^2\sec(a+bx)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)^3dx-\int (c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 4669}$$

$$\frac{2d\int (c+dx)\log\left(1-ie^{i(a+bx)}\right)dx}{b}-\frac{2d\int (c+dx)\log\left(1+ie^{i(a+bx)}\right)dx}{b}+\int (c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)^3dx+\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{ 3011}$$

$$-\frac{2d\left(\frac{i(c+dx)\text{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{id\int \text{PolyLog}(2,-ie^{i(a+bx)})dx}{b}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\text{PolyLog}(2,ie^{i(a+bx)})}{b}-\frac{id\int \text{PolyLog}(2,ie^{i(a+bx)})dx}{b}\right)}{b}+\int (c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)^3dx+\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{ 2720}$$

$$-\frac{2d\left(\frac{i(c+dx)\text{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{d\int e^{-i(a+bx)}\text{PolyLog}(2,-ie^{i(a+bx)})de^{i(a+bx)}}{b^2}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\text{PolyLog}(2,ie^{i(a+bx)})}{b}-\frac{d\int e^{-i(a+bx)}\text{PolyLog}(2,ie^{i(a+bx)})de^{i(a+bx)}}{b^2}\right)}{b}+\int (c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)^3dx+\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \text{ 4674}$$

$$-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+\frac{d^2\int\sec(a+bx)dx}{b^2}+\frac{1}{2}\int(c+dx)^2\sec(a+bx)dx+\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{d(c+dx)\sec(a+bx)}{b^2}+\frac{(c+dx)^2\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+\frac{d^2\int\csc\left(a+bx+\frac{\pi}{2}\right)dx}{b^2}+\frac{1}{2}\int(c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)dx+\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{d(c+dx)\sec(a+bx)}{b^2}+\frac{(c+dx)^2\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+\frac{1}{2}\int(c+dx)^2\csc\left(a+bx+\frac{\pi}{2}\right)dx+\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{d^2\text{arctanh}(\sin(a+bx))}{b^3}-\frac{d(c+dx)\sec(a+bx)}{b^2}+\frac{(c+dx)^2\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{4669}$$

$$\frac{1}{2}\left(-\frac{2d\int(c+dx)\log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{2d\int(c+dx)\log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}\right)-\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b}-\frac{d\int e^{-i(a+bx)}\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}\right)}{b}+\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{d^2\text{arctanh}(\sin(a+bx))}{b^3}-\frac{d(c+dx)\sec(a+bx)}{b^2}+\frac{(c+dx)^2\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{3011}$$

---

3.299. $\int(c+dx)^2\sec(a+bx)\tan^2(a+bx)\,dx$

$$\frac{1}{2}\left(\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{id\int \operatorname{PolyLog}(2,-ie^{i(a+bx)})dx}{b}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,ie^{i(a+bx)})}{b}-\frac{id\int \operatorname{PolyLog}(2,ie^{i(a+bx)}\right.}{b}}{b}\right.$$

$$\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}(2,-ie^{i(a+bx)})de^{i(a+bx)}}{b^2}\right)}{b}+$$
$$\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,ie^{i(a+bx)})}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}(2,ie^{i(a+bx)})de^{i(a+bx)}}{b^2}\right)}{b}+$$
$$\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{d^2\operatorname{arctanh}(\sin(a+bx))}{b^3}-\frac{d(c+dx)\sec(a+bx)}{b^2}+$$
$$\frac{(c+dx)^2\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{2720}$$

$$\frac{1}{2}\left(\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}(2,-ie^{i(a+bx)})de^{i(a+bx)}}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,ie^{i(a+bx)})}{b}-\frac{d\int e^{-i}}{}\right.}{b}\right.$$

$$\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}(2,-ie^{i(a+bx)})de^{i(a+bx)}}{b^2}\right)}{b}+$$
$$\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,ie^{i(a+bx)})}{b}-\frac{d\int e^{-i(a+bx)}\operatorname{PolyLog}(2,ie^{i(a+bx)})de^{i(a+bx)}}{b^2}\right)}{b}+$$
$$\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{d^2\operatorname{arctanh}(\sin(a+bx))}{b^3}-\frac{d(c+dx)\sec(a+bx)}{b^2}+$$
$$\frac{(c+dx)^2\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}\left(-\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{d\operatorname{PolyLog}(3,-ie^{i(a+bx)})}{b^2}\right)}{b}-\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,}{b}\right.}{}\right.$$

$$\frac{2i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{d^2\operatorname{arctanh}(\sin(a+bx))}{b^3}-$$
$$\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-ie^{i(a+bx)})}{b}-\frac{d\operatorname{PolyLog}(3,-ie^{i(a+bx)})}{b^2}\right)}{b}+$$
$$\frac{2d\left(\frac{i(c+dx)\operatorname{PolyLog}(2,ie^{i(a+bx)})}{b}-\frac{d\operatorname{PolyLog}(3,ie^{i(a+bx)})}{b^2}\right)}{b}-\frac{d(c+dx)\sec(a+bx)}{b^2}+$$
$$\frac{(c+dx)^2\tan(a+bx)\sec(a+bx)}{2b}$$

input
```
Int[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

3.299. $\int (c+dx)^2\sec(a+bx)\tan^2(a+bx)\,dx$

output
```
((2*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (d^2*ArcTanh[Sin[a + b*x]]
)/b^3 - (2*d*((I*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b - (d*PolyLo
g[3, (-I)*E^(I*(a + b*x))])/b^2))/b + (2*d*((I*(c + d*x)*PolyLog[2, I*E^(I
*(a + b*x))])/b - (d*PolyLog[3, I*E^(I*(a + b*x))])/b^2))/b + (((-2*I)*(c
+ d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (2*d*((I*(c + d*x)*PolyLog[2, (-I)*E
^(I*(a + b*x))])/b - (d*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^2))/b - (2*d*(
(I*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b - (d*PolyLog[3, I*E^(I*(a +
b*x))])/b^2))/b)/2 - (d*(c + d*x)*Sec[a + b*x])/b^2 + ((c + d*x)^2*Sec[a +
 b*x]*Tan[a + b*x])/(2*b)
```

### 3.299.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

3.299. $\int (c+dx)^2 \sec(a+bx) \tan^2(a+bx) \, dx$

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4913
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.299.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(174) = 348$.

Time = 1.25 (sec) , antiderivative size = 584, normalized size of antiderivative = 3.03

| method | result |
|---|---|
| risch | $-\frac{i d^2 \operatorname{polylog}(2,-i e^{i(xb+a)}) x}{b^2}+\frac{i d^2 a^2 \arctan(e^{i(xb+a)})}{b^3}-\frac{2icda \arctan(e^{i(xb+a)})}{b^2}-\frac{icd \operatorname{polylog}(2,-i e^{i(xb+a)})}{b^2}+\frac{a^2d^2 \ln(1-}{2b}$ |

input
```
int((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-I/b^2*d^2*polylog(2,-I*exp(I*(b*x+a)))*x+I/b^3*d^2*a^2*arctan(exp(I*(b*x+
a)))-2*I/b^2*c*d*a*arctan(exp(I*(b*x+a)))-I/b^2*c*d*polylog(2,-I*exp(I*(b*
x+a)))+1/2/b^3*a^2*d^2*ln(1-I*exp(I*(b*x+a)))-d^2*polylog(3,I*exp(I*(b*x+a
)))/b^3-1/b*c*d*ln(1-I*exp(I*(b*x+a)))*x-1/2/b^3*a^2*d^2*ln(1+I*exp(I*(b*x
+a)))+1/b^2*c*d*ln(1+I*exp(I*(b*x+a)))*a-1/2/b*d^2*ln(1-I*exp(I*(b*x+a)))*
x^2+1/2/b*d^2*ln(1+I*exp(I*(b*x+a)))*x^2+1/b*c*d*ln(1+I*exp(I*(b*x+a)))*x+
d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3-1/b^2*c*d*ln(1-I*exp(I*(b*x+a)))*a+I/
b*c^2*arctan(exp(I*(b*x+a)))+I/b^2*d^2*polylog(2,I*exp(I*(b*x+a)))*x+I/b^2
*c*d*polylog(2,I*exp(I*(b*x+a)))-2*I/b^3*d^2*arctan(exp(I*(b*x+a)))-I/b^2/
(exp(2*I*(b*x+a))+1)^2*(x^2*d^2*b*exp(3*I*(b*x+a))+2*c*d*x*b*exp(3*I*(b*x+
a))+c^2*b*exp(3*I*(b*x+a))-x^2*d^2*b*exp(I*(b*x+a))-2*c*d*x*b*exp(I*(b*x+a
))-2*I*d^2*x*exp(3*I*(b*x+a))-c^2*b*exp(I*(b*x+a))-2*I*d*c*exp(3*I*(b*x+a)
)-2*I*d^2*x*exp(I*(b*x+a))-2*I*c*d*exp(I*(b*x+a)))
```

### 3.299.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(165) = 330$.

Time = 0.30 (sec) , antiderivative size = 795, normalized size of antiderivative = 4.12

$$\int (c+dx)^2 \sec(a+bx)\tan^2(a+bx)\,dx$$
$$= \frac{2\,d^2\cos(bx+a)^2\,\mathrm{polylog}(3, i\,\cos(bx+a)+\sin(bx+a)) - 2\,d^2\cos(bx+a)^2\,\mathrm{polylog}(3, i\,\cos(bx+a)-\mathrm{s}}{}$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*(2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*d^
2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*cos(b*x
 + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*cos(b*x + a)^2*
polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*cos(
b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)
*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) - 2*(I*b*d^2*x + I*b*
c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 2*(I*b*d^2*x +
 I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^2*c^2
- 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x +
 a) + I) + (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2*log(cos(b*
x + a) - I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^
2*d^2)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^
2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*cos(b*x + a) -
 sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos
(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2
*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x
 + a) + 1) - (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2*log(-cos
(b*x + a) + I*sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*co
s(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 4*(b*d^2*x + b*c*d)
*cos(b*x + a) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(b*x + a))/(...
```

### 3.299.6 Sympy [F]

$$\int (c+dx)^2 \sec(a+bx) \tan^2(a+bx) \, dx = \int (c+dx)^2 \tan^2(a+bx) \sec(a+bx) \, dx$$

input
```
integrate((d*x+c)**2*sec(b*x+a)*tan(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*tan(a + b*x)**2*sec(a + b*x), x)
```

### 3.299.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1891 vs. 2(165) = 330.

Time = 0.57 (sec) , antiderivative size = 1891, normalized size of antiderivative = 9.80

$$\int (c+dx)^2 \sec(a+bx)\tan^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(c^2*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - l
og(sin(b*x + a) - 1)) - 2*a*c*d*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log
(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b + a^2*d^2*(2*sin(b*x + a)/(s
in(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b^2 -
4*(2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2 + ((b*x + a)^2
*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(4*b*x + 4*a) + 2*((b*x + a
)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(2*b*x + 2*a) + (I*(b*x
+ a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) - 2*I*d^2)*sin(4*b*x + 4*a) +
 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) - 2*I*d^2)*sin(2*b
*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + 2*((b*x + a)^2*d^2 +
2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2 + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)
*(b*x + a) - 2*d^2)*cos(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d
^2)*(b*x + a) - 2*d^2)*cos(2*b*x + 2*a) + (I*(b*x + a)^2*d^2 + 2*(I*b*c*d
- I*a*d^2)*(b*x + a) - 2*I*d^2)*sin(4*b*x + 4*a) + 2*(I*(b*x + a)^2*d^2 +
2*(I*b*c*d - I*a*d^2)*(b*x + a) - 2*I*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b
*x + a), -sin(b*x + a) + 1) - 4*((b*x + a)^2*d^2 - 2*I*b*c*d + 2*I*a*d^2 +
 2*(b*c*d - (a + I)*d^2)*(b*x + a))*cos(3*b*x + 3*a) + 4*((b*x + a)^2*d^2
+ 2*I*b*c*d - 2*I*a*d^2 + 2*(b*c*d - (a - I)*d^2)*(b*x + a))*cos(b*x + a)
+ 4*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4
*b*x + 4*a) + 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (I*b...
```

### 3.299.8 Giac [F]

$$\int (c+dx)^2 \sec(a+bx)\tan^2(a+bx)\,dx = \int (dx+c)^2 \sec(bx+a)\tan(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*sec(b*x + a)*tan(b*x + a)^2, x)
```

### 3.299.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \sec(a+bx) \tan^2(a+bx) \, dx = \text{Hanged}$$

input
```
int((tan(a + b*x)^2*(c + d*x)^2)/cos(a + b*x),x)
```

output
```
\text{Hanged}
```

# 3.300 $\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx$

3.300.1 Optimal result . . . 2264
3.300.2 Mathematica [B] (warning: unable to verify) . . . 2264
3.300.3 Rubi [A] (verified) . . . 2265
3.300.4 Maple [B] (verified) . . . 2268
3.300.5 Fricas [B] (verification not implemented) . . . 2269
3.300.6 Sympy [F] . . . 2270
3.300.7 Maxima [F] . . . 2270
3.300.8 Giac [F] . . . 2271
3.300.9 Mupad [F(-1)] . . . 2271

## 3.300.1 Optimal result

Integrand size = 20, antiderivative size = 117

$$\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx = \frac{i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b} - \frac{id\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2} + \frac{id\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2} - \frac{d\sec(a+bx)}{2b^2} + \frac{(c+dx)\sec(a+bx)\tan(a+bx)}{2b}$$

output
```
I*(d*x+c)*arctan(exp(I*(b*x+a)))/b-1/2*I*d*polylog(2,-I*exp(I*(b*x+a)))/b^
2+1/2*I*d*polylog(2,I*exp(I*(b*x+a)))/b^2-1/2*d*sec(b*x+a)/b^2+1/2*(d*x+c)
*sec(b*x+a)*tan(b*x+a)/b
```

## 3.300.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 555 vs. 2(117) = 234.

Time = 7.03 (sec) , antiderivative size = 555, normalized size of antiderivative = 4.74

$$\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx = -\frac{c\operatorname{arctanh}(\sin(a+bx))}{2b}$$
$$+\frac{dx\left(a\log\left(1-\tan\left(\frac{1}{2}(a+bx)\right)\right)+i\log\left(1+i\tan\left(\frac{1}{2}(a+bx)\right)\right)\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)\left(-1+\tan\left(\frac{1}{2}(a+bx)\right)\right)\right)\cdots\right.}{}$$
$$+\frac{dx}{4b\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)^2}$$
$$-\frac{d\sin\left(\frac{1}{2}(a+bx)\right)}{2b^2\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)}-\frac{dx}{4b\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right)^2}$$
$$+\frac{d\sin\left(\frac{1}{2}(a+bx)\right)}{2b^2\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right)}+\frac{c\sec(a+bx)\tan(a+bx)}{2b}$$

input
```
Integrate[(c + d*x)*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
-1/2*(c*ArcTanh[Sin[a + b*x]])/b + (d*x*(a*Log[1 - Tan[(a + b*x)/2]] + I*L
og[1 + I*Tan[(a + b*x)/2]]*Log[(-1/2 - I/2)*(-1 + Tan[(a + b*x)/2])] - I*L
og[1 - I*Tan[(a + b*x)/2]]*Log[(-1/2 + I/2)*(-1 + Tan[(a + b*x)/2])] - I*L
og[1 + I*Tan[(a + b*x)/2]]*Log[(1/2 - I/2)*(1 + Tan[(a + b*x)/2])] + I*Log
[1 - I*Tan[(a + b*x)/2]]*Log[(1/2 + I/2)*(1 + Tan[(a + b*x)/2])] - a*Log[1
 + Tan[(a + b*x)/2]] - I*PolyLog[2, ((1 + I) - (1 - I)*Tan[(a + b*x)/2])/2
] + I*PolyLog[2, (-1/2 - I/2)*(I + Tan[(a + b*x)/2])] - I*PolyLog[2, ((1 +
 I) + (1 - I)*Tan[(a + b*x)/2])/2] + I*PolyLog[2, ((1 - I) + (1 + I)*Tan[(
a + b*x)/2])/2]))/(2*b*(a - I*Log[1 - I*Tan[(a + b*x)/2]] + I*Log[1 + I*Ta
n[(a + b*x)/2]])) + (d*x)/(4*b*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])^2) -
(d*Sin[(a + b*x)/2])/(2*b^2*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])) - (d*x)
/(4*b*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])^2) + (d*Sin[(a + b*x)/2])/(2*b
^2*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])) + (c*Sec[a + b*x]*Tan[a + b*x])/
(2*b)
```

### 3.300.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.64, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4913, 3042, 4669, 2715, 2838, 4673, 3042, 4669, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\tan^2(a+bx)\sec(a+bx)\,dx$$

$$\Big\downarrow \; 4913$$

$$\int (c+dx)\sec^3(a+bx)dx-\int (c+dx)\sec(a+bx)dx$$

$$\Big\downarrow \; 3042$$

$$\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)^3dx-\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \; 4669$$

$$\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)^3dx+\frac{d\int \log\left(1-ie^{i(a+bx)}\right)dx}{b}-\frac{d\int \log\left(1+ie^{i(a+bx)}\right)dx}{b}+\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \; 2715$$

$$-\frac{id\int e^{-i(a+bx)}\log\left(1-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}+\frac{id\int e^{-i(a+bx)}\log\left(1+ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}+\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)^3dx+\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}$$

$$\Big\downarrow \; 2838$$

$$\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)^3dx+\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{id\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}+\frac{id\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}$$

$$\Big\downarrow \; 4673$$

$$\frac{1}{2}\int (c+dx)\sec(a+bx)dx+\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{id\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}+\frac{id\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}-\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \; 3042$$

$$\frac{1}{2}\int (c+dx)\csc\left(a+bx+\frac{\pi}{2}\right)dx+\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{id\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}+\frac{id\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}-\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)\tan(a+bx)\sec(a+bx)}{2b}$$

$$\Big\downarrow \; 4669$$

3.300. $\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx$

$$\frac{1}{2}\left(-\frac{d\int \log\left(1-ie^{i(a+bx)}\right)dx}{b}+\frac{d\int \log\left(1+ie^{i(a+bx)}\right)dx}{b}-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}\right)+\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}+\frac{id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}-\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)\tan(a+bx)\sec(a+bx)}{2b}$$

$\big\downarrow$ 2715

$$\frac{1}{2}\left(\frac{id\int e^{-i(a+bx)}\log\left(1-ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{id\int e^{-i(a+bx)}\log\left(1+ie^{i(a+bx)}\right)de^{i(a+bx)}}{b^2}-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}\right.$$
$$\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}+\frac{id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}-\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)\tan(a+bx)\sec(a+bx)}{2b}$$

$\big\downarrow$ 2838

$$\frac{1}{2}\left(-\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}\right)+\frac{2i(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{id\,\mathrm{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}+\frac{id\,\mathrm{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}-\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)\tan(a+bx)\sec(a+bx)}{2b}$$

input
```
Int[(c + d*x)*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

output
```
((2*I)*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b - (I*d*PolyLog[2, (-I)*E^(I*(a
 + b*x))])/b^2 + (I*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2 + (((-2*I)*(c + d
*x)*ArcTan[E^(I*(a + b*x))])/b + (I*d*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^
2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2)/2 - (d*Sec[a + b*x])/(2*b^2)
+ ((c + d*x)*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

### 3.300.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 4913
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

### 3.300.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(98) = 196$.

Time = 0.70 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.28

| method | result |
|---|---|
| risch | $-\frac{i\left(dxb\,\mathrm{e}^{3i(xb+a)}-id\,\mathrm{e}^{3i(xb+a)}+cb\,\mathrm{e}^{3i(xb+a)}-dxb\,\mathrm{e}^{i(xb+a)}-id\,\mathrm{e}^{i(xb+a)}-cb\,\mathrm{e}^{i(xb+a)}\right)}{b^2\left(\mathrm{e}^{2i(xb+a)}+1\right)^2}+\frac{ic\arctan\left(\mathrm{e}^{i(xb+a)}\right)}{b}+\frac{d\ln\left(1+i\mathrm{e}^{i(xb+a)}\right)x}{2b}$ |

input
```
int((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-I/b^2/(exp(2*I*(b*x+a))+1)^2*(d*x*b*exp(3*I*(b*x+a))-I*d*exp(3*I*(b*x+a))
+c*b*exp(3*I*(b*x+a))-d*x*b*exp(I*(b*x+a))-I*d*exp(I*(b*x+a))-c*b*exp(I*(b
*x+a)))+I/b*c*arctan(exp(I*(b*x+a)))+1/2/b*d*ln(1+I*exp(I*(b*x+a)))*x+1/2/
b^2*d*ln(1+I*exp(I*(b*x+a)))*a-1/2/b*d*ln(1-I*exp(I*(b*x+a)))*x-1/2/b^2*d*
ln(1-I*exp(I*(b*x+a)))*a-1/2*I/b^2*d*dilog(1+I*exp(I*(b*x+a)))+1/2*I/b^2*d
*dilog(1-I*exp(I*(b*x+a)))-I/b^2*d*a*arctan(exp(I*(b*x+a)))
```

## 3.300.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 435 vs. 2(93) = 186.

Time = 0.30 (sec) , antiderivative size = 435, normalized size of antiderivative = 3.72

$$\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx$$
$$= \frac{i\,d\cos{(bx+a)}^2\,\mathrm{Li}_2(i\,\cos{(bx+a)}+\sin{(bx+a)})+i\,d\cos{(bx+a)}^2\,\mathrm{Li}_2(i\,\cos{(bx+a)}-\sin{(bx+a)})-i$$

input
```
integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/4*(I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*cos(b*x
 + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(-I
*cos(b*x + a) + sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) -
 sin(b*x + a)) - (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x +
 a) + I) + (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) +
I) - (b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) +
 (b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b*
d*x + a*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x
 + a*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b*c - a*
d)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*c - a*d)*co
s(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 2*d*cos(b*x + a) +
2*(b*d*x + b*c)*sin(b*x + a))/(b^2*cos(b*x + a)^2)
```

### 3.300.6 Sympy [F]

$$\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx = \int (c+dx)\tan^2{(a+bx)}\sec{(a+bx)}\,dx$$

input
```
integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*tan(a + b*x)**2*sec(a + b*x), x)
```

### 3.300.7 Maxima [F]

$$\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx = \int (dx+c)\sec{(bx+a)}\tan{(bx+a)}^2\,dx$$

input
```
integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(4*(d*cos(3*b*x + 3*a) + d*cos(b*x + a) - (b*d*x + b*c)*sin(3*b*x + 3
*a) + (b*d*x + b*c)*sin(b*x + a))*cos(4*b*x + 4*a) + 4*(2*d*cos(2*b*x + 2*
a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a) + d)*cos(3*b*x + 3*a) + 8*(d*cos(b*x
 + a) + (b*d*x + b*c)*sin(b*x + a))*cos(2*b*x + 2*a) + 4*d*cos(b*x + a) +
4*(b^2*d*cos(4*b*x + 4*a)^2 + 4*b^2*d*cos(2*b*x + 2*a)^2 + b^2*d*sin(4*b*x
 + 4*a)^2 + 4*b^2*d*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*b^2*d*sin(2*b*x
+ 2*a)^2 + 4*b^2*d*cos(2*b*x + 2*a) + b^2*d + 2*(2*b^2*d*cos(2*b*x + 2*a)
+ b^2*d)*cos(4*b*x + 4*a))*integrate((x*cos(2*b*x + 2*a)*cos(b*x + a) + x*
sin(2*b*x + 2*a)*sin(b*x + a) + x*cos(b*x + a))/(cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1), x) + (b*c*cos(4*b*x + 4*a)^2 + 4
*b*c*cos(2*b*x + 2*a)^2 + b*c*sin(4*b*x + 4*a)^2 + 4*b*c*sin(4*b*x + 4*a)*
sin(2*b*x + 2*a) + 4*b*c*sin(2*b*x + 2*a)^2 + 4*b*c*cos(2*b*x + 2*a) + b*c
 + 2*(2*b*c*cos(2*b*x + 2*a) + b*c)*cos(4*b*x + 4*a))*log(cos(b*x + a)^2 +
 sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (b*c*cos(4*b*x + 4*a)^2 + 4*b*c*co
s(2*b*x + 2*a)^2 + b*c*sin(4*b*x + 4*a)^2 + 4*b*c*sin(4*b*x + 4*a)*sin(2*b
*x + 2*a) + 4*b*c*sin(2*b*x + 2*a)^2 + 4*b*c*cos(2*b*x + 2*a) + b*c + 2*(2
*b*c*cos(2*b*x + 2*a) + b*c)*cos(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b*
x + a)^2 - 2*sin(b*x + a) + 1) + 4*((b*d*x + b*c)*cos(3*b*x + 3*a) - (b*d*
x + b*c)*cos(b*x + a) + d*sin(3*b*x + 3*a) + d*sin(b*x + a))*sin(4*b*x + 4
*a) - 4*(b*d*x + b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - 2*d*sin(2*b*x...
```

### 3.300.8 Giac [F]

$$\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx = \int (dx+c)\sec(bx+a)\tan(bx+a)^2\,dx$$

input
```
integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)*sec(b*x + a)*tan(b*x + a)^2, x)
```

### 3.300.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx = \text{Hanged}$$

input
```
int((tan(a + b*x)^2*(c + d*x))/cos(a + b*x),x)
```

output
```
\text{Hanged}
```

# 3.301 $\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx$

3.301.1 Optimal result . . . 2272
3.301.2 Mathematica [N/A] . . . 2272
3.301.3 Rubi [N/A] . . . 2273
3.301.4 Maple [N/A] (verified) . . . 2274
3.301.5 Fricas [N/A] . . . 2274
3.301.6 Sympy [N/A] . . . 2275
3.301.7 Maxima [N/A] . . . 2275
3.301.8 Giac [N/A] . . . 2276
3.301.9 Mupad [N/A] . . . 2277

## 3.301.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx = -\mathrm{Int}\left(\frac{\sec(a+bx)}{c+dx},x\right)+\mathrm{Int}\left(\frac{\sec^3(a+bx)}{c+dx},x\right)$$

output
```
-Unintegrable(sec(b*x+a)/(d*x+c),x)+Unintegrable(sec(b*x+a)^3/(d*x+c),x)
```

## 3.301.2 Mathematica [N/A]

Not integrable

Time = 32.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x),x]
```

output
```
Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x), x]
```

## 3.301.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4913, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{ 4913}$$

$$\int \frac{\sec^3(a+bx)}{c+dx}dx-\int \frac{\sec(a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)^3}{c+dx}dx-\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{c+dx}dx$$

$$\Big\downarrow \text{ 4680}$$

$$\int \frac{\sec^3(a+bx)}{c+dx}dx-\int \frac{\sec(a+bx)}{c+dx}dx$$

input
```
Int[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x),x]
```

output
```
$Aborted
```

### 3.301.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4913
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

## 3.301.4 Maple [N/A] (verified)

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sec (xb+a) \tan (xb+a)^2}{dx+c}dx$$

input
```
int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
```

output
```
int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
```

## 3.301.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\sec (bx+a) \tan (bx+a)^2}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)*tan(b*x + a)^2/(d*x + c), x)
```

### 3.301.6 Sympy [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\tan^2{(a+bx)}\sec{(a+bx)}}{c+dx}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(tan(a + b*x)**2*sec(a + b*x)/(c + d*x), x)
```

### 3.301.7 Maxima [N/A]

Not integrable

Time = 2.05 (sec) , antiderivative size = 1331, normalized size of antiderivative = 60.50

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\sec{(bx+a)}\tan{(bx+a)}^2}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
((d*cos(3*b*x + 3*a) + d*cos(b*x + a) + (b*d*x + b*c)*sin(3*b*x + 3*a) - (
b*d*x + b*c)*sin(b*x + a))*cos(4*b*x + 4*a) + (2*d*cos(2*b*x + 2*a) - 2*(b
*d*x + b*c)*sin(2*b*x + 2*a) + d)*cos(3*b*x + 3*a) + 2*(d*cos(b*x + a) - (
b*d*x + b*c)*sin(b*x + a))*cos(2*b*x + 2*a) + d*cos(b*x + a) - (b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*
x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 +
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*c
os(4*b*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)
)*integrate(((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(2*b*x + 2*a
)*cos(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(2*b*x +
 2*a)*sin(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x
 + a))/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3
*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2
+ 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x +
2*a)), x) - ((b*d*x + b*c)*cos(3*b*x + 3*a) - (b*d*x + b*c)*cos(b*x + a) -
 d*sin(3*b*x + 3*a) - d*sin(b*x + a))*sin(4*b*x + 4*a) + (b*d*x + b*c +...
```

### 3.301.8 Giac [N/A]

Not integrable

Time = 5.86 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\sec\left(bx+a\right)\tan\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)*tan(b*x + a)^2/(d*x + c), x)
```

### 3.301.9 Mupad [N/A]

Not integrable

Time = 25.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx = \int \frac{\tan(a+b\,x)^2}{\cos(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(tan(a + b*x)^2/(cos(a + b*x)*(c + d*x)),x)
```

output
```
int(tan(a + b*x)^2/(cos(a + b*x)*(c + d*x)), x)
```

# 3.302 $\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2} dx$

3.302.1 Optimal result . . . 2278
3.302.2 Mathematica [N/A] . . . 2278
3.302.3 Rubi [N/A] . . . 2279
3.302.4 Maple [N/A] (verified) . . . 2280
3.302.5 Fricas [N/A] . . . 2280
3.302.6 Sympy [N/A] . . . 2281
3.302.7 Maxima [N/A] . . . 2281
3.302.8 Giac [N/A] . . . 2282
3.302.9 Mupad [N/A] . . . 2283

## 3.302.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2} dx = -\text{Int}\left(\frac{\sec(a+bx)}{(c+dx)^2}, x\right) + \text{Int}\left(\frac{\sec^3(a+bx)}{(c+dx)^2}, x\right)$$

output
```
-Unintegrable(sec(b*x+a)/(d*x+c)^2,x)+Unintegrable(sec(b*x+a)^3/(d*x+c)^2,
x)
```

## 3.302.2 Mathematica [N/A]

Not integrable

Time = 33.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2} dx = \int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2} dx$$

input
```
Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2, x]
```

### 3.302.3 Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4913, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(a+bx)\sec(a+bx)}{(c+dx)^2} \, dx$$

$$\Big\downarrow \text{4913}$$

$$\int \frac{\sec^3(a+bx)}{(c+dx)^2}dx-\int \frac{\sec(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)^3}{(c+dx)^2}dx-\int \frac{\csc\left(a+bx+\frac{\pi}{2}\right)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{4680}$$

$$\int \frac{\sec^3(a+bx)}{(c+dx)^2}dx-\int \frac{\sec(a+bx)}{(c+dx)^2}dx$$

input
```
Int[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2,x]
```

output
```
$Aborted
```

#### 3.302.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4913
```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

### 3.302.4 Maple [N/A] (verified)

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sec \left(xb+a\right) \tan \left(xb+a\right)^{2}}{\left(dx+c\right)^{2}}dx$$

input
```
int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
```

### 3.302.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec \left(bx+a\right) \tan \left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)*tan(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.302.6 Sympy [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan^2{(a+bx)}\sec{(a+bx)}}{(c+dx)^2}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(tan(a + b*x)**2*sec(a + b*x)/(c + d*x)**2, x)
```

### 3.302.7 Maxima [N/A]

Not integrable

Time = 5.26 (sec) , antiderivative size = 1645, normalized size of antiderivative = 74.77

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec{(bx+a)}\tan{(bx+a)}^2}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
((2*d*cos(3*b*x + 3*a) + 2*d*cos(b*x + a) + (b*d*x + b*c)*sin(3*b*x + 3*a)
 - (b*d*x + b*c)*sin(b*x + a))*cos(4*b*x + 4*a) + 2*(2*d*cos(2*b*x + 2*a)
- (b*d*x + b*c)*sin(2*b*x + 2*a) + d)*cos(3*b*x + 3*a) + 2*(2*d*cos(b*x +
a) - (b*d*x + b*c)*sin(b*x + a))*cos(2*b*x + 2*a) + 2*d*cos(b*x + a) - (b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b
^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^
3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d
^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 + 4
*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a
)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^
2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)
*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*integrate(((b^2*d^2*x^2 + 2*b^2*
c*d*x + b^2*c^2 - 6*d^2)*cos(2*b*x + 2*a)*cos(b*x + a) + (b^2*d^2*x^2 + 2*
b^2*c*d*x + b^2*c^2 - 6*d^2)*sin(2*b*x + 2*a)*sin(b*x + a) + (b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2 - 6*d^2)*cos(b*x + a))/(b^2*d^4*x^4 + 4*b^2*c*d^3*
x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c
*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(2*b*x + 2*a)^2
 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x +...
```

### 3.302.8 Giac [N/A]

Not integrable

Time = 28.89 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec(bx+a)\tan(bx+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)*tan(b*x + a)^2/(d*x + c)^2, x)
```

### 3.302.9 Mupad [N/A]

Not integrable

Time = 25.72 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(a+b\,x)^2}{\cos\left(a+b\,x\right)\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(tan(a + b*x)^2/(cos(a + b*x)*(c + d*x)^2),x)
```

output
```
int(tan(a + b*x)^2/(cos(a + b*x)*(c + d*x)^2), x)
```

# 3.303 $\int (c+dx)^m \tan^3(a+bx)\,dx$

3.303.1 Optimal result . . . . . . . . . . 2284
3.303.2 Mathematica [N/A] . . . . . . . . . . 2284
3.303.3 Rubi [N/A] . . . . . . . . . . 2285
3.303.4 Maple [N/A] (verified) . . . . . . . . . . 2286
3.303.5 Fricas [N/A] . . . . . . . . . . 2286
3.303.6 Sympy [N/A] . . . . . . . . . . 2286
3.303.7 Maxima [N/A] . . . . . . . . . . 2287
3.303.8 Giac [N/A] . . . . . . . . . . 2287
3.303.9 Mupad [N/A] . . . . . . . . . . 2287

## 3.303.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (c+dx)^m \tan^3(a+bx)\,dx = \text{Int}\left((c+dx)^m \tan^3(a+bx),x\right)$$

output
```
Unintegrable((d*x+c)^m*tan(b*x+a)^3,x)
```

## 3.303.2 Mathematica [N/A]

Not integrable

Time = 10.90 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \tan^3(a+bx)\,dx = \int (c+dx)^m \tan^3(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Tan[a + b*x]^3,x]
```

output
```
Integrate[(c + d*x)^m*Tan[a + b*x]^3, x]
```

## 3.303.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(a+bx)(c+dx)^m\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \tan(a+bx)^3(c+dx)^m dx$$
$$\Big\downarrow \text{ 4222}$$
$$\int \tan^3(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Tan[a + b*x]^3,x]
```

output
```
$Aborted
```

### 3.303.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

### 3.303.4 Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \tan(xb+a)^3 \, dx$$

input
```
int((d*x+c)^m*tan(b*x+a)^3,x)
```

output
```
int((d*x+c)^m*tan(b*x+a)^3,x)
```

### 3.303.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \tan^3(a+bx) \, dx = \int (dx+c)^m \tan(bx+a)^3 \, dx$$

input
```
integrate((d*x+c)^m*tan(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*tan(b*x + a)^3, x)
```

### 3.303.6 Sympy [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (c+dx)^m \tan^3(a+bx) \, dx = \int (c+dx)^m \tan^3(a+bx) \, dx$$

input
```
integrate((d*x+c)**m*tan(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**m*tan(a + b*x)**3, x)
```

### 3.303.7 Maxima [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \tan^3(a+bx)\,dx = \int (dx+c)^m \tan(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*tan(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*tan(b*x + a)^3, x)
```

### 3.303.8 Giac [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \tan^3(a+bx)\,dx = \int (dx+c)^m \tan(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*tan(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*tan(b*x + a)^3, x)
```

### 3.303.9 Mupad [N/A]

Not integrable

Time = 25.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \tan^3(a+bx)\,dx = \int \tan(a+b\,x)^3\,(c+d\,x)^m\,dx$$

input
```
int(tan(a + b*x)^3*(c + d*x)^m,x)
```

output
```
int(tan(a + b*x)^3*(c + d*x)^m, x)
```

# 3.304 $\int (c+dx)^3 \tan^3(a+bx)\,dx$


3.304.1 Optimal result . . . . . 2288
3.304.2 Mathematica [B] (warning: unable to verify) . . . . . 2289
3.304.3 Rubi [A] (verified) . . . . . 2290
3.304.4 Maple [B] (verified) . . . . . 2296
3.304.5 Fricas [B] (verification not implemented) . . . . . 2297
3.304.6 Sympy [F] . . . . . 2298
3.304.7 Maxima [B] (verification not implemented) . . . . . 2298
3.304.8 Giac [F] . . . . . 2299
3.304.9 Mupad [F(-1)] . . . . . 2300


## 3.304.1 Optimal result

Integrand size = 16, antiderivative size = 259

$$\int (c+dx)^3 \tan^3(a+bx)\,dx = \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{i(c+dx)^4}{4d} - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id^3\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^4} - \frac{3id(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} + \frac{3d^2(c+dx)\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} + \frac{3id^3\,\mathrm{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4} - \frac{3d(c+dx)^2\tan(a+bx)}{2b^2} + \frac{(c+dx)^3\tan^2(a+bx)}{2b}$$

output
```
3/2*I*d*(d*x+c)^2/b^2+1/2*(d*x+c)^3/b-1/4*I*(d*x+c)^4/d-3*d^2*(d*x+c)*ln(1
+exp(2*I*(b*x+a)))/b^3+(d*x+c)^3*ln(1+exp(2*I*(b*x+a)))/b+3/2*I*d^3*polylo
g(2,-exp(2*I*(b*x+a)))/b^4-3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+a)))/
b^2+3/2*d^2*(d*x+c)*polylog(3,-exp(2*I*(b*x+a)))/b^3+3/4*I*d^3*polylog(4,-
exp(2*I*(b*x+a)))/b^4-3/2*d*(d*x+c)^2*tan(b*x+a)/b^2+1/2*(d*x+c)^3*tan(b*x
+a)^2/b
```

### 3.304.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 814 vs. $2(259) = 518$.

Time = 6.92 (sec) , antiderivative size = 814, normalized size of antiderivative = 3.14

$$\int (c+dx)^3\tan^3(a+bx)\,dx$$
$$= \frac{icd^2e^{-ia}\left(2b^2x^2\left(2bx-3i(1+e^{2ia})\log\left(1+e^{-2i(a+bx)}\right)\right)+6b(1+e^{2ia})\,x\,\text{PolyLog}\left(2,-e^{-2i(a+bx)}\right)-3i(1+\right.}{4b^3}$$
$$+\frac{id^3e^{ia}\left(2b^4e^{-2ia}x^4-4ib^3(1+e^{-2ia})\,x^3\log\left(1+e^{-2i(a+bx)}\right)+6b^2(1+e^{-2ia})\,x^2\,\text{PolyLog}\left(2,-e^{-2i(a+bx)}\right)-\right.}{8b^4}$$
$$+\frac{(c+dx)^3\sec^2(a+bx)}{2b}$$
$$+\frac{c^3\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))}{b\left(\cos^2(a)+\sin^2(a)\right)}$$
$$-\frac{3cd^2\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))}{b^3\left(\cos^2(a)+\sin^2(a)\right)}$$
$$+\frac{3c^2d\csc(a)\left(b^2e^{-i\arctan(\cot(a))}x^2-\frac{\cot(a)\left(ibx(-\pi-2\arctan(\cot(a)))-\pi\log\left(1+e^{-2ibx}\right)-2(bx-\arctan(\cot(a)))\log\left(1-e^{2i(bx-\arctan\right.\right.}}{\right.}}{2b^2\sqrt{\csc^2(a)\left(\cos^2(a)+\right.}}$$
$$-\frac{3d^3\csc(a)\left(b^2e^{-i\arctan(\cot(a))}x^2-\frac{\cot(a)\left(ibx(-\pi-2\arctan(\cot(a)))-\pi\log\left(1+e^{-2ibx}\right)-2(bx-\arctan(\cot(a)))\log\left(1-e^{2i(bx-\arctan\right.\right.}}{\right.}}{2b^4\sqrt{\csc^2(a)\left(\cos^2(a)+\right.}}$$
$$-\frac{3\sec(a)\sec(a+bx)\left(c^2d\sin(bx)+2cd^2x\sin(bx)+d^3x^2\sin(bx)\right)}{2b^2}$$
$$-\frac{1}{4}x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)\tan(a)$$

input
```
Integrate[(c + d*x)^3*Tan[a + b*x]^3,x]
```

output
```
((I/4)*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)
*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))]
- (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*
E^(I*a)) + ((I/8)*d^3*E^(I*a)*((2*b^4*x^4)/E^((2*I)*a) - (4*I)*b^3*(1 + E^
((-2*I)*a))*x^3*Log[1 + E^((-2*I)*(a + b*x))] + 6*b^2*(1 + E^((-2*I)*a))*x
^2*PolyLog[2, -E^((-2*I)*(a + b*x))] - (6*I)*b*(1 + E^((-2*I)*a))*x*PolyLo
g[3, -E^((-2*I)*(a + b*x))] - 3*(1 + E^((-2*I)*a))*PolyLog[4, -E^((-2*I)*(
a + b*x))])*Sec[a])/b^4 + ((c + d*x)^3*Sec[a + b*x]^2)/(2*b) + (c^3*Sec[a]
*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^
2 + Sin[a]^2)) - (3*c*d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[
b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) + (3*c^2*d*Csc[a]*((b^2*x
^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Lo
g[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - A
rcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTa
n[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Co
t[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (3*d^3*Csc
[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a
]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*
I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[
b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]...
```

### 3.304.3 Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.14, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 4203, 3042, 4202, 2620, 3011, 4203, 17, 3042, 4202, 2620, 2715, 2838, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \tan^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3 \tan(a+bx)^3dx$$

$$\Big\downarrow \text{4203}$$

$$-\frac{3d\int (c+dx)^2\tan^2(a+bx)dx}{2b}-\int (c+dx)^3\tan(a+bx)dx+\frac{(c+dx)^3\tan^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$-\int (c+dx)^3\tan(a+bx)dx-\frac{3d\int (c+dx)^2\tan(a+bx)^2dx}{2b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}$$

$$\Big\downarrow \text{4202}$$

$$2i\int \frac{e^{2i(a+bx)}(c+dx)^3}{1+e^{2i(a+bx)}}dx-\frac{3d\int (c+dx)^2\tan(a+bx)^2dx}{2b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\Big\downarrow \text{2620}$$

$$2i\left(\frac{3id\int (c+dx)^2\log\left(1+e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{3d\int (c+dx)^2\tan(a+bx)^2dx}{2b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\Big\downarrow \text{3011}$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int (c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{3d\int (c+dx)^2\tan(a+bx)^2dx}{2b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\Big\downarrow \text{4203}$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int (c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{3d\left(-\frac{2d\int (c+dx)\tan(a+bx)dx}{b}-\int (c+dx)^2dx+\frac{(c+dx)^2\tan(a+bx)}{b}\right)}{2b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\Big\downarrow \text{17}$$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int (c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{3d\left(-\frac{2d\int (c+dx)\tan(a+bx)dx}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$$\Big\downarrow \text{3042}$$

3.304. $\int (c+dx)^3\tan^3(a+bx)\,dx$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{3d\left(-\frac{2d\int(c+dx)\tan(a+bx)dx}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$\downarrow$ 4202

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\int\frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}dx\right)}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$\downarrow$ 2620

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{id\int\log\left(1+e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$\downarrow$ 2715

$$-\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(\frac{d\int e^{-2i(a+bx)}\log\left(1+e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}+2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)+\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$\downarrow$ 2838

3.304. $\int(c+dx)^3\tan^3(a+bx)\,dx$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$\downarrow$ 7163

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\left(\frac{id\int\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i(c+dx)^3\log(1\ldots}{2b}\right.$$

$$-\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$\downarrow$ 2720

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\left(\frac{d\int e^{-2i(a+bx)}\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i(c\ldots}{}\right.$$

$$-\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

$\downarrow$ 7143

---

3.304. $\int(c+dx)^3\tan^3(a+bx)\,dx$

$$2i\left(\frac{3id\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\left(\frac{d\operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{i(c+dx)^3\log\left(1+e^2\right.}{2b}\right.$$

$$\frac{3d\left(-\frac{2d\left(\frac{i(c+dx)^2}{2d}-2i\left(-\frac{d\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)\right)}{b}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{(c+dx)^3}{3d}\right)}{2b}+$$

$$\frac{(c+dx)^3\tan^2(a+bx)}{2b}-\frac{i(c+dx)^4}{4d}$$

input
```
Int[(c + d*x)^3*Tan[a + b*x]^3,x]
```

output
```
((-1/4*I)*(c + d*x)^4)/d + (2*I)*(((-1/2*I)*(c + d*x)^3*Log[1 + E^((2*I)*(
a + b*x))])/b + (((3*I)/2)*d*(((I/2)*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a +
 b*x))])/b - (I*d*(((-1/2*I)*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/b
 + (d*PolyLog[4, -E^((2*I)*(a + b*x))])/(4*b^2)))/b))/b) + ((c + d*x)^3*Ta
n[a + b*x]^2)/(2*b) - (3*d*(-1/3*(c + d*x)^3/d - (2*d*(((I/2)*(c + d*x)^2)
/d - (2*I)*(((-1/2*I)*(c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b - (d*PolyL
og[2, -E^((2*I)*(a + b*x))])/(4*b^2))))/b + ((c + d*x)^2*Tan[a + b*x])/b))
/(2*b)
```

### 3.304.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

## 3.304. $\int (c+dx)^3 \tan^3(a+bx)\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

### 3.304.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 728 vs. 2(224) = 448.

Time = 0.72 (sec) , antiderivative size = 729, normalized size of antiderivative = 2.81

| method | result |
|---|---|
| risch | $-\frac{6id\,c^2xa}{b}-\frac{3ic\,d^2\operatorname{polylog}\left(2,-e^{2i(xb+a)}\right)x}{b^2}+\frac{6ic\,d^2a^2x}{b^2}+ic^3x+\frac{ic^4}{4d}+\frac{c^3\ln\left(e^{2i(xb+a)}+1\right)}{b}-\frac{2c^3\ln\left(e^{i(xb+a)}\right)}{b}-\frac{6c\,d^2}{}$ |

input
```
int((d*x+c)^3*tan(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*c^3*ln(exp(2*I*(b*x+a))+1)+(2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2*e
xp(2*I*(b*x+a))+6*b*c*d^2*x^2*exp(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+a)
)+6*b*c^2*d*x*exp(2*I*(b*x+a))-3*I*c^2*d*exp(2*I*(b*x+a))-3*I*d^3*x^2+2*b*
c^3*exp(2*I*(b*x+a))-6*I*c*d^2*x-3*I*c^2*d)/b^2/(exp(2*I*(b*x+a))+1)^2-3/b
^3*d^2*c*ln(exp(2*I*(b*x+a))+1)+6/b^3*d^2*c*ln(exp(I*(b*x+a)))-3/b^3*d^3*l
n(exp(2*I*(b*x+a))+1)*x-6/b^4*d^3*a*ln(exp(I*(b*x+a)))+3*I/b^2*d^3*x^2+3*I
/b^4*d^3*a^2+3/2*I*d^3*polylog(2,-exp(2*I*(b*x+a)))/b^4-6*I/b*d*c^2*x*a+6*
I/b^2*c*d^2*a^2*x-I*d^2*c*x^3-3/2*I*d*c^2*x^2-6/b^3*c*d^2*a^2*ln(exp(I*(b*
x+a)))+6/b^2*c^2*d*a*ln(exp(I*(b*x+a)))+3/b*d*c^2*ln(exp(2*I*(b*x+a))+1)*x
+3/2/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x+3/2/b^3*c*d^2*polylog(3,-exp(2
*I*(b*x+a)))+1/b*d^3*ln(exp(2*I*(b*x+a))+1)*x^3-3/2*I/b^2*d*c^2*polylog(2,
-exp(2*I*(b*x+a)))-2/b*c^3*ln(exp(I*(b*x+a)))-1/4*I*d^3*x^4-3/2*I/b^2*d^3*
polylog(2,-exp(2*I*(b*x+a)))*x^2+3/b*c*d^2*ln(exp(2*I*(b*x+a))+1)*x^2-3*I/
b^2*d*c^2*a^2+4*I/b^3*c*d^2*a^3-2*I/b^3*d^3*a^3*x+3/4*I*d^3*polylog(4,-exp
(2*I*(b*x+a)))/b^4+2/b^4*d^3*a^3*ln(exp(I*(b*x+a)))-3/2*I/b^4*d^3*a^4+I*c^
3*x+1/4*I/d*c^4-3*I/b^2*c*d^2*polylog(2,-exp(2*I*(b*x+a)))*x+6*I/b^3*d^3*x
*a
```

### 3.304.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 592 vs. 2(217) = 434.

Time = 0.27 (sec) , antiderivative size = 592, normalized size of antiderivative = 2.29

$$\int (c+dx)^3\tan^3(a+bx)\,dx$$
$$= \frac{4\,b^3d^3x^3 + 12\,b^3cd^2x^2 + 12\,b^3c^2dx - 3i\,d^3\mathrm{polylog}\left(4, \frac{\tan(bx+a)^2+2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right) + 3i\,d^3\mathrm{polylog}\left(4, \frac{\tan(bx+a)^2-2}{\tan(bx}\right.}{}$$

input
```
integrate((d*x+c)^3*tan(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/8*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x - 3*I*d^3*polylog(4
, (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 3*I*d^3*
polylog(4, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) +
 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*tan(b*x + a)^
2 - 6*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d + I*d^3)*dilog(2*(I*
tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) - 6*(I*b^2*d^3*x^2 + 2*I*b^2*c
*d^2*x + I*b^2*c^2*d - I*d^3)*dilog(2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^
2 + 1) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 3*b*c*d^2 + 3*(
b^3*c^2*d - b*d^3)*x)*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) +
4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 3*b*c*d^2 + 3*(b^3*c^2*d - b*
d^3)*x)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 6*(b*d^3*x +
b*c*d^2)*polylog(3, (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^
2 + 1)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, (tan(b*x + a)^2 - 2*I*tan(b*x +
 a) - 1)/(tan(b*x + a)^2 + 1)) - 12*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2
*d)*tan(b*x + a))/b^4
```

### 3.304.6 Sympy [F]

$$\int (c+dx)^3\tan^3(a+bx)\,dx = \int (c+dx)^3\tan^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*tan(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**3*tan(a + b*x)**3, x)
```

### 3.304.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2413 vs. $2(217) = 434$.

Time = 0.90 (sec) , antiderivative size = 2413, normalized size of antiderivative = 9.32

$$\int (c+dx)^3\tan^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*tan(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(c^3*(1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1)) - 3*a*c^2*d*(
1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b + 3*a^2*c*d^2*(1/(sin(
b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b^2 - a^3*d^3*(1/(sin(b*x + a)^
2 - 1) - log(sin(b*x + a)^2 - 1))/b^3 + 2*(3*(b*x + a)^4*d^3 + 36*b^2*c^2*
d - 72*a*b*c*d^2 + 36*a^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a)^3 + 18*(b^2
*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a)^2 - 4*(4*(b*x + a)^3*d^3 - 9*b*c
*d^2 + 9*a*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*
d^2 + (a^2 - 1)*d^3)*(b*x + a) + (4*(b*x + a)^3*d^3 - 9*b*c*d^2 + 9*a*d^3
+ 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)
*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 2*(4*(b*x + a)^3*d^3 - 9*b*c*d^2 + 9*a
*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2
 - 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-4*I*(b*x + a)^3*d^3 + 9*I*b*c*d
^2 - 9*I*a*d^3 + 9*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 9*(-I*b^2*c^2*d +
2*I*a*b*c*d^2 + (-I*a^2 + I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*(-4*I*(b
*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + 9*(-I*b*c*d^2 + I*a*d^3)*(b*x +
a)^2 + 9*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*a^2 + I)*d^3)*(b*x + a))*sin(
2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + 3*((b*x +
a)^4*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a)^3 + 6*(b^2*c^2*d - 2*a*b*c*d^2 +
(a^2 - 2)*d^3)*(b*x + a)^2 - 24*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4
*a) + 6*((b*x + a)^4*d^3 + 6*b^2*c^2*d - 12*a*b*c*d^2 + 6*a^2*d^3 + 4*(...
```

### 3.304.8 Giac [F]

$$\int (c+dx)^3 \tan^3(a+bx)\,dx = \int (dx+c)^3 \tan(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^3*tan(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*tan(b*x + a)^3, x)
```

### 3.304.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3\tan^3(a+bx)\,dx=\int \tan(a+b\,x)^3\,(c+d\,x)^3\,dx$$

input
```
int(tan(a + b*x)^3*(c + d*x)^3,x)
```

output
```
int(tan(a + b*x)^3*(c + d*x)^3, x)
```

# 3.305 $\int (c+dx)^2 \tan^3(a+bx)\,dx$

3.305.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2301
3.305.2 Mathematica [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2301
3.305.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2303
3.305.4 Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2306
3.305.5 Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 2307
3.305.6 Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2308
3.305.7 Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 2308
3.305.8 Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2309
3.305.9 Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2309

## 3.305.1 Optimal result

Integrand size = 16, antiderivative size = 169

$$\int (c+dx)^2 \tan^3(a+bx)\,dx = \frac{cdx}{b}+\frac{d^2x^2}{2b}-\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3}-\frac{id(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} + \frac{d^2\,\mathrm{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} - \frac{d(c+dx)\tan(a+bx)}{b^2}+\frac{(c+dx)^2\tan^2(a+bx)}{2b}$$

output
```
c*d*x/b+1/2*d^2*x^2/b-1/3*I*(d*x+c)^3/d+(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b
-d^2*ln(cos(b*x+a))/b^3-I*d*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2+1/2*d
^2*polylog(3,-exp(2*I*(b*x+a)))/b^3-d*(d*x+c)*tan(b*x+a)/b^2+1/2*(d*x+c)^2
*tan(b*x+a)^2/b
```

## 3.305.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 454 vs. 2(169) = 338.

Time = 6.74 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.69

$$\int (c+dx)^2 \tan^3(a+bx)\,dx$$

$$= \frac{id^2e^{-ia}\left(2b^2x^2\left(2bx-3i(1+e^{2ia})\log\left(1+e^{-2i(a+bx)}\right)\right)+6b(1+e^{2ia})\,x\,\text{PolyLog}\left(2,-e^{-2i(a+bx)}\right)-3i(1+e\right.}{12b^3}$$

$$+\frac{(c+dx)^2\sec^2(a+bx)}{2b}$$

$$+\frac{c^2\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))}{b\left(\cos^2(a)+\sin^2(a)\right)}$$

$$-\frac{d^2\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))}{b^3\left(\cos^2(a)+\sin^2(a)\right)}$$

$$+\frac{cd\csc(a)\left(b^2e^{-i\arctan(\cot(a))}x^2-\frac{\cot(a)(ibx(-\pi-2\arctan(\cot(a)))-\pi\log(1+e^{-2ibx})-2(bx-\arctan(\cot(a)))\log(1-e^{2i(bx-\arctan(c}}{\sqrt{}}\right.}{b^2\sqrt{\csc^2(a)\left(\cos^2(a)+s\right.}}$$

$$+\frac{\sec(a)\sec(a+bx)\left(-cd\sin(bx)-d^2x\sin(bx)\right)}{b^2}-\frac{1}{3}x\left(3c^2+3cdx+d^2x^2\right)\tan(a)$$

input
```
Integrate[(c + d*x)^2*Tan[a + b*x]^3,x]
```

output
```
((I/12)*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*
(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] -
 (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E
^(I*a)) + ((c + d*x)^2*Sec[a + b*x]^2)/(2*b) + (c^2*Sec[a]*(Cos[a]*Log[Cos
[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) -
 (d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))
/(b^3*(Cos[a]^2 + Sin[a]^2)) + (c*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]])
 - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] -
2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Lo
g[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[
2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*Sq
rt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Sec[a]*Sec[a + b*x]*(-(c*d*Sin[b*x]
) - d^2*x*Sin[b*x]))/b^2 - (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Tan[a])/3
```

## 3.305. $\int (c+dx)^2 \tan^3(a+bx)\,dx$

### 3.305.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 4203, 3042, 4202, 2620, 3011, 2720, 4203, 17, 3042, 3956, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\tan^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2\tan(a+bx)^3dx$$

$$\Big\downarrow \text{4203}$$

$$-\frac{d\int (c+dx)\tan^2(a+bx)dx}{b}-\int (c+dx)^2\tan(a+bx)dx+\frac{(c+dx)^2\tan^2(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$-\int (c+dx)^2\tan(a+bx)dx-\frac{d\int (c+dx)\tan(a+bx)^2dx}{b}+\frac{(c+dx)^2\tan^2(a+bx)}{2b}$$

$$\Big\downarrow \text{4202}$$

$$2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}dx-\frac{d\int (c+dx)\tan(a+bx)^2dx}{b}+\frac{(c+dx)^2\tan^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

$$\Big\downarrow \text{2620}$$

$$2i\left(\frac{id\int (c+dx)\log\left(1+e^{2i(a+bx)}\right)dx}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{d\int (c+dx)\tan(a+bx)^2dx}{b}+\frac{(c+dx)^2\tan^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

$$\Big\downarrow \text{3011}$$

$$2i\left(\frac{id\left(\frac{i(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int \text{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{2b}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{d\int (c+dx)\tan(a+bx)^2dx}{b}+\frac{(c+dx)^2\tan^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

$$\Big\downarrow \text{2720}$$

$$2i\left(\frac{id\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\int e^{-2i(a+bx)}\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{d\int(c+dx)\tan(a+bx)^2dx}{b}+\frac{(c+dx)^2\tan^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

$\Big\downarrow$ 4203

$$2i\left(\frac{id\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\int e^{-2i(a+bx)}\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{d\left(-\frac{d\int\tan(a+bx)dx}{b}-\int(c+dx)dx+\frac{(c+dx)\tan(a+bx)}{b}\right)}{b}+\frac{(c+dx)^2\tan^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

$\Big\downarrow$ 17

$$2i\left(\frac{id\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\int e^{-2i(a+bx)}\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{d\left(-\frac{d\int\tan(a+bx)dx}{b}+\frac{(c+dx)\tan(a+bx)}{b}-\frac{(c+dx)^2}{2d}\right)}{b}+\frac{(c+dx)^2\tan^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

$\Big\downarrow$ 3042

$$2i\left(\frac{id\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\int e^{-2i(a+bx)}\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{d\left(-\frac{d\int\tan(a+bx)dx}{b}+\frac{(c+dx)\tan(a+bx)}{b}-\frac{(c+dx)^2}{2d}\right)}{b}+\frac{(c+dx)^2\tan^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

$\Big\downarrow$ 3956

$$2i\left(\frac{id\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\int e^{-2i(a+bx)}\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{d\left(\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}-\frac{(c+dx)^2}{2d}\right)}{b}+\frac{(c+dx)^2\tan^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

$\Big\downarrow$ 7143

---

## 3.305. $\int(c+dx)^2\tan^3(a+bx)\,dx$

$$2i\left(\frac{id\left(\frac{i(c+dx)\operatorname{PolyLog}(2,-e^{2i(a+bx)})}{2b}-\frac{d\operatorname{PolyLog}(3,-e^{2i(a+bx)})}{4b^2}\right)}{b}-\frac{i(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)-\frac{d\left(\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}-\frac{(c+dx)^2}{2d}\right)}{b}+\frac{(c+dx)^2\tan^2(a+bx)}{2b}-\frac{i(c+dx)^3}{3d}$$

input
```
Int[(c + d*x)^2*Tan[a + b*x]^3,x]
```

output
```
((-1/3*I)*(c + d*x)^3)/d + (2*I)*(((-1/2*I)*(c + d*x)^2*Log[1 + E^((2*I)*(
a + b*x))])/b + (I*d*(((I/2)*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b
 - (d*PolyLog[3, -E^((2*I)*(a + b*x))])/(4*b^2)))/b) + ((c + d*x)^2*Tan[a
+ b*x]^2)/(2*b) - (d*(-1/2*(c + d*x)^2/d + (d*Log[Cos[a + b*x]])/b^2 + ((c
 + d*x)*Tan[a + b*x])/b))/b
```

### 3.305.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.305.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(153) = 306$.

Time = 0.70 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.42

| method | result |
|---|---|
| risch | $-\frac{2idc\,a^2}{b^2}+\frac{2id^2a^2x}{b^2}-\frac{icd\,\text{polylog}\left(2,-\mathrm{e}^{2i(xb+a)}\right)}{b^2}+\frac{4id^2a^3}{3b^3}+\frac{2b\,d^2x^2\mathrm{e}^{2i(xb+a)}+4bcdx\,\mathrm{e}^{2i(xb+a)}+2b\,c^2\mathrm{e}^{2i(xb+a)}-2id^2x\,\mathrm{e}^{2i(x}}{b^2\left(\mathrm{e}^{2i(xb+a)}+1\right)^2}$ |

input
```
int((d*x+c)^2*tan(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*I/b^2*d*c*a^2+2*I/b^2*d^2*a^2*x-I/b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))+
4/3*I/b^3*d^2*a^3+2*(b*d^2*x^2*exp(2*I*(b*x+a))+2*b*c*d*x*exp(2*I*(b*x+a))
+b*c^2*exp(2*I*(b*x+a))-I*d^2*x*exp(2*I*(b*x+a))-I*c*d*exp(2*I*(b*x+a))-I*
d^2*x-I*d*c)/b^2/(exp(2*I*(b*x+a))+1)^2+I*c^2*x+1/b*d^2*ln(exp(2*I*(b*x+a)
)+1)*x^2-1/3*I*d^2*x^3+4/b^2*c*d*a*ln(exp(I*(b*x+a)))-I/b^2*d^2*polylog(2,
-exp(2*I*(b*x+a)))*x+1/3*I/d*c^3-4*I/b*d*c*x*a-2/b*c^2*ln(exp(I*(b*x+a)))-
2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))-I*d*c*x^2+1/b*c^2*ln(exp(2*I*(b*x+a))+1)+
2/b*c*d*ln(exp(2*I*(b*x+a))+1)*x+1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3-
1/b^3*d^2*ln(exp(2*I*(b*x+a))+1)+2/b^3*d^2*ln(exp(I*(b*x+a)))
```

### 3.305.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(150) = 300$.

Time = 0.26 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.09

$$\int (c+dx)^2\tan^3(a+bx)\,dx = \frac{2\,b^2d^2x^2 + 4\,b^2cdx + d^2\text{polylog}\left(3, \frac{\tan(bx+a)^2+2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right) + d^2\text{polylog}\left(3, \frac{\tan(bx+a)^2-2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right) + 2\,(b^2 \cdots}{\cdots}$$

input
```
integrate((d*x+c)^2*tan(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + d^2*polylog(3, (tan(b*x + a)^2 + 2*I*ta
n(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + d^2*polylog(3, (tan(b*x + a)^2 - 2
*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2)*tan(b*x + a)^2 - 2*(-I*b*d^2*x - I*b*c*d)*dilog(2*(I*tan(b*x +
a) - 1)/(tan(b*x + a)^2 + 1) + 1) - 2*(I*b*d^2*x + I*b*c*d)*dilog(2*(-I*ta
n(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2 - d^2)*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 2*(b^2
*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - d^2)*log(-2*(-I*tan(b*x + a) - 1)/(tan(
b*x + a)^2 + 1)) - 4*(b*d^2*x + b*c*d)*tan(b*x + a))/b^3
```

### 3.305.6 Sympy [F]

$$\int (c+dx)^2 \tan^3(a+bx)\,dx = \int (c+dx)^2 \tan^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*tan(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**2*tan(a + b*x)**3, x)
```

### 3.305.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1228 vs. $2(150) = 300$.

Time = 0.50 (sec) , antiderivative size = 1228, normalized size of antiderivative = 7.27

$$\int (c+dx)^2 \tan^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*tan(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(c^2*(1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1)) - 2*a*c*d*(1/
(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b + a^2*d^2*(1/(sin(b*x +
a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b^2 + 2*(2*(b*x + a)^3*d^2 + 6*(b*c*d
 - a*d^2)*(b*x + a)^2 + 12*b*c*d - 12*a*d^2 - 6*((b*x + a)^2*d^2 + 2*(b*c*
d - a*d^2)*(b*x + a) - d^2 + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a
) - d^2)*cos(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x +
a) - d^2)*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*
(b*x + a) + I*d^2)*sin(4*b*x + 4*a) - 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d
+ I*a*d^2)*(b*x + a) + I*d^2)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a),
cos(2*b*x + 2*a) + 1) + 2*((b*x + a)^3*d^2 + 3*(b*c*d - a*d^2)*(b*x + a)^2
 - 6*(b*x + a)*d^2)*cos(4*b*x + 4*a) + 4*((b*x + a)^3*d^2 + 3*(b*c*d - (a
- I)*d^2)*(b*x + a)^2 + 3*b*c*d - 3*a*d^2 + 3*(2*I*b*c*d + (-2*I*a - 1)*d^
2)*(b*x + a))*cos(2*b*x + 2*a) + 6*(b*c*d + (b*x + a)*d^2 - a*d^2 + (b*c*d
 + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) + 2*(b*c*d + (b*x + a)*d^2 - a*
d^2)*cos(2*b*x + 2*a) + (I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(4*b*x +
4*a) + 2*(I*b*c*d + I*(b*x + a)*d^2 - I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^
(2*I*b*x + 2*I*a)) + 3*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a
) - I*d^2 + (I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a) - I*d^2)*
cos(4*b*x + 4*a) + 2*(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a)
- I*d^2)*cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x +...
```

### 3.305.8 Giac [F]

$$\int (c+dx)^2\tan^3(a+bx)\,dx=\int (dx+c)^2\tan\left(bx+a\right)^3\,dx$$

input
```
integrate((d*x+c)^2*tan(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*tan(b*x + a)^3, x)
```

### 3.305.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2\tan^3(a+bx)\,dx=\int \tan(a+b\,x)^3\,(c+d\,x)^2\,dx$$

input
```
int(tan(a + b*x)^3*(c + d*x)^2,x)
```

output
```
int(tan(a + b*x)^3*(c + d*x)^2, x)
```

# 3.306 $\int (c+dx)\tan^3(a+bx)\,dx$

3.306.1 Optimal result . . . . . . . . . . 2310
3.306.2 Mathematica [B] (verified) . . . . . . . . . . 2310
3.306.3 Rubi [A] (verified) . . . . . . . . . . 2311
3.306.4 Maple [A] (verified) . . . . . . . . . . 2314
3.306.5 Fricas [A] (verification not implemented) . . . . . . . . . . 2314
3.306.6 Sympy [F] . . . . . . . . . . 2315
3.306.7 Maxima [B] (verification not implemented) . . . . . . . . . . 2315
3.306.8 Giac [F] . . . . . . . . . . 2316
3.306.9 Mupad [F(-1)] . . . . . . . . . . 2316

## 3.306.1 Optimal result

Integrand size = 14, antiderivative size = 108

$$\int (c+dx)\tan^3(a+bx)\,dx=\frac{dx}{2b}-\frac{i(c+dx)^2}{2d}+\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{id\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{d\tan(a+bx)}{2b^2}+\frac{(c+dx)\tan^2(a+bx)}{2b}$$

output
```
1/2*d*x/b-1/2*I*(d*x+c)^2/d+(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b-1/2*I*d*polyl
og(2,-exp(2*I*(b*x+a)))/b^2-1/2*d*tan(b*x+a)/b^2+1/2*(d*x+c)*tan(b*x+a)^2/
b
```

## 3.306.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 240 vs. 2(108) = 216.

Time = 6.26 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.22

$$\int (c+dx)\tan^3(a+bx)\,dx=\frac{dx\sec^2(a+bx)}{2b}+\frac{d\csc(a)\left(b^2e^{-i\arctan(\cot(a))}x^2-\frac{\cot(a)\left(ibx(-\pi-2\arctan(\cot(a)))-\pi\log\left(1+e^{-2ibx}\right)-2(bx-\arctan(\cot(a)))\log\left(1-e^{2i(bx-\arctan(\cot(a)))}\right)\cdots\right)}{\sqrt{\cdots}}\right)}{2b^2\sqrt{\csc^2(a)\left(\cos^2(a)+\sin^2(a)\right)}}-\frac{d\sec(a)\sec(a+bx)\sin(bx)}{2b^2}-\frac{1}{2}dx^2\tan(a)+\frac{c\left(2\log(\cos(a+bx))+\tan^2(a+bx)\right)}{2b}$$

input
```
Integrate[(c + d*x)*Tan[a + b*x]^3,x]
```

output
```
(d*x*Sec[a + b*x]^2)/(2*b) + (d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (
Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b
*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Co
s[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E
^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt
[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (d*Sec[a]*Sec[a + b*x]*Sin[b*x])/(2*b^
2) - (d*x^2*Tan[a])/2 + (c*(2*Log[Cos[a + b*x]] + Tan[a + b*x]^2))/(2*b)
```

## 3.306.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3042, 4203, 3042, 3954, 24, 4202, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\tan^3(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)\tan(a+bx)^3dx$$

$$\Big\downarrow \text{ 4203}$$

$$-\int (c+dx)\tan(a+bx)dx-\frac{d\int \tan^2(a+bx)dx}{2b}+\frac{(c+dx)\tan^2(a+bx)}{2b}$$

$$\Big\downarrow \text{ 3042}$$

$$-\int (c+dx)\tan(a+bx)dx-\frac{d\int \tan(a+bx)^2dx}{2b}+\frac{(c+dx)\tan^2(a+bx)}{2b}$$

$$\Big\downarrow \text{ 3954}$$

$$-\int (c+dx)\tan(a+bx)dx-\frac{d\left(\frac{\tan(a+bx)}{b}-\int 1dx\right)}{2b}+\frac{(c+dx)\tan^2(a+bx)}{2b}$$

$$\Big\downarrow \text{ 24}$$

$$-\int (c+dx)\tan(a+bx)dx+\frac{(c+dx)\tan^2(a+bx)}{2b}-\frac{d\left(\frac{\tan(a+bx)}{b}-x\right)}{2b}$$

$$\Big\downarrow \text{4202}$$

$$2i\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}dx+\frac{(c+dx)\tan^2(a+bx)}{2b}-\frac{d\left(\frac{\tan(a+bx)}{b}-x\right)}{2b}-\frac{i(c+dx)^2}{2d}$$

$$\Big\downarrow \text{2620}$$

$$2i\left(\frac{id\int \log\left(1+e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)+\frac{(c+dx)\tan^2(a+bx)}{2b}-\frac{d\left(\frac{\tan(a+bx)}{b}-x\right)}{2b}-\frac{i(c+dx)^2}{2d}$$

$$\Big\downarrow \text{2715}$$

$$2i\left(\frac{d\int e^{-2i(a+bx)}\log\left(1+e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)+\frac{(c+dx)\tan^2(a+bx)}{2b}-\frac{d\left(\frac{\tan(a+bx)}{b}-x\right)}{2b}-\frac{i(c+dx)^2}{2d}$$

$$\Big\downarrow \text{2838}$$

$$2i\left(-\frac{d\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{i(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{2b}\right)+\frac{(c+dx)\tan^2(a+bx)}{2b}-\frac{d\left(\frac{\tan(a+bx)}{b}-x\right)}{2b}-\frac{i(c+dx)^2}{2d}$$

input
```
Int[(c + d*x)*Tan[a + b*x]^3,x]
```

output
```
((-1/2*I)*(c + d*x)^2)/d + (2*I)*(((-1/2*I)*(c + d*x)*Log[1 + E^((2*I)*(a
+ b*x))])/b - (d*PolyLog[2, -E^((2*I)*(a + b*x))])/(4*b^2)) + ((c + d*x)*T
an[a + b*x]^2)/(2*b) - (d*(-x + Tan[a + b*x]/b))/(2*b)
```

3.306. $\int (c+dx)\tan^3(a+bx)\,dx$

### 3.306.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4202
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Simp[2*I   Int[(c + d*x)^m*(E^(2*I*(
e + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGt
Q[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

## 3.306. $\int (c+dx)\tan^3(a+bx)\,dx$

### 3.306.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.69

| method | result |
|---|---|
| risch | $-\frac{id\,x^2}{2}+icx+\frac{2bdx\,e^{2i(xb+a)}-id\,e^{2i(xb+a)}+2bc\,e^{2i(xb+a)}-id}{b^2\left(e^{2i(xb+a)}+1\right)^2}+\frac{c\ln\left(e^{2i(xb+a)}+1\right)}{b}-\frac{2c\ln\left(e^{i(xb+a)}\right)}{b}-\frac{2idxa}{b}-\frac{id\,a^2}{b^2}+$ |

input
```
int((d*x+c)*tan(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*d*x^2+I*c*x+(2*b*d*x*exp(2*I*(b*x+a))-I*d*exp(2*I*(b*x+a))+2*b*c*ex
p(2*I*(b*x+a))-I*d)/b^2/(exp(2*I*(b*x+a))+1)^2+1/b*c*ln(exp(2*I*(b*x+a))+1
)-2/b*c*ln(exp(I*(b*x+a)))-2*I/b*d*x*a-I/b^2*d*a^2+1/b*d*ln(exp(2*I*(b*x+a
))+1)*x-1/2*I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2+2/b^2*d*a*ln(exp(I*(b*x+a
)))
```

### 3.306.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.56

$$\int (c+dx)\tan^3(a+bx)\,dx$$
$$=\frac{2\,bdx+2\,(bdx+bc)\tan\left(bx+a\right)^2+i\,d\mathrm{Li}_2\left(\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)-i\,d\mathrm{Li}_2\left(\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)+2\,(bdx+}{4\,b^2}$$

input
```
integrate((d*x+c)*tan(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(2*b*d*x + 2*(b*d*x + b*c)*tan(b*x + a)^2 + I*d*dilog(2*(I*tan(b*x + a
) - 1)/(tan(b*x + a)^2 + 1) + 1) - I*d*dilog(2*(-I*tan(b*x + a) - 1)/(tan(
b*x + a)^2 + 1) + 1) + 2*(b*d*x + b*c)*log(-2*(I*tan(b*x + a) - 1)/(tan(b*
x + a)^2 + 1)) + 2*(b*d*x + b*c)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a
)^2 + 1)) - 2*d*tan(b*x + a))/b^2
```

### 3.306.6 Sympy [F]

$$\int (c+dx)\tan^3(a+bx)\,dx = \int (c+dx)\tan^3(a+bx)\,dx$$

input
```
integrate((d*x+c)*tan(b*x+a)**3,x)
```

output
```
Integral((c + d*x)*tan(a + b*x)**3, x)
```

### 3.306.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 517 vs. $2(89) = 178$.

Time = 0.44 (sec) , antiderivative size = 517, normalized size of antiderivative = 4.79

$$\int (c+dx)\tan^3(a+bx)\,dx = -\frac{b^2dx^2 + 2\,b^2cx - 2\,(bdx + bc + (bdx + bc)\cos(4\,bx + 4\,a) + 2\,(bdx + bc)\cos(2\,bx + 2\,a) - (-i\,bdx - i\,b}{}$$

input
```
integrate((d*x+c)*tan(b*x+a)^3,x, algorithm="maxima")
```

output
```
-(b^2*d*x^2 + 2*b^2*c*x - 2*(b*d*x + b*c + (b*d*x + b*c)*cos(4*b*x + 4*a)
+ 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (-I*b*d*x - I*b*c)*sin(4*b*x + 4*a) -
 2*(-I*b*d*x - I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*
x + 2*a) + 1) + (b^2*d*x^2 + 2*b^2*c*x)*cos(4*b*x + 4*a) + 2*(b^2*d*x^2 +
2*I*b*c + 2*(b^2*c + I*b*d)*x + d)*cos(2*b*x + 2*a) + (d*cos(4*b*x + 4*a)
+ 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) + 2*I*d*sin(2*b*x + 2*a) + d
)*dilog(-e^(2*I*b*x + 2*I*a)) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos
(4*b*x + 4*a) - 2*(I*b*d*x + I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(4
*b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + s
in(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - (-I*b^2*d*x^2 - 2*I*b^2*c*x)
*sin(4*b*x + 4*a) + 2*(I*b^2*d*x^2 - 2*b*c + 2*(I*b^2*c - b*d)*x + I*d)*si
n(2*b*x + 2*a) + 2*d)/(-2*I*b^2*cos(4*b*x + 4*a) - 4*I*b^2*cos(2*b*x + 2*a
) + 2*b^2*sin(4*b*x + 4*a) + 4*b^2*sin(2*b*x + 2*a) - 2*I*b^2)
```

### 3.306.8 Giac [F]

$$\int (c+dx)\tan^3(a+bx)\,dx = \int (dx+c)\tan{(bx+a)}^3\,dx$$

input
```
integrate((d*x+c)*tan(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)*tan(b*x + a)^3, x)
```

### 3.306.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\tan^3(a+bx)\,dx = \int {\tan(a+b\,x)}^3\,(c+d\,x)\,dx$$

input
```
int(tan(a + b*x)^3*(c + d*x),x)
```

output
```
int(tan(a + b*x)^3*(c + d*x), x)
```

# 3.307 $\int \frac{\tan^3(a+bx)}{c+dx}\,dx$

3.307.1 Optimal result . . . . . 2317
3.307.2 Mathematica [N/A] . . . . . 2317
3.307.3 Rubi [N/A] . . . . . 2318
3.307.4 Maple [N/A] (verified) . . . . . 2319
3.307.5 Fricas [N/A] . . . . . 2319
3.307.6 Sympy [N/A] . . . . . 2319
3.307.7 Maxima [N/A] . . . . . 2320
3.307.8 Giac [N/A] . . . . . 2320
3.307.9 Mupad [N/A] . . . . . 2321

## 3.307.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\tan^3(a+bx)}{c+dx},x\right)$$

output
```
Unintegrable(tan(b*x+a)^3/(d*x+c),x)
```

## 3.307.2 Mathematica [N/A]

Not integrable

Time = 9.48 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx = \int \frac{\tan^3(a+bx)}{c+dx}\,dx$$

input
```
Integrate[Tan[a + b*x]^3/(c + d*x),x]
```

output
```
Integrate[Tan[a + b*x]^3/(c + d*x), x]
```

### 3.307.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(a+bx)}{c+dx} \, dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\tan(a+bx)^3}{c+dx}dx$$
$$\Big\downarrow \; 4222$$
$$\int \frac{\tan^3(a+bx)}{c+dx}dx$$

input
```
Int[Tan[a + b*x]^3/(c + d*x),x]
```

output
```
$Aborted
```

#### 3.307.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Uninteg
rable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

### 3.307.4 Maple [N/A] (verified)

Not integrable

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\tan (xb+a)^3}{dx+c}dx$$

input
```
int(tan(b*x+a)^3/(d*x+c),x)
```

output
```
int(tan(b*x+a)^3/(d*x+c),x)
```

### 3.307.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx=\int \frac{\tan (bx+a)^3}{dx+c}\,dx$$

input
```
integrate(tan(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
integral(tan(b*x + a)^3/(d*x + c), x)
```

### 3.307.6 Sympy [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx=\int \frac{\tan^3{(a+bx)}}{c+dx}\,dx$$

input
```
integrate(tan(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(tan(a + b*x)**3/(c + d*x), x)
```

### 3.307.7 Maxima [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 1077, normalized size of antiderivative = 67.31

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx = \int \frac{\tan{(bx+a)}^3}{dx+c}\,dx$$

input
```
integrate(tan(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output
```
(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
 (2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
+ 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
+ (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*s
in(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*
x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d
^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate(2*(b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2 - d^2)*sin(2*b*x + 2*a)/(b^2*d^3*x^3 + 3*b^2*c*d^
2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x
^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)), x) + (d*cos(2*b*x + 2*a)
+ 2*(b*d*x + b*c)*sin(2*b*x + 2*a) + d)*sin(4*b*x + 4*a) + d*sin(2*b*x + 2
*a))/(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2
*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 +
 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2...
```

### 3.307.8 Giac [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx = \int \frac{\tan{(bx+a)}^3}{dx+c}\,dx$$

input
```
integrate(tan(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output
```
integrate(tan(b*x + a)^3/(d*x + c), x)
```

### 3.307.9 Mupad [N/A]

Not integrable

Time = 25.57 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx=\int \frac{\tan(a+b\,x)^3}{c+d\,x}\,dx$$

input
```
int(tan(a + b*x)^3/(c + d*x),x)
```

output
```
int(tan(a + b*x)^3/(c + d*x), x)
```

# 3.308 $\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx$

3.308.1 Optimal result . . . . . 2322
3.308.2 Mathematica [N/A] . . . . . 2322
3.308.3 Rubi [N/A] . . . . . 2323
3.308.4 Maple [N/A] (verified) . . . . . 2324
3.308.5 Fricas [N/A] . . . . . 2324
3.308.6 Sympy [N/A] . . . . . 2324
3.308.7 Maxima [N/A] . . . . . 2325
3.308.8 Giac [N/A] . . . . . 2325
3.308.9 Mupad [N/A] . . . . . 2326

## 3.308.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\tan^3(a+bx)}{(c+dx)^2},x\right)$$

output
```
Unintegrable(tan(b*x+a)^3/(d*x+c)^2,x)
```

## 3.308.2 Mathematica [N/A]

Not integrable

Time = 9.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[Tan[a + b*x]^3/(c + d*x)^2,x]
```

output
```
Integrate[Tan[a + b*x]^3/(c + d*x)^2, x]
```

### 3.308.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {3042, 4222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2} \, dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\tan(a+bx)^3}{(c+dx)^2} dx$$
$$\Big\downarrow \; 4222$$
$$\int \frac{\tan^3(a+bx)}{(c+dx)^2} dx$$

input
```
Int[Tan[a + b*x]^3/(c + d*x)^2,x]
```

output
```
$Aborted
```

#### 3.308.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4222
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], I^
n*Unintegrable[(c + d*x)^m*Coth[(-I)*(e - Pi/2) - I*f*x]^n, x], I^n*Unintegrable[(c + d*x)^m*Tanh[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2],
 (-1)^n*Unintegrable[(c + d*x)^m*Cot[e - Pi/2 + f*x]^n, x], Unintegrable[(c
 + d*x)^m*Tan[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && Integ
erQ[n]
```

### 3.308.4 Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\tan (xb+a)^3}{(dx+c)^2}dx$$

input
```
int(tan(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int(tan(b*x+a)^3/(d*x+c)^2,x)
```

### 3.308.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan (bx+a)^3}{(dx+c)^2}\,dx$$

input
```
integrate(tan(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(tan(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.308.6 Sympy [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(tan(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(tan(a + b*x)**3/(c + d*x)**2, x)
```

### 3.308.7 Maxima [N/A]

Not integrable

Time = 6.83 (sec) , antiderivative size = 1386, normalized size of antiderivative = 86.62

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(bx+a)^3}{(dx+c)^2}\,dx$$

input
```
integrate(tan(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
 2*((b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
+ 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^
2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2
*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*
x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 +
3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^
3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2
*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + 2*(b^2*d^3*x^3 + 3*
b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a
) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x
+ 2*a))*integrate(2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 3*d^2)*sin(2*b*
x + 2*a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*
x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c
^3*d*x + b^2*c^4)*cos(2*b*x + 2*a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*
b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^4
*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(
2*b*x + 2*a)), x) + 2*(d*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(2*b*x + 2*a)
 + d)*sin(4*b*x + 4*a) + 2*d*sin(2*b*x + 2*a))/(b^2*d^3*x^3 + 3*b^2*c*d...
```

### 3.308.8 Giac [N/A]

Not integrable

Time = 12.46 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\tan(bx+a)^3}{(dx+c)^2}\,dx$$

input
```
integrate(tan(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(tan(b*x + a)^3/(d*x + c)^2, x)
```

### 3.308.9 Mupad [N/A]

Not integrable

Time = 28.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2} \, dx = \int \frac{\tan(a+b\,x)^3}{(c+d\,x)^2} \, dx$$

input
```
int(tan(a + b*x)^3/(c + d*x)^2,x)
```

output
```
int(tan(a + b*x)^3/(c + d*x)^2, x)
```

# 3.309 $\int (c+dx)^m \csc(a+bx) \sec^3(a+bx) \, dx$

3.309.1 Optimal result . . . . . . . . . . 2327
3.309.2 Mathematica [N/A] . . . . . . . . . . 2327
3.309.3 Rubi [N/A] . . . . . . . . . . 2328
3.309.4 Maple [N/A] (verified) . . . . . . . . . . 2328
3.309.5 Fricas [N/A] . . . . . . . . . . 2329
3.309.6 Sympy [F(-1)] . . . . . . . . . . 2329
3.309.7 Maxima [N/A] . . . . . . . . . . 2329
3.309.8 Giac [N/A] . . . . . . . . . . 2330
3.309.9 Mupad [N/A] . . . . . . . . . . 2330

## 3.309.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (c+dx)^m \csc(a+bx) \sec^3(a+bx) \, dx = \text{Int}\big((c+dx)^m \csc(a+bx) \sec^3(a+bx), x\big)$$

output
```
CannotIntegrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x)
```

## 3.309.2 Mathematica [N/A]

Not integrable

Time = 21.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc(a+bx) \sec^3(a+bx) \, dx = \int (c+dx)^m \csc(a+bx) \sec^3(a+bx) \, dx$$

input
```
Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

output
```
Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3, x]
```

## 3.309.3 Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc(a+bx)\sec^3(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{7299}$$

$$\int \csc(a+bx)\sec^3(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

output
```
$Aborted
```

### 3.309.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.309.4 Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \csc(xb+a) \sec(xb+a)^3 \,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x)
```

output
```
int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x)
```

### 3.309.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc(a+bx)\sec^3(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)\sec(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^3, x)
```

### 3.309.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m \csc(a+bx)\sec^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*csc(b*x+a)*sec(b*x+a)**3,x)
```

output
```
Timed out
```

### 3.309.7 Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc(a+bx)\sec^3(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)\sec(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^3, x)
```

### 3.309.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (c+dx)^m \csc(a+bx)\sec^3(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)\sec(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^3, x)
```

### 3.309.9 Mupad [N/A]

Not integrable

Time = 25.79 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int (c+dx)^m \csc(a+bx)\sec^3(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\cos(a+b\,x)^3\,\sin(a+b\,x)}\,dx$$

input
```
int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)),x)
```

output
```
int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)), x)
```

# 3.310 $\int (c+dx)^4 \csc(a+bx) \sec^3(a+bx) \, dx$


3.310.1 Optimal result . . . . . 2332
3.310.2 Mathematica [B] (warning: unable to verify) . . . . . 2333
3.310.3 Rubi [A] (verified) . . . . . 2334
3.310.4 Maple [B] (verified) . . . . . 2337
3.310.5 Fricas [F(-2)] . . . . . 2338
3.310.6 Sympy [F(-1)] . . . . . 2338
3.310.7 Maxima [B] (verification not implemented) . . . . . 2338
3.310.8 Giac [F] . . . . . 2339
3.310.9 Mupad [F(-1)] . . . . . 2340

### 3.310.1 Optimal result

Integrand size = 22, antiderivative size = 399

$$\int (c+dx)^4 \csc(a+bx)\sec^3(a+bx)\,dx = \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4\operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{6id^3(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^4} + \frac{2id(c+dx)^3\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} - \frac{2id(c+dx)^3\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2} - \frac{3d^4\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^5} - \frac{3d^2(c+dx)^2\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3} + \frac{3d^2(c+dx)^2\operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3} - \frac{3id^3(c+dx)\operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4} + \frac{3id^3(c+dx)\operatorname{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4} + \frac{3d^4\operatorname{PolyLog}\left(5,-e^{2i(a+bx)}\right)}{2b^5} - \frac{3d^4\operatorname{PolyLog}\left(5,e^{2i(a+bx)}\right)}{2b^5} - \frac{2d(c+dx)^3\tan(a+bx)}{b^2} + \frac{(c+dx)^4\tan^2(a+bx)}{2b}$$

output
```
2*I*d*(d*x+c)^3*polylog(2,-exp(2*I*(b*x+a)))/b^2+1/2*(d*x+c)^4/b-2*(d*x+c)
^4*arctanh(exp(2*I*(b*x+a)))/b-6*d^2*(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b^3-
2*I*d*(d*x+c)^3*polylog(2,exp(2*I*(b*x+a)))/b^2+6*I*d^3*(d*x+c)*polylog(2,
-exp(2*I*(b*x+a)))/b^4-3*I*d^3*(d*x+c)*polylog(4,-exp(2*I*(b*x+a)))/b^4-3*
d^4*polylog(3,-exp(2*I*(b*x+a)))/b^5-3*d^2*(d*x+c)^2*polylog(3,-exp(2*I*(b
*x+a)))/b^3+3*d^2*(d*x+c)^2*polylog(3,exp(2*I*(b*x+a)))/b^3+2*I*d*(d*x+c)^
3/b^2+3*I*d^3*(d*x+c)*polylog(4,exp(2*I*(b*x+a)))/b^4+3/2*d^4*polylog(5,-e
xp(2*I*(b*x+a)))/b^5-3/2*d^4*polylog(5,exp(2*I*(b*x+a)))/b^5-2*d*(d*x+c)^3
*tan(b*x+a)/b^2+1/2*(d*x+c)^4*tan(b*x+a)^2/b
```

### 3.310.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2227 vs. $2(399) = 798$.

Time = 7.65 (sec) , antiderivative size = 2227, normalized size of antiderivative = 5.58

$$\int (c+dx)^4 \csc(a+bx)\sec^3(a+bx)\,dx = \text{Result too large to show}$$

input
```
Integrate[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

output
```
-((c^2*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2
*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2
*Log[1 + E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, -E^((-I
)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, E^((-I)*(a + b*x))] +
(6*I)*(1 - E^((-2*I)*a))*PolyLog[3, -E^((-I)*(a + b*x))] + (6*I)*(1 - E^((
-2*I)*a))*PolyLog[3, E^((-I)*(a + b*x))]))/b^3) - (c*d^3*E^(I*a)*Csc[a]*((
b^4*x^4)/E^((2*I)*a) + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a
 + b*x))] + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] -
 6*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - 6*b^2*(1 -
 E^((-2*I)*a))*x^2*PolyLog[2, E^((-I)*(a + b*x))] + (12*I)*b*(1 - E^((-2*I
)*a))*x*PolyLog[3, -E^((-I)*(a + b*x))] + (12*I)*b*(1 - E^((-2*I)*a))*x*Po
lyLog[3, E^((-I)*(a + b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, -E^((-I)*(
a + b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, E^((-I)*(a + b*x))]))/b^4 -
(d^4*E^(I*a)*Csc[a]*((2*b^5*x^5)/E^((2*I)*a) + (5*I)*b^4*(1 - E^((-2*I)*a)
)*x^4*Log[1 - E^((-I)*(a + b*x))] + (5*I)*b^4*(1 - E^((-2*I)*a))*x^4*Log[1
 + E^((-I)*(a + b*x))] - 20*b^3*(1 - E^((-2*I)*a))*x^3*PolyLog[2, -E^((-I)
*(a + b*x))] - 20*b^3*(1 - E^((-2*I)*a))*x^3*PolyLog[2, E^((-I)*(a + b*x))
] + (60*I)*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[3, -E^((-I)*(a + b*x))] + (6
0*I)*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[3, E^((-I)*(a + b*x))] + 120*b*(1
- E^((-2*I)*a))*x*PolyLog[4, -E^((-I)*(a + b*x))] + 120*b*(1 - E^((-2*I...
```

### 3.310.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4920, 27, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4\csc(a+bx)\sec^3(a+bx)\,dx$$

$$\Big\downarrow \text{4920}$$

$$-4d\int \frac{1}{2}(c+dx)^3\left(\frac{\tan^2(a+bx)}{b}+\frac{2\log(\tan(a+bx))}{b}\right)dx+\frac{(c+dx)^4\tan^2(a+bx)}{2b}+\frac{(c+dx)^4\log(\tan(a+bx))}{b}$$

$$\Big\downarrow \text{27}$$

$$-2d\int (c+dx)^3\left(\frac{\tan^2(a+bx)}{b}+\frac{2\log(\tan(a+bx))}{b}\right)dx+\frac{(c+dx)^4\tan^2(a+bx)}{2b}+\frac{(c+dx)^4\log(\tan(a+bx))}{b}$$

$\downarrow$ 7292

$$-2d\int \frac{(c+dx)^3\left(\tan^2(a+bx)+2\log(\tan(a+bx))\right)}{b}dx+\frac{(c+dx)^4\tan^2(a+bx)}{2b}+\frac{(c+dx)^4\log(\tan(a+bx))}{b}$$

$\downarrow$ 27

$$-\frac{2d\int (c+dx)^3\left(\tan^2(a+bx)+2\log(\tan(a+bx))\right)dx}{b}+\frac{(c+dx)^4\tan^2(a+bx)}{2b}+\frac{(c+dx)^4\log(\tan(a+bx))}{b}$$

$\downarrow$ 7293

$$-\frac{2d\int \left(\tan^2(a+bx)(c+dx)^3+2\log(\tan(a+bx))(c+dx)^3\right)dx}{b}+\frac{(c+dx)^4\tan^2(a+bx)}{2b}+\frac{(c+dx)^4\log(\tan(a+bx))}{b}$$

$\downarrow$ 2009

$$-\frac{2d\left(\frac{(c+dx)^4\text{arctanh}\left(e^{2i(a+bx)}\right)}{d}+\frac{3d^3\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^4}-\frac{3d^3\text{PolyLog}\left(5,-e^{2i(a+bx)}\right)}{4b^4}+\frac{3d^3\text{PolyLog}\left(5,e^{2i(a+bx)}\right)}{4b^4}-\frac{3id^2(c+dx)\ldots}{}\right.}{}$$

$$\frac{(c+dx)^4\tan^2(a+bx)}{2b}+\frac{(c+dx)^4\log(\tan(a+bx))}{b}$$

input
```
Int[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

3.310. $\int (c+dx)^4\csc(a+bx)\sec^3(a+bx)\,dx$

output
```
((c + d*x)^4*Log[Tan[a + b*x]])/b + ((c + d*x)^4*Tan[a + b*x]^2)/(2*b) - (
2*d*(((-I)*(c + d*x)^3)/b - (c + d*x)^4/(4*d) + ((c + d*x)^4*ArcTanh[E^((2
*I)*(a + b*x))])/d + (3*d*(c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b^2 +
((c + d*x)^4*Log[Tan[a + b*x]])/(2*d) - ((3*I)*d^2*(c + d*x)*PolyLog[2, -E
^((2*I)*(a + b*x))])/b^3 - (I*(c + d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))]
)/b + (I*(c + d*x)^3*PolyLog[2, E^((2*I)*(a + b*x))])/b + (3*d^3*PolyLog[3
, -E^((2*I)*(a + b*x))])/(2*b^4) + (3*d*(c + d*x)^2*PolyLog[3, -E^((2*I)*(
a + b*x))])/(2*b^2) - (3*d*(c + d*x)^2*PolyLog[3, E^((2*I)*(a + b*x))])/(2
*b^2) + (((3*I)/2)*d^2*(c + d*x)*PolyLog[4, -E^((2*I)*(a + b*x))])/b^3 - (
((3*I)/2)*d^2*(c + d*x)*PolyLog[4, E^((2*I)*(a + b*x))])/b^3 - (3*d^3*Poly
Log[5, -E^((2*I)*(a + b*x))])/(4*b^4) + (3*d^3*PolyLog[5, E^((2*I)*(a + b*
x))])/(4*b^4) + ((c + d*x)^3*Tan[a + b*x])/b))/b
```

### 3.310.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### 3.310.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1728 vs. 2(361) = 722.

Time = 1.03 (sec) , antiderivative size = 1729, normalized size of antiderivative = 4.33

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1729 |

input
```
int((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(b*d^4*x^4*exp(2*I*(b*x+a))+4*b*c*d^3*x^3*exp(2*I*(b*x+a))+6*b*c^2*d^2*x
^2*exp(2*I*(b*x+a))+4*b*c^3*d*x*exp(2*I*(b*x+a))-2*I*d^4*x^3*exp(2*I*(b*x+
a))+b*c^4*exp(2*I*(b*x+a))-6*I*c*d^3*x^2*exp(2*I*(b*x+a))-6*I*c^2*d^2*x*ex
p(2*I*(b*x+a))-2*I*d^4*x^3-2*I*c^3*d*exp(2*I*(b*x+a))-6*I*c*d^3*x^2-6*I*c^
2*d^2*x-2*I*c^3*d)/b^2/(exp(2*I*(b*x+a))+1)^2-3*d^4*polylog(3,-exp(2*I*(b*
x+a)))/b^5+24*I/b^3*d^3*c*x*a-12*I/b^2*d^3*c*polylog(2,-exp(I*(b*x+a)))*x^
2+6*I/b^2*d^3*c*polylog(2,-exp(2*I*(b*x+a)))*x^2-12*I/b^2*d^3*c*polylog(2,
exp(I*(b*x+a)))*x^2-12*I/b^2*c^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+6*I/b^2*
c^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-12*I/b^2*c^2*d^2*polylog(2,exp(I*(b
*x+a)))*x-12/b^3*d^3*c*ln(exp(2*I*(b*x+a))+1)*x+4/b*c^3*d*ln(1-exp(I*(b*x+
a)))*x+4/b^4*d^3*c*ln(1-exp(I*(b*x+a)))*a^3+12*I/b^4*d^3*c*a^2+6*I/b^4*d^3
*c*polylog(2,-exp(2*I*(b*x+a)))+2*I/b^2*d^4*polylog(2,-exp(2*I*(b*x+a)))*x
^3+3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5-4*I/b^2*c^3*d*polylog(2,exp(I*
(b*x+a)))+12*I/b^2*d^3*c*x^2+24*I/b^4*d^3*c*polylog(4,-exp(I*(b*x+a)))-3*I
/b^4*d^3*c*polylog(4,-exp(2*I*(b*x+a)))+24*I/b^4*d^3*c*polylog(4,exp(I*(b*
x+a)))+6*I/b^4*d^4*polylog(2,-exp(2*I*(b*x+a)))*x-4*I/b^2*d^4*polylog(2,-e
xp(I*(b*x+a)))*x^3-4*I/b^2*d^4*polylog(2,exp(I*(b*x+a)))*x^3-12*I/b^4*a^2*
d^4*x+24*I/b^4*d^4*polylog(4,-exp(I*(b*x+a)))*x-3*I/b^4*d^4*polylog(4,-exp
(2*I*(b*x+a)))*x+24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-24*d^4*polylog(5
,-exp(I*(b*x+a)))/b^5-24*d^4*polylog(5,exp(I*(b*x+a)))/b^5-1/b^5*d^4*ln...
```

### 3.310.5 Fricas [F(-2)]

Exception generated.

$$\int (c+dx)^4 \csc(a+bx) \sec^3(a+bx)\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   Too
many variables
```

### 3.310.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^4 \csc(a+bx) \sec^3(a+bx)\, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**4*csc(b*x+a)*sec(b*x+a)**3,x)
```

output
```
Timed out
```

### 3.310.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8853 vs. $2(352) = 704$.

Time = 4.27 (sec) , antiderivative size = 8853, normalized size of antiderivative = 22.19

$$\int (c+dx)^4 \csc(a+bx) \sec^3(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(c^4*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x
+ a)^2)) - 4*a*c^3*d*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - l
og(sin(b*x + a)^2))/b + 6*a^2*c^2*d^2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*
x + a)^2 - 1) - log(sin(b*x + a)^2))/b^2 - 4*a^3*c*d^3*(1/(sin(b*x + a)^2
- 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^3 + a^4*d^4*(1/(si
n(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^4 + 2
*(24*b^3*c^3*d - 72*a*b^2*c^2*d^2 + 72*a^2*b*c*d^3 - 24*a^3*d^4 + 4*(3*(b*
x + a)^4*d^4 + 9*b^2*c^2*d^2 - 18*a*b*c*d^3 + 9*a^2*d^4 + 8*(b*c*d^3 - a*d
^4)*(b*x + a)^3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b*x + a)^
2 + 6*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 + 1)*b*c*d^3 - (a^3 + 3*a)*d^4
)*(b*x + a) + (3*(b*x + a)^4*d^4 + 9*b^2*c^2*d^2 - 18*a*b*c*d^3 + 9*a^2*d^
4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2
+ 1)*d^4)*(b*x + a)^2 + 6*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 + 1)*b*c*d
^3 - (a^3 + 3*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) + 2*(3*(b*x + a)^4*d^4 +
 9*b^2*c^2*d^2 - 18*a*b*c*d^3 + 9*a^2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^
3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b*x + a)^2 + 6*(b^3*c^3
*d - 3*a*b^2*c^2*d^2 + 3*(a^2 + 1)*b*c*d^3 - (a^3 + 3*a)*d^4)*(b*x + a))*c
os(2*b*x + 2*a) - (-3*I*(b*x + a)^4*d^4 - 9*I*b^2*c^2*d^2 + 18*I*a*b*c*d^3
 - 9*I*a^2*d^4 + 8*(-I*b*c*d^3 + I*a*d^4)*(b*x + a)^3 + 9*(-I*b^2*c^2*d^2
+ 2*I*a*b*c*d^3 + (-I*a^2 - I)*d^4)*(b*x + a)^2 + 6*(-I*b^3*c^3*d + 3*I...
```

### 3.310.8 Giac [F]

$$\int (c+dx)^4\csc(a+bx)\sec^3(a+bx)\,dx=\int (dx+c)^4\csc\left(bx+a\right)\sec\left(bx+a\right)^3\,dx$$

input
```
integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*csc(b*x + a)*sec(b*x + a)^3, x)
```

### 3.310.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \csc(a+bx) \sec^3(a+bx) \, dx = \text{Hanged}$$

input
```
int((c + d*x)^4/(cos(a + b*x)^3*sin(a + b*x)),x)
```

output
```
\text{Hanged}
```

# 3.311 $\int (c+dx)^3 \csc(a+bx) \sec^3(a+bx) \, dx$


3.311.1 Optimal result . . . . . 2341
3.311.2 Mathematica [B] (warning: unable to verify) . . . . . 2342
3.311.3 Rubi [A] (verified) . . . . . 2343
3.311.4 Maple [B] (verified) . . . . . 2345
3.311.5 Fricas [B] (verification not implemented) . . . . . 2346
3.311.6 Sympy [F] . . . . . 2346
3.311.7 Maxima [B] (verification not implemented) . . . . . 2347
3.311.8 Giac [F] . . . . . 2347
3.311.9 Mupad [F(-1)] . . . . . 2348


## 3.311.1 Optimal result

Integrand size = 22, antiderivative size = 325

$$\begin{aligned}\int (c+dx)^3 \csc(a+bx) \sec^3(a+bx) \, dx =& \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} \\ &- \frac{2(c+dx)^3 \text{arctanh}\left(e^{2i(a+bx)}\right)}{b} \\ &- \frac{3d^2(c+dx) \log\left(1+e^{2i(a+bx)}\right)}{b^3} \\ &+ \frac{3id^3 \text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^4} \\ &+ \frac{3id(c+dx)^2 \text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} \\ &- \frac{3id(c+dx)^2 \text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2} \\ &- \frac{3d^2(c+dx) \text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} \\ &+ \frac{3d^2(c+dx) \text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3} \\ &- \frac{3id^3 \text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4} \\ &+ \frac{3id^3 \text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{4b^4} \\ &- \frac{3d(c+dx)^2 \tan(a+bx)}{2b^2} \\ &+ \frac{(c+dx)^3 \tan^2(a+bx)}{2b}\end{aligned}$$

output
```
3/2*I*d*(d*x+c)^2/b^2+1/2*(d*x+c)^3/b-2*(d*x+c)^3*arctanh(exp(2*I*(b*x+a))
)/b-3*d^2*(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b^3+3/2*I*d^3*polylog(2,-exp(2*I*
(b*x+a)))/b^4+3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+a)))/b^2-3/2*I*d*(
d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*x+c)*polylog(3,-exp(2*
I*(b*x+a)))/b^3+3/2*d^2*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/b^3-3/4*I*d^3*
polylog(4,-exp(2*I*(b*x+a)))/b^4+3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^4
-3/2*d*(d*x+c)^2*tan(b*x+a)/b^2+1/2*(d*x+c)^3*tan(b*x+a)^2/b
```

### 3.311.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1535 vs. $2(325) = 650$.

Time = 6.93 (sec) , antiderivative size = 1535, normalized size of antiderivative = 4.72

$$\int (c+dx)^3 \csc(a+bx) \sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
Integrate[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

output
```
-1/2*(c*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-
2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^
2*Log[1 + E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, -E^((-
I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, E^((-I)*(a + b*x))] +
 (6*I)*(1 - E^((-2*I)*a))*PolyLog[3, -E^((-I)*(a + b*x))] + (6*I)*(1 - E^(
(-2*I)*a))*PolyLog[3, E^((-I)*(a + b*x))]))/b^3 - (d^3*E^(I*a)*Csc[a]*((b^
4*x^4)/E^((2*I)*a) + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a +
 b*x))] + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - 6
*b^2*(1 - E^((-2*I)*a))*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - 6*b^2*(1 - E
^((-2*I)*a))*x^2*PolyLog[2, E^((-I)*(a + b*x))] + (12*I)*b*(1 - E^((-2*I)*
a))*x*PolyLog[3, -E^((-I)*(a + b*x))] + (12*I)*b*(1 - E^((-2*I)*a))*x*Poly
Log[3, E^((-I)*(a + b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, -E^((-I)*(a
+ b*x))] + 12*(1 - E^((-2*I)*a))*PolyLog[4, E^((-I)*(a + b*x))]))/(4*b^4)
+ (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Csc[a]*Sec[a])/4 - ((I/4)
*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a +
b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (3*I
)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(I*a
)) - ((I/8)*d^3*E^(I*a)*((2*b^4*x^4)/E^((2*I)*a) - (4*I)*b^3*(1 + E^((-2*I
)*a))*x^3*Log[1 + E^((-2*I)*(a + b*x))] + 6*b^2*(1 + E^((-2*I)*a))*x^2*Pol
yLog[2, -E^((-2*I)*(a + b*x))] - (6*I)*b*(1 + E^((-2*I)*a))*x*PolyLog[3...
```

### 3.311.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4920, 27, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \csc(a+bx)\sec^3(a+bx)\,dx$$

$$\Big\downarrow \text{4920}$$

$$-3d\int \frac{1}{2}(c+dx)^2\left(\frac{\tan^2(a+bx)}{b}+\frac{2\log(\tan(a+bx))}{b}\right)dx+\frac{(c+dx)^3\tan^2(a+bx)}{2b}+\frac{(c+dx)^3\log(\tan(a+bx))}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3}{2}d\int (c+dx)^2\left(\frac{\tan^2(a+bx)}{b}+\frac{2\log(\tan(a+bx))}{b}\right)dx+\frac{(c+dx)^3\tan^2(a+bx)}{2b}+\frac{(c+dx)^3\log(\tan(a+bx))}{b}$$

$$\Big\downarrow \text{7292}$$

$$-\frac{3}{2}d\int \frac{(c+dx)^2\left(\tan^2(a+bx)+2\log(\tan(a+bx))\right)}{b}dx+\frac{(c+dx)^3\tan^2(a+bx)}{2b}+\frac{(c+dx)^3\log(\tan(a+bx))}{b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3d\int (c+dx)^2\left(\tan^2(a+bx)+2\log(\tan(a+bx))\right)dx}{2b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}+\frac{(c+dx)^3\log(\tan(a+bx))}{b}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{3d\int \left(\tan^2(a+bx)(c+dx)^2+2\log(\tan(a+bx))(c+dx)^2\right)dx}{2b}+\frac{(c+dx)^3\tan^2(a+bx)}{2b}+\frac{(c+dx)^3\log(\tan(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3d\left(\frac{4(c+dx)^3\text{arctanh}(e^{2i(a+bx)})}{3d}-\frac{id^2\,\text{PolyLog}(2,-e^{2i(a+bx)})}{b^3}+\frac{id^2\,\text{PolyLog}(4,-e^{2i(a+bx)})}{2b^3}-\frac{id^2\,\text{PolyLog}(4,e^{2i(a+bx)})}{2b^3}+\frac{d(c+dx)\,\text{P}}{}\right.}{}$$

$$\frac{(c+dx)^3\tan^2(a+bx)}{2b}+\frac{(c+dx)^3\log(\tan(a+bx))}{b}$$

input
```
Int[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

output
```
((c + d*x)^3*Log[Tan[a + b*x]])/b + ((c + d*x)^3*Tan[a + b*x]^2)/(2*b) - (
3*d*(((-I)*(c + d*x)^2)/b - (c + d*x)^3/(3*d) + (4*(c + d*x)^3*ArcTanh[E^(
(2*I)*(a + b*x))])/(3*d) + (2*d*(c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b^
2 + (2*(c + d*x)^3*Log[Tan[a + b*x]])/(3*d) - (I*d^2*PolyLog[2, -E^((2*I)*
(a + b*x))])/b^3 - (I*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b + (I
*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b + (d*(c + d*x)*PolyLog[3,
-E^((2*I)*(a + b*x))])/b^2 - (d*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])
/b^2 + ((I/2)*d^2*PolyLog[4, -E^((2*I)*(a + b*x))])/b^3 - ((I/2)*d^2*PolyL
og[4, E^((2*I)*(a + b*x))])/b^3 + ((c + d*x)^2*Tan[a + b*x])/b))/(2*b)
```

### 3.311.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### 3.311.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1114 vs. $2(279) = 558$.

Time = 0.92 (sec) , antiderivative size = 1115, normalized size of antiderivative = 3.43

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1115 |

input
```
int((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*e
xp(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-
3*I*c^2*d*exp(2*I*(b*x+a))-3*I*d^3*x^2+2*b*c^3*exp(2*I*(b*x+a))-6*I*c*d^2*
x-3*I*c^2*d)/b^2/(exp(2*I*(b*x+a))+1)^2-3/2/b^3*d^3*polylog(3,-exp(2*I*(b*
x+a)))*x-3/2/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))-1/b*d^3*ln(exp(2*I*(b*
x+a))+1)*x^3-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4-3/b^3*d^2*c*ln(exp
(2*I*(b*x+a))+1)+6/b^3*d^2*c*ln(exp(I*(b*x+a)))-3/b^3*d^3*ln(exp(2*I*(b*x+
a))+1)*x-6/b^4*d^3*a*ln(exp(I*(b*x+a)))+3*I/b^2*d^3*x^2+3*I/b^4*d^3*a^2+3/
2*I*d^3*polylog(2,-exp(2*I*(b*x+a)))/b^4-1/b*c^3*ln(exp(2*I*(b*x+a))+1)-6*
I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x-6*I/b^2*c*d^2*polylog(2,-exp(I*(b*
x+a)))*x+3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)-3/b^2*c^2*d*a*ln(exp(I*(b*x+
a))-1)+3/b^2*d*c^2*ln(1-exp(I*(b*x+a)))*a-3/b^3*c*d^2*ln(1-exp(I*(b*x+a)))
*a^2+3*I/b^2*c*d^2*polylog(2,-exp(2*I*(b*x+a)))*x+1/b*c^3*ln(exp(I*(b*x+a)
)+1)+1/b*c^3*ln(exp(I*(b*x+a))-1)-3/b*c*d^2*ln(exp(2*I*(b*x+a))+1)*x^2-3/b
*c^2*d*ln(exp(2*I*(b*x+a))+1)*x+3/b*d*c^2*ln(1-exp(I*(b*x+a)))*x+3/b*d*c^2
*ln(exp(I*(b*x+a))+1)*x+3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2+3/b*c*d^2*ln(ex
p(I*(b*x+a))+1)*x^2-3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-3*I/b^2*d*c
^2*polylog(2,exp(I*(b*x+a)))-3*I/b^2*d*c^2*polylog(2,-exp(I*(b*x+a)))-3*I/
b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+1
/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x...
```

### 3.311.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2268 vs. $2(270) = 540$.

Time = 0.42 (sec) , antiderivative size = 2268, normalized size of antiderivative = 6.98

$$\int (c+dx)^3 \csc(a+bx)\sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 + 6*I*d^3*cos
(b*x + a)^2*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*cos(b*x +
a)^2*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*cos(b*x + a)^2*po
lylog(4, I*cos(b*x + a) + sin(b*x + a)) - 6*I*d^3*cos(b*x + a)^2*polylog(4
, I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*cos(b*x + a)^2*polylog(4, -I*co
s(b*x + a) + sin(b*x + a)) + 6*I*d^3*cos(b*x + a)^2*polylog(4, -I*cos(b*x
+ a) - sin(b*x + a)) - 6*I*d^3*cos(b*x + a)^2*polylog(4, -cos(b*x + a) + I
*sin(b*x + a)) + 6*I*d^3*cos(b*x + a)^2*polylog(4, -cos(b*x + a) - I*sin(b
*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*cos(b*x + a)^
2*dilog(cos(b*x + a) + I*sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2
*x - I*b^2*c^2*d)*cos(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*
(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d + I*d^3)*cos(b*x + a)^2*dil
og(I*cos(b*x + a) + sin(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x -
I*b^2*c^2*d - I*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) -
 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d - I*d^3)*cos(b*x + a)^2
*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*
x + I*b^2*c^2*d + I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x +
a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*cos(b*x + a)^2*di
log(-cos(b*x + a) + I*sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x +
 I*b^2*c^2*d)*cos(b*x + a)^2*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (b...
```

### 3.311.6 Sympy [F]

$$\int (c+dx)^3 \csc(a+bx)\sec^3(a+bx)\,dx = \int (c+dx)^3 \csc(a+bx)\sec^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*csc(b*x+a)*sec(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**3*csc(a + b*x)*sec(a + b*x)**3, x)
```

### 3.311.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4954 vs. 2(270) = 540.

Time = 1.41 (sec) , antiderivative size = 4954, normalized size of antiderivative = 15.24

$$\int (c+dx)^3 \csc(a+bx)\sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(c^3*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x
+ a)^2)) - 3*a*c^2*d*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - l
og(sin(b*x + a)^2))/b + 3*a^2*c*d^2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x
+ a)^2 - 1) - log(sin(b*x + a)^2))/b^2 - a^3*d^3*(1/(sin(b*x + a)^2 - 1) +
 log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^3 + 2*(18*b^2*c^2*d - 36
*a*b*c*d^2 + 18*a^2*d^3 + 2*(4*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 9*(
b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)
*(b*x + a) + (4*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 9*(b*c*d^2 - a*d^3
)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos
(4*b*x + 4*a) + 2*(4*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 9*(b*c*d^2 -
a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a)
)*cos(2*b*x + 2*a) - (-4*I*(b*x + a)^3*d^3 - 9*I*b*c*d^2 + 9*I*a*d^3 + 9*(
-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 9*(-I*b^2*c^2*d + 2*I*a*b*c*d^2 + (-I*
a^2 - I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*(-4*I*(b*x + a)^3*d^3 - 9*I*
b*c*d^2 + 9*I*a*d^3 + 9*(-I*b*c*d^2 + I*a*d^3)*(b*x + a)^2 + 9*(-I*b^2*c^2
*d + 2*I*a*b*c*d^2 + (-I*a^2 - I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan
2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 6*((b*x + a)^3*d^3 + 3*(b*c*d^
2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a) +
 ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b
*c*d^2 + a^2*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 2*((b*x + a)^3*d^3 + 3*...
```

### 3.311.8 Giac [F]

$$\int (c+dx)^3 \csc(a+bx)\sec^3(a+bx)\,dx = \int (dx+c)^3 \csc(bx+a)\sec(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*csc(b*x + a)*sec(b*x + a)^3, x)
```

### 3.311.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc(a+bx) \sec^3(a+bx)\, dx = \text{Hanged}$$

input
```
int((c + d*x)^3/(cos(a + b*x)^3*sin(a + b*x)),x)
```

output
```
\text{Hanged}
```

# 3.312 $\int (c+dx)^2 \csc(a+bx)\sec^3(a+bx)\,dx$


3.312.1 Optimal result . . . 2349
3.312.2 Mathematica [B] (warning: unable to verify) . . . 2350
3.312.3 Rubi [A] (verified) . . . 2351
3.312.4 Maple [B] (verified) . . . 2353
3.312.5 Fricas [B] (verification not implemented) . . . 2354
3.312.6 Sympy [F] . . . 2355
3.312.7 Maxima [B] (verification not implemented) . . . 2356
3.312.8 Giac [F] . . . 2356
3.312.9 Mupad [F(-1)] . . . 2357


## 3.312.1 Optimal result

Integrand size = 22, antiderivative size = 201

$$\int (c+dx)^2 \csc(a+bx)\sec^3(a+bx)\,dx = \frac{cdx}{b}+\frac{d^2x^2}{2b}-\frac{2(c+dx)^2\operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3} + \frac{id(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} - \frac{id(c+dx)\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2} - \frac{d^2\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} + \frac{d^2\operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3} - \frac{d(c+dx)\tan(a+bx)}{b^2}+\frac{(c+dx)^2\tan^2(a+bx)}{2b}$$

output
```
c*d*x/b+1/2*d^2*x^2/b-2*(d*x+c)^2*arctanh(exp(2*I*(b*x+a)))/b-d^2*ln(cos(b
*x+a))/b^3+I*d*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2-I*d*(d*x+c)*polylo
g(2,exp(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+1/2*d^2
*polylog(3,exp(2*I*(b*x+a)))/b^3-d*(d*x+c)*tan(b*x+a)/b^2+1/2*(d*x+c)^2*ta
n(b*x+a)^2/b
```

### 3.312.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 883 vs. 2(201) = 402.

Time = 6.65 (sec) , antiderivative size = 883, normalized size of antiderivative = 4.39

$$\int (c+dx)^2\csc(a+bx)\sec^3(a+bx)\,dx =$$
$$-\frac{d^2e^{ia}\csc(a)\left(2b^3e^{-2ia}x^3+3ib^2(1-e^{-2ia})\,x^2\log\left(1-e^{-i(a+bx)}\right)+3ib^2(1-e^{-2ia})\,x^2\log\left(1+e^{-i(a+bx)}\right)-\right.}{}$$
$$+\frac{1}{3}x\left(3c^2+3cdx+d^2x^2\right)\csc(a)\sec(a)$$
$$-\frac{id^2e^{-ia}\left(2b^2x^2\left(2bx-3i(1+e^{2ia})\log\left(1+e^{-2i(a+bx)}\right)\right)+6b(1+e^{2ia})\,x\,\mathrm{PolyLog}\left(2,-e^{-2i(a+bx)}\right)-3i(1+\right.}{12b^3}$$
$$+\frac{(c+dx)^2\sec^2(a+bx)}{2b}$$
$$-\frac{c^2\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))}{b\left(\cos^2(a)+\sin^2(a)\right)}$$
$$-\frac{d^2\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))}{b^3\left(\cos^2(a)+\sin^2(a)\right)}$$
$$+\frac{c^2\csc(a)(-bx\cos(a)+\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a))}{b\left(\cos^2(a)+\sin^2(a)\right)}$$
$$-\frac{cd\csc(a)\left(b^2e^{-i\arctan(\cot(a))}x^2-\frac{\cot(a)\left(ibx(-\pi-2\arctan(\cot(a)))-\pi\log\left(1+e^{-2ibx}\right)-2(bx-\arctan(\cot(a)))\log\left(1-e^{2i(bx-\arctan(\right.\right.}{}\right.}{b^2\sqrt{\csc^2(a)\left(\cos^2(a)+\mathrm{s}\right.}}$$
$$+\frac{\sec(a)\sec(a+bx)\left(-cd\sin(bx)-d^2x\sin(bx)\right)}{b^2}$$
$$-\frac{cd\csc(a)\sec(a)\left(b^2e^{i\arctan(\tan(a))}x^2+\frac{\left(ibx(-\pi+2\arctan(\tan(a)))-\pi\log\left(1+e^{-2ibx}\right)-2(bx+\arctan(\tan(a)))\log\left(1-e^{2i(bx+\arctan\right.\right.}{}\right.}{b^2\sqrt{\sec^2(a)\left(\cos^2(a)+\mathrm{s}\right.}}$$

input
```
Integrate[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

output
```
-1/6*(d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*
I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*
Log[1 + E^((-I)*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, -E^((-I)
*(a + b*x))] - 6*b*(1 - E^((-2*I)*a))*x*PolyLog[2, E^((-I)*(a + b*x))] + (
6*I)*(1 - E^((-2*I)*a))*PolyLog[3, -E^((-I)*(a + b*x))] + (6*I)*(1 - E^((-
2*I)*a))*PolyLog[3, E^((-I)*(a + b*x))]))/b^3 + (x*(3*c^2 + 3*c*d*x + d^2*
x^2)*Csc[a]*Sec[a])/3 - ((I/12)*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I
)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2,
-E^((-2*I)*(a + b*x))] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a
+ b*x))])*Sec[a])/(b^3*E^(I*a)) + ((c + d*x)^2*Sec[a + b*x]^2)/(2*b) - (c^
2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*
(Cos[a]^2 + Sin[a]^2)) - (d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*
Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) + (c^2*Csc[a]*(-(b*x*
Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + S
in[a]^2)) - (c*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(
-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot
[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*Arc
Tan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x -
ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*Sqrt[Csc[a]^2*(Cos[a]
^2 + Sin[a]^2)]) + (Sec[a]*Sec[a + b*x]*(-(c*d*Sin[b*x]) - d^2*x*Sin[b*...
```

### 3.312.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4920, 27, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\csc(a+bx)\sec^3(a+bx)\,dx$$

$$\Big\downarrow \text{4920}$$

$$-2d\int \frac{1}{2}(c+dx)\left(\frac{\tan^2(a+bx)}{b}+\frac{2\log(\tan(a+bx))}{b}\right)dx+\frac{(c+dx)^2\tan^2(a+bx)}{2b}+\frac{(c+dx)^2\log(\tan(a+bx))}{b}$$

$$\Big\downarrow \text{27}$$

$$-d\int (c+dx)\left(\frac{\tan^2(a+bx)}{b}+\frac{2\log(\tan(a+bx))}{b}\right)dx+\frac{(c+dx)^2\tan^2(a+bx)}{2b}+\frac{(c+dx)^2\log(\tan(a+bx))}{b}$$

$\big\downarrow$ 7292

$$-d\int \frac{(c+dx)\left(\tan^2(a+bx)+2\log(\tan(a+bx))\right)}{b}dx+\frac{(c+dx)^2\tan^2(a+bx)}{2b}+\frac{(c+dx)^2\log(\tan(a+bx))}{b}$$

$\big\downarrow$ 27

$$-\frac{d\int (c+dx)\left(\tan^2(a+bx)+2\log(\tan(a+bx))\right)dx}{b}+\frac{(c+dx)^2\tan^2(a+bx)}{2b}+\frac{(c+dx)^2\log(\tan(a+bx))}{b}$$

$\big\downarrow$ 7293

$$-\frac{d\int \left((c+dx)\tan^2(a+bx)+2(c+dx)\log(\tan(a+bx))\right)dx}{b}+\frac{(c+dx)^2\tan^2(a+bx)}{2b}+\frac{(c+dx)^2\log(\tan(a+bx))}{b}$$

$\big\downarrow$ 2009

$$-\frac{d\left(\frac{2(c+dx)^2\text{arctanh}\left(e^{2i(a+bx)}\right)}{d}+\frac{d\,\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^2}-\frac{d\,\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^2}+\frac{d\log(\cos(a+bx))}{b^2}-\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{2i(\cdots}\right.}{b}\right.}{b}$$
$$\frac{(c+dx)^2\tan^2(a+bx)}{2b}+\frac{(c+dx)^2\log(\tan(a+bx))}{b}$$

input
```
Int[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

output
```
((c + d*x)^2*Log[Tan[a + b*x]])/b + ((c + d*x)^2*Tan[a + b*x]^2)/(2*b) - (
d*(-1/2*(c + d*x)^2/d + (2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/d + (
d*Log[Cos[a + b*x]])/b^2 + ((c + d*x)^2*Log[Tan[a + b*x]])/d - (I*(c + d*x
)*PolyLog[2, -E^((2*I)*(a + b*x))])/b + (I*(c + d*x)*PolyLog[2, E^((2*I)*(
a + b*x))])/b + (d*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^2) - (d*PolyLog[
3, E^((2*I)*(a + b*x))])/(2*b^2) + ((c + d*x)*Tan[a + b*x])/b))/b
```

3.312. $\int (c+dx)^2\csc(a+bx)\sec^3(a+bx)\,dx$

### 3.312.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## 3.312.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(181) = 362$.

Time = 0.84 (sec) , antiderivative size = 614, normalized size of antiderivative = 3.05

| method | result |
|---|---|
| risch | $-\frac{2 i d^{2} \operatorname{polylog}\left(2, \mathrm{e}^{i(x b+a)}\right) x}{b^{2}}-\frac{2 i d c \operatorname{polylog}\left(2, \mathrm{e}^{i(x b+a)}\right)}{b^{2}}+\frac{2 d c \ln \left(1-\mathrm{e}^{i(x b+a)}\right) x}{b}+\frac{2 d c \ln \left(1-\mathrm{e}^{i(x b+a)}\right) a}{b^{2}}+\frac{2 d^{2} \operatorname{polylog}\left(3, \mathrm{e}^{i(x b}\right.}{b^{3}}$ |

input
```
int((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b*d^2*ln(exp(2*I*(b*x+a))+1)*x^2+1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)-1/b
^3*d^2*ln(exp(2*I*(b*x+a))+1)+2/b^3*d^2*ln(exp(I*(b*x+a)))+I/b^2*d^2*polyl
og(2,-exp(2*I*(b*x+a)))*x-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x-2*I/b^2*
d^2*polylog(2,-exp(I*(b*x+a)))*x+1/b*c^2*ln(exp(I*(b*x+a))+1)+1/b*c^2*ln(e
xp(I*(b*x+a))-1)-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+1/b*d^2*ln(1-exp(I*(b*
x+a)))*x^2+1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+2*(b*d^2*x^2*exp(2*I*(b*x+a))+
2*b*c*d*x*exp(2*I*(b*x+a))+b*c^2*exp(2*I*(b*x+a))-I*d^2*x*exp(2*I*(b*x+a))
-I*c*d*exp(2*I*(b*x+a))-I*d^2*x-I*d*c)/b^2/(exp(2*I*(b*x+a))+1)^2-1/b*c^2*
ln(exp(2*I*(b*x+a))+1)+2/b*d*c*ln(exp(I*(b*x+a))+1)*x+2/b^2*d*c*ln(1-exp(I
*(b*x+a)))*a-2/b^2*c*d*a*ln(exp(I*(b*x+a))-1)+2/b*d*c*ln(1-exp(I*(b*x+a)))
*x-2*I/b^2*d*c*polylog(2,exp(I*(b*x+a)))-2*I/b^2*d*c*polylog(2,-exp(I*(b*x
+a)))-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3-2/b*c*d*ln(exp(2*I*(b*x+a))
+1)*x+2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp(I*(b*x+a)))
/b^3+I/b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))
```

### 3.312.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1404 vs. $2(177) = 354$.

Time = 0.36 (sec) , antiderivative size = 1404, normalized size of antiderivative = 6.99

$$\int (c+dx)^2 \csc(a+bx) \sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*d^2*cos(b*x + a)^2*polylog(3, cos(b*x +
 a) + I*sin(b*x + a)) + 2*d^2*cos(b*x + a)^2*polylog(3, cos(b*x + a) - I*s
in(b*x + a)) - 2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x +
a)) - 2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 2*d
^2*cos(b*x + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*cos(b
*x + a)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*cos(b*x + a)^
2*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*cos(b*x + a)^2*polylo
g(3, -cos(b*x + a) - I*sin(b*x + a)) + b^2*c^2 - 2*(I*b*d^2*x + I*b*c*d)*c
os(b*x + a)^2*dilog(cos(b*x + a) + I*sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c
*d)*cos(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a)) - 2*(I*b*d^2*x + I
*b*c*d)*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) - 2*(-I*b*d^2*
x - I*b*c*d)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) - 2*(-I*b
*d^2*x - I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 2
*(I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)
) - 2*(-I*b*d^2*x - I*b*c*d)*cos(b*x + a)^2*dilog(-cos(b*x + a) + I*sin(b*
x + a)) - 2*(I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2*dilog(-cos(b*x + a) - I*s
in(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a)^2*log(co
s(b*x + a) + I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*c
os(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^
2*c*d*x + b^2*c^2)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + 1...
```

## 3.312.6 Sympy [F]

$$\int (c+dx)^2 \csc(a+bx)\sec^3(a+bx)\,dx = \int (c+dx)^2 \csc(a+bx)\sec^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*csc(b*x+a)*sec(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**2*csc(a + b*x)*sec(a + b*x)**3, x)
```

### 3.312.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2447 vs. $2(177) = 354$.

Time = 0.63 (sec) , antiderivative size = 2447, normalized size of antiderivative = 12.17

$$\int (c+dx)^2 \csc(a+bx)\sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(c^2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x
+ a)^2)) - 2*a*c*d*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log
(sin(b*x + a)^2))/b + a^2*d^2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2
 - 1) - log(sin(b*x + a)^2))/b^2 - 2*(4*(b*x + a)*d^2*cos(4*b*x + 4*a) + 4
*I*(b*x + a)*d^2*sin(4*b*x + 4*a) - 4*b*c*d + 4*a*d^2 - 2*((b*x + a)^2*d^2
 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2 + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2
)*(b*x + a) + d^2)*cos(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^
2)*(b*x + a) + d^2)*cos(2*b*x + 2*a) - (-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d +
 I*a*d^2)*(b*x + a) - I*d^2)*sin(4*b*x + 4*a) - 2*(-I*(b*x + a)^2*d^2 + 2*
(-I*b*c*d + I*a*d^2)*(b*x + a) - I*d^2)*sin(2*b*x + 2*a))*arctan2(sin(2*b*
x + 2*a), cos(2*b*x + 2*a) + 1) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(
b*x + a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a
) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (
I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a) + 2*
(I*(b*x + a)^2*d^2 + 2*(I*b*c*d - I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*ar
ctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*
d^2)*(b*x + a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x
 + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*
a) - (-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*sin(4*b*x + 4
*a) - 2*(-I*(b*x + a)^2*d^2 + 2*(-I*b*c*d + I*a*d^2)*(b*x + a))*sin(2*b...
```

### 3.312.8 Giac [F]

$$\int (c+dx)^2 \csc(a+bx)\sec^3(a+bx)\,dx = \int (dx+c)^2 \csc(bx+a)\sec(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csc(b*x + a)*sec(b*x + a)^3, x)
```

### 3.312.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc(a+bx) \sec^3(a+bx) \, dx = \text{Hanged}$$

input
```
int((c + d*x)^2/(cos(a + b*x)^3*sin(a + b*x)),x)
```

output
```
\text{Hanged}
```

# 3.313 $\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx$

3.313.1 Optimal result . . . . . . . . . . 2358
3.313.2 Mathematica [A] (verified) . . . . . . . . . . 2359
3.313.3 Rubi [A] (verified) . . . . . . . . . . 2359
3.313.4 Maple [B] (verified) . . . . . . . . . . 2360
3.313.5 Fricas [B] (verification not implemented) . . . . . . . . . . 2361
3.313.6 Sympy [F] . . . . . . . . . . 2362
3.313.7 Maxima [B] (verification not implemented) . . . . . . . . . . 2362
3.313.8 Giac [F] . . . . . . . . . . 2363
3.313.9 Mupad [F(-1)] . . . . . . . . . . 2363

## 3.313.1 Optimal result

Integrand size = 20, antiderivative size = 139

$$\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx = \frac{dx}{2b} - \frac{2dx\,\text{arctanh}\left(e^{2ia+2ibx}\right)}{b} + \frac{c\log(\tan(a+bx))}{b} + \frac{id\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} - \frac{id\,\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2} - \frac{d\tan(a+bx)}{2b^2} + \frac{c\tan^2(a+bx)}{2b} + \frac{dx\tan^2(a+bx)}{2b}$$

output
```
1/2*d*x/b-2*d*x*arctanh(exp(2*I*a+2*I*b*x))/b+c*ln(tan(b*x+a))/b+1/2*I*d*p
olylog(2,-exp(2*I*(b*x+a)))/b^2-1/2*I*d*polylog(2,exp(2*I*(b*x+a)))/b^2-1/
2*d*tan(b*x+a)/b^2+1/2*c*tan(b*x+a)^2/b+1/2*d*x*tan(b*x+a)^2/b
```

### 3.313.2 Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.61

$$\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx$$
$$= -\frac{c\log(\cos(a+bx))}{b}+\frac{ad\log(\cos(a+bx))}{b^2}+\frac{c\log(\sin(a+bx))}{b} - \frac{ad(\log(\cos(a+bx))+\log(\tan(a+bx)))}{b^2} + \frac{d\left(\frac{1}{2}i(a+bx)^2-(a+bx)\log\left(1+e^{2i(a+bx)}\right)+\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)\right)}{b^2} + \frac{d\left((a+bx)\log\left(1-e^{2i(a+bx)}\right)-\frac{1}{2}i\left((a+bx)^2+\text{PolyLog}\left(2,e^{2i(a+bx)}\right)\right)\right)}{b^2} + \frac{c\sec^2(a+bx)}{2b}+\frac{dx\sec^2(a+bx)}{2b}-\frac{d\tan(a+bx)}{2b^2}$$

input
```
Integrate[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

output
```
-((c*Log[Cos[a + b*x]])/b) + (a*d*Log[Cos[a + b*x]])/b^2 + (c*Log[Sin[a +
b*x]])/b - (a*d*(Log[Cos[a + b*x]] + Log[Tan[a + b*x]]))/b^2 + (d*((I/2)*(
a + b*x)^2 - (a + b*x)*Log[1 + E^((2*I)*(a + b*x))] + (I/2)*PolyLog[2, -E^
((2*I)*(a + b*x))]))/b^2 + (d*((a + b*x)*Log[1 - E^((2*I)*(a + b*x))] - (I
/2)*((a + b*x)^2 + PolyLog[2, E^((2*I)*(a + b*x))])))/b^2 + (c*Sec[a + b*x
]^2)/(2*b) + (d*x*Sec[a + b*x]^2)/(2*b) - (d*Tan[a + b*x])/(2*b^2)
```

### 3.313.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4920, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx$$
$$\Big\downarrow \; 4920$$
$$-d\int \left(\frac{\tan^2(a+bx)}{2b}+\frac{\log(\tan(a+bx))}{b}\right)dx+\frac{(c+dx)\tan^2(a+bx)}{2b}+\frac{(c+dx)\log(\tan(a+bx))}{b}$$

$\downarrow$ 2009

$$-d\left(\frac{2x\text{arctanh}\left(e^{2i(a+bx)}\right)}{b}-\frac{i\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}+\frac{i\,\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}+\frac{\tan(a+bx)}{2b^2}+\frac{x\log(\tan(a+b\right.$$
$$\frac{(c+dx)\tan^2(a+bx)}{2b}+\frac{(c+dx)\log(\tan(a+bx))}{b}$$

input
```
Int[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

output
```
((c + d*x)*Log[Tan[a + b*x]])/b + ((c + d*x)*Tan[a + b*x]^2)/(2*b) - d*(-1
/2*x/b + (2*x*ArcTanh[E^((2*I)*(a + b*x))])/b + (x*Log[Tan[a + b*x]])/b -
((I/2)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 + ((I/2)*PolyLog[2, E^((2*I)*
(a + b*x))])/b^2 + Tan[a + b*x]/(2*b^2))
```

### 3.313.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

## 3.313.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(118) = 236.

Time = 0.61 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.94

| method | result |
|---|---|
| risch | $\frac{2bdx\,e^{2i(xb+a)}-id\,e^{2i(xb+a)}+2bc\,e^{2i(xb+a)}-id}{b^2\left(e^{2i(xb+a)}+1\right)^2}+\frac{c\ln\left(e^{i(xb+a)}+1\right)}{b}-\frac{c\ln\left(e^{2i(xb+a)}+1\right)}{b}+\frac{c\ln\left(e^{i(xb+a)}-1\right)}{b}+\frac{d\ln\left(e^{i(xb+a)}+\right.}{b}$ |

input
```
int((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(2*b*d*x*exp(2*I*(b*x+a))-I*d*exp(2*I*(b*x+a))+2*b*c*exp(2*I*(b*x+a))-I*d)
/b^2/(exp(2*I*(b*x+a))+1)^2+1/b*c*ln(exp(I*(b*x+a))+1)-1/b*c*ln(exp(2*I*(b
*x+a))+1)+1/b*c*ln(exp(I*(b*x+a))-1)+1/b*d*ln(exp(I*(b*x+a))+1)*x-I*d*poly
log(2,-exp(I*(b*x+a)))/b^2-1/b*d*ln(exp(2*I*(b*x+a))+1)*x+1/2*I*d*polylog(
2,-exp(2*I*(b*x+a)))/b^2+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-exp(I*(
b*x+a)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(b*x+a))-1
)
```

## 3.313.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 760 vs. 2(112) = 224.

Time = 0.32 (sec) , antiderivative size = 760, normalized size of antiderivative = 5.47

$$\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(-I*d*cos(b*x + a)^2*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d*cos(b*
x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(I
*cos(b*x + a) + sin(b*x + a)) + I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) -
sin(b*x + a)) + I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) -
 I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) + I*d*cos(b*x +
a)^2*dilog(-cos(b*x + a) + I*sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(-cos
(b*x + a) - I*sin(b*x + a)) + (b*d*x + b*c)*cos(b*x + a)^2*log(cos(b*x + a
) + I*sin(b*x + a) + 1) - (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*
sin(b*x + a) + I) + (b*d*x + b*c)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(
b*x + a) + 1) - (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x +
a) + I) - (b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) +
 1) - (b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1)
- (b*d*x + a*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (
b*d*x + a*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c
 - a*d)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) +
 (b*c - a*d)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1
/2) + (b*d*x + a*d)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + 1)
 - (b*c - a*d)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b
*d*x + a*d)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - (b*c
- a*d)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) + b*d*x -...
```

### 3.313.6 Sympy [F]

$$\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx=\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)**3,x)
```

output
```
Integral((c + d*x)*csc(a + b*x)*sec(a + b*x)**3, x)
```

### 3.313.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1028 vs. $2(112) = 224$.

Time = 0.47 (sec) , antiderivative size = 1028, normalized size of antiderivative = 7.40

$$\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
-(2*(b*d*x + b*c + (b*d*x + b*c)*cos(4*b*x + 4*a) + 2*(b*d*x + b*c)*cos(2*
b*x + 2*a) - (-I*b*d*x - I*b*c)*sin(4*b*x + 4*a) - 2*(-I*b*d*x - I*b*c)*si
n(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 2*(b*d*x
 + b*c + (b*d*x + b*c)*cos(4*b*x + 4*a) + 2*(b*d*x + b*c)*cos(2*b*x + 2*a)
 + (I*b*d*x + I*b*c)*sin(4*b*x + 4*a) + 2*(I*b*d*x + I*b*c)*sin(2*b*x + 2*
a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*(b*c*cos(4*b*x + 4*a) + 2*
b*c*cos(2*b*x + 2*a) + I*b*c*sin(4*b*x + 4*a) + 2*I*b*c*sin(2*b*x + 2*a) +
 b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 2*(b*d*x*cos(4*b*x + 4*a)
+ 2*b*d*x*cos(2*b*x + 2*a) + I*b*d*x*sin(4*b*x + 4*a) + 2*I*b*d*x*sin(2*b*
x + 2*a) + b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*(-2*I*b*d*x
 - 2*I*b*c - d)*cos(2*b*x + 2*a) - (d*cos(4*b*x + 4*a) + 2*d*cos(2*b*x + 2
*a) + I*d*sin(4*b*x + 4*a) + 2*I*d*sin(2*b*x + 2*a) + d)*dilog(-e^(2*I*b*x
 + 2*I*a)) + 2*(d*cos(4*b*x + 4*a) + 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x
+ 4*a) + 2*I*d*sin(2*b*x + 2*a) + d)*dilog(-e^(I*b*x + I*a)) + 2*(d*cos(4*
b*x + 4*a) + 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) + 2*I*d*sin(2*b*x
 + 2*a) + d)*dilog(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*
c)*cos(4*b*x + 4*a) - 2*(I*b*d*x + I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)
*sin(4*b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)
^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (I*b*d*x + I*b*c + (I*
b*d*x + I*b*c)*cos(4*b*x + 4*a) - 2*(-I*b*d*x - I*b*c)*cos(2*b*x + 2*a)...
```

### 3.313.8 Giac [F]

$$\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx = \int (dx+c)\csc(bx+a)\sec(bx+a)^3\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)*csc(b*x + a)*sec(b*x + a)^3, x)
```

### 3.313.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx = \text{Hanged}$$

input
```
int((c + d*x)/(cos(a + b*x)^3*sin(a + b*x)),x)
```

output
```
\text{Hanged}
```

# 3.314 $\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$

3.314.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2364
3.314.2 Mathematica [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2364
3.314.3 Rubi [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2365
3.314.4 Maple [N/A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 2365
3.314.5 Fricas [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2366
3.314.6 Sympy [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2366
3.314.7 Maxima [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2366
3.314.8 Giac [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2367
3.314.9 Mupad [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2368

## 3.314.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \mathrm{Int}\left(\frac{\csc(a+bx)\sec^3(a+bx)}{c+dx},x\right)$$

output
```
CannotIntegrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x)
```

## 3.314.2 Mathematica [N/A]

Not integrable

Time = 7.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x),x]
```

output
```
Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x), x]
```

## 3.314.3 Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}dx$$

input
```
Int[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x),x]
```

output
```
$Aborted
```

### 3.314.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.314.4 Maple [N/A] (verified)

Not integrable

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\csc(xb+a)\sec(xb+a)^3}{dx+c}dx$$

input
```
int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x)
```

output
```
int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x)
```

### 3.314.5 Fricas [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)^3}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)*sec(b*x + a)^3/(d*x + c), x)
```

### 3.314.6 Sympy [N/A]

Not integrable

Time = 2.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(csc(a + b*x)*sec(a + b*x)**3/(c + d*x), x)
```

### 3.314.7 Maxima [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 1865, normalized size of antiderivative = 84.77

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)^3}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output
```
(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
 (2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
+ 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
+ (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*s
in(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*
x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d
^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate(2*(b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2 + d^2)*sin(2*b*x + 2*a)/(b^2*d^3*x^3 + 3*b^2*c*d^
2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x
^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)), x) - (b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^
2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*
x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b...
```

### 3.314.8 Giac [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc\left(bx+a\right)\sec\left(bx+a\right)^3}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)*sec(b*x + a)^3/(d*x + c), x)
```

### 3.314.9 Mupad [N/A]

Not integrable

Time = 25.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)), x)
```

# 3.315 $\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$

3.315.1 Optimal result . . . 2369
3.315.2 Mathematica [N/A] . . . 2369
3.315.3 Rubi [N/A] . . . 2370
3.315.4 Maple [N/A] (verified) . . . 2370
3.315.5 Fricas [N/A] . . . 2371
3.315.6 Sympy [N/A] . . . 2371
3.315.7 Maxima [N/A] . . . 2371
3.315.8 Giac [F(-1)] . . . 2372
3.315.9 Mupad [N/A] . . . 2373

## 3.315.1 Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2},x\right)$$

output
```
CannotIntegrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x)
```

## 3.315.2 Mathematica [N/A]

Not integrable

Time = 5.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2,x]
```

output
```
Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

## 3.315.3 Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2} \, dx$$
$$\Big\downarrow \text{ 7299}$$
$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}dx$$

input
```
Int[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2,x]
```

output
```
$Aborted
```

### 3.315.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.315.4 Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\csc \left(x b+a\right) \sec \left(x b+a\right)^{3}}{\left(d x+c\right)^{2}}d x$$

input
```
int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x)
```

### 3.315.5 Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)^3}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)*sec(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.315.6 Sympy [N/A]

Not integrable

Time = 4.87 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)*sec(a + b*x)**3/(c + d*x)**2, x)
```

### 3.315.7 Maxima [N/A]

Not integrable

Time = 5.43 (sec) , antiderivative size = 2516, normalized size of antiderivative = 114.36

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(bx+a)\sec(bx+a)^3}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
(4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
 2*((b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
+ 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^
2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2
*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*
x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 +
3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^
3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2
*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + 2*(b^2*d^3*x^3 + 3*
b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a
) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x
+ 2*a))*integrate(2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 3*d^2)*sin(2*b*
x + 2*a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*
x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c
^3*d*x + b^2*c^4)*cos(2*b*x + 2*a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*
b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^4
*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(
2*b*x + 2*a)), x) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c
^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*...
```

### 3.315.8 Giac [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

### 3.315.9 Mupad [N/A]

Not integrable

Time = 25.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^2),x)
```

output
```
int(1/(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^2), x)
```

# 3.316 $\int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx$

3.316.1 Optimal result . . . . . . . . . . 2374
3.316.2 Mathematica [N/A] . . . . . . . . . . 2374
3.316.3 Rubi [N/A] . . . . . . . . . . 2375
3.316.4 Maple [N/A] (verified) . . . . . . . . . . 2375
3.316.5 Fricas [N/A] . . . . . . . . . . 2376
3.316.6 Sympy [F(-1)] . . . . . . . . . . 2376
3.316.7 Maxima [N/A] . . . . . . . . . . 2376
3.316.8 Giac [N/A] . . . . . . . . . . 2377
3.316.9 Mupad [N/A] . . . . . . . . . . 2377

## 3.316.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx = \text{Int}\big((c+dx)^m \csc^2(a+bx)\sec^3(a+bx),x\big)$$

output
```
CannotIntegrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

## 3.316.2 Mathematica [N/A]

Not integrable

Time = 25.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx = \int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

output
```
Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3, x]
```

## 3.316.3 Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^2(a+bx)\sec^3(a+bx)(c+dx)^m\,dx$$
$$\Big\downarrow \ 7299$$
$$\int \csc^2(a+bx)\sec^3(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

output
```
$Aborted
```

### 3.316.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.316.4 Maple [N/A] (verified)

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \csc(xb+a)^2 \sec(xb+a)^3\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

output
```
int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

### 3.316.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)^3 \,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

### 3.316.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*csc(b*x+a)**2*sec(b*x+a)**3,x)
```

output
```
Timed out
```

### 3.316.7 Maxima [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)^3 \,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

### 3.316.8 Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^2(a+bx) \sec^3(a+bx)\,dx = \int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

### 3.316.9 Mupad [N/A]

Not integrable

Time = 25.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^2(a+bx) \sec^3(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\cos(a+b\,x)^3 \sin(a+b\,x)^2}\,dx$$

input
```
int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)^2),x)
```

output
```
int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)^2), x)
```

# 3.317 $\int (c+dx)^3 \csc^2(a+bx) \sec^3(a+bx)\,dx$

3.317.1 Optimal result . . . 2378
3.317.2 Mathematica [A] (verified) . . . 2379
3.317.3 Rubi [A] (verified) . . . 2380
3.317.4 Maple [B] (verified) . . . 2382
3.317.5 Fricas [B] (verification not implemented) . . . 2383
3.317.6 Sympy [F(-1)] . . . 2384
3.317.7 Maxima [B] (verification not implemented) . . . 2385
3.317.8 Giac [F] . . . 2385
3.317.9 Mupad [F(-1)] . . . 2386

## 3.317.1 Optimal result

Integrand size = 24, antiderivative size = 486

$$
\begin{aligned}
&\int (c+dx)^3 \csc^2(a+bx) \sec^3(a+bx)\,dx \\
&= -\frac{6id^2(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{b} \\
&\quad - \frac{6d(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^3\csc(a+bx)}{2b} \\
&\quad + \frac{6id^2(c+dx)\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3} + \frac{3id^3\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^4} \\
&\quad + \frac{9id(c+dx)^2\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2} - \frac{3id^3\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^4} \\
&\quad - \frac{9id(c+dx)^2\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2} - \frac{6id^2(c+dx)\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3} \\
&\quad - \frac{6d^3\,\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^4} - \frac{9d^2(c+dx)\,\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} \\
&\quad + \frac{9d^2(c+dx)\,\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3} + \frac{6d^3\,\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^4} \\
&\quad - \frac{9id^3\,\text{PolyLog}\left(4,-ie^{i(a+bx)}\right)}{b^4} + \frac{9id^3\,\text{PolyLog}\left(4,ie^{i(a+bx)}\right)}{b^4} \\
&\quad - \frac{3d(c+dx)^2\sec(a+bx)}{2b^2} + \frac{(c+dx)^3\csc(a+bx)\sec^2(a+bx)}{2b}
\end{aligned}
$$

output
```
3*I*d^3*polylog(2,-I*exp(I*(b*x+a)))/b^4-3*I*(d*x+c)^3*arctan(exp(I*(b*x+a
)))/b-6*d*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b^2-3/2*(d*x+c)^3*csc(b*x+a)/b
-9*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4+9*I*d^3*polylog(4,I*exp(I*(b*x+a
)))/b^4-6*I*d^2*(d*x+c)*arctan(exp(I*(b*x+a)))/b^3-3*I*d^3*polylog(2,I*exp
(I*(b*x+a)))/b^4-9/2*I*d*(d*x+c)^2*polylog(2,I*exp(I*(b*x+a)))/b^2+9/2*I*d
*(d*x+c)^2*polylog(2,-I*exp(I*(b*x+a)))/b^2-6*d^3*polylog(3,-exp(I*(b*x+a)
))/b^4-9*d^2*(d*x+c)*polylog(3,-I*exp(I*(b*x+a)))/b^3+9*d^2*(d*x+c)*polylo
g(3,I*exp(I*(b*x+a)))/b^3+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4+6*I*d^2*(d*x
+c)*polylog(2,-exp(I*(b*x+a)))/b^3-6*I*d^2*(d*x+c)*polylog(2,exp(I*(b*x+a)
))/b^3-3/2*d*(d*x+c)^2*sec(b*x+a)/b^2+1/2*(d*x+c)^3*csc(b*x+a)*sec(b*x+a)^
2/b
```

### 3.317.2 Mathematica [A] (verified)

Time = 7.51 (sec) , antiderivative size = 819, normalized size of antiderivative = 1.69

$$\int (c+dx)^3 \csc^2(a+bx)\sec^3(a+bx)\,dx$$
$$= \frac{3d\Big((c+dx)^2\log\left(1-e^{i(a+bx)}\right)-(c+dx)^2\log\left(1+e^{i(a+bx)}\right)+\frac{2id\left(b(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)+id\operatorname{PolyLog}\left(3,-e^{i(a+\cdots}\right.\right.}{b^2}}{b^2}$$
$$-\frac{3\left(2ib^3c^3\arctan\left(e^{i(a+bx)}\right)+4ibcd^2\arctan\left(e^{i(a+bx)}\right)-3b^3c^2dx\log\left(1-ie^{i(a+bx)}\right)-2bd^3x\log\left(1-ie^{i(a+b\cdots}\right.\right.}{}$$
$$-\frac{\csc(a+bx)\sec^2(a+bx)\left(bc^3+3bc^2dx+3bcd^2x^2+bd^3x^3+3bc^3\cos(2a+2bx)+9bc^2dx\cos(2a+2bx)\cdots\right.}{}$$

input
```
Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

output
```
(3*d*((c + d*x)^2*Log[1 - E^(I*(a + b*x))] - (c + d*x)^2*Log[1 + E^(I*(a +
 b*x))] + ((2*I)*d*(b*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] + I*d*PolyLog
[3, -E^(I*(a + b*x))]))/b^2 + (2*d*((-I)*b*(c + d*x)*PolyLog[2, E^(I*(a +
b*x))] + d*PolyLog[3, E^(I*(a + b*x))]))/b^2))/b^2 - (3*((2*I)*b^3*c^3*Arc
Tan[E^(I*(a + b*x))] + (4*I)*b*c*d^2*ArcTan[E^(I*(a + b*x))] - 3*b^3*c^2*d
*x*Log[1 - I*E^(I*(a + b*x))] - 2*b*d^3*x*Log[1 - I*E^(I*(a + b*x))] - 3*b
^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 - I*E^(I*(a +
b*x))] + 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] + 2*b*d^3*x*Log[1 + I*E^
(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + b^3*d^3*x^3*
Log[1 + I*E^(I*(a + b*x))] - I*d*(2*d^2 + 3*b^2*(c + d*x)^2)*PolyLog[2, (-
I)*E^(I*(a + b*x))] + I*d*(2*d^2 + 3*b^2*(c + d*x)^2)*PolyLog[2, I*E^(I*(a
 + b*x))] + 6*b*c*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog
[3, (-I)*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, I*E^(I*(a + b*x))] - 6*b*
d^3*x*PolyLog[3, I*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a +
b*x))] - (6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))]))/(2*b^4) - (Csc[a + b*x]
*Sec[a + b*x]^2*(b*c^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2 + b*d^3*x^3 + 3*b*c^3
*Cos[2*a + 2*b*x] + 9*b*c^2*d*x*Cos[2*a + 2*b*x] + 9*b*c*d^2*x^2*Cos[2*a +
 2*b*x] + 3*b*d^3*x^3*Cos[2*a + 2*b*x] + 3*c^2*d*Sin[2*a + 2*b*x] + 6*c*d^
2*x*Sin[2*a + 2*b*x] + 3*d^3*x^2*Sin[2*a + 2*b*x]))/(4*b^2)
```

### 3.317.3 Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4920, 27, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \csc^2(a+bx)\sec^3(a+bx)\,dx$$

$$\Big\downarrow \text{ 4920}$$

$$-3d\int \frac{1}{2}(c+dx)^2\left(\frac{\csc(a+bx)\sec^2(a+bx)}{b}+\frac{3\text{arctanh}(\sin(a+bx))}{b}-\frac{3\csc(a+bx)}{b}\right)dx+\frac{3(c+dx)^3\text{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)^3\csc(a+bx)}{2b}+\frac{(c+dx)^3\csc(a+bx)\sec^2(a+bx)}{2b}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3}{2}d\int (c+dx)^2\left(\frac{\csc(a+bx)\sec^2(a+bx)}{b}+\frac{3\text{arctanh}(\sin(a+bx))}{b}-\frac{3\csc(a+bx)}{b}\right)dx+\frac{3(c+dx)^3\text{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)^3\csc(a+bx)}{2b}+\frac{(c+dx)^3\csc(a+bx)\sec^2(a+bx)}{2b}$$

$$\Big\downarrow \; 7292$$

$$-\frac{3}{2}d\int \frac{(c+dx)^2\left(\csc(a+bx)\sec^2(a+bx)+3\text{arctanh}(\sin(a+bx))-3\csc(a+bx)\right)}{b}dx+\frac{3(c+dx)^3\text{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)^3\csc(a+bx)}{2b}+\frac{(c+dx)^3\csc(a+bx)\sec^2(a+bx)}{2b}$$

$$\Big\downarrow \; 27$$

$$-\frac{3d\int (c+dx)^2\left(\csc(a+bx)\sec^2(a+bx)+3\text{arctanh}(\sin(a+bx))-3\csc(a+bx)\right)dx}{2b}+\frac{3(c+dx)^3\text{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)^3\csc(a+bx)}{2b}+\frac{(c+dx)^3\csc(a+bx)\sec^2(a+bx)}{2b}$$

$$\Big\downarrow \; 7293$$

$$-\frac{3d\int \left(\csc(a+bx)\sec^2(a+bx)(c+dx)^2+3(\text{arctanh}(\sin(a+bx))-\csc(a+bx))(c+dx)^2\right)dx}{2b}+\frac{3(c+dx)^3\text{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)^3\csc(a+bx)}{2b}+\frac{(c+dx)^3\csc(a+bx)\sec^2(a+bx)}{2b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{3d\left(\frac{4id(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b^2}+\frac{2i(c+dx)^3\arctan\left(e^{i(a+bx)}\right)}{d}+\frac{4(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{(c+dx)^3\text{arctanh}(\sin(a+bx))}{d}-\frac{2id^2}{\ldots}\right.}{\ldots}$$

$$\frac{3(c+dx)^3\text{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)^3\csc(a+bx)}{2b}+\frac{(c+dx)^3\csc(a+bx)\sec^2(a+bx)}{2b}$$

input

```
Int[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

output

```
(3*(c + d*x)^3*ArcTanh[Sin[a + b*x]])/(2*b) - (3*(c + d*x)^3*Csc[a + b*x])
/(2*b) + ((c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^2)/(2*b) - (3*d*(((4*I)*d*
(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 + ((2*I)*(c + d*x)^3*ArcTan[E^(I*(a
 + b*x))])/d + (4*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b + ((c + d*x)^3*A
rcTanh[Sin[a + b*x]])/d - ((4*I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])
/b^2 - ((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 - ((3*I)*(c + d*x)
^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b + ((2*I)*d^2*PolyLog[2, I*E^(I*(a +
 b*x))])/b^3 + ((3*I)*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b + ((4*I
)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 + (4*d^2*PolyLog[3, -E^(I*(
a + b*x))])/b^3 + (6*d*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^2 - (
6*d*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^2 - (4*d^2*PolyLog[3, E^(I*
(a + b*x))])/b^3 + ((6*I)*d^2*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^3 - ((6*
I)*d^2*PolyLog[4, I*E^(I*(a + b*x))])/b^3 + ((c + d*x)^2*Sec[a + b*x])/b))
/(2*b)
```

## 3.317. $\int (c+dx)^3\csc^2(a+bx)\sec^3(a+bx)\,dx$

### 3.317.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## 3.317.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1628 vs. $2(428) = 856$.

Time = 1.36 (sec) , antiderivative size = 1629, normalized size of antiderivative = 3.35

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1629 |

input
```
int((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-I/b^2/(exp(2*I*(b*x+a))+1)^2/(exp(2*I*(b*x+a))-1)*(3*d^3*x^3*b*exp(5*I*(b
*x+a))+9*c*d^2*x^2*b*exp(5*I*(b*x+a))+9*c^2*d*x*b*exp(5*I*(b*x+a))+2*d^3*x
^3*b*exp(3*I*(b*x+a))+3*c^3*b*exp(5*I*(b*x+a))+6*c*d^2*x^2*b*exp(3*I*(b*x+
a))-3*I*d^3*x^2*exp(5*I*(b*x+a))+6*c^2*d*x*b*exp(3*I*(b*x+a))+3*d^3*x^3*b*
exp(I*(b*x+a))+3*I*c^2*d*exp(I*(b*x+a))+2*c^3*b*exp(3*I*(b*x+a))+9*c*d^2*x
^2*b*exp(I*(b*x+a))+3*I*d^3*x^2*exp(I*(b*x+a))+9*c^2*d*x*b*exp(I*(b*x+a))+
3*c^3*b*exp(I*(b*x+a))-6*I*c*d^2*x*exp(5*I*(b*x+a))+6*I*c*d^2*x*exp(I*(b*x
+a))-3*I*c^2*d*exp(5*I*(b*x+a)))-9*I/b^3*a^2*c*d^2*arctan(exp(I*(b*x+a)))-
3*I*d^3*polylog(2,I*exp(I*(b*x+a)))/b^4-9*I*d^3*polylog(4,-I*exp(I*(b*x+a)
))/b^4+9/b^3*c*d^2*polylog(3,I*exp(I*(b*x+a)))+3/b^3*d^3*ln(1-I*exp(I*(b*x
+a)))*x+3/b^4*d^3*ln(1-I*exp(I*(b*x+a)))*a+3/b^2*d^3*ln(1-exp(I*(b*x+a)))*
x^2+3/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^2-3/2/b*d^3*ln(1+I*exp(I*(b*x+a)))*x^
3-3/b^2*d^3*ln(exp(I*(b*x+a))+1)*x^2+3/2/b*d^3*ln(1-I*exp(I*(b*x+a)))*x^3+
9/b^3*d^3*polylog(3,I*exp(I*(b*x+a)))*x+3/b^4*d^3*a^2*ln(exp(I*(b*x+a))-1)
-3/b^2*c^2*d*ln(exp(I*(b*x+a))+1)+3/b^2*c^2*d*ln(exp(I*(b*x+a))-1)+3*I*d^3
*polylog(2,-I*exp(I*(b*x+a)))/b^4+9*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4-
9*I/b^2*c*d^2*polylog(2,I*exp(I*(b*x+a)))*x+9*I/b^2*c*d^2*polylog(2,-I*exp
(I*(b*x+a)))*x+9*I/b^2*a*c^2*d*arctan(exp(I*(b*x+a)))-3/b^3*d^3*ln(1+I*exp
(I*(b*x+a)))*x-3/b^4*d^3*ln(1+I*exp(I*(b*x+a)))*a-3/2/b^4*a^3*d^3*ln(1+I*e
xp(I*(b*x+a)))-9/b^3*c*d^2*polylog(3,-I*exp(I*(b*x+a)))+3/2/b^4*a^3*d^3...
```

## 3.317.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2226 vs. 2(404) = 808.

Time = 0.43 (sec) , antiderivative size = 2226, normalized size of antiderivative = 4.58

$$\int (c+dx)^3 \csc^2(a+bx) \sec^3(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 18*I*d^3*cos(b*x +
a)^2*polylog(4, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + 18*I*d^3*cos
(b*x + a)^2*polylog(4, I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - 18*I*
d^3*cos(b*x + a)^2*polylog(4, -I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a)
 - 18*I*d^3*cos(b*x + a)^2*polylog(4, -I*cos(b*x + a) - sin(b*x + a))*sin(
b*x + a) + 12*d^3*cos(b*x + a)^2*polylog(3, cos(b*x + a) + I*sin(b*x + a))
*sin(b*x + a) + 12*d^3*cos(b*x + a)^2*polylog(3, cos(b*x + a) - I*sin(b*x
+ a))*sin(b*x + a) - 12*d^3*cos(b*x + a)^2*polylog(3, -cos(b*x + a) + I*si
n(b*x + a))*sin(b*x + a) - 12*d^3*cos(b*x + a)^2*polylog(3, -cos(b*x + a)
- I*sin(b*x + a))*sin(b*x + a) + 2*b^3*c^3 - 12*(I*b*d^3*x + I*b*c*d^2)*co
s(b*x + a)^2*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 12*(-I*b*
d^3*x - I*b*c*d^2)*cos(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a))*sin
(b*x + a) - 3*(3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 2*I*d^3
)*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 3*(3*
I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 2*I*d^3)*cos(b*x + a)^2*
dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - 3*(-3*I*b^2*d^3*x^2 -
6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 2*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x
 + a) + sin(b*x + a))*sin(b*x + a) - 3*(-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x
 - 3*I*b^2*c^2*d - 2*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x
 + a))*sin(b*x + a) - 12*(I*b*d^3*x + I*b*c*d^2)*cos(b*x + a)^2*dilog(-...
```

## 3.317.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^2(a+bx) \sec^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**3*csc(b*x+a)**2*sec(b*x+a)**3,x)
```

output
```
Timed out
```

### 3.317.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 8046 vs. 2(404) = 808.

Time = 3.83 (sec) , antiderivative size = 8046, normalized size of antiderivative = 16.56

$$\int (c+dx)^3 \csc^2(a+bx) \sec^3(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(c^3*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*lo
g(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1)) - 3*a*c^2*d*(2*(3*sin(b*x +
 a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log(sin(b*x + a) + 1) + 3*l
og(sin(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x +
 a)^3 - sin(b*x + a)) - 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))
/b^2 - a^3*d^3*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) -
 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b^3 - 4*(6*((b*x + a)^
3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2
*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a) - ((b*x + a)^3*d^3 + 2*b*c*d
^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^
2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(6*b*x + 6*a) - ((b*x + a)^3*d^3 + 2*b*
c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c
*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + ((b*x + a)^3*d^3 + 2
*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*
b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (I*(b*x + a)^3*d^
3 + 2*I*b*c*d^2 - 2*I*a*d^3 + 3*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + (3*I*b
^2*c^2*d - 6*I*a*b*c*d^2 + (3*I*a^2 + 2*I)*d^3)*(b*x + a))*sin(6*b*x + 6*a
) - (I*(b*x + a)^3*d^3 + 2*I*b*c*d^2 - 2*I*a*d^3 + 3*(I*b*c*d^2 - I*a*d^3)
*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + (3*I*a^2 + 2*I)*d^3)*(b*x
+ a))*sin(4*b*x + 4*a) - (-I*(b*x + a)^3*d^3 - 2*I*b*c*d^2 + 2*I*a*d^3 ...
```

### 3.317.8 Giac [F]

$$\int (c+dx)^3 \csc^2(a+bx) \sec^3(a+bx) \, dx = \int (dx+c)^3 \csc(bx+a)^2 \sec(bx+a)^3 \, dx$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

### 3.317.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^2(a+bx) \sec^3(a+bx) \, dx = \text{Hanged}$$

input
```
int((c + d*x)^3/(cos(a + b*x)^3*sin(a + b*x)^2),x)
```

output
```
\text{Hanged}
```

# 3.318 $\int (c+dx)^2 \csc^2(a+bx) \sec^3(a+bx)\,dx$

3.318.1 Optimal result . . . . . 2388
3.318.2 Mathematica [B] (warning: unable to verify) . . . . . 2389
3.318.3 Rubi [A] (verified) . . . . . 2390
3.318.4 Maple [B] (verified) . . . . . 2392
3.318.5 Fricas [B] (verification not implemented) . . . . . 2393
3.318.6 Sympy [F] . . . . . 2394
3.318.7 Maxima [B] (verification not implemented) . . . . . 2395
3.318.8 Giac [F] . . . . . 2395
3.318.9 Mupad [F(-1)] . . . . . 2396

### 3.318.1 Optimal result

Integrand size = 24, antiderivative size = 341

$$\int (c+dx)^2 \csc^2(a+bx)\sec^3(a+bx)\,dx = -\frac{3i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b} + \frac{2d^2x\text{arctanh}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b^2} - \frac{d^2x\text{arctanh}(\cos(a+bx))}{b^2} + \frac{d(c+dx)\text{arctanh}(\cos(a+bx))}{b^2} + \frac{d^2\text{arctanh}(\sin(a+bx))}{b^3} - \frac{3(c+dx)^2\csc(a+bx)}{2b} + \frac{2id^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3} + \frac{3id(c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2} - \frac{3id(c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2} - \frac{2id^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3} - \frac{3d^2\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3} + \frac{3d^2\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3} - \frac{d(c+dx)\sec(a+bx)}{b^2} + \frac{(c+dx)^2\csc(a+bx)\sec^2(a+bx)}{2b}$$

output
```
-3*I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b+2*d^2*x*arctanh(exp(I*(b*x+a)))/b^
2-6*d*(d*x+c)*arctanh(exp(I*(b*x+a)))/b^2-d^2*x*arctanh(cos(b*x+a))/b^2+d*
(d*x+c)*arctanh(cos(b*x+a))/b^2+d^2*arctanh(sin(b*x+a))/b^3-3/2*(d*x+c)^2*
csc(b*x+a)/b+2*I*d^2*polylog(2,-exp(I*(b*x+a)))/b^3+3*I*d*(d*x+c)*polylog(
2,-I*exp(I*(b*x+a)))/b^2-3*I*d*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^2-2*I
*d^2*polylog(2,exp(I*(b*x+a)))/b^3-3*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+
3*d^2*polylog(3,I*exp(I*(b*x+a)))/b^3-d*(d*x+c)*sec(b*x+a)/b^2+1/2*(d*x+c)
^2*csc(b*x+a)*sec(b*x+a)^2/b
```

## 3.318.2 Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 889 vs. $2(341) = 682$.

Time = 7.33 (sec) , antiderivative size = 889, normalized size of antiderivative = 2.61

$$\int (c+dx)^2\csc^2(a+bx)\sec^3(a+bx)\,dx =$$

$$-\frac{6ib^2c^2\arctan\left(e^{i(a+bx)}\right)+4id^2\arctan\left(e^{i(a+bx)}\right)-6b^2cdx\log\left(1-ie^{i(a+bx)}\right)-3b^2d^2x^2\log\left(1-ie^{i(a+bx)}\right)}{}$$

$$-\frac{(c+dx)\csc(a)\sec(a)(bc\cos(a)+bdx\cos(a)+d\sin(a))}{b^2}$$

$$+\frac{4icd\arctan\left(\frac{i\cos(a)-i\sin(a)\tan\left(\frac{bx}{2}\right)}{\sqrt{\cos^2(a)+\sin^2(a)}}\right)}{b^2\sqrt{\cos^2(a)+\sin^2(a)}}$$

$$+\frac{\sec\left(\frac{a}{2}\right)\sec\left(\frac{a}{2}+\frac{bx}{2}\right)\left(-c^2\sin\left(\frac{bx}{2}\right)-2cdx\sin\left(\frac{bx}{2}\right)-d^2x^2\sin\left(\frac{bx}{2}\right)\right)}{2b}$$

$$+\frac{\csc\left(\frac{a}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)\left(c^2\sin\left(\frac{bx}{2}\right)+2cdx\sin\left(\frac{bx}{2}\right)+d^2x^2\sin\left(\frac{bx}{2}\right)\right)}{2b}$$

$$+\frac{c^2+2cdx+d^2x^2}{4b\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)^2}+\frac{-cd\sin\left(\frac{bx}{2}\right)-d^2x\sin\left(\frac{bx}{2}\right)}{b^2\left(\cos\left(\frac{a}{2}\right)-\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}$$

$$+\frac{-c^2-2cdx-d^2x^2}{4b\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)+\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)^2}+\frac{cd\sin\left(\frac{bx}{2}\right)+d^2x\sin\left(\frac{bx}{2}\right)}{b^2\left(\cos\left(\frac{a}{2}\right)+\sin\left(\frac{a}{2}\right)\right)\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)+\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}$$

$$+\frac{2d^2\left(-\frac{2\arctan(\tan(a))\operatorname{arctanh}\left(\frac{-\cos(a)+\sin(a)\tan\left(\frac{bx}{2}\right)}{\sqrt{\cos^2(a)+\sin^2(a)}}\right)}{\sqrt{\cos^2(a)+\sin^2(a)}}+\frac{((bx+\arctan(\tan(a)))(\log(1-e^{i(bx+\arctan(\tan(a)))})-\log(1+e^{i(bx+\arctan}}{}\right.}{b^3}$$

input
```
Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

output
```
-1/2*((6*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + (4*I)*d^2*ArcTan[E^(I*(a + b
*x))] - 6*b^2*c*d*x*Log[1 - I*E^(I*(a + b*x))] - 3*b^2*d^2*x^2*Log[1 - I*E
^(I*(a + b*x))] + 6*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x))] + 3*b^2*d^2*x^2*L
og[1 + I*E^(I*(a + b*x))] - (6*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a +
b*x))] + (6*I)*b*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))] + 6*d^2*PolyLog
[3, (-I)*E^(I*(a + b*x))] - 6*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - ((c
 + d*x)*Csc[a]*Sec[a]*(b*c*Cos[a] + b*d*x*Cos[a] + d*Sin[a]))/b^2 + ((4*I)
*c*d*ArcTan[(I*Cos[a] - I*Sin[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]])
/(b^2*Sqrt[Cos[a]^2 + Sin[a]^2]) + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c^2*Sin
[(b*x)/2]) - 2*c*d*x*Sin[(b*x)/2] - d^2*x^2*Sin[(b*x)/2]))/(2*b) + (Csc[a/
2]*Csc[a/2 + (b*x)/2]*(c^2*Sin[(b*x)/2] + 2*c*d*x*Sin[(b*x)/2] + d^2*x^2*S
in[(b*x)/2]))/(2*b) + (c^2 + 2*c*d*x + d^2*x^2)/(4*b*(Cos[a/2 + (b*x)/2] -
 Sin[a/2 + (b*x)/2])^2) + (-(c*d*Sin[(b*x)/2]) - d^2*x*Sin[(b*x)/2])/(b^2*
(Cos[a/2] - Sin[a/2])*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) + (-c^2 -
 2*c*d*x - d^2*x^2)/(4*b*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2])^2) + (c
*d*Sin[(b*x)/2] + d^2*x*Sin[(b*x)/2])/(b^2*(Cos[a/2] + Sin[a/2])*(Cos[a/2
+ (b*x)/2] + Sin[a/2 + (b*x)/2])) + (2*d^2*((-2*ArcTan[Tan[a]]*ArcTanh[(-C
os[a] + Sin[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]])/Sqrt[Cos[a]^2 + S
in[a]^2] + (((b*x + ArcTan[Tan[a]])*(Log[1 - E^(I*(b*x + ArcTan[Tan[a]]))]
 - Log[1 + E^(I*(b*x + ArcTan[Tan[a]]))]) + I*(PolyLog[2, -E^(I*(b*x + ...
```

## 3.318.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4920, 27, 7292, 27, 7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \csc^2(a+bx)\sec^3(a+bx)\,dx$$

$$\Big\downarrow \text{4920}$$

$$-2d\int \frac{1}{2}(c+dx)\left(\frac{\csc(a+bx)\sec^2(a+bx)}{b}+\frac{3\text{arctanh}(\sin(a+bx))}{b}-\frac{3\csc(a+bx)}{b}\right)dx+\frac{3(c+dx)^2\text{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)^2\csc(a+bx)}{2b}+\frac{(c+dx)^2\csc(a+bx)\sec^2(a+bx)}{2b}$$

$$\Big\downarrow \text{27}$$

$$-d\int (c+dx)\left(\frac{\csc(a+bx)\sec^2(a+bx)}{b}+\frac{3\text{arctanh}(\sin(a+bx))}{b}-\frac{3\csc(a+bx)}{b}\right)dx+\frac{3(c+dx)^2\text{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)^2\csc(a+bx)}{2b}+\frac{(c+dx)^2\csc(a+bx)\sec^2(a+bx)}{2b}$$

$$\Big\downarrow \text{7292}$$

$$-d\int \frac{(c+dx)\left(\csc(a+bx)\sec^2(a+bx)+3\text{arctanh}(\sin(a+bx))-3\csc(a+bx)\right)}{b}dx+\frac{3(c+dx)^2\text{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)^2\csc(a+bx)}{2b}+\frac{(c+dx)^2\csc(a+bx)\sec^2(a+bx)}{2b}$$

$$\Big\downarrow \text{27}$$

$$-\frac{d\int (c+dx)\left(\csc(a+bx)\sec^2(a+bx)+3\text{arctanh}(\sin(a+bx))-3\csc(a+bx)\right)dx}{b}+\frac{3(c+dx)^2\text{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)^2\csc(a+bx)}{2b}+\frac{(c+dx)^2\csc(a+bx)\sec^2(a+bx)}{2b}$$

$$\Big\downarrow \text{7293}$$

$$-\frac{d\int \left((c+dx)\csc(a+bx)\sec^2(a+bx)+3(c+dx)(\text{arctanh}(\sin(a+bx))-\csc(a+bx))\right)dx}{b}+\frac{3(c+dx)^2\text{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)^2\csc(a+bx)}{2b}+\frac{(c+dx)^2\csc(a+bx)\sec^2(a+bx)}{2b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\left(\frac{3i(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{d}-\frac{d\text{arctanh}(\sin(a+bx))}{b^2}+\frac{6(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{3(c+dx)^2\text{arctanh}(\sin(a+bx))}{2d}-\frac{(c+dx)\text{arc}}{}\right.}{}$$

$$\frac{3(c+dx)^2\text{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)^2\csc(a+bx)}{2b}+\frac{(c+dx)^2\csc(a+bx)\sec^2(a+bx)}{2b}$$

input
```
Int[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

output
```
(3*(c + d*x)^2*ArcTanh[Sin[a + b*x]])/(2*b) - (3*(c + d*x)^2*Csc[a + b*x])
/(2*b) + ((c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^2)/(2*b) - (d*(((3*I)*(c +
 d*x)^2*ArcTan[E^(I*(a + b*x))])/d - (2*d*x*ArcTanh[E^(I*(a + b*x))])/b +
(6*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b + (d*x*ArcTanh[Cos[a + b*x]])/b -
 ((c + d*x)*ArcTanh[Cos[a + b*x]])/b - (d*ArcTanh[Sin[a + b*x]])/b^2 + (3*
(c + d*x)^2*ArcTanh[Sin[a + b*x]])/(2*d) - ((2*I)*d*PolyLog[2, -E^(I*(a +
b*x))])/b^2 - ((3*I)*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b + ((3*I
)*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b + ((2*I)*d*PolyLog[2, E^(I*(a
 + b*x))])/b^2 + (3*d*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^2 - (3*d*PolyLog
[3, I*E^(I*(a + b*x))])/b^2 + ((c + d*x)*Sec[a + b*x])/b))/b
```

3.318. $\int (c+dx)^2\csc^2(a+bx)\sec^3(a+bx)\,dx$

### 3.318.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

## 3.318.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 769 vs. $2(310) = 620$.

Time = 1.05 (sec) , antiderivative size = 770, normalized size of antiderivative = 2.26

| method | result |
|---|---|
| risch | $\frac{3d^2\ln\left(1-i\,\mathrm{e}^{i(xb+a)}\right)x^2}{2b}-\frac{3d^2\ln\left(1+i\,\mathrm{e}^{i(xb+a)}\right)x^2}{2b}-\frac{2id^2\arctan\left(\mathrm{e}^{i(xb+a)}\right)}{b^3}-\frac{2dc\ln\left(\mathrm{e}^{i(xb+a)}+1\right)}{b^2}+\frac{3id^2\,\mathrm{polylog}\left(2,-i\,\mathrm{e}^{i(xb+a)}\right)}{b^2}$ |

input
```
int((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*d^2/b^3*a*ln(exp(I*(b*x+a))-1)-3*I/b*c^2*arctan(exp(I*(b*x+a)))+3/2/b*d
^2*ln(1-I*exp(I*(b*x+a)))*x^2-3/2/b*d^2*ln(1+I*exp(I*(b*x+a)))*x^2-2*d/b^2
*c*ln(exp(I*(b*x+a))+1)+2*d/b^2*c*ln(exp(I*(b*x+a))-1)-2*d^2/b^2*ln(exp(I*
(b*x+a))+1)*x-3*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+3*d^2*polylog(3,I*exp
(I*(b*x+a)))/b^3+6*I/b^2*c*d*a*arctan(exp(I*(b*x+a)))-2*I/b^3*d^2*arctan(e
xp(I*(b*x+a)))+2*I/b^3*d^2*dilog(exp(I*(b*x+a)))+2*I/b^3*d^2*dilog(exp(I*(
b*x+a))+1)-3*I/b^3*d^2*a^2*arctan(exp(I*(b*x+a)))-3*I/b^2*d^2*polylog(2,I*
exp(I*(b*x+a)))*x-3*I/b^2*c*d*polylog(2,I*exp(I*(b*x+a)))+3*I/b^2*c*d*poly
log(2,-I*exp(I*(b*x+a)))+3*I/b^2*d^2*polylog(2,-I*exp(I*(b*x+a)))*x+3/b^2*
c*d*ln(1-I*exp(I*(b*x+a)))*a-3/b^2*c*d*ln(1+I*exp(I*(b*x+a)))*a-3/b*c*d*ln
(1+I*exp(I*(b*x+a)))*x+3/b*c*d*ln(1-I*exp(I*(b*x+a)))*x-I/b^2/(exp(2*I*(b*
x+a))+1)^2/(exp(2*I*(b*x+a))-1)*(3*x^2*d^2*b*exp(5*I*(b*x+a))+6*c*d*x*b*ex
p(5*I*(b*x+a))+3*c^2*b*exp(5*I*(b*x+a))+2*x^2*d^2*b*exp(3*I*(b*x+a))+4*c*d
*x*b*exp(3*I*(b*x+a))-2*I*d^2*x*exp(5*I*(b*x+a))+2*c^2*b*exp(3*I*(b*x+a))+
3*x^2*d^2*b*exp(I*(b*x+a))-2*I*c*d*exp(5*I*(b*x+a))+6*c*d*x*b*exp(I*(b*x+a
))+3*c^2*b*exp(I*(b*x+a))+2*I*d^2*x*exp(I*(b*x+a))+2*I*d*c*exp(I*(b*x+a)))
-3/2/b^3*a^2*d^2*ln(1-I*exp(I*(b*x+a)))+3/2/b^3*a^2*d^2*ln(1+I*exp(I*(b*x+
a)))
```

### 3.318.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1366 vs. 2(297) = 594.

Time = 0.37 (sec) , antiderivative size = 1366, normalized size of antiderivative = 4.01

$$\int (c+dx)^2 \csc^2(a+bx) \sec^3(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(2*b^2*d^2*x^2 - 4*I*d^2*cos(b*x + a)^2*dilog(cos(b*x + a) + I*sin(b*x
 + a))*sin(b*x + a) + 4*I*d^2*cos(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*
x + a))*sin(b*x + a) - 4*I*d^2*cos(b*x + a)^2*dilog(-cos(b*x + a) + I*sin(
b*x + a))*sin(b*x + a) + 4*I*d^2*cos(b*x + a)^2*dilog(-cos(b*x + a) - I*si
n(b*x + a))*sin(b*x + a) - 6*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a)
+ sin(b*x + a))*sin(b*x + a) + 6*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x +
 a) - sin(b*x + a))*sin(b*x + a) - 6*d^2*cos(b*x + a)^2*polylog(3, -I*cos(
b*x + a) + sin(b*x + a))*sin(b*x + a) + 6*d^2*cos(b*x + a)^2*polylog(3, -I
*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) + 4*b^2*c*d*x - 6*(I*b*d^2*x +
I*b*c*d)*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a)
- 6*(I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x +
a))*sin(b*x + a) - 6*(-I*b*d^2*x - I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*
x + a) + sin(b*x + a))*sin(b*x + a) - 6*(-I*b*d^2*x - I*b*c*d)*cos(b*x + a
)^2*dilog(-I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - 4*(b*d^2*x + b*c*
d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + (3
*b^2*c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) +
I*sin(b*x + a) + I)*sin(b*x + a) - 4*(b*d^2*x + b*c*d)*cos(b*x + a)^2*log(
cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - (3*b^2*c^2 - 6*a*b*c*d +
 (3*a^2 + 2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I)*si
n(b*x + a) + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*...
```

### 3.318.6 Sympy [F]

$$\int (c+dx)^2\csc^2(a+bx)\sec^3(a+bx)\,dx=\int (c+dx)^2\csc^2(a+bx)\sec^3(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*csc(b*x+a)**2*sec(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**2*csc(a + b*x)**2*sec(a + b*x)**3, x)
```

### 3.318.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3828 vs. $2(297) = 594$.

Time = 1.12 (sec) , antiderivative size = 3828, normalized size of antiderivative = 11.23

$$\int (c+dx)^2 \csc^2(a+bx) \sec^3(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/4*(c^2*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*lo
g(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1)) - 2*a*c*d*(2*(3*sin(b*x + a
)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log(sin(b*x + a) + 1) + 3*log
(sin(b*x + a) - 1))/b + a^2*d^2*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3
- sin(b*x + a)) - 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b^2 -
 4*(2*(3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 - (3*(b*x +
 a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(6*b*x + 6*a) - (3*(b*
x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(4*b*x + 4*a) + (3*
(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(2*b*x + 2*a) -
(3*I*(b*x + a)^2*d^2 + 6*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(6*b*
x + 6*a) - (3*I*(b*x + a)^2*d^2 + 6*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^
2)*sin(4*b*x + 4*a) - (-3*I*(b*x + a)^2*d^2 + 6*(-I*b*c*d + I*a*d^2)*(b*x
+ a) - 2*I*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1)
+ 2*(3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 - (3*(b*x + a
)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(6*b*x + 6*a) - (3*(b*x
+ a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(4*b*x + 4*a) + (3*(b
*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(2*b*x + 2*a) - (3
*I*(b*x + a)^2*d^2 + 6*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(6*b*x
+ 6*a) - (3*I*(b*x + a)^2*d^2 + 6*(I*b*c*d - I*a*d^2)*(b*x + a) + 2*I*d^2)
*sin(4*b*x + 4*a) - (-3*I*(b*x + a)^2*d^2 + 6*(-I*b*c*d + I*a*d^2)*(b*x...
```

### 3.318.8 Giac [F]

$$\int (c+dx)^2 \csc^2(a+bx) \sec^3(a+bx) \, dx = \int (dx+c)^2 \csc(bx+a)^2 \sec(bx+a)^3 \, dx$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

### 3.318.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc^2(a+bx) \sec^3(a+bx) \, dx = \text{Hanged}$$

input
```
int((c + d*x)^2/(cos(a + b*x)^3*sin(a + b*x)^2),x)
```

output
```
\text{Hanged}
```

# 3.319 $\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx$

3.319.1 Optimal result . . . 2397
3.319.2 Mathematica [C] (warning: unable to verify) . . . 2398
3.319.3 Rubi [A] (verified) . . . 2399
3.319.4 Maple [B] (verified) . . . 2400
3.319.5 Fricas [B] (verification not implemented) . . . 2401
3.319.6 Sympy [F] . . . 2402
3.319.7 Maxima [F] . . . 2402
3.319.8 Giac [F] . . . 2403
3.319.9 Mupad [F(-1)] . . . 2403

## 3.319.1 Optimal result

Integrand size = 22, antiderivative size = 162

$$\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx = -\frac{3idx\arctan\left(e^{i(a+bx)}\right)}{b}-\frac{d\text{arctanh}(\cos(a+bx))}{b^2}+\frac{3c\text{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)\csc(a+bx)}{2b}+\frac{3id\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2}-\frac{3id\,\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2}-\frac{d\sec(a+bx)}{2b^2}+\frac{(c+dx)\csc(a+bx)\sec^2(a+bx)}{2b}$$

output
```
-3*I*d*x*arctan(exp(I*(b*x+a)))/b-d*arctanh(cos(b*x+a))/b^2+3/2*c*arctanh(
sin(b*x+a))/b-3/2*(d*x+c)*csc(b*x+a)/b+3/2*I*d*polylog(2,-I*exp(I*(b*x+a))
)/b^2-3/2*I*d*polylog(2,I*exp(I*(b*x+a)))/b^2-1/2*d*sec(b*x+a)/b^2+1/2*(d*
x+c)*csc(b*x+a)*sec(b*x+a)^2/b
```

### 3.319.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.96 (sec) , antiderivative size = 669, normalized size of antiderivative = 4.13

$$\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx$$
$$= \frac{d\left(a\cos\left(\frac{1}{2}(a+bx)\right)-(a+bx)\cos\left(\frac{1}{2}(a+bx)\right)\right)\csc\left(\frac{1}{2}(a+bx)\right)}{2b^2}$$
$$-\frac{c\csc(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},2,\frac{1}{2},\sin^2(a+bx)\right)}{b}$$
$$-\frac{d\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b^2}+\frac{d\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b^2}$$
$$-\frac{3dx\left(a\log\left(1-\tan\left(\frac{1}{2}(a+bx)\right)\right)-a\log\left(1+\tan\left(\frac{1}{2}(a+bx)\right)\right)-i\left(\log\left(1+i\tan\left(\frac{1}{2}(a+bx)\right)\right)\log\left(\left(\frac{1}{2}-\right.\right.\right.\right.}{}$$
$$+\frac{dx}{4b\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)^2}-\frac{d\sin\left(\frac{1}{2}(a+bx)\right)}{2b^2\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)}$$
$$-\frac{dx}{4b\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right)^2}+\frac{d\sin\left(\frac{1}{2}(a+bx)\right)}{2b^2\left(\cos\left(\frac{1}{2}(a+bx)\right)+\sin\left(\frac{1}{2}(a+bx)\right)\right)}$$
$$+\frac{d\sec\left(\frac{1}{2}(a+bx)\right)\left(a\sin\left(\frac{1}{2}(a+bx)\right)-(a+bx)\sin\left(\frac{1}{2}(a+bx)\right)\right)}{2b^2}$$

input
```
Integrate[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

output
```
(d*(a*Cos[(a + b*x)/2] - (a + b*x)*Cos[(a + b*x)/2])*Csc[(a + b*x)/2])/(2*
b^2) - (c*Csc[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, Sin[a + b*x]^2])/b
- (d*Log[Cos[(a + b*x)/2]])/b^2 + (d*Log[Sin[(a + b*x)/2]])/b^2 - (3*d*x*(
a*Log[1 - Tan[(a + b*x)/2]] - a*Log[1 + Tan[(a + b*x)/2]] - I*(Log[1 + I*T
an[(a + b*x)/2]]*Log[(1/2 - I/2)*(1 + Tan[(a + b*x)/2])] + PolyLog[2, ((1
+ I) - (1 - I)*Tan[(a + b*x)/2])/2]) + I*(Log[1 - I*Tan[(a + b*x)/2]]*Log[
(1/2 + I/2)*(1 + Tan[(a + b*x)/2])] + PolyLog[2, (-1/2 - I/2)*(I + Tan[(a
+ b*x)/2])]) - I*(Log[1 - I*Tan[(a + b*x)/2]]*Log[(-1/2 + I/2)*(-1 + Tan[(
a + b*x)/2])] + PolyLog[2, ((1 + I) + (1 - I)*Tan[(a + b*x)/2])/2]) + I*(L
og[1 + I*Tan[(a + b*x)/2]]*Log[(-1/2 - I/2)*(-1 + Tan[(a + b*x)/2])] + Pol
yLog[2, ((1 - I) + (1 + I)*Tan[(a + b*x)/2])/2])))/(2*b*(a - I*Log[1 - I*T
an[(a + b*x)/2]] + I*Log[1 + I*Tan[(a + b*x)/2]])) + (d*x)/(4*b*(Cos[(a +
b*x)/2] - Sin[(a + b*x)/2])^2) - (d*Sin[(a + b*x)/2])/(2*b^2*(Cos[(a + b*x
)/2] - Sin[(a + b*x)/2])) - (d*x)/(4*b*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2
])^2) + (d*Sin[(a + b*x)/2])/(2*b^2*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2]))
 + (d*Sec[(a + b*x)/2]*(a*Sin[(a + b*x)/2] - (a + b*x)*Sin[(a + b*x)/2]))/
(2*b^2)
```

### 3.319.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4920, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx$$

$$\Big\downarrow \ 4920$$

$$-d\int \left(\frac{\csc(a+bx)\sec^2(a+bx)}{2b}+\frac{3\text{arctanh}(\sin(a+bx))}{2b}-\frac{3\csc(a+bx)}{2b}\right)dx+\frac{3(c+dx)\text{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)\csc(a+bx)}{2b}+\frac{(c+dx)\csc(a+bx)\sec^2(a+bx)}{2b}$$

$$\Big\downarrow \ 2009$$

$$-d\left(\frac{3ix\arctan\left(e^{i(a+bx)}\right)}{b}+\frac{\text{arctanh}(\cos(a+bx))}{b^2}+\frac{3x\text{arctanh}(\sin(a+bx))}{2b}-\frac{3i\,\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2}+\frac{3i\,\text{Po}\ldots\right.$$
$$\frac{3(c+dx)\text{arctanh}(\sin(a+bx))}{2b}-\frac{3(c+dx)\csc(a+bx)}{2b}+\frac{(c+dx)\csc(a+bx)\sec^2(a+bx)}{2b}$$

input
```
Int[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

output
```
(3*(c + d*x)*ArcTanh[Sin[a + b*x]])/(2*b) - (3*(c + d*x)*Csc[a + b*x])/(2*
b) + ((c + d*x)*Csc[a + b*x]*Sec[a + b*x]^2)/(2*b) - d*(((3*I)*x*ArcTan[E^
(I*(a + b*x))])/b + ArcTanh[Cos[a + b*x]]/b^2 + (3*x*ArcTanh[Sin[a + b*x]]
)/(2*b) - (((3*I)/2)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 + (((3*I)/2)*Po
lyLog[2, I*E^(I*(a + b*x))])/b^2 + Sec[a + b*x]/(2*b^2))
```

### 3.319.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4920
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Simp[(c + d*x)^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

## 3.319.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(139) = 278.

Time = 0.90 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.12

| method | result |
|---|---|
| risch | $-\frac{i\left(3dxb\,e^{5i(xb+a)}+3cb\,e^{5i(xb+a)}+2dxb\,e^{3i(xb+a)}+2cb\,e^{3i(xb+a)}-id\,e^{5i(xb+a)}+3dxb\,e^{i(xb+a)}+3cb\,e^{i(xb+a)}+id\,e^{i(xb+a)}\right)}{b^2\left(e^{2i(xb+a)}+1\right)^2\left(e^{2i(xb+a)}-1\right)}-\frac{3ic\,\mathrm{arc}$ |

input
```
int((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-I/b^2/(exp(2*I*(b*x+a))+1)^2/(exp(2*I*(b*x+a))-1)*(3*d*x*b*exp(5*I*(b*x+a
))+3*c*b*exp(5*I*(b*x+a))+2*d*x*b*exp(3*I*(b*x+a))+2*c*b*exp(3*I*(b*x+a))-
I*d*exp(5*I*(b*x+a))+3*d*x*b*exp(I*(b*x+a))+3*c*b*exp(I*(b*x+a))+I*d*exp(I
*(b*x+a)))-3*I/b*c*arctan(exp(I*(b*x+a)))+3*I/b^2*a*d*arctan(exp(I*(b*x+a)
))-d/b^2*ln(exp(I*(b*x+a))+1)+d/b^2*ln(exp(I*(b*x+a))-1)-3/2/b^2*d*ln(1+I*
exp(I*(b*x+a)))*a-3/2/b*d*ln(1+I*exp(I*(b*x+a)))*x+3/2/b*d*ln(1-I*exp(I*(b
*x+a)))*x+3/2/b^2*d*ln(1-I*exp(I*(b*x+a)))*a+3/2*I/b^2*d*dilog(1+I*exp(I*(
b*x+a)))-3/2*I/b^2*d*dilog(1-I*exp(I*(b*x+a)))
```

## 3.319.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(132) = 264$.

Time = 0.32 (sec) , antiderivative size = 592, normalized size of antiderivative = 3.65

$$\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx$$
$$= \frac{-3i\,d\cos{(bx+a)}^2\,\mathrm{Li}_2(i\,\cos{(bx+a)}+\sin{(bx+a)})\sin{(bx+a)}-3i\,d\cos{(bx+a)}^2\,\mathrm{Li}_2(i\,\cos{(bx+a)}-\ldots}{}$$

input
```
integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(-3*I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x +
a) - 3*I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a
) + 3*I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a
) + 3*I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a
) + 3*(b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + I)*si
n(b*x + a) - 3*(b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a
) + I)*sin(b*x + a) - 2*d*cos(b*x + a)^2*log(1/2*cos(b*x + a) + 1/2)*sin(b
*x + a) + 3*(b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a)
 + 1)*sin(b*x + a) - 3*(b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) - s
in(b*x + a) + 1)*sin(b*x + a) + 3*(b*d*x + a*d)*cos(b*x + a)^2*log(-I*cos(
b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) - 3*(b*d*x + a*d)*cos(b*x + a)^2
*log(-I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) + 2*d*cos(b*x + a)^2
*log(-1/2*cos(b*x + a) + 1/2)*sin(b*x + a) + 3*(b*c - a*d)*cos(b*x + a)^2*
log(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) - 3*(b*c - a*d)*cos(b
*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + 2*b*d*x -
 6*(b*d*x + b*c)*cos(b*x + a)^2 - 2*d*cos(b*x + a)*sin(b*x + a) + 2*b*c)/(
b^2*cos(b*x + a)^2*sin(b*x + a))
```

## 3.319.6 Sympy [F]

$$\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx=\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)**2*sec(b*x+a)**3,x)
```

output
```
Integral((c + d*x)*csc(a + b*x)**2*sec(a + b*x)**3, x)
```

## 3.319.7 Maxima [F]

$$\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx=\int (dx+c)\csc(bx+a)^2\sec(bx+a)^3\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*(8*(b*d*x + b*c)*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) - 4*(d*cos(5*b*x +
5*a) - d*cos(b*x + a) - 3*(b*d*x + b*c)*sin(5*b*x + 5*a) - 2*(b*d*x + b*c)
*sin(3*b*x + 3*a) - 3*(b*d*x + b*c)*sin(b*x + a))*cos(6*b*x + 6*a) - 4*(d*
cos(4*b*x + 4*a) - d*cos(2*b*x + 2*a) + 3*(b*d*x + b*c)*sin(4*b*x + 4*a) -
 3*(b*d*x + b*c)*sin(2*b*x + 2*a) - d)*cos(5*b*x + 5*a) + 4*(d*cos(b*x + a
) + 2*(b*d*x + b*c)*sin(3*b*x + 3*a) + 3*(b*d*x + b*c)*sin(b*x + a))*cos(4
*b*x + 4*a) - 4*(d*cos(b*x + a) + 3*(b*d*x + b*c)*sin(b*x + a))*cos(2*b*x
+ 2*a) - 4*d*cos(b*x + a) + 12*(b^2*d*cos(6*b*x + 6*a)^2 + b^2*d*cos(4*b*x
 + 4*a)^2 + b^2*d*cos(2*b*x + 2*a)^2 + b^2*d*sin(6*b*x + 6*a)^2 + b^2*d*si
n(4*b*x + 4*a)^2 - 2*b^2*d*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + b^2*d*sin(2
*b*x + 2*a)^2 + 2*b^2*d*cos(2*b*x + 2*a) + b^2*d + 2*(b^2*d*cos(4*b*x + 4*
a) - b^2*d*cos(2*b*x + 2*a) - b^2*d)*cos(6*b*x + 6*a) - 2*(b^2*d*cos(2*b*x
 + 2*a) + b^2*d)*cos(4*b*x + 4*a) + 2*(b^2*d*sin(4*b*x + 4*a) - b^2*d*sin(
2*b*x + 2*a))*sin(6*b*x + 6*a))*integrate((x*cos(2*b*x + 2*a)*cos(b*x + a)
 + x*sin(2*b*x + 2*a)*sin(b*x + a) + x*cos(b*x + a))/(cos(2*b*x + 2*a)^2 +
 sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1), x) + 3*(b*c*cos(6*b*x + 6*a
)^2 + b*c*cos(4*b*x + 4*a)^2 + b*c*cos(2*b*x + 2*a)^2 + b*c*sin(6*b*x + 6*
a)^2 + b*c*sin(4*b*x + 4*a)^2 - 2*b*c*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) +
b*c*sin(2*b*x + 2*a)^2 + 2*b*c*cos(2*b*x + 2*a) + b*c + 2*(b*c*cos(4*b*x +
 4*a) - b*c*cos(2*b*x + 2*a) - b*c)*cos(6*b*x + 6*a) - 2*(b*c*cos(2*b*x...
```

### 3.319.8 Giac [F]

$$\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx=\int (dx+c)\csc{(bx+a)}^2\sec{(bx+a)}^3\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

### 3.319.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx=\text{Hanged}$$

input
```
int((c + d*x)/(cos(a + b*x)^3*sin(a + b*x)^2),x)
```

output
```
\text{Hanged}
```

# 3.320 $\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$

3.320.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2404
3.320.2 Mathematica [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2404
3.320.3 Rubi [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2405
3.320.4 Maple [N/A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2405
3.320.5 Fricas [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2406
3.320.6 Sympy [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2406
3.320.7 Maxima [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2406
3.320.8 Giac [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2407
3.320.9 Mupad [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2408

## 3.320.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx},x\right)$$

output
```
CannotIntegrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x)
```

## 3.320.2 Mathematica [N/A]

Not integrable

Time = 14.98 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x),x]
```

output
```
Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x), x]
```

## 3.320.3 Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}dx$$

input
```
Int[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x),x]
```

output
```
$Aborted
```

### 3.320.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.320.4 Maple [N/A] (verified)

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc(xb+a)^2\sec(xb+a)^3}{dx+c}dx$$

input
```
int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x)
```

output
```
int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x)
```

### 3.320.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^2\sec(bx+a)^3}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^2*sec(b*x + a)^3/(d*x + c), x)
```

### 3.320.6 Sympy [N/A]

Not integrable

Time = 7.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

input
```
integrate(csc(b*x+a)**2*sec(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(csc(a + b*x)**2*sec(a + b*x)**3/(c + d*x), x)
```

### 3.320.7 Maxima [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 3695, normalized size of antiderivative = 153.96

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^2\sec(bx+a)^3}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output
```
(2*(b*d*x + b*c)*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) + (d*cos(5*b*x + 5*a) -
 d*cos(b*x + a) + 3*(b*d*x + b*c)*sin(5*b*x + 5*a) + 2*(b*d*x + b*c)*sin(3
*b*x + 3*a) + 3*(b*d*x + b*c)*sin(b*x + a))*cos(6*b*x + 6*a) + (d*cos(4*b*
x + 4*a) - d*cos(2*b*x + 2*a) - 3*(b*d*x + b*c)*sin(4*b*x + 4*a) + 3*(b*d*
x + b*c)*sin(2*b*x + 2*a) - d)*cos(5*b*x + 5*a) - (d*cos(b*x + a) - 2*(b*d
*x + b*c)*sin(3*b*x + 3*a) - 3*(b*d*x + b*c)*sin(b*x + a))*cos(4*b*x + 4*a
) + (d*cos(b*x + a) - 3*(b*d*x + b*c)*sin(b*x + a))*cos(2*b*x + 2*a) + d*c
os(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*
c*d*x + b^2*c^2)*cos(6*b*x + 6*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*cos(4*b*x + 4*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2
*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(6*b*x + 6*a)^2 + (b^2*d^
2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 - 2*(b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + (b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*
c^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a) + (b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(6*b*x + 6*a) - 2*(b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2
*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*co
s(2*b*x + 2*a) + 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)
 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a))*sin(6*b*x + ...
```

## 3.320.8 Giac [N/A]

Not integrable

Time = 117.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^2\sec(bx+a)^3}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sec(b*x + a)^3/(d*x + c), x)
```

### 3.320.9 Mupad [N/A]

Not integrable

Time = 25.83 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^2\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)), x)
```

# 3.321 $\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$

3.321.1 Optimal result . . . 2409
3.321.2 Mathematica [N/A] . . . 2409
3.321.3 Rubi [N/A] . . . 2410
3.321.4 Maple [N/A] (verified) . . . 2410
3.321.5 Fricas [N/A] . . . 2411
3.321.6 Sympy [N/A] . . . 2411
3.321.7 Maxima [N/A] . . . 2411
3.321.8 Giac [F(-1)] . . . 2412
3.321.9 Mupad [N/A] . . . 2413

## 3.321.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2},x\right)$$

output
```
CannotIntegrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x)
```

## 3.321.2 Mathematica [N/A]

Not integrable

Time = 18.72 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2,x]
```

output
```
Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

## 3.321.3 Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$
$$\Big\downarrow \; 7299$$
$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}dx$$

input
```
Int[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2,x]
```

output
```
$Aborted
```

### 3.321.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.321.4 Maple [N/A] (verified)

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc\left(xb+a\right)^{2}\sec\left(xb+a\right)^{3}}{\left(dx+c\right)^{2}}dx$$

input
```
int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x)
```

### 3.321.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc{(bx+a)}^2\sec{(bx+a)}^3}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^2*sec(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.321.6 Sympy [N/A]

Not integrable

Time = 14.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^2{(a+bx)}\sec^3{(a+bx)}}{(c+dx)^2}\,dx$$

input
```
integrate(csc(b*x+a)**2*sec(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)**2*sec(a + b*x)**3/(c + d*x)**2, x)
```

### 3.321.7 Maxima [N/A]

Not integrable

Time = 7.40 (sec) , antiderivative size = 4747, normalized size of antiderivative = 197.79

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc{(bx+a)}^2\sec{(bx+a)}^3}{(dx+c)^2}\,dx$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
(2*(b*d*x + b*c)*cos(3*b*x + 3*a)*sin(2*b*x + 2*a) + (2*d*cos(5*b*x + 5*a)
 - 2*d*cos(b*x + a) + 3*(b*d*x + b*c)*sin(5*b*x + 5*a) + 2*(b*d*x + b*c)*s
in(3*b*x + 3*a) + 3*(b*d*x + b*c)*sin(b*x + a))*cos(6*b*x + 6*a) + (2*d*co
s(4*b*x + 4*a) - 2*d*cos(2*b*x + 2*a) - 3*(b*d*x + b*c)*sin(4*b*x + 4*a) +
 3*(b*d*x + b*c)*sin(2*b*x + 2*a) - 2*d)*cos(5*b*x + 5*a) - (2*d*cos(b*x +
 a) - 2*(b*d*x + b*c)*sin(3*b*x + 3*a) - 3*(b*d*x + b*c)*sin(b*x + a))*cos
(4*b*x + 4*a) + (2*d*cos(b*x + a) - 3*(b*d*x + b*c)*sin(b*x + a))*cos(2*b*
x + 2*a) + 2*d*cos(b*x + a) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*c
os(6*b*x + 6*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c
^3)*cos(4*b*x + 4*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3)*sin(6*b*x + 6*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*
c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 +
 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + (b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 - 2*(b^2*
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - (b^2*d^3*x^3 + 3*b^2
*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a) + (b^2*d^3*x^3 + 3*
b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(6*b*x + 6*a
) - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d...
```

### 3.321.8 Giac [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

### 3.321.9 Mupad [N/A]

Not integrable

Time = 25.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{1}{\cos(a+b\,x)^3\,\sin(a+b\,x)^2\,(c+d\,x)^2}\,dx$$

input
```
int(1/(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^2),x)
```

output
```
int(1/(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^2), x)
```

# 3.322 $\int (c+dx)^m \csc^3(a+bx) \sec^3(a+bx) \, dx$

3.322.1 Optimal result . . . 2414
3.322.2 Mathematica [N/A] . . . 2414
3.322.3 Rubi [N/A] . . . 2415
3.322.4 Maple [N/A] (verified) . . . 2415
3.322.5 Fricas [N/A] . . . 2416
3.322.6 Sympy [F(-1)] . . . 2416
3.322.7 Maxima [N/A] . . . 2416
3.322.8 Giac [N/A] . . . 2417
3.322.9 Mupad [N/A] . . . 2417

## 3.322.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int (c+dx)^m \csc^3(a+bx) \sec^3(a+bx) \, dx = \text{Int}\big((c+dx)^m \csc^3(a+bx) \sec^3(a+bx), x\big)$$

output
```
CannotIntegrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

## 3.322.2 Mathematica [N/A]

Not integrable

Time = 28.98 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^3(a+bx) \sec^3(a+bx) \, dx = \int (c+dx)^m \csc^3(a+bx) \sec^3(a+bx) \, dx$$

input
```
Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

output
```
Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3, x]
```

## 3.322.3 Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {7299}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \csc^3(a+bx)\sec^3(a+bx)(c+dx)^m\,dx$$
$$\Big\downarrow \; 7299$$
$$\int \csc^3(a+bx)\sec^3(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

output
```
$Aborted
```

### 3.322.3.1 Defintions of rubi rules used

rule 7299
```
Int[u_, x_] :> CannotIntegrate[u, x]
```

## 3.322.4 Maple [N/A] (verified)

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \csc(xb+a)^3 \sec(xb+a)^3\,dx$$

input
```
int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

output
```
int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

### 3.322.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^3(a+bx) \sec^3(a+bx) \, dx = \int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)^3 \, dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

### 3.322.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m \csc^3(a+bx) \sec^3(a+bx) \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*csc(b*x+a)**3*sec(b*x+a)**3,x)
```

output
```
Timed out
```

### 3.322.7 Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^3(a+bx) \sec^3(a+bx) \, dx = \int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)^3 \, dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

### 3.322.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^3(a+bx) \sec^3(a+bx)\, dx = \int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)^3\, dx$$

input
```
integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

### 3.322.9 Mupad [N/A]

Not integrable

Time = 25.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (c+dx)^m \csc^3(a+bx) \sec^3(a+bx)\, dx = \int \frac{(c+d\,x)^m}{\cos(a+b\,x)^3 \sin(a+b\,x)^3}\, dx$$

input
```
int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)^3),x)
```

output
```
int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)^3), x)
```

# 3.323 $\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx) \, dx$

3.323.1 Optimal result . . . 2418
3.323.2 Mathematica [A] (verified) . . . 2419
3.323.3 Rubi [A] (verified) . . . 2419
3.323.4 Maple [B] (verified) . . . 2423
3.323.5 Fricas [B] (verification not implemented) . . . 2424
3.323.6 Sympy [F(-1)] . . . 2425
3.323.7 Maxima [B] (verification not implemented) . . . 2426
3.323.8 Giac [F] . . . 2426
3.323.9 Mupad [F(-1)] . . . 2427

## 3.323.1 Optimal result

Integrand size = 24, antiderivative size = 318

$$\begin{aligned}
&\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx) \, dx \\
&= -\frac{6d^2(c+dx)\text{arctanh}\left(e^{2i(a+bx)}\right)}{b^3} - \frac{4(c+dx)^3\text{arctanh}\left(e^{2i(a+bx)}\right)}{b} \\
&\quad - \frac{3d(c+dx)^2 \csc(2a+2bx)}{b^2} - \frac{2(c+dx)^3 \cot(2a+2bx) \csc(2a+2bx)}{b} \\
&\quad + \frac{3id^3 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^4} + \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} \\
&\quad - \frac{3id^3 \operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^4} - \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2} \\
&\quad - \frac{3d^2(c+dx) \operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3} + \frac{3d^2(c+dx) \operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3} \\
&\quad - \frac{3id^3 \operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{2b^4} + \frac{3id^3 \operatorname{PolyLog}\left(4,e^{2i(a+bx)}\right)}{2b^4}
\end{aligned}$$

output
```
-6*d^2*(d*x+c)*arctanh(exp(2*I*(b*x+a)))/b^3-4*(d*x+c)^3*arctanh(exp(2*I*(
b*x+a)))/b-3*d*(d*x+c)^2*csc(2*b*x+2*a)/b^2-2*(d*x+c)^3*cot(2*b*x+2*a)*csc
(2*b*x+2*a)/b+3/2*I*d^3*polylog(2,-exp(2*I*(b*x+a)))/b^4+3*I*d*(d*x+c)^2*p
olylog(2,-exp(2*I*(b*x+a)))/b^2-3/2*I*d^3*polylog(2,exp(2*I*(b*x+a)))/b^4-
3*I*d*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2-3*d^2*(d*x+c)*polylog(3,-e
xp(2*I*(b*x+a)))/b^3+3*d^2*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/b^3-3/2*I*d
^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+3/2*I*d^3*polylog(4,exp(2*I*(b*x+a)))/
b^4
```

### 3.323.2 Mathematica [A] (verified)

Time = 6.27 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.52

$$\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx)\,dx =$$
$$-\frac{8b^3c^3\operatorname{arctanh}\left(e^{2i(a+bx)}\right)+12bcd^2\operatorname{arctanh}\left(e^{2i(a+bx)}\right)+2b^2(c+dx)^2(3d+2b(c+dx)\cot(2(a+bx)))\csc($$

input
```
Integrate[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

output
```
-1/2*(8*b^3*c^3*ArcTanh[E^((2*I)*(a + b*x))] + 12*b*c*d^2*ArcTanh[E^((2*I)
*(a + b*x))] + 2*b^2*(c + d*x)^2*(3*d + 2*b*(c + d*x)*Cot[2*(a + b*x)])*Cs
c[2*(a + b*x)] - 12*b^3*c^2*d*x*Log[1 - E^((2*I)*(a + b*x))] - 6*b*d^3*x*L
og[1 - E^((2*I)*(a + b*x))] - 12*b^3*c*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))
] - 4*b^3*d^3*x^3*Log[1 - E^((2*I)*(a + b*x))] + 12*b^3*c^2*d*x*Log[1 + E^
((2*I)*(a + b*x))] + 6*b*d^3*x*Log[1 + E^((2*I)*(a + b*x))] + 12*b^3*c*d^2
*x^2*Log[1 + E^((2*I)*(a + b*x))] + 4*b^3*d^3*x^3*Log[1 + E^((2*I)*(a + b*
x))] - (3*I)*d*(d^2 + 2*b^2*(c + d*x)^2)*PolyLog[2, -E^((2*I)*(a + b*x))]
+ (3*I)*d*(d^2 + 2*b^2*(c + d*x)^2)*PolyLog[2, E^((2*I)*(a + b*x))] + 6*b*
c*d^2*PolyLog[3, -E^((2*I)*(a + b*x))] + 6*b*d^3*x*PolyLog[3, -E^((2*I)*(a
 + b*x))] - 6*b*c*d^2*PolyLog[3, E^((2*I)*(a + b*x))] - 6*b*d^3*x*PolyLog[
3, E^((2*I)*(a + b*x))] + (3*I)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))] - (3*
I)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4
```

### 3.323.3 Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {4919, 3042, 4674, 3042, 4671, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx)\,dx$$
$$\Big\downarrow \text{ 4919}$$
$$8\int (c+dx)^3 \csc^3(2a+2bx)dx$$
$$\Big\downarrow \text{ 3042}$$

$$8\int (c+dx)^3\csc(2a+2bx)^3dx$$

$\Big\downarrow$ 4674

$$8\left(\frac{3d^2\int (c+dx)\csc(2a+2bx)dx}{4b^2}+\frac{1}{2}\int (c+dx)^3\csc(2a+2bx)dx-\frac{3d(c+dx)^2\csc(2a+2bx)}{8b^2}-\frac{(c+dx)^3\cot(2\right.$$

$\Big\downarrow$ 3042

$$8\left(\frac{3d^2\int (c+dx)\csc(2a+2bx)dx}{4b^2}+\frac{1}{2}\int (c+dx)^3\csc(2a+2bx)dx-\frac{3d(c+dx)^2\csc(2a+2bx)}{8b^2}-\frac{(c+dx)^3\cot(2\right.$$

$\Big\downarrow$ 4671

$$8\left(\frac{3d^2\left(-\frac{d\int \log\left(1-e^{2i(a+bx)}\right)dx}{2b}+\frac{d\int \log\left(1+e^{2i(a+bx)}\right)dx}{2b}-\frac{(c+dx)\text{arctanh}\left(e^{2i(a+bx)}\right)}{b}\right)}{4b^2}+\frac{1}{2}\left(-\frac{3d\int (c+dx)^2\log\left(1-e^{2i(\right.}{2b}\right.\right.$$

$\Big\downarrow$ 2715

$$8\left(\frac{3d^2\left(\frac{id\int e^{-2i(a+bx)}\log\left(1-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{id\int e^{-2i(a+bx)}\log\left(1+e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{(c+dx)\text{arctanh}\left(e^{2i(a+bx)}\right)}{b}\right)}{4b^2}+\frac{1}{2}\left(\right.\right.$$

$\Big\downarrow$ 2838

$$8\left(\frac{1}{2}\left(-\frac{3d\int (c+dx)^2\log\left(1-e^{2i(a+bx)}\right)dx}{2b}+\frac{3d\int (c+dx)^2\log\left(1+e^{2i(a+bx)}\right)dx}{2b}-\frac{(c+dx)^3\text{arctanh}\left(e^{2i(a+bx)}\right)}{b}\right.\right.$$

$\Big\downarrow$ 3011

$$8\left(\frac{1}{2}\left(\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\int (c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)dx}{b}\right)}{2b}-\frac{3d\left(\frac{i(c+dx)^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b}-\frac{id\int (c+d\right.}{2b}\right.\right.$$

$\Big\downarrow$ 7163

3.323. $\int (c+dx)^3\csc^3(a+bx)\sec^3(a+bx)\,dx$

$$8\left(\frac{1}{2}\left(\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\left(\frac{id\int\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)dx}{2b}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\frac{3d\left(\frac{i(c+dx)^2\operatorname{Poly}\cdots}{}\right.}{}\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$8\left(\frac{1}{2}\left(\frac{3d\left(\frac{i(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{id\left(\frac{d\int e^{-2i(a+bx)}\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{i(c+dx)\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b}\right)}{b}\right)}{2b}-\cdots\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$8\left(\frac{3d^2\left(-\frac{(c+dx)\operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b}+\frac{id\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{id\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{4b^2}\right)}{4b^2}+\frac{1}{2}\left(-\frac{(c+dx)^3\operatorname{arctanh}\left(e^{2i(a+\cdots}\right.}{b}\right.\right.$$

input
```
Int[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

output
```
8*((-3*d*(c + d*x)^2*Csc[2*a + 2*b*x])/(8*b^2) - ((c + d*x)^3*Cot[2*a + 2*
b*x]*Csc[2*a + 2*b*x])/(4*b) + (3*d^2*(-(((c + d*x)*ArcTanh[E^((2*I)*(a +
b*x))])/b) + ((I/4)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - ((I/4)*d*Pol
yLog[2, E^((2*I)*(a + b*x))])/b^2))/(4*b^2) + (-(((c + d*x)^3*ArcTanh[E^((
2*I)*(a + b*x))])/b) + (3*d*(((I/2)*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a +
b*x))])/b - (I*d*(((-1/2*I)*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/b
+ (d*PolyLog[4, -E^((2*I)*(a + b*x))])/(4*b^2)))/b))/(2*b) - (3*d*(((I/2)*
(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b - (I*d*(((-1/2*I)*(c + d*x)
*PolyLog[3, E^((2*I)*(a + b*x))])/b + (d*PolyLog[4, E^((2*I)*(a + b*x))])/
(4*b^2)))/b))/(2*b))/2)
```

## 3.323. $\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx)\,dx$

### 3.323.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

## 3.323. $\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx)\,dx$

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

### 3.323.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1328 vs. $2(284) = 568$.

Time = 1.22 (sec) , antiderivative size = 1329, normalized size of antiderivative = 4.18

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1329 |

input
```
int((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/2*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4-3*I*d^3*polylog(2,exp(I*(b*x+a
)))/b^4-3/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x-3/b^3*c*d^2*polylog(3,-ex
p(2*I*(b*x+a)))-2/b*d^3*ln(exp(2*I*(b*x+a))+1)*x^3-3/b^3*d^2*c*ln(exp(2*I*
(b*x+a))+1)-3/b^3*d^3*ln(exp(2*I*(b*x+a))+1)*x+2/b^2/(exp(2*I*(b*x+a))+1)^
2/(exp(2*I*(b*x+a))-1)^2*(2*d^3*x^3*b*exp(6*I*(b*x+a))+6*c*d^2*x^2*b*exp(6
*I*(b*x+a))+6*c^2*d*x*b*exp(6*I*(b*x+a))+2*c^3*b*exp(6*I*(b*x+a))-3*I*d^3*
x^2*exp(6*I*(b*x+a))+2*b*d^3*x^3*exp(2*I*(b*x+a))-6*I*c*d^2*x*exp(6*I*(b*x
+a))+6*b*c*d^2*x^2*exp(2*I*(b*x+a))-3*I*c^2*d*exp(6*I*(b*x+a))+6*b*c^2*d*x
*exp(2*I*(b*x+a))+2*b*c^3*exp(2*I*(b*x+a))+3*I*d^3*x^2*exp(2*I*(b*x+a))+6*
I*c*d^2*x*exp(2*I*(b*x+a))+3*I*c^2*d*exp(2*I*(b*x+a)))+3/2*I*d^3*polylog(2
,-exp(2*I*(b*x+a)))/b^4-2/b*c^3*ln(exp(2*I*(b*x+a))+1)+6/b^3*c*d^2*a^2*ln(
exp(I*(b*x+a))-1)-6/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)+6/b^2*d*c^2*ln(1-exp(
I*(b*x+a)))*a-6/b^3*c*d^2*ln(1-exp(I*(b*x+a)))*a^2-12*I/b^2*c*d^2*polylog(
2,-exp(I*(b*x+a)))*x+6*I/b^2*c*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-12*I/b^2
*c*d^2*polylog(2,exp(I*(b*x+a)))*x+2/b*c^3*ln(exp(I*(b*x+a))+1)+2/b*c^3*ln
(exp(I*(b*x+a))-1)-6/b*c*d^2*ln(exp(2*I*(b*x+a))+1)*x^2-6/b*c^2*d*ln(exp(2
*I*(b*x+a))+1)*x+3/b^3*c*d^2*ln(exp(I*(b*x+a))+1)+3/b^3*c*d^2*ln(exp(I*(b*
x+a))-1)+3/b^3*d^3*ln(exp(I*(b*x+a))+1)*x+3/b^3*d^3*ln(1-exp(I*(b*x+a)))*x
+12*I/b^4*d^3*polylog(4,-exp(I*(b*x+a)))+12*I/b^4*d^3*polylog(4,exp(I*(b*x
+a)))-3*I/b^4*d^3*polylog(2,-exp(I*(b*x+a)))+6/b*d*c^2*ln(1-exp(I*(b*x+...
```

### 3.323.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4193 vs. 2(274) = 548.

Time = 0.47 (sec) , antiderivative size = 4193, normalized size of antiderivative = 13.19

$$\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 2*(b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a)^2 - 3*(b^2*
d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x + a) + 3*((2*I*b
^2*d^3*x^2 + 4*I*b^2*c*d^2*x + 2*I*b^2*c^2*d + I*d^3)*cos(b*x + a)^4 + (-2
*I*b^2*d^3*x^2 - 4*I*b^2*c*d^2*x - 2*I*b^2*c^2*d - I*d^3)*cos(b*x + a)^2)*
dilog(cos(b*x + a) + I*sin(b*x + a)) + 3*((-2*I*b^2*d^3*x^2 - 4*I*b^2*c*d^
2*x - 2*I*b^2*c^2*d - I*d^3)*cos(b*x + a)^4 + (2*I*b^2*d^3*x^2 + 4*I*b^2*c
*d^2*x + 2*I*b^2*c^2*d + I*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) - I*sin
(b*x + a)) + 3*((2*I*b^2*d^3*x^2 + 4*I*b^2*c*d^2*x + 2*I*b^2*c^2*d + I*d^3
)*cos(b*x + a)^4 + (-2*I*b^2*d^3*x^2 - 4*I*b^2*c*d^2*x - 2*I*b^2*c^2*d - I
*d^3)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) + sin(b*x + a)) + 3*((-2*I*b^2*
d^3*x^2 - 4*I*b^2*c*d^2*x - 2*I*b^2*c^2*d - I*d^3)*cos(b*x + a)^4 + (2*I*b
^2*d^3*x^2 + 4*I*b^2*c*d^2*x + 2*I*b^2*c^2*d + I*d^3)*cos(b*x + a)^2)*dilo
g(I*cos(b*x + a) - sin(b*x + a)) + 3*((-2*I*b^2*d^3*x^2 - 4*I*b^2*c*d^2*x
- 2*I*b^2*c^2*d - I*d^3)*cos(b*x + a)^4 + (2*I*b^2*d^3*x^2 + 4*I*b^2*c*d^2
*x + 2*I*b^2*c^2*d + I*d^3)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) + sin(b*
x + a)) + 3*((2*I*b^2*d^3*x^2 + 4*I*b^2*c*d^2*x + 2*I*b^2*c^2*d + I*d^3)*c
os(b*x + a)^4 + (-2*I*b^2*d^3*x^2 - 4*I*b^2*c*d^2*x - 2*I*b^2*c^2*d - I*d^
3)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 3*((-2*I*b^2*d^
3*x^2 - 4*I*b^2*c*d^2*x - 2*I*b^2*c^2*d - I*d^3)*cos(b*x + a)^4 + (2*I*...
```

### 3.323.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx)\, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**3*csc(b*x+a)**3*sec(b*x+a)**3,x)
```

output
```
Timed out
```

### 3.323.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 5646 vs. 2(274) = 548.

Time = 2.42 (sec) , antiderivative size = 5646, normalized size of antiderivative = 17.75

$$\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(c^3*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*lo
g(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2)) - 3*a*c^2*d*((2*sin(b*x + a
)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2
*log(sin(b*x + a)^2))/b + 3*a^2*c*d^2*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a
)^4 - sin(b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))
/b^2 - a^3*d^3*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) +
 2*log(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b^3 + 2*(2*(8*(b*x + a
)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(2*b^
2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a) + (8*(b*x + a)^3*d^3 +
9*b*c*d^2 - 9*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(2*b^2*c^2*d -
4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*cos(8*b*x + 8*a) - 2*(8*(b*x + a
)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(2*b^
2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - (-8
*I*(b*x + a)^3*d^3 - 9*I*b*c*d^2 + 9*I*a*d^3 + 18*(-I*b*c*d^2 + I*a*d^3)*(
b*x + a)^2 + 9*(-2*I*b^2*c^2*d + 4*I*a*b*c*d^2 + (-2*I*a^2 - I)*d^3)*(b*x
+ a))*sin(8*b*x + 8*a) - 2*(8*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3
+ 18*(I*b*c*d^2 - I*a*d^3)*(b*x + a)^2 + 9*(2*I*b^2*c^2*d - 4*I*a*b*c*d^2
+ (2*I*a^2 + I)*d^3)*(b*x + a))*sin(4*b*x + 4*a))*arctan2(sin(2*b*x + 2*a)
, cos(2*b*x + 2*a) + 1) - 6*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*(
b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + ...
```

### 3.323.8 Giac [F]

$$\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx)\, dx = \int (dx+c)^3 \csc(bx+a)^3 \sec(bx+a)^3\, dx$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

### 3.323.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx) \, dx = \text{Hanged}$$

input
```
int((c + d*x)^3/(cos(a + b*x)^3*sin(a + b*x)^3),x)
```

output
```
\text{Hanged}
```

# 3.324 $\int (c+dx)^2 \csc^3(a+bx) \sec^3(a+bx) \, dx$

3.324.1 Optimal result . . . 2428
3.324.2 Mathematica [B] (verified) . . . 2429
3.324.3 Rubi [A] (verified) . . . 2429
3.324.4 Maple [B] (verified) . . . 2432
3.324.5 Fricas [B] (verification not implemented) . . . 2433
3.324.6 Sympy [F(-1)] . . . 2434
3.324.7 Maxima [B] (verification not implemented) . . . 2435
3.324.8 Giac [F] . . . 2435
3.324.9 Mupad [F(-1)] . . . 2436

## 3.324.1 Optimal result

Integrand size = 24, antiderivative size = 190

$$\begin{aligned}
&\int (c+dx)^2 \csc^3(a+bx) \sec^3(a+bx) \, dx \\
&= -\frac{4(c+dx)^2 \operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2 \operatorname{arctanh}(\cos(2a+2bx))}{b^3} \\
&\quad - \frac{2d(c+dx)\csc(2a+2bx)}{b^2} - \frac{2(c+dx)^2 \cot(2a+2bx)\csc(2a+2bx)}{b} \\
&\quad + \frac{2id(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} - \frac{2id(c+dx)\operatorname{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2} \\
&\quad - \frac{d^2 \operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3} + \frac{d^2 \operatorname{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3}
\end{aligned}$$

output
```
-4*(d*x+c)^2*arctanh(exp(2*I*(b*x+a)))/b-d^2*arctanh(cos(2*b*x+2*a))/b^3-2
*d*(d*x+c)*csc(2*b*x+2*a)/b^2-2*(d*x+c)^2*cot(2*b*x+2*a)*csc(2*b*x+2*a)/b+
2*I*d*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2-2*I*d*(d*x+c)*polylog(2,exp
(2*I*(b*x+a)))/b^2-d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+d^2*polylog(3,exp(
2*I*(b*x+a)))/b^3
```

### 3.324.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 381 vs. 2(190) = 380.

Time = 7.57 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.01

$$\int (c+dx)^2 \csc^3(a+bx) \sec^3(a+bx)\, dx$$

$$= 8\left(-\frac{d(c+dx)\csc(2a)}{4b^2} + \frac{(-c^2-2cdx-d^2x^2)\csc^2(a+bx)}{16b} - \frac{4b^2c^2\text{arctanh}\left(e^{2i(a+bx)}\right) + 2d^2\text{arctanh}\left(e^{2i(a+bx)}\right) - 4b^2cdx\log\left(1-e^{2i(a+bx)}\right) - 2b^2d^2x^2\log\left(1-e^{2i(a+bx)}\right.}{} + \frac{(c^2+2cdx+d^2x^2)\sec^2(a+bx)}{16b} + \frac{\sec(a)\sec(a+bx)(-cd\sin(bx)-d^2x\sin(bx))}{8b^2} + \frac{\csc(a)\csc(a+bx)(cd\sin(bx)+d^2x\sin(bx))}{8b^2}\right)$$

input
```
Integrate[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

output
```
8*(-1/4*(d*(c + d*x)*Csc[2*a])/b^2 + ((-c^2 - 2*c*d*x - d^2*x^2)*Csc[a + b
*x]^2)/(16*b) - (4*b^2*c^2*ArcTanh[E^((2*I)*(a + b*x))] + 2*d^2*ArcTanh[E^
((2*I)*(a + b*x))] - 4*b^2*c*d*x*Log[1 - E^((2*I)*(a + b*x))] - 2*b^2*d^2*
x^2*Log[1 - E^((2*I)*(a + b*x))] + 4*b^2*c*d*x*Log[1 + E^((2*I)*(a + b*x))
] + 2*b^2*d^2*x^2*Log[1 + E^((2*I)*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyL
og[2, -E^((2*I)*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, E^((2*I)*(a +
 b*x))] + d^2*PolyLog[3, -E^((2*I)*(a + b*x))] - d^2*PolyLog[3, E^((2*I)*(
a + b*x))])/(8*b^3) + ((c^2 + 2*c*d*x + d^2*x^2)*Sec[a + b*x]^2)/(16*b) +
(Sec[a]*Sec[a + b*x]*(-(c*d*Sin[b*x]) - d^2*x*Sin[b*x]))/(8*b^2) + (Csc[a]
*Csc[a + b*x]*(c*d*Sin[b*x] + d^2*x*Sin[b*x]))/(8*b^2))
```

### 3.324.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4919, 3042, 4674, 3042, 4257, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \csc^3(a+bx) \sec^3(a+bx)\,dx$$

$$\Big\downarrow \; 4919$$

$$8\int (c+dx)^2 \csc^3(2a+2bx)dx$$

$$\Big\downarrow \; 3042$$

$$8\int (c+dx)^2 \csc(2a+2bx)^3dx$$

$$\Big\downarrow \; 4674$$

$$8\left(\frac{d^2\int \csc(2a+2bx)dx}{4b^2}+\frac{1}{2}\int (c+dx)^2\csc(2a+2bx)dx-\frac{d(c+dx)\csc(2a+2bx)}{4b^2}-\frac{(c+dx)^2\cot(2a+2bx)\csc}{4b}\right.$$

$$\Big\downarrow \; 3042$$

$$8\left(\frac{d^2\int \csc(2a+2bx)dx}{4b^2}+\frac{1}{2}\int (c+dx)^2\csc(2a+2bx)dx-\frac{d(c+dx)\csc(2a+2bx)}{4b^2}-\frac{(c+dx)^2\cot(2a+2bx)\csc}{4b}\right.$$

$$\Big\downarrow \; 4257$$

$$8\left(\frac{1}{2}\int (c+dx)^2\csc(2a+2bx)dx-\frac{d^2\text{arctanh}(\cos(2a+2bx))}{8b^3}-\frac{d(c+dx)\csc(2a+2bx)}{4b^2}-\frac{(c+dx)^2\cot(2a+2b}{4b}\right.$$

$$\Big\downarrow \; 4671$$

$$8\left(\frac{1}{2}\left(-\frac{d\int (c+dx)\log\left(1-e^{2i(a+bx)}\right)dx}{b}+\frac{d\int (c+dx)\log\left(1+e^{2i(a+bx)}\right)dx}{b}-\frac{(c+dx)^2\text{arctanh}\left(e^{2i(a+bx)}\right)}{b}\right)-\right.$$

$$\Big\downarrow \; 3011$$

$$8\left(\frac{1}{2}\left(\frac{d\left(\frac{i(c+dx)\text{PolyLog}(2,-e^{2i(a+bx)})}{2b}-\frac{id\int \text{PolyLog}(2,-e^{2i(a+bx)})dx}{2b}\right)}{b}-\frac{d\left(\frac{i(c+dx)\text{PolyLog}(2,e^{2i(a+bx)})}{2b}-\frac{id\int \text{PolyLog}(2,e^{2i(a+}}{2b}\right.}{b}\right.\right.$$

$$\Big\downarrow \; 2720$$

$$8\left(\frac{1}{2}\left(\frac{d\left(\frac{i(c+dx)\text{PolyLog}(2,-e^{2i(a+bx)})}{2b}-\frac{d\int e^{-2i(a+bx)}\text{PolyLog}(2,-e^{2i(a+bx)})de^{2i(a+bx)}}{4b^2}\right)}{b}-\frac{d\left(\frac{i(c+dx)\text{PolyLog}(2,e^{2i(a+bx)})}{2b}-\frac{d\int}{}\right.}{}\right.\right.$$

$$\Big\downarrow \; 7143$$

3.324. $\int (c+dx)^2 \csc^3(a+bx) \sec^3(a+bx)\,dx$

$$8\left(-\frac{d^2\text{arctanh}(\cos(2a+2bx))}{8b^3}+\frac{1}{2}\left(-\frac{(c+dx)^2\text{arctanh}\left(e^{2i(a+bx)}\right)}{b}+\frac{d\left(\frac{i(c+dx)\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b}-\frac{d\,\text{PolyLog}(3,-}{4b^2}\right.}{b}\right.\right.$$

input
```
Int[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

output
```
8*(-1/8*(d^2*ArcTanh[Cos[2*a + 2*b*x]])/b^3 - (d*(c + d*x)*Csc[2*a + 2*b*x
])/(4*b^2) - ((c + d*x)^2*Cot[2*a + 2*b*x]*Csc[2*a + 2*b*x])/(4*b) + (-(((
c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/b) + (d*(((I/2)*(c + d*x)*PolyLog
[2, -E^((2*I)*(a + b*x))])/b - (d*PolyLog[3, -E^((2*I)*(a + b*x))])/(4*b^2
)))/b - (d*(((I/2)*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b - (d*PolyL
og[3, E^((2*I)*(a + b*x))])/(4*b^2)))/b)/2)
```

### 3.324.3.1 Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.324.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 715 vs. $2(178) = 356$.

Time = 0.96 (sec) , antiderivative size = 716, normalized size of antiderivative = 3.77

| method | result |
|---|---|
| risch | $-\frac{4id^2 \operatorname{polylog}\left(2,\mathrm{e}^{i(xb+a)}\right)x}{b^2}-\frac{4idc \operatorname{polylog}\left(2,\mathrm{e}^{i(xb+a)}\right)}{b^2}+\frac{4dc\ln\left(1-\mathrm{e}^{i(xb+a)}\right)x}{b}+\frac{4dc\ln\left(1-\mathrm{e}^{i(xb+a)}\right)a}{b^2}+\frac{4d^2 \operatorname{polylog}\left(3,\mathrm{e}^{i(xb}\right.}{b^3}$ |

input
```
int((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/b*d^2*ln(exp(2*I*(b*x+a))+1)*x^2+2/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)-1/b
^3*d^2*ln(exp(2*I*(b*x+a))+1)+2/b*c^2*ln(exp(I*(b*x+a))+1)+2/b*c^2*ln(exp(
I*(b*x+a))-1)-2/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+2/b*d^2*ln(1-exp(I*(b*x+a
)))*x^2+2/b*d^2*ln(exp(I*(b*x+a))+1)*x^2-2/b*c^2*ln(exp(2*I*(b*x+a))+1)+4/
b*d*c*ln(exp(I*(b*x+a))+1)*x+4/b^2*d*c*ln(1-exp(I*(b*x+a)))*a-4/b^2*c*d*a*
ln(exp(I*(b*x+a))-1)+4/b*d*c*ln(1-exp(I*(b*x+a)))*x-d^2*polylog(3,-exp(2*I
*(b*x+a)))/b^3-4/b*c*d*ln(exp(2*I*(b*x+a))+1)*x+4*d^2*polylog(3,-exp(I*(b*
x+a)))/b^3+4*d^2*polylog(3,exp(I*(b*x+a)))/b^3+1/b^3*d^2*ln(exp(I*(b*x+a))
+1)+1/b^3*d^2*ln(exp(I*(b*x+a))-1)+4/b^2/(exp(2*I*(b*x+a))+1)^2/(exp(2*I*(
b*x+a))-1)^2*(x^2*d^2*b*exp(6*I*(b*x+a))+2*c*d*x*b*exp(6*I*(b*x+a))+c^2*b*
exp(6*I*(b*x+a))-I*d^2*x*exp(6*I*(b*x+a))+b*d^2*x^2*exp(2*I*(b*x+a))-I*c*d
*exp(6*I*(b*x+a))+2*b*c*d*x*exp(2*I*(b*x+a))+b*c^2*exp(2*I*(b*x+a))+I*d^2*
x*exp(2*I*(b*x+a))+I*c*d*exp(2*I*(b*x+a)))+2*I/b^2*d^2*polylog(2,-exp(2*I*
(b*x+a)))*x-4*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x-4*I/b^2*c*d*polylog(2,
-exp(I*(b*x+a)))-4*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+2*I/b^2*c*d*poly
log(2,-exp(2*I*(b*x+a)))-4*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))
```

### 3.324.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2387 vs. $2(174) = 348$.

Time = 0.40 (sec) , antiderivative size = 2387, normalized size of antiderivative = 12.56

$$\int (c+dx)^2 \csc^3(a+bx) \sec^3(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*cos(b*x + a)^2 - 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) +
 4*((I*b*d^2*x + I*b*c*d)*cos(b*x + a)^4 + (-I*b*d^2*x - I*b*c*d)*cos(b*x
+ a)^2)*dilog(cos(b*x + a) + I*sin(b*x + a)) + 4*((-I*b*d^2*x - I*b*c*d)*c
os(b*x + a)^4 + (I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2)*dilog(cos(b*x + a) -
 I*sin(b*x + a)) + 4*((I*b*d^2*x + I*b*c*d)*cos(b*x + a)^4 + (-I*b*d^2*x -
 I*b*c*d)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) + sin(b*x + a)) + 4*((-I*b*
d^2*x - I*b*c*d)*cos(b*x + a)^4 + (I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2)*di
log(I*cos(b*x + a) - sin(b*x + a)) + 4*((-I*b*d^2*x - I*b*c*d)*cos(b*x + a
)^4 + (I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) + sin(b*
x + a)) + 4*((I*b*d^2*x + I*b*c*d)*cos(b*x + a)^4 + (-I*b*d^2*x - I*b*c*d)
*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 4*((-I*b*d^2*x -
I*b*c*d)*cos(b*x + a)^4 + (I*b*d^2*x + I*b*c*d)*cos(b*x + a)^2)*dilog(-cos
(b*x + a) + I*sin(b*x + a)) + 4*((I*b*d^2*x + I*b*c*d)*cos(b*x + a)^4 + (-
I*b*d^2*x - I*b*c*d)*cos(b*x + a)^2)*dilog(-cos(b*x + a) - I*sin(b*x + a))
 - ((2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*cos(b*x + a)^4 - (2*b^
2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*cos(b*x + a)^2)*log(cos(b*x + a
) + I*sin(b*x + a) + 1) + ((2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b
*x + a)^4 - (2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^2)*log(
cos(b*x + a) + I*sin(b*x + a) + I) - ((2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*...
```

### 3.324.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc^3(a+bx) \sec^3(a+bx) \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*csc(b*x+a)**3*sec(b*x+a)**3,x)
```

output
```
Timed out
```

### 3.324.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2728 vs. 2(174) = 348.

Time = 0.76 (sec) , antiderivative size = 2728, normalized size of antiderivative = 14.36

$$\int (c+dx)^2 \csc^3(a+bx)\sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/2*(c^2*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*lo
g(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2)) - 2*a*c*d*((2*sin(b*x + a)^
2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2*l
og(sin(b*x + a)^2))/b + a^2*d^2*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 -
sin(b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b^2 +
 2*(2*(2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2 + (2*(b*x + a
)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*cos(8*b*x + 8*a) - 2*(2*(b*x
+ a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*cos(4*b*x + 4*a) - (-2*I*(
b*x + a)^2*d^2 + 4*(-I*b*c*d + I*a*d^2)*(b*x + a) - I*d^2)*sin(8*b*x + 8*a
) - 2*(2*I*(b*x + a)^2*d^2 + 4*(I*b*c*d - I*a*d^2)*(b*x + a) + I*d^2)*sin(
4*b*x + 4*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 2*(2*(b*x
+ a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2 + (2*(b*x + a)^2*d^2 + 4*(b
*c*d - a*d^2)*(b*x + a) + d^2)*cos(8*b*x + 8*a) - 2*(2*(b*x + a)^2*d^2 + 4
*(b*c*d - a*d^2)*(b*x + a) + d^2)*cos(4*b*x + 4*a) + (2*I*(b*x + a)^2*d^2
+ 4*(I*b*c*d - I*a*d^2)*(b*x + a) + I*d^2)*sin(8*b*x + 8*a) + 2*(-2*I*(b*x
 + a)^2*d^2 + 4*(-I*b*c*d + I*a*d^2)*(b*x + a) - I*d^2)*sin(4*b*x + 4*a))*
arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*(d^2*cos(8*b*x + 8*a) - 2*d^2*
cos(4*b*x + 4*a) + I*d^2*sin(8*b*x + 8*a) - 2*I*d^2*sin(4*b*x + 4*a) + d^2
)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 4*((b*x + a)^2*d^2 + 2*(b*c*d
- a*d^2)*(b*x + a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*co...
```

### 3.324.8 Giac [F]

$$\int (c+dx)^2 \csc^3(a+bx)\sec^3(a+bx)\,dx = \int (dx+c)^2 \csc(bx+a)^3 \sec(bx+a)^3\, dx$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

### 3.324.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc^3(a+bx) \sec^3(a+bx) \, dx = \text{Hanged}$$

input
```
int((c + d*x)^2/(cos(a + b*x)^3*sin(a + b*x)^3),x)
```

output
```
\text{Hanged}
```

# 3.325 $\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx$


3.325.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2437
3.325.2 Mathematica [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 2437
3.325.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2438
3.325.4 Maple [B] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2440
3.325.5 Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . 2441
3.325.6 Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2442
3.325.7 Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . . 2442
3.325.8 Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2443
3.325.9 Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2443


## 3.325.1 Optimal result

Integrand size = 22, antiderivative size = 110

$$\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx = -\frac{4(c+dx)\operatorname{arctanh}\left(e^{2i(a+bx)}\right)}{b} - \frac{d\csc(2a+2bx)}{b^2} - \frac{2(c+dx)\cot(2a+2bx)\csc(2a+2bx)}{b} + \frac{id\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2} - \frac{id\,\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}$$

output

```
-4*(d*x+c)*arctanh(exp(2*I*(b*x+a)))/b-d*csc(2*b*x+2*a)/b^2-2*(d*x+c)*cot(
2*b*x+2*a)*csc(2*b*x+2*a)/b+I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2-I*d*polyl
og(2,exp(2*I*(b*x+a)))/b^2
```

## 3.325.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 236 vs. 2(110) = 220.

Time = 1.89 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.15

$$\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx$$
$$= -\frac{d\cot(a+bx)}{2b^2} - \frac{c\csc^2(a+bx)}{2b} + \frac{d(2a-2(a+bx))\csc^2(a+bx)}{4b^2} - \frac{2c\log(\cos(a+bx))}{b} + \frac{2c\log(\sin(a+bx))}{b} - \frac{2ad\log(\tan(a+bx))}{b^2} + \frac{d\left(2(a+bx)\left(\log\left(1-e^{2i(a+bx)}\right)-\log\left(1+e^{2i(a+bx)}\right)\right)+i\left(\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)-\text{PolyLog}\left(2,e^{2i(a+bx)}\right.\right.\right.}{b^2} + \frac{c\sec^2(a+bx)}{2b} + \frac{d(-2a+2(a+bx))\sec^2(a+bx)}{4b^2} - \frac{d\tan(a+bx)}{2b^2}$$

input
```
Integrate[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

output
```
-1/2*(d*Cot[a + b*x])/b^2 - (c*Csc[a + b*x]^2)/(2*b) + (d*(2*a - 2*(a + b*
x))*Csc[a + b*x]^2)/(4*b^2) - (2*c*Log[Cos[a + b*x]])/b + (2*c*Log[Sin[a +
 b*x]])/b - (2*a*d*Log[Tan[a + b*x]])/b^2 + (d*(2*(a + b*x)*(Log[1 - E^((2
*I)*(a + b*x))] - Log[1 + E^((2*I)*(a + b*x))]) + I*(PolyLog[2, -E^((2*I)*
(a + b*x))] - PolyLog[2, E^((2*I)*(a + b*x))])))/b^2 + (c*Sec[a + b*x]^2)/
(2*b) + (d*(-2*a + 2*(a + b*x))*Sec[a + b*x]^2)/(4*b^2) - (d*Tan[a + b*x])
/(2*b^2)
```

### 3.325.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {4919, 3042, 4673, 3042, 4671, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx$$
$$\Big\downarrow \; 4919$$
$$8\int (c+dx)\csc^3(2a+2bx)dx$$
$$\Big\downarrow \; 3042$$
$$8\int (c+dx)\csc(2a+2bx)^3dx$$

$$\Big\downarrow \text{4673}$$

$$8\left(\frac{1}{2}\int (c+dx)\csc(2a+2bx)dx-\frac{d\csc(2a+2bx)}{8b^2}-\frac{(c+dx)\cot(2a+2bx)\csc(2a+2bx)}{4b}\right)$$

$$\Big\downarrow \text{3042}$$

$$8\left(\frac{1}{2}\int (c+dx)\csc(2a+2bx)dx-\frac{d\csc(2a+2bx)}{8b^2}-\frac{(c+dx)\cot(2a+2bx)\csc(2a+2bx)}{4b}\right)$$

$$\Big\downarrow \text{4671}$$

$$8\left(\frac{1}{2}\left(-\frac{d\int \log\left(1-e^{2i(a+bx)}\right)dx}{2b}+\frac{d\int \log\left(1+e^{2i(a+bx)}\right)dx}{2b}-\frac{(c+dx)\text{arctanh}\left(e^{2i(a+bx)}\right)}{b}\right)-\frac{d\csc(2a+2bx)}{8b^2}-\right.$$

$$\Big\downarrow \text{2715}$$

$$8\left(\frac{1}{2}\left(\frac{id\int e^{-2i(a+bx)}\log\left(1-e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{id\int e^{-2i(a+bx)}\log\left(1+e^{2i(a+bx)}\right)de^{2i(a+bx)}}{4b^2}-\frac{(c+dx)\text{arctan}}{b}\right.\right.$$

$$\Big\downarrow \text{2838}$$

$$8\left(\frac{1}{2}\left(-\frac{(c+dx)\text{arctanh}\left(e^{2i(a+bx)}\right)}{b}+\frac{id\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{4b^2}-\frac{id\,\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{4b^2}\right)-\frac{d\csc(2a+2bx)}{8b^2}\right.$$

input
```
Int[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

output
```
8*(-1/8*(d*Csc[2*a + 2*b*x])/b^2 - ((c + d*x)*Cot[2*a + 2*b*x]*Csc[2*a + 2
*b*x])/(4*b) + (-(((c + d*x)*ArcTanh[E^((2*I)*(a + b*x))])/b) + ((I/4)*d*P
olyLog[2, -E^((2*I)*(a + b*x))])/b^2 - ((I/4)*d*PolyLog[2, E^((2*I)*(a + b
*x))])/b^2)/2)
```

### 3.325.3.1 Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.325. $\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx$

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

### 3.325.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(102) = 204$.

Time = 0.80 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.95

| method | result |
|---|---|
| risch | $\frac{4dxb\,\mathrm{e}^{6i(xb+a)}+4cb\,\mathrm{e}^{6i(xb+a)}-2id\,\mathrm{e}^{6i(xb+a)}+4bdx\,\mathrm{e}^{2i(xb+a)}+4bc\,\mathrm{e}^{2i(xb+a)}+2id\,\mathrm{e}^{2i(xb+a)}}{b^{2}\left(\mathrm{e}^{2i(xb+a)}-1\right)^{2}\left(\mathrm{e}^{2i(xb+a)}+1\right)^{2}}+\frac{2c\ln\left(\mathrm{e}^{i(xb+a)}+1\right)}{b}-\frac{2c\ln\left(\mathrm{e}^{2i(xb+a)}\right.}{b}$ |

input
```
int((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
2/b^2/(exp(2*I*(b*x+a))-1)^2/(exp(2*I*(b*x+a))+1)^2*(2*d*x*b*exp(6*I*(b*x+
a))+2*c*b*exp(6*I*(b*x+a))-I*d*exp(6*I*(b*x+a))+2*b*d*x*exp(2*I*(b*x+a))+2
*b*c*exp(2*I*(b*x+a))+I*d*exp(2*I*(b*x+a)))+2/b*c*ln(exp(I*(b*x+a))+1)-2/b
*c*ln(exp(2*I*(b*x+a))+1)+2/b*c*ln(exp(I*(b*x+a))-1)+2/b*d*ln(exp(I*(b*x+a
))+1)*x-2*I*d*polylog(2,-exp(I*(b*x+a)))/b^2-2/b*d*ln(exp(2*I*(b*x+a))+1)*
x+I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2+2/b*d*ln(1-exp(I*(b*x+a)))*x+2/b^2*
d*ln(1-exp(I*(b*x+a)))*a-2*I/b^2*d*polylog(2,exp(I*(b*x+a)))-2/b^2*d*a*ln(
exp(I*(b*x+a))-1)
```

### 3.325.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1193 vs. $2(98) = 196$.

Time = 0.33 (sec) , antiderivative size = 1193, normalized size of antiderivative = 10.85

$$\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/2*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 - d*cos(b*x + a)*sin(b*x + a)
 + b*c + 2*(I*d*cos(b*x + a)^4 - I*d*cos(b*x + a)^2)*dilog(cos(b*x + a) +
I*sin(b*x + a)) + 2*(-I*d*cos(b*x + a)^4 + I*d*cos(b*x + a)^2)*dilog(cos(b
*x + a) - I*sin(b*x + a)) + 2*(I*d*cos(b*x + a)^4 - I*d*cos(b*x + a)^2)*di
log(I*cos(b*x + a) + sin(b*x + a)) + 2*(-I*d*cos(b*x + a)^4 + I*d*cos(b*x
+ a)^2)*dilog(I*cos(b*x + a) - sin(b*x + a)) + 2*(-I*d*cos(b*x + a)^4 + I*
d*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + 2*(I*d*cos(b*x +
 a)^4 - I*d*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 2*(-I*
d*cos(b*x + a)^4 + I*d*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a
)) + 2*(I*d*cos(b*x + a)^4 - I*d*cos(b*x + a)^2)*dilog(-cos(b*x + a) - I*s
in(b*x + a)) - 2*((b*d*x + b*c)*cos(b*x + a)^4 - (b*d*x + b*c)*cos(b*x + a
)^2)*log(cos(b*x + a) + I*sin(b*x + a) + 1) + 2*((b*c - a*d)*cos(b*x + a)^
4 - (b*c - a*d)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a) + I) - 2
*((b*d*x + b*c)*cos(b*x + a)^4 - (b*d*x + b*c)*cos(b*x + a)^2)*log(cos(b*x
 + a) - I*sin(b*x + a) + 1) + 2*((b*c - a*d)*cos(b*x + a)^4 - (b*c - a*d)*
cos(b*x + a)^2)*log(cos(b*x + a) - I*sin(b*x + a) + I) + 2*((b*d*x + a*d)*
cos(b*x + a)^4 - (b*d*x + a*d)*cos(b*x + a)^2)*log(I*cos(b*x + a) + sin(b*
x + a) + 1) + 2*((b*d*x + a*d)*cos(b*x + a)^4 - (b*d*x + a*d)*cos(b*x + a)
^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + 2*((b*d*x + a*d)*cos(b*x + a)
^4 - (b*d*x + a*d)*cos(b*x + a)^2)*log(-I*cos(b*x + a) + sin(b*x + a) +...
```

### 3.325.6 Sympy [F]

$$\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx=\int (c+dx)\csc^3{(a+bx)}\sec^3{(a+bx)}\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)**3*sec(b*x+a)**3,x)
```

output
```
Integral((c + d*x)*csc(a + b*x)**3*sec(a + b*x)**3, x)
```

### 3.325.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1070 vs. $2(98) = 196$.

Time = 0.51 (sec) , antiderivative size = 1070, normalized size of antiderivative = 9.73

$$\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

output
```
-(2*(b*d*x + b*c + (b*d*x + b*c)*cos(8*b*x + 8*a) - 2*(b*d*x + b*c)*cos(4*
b*x + 4*a) - (-I*b*d*x - I*b*c)*sin(8*b*x + 8*a) - 2*(I*b*d*x + I*b*c)*sin
(4*b*x + 4*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 2*(b*d*x
+ b*c + (b*d*x + b*c)*cos(8*b*x + 8*a) - 2*(b*d*x + b*c)*cos(4*b*x + 4*a)
+ (I*b*d*x + I*b*c)*sin(8*b*x + 8*a) + 2*(-I*b*d*x - I*b*c)*sin(4*b*x + 4*
a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*(b*c*cos(8*b*x + 8*a) - 2*
b*c*cos(4*b*x + 4*a) + I*b*c*sin(8*b*x + 8*a) - 2*I*b*c*sin(4*b*x + 4*a) +
 b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + 2*(b*d*x*cos(8*b*x + 8*a)
- 2*b*d*x*cos(4*b*x + 4*a) + I*b*d*x*sin(8*b*x + 8*a) - 2*I*b*d*x*sin(4*b*
x + 4*a) + b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*(-2*I*b*d*x
 - 2*I*b*c - d)*cos(6*b*x + 6*a) - 2*(-2*I*b*d*x - 2*I*b*c + d)*cos(2*b*x
+ 2*a) - (d*cos(8*b*x + 8*a) - 2*d*cos(4*b*x + 4*a) + I*d*sin(8*b*x + 8*a)
 - 2*I*d*sin(4*b*x + 4*a) + d)*dilog(-e^(2*I*b*x + 2*I*a)) + 2*(d*cos(8*b*
x + 8*a) - 2*d*cos(4*b*x + 4*a) + I*d*sin(8*b*x + 8*a) - 2*I*d*sin(4*b*x +
 4*a) + d)*dilog(-e^(I*b*x + I*a)) + 2*(d*cos(8*b*x + 8*a) - 2*d*cos(4*b*x
 + 4*a) + I*d*sin(8*b*x + 8*a) - 2*I*d*sin(4*b*x + 4*a) + d)*dilog(e^(I*b*
x + I*a)) + (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(8*b*x + 8*a) - 2*(-
I*b*d*x - I*b*c)*cos(4*b*x + 4*a) + (b*d*x + b*c)*sin(8*b*x + 8*a) - 2*(b*
d*x + b*c)*sin(4*b*x + 4*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 +
 2*cos(2*b*x + 2*a) + 1) + (I*b*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(8*b...
```

### 3.325.8 Giac [F]

$$\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx=\int (dx+c)\csc(bx+a)^3\sec(bx+a)^3\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

### 3.325.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx=\text{Hanged}$$

input
```
int((c + d*x)/(cos(a + b*x)^3*sin(a + b*x)^3),x)
```

output
```
\text{Hanged}
```

# 3.326 $\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$

3.326.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2444
3.326.2 Mathematica [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2444
3.326.3 Rubi [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2445
3.326.4 Maple [N/A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2446
3.326.5 Fricas [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2446
3.326.6 Sympy [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2446
3.326.7 Maxima [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2447
3.326.8 Giac [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2448
3.326.9 Mupad [N/A] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2448

## 3.326.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx = 8\text{Int}\left(\frac{\csc^3(2a+2bx)}{c+dx},x\right)$$

output
```
8*Unintegrable(csc(2*b*x+2*a)^3/(d*x+c),x)
```

## 3.326.2 Mathematica [N/A]

Not integrable

Time = 21.76 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

input
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x),x]
```

output
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x), x]
```

## 3.326.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4919, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \; 4919$$

$$8\int \frac{\csc^3(2a+2bx)}{c+dx}dx$$

$$\Big\downarrow \; 3042$$

$$8\int \frac{\csc(2a+2bx)^3}{c+dx}dx$$

$$\Big\downarrow \; 4680$$

$$8\int \frac{\csc^3(2a+2bx)}{c+dx}dx$$

input
```
Int[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x),x]
```

output
```
$Aborted
```

### 3.326.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

### 3.326.4 Maple [N/A] (verified)

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc (xb+a)^3 \sec (xb+a)^3}{dx+c} dx$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x)
```

output
```
int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x)
```

### 3.326.5 Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx) \sec^3(a+bx)}{c+dx} dx = \int \frac{\csc (bx+a)^3 \sec (bx+a)^3}{dx+c} dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^3*sec(b*x + a)^3/(d*x + c), x)
```

### 3.326.6 Sympy [N/A]

Not integrable

Time = 20.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\csc^3(a+bx) \sec^3(a+bx)}{c+dx} dx = \int \frac{\csc^3 (a+bx) \sec^3 (a+bx)}{c+dx} dx$$

input
```
integrate(csc(b*x+a)**3*sec(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(csc(a + b*x)**3*sec(a + b*x)**3/(c + d*x), x)
```

### 3.326.7 Maxima [N/A]

Not integrable

Time = 3.93 (sec) , antiderivative size = 2410, normalized size of antiderivative = 100.42

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)^3}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output
```
(2*(2*(b*d*x + b*c)*cos(6*b*x + 6*a) + 2*(b*d*x + b*c)*cos(2*b*x + 2*a) -
d*sin(6*b*x + 6*a) + d*sin(2*b*x + 2*a))*cos(8*b*x + 8*a) + 4*(b*d*x + b*c
 - 2*(b*d*x + b*c)*cos(4*b*x + 4*a) - d*sin(4*b*x + 4*a))*cos(6*b*x + 6*a)
 - 4*(2*(b*d*x + b*c)*cos(2*b*x + 2*a) + d*sin(2*b*x + 2*a))*cos(4*b*x + 4
*a) + 4*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*
c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(8*b*x + 8*a)^2 + 4*(b^2*d^
2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + (b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*sin(8*b*x + 8*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2)*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*
c^2)*sin(4*b*x + 4*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a))*cos(8*b*x + 8*a) - 4*(b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a))*integrate(2*(2*b^2*d
^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*sin(2*b*x + 2*a)/(b^2*d^3*x^3 + 3*
b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 +
 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2
*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)), x) - (b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(8*b*x
 + 8*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(8*b*x + 8*a)^2 - 4*(b^2*d^2*x^...
```

### 3.326.8 Giac [N/A]

Not integrable

Time = 3.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc(bx+a)^3\sec(bx+a)^3}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^3*sec(b*x + a)^3/(d*x + c), x)
```

### 3.326.9 Mupad [N/A]

Not integrable

Time = 26.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{1}{\cos(a+b\,x)^3\sin(a+b\,x)^3\,(c+d\,x)}\,dx$$

input
```
int(1/(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)),x)
```

output
```
int(1/(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)), x)
```

# 3.327 $\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$

3.327.1 Optimal result . . . 2449
3.327.2 Mathematica [N/A] . . . 2449
3.327.3 Rubi [N/A] . . . 2450
3.327.4 Maple [N/A] (verified) . . . 2451
3.327.5 Fricas [N/A] . . . 2451
3.327.6 Sympy [N/A] . . . 2451
3.327.7 Maxima [N/A] . . . 2452
3.327.8 Giac [F(-1)] . . . 2453
3.327.9 Mupad [N/A] . . . 2453

## 3.327.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = 8\text{Int}\left(\frac{\csc^3(2a+2bx)}{(c+dx)^2},x\right)$$

output
```
8*Unintegrable(csc(2*b*x+2*a)^3/(d*x+c)^2,x)
```

## 3.327.2 Mathematica [N/A]

Not integrable

Time = 24.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x)^2,x]
```

output
```
Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

### 3.327.3 Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4919, 3042, 4680}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{4919}$$

$$8\int \frac{\csc^3(2a+2bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$8\int \frac{\csc(2a+2bx)^3}{(c+dx)^2}dx$$

$$\Big\downarrow \text{4680}$$

$$8\int \frac{\csc^3(2a+2bx)}{(c+dx)^2}dx$$

input
```
Int[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x)^2,x]
```

output
```
$Aborted
```

#### 3.327.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4680
```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
Simp[If[MatchQ[f, (f1_.)*(Complex[0, j_])], If[MatchQ[e, (e1_.) + Pi/2], Un
integrable[(c + d*x)^m*Sech[I*(e - Pi/2) + I*f*x]^n, x], (-I)^n*Unintegrabl
e[(c + d*x)^m*Csch[(-I)*e - I*f*x]^n, x]], If[MatchQ[e, (e1_.) + Pi/2], Uni
ntegrable[(c + d*x)^m*Sec[e - Pi/2 + f*x]^n, x], Unintegrable[(c + d*x)^m*C
sc[e + f*x]^n, x]]], x] /; FreeQ[{c, d, e, f, m, n}, x] && IntegerQ[n]
```

rule 4919
```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Simp[2^n   Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n
, x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

### 3.327.4 Maple [N/A] (verified)

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc(xb+a)^3 \sec(xb+a)^3}{(dx+c)^2} dx$$

input
```
int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x)
```

### 3.327.5 Fricas [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2} dx = \int \frac{\csc(bx+a)^3 \sec(bx+a)^3}{(dx+c)^2} dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^3*sec(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.327.6 Sympy [N/A]

Not integrable

Time = 41.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2} dx = \int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2} dx$$

input
```
integrate(csc(b*x+a)**3*sec(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(csc(a + b*x)**3*sec(a + b*x)**3/(c + d*x)**2, x)
```

### 3.327.7 Maxima [N/A]

Not integrable

Time = 13.68 (sec) , antiderivative size = 3083, normalized size of antiderivative = 128.46

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)^3\sec\left(bx+a\right)^3}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
(4*((b*d*x + b*c)*cos(6*b*x + 6*a) + (b*d*x + b*c)*cos(2*b*x + 2*a) - d*si
n(6*b*x + 6*a) + d*sin(2*b*x + 2*a))*cos(8*b*x + 8*a) + 4*(b*d*x + b*c - 2
*(b*d*x + b*c)*cos(4*b*x + 4*a) - 2*d*sin(4*b*x + 4*a))*cos(6*b*x + 6*a) -
 8*((b*d*x + b*c)*cos(2*b*x + 2*a) + d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
+ 4*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^
2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2
*c^3)*cos(8*b*x + 8*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*
x + b^2*c^3)*cos(4*b*x + 4*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*sin(8*b*x + 8*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 +
3*b^2*c^2*d*x + b^2*c^3)*sin(8*b*x + 8*a)*sin(4*b*x + 4*a) + 4*(b^2*d^3*x^
3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 + 2*(b^2
*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*
b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a))*cos(8*b*x + 8*a
) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x
+ 4*a))*integrate(2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 3*d^2)*sin(
2*b*x + 2*a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^
3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b
^2*c^3*d*x + b^2*c^4)*cos(2*b*x + 2*a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3
+ 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*sin(2*b*x + 2*a)^2 + 2*(b^2
*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^...
```

### 3.327.8 Giac [F(-1)]

Timed out.

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

### 3.327.9 Mupad [N/A]

Not integrable

Time = 26.57 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{1}{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^3\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^2),x)
```

output
```
int(1/(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^2), x)
```

# 3.328 $\int x\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx$

3.328.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2454
3.328.2 Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 2454
3.328.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2455
3.328.4 Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2457
3.328.5 Fricas [F(-2)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2457
3.328.6 Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2457
3.328.7 Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2458
3.328.8 Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2458
3.328.9 Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2458

## 3.328.1 Optimal result

Integrand size = 18, antiderivative size = 83

$$\int x\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx=-\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}+\frac{20\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{147b^2}+\frac{20\sqrt{\cos(a+bx)}\sin(a+bx)}{147b^2}+\frac{4\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)}{49b^2}$$

output
```
-2/7*x*cos(b*x+a)^(7/2)/b+20/147*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/
2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))/b^2+4/49*cos(b*x+a)^(5/2)*sin
(b*x+a)/b^2+20/147*sin(b*x+a)*cos(b*x+a)^(1/2)/b^2
```

## 3.328.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int x\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx$$
$$=\frac{40\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+\sqrt{\cos(a+bx)}(-63bx\cos(a+bx)-21bx\cos(3(a+bx))+46\sin(a+bx)+\cdots}{294b^2}$$

input
```
Integrate[x*Cos[a + b*x]^(5/2)*Sin[a + b*x],x]
```

output
```
(40*EllipticF[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*(-63*b*x*Cos[a + b*x] -
 21*b*x*Cos[3*(a + b*x)] + 46*Sin[a + b*x] + 6*Sin[3*(a + b*x)]))/(294*b^2
)
```

## 3.328.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3925, 3042, 3115, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin(a+bx)\cos^{\frac{5}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{3925}$$

$$\frac{2\int \cos^{\frac{7}{2}}(a+bx)dx}{7b}-\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \sin\left(a+bx+\frac{\pi}{2}\right)^{7/2}dx}{7b}-\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{2\left(\frac{5}{7}\int \cos^{\frac{3}{2}}(a+bx)dx+\frac{2\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{7b}\right)}{7b}-\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{5}{7}\int \sin\left(a+bx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{7b}\right)}{7b}-\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{2\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(a+bx)}}dx+\frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}\right)+\frac{2\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{7b}\right)}{7b}-\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}\right)+\frac{2\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{7b}\right)}{7b}-\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\left(\frac{2\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{7b}+\frac{5}{7}\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b}+\frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}\right)\right)}{7b}-\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}$$

input
```
Int[x*Cos[a + b*x]^(5/2)*Sin[a + b*x],x]
```

output
```
(-2*x*Cos[a + b*x]^(7/2))/(7*b) + (2*((2*Cos[a + b*x]^(5/2)*Sin[a + b*x])/
(7*b) + (5*((2*EllipticF[(a + b*x)/2, 2])/(3*b) + (2*Sqrt[Cos[a + b*x]]*Si
n[a + b*x])/(3*b)))/7))/(7*b)
```

### 3.328.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3925
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cos[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

### 3.328.4 Maple [F]

$$\int x \cos(xb+a)^{\frac{5}{2}} \sin(xb+a)\,dx$$

input
```
int(x*cos(b*x+a)^(5/2)*sin(b*x+a),x)
```

output
```
int(x*cos(b*x+a)^(5/2)*sin(b*x+a),x)
```

### 3.328.5 Fricas [F(-2)]

Exception generated.

$$\int x \cos^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

### 3.328.6 Sympy [F(-1)]

Timed out.

$$\int x \cos^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x*cos(b*x+a)**(5/2)*sin(b*x+a),x)
```

output
```
Timed out
```

### 3.328.7 Maxima [F]

$$\int x\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx=\int x\cos\left(bx+a\right)^{\frac{5}{2}}\sin\left(bx+a\right)\,dx$$

input
```
integrate(x*cos(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)^(5/2)*sin(b*x + a), x)
```

### 3.328.8 Giac [F]

$$\int x\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx=\int x\cos\left(bx+a\right)^{\frac{5}{2}}\sin\left(bx+a\right)\,dx$$

input
```
integrate(x*cos(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)^(5/2)*sin(b*x + a), x)
```

### 3.328.9 Mupad [F(-1)]

Timed out.

$$\int x\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx=\int x\cos\left(a+b\,x\right)^{5/2}\,\sin\left(a+b\,x\right)\,dx$$

input
```
int(x*cos(a + b*x)^(5/2)*sin(a + b*x),x)
```

output
```
int(x*cos(a + b*x)^(5/2)*sin(a + b*x), x)
```

# 3.329 $\int x \cos^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx$

3.329.1 Optimal result . . . . . 2459
3.329.2 Mathematica [A] (verified) . . . . . 2459
3.329.3 Rubi [A] (verified) . . . . . 2460
3.329.4 Maple [F] . . . . . 2461
3.329.5 Fricas [F(-2)] . . . . . 2461
3.329.6 Sympy [F(-1)] . . . . . 2462
3.329.7 Maxima [F] . . . . . 2462
3.329.8 Giac [F] . . . . . 2462
3.329.9 Mupad [F(-1)] . . . . . 2463

## 3.329.1 Optimal result

Integrand size = 18, antiderivative size = 60

$$\int x \cos^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx = -\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}+\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2}+\frac{4\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)}{25b^2}$$

output
```
-2/5*x*cos(b*x+a)^(5/2)/b+12/25*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2
*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b^2+4/25*cos(b*x+a)^(3/2)*sin(
b*x+a)/b^2
```

## 3.329.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int x \cos^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx = -\frac{2\left(-6E\left(\frac{1}{2}(a+bx)\middle|2\right)+\cos^{\frac{3}{2}}(a+bx)(5bx\cos(a+bx)-2\sin(a+bx))\right)}{25b^2}$$

input
```
Integrate[x*Cos[a + b*x]^(3/2)*Sin[a + b*x],x]
```

output
```
(-2*(-6*EllipticE[(a + b*x)/2, 2] + Cos[a + b*x]^(3/2)*(5*b*x*Cos[a + b*x]
 - 2*Sin[a + b*x])))/(25*b^2)
```

### 3.329.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3925, 3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sin(a+bx)\cos^{\frac{3}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{3925}$$

$$\frac{2\int \cos^{\frac{5}{2}}(a+bx)dx}{5b}-\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \sin\left(a+bx+\frac{\pi}{2}\right)^{5/2}dx}{5b}-\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{2\left(\frac{3}{5}\int \sqrt{\cos(a+bx)}dx+\frac{2\sin(a+bx)\cos^{\frac{3}{2}}(a+bx)}{5b}\right)}{5b}-\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{3}{5}\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx+\frac{2\sin(a+bx)\cos^{\frac{3}{2}}(a+bx)}{5b}\right)}{5b}-\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\left(\frac{6E\left(\frac{1}{2}(a+bx)|2\right)}{5b}+\frac{2\sin(a+bx)\cos^{\frac{3}{2}}(a+bx)}{5b}\right)}{5b}-\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}$$

input
```
Int[x*Cos[a + b*x]^(3/2)*Sin[a + b*x],x]
```

output
```
(-2*x*Cos[a + b*x]^(5/2))/(5*b) + (2*((6*EllipticE[(a + b*x)/2, 2])/(5*b)
+ (2*Cos[a + b*x]^(3/2)*Sin[a + b*x])/(5*b)))/(5*b)
```

### 3.329.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3925
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cos[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

## 3.329.4 Maple [F]

$$\int x \cos (xb+a)^{\frac{3}{2}} \sin (xb+a)\, dx$$

input
```
int(x*cos(b*x+a)^(3/2)*sin(b*x+a),x)
```

output
```
int(x*cos(b*x+a)^(3/2)*sin(b*x+a),x)
```

## 3.329.5 Fricas [F(-2)]

Exception generated.

$$\int x \cos^{\frac{3}{2}}(a+bx) \sin(a+bx)\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

### 3.329.6 Sympy [F(-1)]

Timed out.

$$\int x\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x*cos(b*x+a)**(3/2)*sin(b*x+a),x)
```

output
```
Timed out
```

### 3.329.7 Maxima [F]

$$\int x\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx = \int x\cos\left(bx+a\right)^{\frac{3}{2}}\sin\left(bx+a\right)\,dx$$

input
```
integrate(x*cos(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)^(3/2)*sin(b*x + a), x)
```

### 3.329.8 Giac [F]

$$\int x\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx = \int x\cos\left(bx+a\right)^{\frac{3}{2}}\sin\left(bx+a\right)\,dx$$

input
```
integrate(x*cos(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)^(3/2)*sin(b*x + a), x)
```

### 3.329.9 Mupad [F(-1)]

Timed out.

$$\int x\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx=\int x\cos\left(a+b\,x\right)^{3/2}\,\sin\left(a+b\,x\right)\,dx$$

input
```
int(x*cos(a + b*x)^(3/2)*sin(a + b*x),x)
```

output
```
int(x*cos(a + b*x)^(3/2)*sin(a + b*x), x)
```

# 3.330 $\int x\sqrt{\cos(a+bx)}\sin(a+bx)\,dx$

3.330.1 Optimal result . . . 2464
3.330.2 Mathematica [A] (verified) . . . 2464
3.330.3 Rubi [A] (verified) . . . 2465
3.330.4 Maple [F] . . . 2466
3.330.5 Fricas [F(-2)] . . . 2466
3.330.6 Sympy [F] . . . 2467
3.330.7 Maxima [F] . . . 2467
3.330.8 Giac [F] . . . 2467
3.330.9 Mupad [F(-1)] . . . 2468

## 3.330.1 Optimal result

Integrand size = 18, antiderivative size = 60

$$\int x\sqrt{\cos(a+bx)}\sin(a+bx)\,dx=-\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}+\frac{4\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{9b^2}+\frac{4\sqrt{\cos(a+bx)}\sin(a+bx)}{9b^2}$$

output
```
-2/3*x*cos(b*x+a)^(3/2)/b+4/9*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b
*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))/b^2+4/9*sin(b*x+a)*cos(b*x+a)^(1
/2)/b^2
```

## 3.330.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.87

$$\int x\sqrt{\cos(a+bx)}\sin(a+bx)\,dx=\frac{4\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+2\sqrt{\cos(a+bx)}(-3bx\cos(a+bx)+2\sin(a+bx))}{9b^2}$$

input
```
Integrate[x*Sqrt[Cos[a + b*x]]*Sin[a + b*x],x]
```

output
```
(4*EllipticF[(a + b*x)/2, 2] + 2*Sqrt[Cos[a + b*x]]*(-3*b*x*Cos[a + b*x] +
 2*Sin[a + b*x]))/(9*b^2)
```

### 3.330.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3925, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin(a+bx)\sqrt{\cos(a+bx)}\,dx$$

$$\Big\downarrow \text{3925}$$

$$\frac{2\int \cos^{\frac{3}{2}}(a+bx)dx}{3b}-\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \sin\left(a+bx+\frac{\pi}{2}\right)^{3/2}dx}{3b}-\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{2\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(a+bx)}}dx+\frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}\right)}{3b}-\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}\right)}{3b}-\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b}+\frac{2\sin(a+bx)\sqrt{\cos(a+bx)}}{3b}\right)}{3b}-\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}$$

input
```
Int[x*Sqrt[Cos[a + b*x]]*Sin[a + b*x],x]
```

output
```
(-2*x*Cos[a + b*x]^(3/2))/(3*b) + (2*((2*EllipticF[(a + b*x)/2, 2])/(3*b)
+ (2*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(3*b)))/(3*b)
```

### 3.330.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3925
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cos[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

## 3.330.4 Maple [F]

$$\int x \sin (xb+a) \sqrt{\cos (xb+a)}dx$$

input
```
int(x*sin(b*x+a)*cos(b*x+a)^(1/2),x)
```

output
```
int(x*sin(b*x+a)*cos(b*x+a)^(1/2),x)
```

## 3.330.5 Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{\cos(a+bx)}\sin(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sin(b*x+a)*cos(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

### 3.330.6 Sympy [F]

$$\int x\sqrt{\cos(a+bx)}\sin(a+bx)\,dx = \int x\sin{(a+bx)}\sqrt{\cos{(a+bx)}}\,dx$$

input
```
integrate(x*sin(b*x+a)*cos(b*x+a)**(1/2),x)
```

output
```
Integral(x*sin(a + b*x)*sqrt(cos(a + b*x)), x)
```

### 3.330.7 Maxima [F]

$$\int x\sqrt{\cos(a+bx)}\sin(a+bx)\,dx = \int x\sqrt{\cos{(bx+a)}}\sin{(bx+a)}\;dx$$

input
```
integrate(x*sin(b*x+a)*cos(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*sqrt(cos(b*x + a))*sin(b*x + a), x)
```

### 3.330.8 Giac [F]

$$\int x\sqrt{\cos(a+bx)}\sin(a+bx)\,dx = \int x\sqrt{\cos{(bx+a)}}\sin{(bx+a)}\;dx$$

input
```
integrate(x*sin(b*x+a)*cos(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*sqrt(cos(b*x + a))*sin(b*x + a), x)
```

### 3.330.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{\cos(a+bx)}\sin(a+bx)\,dx = \int x\,\sqrt{\cos\left(a+b\,x\right)}\,\sin\left(a+b\,x\right)\,dx$$

input
```
int(x*cos(a + b*x)^(1/2)*sin(a + b*x),x)
```

output
```
int(x*cos(a + b*x)^(1/2)*sin(a + b*x), x)
```

# 3.331 $\int \frac{x\sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx$

3.331.1 Optimal result . . . . . . . . . . 2469
3.331.2 Mathematica [C] (verified) . . . . . . . . . . 2469
3.331.3 Rubi [A] (verified) . . . . . . . . . . 2470
3.331.4 Maple [C] (verified) . . . . . . . . . . 2471
3.331.5 Fricas [F(-2)] . . . . . . . . . . 2471
3.331.6 Sympy [F] . . . . . . . . . . 2472
3.331.7 Maxima [F] . . . . . . . . . . 2472
3.331.8 Giac [F] . . . . . . . . . . 2472
3.331.9 Mupad [F(-1)] . . . . . . . . . . 2473

## 3.331.1 Optimal result

Integrand size = 18, antiderivative size = 33

$$\int \frac{x\sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx=-\frac{2x\sqrt{\cos(a+bx)}}{b}+\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}$$

output
```
4*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*
b*x),2^(1/2))/b^2-2*x*cos(b*x+a)^(1/2)/b
```

## 3.331.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.69 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.00

$$\int \frac{x\sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx=$$
$$-\frac{2\sqrt{\cos(a+bx)}\left(bx-2\tan(a+bx)+\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\tan^2(a+bx)\right)\sqrt[4]{\sec^2(a+bx)}\tan(\right.}{b^2}$$

input
```
Integrate[(x*Sin[a + b*x])/Sqrt[Cos[a + b*x]],x]
```

output
```
(-2*Sqrt[Cos[a + b*x]]*(b*x - 2*Tan[a + b*x] + Hypergeometric2F1[1/4, 1/2,
 3/2, -Tan[a + b*x]^2]*(Sec[a + b*x]^2)^(1/4)*Tan[a + b*x]))/b^2
```

## 3.331.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3925, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sin(a+bx)}{\sqrt{\cos(a+bx)}}dx$$

$$\Big\downarrow \text{3925}$$

$$\frac{2\int \sqrt{\cos(a+bx)}dx}{b}-\frac{2x\sqrt{\cos(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{b}-\frac{2x\sqrt{\cos(a+bx)}}{b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}-\frac{2x\sqrt{\cos(a+bx)}}{b}$$

input
```
Int[(x*Sin[a + b*x])/Sqrt[Cos[a + b*x]],x]
```

output
```
(-2*x*Sqrt[Cos[a + b*x]])/b + (4*EllipticE[(a + b*x)/2, 2])/b^2
```

### 3.331.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3925
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cos[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

### 3.331.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 310, normalized size of antiderivative = 9.39

| method | result |
|---|---|
| risch | $-\frac{(xb+2i)\left(\mathrm{e}^{2i(xb+a)}+1\right)\sqrt{2}\,\mathrm{e}^{-i(xb+a)}}{b^2\sqrt{\left(\mathrm{e}^{2i(xb+a)}+1\right)\mathrm{e}^{-i(xb+a)}}} - \frac{2i\left(-\frac{2\left(\mathrm{e}^{2i(xb+a)}+1\right)}{\sqrt{\left(\mathrm{e}^{2i(xb+a)}+1\right)\mathrm{e}^{i(xb+a)}}}+\frac{i\sqrt{-i\left(i+\mathrm{e}^{i(xb+a)}\right)}\sqrt{2}\sqrt{i\left(\mathrm{e}^{i(xb+a)}-i\right)}\sqrt{i\mathrm{e}^{i(xb+a)}}\left(-2i\,\mathrm{Elli}\right.}{\sqrt{\mathrm{e}^{3i(x}}}\right.}{b^2\sqrt{(\mathrm{e}}}$ |

input
```
int(x*sin(b*x+a)/cos(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(x*b+2*I)*(exp(I*(b*x+a))^2+1)/b^2*2^(1/2)/((exp(I*(b*x+a))^2+1)/exp(I*(b
*x+a)))^(1/2)/exp(I*(b*x+a))-2*I/b^2*(-2*(exp(I*(b*x+a))^2+1)/((exp(I*(b*x
+a))^2+1)*exp(I*(b*x+a)))^(1/2)+I*(-I*(I+exp(I*(b*x+a))))^(1/2)*2^(1/2)*(I
*(exp(I*(b*x+a))-I))^(1/2)*(I*exp(I*(b*x+a)))^(1/2)/(exp(I*(b*x+a))^3+exp(
I*(b*x+a)))^(1/2)*(-2*I*EllipticE((-I*(I+exp(I*(b*x+a))))^(1/2),1/2*2^(1/2
))+I*EllipticF((-I*(I+exp(I*(b*x+a))))^(1/2),1/2*2^(1/2))))*2^(1/2)/((exp(
I*(b*x+a))^2+1)/exp(I*(b*x+a)))^(1/2)*((exp(I*(b*x+a))^2+1)*exp(I*(b*x+a))
)^(1/2)/exp(I*(b*x+a))
```

### 3.331.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

### 3.331.6 Sympy [F]

$$\int \frac{x\sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{x\sin{(a+bx)}}{\sqrt{\cos{(a+bx)}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)**(1/2),x)
```

output
```
Integral(x*sin(a + b*x)/sqrt(cos(a + b*x)), x)
```

### 3.331.7 Maxima [F]

$$\int \frac{x\sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{x\sin{(bx+a)}}{\sqrt{\cos{(bx+a)}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*sin(b*x + a)/sqrt(cos(b*x + a)), x)
```

### 3.331.8 Giac [F]

$$\int \frac{x\sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx = \int \frac{x\sin{(bx+a)}}{\sqrt{\cos{(bx+a)}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*sin(b*x + a)/sqrt(cos(b*x + a)), x)
```

### 3.331.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx=\int \frac{x\,\sin\left(a+b\,x\right)}{\sqrt{\cos\left(a+b\,x\right)}}\,dx$$

input
```
int((x*sin(a + b*x))/cos(a + b*x)^(1/2),x)
```

output
```
int((x*sin(a + b*x))/cos(a + b*x)^(1/2), x)
```

# 3.332 $\int \frac{x\sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx$

3.332.1 Optimal result . . . 2474
3.332.2 Mathematica [A] (verified) . . . 2474
3.332.3 Rubi [A] (verified) . . . 2475
3.332.4 Maple [F] . . . 2476
3.332.5 Fricas [F(-2)] . . . 2476
3.332.6 Sympy [F] . . . 2476
3.332.7 Maxima [F] . . . 2477
3.332.8 Giac [F] . . . 2477
3.332.9 Mupad [F(-1)] . . . 2477

## 3.332.1 Optimal result

Integrand size = 18, antiderivative size = 33

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \frac{2x}{b\sqrt{\cos(a+bx)}} - \frac{4\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b^2}$$

output
```
-4*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2
*b*x),2^(1/2))/b^2+2*x/b/cos(b*x+a)^(1/2)
```

## 3.332.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \frac{2x}{b\sqrt{\cos(a+bx)}} - \frac{4\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b^2}$$

input
```
Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(3/2),x]
```

output
```
(2*x)/(b*Sqrt[Cos[a + b*x]]) - (4*EllipticF[(a + b*x)/2, 2])/b^2
```

### 3.332.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3925, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx$$

$$\Big\downarrow \text{3925}$$

$$\frac{2x}{b\sqrt{\cos(a+bx)}}-\frac{2\int \frac{1}{\sqrt{\cos(a+bx)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{b\sqrt{\cos(a+bx)}}-\frac{2\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2x}{b\sqrt{\cos(a+bx)}}-\frac{4\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b^2}$$

input
```
Int[(x*Sin[a + b*x])/Cos[a + b*x]^(3/2),x]
```

output
```
(2*x)/(b*Sqrt[Cos[a + b*x]]) - (4*EllipticF[(a + b*x)/2, 2])/b^2
```

#### 3.332.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3925
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cos[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

### 3.332.4 Maple [F]

$$\int \frac{x\sin\left(xb+a\right)}{\cos\left(xb+a\right)^{\frac{3}{2}}}dx$$

input
```
int(x*sin(b*x+a)/cos(b*x+a)^(3/2),x)
```

output
```
int(x*sin(b*x+a)/cos(b*x+a)^(3/2),x)
```

### 3.332.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.332.6 Sympy [F]

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\sin\left(a+bx\right)}{\cos^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)**(3/2),x)
```

output
```
Integral(x*sin(a + b*x)/cos(a + b*x)**(3/2), x)
```

### 3.332.7 Maxima [F]

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x*sin(b*x + a)/cos(b*x + a)^(3/2), x)
```

### 3.332.8 Giac [F]

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*sin(b*x + a)/cos(b*x + a)^(3/2), x)
```

### 3.332.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\,\sin(a+b\,x)}{\cos(a+b\,x)^{3/2}}\,dx$$

input
```
int((x*sin(a + b*x))/cos(a + b*x)^(3/2),x)
```

output
```
int((x*sin(a + b*x))/cos(a + b*x)^(3/2), x)
```

# 3.333 $\int \frac{x\sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx$

3.333.1 Optimal result . . . . . 2478
3.333.2 Mathematica [A] (verified) . . . . . 2478
3.333.3 Rubi [A] (verified) . . . . . 2479
3.333.4 Maple [F] . . . . . 2480
3.333.5 Fricas [F(-2)] . . . . . 2480
3.333.6 Sympy [F(-1)] . . . . . 2481
3.333.7 Maxima [F] . . . . . 2481
3.333.8 Giac [F] . . . . . 2481
3.333.9 Mupad [F(-1)] . . . . . 2482

## 3.333.1 Optimal result

Integrand size = 18, antiderivative size = 60

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx=\frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)}+\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b^2}-\frac{4\sin(a+bx)}{3b^2\sqrt{\cos(a+bx)}}$$

output
```
2/3*x/b/cos(b*x+a)^(3/2)+4/3*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*
x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b^2-4/3*sin(b*x+a)/b^2/cos(b*x+a)
^(1/2)
```

## 3.333.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx=\frac{2\left(bx+2\cos^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right)-\sin(2(a+bx))\right)}{3b^2\cos^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(5/2),x]
```

output
```
(2*(b*x + 2*Cos[a + b*x]^(3/2)*EllipticE[(a + b*x)/2, 2] - Sin[2*(a + b*x)
]))/(3*b^2*Cos[a + b*x]^(3/2))
```

### 3.333.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3925, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3925}$$

$$\frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)}-\frac{2\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}dx}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)}-\frac{2\int \frac{1}{\sin\left(a+bx+\frac{\pi}{2}\right)^{3/2}}dx}{3b}$$

$$\Big\downarrow \text{3116}$$

$$\frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)}-\frac{2\left(\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\int \sqrt{\cos(a+bx)}dx\right)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)}-\frac{2\left(\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx\right)}{3b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)}-\frac{2\left(\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\frac{2E\left(\frac{1}{2}(a+bx)|2\right)}{b}\right)}{3b}$$

input
```
Int[(x*Sin[a + b*x])/Cos[a + b*x]^(5/2),x]
```

output
```
(2*x)/(3*b*Cos[a + b*x]^(3/2)) - (2*((-2*EllipticE[(a + b*x)/2, 2])/b + (2
*Sin[a + b*x])/(b*Sqrt[Cos[a + b*x]])))/(3*b)
```

### 3.333.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3925
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cos[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

## 3.333.4 Maple [F]

$$\int \frac{x \sin (xb + a)}{\cos (xb + a)^{\frac{5}{2}}}dx$$

input
```
int(x*sin(b*x+a)/cos(b*x+a)^(5/2),x)
```

output
```
int(x*sin(b*x+a)/cos(b*x+a)^(5/2),x)
```

## 3.333.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x \sin(a + bx)}{\cos^{\frac{5}{2}}(a + bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.333.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)**(5/2),x)
```

output
```
Timed out
```

### 3.333.7 Maxima [F]

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\sin{(bx+a)}}{\cos{(bx+a)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x*sin(b*x + a)/cos(b*x + a)^(5/2), x)
```

### 3.333.8 Giac [F]

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\sin{(bx+a)}}{\cos{(bx+a)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(x*sin(b*x + a)/cos(b*x + a)^(5/2), x)
```

### 3.333.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\,\sin\left(a+b\,x\right)}{{\cos\left(a+b\,x\right)}^{5/2}}\,dx$$

input
```
int((x*sin(a + b*x))/cos(a + b*x)^(5/2),x)
```

output
```
int((x*sin(a + b*x))/cos(a + b*x)^(5/2), x)
```

# 3.334 $\int \frac{x \sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)} dx$

3.334.1 Optimal result . . . 2483
3.334.2 Mathematica [A] (verified) . . . 2483
3.334.3 Rubi [A] (verified) . . . 2484
3.334.4 Maple [F] . . . 2485
3.334.5 Fricas [F(-2)] . . . 2485
3.334.6 Sympy [F(-1)] . . . 2486
3.334.7 Maxima [F] . . . 2486
3.334.8 Giac [F] . . . 2486
3.334.9 Mupad [F(-1)] . . . 2487

## 3.334.1 Optimal result

Integrand size = 18, antiderivative size = 60

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)} dx = \frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)} - \frac{4\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{15b^2} - \frac{4\sin(a+bx)}{15b^2\cos^{\frac{3}{2}}(a+bx)}$$

output
```
2/5*x/b/cos(b*x+a)^(5/2)-4/15*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b
*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))/b^2-4/15*sin(b*x+a)/b^2/cos(b*x+
a)^(3/2)
```

## 3.334.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)} dx = -\frac{2\left(-3bx + 2\cos^{\frac{5}{2}}(a+bx)\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right) + \sin(2(a+bx))\right)}{15b^2\cos^{\frac{5}{2}}(a+bx)}$$

input
```
Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(7/2),x]
```

output
```
(-2*(-3*b*x + 2*Cos[a + b*x]^(5/2)*EllipticF[(a + b*x)/2, 2] + Sin[2*(a +
b*x)]))/(15*b^2*Cos[a + b*x]^(5/2))
```

### 3.334.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3925, 3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3925}$$

$$\frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{2\int \frac{1}{\cos^{\frac{5}{2}}(a+bx)}dx}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{2\int \frac{1}{\sin\left(a+bx+\frac{\pi}{2}\right)^{5/2}}dx}{5b}$$

$$\Big\downarrow \text{3116}$$

$$\frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{2\left(\frac{1}{3}\int \frac{1}{\sqrt{\cos(a+bx)}}dx+\frac{2\sin(a+bx)}{3b\cos^{\frac{3}{2}}(a+bx)}\right)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{2\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(a+bx)}{3b\cos^{\frac{3}{2}}(a+bx)}\right)}{5b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{2\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b}+\frac{2\sin(a+bx)}{3b\cos^{\frac{3}{2}}(a+bx)}\right)}{5b}$$

input
```
Int[(x*Sin[a + b*x])/Cos[a + b*x]^(7/2),x]
```

output
```
(2*x)/(5*b*Cos[a + b*x]^(5/2)) - (2*((2*EllipticF[(a + b*x)/2, 2])/(3*b) +
 (2*Sin[a + b*x])/(3*b*Cos[a + b*x]^(3/2))))/(5*b)
```

### 3.334.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3925
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cos[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

## 3.334.4 Maple [F]

$$\int \frac{x \sin \left(x b +a \right)}{\cos \left(x b +a \right)^{\frac{7}{2}}}d x$$

input
```
int(x*sin(b*x+a)/cos(b*x+a)^(7/2),x)
```

output
```
int(x*sin(b*x+a)/cos(b*x+a)^(7/2),x)
```

## 3.334.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

### 3.334.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)**(7/2),x)
```

output
```
Timed out
```

### 3.334.7 Maxima [F]

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(x*sin(b*x + a)/cos(b*x + a)^(7/2), x)
```

### 3.334.8 Giac [F]

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(x*sin(b*x + a)/cos(b*x + a)^(7/2), x)
```

### 3.334.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{x\,\sin\left(a+b\,x\right)}{{\cos\left(a+b\,x\right)}^{7/2}}\,dx$$

input
```
int((x*sin(a + b*x))/cos(a + b*x)^(7/2),x)
```

output
```
int((x*sin(a + b*x))/cos(a + b*x)^(7/2), x)
```

# 3.335 $\int \frac{x\sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\,dx$


3.335.1 Optimal result . . . 2488
3.335.2 Mathematica [A] (verified) . . . 2488
3.335.3 Rubi [A] (verified) . . . 2489
3.335.4 Maple [F] . . . 2491
3.335.5 Fricas [F(-2)] . . . 2491
3.335.6 Sympy [F(-1)] . . . 2491
3.335.7 Maxima [F] . . . 2492
3.335.8 Giac [F] . . . 2492
3.335.9 Mupad [F(-1)] . . . 2492


## 3.335.1 Optimal result

Integrand size = 18, antiderivative size = 83

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\,dx=\frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)}+\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{35b^2}-\frac{4\sin(a+bx)}{35b^2\cos^{\frac{5}{2}}(a+bx)}-\frac{12\sin(a+bx)}{35b^2\sqrt{\cos(a+bx)}}$$

output
```
2/7*x/b/cos(b*x+a)^(7/2)+12/35*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*
b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))/b^2-4/35*sin(b*x+a)/b^2/cos(b*x
+a)^(5/2)-12/35*sin(b*x+a)/b^2/cos(b*x+a)^(1/2)
```

## 3.335.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.78

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\,dx=\frac{20bx+24\cos^{\frac{7}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right)-10\sin(2(a+bx))-3\sin(4(a+bx))}{70b^2\cos^{\frac{7}{2}}(a+bx)}$$

input
```
Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(9/2),x]
```

output
```
(20*b*x + 24*Cos[a + b*x]^(7/2)*EllipticE[(a + b*x)/2, 2] - 10*Sin[2*(a +
b*x)] - 3*Sin[4*(a + b*x)])/(70*b^2*Cos[a + b*x]^(7/2))
```

### 3.335.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3925, 3042, 3116, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{ 3925}$$

$$\frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)}-\frac{2\int \frac{1}{\cos^{\frac{7}{2}}(a+bx)}dx}{7b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)}-\frac{2\int \frac{1}{\sin\left(a+bx+\frac{\pi}{2}\right)^{7/2}}dx}{7b}$$

$$\Big\downarrow \text{ 3116}$$

$$\frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{3}{5}\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}dx+\frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{3}{5}\int \frac{1}{\sin\left(a+bx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

$$\Big\downarrow \text{ 3116}$$

$$\frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{3}{5}\left(\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\int \sqrt{\cos(a+bx)}dx\right)+\frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{3}{5}\left(\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\int\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx\right)+\frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

$$\Big\downarrow \ 3119$$

$$\frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{2\sin(a+bx)}{5b\cos^{\frac{5}{2}}(a+bx)}+\frac{3}{5}\left(\frac{2\sin(a+bx)}{b\sqrt{\cos(a+bx)}}-\frac{2E\left(\frac{1}{2}(a+bx)|2\right)}{b}\right)\right)}{7b}$$

input
```
Int[(x*Sin[a + b*x])/Cos[a + b*x]^(9/2),x]
```

output
```
(2*x)/(7*b*Cos[a + b*x]^(7/2)) - (2*((2*Sin[a + b*x])/(5*b*Cos[a + b*x]^(5
/2)) + (3*((-2*EllipticE[(a + b*x)/2, 2])/b + (2*Sin[a + b*x])/(b*Sqrt[Cos
[a + b*x]])))/5))/(7*b)
```

### 3.335.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3925
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> Simp[(-x^(m - n + 1))*(Cos[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] + Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Cos[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.335. $\int \frac{x\sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\,dx$

### 3.335.4 Maple [F]

$$\int \frac{x \sin(xb+a)}{\cos(xb+a)^{\frac{9}{2}}} dx$$

input
```
int(x*sin(b*x+a)/cos(b*x+a)^(9/2),x)
```

output
```
int(x*sin(b*x+a)/cos(b*x+a)^(9/2),x)
```

### 3.335.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(9/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.335.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x \sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)} dx = \text{Timed out}$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)**(9/2),x)
```

output
```
Timed out
```

### 3.335.7 Maxima [F]

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\,dx = \int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x*sin(b*x + a)/cos(b*x + a)^(9/2), x)
```

### 3.335.8 Giac [F]

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\,dx = \int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/cos(b*x+a)^(9/2),x, algorithm="giac")
```

output
```
integrate(x*sin(b*x + a)/cos(b*x + a)^(9/2), x)
```

### 3.335.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\,dx = \int \frac{x\,\sin(a+b\,x)}{\cos(a+b\,x)^{9/2}}\,dx$$

input
```
int((x*sin(a + b*x))/cos(a + b*x)^(9/2),x)
```

output
```
int((x*sin(a + b*x))/cos(a + b*x)^(9/2), x)
```

# 3.336 $\int x \sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx$

3.336.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2493
3.336.2 Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 2493
3.336.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2494
3.336.4 Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2496
3.336.5 Fricas [F(-2)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2496
3.336.6 Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2497
3.336.7 Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2497
3.336.8 Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2497
3.336.9 Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2498

## 3.336.1 Optimal result

Integrand size = 18, antiderivative size = 103

$$\int x \sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx = \frac{12\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{35b^2} + \frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{12\sqrt{\sec(a+bx)}\sin(a+bx)}{35b^2} - \frac{4\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)}{35b^2}$$

output
```
2/7*x*sec(b*x+a)^(7/2)/b-4/35*sec(b*x+a)^(5/2)*sin(b*x+a)/b^2-12/35*sin(b*
x+a)*sec(b*x+a)^(1/2)/b^2+12/35*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2
*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1
/2)/b^2
```

## 3.336.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.63

$$\int x \sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx = \frac{\sec^{\frac{7}{2}}(a+bx)\left(20bx+24\cos^{\frac{7}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right)-10\sin(2(a+bx))-3\sin(4(a+bx))\right)}{70b^2}$$

input
```
Integrate[x*Sec[a + b*x]^(9/2)*Sin[a + b*x],x]
```

output
```
(Sec[a + b*x]^(7/2)*(20*b*x + 24*Cos[a + b*x]^(7/2)*EllipticE[(a + b*x)/2,
 2] - 10*Sin[2*(a + b*x)] - 3*Sin[4*(a + b*x)]))/(70*b^2)
```

## 3.336.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4700, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin(a+bx) \sec^{\frac{9}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{4700}$$

$$\frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\int \sec^{\frac{7}{2}}(a+bx)dx}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\int \csc\left(a+bx+\frac{\pi}{2}\right)^{7/2}dx}{7b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\left(\frac{3}{5}\int \sec^{\frac{3}{2}}(a+bx)dx+\frac{2\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\left(\frac{3}{5}\int \csc\left(a+bx+\frac{\pi}{2}\right)^{3/2}dx+\frac{2\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\left(\frac{3}{5}\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b}-\int \frac{1}{\sqrt{\sec(a+bx)}}dx\right)+\frac{2\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\left(\frac{3}{5}\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b}-\int\frac{1}{\sqrt{\csc\left(a+bx+\frac{\pi}{2}\right)}}dx\right)+\frac{2\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b}$$

$\Big\downarrow$ 4258

$$\frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\left(\frac{3}{5}\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b}-\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int\sqrt{\cos(a+bx)}dx\right)+\frac{2\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b}$$

$\Big\downarrow$ 3042

$$\frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\left(\frac{3}{5}\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b}-\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx\right)+\frac{2\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b}$$

$\Big\downarrow$ 3119

$$\frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\left(\frac{2\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{5b}+\frac{3}{5}\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b}-\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)|2\right)}{b}\right)\right)}{7b}$$

input
```
Int[x*Sec[a + b*x]^(9/2)*Sin[a + b*x],x]
```

output
```
(2*x*Sec[a + b*x]^(7/2))/(7*b) - (2*((2*Sec[a + b*x]^(5/2)*Sin[a + b*x])/(
5*b) + (3*((-2*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b
*x]])/b + (2*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/b))/5))/(7*b)
```

### 3.336.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.336. $\int x\sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx$

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4700
```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^(p - 1)/(b*n*(p - 1))
), x] - Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Sec[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

## 3.336.4 Maple [F]

$$\int x \sec \left(x b+a\right)^{\frac{9}{2}} \sin \left(x b+a\right)dx$$

input
```
int(x*sec(b*x+a)^(9/2)*sin(b*x+a),x)
```

output
```
int(x*sec(b*x+a)^(9/2)*sin(b*x+a),x)
```

## 3.336.5 Fricas [F(-2)]

Exception generated.

$$\int x \sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sec(b*x+a)^(9/2)*sin(b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.336.6 Sympy [F(-1)]

Timed out.

$$\int x\sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x*sec(b*x+a)**(9/2)*sin(b*x+a),x)
```

output
```
Timed out
```

### 3.336.7 Maxima [F]

$$\int x\sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx = \int x\sec\left(bx+a\right)^{\frac{9}{2}}\sin\left(bx+a\right)\,dx$$

input
```
integrate(x*sec(b*x+a)^(9/2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*sec(b*x + a)^(9/2)*sin(b*x + a), x)
```

### 3.336.8 Giac [F]

$$\int x\sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx = \int x\sec\left(bx+a\right)^{\frac{9}{2}}\sin\left(bx+a\right)\,dx$$

input
```
integrate(x*sec(b*x+a)^(9/2)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*sec(b*x + a)^(9/2)*sin(b*x + a), x)
```

### 3.336.9 Mupad [F(-1)]

Timed out.

$$\int x\sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx=\int x\,\sin\left(a+b\,x\right)\,{\left(\frac{1}{\cos\left(a+b\,x\right)}\right)}^{9/2}\,dx$$

input
```
int(x*sin(a + b*x)*(1/cos(a + b*x))^(9/2),x)
```

output
```
int(x*sin(a + b*x)*(1/cos(a + b*x))^(9/2), x)
```

# 3.337 $\int x \sec^{\frac{7}{2}}(a+bx) \sin(a+bx)\,dx$


3.337.1 Optimal result . . . 2499
3.337.2 Mathematica [A] (verified) . . . 2499
3.337.3 Rubi [A] (verified) . . . 2500
3.337.4 Maple [F] . . . 2502
3.337.5 Fricas [F(-2)] . . . 2502
3.337.6 Sympy [F(-1)] . . . 2502
3.337.7 Maxima [F] . . . 2503
3.337.8 Giac [F] . . . 2503
3.337.9 Mupad [F(-1)] . . . 2503


## 3.337.1 Optimal result

Integrand size = 18, antiderivative size = 80

$$\int x \sec^{\frac{7}{2}}(a+bx) \sin(a+bx)\,dx = -\frac{4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{\sec(a+bx)}}{15b^2} + \frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b} - \frac{4\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)}{15b^2}$$

output
```
2/5*x*sec(b*x+a)^(5/2)/b-4/15*sec(b*x+a)^(3/2)*sin(b*x+a)/b^2-4/15*(cos(1/
2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1
/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^2
```

## 3.337.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.76

$$\int x \sec^{\frac{7}{2}}(a+bx) \sin(a+bx)\,dx = \frac{2\sqrt{\sec(a+bx)}\left(-2\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right) + 3bx\sec^2(a+bx) - 2\tan(a+bx)\right)}{15b^2}$$

input
```
Integrate[x*Sec[a + b*x]^(7/2)*Sin[a + b*x],x]
```

output
```
(2*Sqrt[Sec[a + b*x]]*(-2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 3
*b*x*Sec[a + b*x]^2 - 2*Tan[a + b*x]))/(15*b^2)
```

### 3.337.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {4700, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin(a+bx)\sec^{\frac{7}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{4700}$$

$$\frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\int \sec^{\frac{5}{2}}(a+bx)dx}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\int \csc\left(a+bx+\frac{\pi}{2}\right)^{5/2}dx}{5b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\left(\frac{1}{3}\int \sqrt{\sec(a+bx)}dx+\frac{2\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{3b}\right)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\left(\frac{1}{3}\int \sqrt{\csc\left(a+bx+\frac{\pi}{2}\right)}dx+\frac{2\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{3b}\right)}{5b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\left(\frac{1}{3}\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}dx+\frac{2\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{3b}\right)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\left(\frac{1}{3}\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{3b}\right)}{5b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\left(\frac{2\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b}\right)}{5b}$$

input
```
Int[x*Sec[a + b*x]^(7/2)*Sin[a + b*x],x]
```

output
```
(2*x*Sec[a + b*x]^(5/2))/(5*b) - (2*((2*Sqrt[Cos[a + b*x]]*EllipticF[(a +
b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(3*b) + (2*Sec[a + b*x]^(3/2)*Sin[a + b*x])
/(3*b)))/(5*b)
```

### 3.337.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4700
```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^(p - 1)/(b*n*(p - 1))
), x] - Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Sec[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

### 3.337.4 Maple [F]

$$\int x \sec(xb+a)^{\frac{7}{2}} \sin(xb+a)\, dx$$

input
```
int(x*sec(b*x+a)^(7/2)*sin(b*x+a),x)
```

output
```
int(x*sec(b*x+a)^(7/2)*sin(b*x+a),x)
```

### 3.337.5 Fricas [F(-2)]

Exception generated.

$$\int x \sec^{\frac{7}{2}}(a+bx)\sin(a+bx)\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sec(b*x+a)^(7/2)*sin(b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.337.6 Sympy [F(-1)]

Timed out.

$$\int x \sec^{\frac{7}{2}}(a+bx)\sin(a+bx)\, dx = \text{Timed out}$$

input
```
integrate(x*sec(b*x+a)**(7/2)*sin(b*x+a),x)
```

output
```
Timed out
```

### 3.337.7 Maxima [F]

$$\int x\sec^{\frac{7}{2}}(a+bx)\sin(a+bx)\,dx=\int x\sec\left(bx+a\right)^{\frac{7}{2}}\sin\left(bx+a\right)\,dx$$

input
```
integrate(x*sec(b*x+a)^(7/2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*sec(b*x + a)^(7/2)*sin(b*x + a), x)
```

### 3.337.8 Giac [F]

$$\int x\sec^{\frac{7}{2}}(a+bx)\sin(a+bx)\,dx=\int x\sec\left(bx+a\right)^{\frac{7}{2}}\sin\left(bx+a\right)\,dx$$

input
```
integrate(x*sec(b*x+a)^(7/2)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*sec(b*x + a)^(7/2)*sin(b*x + a), x)
```

### 3.337.9 Mupad [F(-1)]

Timed out.

$$\int x\sec^{\frac{7}{2}}(a+bx)\sin(a+bx)\,dx=\int x\,\sin\left(a+b\,x\right)\,\left(\frac{1}{\cos\left(a+b\,x\right)}\right)^{7/2}\,dx$$

input
```
int(x*sin(a + b*x)*(1/cos(a + b*x))^(7/2),x)
```

output
```
int(x*sin(a + b*x)*(1/cos(a + b*x))^(7/2), x)
```

# 3.338 $\int x\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx$

3.338.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2504
3.338.2 Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 2504
3.338.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2505
3.338.4 Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2507
3.338.5 Fricas [F(-2)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2507
3.338.6 Sympy [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2507
3.338.7 Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2508
3.338.8 Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2508
3.338.9 Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2508

## 3.338.1 Optimal result

Integrand size = 18, antiderivative size = 80

$$\int x\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx=\frac{4\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{3b^2}+\frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b}-\frac{4\sqrt{\sec(a+bx)}\sin(a+bx)}{3b^2}$$

output
```
2/3*x*sec(b*x+a)^(3/2)/b-4/3*sin(b*x+a)*sec(b*x+a)^(1/2)/b^2+4/3*(cos(1/2*
a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^(1/2
))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^2
```

## 3.338.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int x\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx=\frac{2\sec^{\frac{3}{2}}(a+bx)\left(bx+2\cos^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right)-\sin(2(a+bx))\right)}{3b^2}$$

input
```
Integrate[x*Sec[a + b*x]^(5/2)*Sin[a + b*x],x]
```

output
```
(2*Sec[a + b*x]^(3/2)*(b*x + 2*Cos[a + b*x]^(3/2)*EllipticE[(a + b*x)/2, 2
] - Sin[2*(a + b*x)]))/(3*b^2)
```

### 3.338.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {4700, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{4700}$$

$$\frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\int \sec^{\frac{3}{2}}(a+bx)dx}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\int \csc\left(a+bx+\frac{\pi}{2}\right)^{3/2}dx}{3b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b}-\int \frac{1}{\sqrt{\sec(a+bx)}}dx\right)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b}-\int \frac{1}{\sqrt{\csc\left(a+bx+\frac{\pi}{2}\right)}}dx\right)}{3b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b}-\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \sqrt{\cos(a+bx)}dx\right)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b}-\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx\right)}{3b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\left(\frac{2\sin(a+bx)\sqrt{\sec(a+bx)}}{b}-\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)|2\right)}{b}\right)}{3b}$$

input
```
Int[x*Sec[a + b*x]^(5/2)*Sin[a + b*x],x]
```

output
```
(2*x*Sec[a + b*x]^(3/2))/(3*b) - (2*((-2*Sqrt[Cos[a + b*x]]*EllipticE[(a +
 b*x)/2, 2]*Sqrt[Sec[a + b*x]])/b + (2*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/b)
)/(3*b)
```

### 3.338.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4700
```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^(p - 1)/(b*n*(p - 1))
), x] - Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Sec[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

3.338. $\int x \sec^{\frac{5}{2}}(a+bx) \sin(a+bx)\, dx$

### 3.338.4 Maple [F]

$$\int x \sec (xb+a)^{\frac{5}{2}} \sin (xb+a)\,dx$$

input
```
int(x*sec(b*x+a)^(5/2)*sin(b*x+a),x)
```

output
```
int(x*sec(b*x+a)^(5/2)*sin(b*x+a),x)
```

### 3.338.5 Fricas [F(-2)]

Exception generated.

$$\int x \sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sec(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.338.6 Sympy [F(-1)]

Timed out.

$$\int x \sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x*sec(b*x+a)**(5/2)*sin(b*x+a),x)
```

output
```
Timed out
```

### 3.338.7 Maxima [F]

$$\int x\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx=\int x\sec\left(bx+a\right)^{\frac{5}{2}}\sin\left(bx+a\right)\,dx$$

input
```
integrate(x*sec(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*sec(b*x + a)^(5/2)*sin(b*x + a), x)
```

### 3.338.8 Giac [F]

$$\int x\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx=\int x\sec\left(bx+a\right)^{\frac{5}{2}}\sin\left(bx+a\right)\,dx$$

input
```
integrate(x*sec(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*sec(b*x + a)^(5/2)*sin(b*x + a), x)
```

### 3.338.9 Mupad [F(-1)]

Timed out.

$$\int x\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx=\int x\,\sin\left(a+b\,x\right)\,\left(\frac{1}{\cos\left(a+b\,x\right)}\right)^{5/2}\,dx$$

input
```
int(x*sin(a + b*x)*(1/cos(a + b*x))^(5/2),x)
```

output
```
int(x*sin(a + b*x)*(1/cos(a + b*x))^(5/2), x)
```

# 3.339 $\int x \sec^{\frac{3}{2}}(a+bx) \sin(a+bx) \, dx$

3.339.1 Optimal result . . . . . 2509
3.339.2 Mathematica [A] (verified) . . . . . 2509
3.339.3 Rubi [A] (verified) . . . . . 2510
3.339.4 Maple [F] . . . . . 2511
3.339.5 Fricas [F(-2)] . . . . . 2511
3.339.6 Sympy [F(-1)] . . . . . 2512
3.339.7 Maxima [F] . . . . . 2512
3.339.8 Giac [F] . . . . . 2512
3.339.9 Mupad [F(-1)] . . . . . 2513

## 3.339.1 Optimal result

Integrand size = 18, antiderivative size = 53

$$\int x \sec^{\frac{3}{2}}(a+bx) \sin(a+bx) \, dx = \frac{2x\sqrt{\sec(a+bx)}}{b} - \frac{4\sqrt{\cos(a+bx)} \, \text{EllipticF}\left(\frac{1}{2}(a+bx),2\right) \sqrt{\sec(a+bx)}}{b^2}$$

output
```
2*x*sec(b*x+a)^(1/2)/b-4*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*E
llipticF(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^2
```

## 3.339.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int x \sec^{\frac{3}{2}}(a+bx) \sin(a+bx) \, dx = \frac{2\left(bx - 2\sqrt{\cos(a+bx)} \, \text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\right) \sqrt{\sec(a+bx)}}{b^2}$$

input
```
Integrate[x*Sec[a + b*x]^(3/2)*Sin[a + b*x],x]
```

output
```
(2*(b*x - 2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])*Sqrt[Sec[a + b*x
]])/b^2
```

### 3.339.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4700, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{4700}$$

$$\frac{2x\sqrt{\sec(a+bx)}}{b}-\frac{2\int \sqrt{\sec(a+bx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\sqrt{\sec(a+bx)}}{b}-\frac{2\int \sqrt{\csc\left(a+bx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2x\sqrt{\sec(a+bx)}}{b}-\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\sqrt{\sec(a+bx)}}{b}-\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2x\sqrt{\sec(a+bx)}}{b}-\frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{b^2}$$

input
```
Int[x*Sec[a + b*x]^(3/2)*Sin[a + b*x],x]
```

output
```
(2*x*Sqrt[Sec[a + b*x]])/b - (4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2,
2]*Sqrt[Sec[a + b*x]])/b^2
```

#### 3.339.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4700
```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^(p - 1)/(b*n*(p - 1))
), x] - Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Sec[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

### 3.339.4 Maple [F]

$$\int x \sec(xb+a)^{\frac{3}{2}} \sin(xb+a)\,dx$$

input
```
int(x*sec(b*x+a)^(3/2)*sin(b*x+a),x)
```

output
```
int(x*sec(b*x+a)^(3/2)*sin(b*x+a),x)
```

### 3.339.5 Fricas [F(-2)]

Exception generated.

$$\int x \sec^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sec(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

### 3.339.6 Sympy [F(-1)]

Timed out.

$$\int x\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx=\text{Timed out}$$

input
```
integrate(x*sec(b*x+a)**(3/2)*sin(b*x+a),x)
```

output
```
Timed out
```

### 3.339.7 Maxima [F]

$$\int x\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx=\int x\sec\left(bx+a\right)^{\frac{3}{2}}\sin\left(bx+a\right)\,dx$$

input
```
integrate(x*sec(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="maxima")
```

output
```
integrate(x*sec(b*x + a)^(3/2)*sin(b*x + a), x)
```

### 3.339.8 Giac [F]

$$\int x\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx=\int x\sec\left(bx+a\right)^{\frac{3}{2}}\sin\left(bx+a\right)\,dx$$

input
```
integrate(x*sec(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="giac")
```

output
```
integrate(x*sec(b*x + a)^(3/2)*sin(b*x + a), x)
```

### 3.339.9 Mupad [F(-1)]

Timed out.

$$\int x\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx=\int x\,\sin\left(a+b\,x\right)\,{\left(\frac{1}{\cos\left(a+b\,x\right)}\right)}^{3/2}\,dx$$

input
```
int(x*sin(a + b*x)*(1/cos(a + b*x))^(3/2),x)
```

output
```
int(x*sin(a + b*x)*(1/cos(a + b*x))^(3/2), x)
```

# 3.340 $\int x\sqrt{\sec(a+bx)}\sin(a+bx)\,dx$

3.340.1 Optimal result . . . 2514
3.340.2 Mathematica [C] (verified) . . . 2514
3.340.3 Rubi [A] (verified) . . . 2515
3.340.4 Maple [C] (verified) . . . 2516
3.340.5 Fricas [F(-2)] . . . 2517
3.340.6 Sympy [F] . . . 2517
3.340.7 Maxima [F] . . . 2518
3.340.8 Giac [F] . . . 2518
3.340.9 Mupad [F(-1)] . . . 2518

## 3.340.1 Optimal result

Integrand size = 18, antiderivative size = 53

$$\int x\sqrt{\sec(a+bx)}\sin(a+bx)\,dx=-\frac{2x}{b\sqrt{\sec(a+bx)}}+\frac{4\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b^2}$$

output
```
-2*x/b/sec(b*x+a)^(1/2)+4*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*
EllipticE(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^
2
```

## 3.340.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.64 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int x\sqrt{\sec(a+bx)}\sin(a+bx)\,dx=-\frac{2\left(bx-2\tan(a+bx)+\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\tan^2(a+bx)\right)\sqrt[4]{\sec^2(a+bx)}\tan(a+bx)\right)}{b^2\sqrt{\sec(a+bx)}}$$

input
```
Integrate[x*Sqrt[Sec[a + b*x]]*Sin[a + b*x],x]
```

output
```
(-2*(b*x - 2*Tan[a + b*x] + Hypergeometric2F1[1/4, 1/2, 3/2, -Tan[a + b*x]
^2]*(Sec[a + b*x]^2)^(1/4)*Tan[a + b*x]))/(b^2*Sqrt[Sec[a + b*x]])
```

### 3.340.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4700, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sin(a+bx)\sqrt{\sec(a+bx)}\,dx$$

$$\Big\downarrow \text{4700}$$

$$\frac{2\int \frac{1}{\sqrt{\sec(a+bx)}}dx}{b}-\frac{2x}{b\sqrt{\sec(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{1}{\sqrt{\csc\left(a+bx+\frac{\pi}{2}\right)}}dx}{b}-\frac{2x}{b\sqrt{\sec(a+bx)}}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \sqrt{\cos(a+bx)}dx}{b}-\frac{2x}{b\sqrt{\sec(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx}{b}-\frac{2x}{b\sqrt{\sec(a+bx)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{4\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}-\frac{2x}{b\sqrt{\sec(a+bx)}}$$

input
```
Int[x*Sqrt[Sec[a + b*x]]*Sin[a + b*x],x]
```

output
```
(-2*x)/(b*Sqrt[Sec[a + b*x]]) + (4*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/
2, 2]*Sqrt[Sec[a + b*x]])/b^2
```

### 3.340.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4700
```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^(p - 1)/(b*n*(p - 1))
), x] - Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Sec[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

## 3.340.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 310, normalized size of antiderivative = 5.85

| method | result |
|---|---|
| risch | $-\frac{(xb+2i)\left(e^{2i(xb+a)}+1\right)\sqrt{2}\sqrt{\frac{e^{i(xb+a)}}{e^{2i(xb+a)}+1}}\,e^{-i(xb+a)}}{b^2}-\frac{2i\left(-\frac{2\left(e^{2i(xb+a)}+1\right)}{\sqrt{\left(e^{2i(xb+a)}+1\right)e^{i(xb+a)}}}+\frac{i\sqrt{-i\left(i+e^{i(xb+a)}\right)}\sqrt{2}\sqrt{i\left(e^{i(xb+a)}-i\right)}\sqrt{ie^{i}}}{}\right.}{}$ |

input
```
int(x*sin(b*x+a)*sec(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(x*b+2*I)*(exp(I*(b*x+a))^2+1)/b^2*2^(1/2)*(exp(I*(b*x+a))/(exp(I*(b*x+a)
)^2+1))^(1/2)/exp(I*(b*x+a))-2*I/b^2*(-2*(exp(I*(b*x+a))^2+1)/((exp(I*(b*x
+a))^2+1)*exp(I*(b*x+a)))^(1/2)+I*(-I*(I+exp(I*(b*x+a))))^(1/2)*2^(1/2)*(I
*(exp(I*(b*x+a))-I))^(1/2)*(I*exp(I*(b*x+a)))^(1/2)/(exp(I*(b*x+a))^3+exp(
I*(b*x+a)))^(1/2)*(-2*I*EllipticE((-I*(I+exp(I*(b*x+a))))^(1/2),1/2*2^(1/2
))+I*EllipticF((-I*(I+exp(I*(b*x+a))))^(1/2),1/2*2^(1/2))))*2^(1/2)*(exp(I
*(b*x+a))/(exp(I*(b*x+a))^2+1))^(1/2)*((exp(I*(b*x+a))^2+1)*exp(I*(b*x+a))
)^(1/2)/exp(I*(b*x+a))
```

### 3.340.5 Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{\sec(a+bx)}\sin(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sin(b*x+a)*sec(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

### 3.340.6 Sympy [F]

$$\int x\sqrt{\sec(a+bx)}\sin(a+bx)\,dx = \int x\sin{(a+bx)}\sqrt{\sec{(a+bx)}}\,dx$$

input
```
integrate(x*sin(b*x+a)*sec(b*x+a)**(1/2),x)
```

output
```
Integral(x*sin(a + b*x)*sqrt(sec(a + b*x)), x)
```

### 3.340.7 Maxima [F]

$$\int x\sqrt{\sec(a+bx)}\sin(a+bx)\,dx = \int x\sqrt{\sec(bx+a)}\sin(bx+a)\,dx$$

input
```
integrate(x*sin(b*x+a)*sec(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*sqrt(sec(b*x + a))*sin(b*x + a), x)
```

### 3.340.8 Giac [F]

$$\int x\sqrt{\sec(a+bx)}\sin(a+bx)\,dx = \int x\sqrt{\sec(bx+a)}\sin(bx+a)\,dx$$

input
```
integrate(x*sin(b*x+a)*sec(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*sqrt(sec(b*x + a))*sin(b*x + a), x)
```

### 3.340.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{\sec(a+bx)}\sin(a+bx)\,dx = \int x\,\sin(a+b\,x)\,\sqrt{\frac{1}{\cos(a+b\,x)}}\,dx$$

input
```
int(x*sin(a + b*x)*(1/cos(a + b*x))^(1/2),x)
```

output
```
int(x*sin(a + b*x)*(1/cos(a + b*x))^(1/2), x)
```

# 3.341 $\int \frac{x\sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx$

3.341.1 Optimal result . . . 2519
3.341.2 Mathematica [A] (verified) . . . 2519
3.341.3 Rubi [A] (verified) . . . 2520
3.341.4 Maple [F] . . . 2522
3.341.5 Fricas [F(-2)] . . . 2522
3.341.6 Sympy [F] . . . 2522
3.341.7 Maxima [F] . . . 2523
3.341.8 Giac [F] . . . 2523
3.341.9 Mupad [F(-1)] . . . 2523

## 3.341.1 Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \frac{x\sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx = -\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)} + \frac{4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{\sec(a+bx)}}{9b^2} + \frac{4\sin(a+bx)}{9b^2\sqrt{\sec(a+bx)}}$$

output
```
-2/3*x/b/sec(b*x+a)^(3/2)+4/9*sin(b*x+a)/b^2/sec(b*x+a)^(1/2)+4/9*(cos(1/2
*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/
2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^2
```

## 3.341.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.79

$$\int \frac{x\sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx = \frac{\sqrt{\sec(a+bx)}\left(-6bx\cos^2(a+bx)+4\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)+2\sin(2(a+bx))\right)}{9b^2}$$

input
```
Integrate[(x*Sin[a + b*x])/Sqrt[Sec[a + b*x]],x]
```

output
```
(Sqrt[Sec[a + b*x]]*(-6*b*x*Cos[a + b*x]^2 + 4*Sqrt[Cos[a + b*x]]*Elliptic
F[(a + b*x)/2, 2] + 2*Sin[2*(a + b*x)]))/(9*b^2)
```

## 3.341.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {4700, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sin(a+bx)}{\sqrt{\sec(a+bx)}}dx$$

$$\Big\downarrow \text{4700}$$

$$\frac{2\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}dx}{3b}-\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{1}{\csc\left(a+bx+\frac{\pi}{2}\right)^{3/2}}dx}{3b}-\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{4256}$$

$$\frac{2\left(\frac{1}{3}\int \sqrt{\sec(a+bx)}dx+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}\right)}{3b}-\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{1}{3}\int \sqrt{\csc\left(a+bx+\frac{\pi}{2}\right)}dx+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}\right)}{3b}-\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2\left(\frac{1}{3}\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int \frac{1}{\sqrt{\cos(a+bx)}}dx+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}\right)}{3b}-\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{1}{3}\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int\frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}\right)}{3b}-\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\left(\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}+\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b}\right)}{3b}-\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}$$

input
```
Int[(x*Sin[a + b*x])/Sqrt[Sec[a + b*x]],x]
```

output
```
(-2*x)/(3*b*Sec[a + b*x]^(3/2)) + (2*((2*Sqrt[Cos[a + b*x]]*EllipticF[(a +
 b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(3*b) + (2*Sin[a + b*x])/(3*b*Sqrt[Sec[a +
 b*x]])))/(3*b)
```

### 3.341.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.341. $\int \frac{x\sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx$

rule 4700
```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^(p - 1)/(b*n*(p - 1))
), x] - Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Sec[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

### 3.341.4 Maple [F]

$$\int \frac{x\sin(xb+a)}{\sqrt{\sec(xb+a)}}dx$$

input
```
int(x*sin(b*x+a)/sec(b*x+a)^(1/2),x)
```

output
```
int(x*sin(b*x+a)/sec(b*x+a)^(1/2),x)
```

### 3.341.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sin(b*x+a)/sec(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

### 3.341.6 Sympy [F]

$$\int \frac{x\sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx = \int \frac{x\sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx$$

input
```
integrate(x*sin(b*x+a)/sec(b*x+a)**(1/2),x)
```

output
```
Integral(x*sin(a + b*x)/sqrt(sec(a + b*x)), x)
```

### 3.341.7 Maxima [F]

$$\int \frac{x\sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx = \int \frac{x\sin(bx+a)}{\sqrt{\sec(bx+a)}}\,dx$$

input
```
integrate(x*sin(b*x+a)/sec(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*sin(b*x + a)/sqrt(sec(b*x + a)), x)
```

### 3.341.8 Giac [F]

$$\int \frac{x\sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx = \int \frac{x\sin(bx+a)}{\sqrt{\sec(bx+a)}}\,dx$$

input
```
integrate(x*sin(b*x+a)/sec(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*sin(b*x + a)/sqrt(sec(b*x + a)), x)
```

### 3.341.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx = \int \frac{x\,\sin(a+b\,x)}{\sqrt{\frac{1}{\cos(a+b\,x)}}}\,dx$$

input
```
int((x*sin(a + b*x))/(1/cos(a + b*x))^(1/2),x)
```

output
```
int((x*sin(a + b*x))/(1/cos(a + b*x))^(1/2), x)
```

# 3.342 $\int \frac{x\sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx$

3.342.1 Optimal result . . . 2524
3.342.2 Mathematica [C] (verified) . . . 2524
3.342.3 Rubi [A] (verified) . . . 2525
3.342.4 Maple [F] . . . 2527
3.342.5 Fricas [F(-2)] . . . 2527
3.342.6 Sympy [F] . . . 2527
3.342.7 Maxima [F] . . . 2528
3.342.8 Giac [F] . . . 2528
3.342.9 Mupad [F(-1)] . . . 2528

## 3.342.1 Optimal result

Integrand size = 18, antiderivative size = 80

$$\int \frac{x\sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)} + \frac{12\sqrt{\cos(a+bx)}E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{25b^2} + \frac{4\sin(a+bx)}{25b^2\sec^{\frac{3}{2}}(a+bx)}$$

output

```
-2/5*x/b/sec(b*x+a)^(5/2)+4/25*sin(b*x+a)/b^2/sec(b*x+a)^(3/2)+12/25*(cos(
1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+1/2*b*x)*EllipticE(sin(1/2*a+1/2*b*x),2^
(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^2
```

## 3.342.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.83 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.21

$$\int \frac{x\sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx = \frac{-5bx-5bx\cos(2(a+bx))+\sec(a+bx)\sin(3(a+bx))+13\tan(a+bx)-6\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2}\right.}{25b^2\sqrt{\sec(a+bx)}}$$

input
```
Integrate[(x*Sin[a + b*x])/Sec[a + b*x]^(3/2),x]
```

output
```
(-5*b*x - 5*b*x*Cos[2*(a + b*x)] + Sec[a + b*x]*Sin[3*(a + b*x)] + 13*Tan[
a + b*x] - 6*Hypergeometric2F1[1/4, 1/2, 3/2, -Tan[a + b*x]^2]*(Sec[a + b*
x]^2)^(1/4)*Tan[a + b*x])/(25*b^2*Sqrt[Sec[a + b*x]])
```

## 3.342.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {4700, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{4700}$$

$$\frac{2\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)}dx}{5b}-\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{1}{\csc\left(a+bx+\frac{\pi}{2}\right)^{5/2}}dx}{5b}-\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{4256}$$

$$\frac{2\left(\frac{3}{5}\int \frac{1}{\sqrt{\sec(a+bx)}}dx+\frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)}\right)}{5b}-\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)}\right)}{5b}-\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2\left(\frac{3}{5}\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int\sqrt{\cos(a+bx)}dx+\frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)}\right)}{5b}-\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{3}{5}\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}dx+\frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)}\right)}{5b}-\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\left(\frac{2\sin(a+bx)}{5b\sec^{\frac{3}{2}}(a+bx)}+\frac{6\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}E\left(\frac{1}{2}(a+bx)|2\right)}{5b}\right)}{5b}-\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}$$

input
```
Int[(x*Sin[a + b*x])/Sec[a + b*x]^(3/2),x]
```

output
```
(-2*x)/(5*b*Sec[a + b*x]^(5/2)) + (2*((6*Sqrt[Cos[a + b*x]]*EllipticE[(a +
 b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(5*b) + (2*Sin[a + b*x])/(5*b*Sec[a + b*x]
^(3/2))))/(5*b)
```

### 3.342.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.342. $\int \frac{x\sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx$

rule 4700
```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^(p - 1)/(b*n*(p - 1))
), x] - Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Sec[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

### 3.342.4 Maple [F]

$$\int \frac{x\sin \left(xb+a\right)}{\sec \left(xb+a\right)^{\frac{3}{2}}}dx$$

input
```
int(x*sin(b*x+a)/sec(b*x+a)^(3/2),x)
```

output
```
int(x*sin(b*x+a)/sec(b*x+a)^(3/2),x)
```

### 3.342.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sin(b*x+a)/sec(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

### 3.342.6 Sympy [F]

$$\int \frac{x\sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\sin\left(a+bx\right)}{\sec^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

input
```
integrate(x*sin(b*x+a)/sec(b*x+a)**(3/2),x)
```

output
```
Integral(x*sin(a + b*x)/sec(a + b*x)**(3/2), x)
```

### 3.342.7 Maxima [F]

$$\int \frac{x\sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\sin(bx+a)}{\sec(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/sec(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x*sin(b*x + a)/sec(b*x + a)^(3/2), x)
```

### 3.342.8 Giac [F]

$$\int \frac{x\sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\sin(bx+a)}{\sec(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/sec(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*sin(b*x + a)/sec(b*x + a)^(3/2), x)
```

### 3.342.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\,\sin(a+b\,x)}{\left(\frac{1}{\cos(a+b\,x)}\right)^{3/2}}\,dx$$

input
```
int((x*sin(a + b*x))/(1/cos(a + b*x))^(3/2),x)
```

output
```
int((x*sin(a + b*x))/(1/cos(a + b*x))^(3/2), x)
```

# 3.343 $\int \frac{x\sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx$

3.343.1 Optimal result . . . 2529
3.343.2 Mathematica [A] (verified) . . . 2529
3.343.3 Rubi [A] (verified) . . . 2530
3.343.4 Maple [F] . . . 2532
3.343.5 Fricas [F(-2)] . . . 2532
3.343.6 Sympy [F] . . . 2533
3.343.7 Maxima [F] . . . 2533
3.343.8 Giac [F] . . . 2533
3.343.9 Mupad [F(-1)] . . . 2534

## 3.343.1 Optimal result

Integrand size = 18, antiderivative size = 103

$$\int \frac{x\sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)} + \frac{20\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)\sqrt{\sec(a+bx)}}{147b^2} + \frac{4\sin(a+bx)}{49b^2\sec^{\frac{5}{2}}(a+bx)} + \frac{20\sin(a+bx)}{147b^2\sqrt{\sec(a+bx)}}$$

output
```
-2/7*x/b/sec(b*x+a)^(7/2)+4/49*sin(b*x+a)/b^2/sec(b*x+a)^(5/2)+20/147*sin(
b*x+a)/b^2/sec(b*x+a)^(1/2)+20/147*(cos(1/2*a+1/2*b*x)^2)^(1/2)/cos(1/2*a+
1/2*b*x)*EllipticF(sin(1/2*a+1/2*b*x),2^(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a)
^(1/2)/b^2
```

## 3.343.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int \frac{x\sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx = \frac{\sqrt{\sec(a+bx)}\Big(-63bx-84bx\cos(2(a+bx))-21bx\cos(4(a+bx))+80\sqrt{\cos(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx\right.}{588b^2}$$

input
```
Integrate[(x*Sin[a + b*x])/Sec[a + b*x]^(5/2),x]
```

output
```
(Sqrt[Sec[a + b*x]]*(-63*b*x - 84*b*x*Cos[2*(a + b*x)] - 21*b*x*Cos[4*(a +
 b*x)] + 80*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 52*Sin[2*(a + b
*x)] + 6*Sin[4*(a + b*x)]))/(588*b^2)
```

## 3.343.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4700, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{4700}$$

$$\frac{2\int \frac{1}{\sec^{\frac{7}{2}}(a+bx)}dx}{7b}-\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{1}{\csc\left(a+bx+\frac{\pi}{2}\right)^{7/2}}dx}{7b}-\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}$$

$$\Big\downarrow \text{4256}$$

$$\frac{2\left(\frac{5}{7}\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}dx+\frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}\right)}{7b}-\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{5}{7}\int \frac{1}{\csc\left(a+bx+\frac{\pi}{2}\right)^{3/2}}dx+\frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}\right)}{7b}-\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}$$

$$\Big\downarrow \text{4256}$$

$$\frac{2\left(\frac{5}{7}\left(\frac{1}{3}\int \sqrt{\sec(a+bx)}dx+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}\right)+\frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}\right)}{7b}-\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{5}{7}\left(\frac{1}{3}\int\sqrt{\csc\left(a+bx+\frac{\pi}{2}\right)}dx+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}\right)+\frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}\right)}{7b}-\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int\frac{1}{\sqrt{\cos(a+bx)}}dx+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}\right)+\frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}\right)}{7b}-\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\int\frac{1}{\sqrt{\sin\left(a+bx+\frac{\pi}{2}\right)}}dx+\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}\right)+\frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}\right)}{7b}-\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\left(\frac{2\sin(a+bx)}{7b\sec^{\frac{5}{2}}(a+bx)}+\frac{5}{7}\left(\frac{2\sin(a+bx)}{3b\sqrt{\sec(a+bx)}}+\frac{2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}(a+bx),2\right)}{3b}\right)\right)}{7b}-\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)}$$

input
```
Int[(x*Sin[a + b*x])/Sec[a + b*x]^(5/2),x]
```

output
```
(-2*x)/(7*b*Sec[a + b*x]^(7/2)) + (2*((2*Sin[a + b*x])/(7*b*Sec[a + b*x]^(
5/2)) + (5*((2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b
*x]])/(3*b) + (2*Sin[a + b*x])/(3*b*Sqrt[Sec[a + b*x]])))/7))/(7*b)
```

### 3.343.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

---

3.343. $\int \frac{x\sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx$

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4700
```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[x^(m - n + 1)*(Sec[a + b*x^n]^(p - 1)/(b*n*(p - 1))
), x] - Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Sec[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

### 3.343.4 Maple [F]

$$\int \frac{x \sin(xb + a)}{\sec(xb + a)^{\frac{5}{2}}} dx$$

input
```
int(x*sin(b*x+a)/sec(b*x+a)^(5/2),x)
```

output
```
int(x*sin(b*x+a)/sec(b*x+a)^(5/2),x)
```

### 3.343.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x \sin(a + bx)}{\sec^{\frac{5}{2}}(a + bx)} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*sin(b*x+a)/sec(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

### 3.343.6 Sympy [F]

$$\int \frac{x\sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\sin\left(a+bx\right)}{\sec^{\frac{5}{2}}\left(a+bx\right)}\,dx$$

input
```
integrate(x*sin(b*x+a)/sec(b*x+a)**(5/2),x)
```

output
```
Integral(x*sin(a + b*x)/sec(a + b*x)**(5/2), x)
```

### 3.343.7 Maxima [F]

$$\int \frac{x\sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\sin\left(bx+a\right)}{\sec\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/sec(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x*sin(b*x + a)/sec(b*x + a)^(5/2), x)
```

### 3.343.8 Giac [F]

$$\int \frac{x\sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\sin\left(bx+a\right)}{\sec\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*sin(b*x+a)/sec(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(x*sin(b*x + a)/sec(b*x + a)^(5/2), x)
```

### 3.343.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\,\sin\left(a+b\,x\right)}{\left(\frac{1}{\cos(a+b\,x)}\right)^{5/2}}\,dx$$

input
```
int((x*sin(a + b*x))/(1/cos(a + b*x))^(5/2),x)
```

output
```
int((x*sin(a + b*x))/(1/cos(a + b*x))^(5/2), x)
```

# 3.344 $\int x\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)\,dx$


3.344.1 Optimal result . . . . . . . . . . 2535
3.344.2 Mathematica [A] (verified) . . . . . . . . . . 2535
3.344.3 Rubi [A] (verified) . . . . . . . . . . 2536
3.344.4 Maple [F] . . . . . . . . . . 2538
3.344.5 Fricas [F(-2)] . . . . . . . . . . 2538
3.344.6 Sympy [F(-1)] . . . . . . . . . . 2538
3.344.7 Maxima [F] . . . . . . . . . . 2539
3.344.8 Giac [F] . . . . . . . . . . 2539
3.344.9 Mupad [F(-1)] . . . . . . . . . . 2539


## 3.344.1 Optimal result

Integrand size = 18, antiderivative size = 88

$$\int x\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)\,dx=-\frac{20\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)}{147b^2}+\frac{20\cos(a+bx)\sqrt{\sin(a+bx)}}{147b^2}+\frac{4\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)}{49b^2}+\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b}$$

output
```
20/147*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Ellip
ticF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2+4/49*cos(b*x+a)*sin(b*x+a)^(5/
2)/b^2+2/7*x*sin(b*x+a)^(7/2)/b+20/147*cos(b*x+a)*sin(b*x+a)^(1/2)/b^2
```

## 3.344.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.76

$$\int x\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)\,dx=\frac{40\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)+\sqrt{\sin(a+bx)}(46\cos(a+bx)-6\cos(3(a+bx))+84bx\sin^3(a+bx}{294b^2}$$

input
```
Integrate[x*Cos[a + b*x]*Sin[a + b*x]^(5/2),x]
```

output
```
(40*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + Sqrt[Sin[a + b*x]]*(46*Cos[a + b
*x] - 6*Cos[3*(a + b*x)] + 84*b*x*Sin[a + b*x]^3))/(294*b^2)
```

### 3.344.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3924, 3042, 3115, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \text{ 3924}$$

$$\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\int\sin^{\frac{7}{2}}(a+bx)dx}{7b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\int\sin(a+bx)^{7/2}dx}{7b}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\left(\frac{5}{7}\int\sin^{\frac{3}{2}}(a+bx)dx-\frac{2\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{7b}\right)}{7b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\left(\frac{5}{7}\int\sin(a+bx)^{3/2}dx-\frac{2\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{7b}\right)}{7b}$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin(a+bx)}}dx-\frac{2\sqrt{\sin(a+bx)}\cos(a+bx)}{3b}\right)-\frac{2\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{7b}\right)}{7b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\sin(a+bx)}}dx-\frac{2\sqrt{\sin(a+bx)}\cos(a+bx)}{3b}\right)-\frac{2\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{7b}\right)}{7b}$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b}-\frac{2\left(\frac{5}{7}\left(\frac{2\,\mathrm{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{3b}-\frac{2\sqrt{\sin(a+bx)}\cos(a+bx)}{3b}\right)-\frac{2\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{7b}\right)}{7b}$$

input
```
Int[x*Cos[a + b*x]*Sin[a + b*x]^(5/2),x]
```

output
```
(2*x*Sin[a + b*x]^(7/2))/(7*b) - (2*((5*((2*EllipticF[(a - Pi/2 + b*x)/2,
2])/(3*b) - (2*Cos[a + b*x]*Sqrt[Sin[a + b*x]])/(3*b)))/7 - (2*Cos[a + b*x
]*Sin[a + b*x]^(5/2))/(7*b)))/(7*b)
```

### 3.344.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

### 3.344.4 Maple [F]

$$\int x\cos(xb+a)\sin(xb+a)^{\frac{5}{2}}\,dx$$

input
```
int(x*cos(b*x+a)*sin(b*x+a)^(5/2),x)
```

output
```
int(x*cos(b*x+a)*sin(b*x+a)^(5/2),x)
```

### 3.344.5 Fricas [F(-2)]

Exception generated.

$$\int x\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)*sin(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

### 3.344.6 Sympy [F(-1)]

Timed out.

$$\int x\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x*cos(b*x+a)*sin(b*x+a)**(5/2),x)
```

output
```
Timed out
```

### 3.344.7 Maxima [F]

$$\int x\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)\,dx=\int x\cos\left(bx+a\right)\sin\left(bx+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x*cos(b*x+a)*sin(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)*sin(b*x + a)^(5/2), x)
```

### 3.344.8 Giac [F]

$$\int x\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)\,dx=\int x\cos\left(bx+a\right)\sin\left(bx+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x*cos(b*x+a)*sin(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)*sin(b*x + a)^(5/2), x)
```

### 3.344.9 Mupad [F(-1)]

Timed out.

$$\int x\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)\,dx=\int x\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^{5/2}\,dx$$

input
```
int(x*cos(a + b*x)*sin(a + b*x)^(5/2),x)
```

output
```
int(x*cos(a + b*x)*sin(a + b*x)^(5/2), x)
```

# 3.345 $\int x \cos(a+bx) \sin^{\frac{3}{2}}(a+bx)\, dx$

3.345.1 Optimal result . . . . . . . . . . 2540
3.345.2 Mathematica [C] (verified) . . . . . . . . . . 2540
3.345.3 Rubi [A] (verified) . . . . . . . . . . 2541
3.345.4 Maple [F] . . . . . . . . . . 2542
3.345.5 Fricas [F(-2)] . . . . . . . . . . 2543
3.345.6 Sympy [F] . . . . . . . . . . 2543
3.345.7 Maxima [F] . . . . . . . . . . 2543
3.345.8 Giac [F] . . . . . . . . . . 2544
3.345.9 Mupad [F(-1)] . . . . . . . . . . 2544

## 3.345.1 Optimal result

Integrand size = 18, antiderivative size = 65

$$\int x \cos(a+bx) \sin^{\frac{3}{2}}(a+bx)\, dx = -\frac{12E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{25b^2} + \frac{4\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)}{25b^2} + \frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}$$

output
```
12/25*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Ellipt
icE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2+4/25*cos(b*x+a)*sin(b*x+a)^(3/2
)/b^2+2/5*x*sin(b*x+a)^(5/2)/b
```

## 3.345.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 2.16 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.66

$$\int x \cos(a+bx) \sin^{\frac{3}{2}}(a+bx)\, dx = \frac{\sqrt{\sin(a+bx)}\left(5bx - 5bx\cos(2(a+bx)) + 2\sin(2(a+bx)) - 12\tan\left(\frac{1}{2}(a+bx)\right) + 4\,\text{Hypergeometric2F1}\right.}{25b^2}$$

input
```
Integrate[x*Cos[a + b*x]*Sin[a + b*x]^(3/2),x]
```

output
```
(Sqrt[Sin[a + b*x]]*(5*b*x - 5*b*x*Cos[2*(a + b*x)] + 2*Sin[2*(a + b*x)] -
 12*Tan[(a + b*x)/2] + 4*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[(a + b*x)/2
]^2]*Sqrt[Sec[(a + b*x)/2]^2]*Tan[(a + b*x)/2]))/(25*b^2)
```

## 3.345.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3924, 3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \sin^{\frac{3}{2}}(a+bx)\cos(a+bx)\,dx$$

$$\Big\downarrow \; 3924$$

$$\frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\int \sin^{\frac{5}{2}}(a+bx)dx}{5b}$$

$$\Big\downarrow \; 3042$$

$$\frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\int \sin(a+bx)^{5/2}dx}{5b}$$

$$\Big\downarrow \; 3115$$

$$\frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\left(\frac{3}{5}\int \sqrt{\sin(a+bx)}dx-\frac{2\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)}{5b}\right)}{5b}$$

$$\Big\downarrow \; 3042$$

$$\frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\left(\frac{3}{5}\int \sqrt{\sin(a+bx)}dx-\frac{2\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)}{5b}\right)}{5b}$$

$$\Big\downarrow \; 3119$$

$$\frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\left(\frac{6E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)|2\right)}{5b}-\frac{2\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)}{5b}\right)}{5b}$$

input
```
Int[x*Cos[a + b*x]*Sin[a + b*x]^(3/2),x]
```

output
```
(2*x*Sin[a + b*x]^(5/2))/(5*b) - (2*((6*EllipticE[(a - Pi/2 + b*x)/2, 2])/
(5*b) - (2*Cos[a + b*x]*Sin[a + b*x]^(3/2))/(5*b)))/(5*b)
```

### 3.345.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

## 3.345.4 Maple [F]

$$\int x\cos\left(xb+a\right)\sin\left(xb+a\right)^{\frac{3}{2}}dx$$

input
```
int(x*cos(b*x+a)*sin(b*x+a)^(3/2),x)
```

output
```
int(x*cos(b*x+a)*sin(b*x+a)^(3/2),x)
```

### 3.345.5 Fricas [F(-2)]

Exception generated.

$$\int x\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)*sin(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

### 3.345.6 Sympy [F]

$$\int x\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)\,dx = \int x\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)\,dx$$

input
```
integrate(x*cos(b*x+a)*sin(b*x+a)**(3/2),x)
```

output
```
Integral(x*sin(a + b*x)**(3/2)*cos(a + b*x), x)
```

### 3.345.7 Maxima [F]

$$\int x\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)\,dx = \int x\cos(bx+a)\sin(bx+a)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*cos(b*x+a)*sin(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)*sin(b*x + a)^(3/2), x)
```

### 3.345.8 Giac [F]

$$\int x\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)\,dx=\int x\cos\left(bx+a\right)\sin\left(bx+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*cos(b*x+a)*sin(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)*sin(b*x + a)^(3/2), x)
```

### 3.345.9 Mupad [F(-1)]

Timed out.

$$\int x\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)\,dx=\int x\,\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^{3/2}\,dx$$

input
```
int(x*cos(a + b*x)*sin(a + b*x)^(3/2),x)
```

output
```
int(x*cos(a + b*x)*sin(a + b*x)^(3/2), x)
```

# 3.346 $\int x\cos(a+bx)\sqrt{\sin(a+bx)}\,dx$

3.346.1 Optimal result . . . 2545
3.346.2 Mathematica [A] (verified) . . . 2545
3.346.3 Rubi [A] (verified) . . . 2546
3.346.4 Maple [F] . . . 2547
3.346.5 Fricas [F(-2)] . . . 2547
3.346.6 Sympy [F] . . . 2548
3.346.7 Maxima [F] . . . 2548
3.346.8 Giac [F] . . . 2548
3.346.9 Mupad [F(-1)] . . . 2549

## 3.346.1 Optimal result

Integrand size = 18, antiderivative size = 65

$$\int x\cos(a+bx)\sqrt{\sin(a+bx)}\,dx=-\frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)}{9b^2}+\frac{4\cos(a+bx)\sqrt{\sin(a+bx)}}{9b^2}+\frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b}$$

output
```
4/9*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Elliptic
F(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2+2/3*x*sin(b*x+a)^(3/2)/b+4/9*cos(
b*x+a)*sin(b*x+a)^(1/2)/b^2
```

## 3.346.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int x\cos(a+bx)\sqrt{\sin(a+bx)}\,dx=\frac{4\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)+2\sqrt{\sin(a+bx)}(2\cos(a+bx)+3bx\sin(a+bx))}{9b^2}$$

input
```
Integrate[x*Cos[a + b*x]*Sqrt[Sin[a + b*x]],x]
```

output
```
(4*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + 2*Sqrt[Sin[a + b*x]]*(2*Cos[a + b
*x] + 3*b*x*Sin[a + b*x]))/(9*b^2)
```

### 3.346.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3924, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\sin(a+bx)}\cos(a+bx)\,dx$$

$$\Big\downarrow \text{3924}$$

$$\frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\int \sin^{\frac{3}{2}}(a+bx)dx}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\int \sin(a+bx)^{3/2}dx}{3b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(a+bx)}}dx-\frac{2\sqrt{\sin(a+bx)}\cos(a+bx)}{3b}\right)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(a+bx)}}dx-\frac{2\sqrt{\sin(a+bx)}\cos(a+bx)}{3b}\right)}{3b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{3b}-\frac{2\sqrt{\sin(a+bx)}\cos(a+bx)}{3b}\right)}{3b}$$

input
```
Int[x*Cos[a + b*x]*Sqrt[Sin[a + b*x]],x]
```

output
```
(-2*((2*EllipticF[(a - Pi/2 + b*x)/2, 2])/(3*b) - (2*Cos[a + b*x]*Sqrt[Sin
[a + b*x]])/(3*b)))/(3*b) + (2*x*Sin[a + b*x]^(3/2))/(3*b)
```

### 3.346.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

## 3.346.4 Maple [F]

$$\int x\cos\left(xb+a\right)\sqrt{\sin\left(xb+a\right)}dx$$

input
```
int(x*cos(b*x+a)*sin(b*x+a)^(1/2),x)
```

output
```
int(x*cos(b*x+a)*sin(b*x+a)^(1/2),x)
```

## 3.346.5 Fricas [F(-2)]

Exception generated.

$$\int x\cos(a+bx)\sqrt{\sin(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)*sin(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

### 3.346.6 Sympy [F]

$$\int x\cos(a+bx)\sqrt{\sin(a+bx)}\,dx=\int x\sqrt{\sin{(a+bx)}}\cos{(a+bx)}\,dx$$

input
```
integrate(x*cos(b*x+a)*sin(b*x+a)**(1/2),x)
```

output
```
Integral(x*sqrt(sin(a + b*x))*cos(a + b*x), x)
```

### 3.346.7 Maxima [F]

$$\int x\cos(a+bx)\sqrt{\sin(a+bx)}\,dx=\int x\cos{(bx+a)}\sqrt{\sin{(bx+a)}}\,dx$$

input
```
integrate(x*cos(b*x+a)*sin(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)*sqrt(sin(b*x + a)), x)
```

### 3.346.8 Giac [F]

$$\int x\cos(a+bx)\sqrt{\sin(a+bx)}\,dx=\int x\cos{(bx+a)}\sqrt{\sin{(bx+a)}}\,dx$$

input
```
integrate(x*cos(b*x+a)*sin(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)*sqrt(sin(b*x + a)), x)
```

### 3.346.9 Mupad [F(-1)]

Timed out.

$$\int x\cos(a+bx)\sqrt{\sin(a+bx)}\,dx = \int x\,\cos\left(a+b\,x\right)\,\sqrt{\sin\left(a+b\,x\right)}\,dx$$

input
```
int(x*cos(a + b*x)*sin(a + b*x)^(1/2),x)
```

output
```
int(x*cos(a + b*x)*sin(a + b*x)^(1/2), x)
```

# 3.347 $\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx$

3.347.1 Optimal result . . . 2550
3.347.2 Mathematica [C] (verified) . . . 2550
3.347.3 Rubi [A] (verified) . . . 2551
3.347.4 Maple [C] (verified) . . . 2552
3.347.5 Fricas [F(-2)] . . . 2552
3.347.6 Sympy [F] . . . 2553
3.347.7 Maxima [F] . . . 2553
3.347.8 Giac [F] . . . 2553
3.347.9 Mupad [F(-1)] . . . 2554

## 3.347.1 Optimal result

Integrand size = 18, antiderivative size = 38

$$\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx = -\frac{4E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{b^2}+\frac{2x\sqrt{\sin(a+bx)}}{b}$$

output
```
4*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(
cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2+2*x*sin(b*x+a)^(1/2)/b
```

## 3.347.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 6.57 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.26

$$\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx = \frac{2\sqrt{\sin(a+bx)}\left(3bx-6\tan\left(\frac{1}{2}(a+bx)\right)+2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)\sqrt{\sec^2\left(\frac{1}{2}(a\right.}\right.}{3b^2}$$

input
```
Integrate[(x*Cos[a + b*x])/Sqrt[Sin[a + b*x]],x]
```

output
```
(2*Sqrt[Sin[a + b*x]]*(3*b*x - 6*Tan[(a + b*x)/2] + 2*Hypergeometric2F1[1/
2, 3/4, 7/4, -Tan[(a + b*x)/2]^2]*Sqrt[Sec[(a + b*x)/2]^2]*Tan[(a + b*x)/2
]))/(3*b^2)
```

### 3.347.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3924, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}dx$$

$$\Big\downarrow \text{3924}$$

$$\frac{2x\sqrt{\sin(a+bx)}}{b}-\frac{2\int \sqrt{\sin(a+bx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x\sqrt{\sin(a+bx)}}{b}-\frac{2\int \sqrt{\sin(a+bx)}dx}{b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2x\sqrt{\sin(a+bx)}}{b}-\frac{4E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}$$

input
```
Int[(x*Cos[a + b*x])/Sqrt[Sin[a + b*x]],x]
```

output
```
(-4*EllipticE[(a - Pi/2 + b*x)/2, 2])/b^2 + (2*x*Sqrt[Sin[a + b*x]])/b
```

#### 3.347.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

### 3.347.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 308, normalized size of antiderivative = 8.11

| method | result |
|---|---|
| risch | $-\frac{i(xb+2i)\left(e^{2i(xb+a)}-1\right)\sqrt{2}\,e^{-i(xb+a)}}{b^2\sqrt{-i\left(e^{2i(xb+a)}-1\right)e^{-i(xb+a)}}} - \frac{2\left(\frac{2i\left(i-ie^{2i(xb+a)}\right)}{\sqrt{e^{i(xb+a)}\left(i-ie^{2i(xb+a)}\right)}} - \frac{\sqrt{e^{i(xb+a)}+1}\,\sqrt{-2\,e^{i(xb+a)}+2}\,\sqrt{-e^{i(xb+a)}}\left(-2\,\text{EllipticE}\left(\sqrt{e\ldots}\right.\right.}{\sqrt{-ie^{3i(xb+a)}+ie^{i(\ldots}}}\right.}{b^2\sqrt{-i\left(e^{2i(xb+\ldots}\right.}}$ |

input
```
int(x*cos(b*x+a)/sin(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*(x*b+2*I)*(exp(I*(b*x+a))^2-1)/b^2*2^(1/2)/(-I*(exp(I*(b*x+a))^2-1)/exp
(I*(b*x+a)))^(1/2)/exp(I*(b*x+a))-2/b^2*(2*I*(I-I*exp(I*(b*x+a))^2)/(exp(I
*(b*x+a))*(I-I*exp(I*(b*x+a))^2))^(1/2)-(exp(I*(b*x+a))+1)^(1/2)*(-2*exp(I
*(b*x+a))+2)^(1/2)*(-exp(I*(b*x+a)))^(1/2)/(-I*exp(I*(b*x+a))^3+I*exp(I*(b
*x+a)))^(1/2)*(-2*EllipticE((exp(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))+Elliptic
F((exp(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))))*2^(1/2)/(-I*(exp(I*(b*x+a))^2-1)
/exp(I*(b*x+a)))^(1/2)*(-I*(exp(I*(b*x+a))^2-1)*exp(I*(b*x+a)))^(1/2)/exp(
I*(b*x+a))
```

### 3.347.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

### 3.347.6 Sympy [F]

$$\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx = \int \frac{x\cos\left(a+bx\right)}{\sqrt{\sin\left(a+bx\right)}}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)**(1/2),x)
```

output
```
Integral(x*cos(a + b*x)/sqrt(sin(a + b*x)), x)
```

### 3.347.7 Maxima [F]

$$\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx = \int \frac{x\cos\left(bx+a\right)}{\sqrt{\sin\left(bx+a\right)}}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)/sqrt(sin(b*x + a)), x)
```

### 3.347.8 Giac [F]

$$\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx = \int \frac{x\cos\left(bx+a\right)}{\sqrt{\sin\left(bx+a\right)}}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)/sqrt(sin(b*x + a)), x)
```

### 3.347.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x \cos(a+bx)}{\sqrt{\sin(a+bx)}} \, dx = \int \frac{x \cos(a+b\,x)}{\sqrt{\sin(a+b\,x)}} \, dx$$

input
```
int((x*cos(a + b*x))/sin(a + b*x)^(1/2),x)
```

output
```
int((x*cos(a + b*x))/sin(a + b*x)^(1/2), x)
```

# 3.348 $\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx$

3.348.1 Optimal result . . . 2555
3.348.2 Mathematica [A] (verified) . . . 2555
3.348.3 Rubi [A] (verified) . . . 2556
3.348.4 Maple [F] . . . 2557
3.348.5 Fricas [F(-2)] . . . 2557
3.348.6 Sympy [F] . . . 2557
3.348.7 Maxima [F] . . . 2558
3.348.8 Giac [F] . . . 2558
3.348.9 Mupad [F(-1)] . . . 2558

## 3.348.1 Optimal result

Integrand size = 18, antiderivative size = 38

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)}{b^2}-\frac{2x}{b\sqrt{\sin(a+bx)}}$$

output
```
-4*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF
(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2-2*x/b/sin(b*x+a)^(1/2)
```

## 3.348.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\left(-2\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)-\frac{bx}{\sqrt{\sin(a+bx)}}\right)}{b^2}$$

input
```
Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(3/2),x]
```

output
```
(2*(-2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] - (b*x)/Sqrt[Sin[a + b*x]]))/b^
2
```

## 3.348.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3924, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3924}$$

$$\frac{2\int \frac{1}{\sqrt{\sin(a+bx)}}dx}{b}-\frac{2x}{b\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{1}{\sqrt{\sin(a+bx)}}dx}{b}-\frac{2x}{b\sqrt{\sin(a+bx)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{b^2}-\frac{2x}{b\sqrt{\sin(a+bx)}}$$

input
```
Int[(x*Cos[a + b*x])/Sin[a + b*x]^(3/2),x]
```

output
```
(4*EllipticF[(a - Pi/2 + b*x)/2, 2])/b^2 - (2*x)/(b*Sqrt[Sin[a + b*x]])
```

### 3.348.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

### 3.348.4 Maple [F]

$$\int \frac{x\cos\left(xb+a\right)}{\sin\left(xb+a\right)^{\frac{3}{2}}}dx$$

input
```
int(x*cos(b*x+a)/sin(b*x+a)^(3/2),x)
```

output
```
int(x*cos(b*x+a)/sin(b*x+a)^(3/2),x)
```

### 3.348.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.348.6 Sympy [F]

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\cos\left(a+bx\right)}{\sin^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)**(3/2),x)
```

output
```
Integral(x*cos(a + b*x)/sin(a + b*x)**(3/2), x)
```

### 3.348.7 Maxima [F]

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)/sin(b*x + a)^(3/2), x)
```

### 3.348.8 Giac [F]

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)/sin(b*x + a)^(3/2), x)
```

### 3.348.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\,\cos(a+b\,x)}{\sin(a+b\,x)^{3/2}}\,dx$$

input
```
int((x*cos(a + b*x))/sin(a + b*x)^(3/2),x)
```

output
```
int((x*cos(a + b*x))/sin(a + b*x)^(3/2), x)
```

# 3.349 $\int \frac{x \cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx$

3.349.1 Optimal result . . . 2559
3.349.2 Mathematica [A] (verified) . . . 2559
3.349.3 Rubi [A] (verified) . . . 2560
3.349.4 Maple [F] . . . 2561
3.349.5 Fricas [F(-2)] . . . 2561
3.349.6 Sympy [F] . . . 2562
3.349.7 Maxima [F] . . . 2562
3.349.8 Giac [F] . . . 2562
3.349.9 Mupad [F(-1)] . . . 2563

## 3.349.1 Optimal result

Integrand size = 18, antiderivative size = 65

$$\int \frac{x \cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = -\frac{4E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{3b^2} - \frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)} - \frac{4\cos(a+bx)}{3b^2\sqrt{\sin(a+bx)}}$$

output
```
4/3*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Elliptic
E(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2-2/3*x/b/sin(b*x+a)^(3/2)-4/3*cos(
b*x+a)/b^2/sin(b*x+a)^(1/2)
```

## 3.349.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \frac{x \cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2\left(bx-2E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sin^{\frac{3}{2}}(a+bx)+\sin(2(a+bx))\right)}{3b^2\sin^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(5/2),x]
```

output
```
(-2*(b*x - 2*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(3/2) + Sin[
2*(a + b*x)]))/(3*b^2*Sin[a + b*x]^(3/2))
```

### 3.349.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3924, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3924}$$

$$\frac{2\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}dx}{3b}-\frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{1}{\sin(a+bx)^{3/2}}dx}{3b}-\frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{2\left(-\int \sqrt{\sin(a+bx)}dx-\frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{3b}-\frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(-\int \sqrt{\sin(a+bx)}dx-\frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{3b}-\frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\left(-\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)|2\right)}{b}-\frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}\right)}{3b}-\frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}$$

input
```
Int[(x*Cos[a + b*x])/Sin[a + b*x]^(5/2),x]
```

output
```
(2*((-2*EllipticE[(a - Pi/2 + b*x)/2, 2])/b - (2*Cos[a + b*x])/(b*Sqrt[Sin
[a + b*x]])))/(3*b) - (2*x)/(3*b*Sin[a + b*x]^(3/2))
```

### 3.349.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

## 3.349.4 Maple [F]

$$\int \frac{x \cos \left(xb+a\right)}{\sin \left(xb+a\right)^{\frac{5}{2}}}dx$$

input
```
int(x*cos(b*x+a)/sin(b*x+a)^(5/2),x)
```

output
```
int(x*cos(b*x+a)/sin(b*x+a)^(5/2),x)
```

## 3.349.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x \cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

### 3.349.6 Sympy [F]

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\cos\left(a+bx\right)}{\sin^{\frac{5}{2}}\left(a+bx\right)}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)**(5/2),x)
```

output
```
Integral(x*cos(a + b*x)/sin(a + b*x)**(5/2), x)
```

### 3.349.7 Maxima [F]

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\cos\left(bx+a\right)}{\sin\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)/sin(b*x + a)^(5/2), x)
```

### 3.349.8 Giac [F]

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\cos\left(bx+a\right)}{\sin\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)/sin(b*x + a)^(5/2), x)
```

### 3.349.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\,\cos\left(a+b\,x\right)}{\sin\left(a+b\,x\right)^{5/2}}\,dx$$

input
```
int((x*cos(a + b*x))/sin(a + b*x)^(5/2),x)
```

output
```
int((x*cos(a + b*x))/sin(a + b*x)^(5/2), x)
```

# 3.350 $\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx$

3.350.1 Optimal result . . . 2564
3.350.2 Mathematica [A] (verified) . . . 2564
3.350.3 Rubi [A] (verified) . . . 2565
3.350.4 Maple [F] . . . 2566
3.350.5 Fricas [F(-2)] . . . 2566
3.350.6 Sympy [F] . . . 2567
3.350.7 Maxima [F] . . . 2567
3.350.8 Giac [F] . . . 2567
3.350.9 Mupad [F(-1)] . . . 2568

## 3.350.1 Optimal result

Integrand size = 18, antiderivative size = 65

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx=\frac{4\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)}{15b^2}-\frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)}-\frac{4\cos(a+bx)}{15b^2\sin^{\frac{3}{2}}(a+bx)}$$

output
```
-4/15*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Ellipt
icF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2-2/5*x/b/sin(b*x+a)^(5/2)-4/15*c
os(b*x+a)/b^2/sin(b*x+a)^(3/2)
```

## 3.350.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx=-\frac{2\left(3bx+2\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)\sin^{\frac{5}{2}}(a+bx)+\sin(2(a+bx))\right)}{15b^2\sin^{\frac{5}{2}}(a+bx)}$$

input
```
Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(7/2),x]
```

output
```
(-2*(3*b*x + 2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(5/2) + Si
n[2*(a + b*x)]))/(15*b^2*Sin[a + b*x]^(5/2))
```

### 3.350.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3924, 3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{3924}$$

$$\frac{2\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}dx}{5b}-\frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{1}{\sin(a+bx)^{5/2}}dx}{5b}-\frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3116}$$

$$\frac{2\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(a+bx)}}dx-\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}\right)}{5b}-\frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{1}{3}\int \frac{1}{\sqrt{\sin(a+bx)}}dx-\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}\right)}{5b}-\frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\left(\frac{2\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{3b}-\frac{2\cos(a+bx)}{3b\sin^{\frac{3}{2}}(a+bx)}\right)}{5b}-\frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)}$$

input
```
Int[(x*Cos[a + b*x])/Sin[a + b*x]^(7/2),x]
```

output
```
(2*((2*EllipticF[(a - Pi/2 + b*x)/2, 2])/(3*b) - (2*Cos[a + b*x])/(3*b*Sin
[a + b*x]^(3/2))))/(5*b) - (2*x)/(5*b*Sin[a + b*x]^(5/2))
```

### 3.350.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

## 3.350.4 Maple [F]

$$\int \frac{x \cos (xb+a)}{\sin (xb+a)^{\frac{7}{2}}}dx$$

input
```
int(x*cos(b*x+a)/sin(b*x+a)^(7/2),x)
```

output
```
int(x*cos(b*x+a)/sin(b*x+a)^(7/2),x)
```

## 3.350.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

### 3.350.6 Sympy [F]

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{x\cos{(a+bx)}}{\sin^{\frac{7}{2}}{(a+bx)}}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)**(7/2),x)
```

output
```
Integral(x*cos(a + b*x)/sin(a + b*x)**(7/2), x)
```

### 3.350.7 Maxima [F]

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{x\cos{(bx+a)}}{\sin{(bx+a)}^{\frac{7}{2}}}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)/sin(b*x + a)^(7/2), x)
```

### 3.350.8 Giac [F]

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{x\cos{(bx+a)}}{\sin{(bx+a)}^{\frac{7}{2}}}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)/sin(b*x + a)^(7/2), x)
```

### 3.350.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{x\ \cos\left(a+b\,x\right)}{{\sin\left(a+b\,x\right)}^{7/2}}\,dx$$

input
```
int((x*cos(a + b*x))/sin(a + b*x)^(7/2),x)
```

output
```
int((x*cos(a + b*x))/sin(a + b*x)^(7/2), x)
```

# 3.351 $\int \frac{x\cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx$

3.351.1 Optimal result . . . 2569
3.351.2 Mathematica [A] (verified) . . . 2569
3.351.3 Rubi [A] (verified) . . . 2570
3.351.4 Maple [F] . . . 2572
3.351.5 Fricas [F(-2)] . . . 2572
3.351.6 Sympy [F(-1)] . . . 2572
3.351.7 Maxima [F] . . . 2573
3.351.8 Giac [F] . . . 2573
3.351.9 Mupad [F(-1)] . . . 2573

## 3.351.1 Optimal result

Integrand size = 18, antiderivative size = 88

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx = -\frac{12E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{35b^2}-\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}-\frac{4\cos(a+bx)}{35b^2\sin^{\frac{5}{2}}(a+bx)}-\frac{12\cos(a+bx)}{35b^2\sqrt{\sin(a+bx)}}$$

output
```
12/35*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Ellipt
icE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2-2/7*x/b/sin(b*x+a)^(7/2)-4/35*c
os(b*x+a)/b^2/sin(b*x+a)^(5/2)-12/35*cos(b*x+a)/b^2/sin(b*x+a)^(1/2)
```

## 3.351.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.83

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx = -\frac{2\left(5bx+6\cos(a+bx)\sin^3(a+bx)-6E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sin^{\frac{7}{2}}(a+bx)+\sin(2(a+bx))\right)}{35b^2\sin^{\frac{7}{2}}(a+bx)}$$

input
```
Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(9/2),x]
```

output
```
(-2*(5*b*x + 6*Cos[a + b*x]*Sin[a + b*x]^3 - 6*EllipticE[(-2*a + Pi - 2*b*
x)/4, 2]*Sin[a + b*x]^(7/2) + Sin[2*(a + b*x)]))/(35*b^2*Sin[a + b*x]^(7/2
))
```

## 3.351.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3924, 3042, 3116, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx$$

$$\Big\downarrow \text{ 3924}$$

$$\frac{2\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}dx}{7b}-\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\int \frac{1}{\sin(a+bx)^{7/2}}dx}{7b}-\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}$$

$$\Big\downarrow \text{ 3116}$$

$$\frac{2\left(\frac{3}{5}\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}dx-\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}\right)}{7b}-\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\left(\frac{3}{5}\int \frac{1}{\sin(a+bx)^{3/2}}dx-\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}\right)}{7b}-\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}$$

$$\Big\downarrow \text{ 3116}$$

$$\frac{2\left(\frac{3}{5}\left(-\int \sqrt{\sin(a+bx)}dx-\frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}\right)-\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}\right)}{7b}-\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\left(\frac{3}{5}\left(-\int\sqrt{\sin(a+bx)}dx-\frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}\right)-\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}\right)}{7b}-\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\left(\frac{3}{5}\left(-\frac{2E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)|2\right)}{b}-\frac{2\cos(a+bx)}{b\sqrt{\sin(a+bx)}}\right)-\frac{2\cos(a+bx)}{5b\sin^{\frac{5}{2}}(a+bx)}\right)}{7b}-\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}$$

input
```
Int[(x*Cos[a + b*x])/Sin[a + b*x]^(9/2),x]
```

output
```
(2*((3*((-2*EllipticE[(a - Pi/2 + b*x)/2, 2])/b - (2*Cos[a + b*x])/(b*Sqrt
[Sin[a + b*x]])))/5 - (2*Cos[a + b*x])/(5*b*Sin[a + b*x]^(5/2))))/(7*b) -
(2*x)/(7*b*Sin[a + b*x]^(7/2))
```

### 3.351.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3924
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)
)), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sin[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

3.351. $\int \frac{x\cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx$

### 3.351.4 Maple [F]

$$\int \frac{x \cos (xb + a)}{\sin (xb + a)^{\frac{9}{2}}} dx$$

input
```
int(x*cos(b*x+a)/sin(b*x+a)^(9/2),x)
```

output
```
int(x*cos(b*x+a)/sin(b*x+a)^(9/2),x)
```

### 3.351.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x \cos(a + bx)}{\sin^{\frac{9}{2}}(a + bx)} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(9/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.351.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x \cos(a + bx)}{\sin^{\frac{9}{2}}(a + bx)} dx = \text{Timed out}$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)**(9/2),x)
```

output
```
Timed out
```

### 3.351.7 Maxima [F]

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx = \int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)/sin(b*x + a)^(9/2), x)
```

### 3.351.8 Giac [F]

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx = \int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{9}{2}}}\,dx$$

input
```
integrate(x*cos(b*x+a)/sin(b*x+a)^(9/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)/sin(b*x + a)^(9/2), x)
```

### 3.351.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx = \int \frac{x\,\cos(a+b\,x)}{\sin(a+b\,x)^{9/2}}\,dx$$

input
```
int((x*cos(a + b*x))/sin(a + b*x)^(9/2),x)
```

output
```
int((x*cos(a + b*x))/sin(a + b*x)^(9/2), x)
```

# 3.352 $\int x\cos(a+bx)\csc^{\frac{9}{2}}(a+bx)\,dx$


3.352.1 Optimal result . . . . . 2574
3.352.2 Mathematica [A] (verified) . . . . . 2574
3.352.3 Rubi [A] (verified) . . . . . 2575
3.352.4 Maple [F] . . . . . 2577
3.352.5 Fricas [F(-2)] . . . . . 2577
3.352.6 Sympy [F(-1)] . . . . . 2578
3.352.7 Maxima [F] . . . . . 2578
3.352.8 Giac [F] . . . . . 2578
3.352.9 Mupad [F(-1)] . . . . . 2579


## 3.352.1 Optimal result

Integrand size = 18, antiderivative size = 108

$$\int x\cos(a+bx)\csc^{\frac{9}{2}}(a+bx)\,dx=-\frac{12\cos(a+bx)\sqrt{\csc(a+bx)}}{35b^2}-\frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}-\frac{12\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{35b^2}$$

output
```
-4/35*cos(b*x+a)*csc(b*x+a)^(5/2)/b^2-2/7*x*csc(b*x+a)^(7/2)/b-12/35*cos(b
*x+a)*csc(b*x+a)^(1/2)/b^2+12/35*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1
/2*a+1/4*Pi+1/2*b*x)*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+
a)^(1/2)*sin(b*x+a)^(1/2)/b^2
```

## 3.352.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.68

$$\int x\cos(a+bx)\csc^{\frac{9}{2}}(a+bx)\,dx=-\frac{2\csc^{\frac{7}{2}}(a+bx)\left(5bx+6\cos(a+bx)\sin^3(a+bx)-6E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sin^{\frac{7}{2}}(a+bx)+\sin(2(a+\right.}{35b^2}$$

input
```
Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(9/2),x]
```

output
```
(-2*Csc[a + b*x]^(7/2)*(5*b*x + 6*Cos[a + b*x]*Sin[a + b*x]^3 - 6*Elliptic
E[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(7/2) + Sin[2*(a + b*x)]))/(35*b^
2)
```

## 3.352.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4701, 3042, 4255, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos(a+bx)\csc^{\frac{9}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{4701}$$

$$\frac{2\int \csc^{\frac{7}{2}}(a+bx)dx}{7b}-\frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \csc(a+bx)^{7/2}dx}{7b}-\frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{2\left(\frac{3}{5}\int \csc^{\frac{3}{2}}(a+bx)dx-\frac{2\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b}-\frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{3}{5}\int \csc(a+bx)^{3/2}dx-\frac{2\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b}-\frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{2\left(\frac{3}{5}\left(-\int \frac{1}{\sqrt{\csc(a+bx)}}dx-\frac{2\cos(a+bx)\sqrt{\csc(a+bx)}}{b}\right)-\frac{2\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b}-\frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{3}{5}\left(-\int \frac{1}{\sqrt{\csc(a+bx)}}dx-\frac{2\cos(a+bx)\sqrt{\csc(a+bx)}}{b}\right)-\frac{2\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b}-\frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2\left(\frac{3}{5}\left(-\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \sqrt{\sin(a+bx)}dx-\frac{2\cos(a+bx)\sqrt{\csc(a+bx)}}{b}\right)-\frac{2\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b}-\frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{3}{5}\left(-\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \sqrt{\sin(a+bx)}dx-\frac{2\cos(a+bx)\sqrt{\csc(a+bx)}}{b}\right)-\frac{2\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b}-\frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\left(\frac{3}{5}\left(-\frac{2\cos(a+bx)\sqrt{\csc(a+bx)}}{b}-\frac{2\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)|2\right)}{b}\right)-\frac{2\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{5b}\right)}{7b}-\frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}$$

input
```
Int[x*Cos[a + b*x]*Csc[a + b*x]^(9/2),x]
```

output
```
(-2*x*Csc[a + b*x]^(7/2))/(7*b) + (2*((-2*Cos[a + b*x]*Csc[a + b*x]^(5/2))
/(5*b) + (3*((-2*Cos[a + b*x]*Sqrt[Csc[a + b*x]])/b - (2*Sqrt[Csc[a + b*x]
]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/b))/5))/(7*b)
```

### 3.352.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

3.352. $\int x\cos(a+bx)\csc^{\frac{9}{2}}(a+bx)\,dx$

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4701
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Csc[a + b*x^n]^(p
 - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ
[p, 1]
```

## 3.352.4 Maple [F]

$$\int x\cos(xb+a)\csc(xb+a)^{\frac{9}{2}}\,dx$$

input
```
int(x*cos(b*x+a)*csc(b*x+a)^(9/2),x)
```

output
```
int(x*cos(b*x+a)*csc(b*x+a)^(9/2),x)
```

## 3.352.5 Fricas [F(-2)]

Exception generated.

$$\int x\cos(a+bx)\csc^{\frac{9}{2}}(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(9/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.352.6 Sympy [F(-1)]

Timed out.

$$\int x\cos(a+bx)\csc^{\frac{9}{2}}(a+bx)\,dx=\text{Timed out}$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)**(9/2),x)
```

output
```
Timed out
```

### 3.352.7 Maxima [F]

$$\int x\cos(a+bx)\csc^{\frac{9}{2}}(a+bx)\,dx=\int x\cos(bx+a)\csc(bx+a)^{\frac{9}{2}}\,dx$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(9/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)*csc(b*x + a)^(9/2), x)
```

### 3.352.8 Giac [F]

$$\int x\cos(a+bx)\csc^{\frac{9}{2}}(a+bx)\,dx=\int x\cos(bx+a)\csc(bx+a)^{\frac{9}{2}}\,dx$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(9/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)*csc(b*x + a)^(9/2), x)
```

### 3.352.9 Mupad [F(-1)]

Timed out.

$$\int x\cos(a+bx)\csc^{\frac{9}{2}}(a+bx)\,dx=\int x\,\cos\left(a+b\,x\right)\,\left(\frac{1}{\sin\left(a+b\,x\right)}\right)^{9/2}\,dx$$

input
```
int(x*cos(a + b*x)*(1/sin(a + b*x))^(9/2),x)
```

output
```
int(x*cos(a + b*x)*(1/sin(a + b*x))^(9/2), x)
```

# 3.353 $\int x\cos(a+bx)\csc^{\frac{7}{2}}(a+bx)\,dx$


3.353.1 Optimal result . . . . . . . . . . 2580
3.353.2 Mathematica [A] (verified) . . . . . . . . . . 2580
3.353.3 Rubi [A] (verified) . . . . . . . . . . 2581
3.353.4 Maple [F] . . . . . . . . . . 2583
3.353.5 Fricas [F(-2)] . . . . . . . . . . 2583
3.353.6 Sympy [F(-1)] . . . . . . . . . . 2583
3.353.7 Maxima [F] . . . . . . . . . . 2584
3.353.8 Giac [F] . . . . . . . . . . 2584
3.353.9 Mupad [F(-1)] . . . . . . . . . . 2584


## 3.353.1 Optimal result

Integrand size = 18, antiderivative size = 85

$$\int x\cos(a+bx)\csc^{\frac{7}{2}}(a+bx)\,dx = -\frac{4\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}+\frac{4\sqrt{\csc(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)\sqrt{\sin(a+bx)}}{15b^2}$$

output
```
-4/15*cos(b*x+a)*csc(b*x+a)^(3/2)/b^2-2/5*x*csc(b*x+a)^(5/2)/b-4/15*(sin(1
/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(cos(1/2*
a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b^2
```

## 3.353.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int x\cos(a+bx)\csc^{\frac{7}{2}}(a+bx)\,dx = -\frac{2\sqrt{\csc(a+bx)}\left(2\cot(a+bx)+3bx\csc^2(a+bx)+2\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)\sqrt{\sin(a+bx)}\right)}{15b^2}$$

input
```
Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(7/2),x]
```

output
```
(-2*Sqrt[Csc[a + b*x]]*(2*Cot[a + b*x] + 3*b*x*Csc[a + b*x]^2 + 2*Elliptic
F[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]]))/(15*b^2)
```

### 3.353.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {4701, 3042, 4255, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos(a+bx)\csc^{\frac{7}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{4701}$$

$$\frac{2\int \csc^{\frac{5}{2}}(a+bx)dx}{5b}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \csc(a+bx)^{5/2}dx}{5b}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{2\left(\frac{1}{3}\int \sqrt{\csc(a+bx)}dx-\frac{2\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{3b}\right)}{5b}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{1}{3}\int \sqrt{\csc(a+bx)}dx-\frac{2\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{3b}\right)}{5b}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2\left(\frac{1}{3}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}dx-\frac{2\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{3b}\right)}{5b}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{1}{3}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}dx-\frac{2\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{3b}\right)}{5b}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\left(\frac{2\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{3b}-\frac{2\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{3b}\right)}{5b}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}$$

input
```
Int[x*Cos[a + b*x]*Csc[a + b*x]^(7/2),x]
```

output
```
(-2*x*Csc[a + b*x]^(5/2))/(5*b) + (2*((-2*Cos[a + b*x]*Csc[a + b*x]^(3/2))
/(3*b) + (2*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a
 + b*x]])/(3*b)))/(5*b)
```

### 3.353.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4701
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Csc[a + b*x^n]^(p
 - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ
[p, 1]
```

### 3.353.4 Maple [F]

$$\int x\cos(xb+a)\csc(xb+a)^{\frac{7}{2}}dx$$

input
```
int(x*cos(b*x+a)*csc(b*x+a)^(7/2),x)
```

output
```
int(x*cos(b*x+a)*csc(b*x+a)^(7/2),x)
```

### 3.353.5 Fricas [F(-2)]

Exception generated.

$$\int x\cos(a+bx)\csc^{\frac{7}{2}}(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.353.6 Sympy [F(-1)]

Timed out.

$$\int x\cos(a+bx)\csc^{\frac{7}{2}}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)**(7/2),x)
```

output
```
Timed out
```

### 3.353.7 Maxima [F]

$$\int x\cos(a+bx)\csc^{\frac{7}{2}}(a+bx)\,dx=\int x\cos\left(bx+a\right)\csc\left(bx+a\right)^{\frac{7}{2}}\,dx$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)*csc(b*x + a)^(7/2), x)
```

### 3.353.8 Giac [F]

$$\int x\cos(a+bx)\csc^{\frac{7}{2}}(a+bx)\,dx=\int x\cos\left(bx+a\right)\csc\left(bx+a\right)^{\frac{7}{2}}\,dx$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)*csc(b*x + a)^(7/2), x)
```

### 3.353.9 Mupad [F(-1)]

Timed out.

$$\int x\cos(a+bx)\csc^{\frac{7}{2}}(a+bx)\,dx=\int x\,\cos\left(a+b\,x\right)\,{\left(\frac{1}{\sin\left(a+b\,x\right)}\right)}^{7/2}\,dx$$

input
```
int(x*cos(a + b*x)*(1/sin(a + b*x))^(7/2),x)
```

output
```
int(x*cos(a + b*x)*(1/sin(a + b*x))^(7/2), x)
```

# 3.354 $\int x\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)\,dx$

3.354.1 Optimal result . . . . . 2585
3.354.2 Mathematica [A] (verified) . . . . . 2585
3.354.3 Rubi [A] (verified) . . . . . 2586
3.354.4 Maple [F] . . . . . 2587
3.354.5 Fricas [F(-2)] . . . . . 2588
3.354.6 Sympy [F(-1)] . . . . . 2588
3.354.7 Maxima [F] . . . . . 2588
3.354.8 Giac [F] . . . . . 2589
3.354.9 Mupad [F(-1)] . . . . . 2589

## 3.354.1 Optimal result

Integrand size = 18, antiderivative size = 85

$$\int x\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)\,dx=-\frac{4\cos(a+bx)\sqrt{\csc(a+bx)}}{3b^2}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}-\frac{4\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{3b^2}$$

output
```
-2/3*x*csc(b*x+a)^(3/2)/b-4/3*cos(b*x+a)*csc(b*x+a)^(1/2)/b^2+4/3*(sin(1/2
*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(cos(1/2*a+
1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b^2
```

## 3.354.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.66

$$\int x\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)\,dx=-\frac{2\csc^{\frac{3}{2}}(a+bx)\left(bx-2E\left(\frac{1}{4}(-2a+\pi-2bx)\middle|2\right)\sin^{\frac{3}{2}}(a+bx)+\sin(2(a+bx))\right)}{3b^2}$$

input
```
Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(5/2),x]
```

output
```
(-2*Csc[a + b*x]^(3/2)*(b*x - 2*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a
+ b*x]^(3/2) + Sin[2*(a + b*x)]))/(3*b^2)
```

### 3.354.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {4701, 3042, 4255, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{4701}$$

$$\frac{2\int \csc^{\frac{3}{2}}(a+bx)dx}{3b}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \csc(a+bx)^{3/2}dx}{3b}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}$$

$$\Big\downarrow \text{4255}$$

$$\frac{2\left(-\int \frac{1}{\sqrt{\csc(a+bx)}}dx-\frac{2\cos(a+bx)\sqrt{\csc(a+bx)}}{b}\right)}{3b}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(-\int \frac{1}{\sqrt{\csc(a+bx)}}dx-\frac{2\cos(a+bx)\sqrt{\csc(a+bx)}}{b}\right)}{3b}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2\left(-\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \sqrt{\sin(a+bx)}dx-\frac{2\cos(a+bx)\sqrt{\csc(a+bx)}}{b}\right)}{3b}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(-\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \sqrt{\sin(a+bx)}dx-\frac{2\cos(a+bx)\sqrt{\csc(a+bx)}}{b}\right)}{3b}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\left(-\frac{2\cos(a+bx)\sqrt{\csc(a+bx)}}{b}-\frac{2\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)|2\right)}{b}\right)}{3b}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}$$

input
```
Int[x*Cos[a + b*x]*Csc[a + b*x]^(5/2),x]
```

3.354. $\int x\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)\,dx$

output
```
(-2*x*Csc[a + b*x]^(3/2))/(3*b) + (2*((-2*Cos[a + b*x]*Sqrt[Csc[a + b*x]])
/b - (2*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b
*x]])/b))/(3*b)
```

### 3.354.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4701
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Csc[a + b*x^n]^(p
 - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ
[p, 1]
```

## 3.354.4 Maple [F]

$$\int x\cos\left(xb+a\right)\csc\left(xb+a\right)^{\frac{5}{2}}dx$$

input
```
int(x*cos(b*x+a)*csc(b*x+a)^(5/2),x)
```

output
```
int(x*cos(b*x+a)*csc(b*x+a)^(5/2),x)
```

### 3.354.5 Fricas [F(-2)]

Exception generated.

$$\int x\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

### 3.354.6 Sympy [F(-1)]

Timed out.

$$\int x\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)**(5/2),x)
```

output
```
Timed out
```

### 3.354.7 Maxima [F]

$$\int x\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)\,dx = \int x\cos(bx+a)\csc(bx+a)^{\frac{5}{2}}\,dx$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)*csc(b*x + a)^(5/2), x)
```

### 3.354.8 Giac [F]

$$\int x\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)\,dx=\int x\cos(bx+a)\csc(bx+a)^{\frac{5}{2}}\,dx$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)*csc(b*x + a)^(5/2), x)
```

### 3.354.9 Mupad [F(-1)]

Timed out.

$$\int x\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)\,dx=\int x\,\cos(a+b\,x)\,\left(\frac{1}{\sin(a+b\,x)}\right)^{5/2}\,dx$$

input
```
int(x*cos(a + b*x)*(1/sin(a + b*x))^(5/2),x)
```

output
```
int(x*cos(a + b*x)*(1/sin(a + b*x))^(5/2), x)
```

# 3.355 $\int x\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)\,dx$

3.355.1 Optimal result . . . 2590
3.355.2 Mathematica [A] (verified) . . . 2590
3.355.3 Rubi [A] (verified) . . . 2591
3.355.4 Maple [F] . . . 2592
3.355.5 Fricas [F(-2)] . . . 2592
3.355.6 Sympy [F(-1)] . . . 2593
3.355.7 Maxima [F] . . . 2593
3.355.8 Giac [F] . . . 2593
3.355.9 Mupad [F(-1)] . . . 2594

## 3.355.1 Optimal result

Integrand size = 18, antiderivative size = 58

$$\int x\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)\,dx = -\frac{2x\sqrt{\csc(a+bx)}}{b}+\frac{4\sqrt{\csc(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)\sqrt{\sin(a+bx)}}{b^2}$$

output
```
-2*x*csc(b*x+a)^(1/2)/b-4*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/
4*Pi+1/2*b*x)*EllipticF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2
)*sin(b*x+a)^(1/2)/b^2
```

## 3.355.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.79

$$\int x\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)\,dx = -\frac{2\sqrt{\csc(a+bx)}\left(bx+2\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)\sqrt{\sin(a+bx)}\right)}{b^2}$$

input
```
Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(3/2),x]
```

output
```
(-2*Sqrt[Csc[a + b*x]]*(b*x + 2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[S
in[a + b*x]]))/b^2
```

### 3.355.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {4701, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{4701}$$

$$\frac{2\int\sqrt{\csc(a+bx)}dx}{b}-\frac{2x\sqrt{\csc(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int\sqrt{\csc(a+bx)}dx}{b}-\frac{2x\sqrt{\csc(a+bx)}}{b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int\frac{1}{\sqrt{\sin(a+bx)}}dx}{b}-\frac{2x\sqrt{\csc(a+bx)}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int\frac{1}{\sqrt{\sin(a+bx)}}dx}{b}-\frac{2x\sqrt{\csc(a+bx)}}{b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{b^2}-\frac{2x\sqrt{\csc(a+bx)}}{b}$$

input
```
Int[x*Cos[a + b*x]*Csc[a + b*x]^(3/2),x]
```

output
```
(-2*x*Sqrt[Csc[a + b*x]])/b + (4*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 +
b*x)/2, 2]*Sqrt[Sin[a + b*x]])/b^2
```

#### 3.355.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4701
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Csc[a + b*x^n]^(p
 - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ
[p, 1]
```

### 3.355.4 Maple [F]

$$\int x\cos(xb+a)\csc(xb+a)^{\frac{3}{2}}\,dx$$

input
```
int(x*cos(b*x+a)*csc(b*x+a)^(3/2),x)
```

output
```
int(x*cos(b*x+a)*csc(b*x+a)^(3/2),x)
```

### 3.355.5 Fricas [F(-2)]

Exception generated.

$$\int x\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

### 3.355.6 Sympy [F(-1)]

Timed out.

$$\int x\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)**(3/2),x)
```

output
```
Timed out
```

### 3.355.7 Maxima [F]

$$\int x\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)\,dx = \int x\cos(bx+a)\csc(bx+a)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)*csc(b*x + a)^(3/2), x)
```

### 3.355.8 Giac [F]

$$\int x\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)\,dx = \int x\cos(bx+a)\csc(bx+a)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)*csc(b*x + a)^(3/2), x)
```

### 3.355.9 Mupad [F(-1)]

Timed out.

$$\int x\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)\,dx = \int x\,\cos(a+b\,x)\,\left(\frac{1}{\sin(a+b\,x)}\right)^{3/2}\,dx$$

input
```
int(x*cos(a + b*x)*(1/sin(a + b*x))^(3/2),x)
```

output
```
int(x*cos(a + b*x)*(1/sin(a + b*x))^(3/2), x)
```

# 3.356 $\int x \cos(a+bx)\sqrt{\csc(a+bx)}\,dx$

3.356.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2595
3.356.2 Mathematica [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . 2595
3.356.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2596
3.356.4 Maple [C] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . 2597
3.356.5 Fricas [F(-2)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2598
3.356.6 Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2598
3.356.7 Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2599
3.356.8 Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2599
3.356.9 Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2599

## 3.356.1 Optimal result

Integrand size = 18, antiderivative size = 58

$$\int x \cos(a+bx)\sqrt{\csc(a+bx)}\,dx = \frac{2x}{b\sqrt{\csc(a+bx)}} - \frac{4\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{b^2}$$

output
```
2*x/b/csc(b*x+a)^(1/2)+4*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4
*Pi+1/2*b*x)*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)
*sin(b*x+a)^(1/2)/b^2
```

## 3.356.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.87 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.83

$$\int x \cos(a+bx)\sqrt{\csc(a+bx)}\,dx$$
$$= \frac{4\cos\left(\frac{1}{2}(a+bx)\right)\sqrt{\csc(a+bx)}\sin\left(\frac{1}{2}(a+bx)\right)\left(3bx-6\tan\left(\frac{1}{2}(a+bx)\right)+2\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4}\right.\right.}{3b^2}$$

input
```
Integrate[x*Cos[a + b*x]*Sqrt[Csc[a + b*x]],x]
```

output
```
(4*Cos[(a + b*x)/2]*Sqrt[Csc[a + b*x]]*Sin[(a + b*x)/2]*(3*b*x - 6*Tan[(a
+ b*x)/2] + 2*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[(a + b*x)/2]^2]*Sqrt[S
ec[(a + b*x)/2]^2]*Tan[(a + b*x)/2]))/(3*b^2)
```

### 3.356.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {4701, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx$$

$$\Big\downarrow \text{4701}$$

$$\frac{2x}{b\sqrt{\csc(a+bx)}}-\frac{2\int \frac{1}{\sqrt{\csc(a+bx)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{b\sqrt{\csc(a+bx)}}-\frac{2\int \frac{1}{\sqrt{\csc(a+bx)}}dx}{b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2x}{b\sqrt{\csc(a+bx)}}-\frac{2\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \sqrt{\sin(a+bx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{b\sqrt{\csc(a+bx)}}-\frac{2\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \sqrt{\sin(a+bx)}dx}{b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2x}{b\sqrt{\csc(a+bx)}}-\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}$$

input
```
Int[x*Cos[a + b*x]*Sqrt[Csc[a + b*x]],x]
```

output
```
(2*x)/(b*Sqrt[Csc[a + b*x]]) - (4*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 +
 b*x)/2, 2]*Sqrt[Sin[a + b*x]])/b^2
```

### 3.356.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4701
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Csc[a + b*x^n]^(p
 - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ
[p, 1]
```

## 3.356.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 308, normalized size of antiderivative = 5.31

| method | result |
|---|---|
| risch | $-\frac{i(xb+2i)\left(\mathrm{e}^{2i(xb+a)}-1\right)\sqrt{2}\,\sqrt{\frac{i\mathrm{e}^{i(xb+a)}}{\mathrm{e}^{2i(xb+a)}-1}}\,\mathrm{e}^{-i(xb+a)}}{b^{2}}-\frac{2\left(-\frac{2i\left(-i+i\mathrm{e}^{2i(xb+a)}\right)}{\sqrt{\mathrm{e}^{i(xb+a)}\left(-i+i\mathrm{e}^{2i(xb+a)}\right)}}-\frac{\sqrt{\mathrm{e}^{i(xb+a)}+1}\,\sqrt{-2\,\mathrm{e}^{i(xb+a)}+2}\,\sqrt{-\mathrm{e}^{i(xb+}}}{}\right.}{}$ |

input
```
int(x*cos(b*x+a)*csc(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*(x*b+2*I)*(exp(I*(b*x+a))^2-1)/b^2*2^(1/2)*(I*exp(I*(b*x+a))/(exp(I*(b*
x+a))^2-1))^(1/2)/exp(I*(b*x+a))-2/b^2*(-2*I*(-I+I*exp(I*(b*x+a))^2)/(exp(
I*(b*x+a))*(-I+I*exp(I*(b*x+a))^2))^(1/2)-(exp(I*(b*x+a))+1)^(1/2)*(-2*exp
(I*(b*x+a))+2)^(1/2)*(-exp(I*(b*x+a)))^(1/2)/(I*exp(I*(b*x+a))^3-I*exp(I*(
b*x+a)))^(1/2)*(-2*EllipticE((exp(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))+Ellipti
cF((exp(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))))*2^(1/2)*(I*exp(I*(b*x+a))/(exp(
I*(b*x+a))^2-1))^(1/2)*(I*exp(I*(b*x+a))*(exp(I*(b*x+a))^2-1))^(1/2)/exp(I
*(b*x+a))
```

### 3.356.5 Fricas [F(-2)]

Exception generated.

$$\int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

### 3.356.6 Sympy [F]

$$\int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx = \int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)**(1/2),x)
```

output
```
Integral(x*cos(a + b*x)*sqrt(csc(a + b*x)), x)
```

### 3.356.7 Maxima [F]

$$\int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx=\int x\cos\left(bx+a\right)\sqrt{\csc\left(bx+a\right)}\,dx$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)*sqrt(csc(b*x + a)), x)
```

### 3.356.8 Giac [F]

$$\int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx=\int x\cos\left(bx+a\right)\sqrt{\csc\left(bx+a\right)}\,dx$$

input
```
integrate(x*cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)*sqrt(csc(b*x + a)), x)
```

### 3.356.9 Mupad [F(-1)]

Timed out.

$$\int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx=\int x\,\cos\left(a+b\,x\right)\,\sqrt{\frac{1}{\sin\left(a+b\,x\right)}}\,dx$$

input
```
int(x*cos(a + b*x)*(1/sin(a + b*x))^(1/2),x)
```

output
```
int(x*cos(a + b*x)*(1/sin(a + b*x))^(1/2), x)
```

# 3.357 $\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

3.357.1 Optimal result . . . 2600
3.357.2 Mathematica [A] (verified) . . . 2600
3.357.3 Rubi [A] (verified) . . . 2601
3.357.4 Maple [F] . . . 2603
3.357.5 Fricas [F(-2)] . . . 2603
3.357.6 Sympy [F] . . . 2603
3.357.7 Maxima [F] . . . 2604
3.357.8 Giac [F] . . . 2604
3.357.9 Mupad [F(-1)] . . . 2604

## 3.357.1 Optimal result

Integrand size = 18, antiderivative size = 85

$$\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} + \frac{4\cos(a+bx)}{9b^2\sqrt{\csc(a+bx)}} - \frac{4\sqrt{\csc(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)\sqrt{\sin(a+bx)}}{9b^2}$$

output
```
2/3*x/b/csc(b*x+a)^(3/2)+4/9*cos(b*x+a)/b^2/csc(b*x+a)^(1/2)+4/9*(sin(1/2*
a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(cos(1/2*a+1
/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b^2
```

## 3.357.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \frac{2\sqrt{\csc(a+bx)}\left(2\,\text{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)\sqrt{\sin(a+bx)}+3bx\sin^2(a+bx)+\sin(2(a+bx))\right)}{9b^2}$$

input
```
Integrate[(x*Cos[a + b*x])/Sqrt[Csc[a + b*x]],x]
```

output
```
(2*Sqrt[Csc[a + b*x]]*(2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a +
b*x]] + 3*b*x*Sin[a + b*x]^2 + Sin[2*(a + b*x)]))/(9*b^2)
```

### 3.357.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {4701, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$

$$\Big\downarrow \text{4701}$$

$$\frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)}-\frac{2\int \frac{1}{\csc^{\frac{3}{2}}(a+bx)}dx}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)}-\frac{2\int \frac{1}{\csc(a+bx)^{3/2}}dx}{3b}$$

$$\Big\downarrow \text{4256}$$

$$\frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)}-\frac{2\left(\frac{1}{3}\int \sqrt{\csc(a+bx)}dx-\frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}\right)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)}-\frac{2\left(\frac{1}{3}\int \sqrt{\csc(a+bx)}dx-\frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}\right)}{3b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)}-\frac{2\left(\frac{1}{3}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}dx-\frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}\right)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)}-\frac{2\left(\frac{1}{3}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int \frac{1}{\sqrt{\sin(a+bx)}}dx-\frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}\right)}{3b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)}-\frac{2\left(\frac{2\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,\text{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{3b}-\frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}\right)}{3b}$$

input
```
Int[(x*Cos[a + b*x])/Sqrt[Csc[a + b*x]],x]
```

output
```
(2*x)/(3*b*Csc[a + b*x]^(3/2)) - (2*((-2*Cos[a + b*x])/(3*b*Sqrt[Csc[a + b
*x]]) + (2*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a
+ b*x]])/(3*b)))/(3*b)
```

### 3.357.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4701
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Csc[a + b*x^n]^(p
 - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ
[p, 1]
```

### 3.357.4 Maple [F]

$$\int \frac{x\cos(xb+a)}{\sqrt{\csc(xb+a)}}dx$$

input
```
int(x*cos(b*x+a)/csc(b*x+a)^(1/2),x)
```

output
```
int(x*cos(b*x+a)/csc(b*x+a)^(1/2),x)
```

### 3.357.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

### 3.357.6 Sympy [F]

$$\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$

input
```
integrate(x*cos(b*x+a)/csc(b*x+a)**(1/2),x)
```

output
```
Integral(x*cos(a + b*x)/sqrt(csc(a + b*x)), x)
```

### 3.357.7 Maxima [F]

$$\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{x\cos\left(bx+a\right)}{\sqrt{\csc\left(bx+a\right)}}\,dx$$

input
```
integrate(x*cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)/sqrt(csc(b*x + a)), x)
```

### 3.357.8 Giac [F]

$$\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{x\cos\left(bx+a\right)}{\sqrt{\csc\left(bx+a\right)}}\,dx$$

input
```
integrate(x*cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)/sqrt(csc(b*x + a)), x)
```

### 3.357.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx = \int \frac{x\,\cos\left(a+b\,x\right)}{\sqrt{\frac{1}{\sin(a+b\,x)}}}\,dx$$

input
```
int((x*cos(a + b*x))/(1/sin(a + b*x))^(1/2),x)
```

output
```
int((x*cos(a + b*x))/(1/sin(a + b*x))^(1/2), x)
```

# 3.358 $\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx$

3.358.1 Optimal result . . . 2605
3.358.2 Mathematica [C] (verified) . . . 2605
3.358.3 Rubi [A] (verified) . . . 2606
3.358.4 Maple [F] . . . 2608
3.358.5 Fricas [F(-2)] . . . 2608
3.358.6 Sympy [F] . . . 2608
3.358.7 Maxima [F] . . . 2609
3.358.8 Giac [F] . . . 2609
3.358.9 Mupad [F(-1)] . . . 2609

## 3.358.1 Optimal result

Integrand size = 18, antiderivative size = 85

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx = \frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)}+\frac{4\cos(a+bx)}{25b^2\csc^{\frac{3}{2}}(a+bx)} - \frac{12\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{25b^2}$$

output
```
2/5*x/b/csc(b*x+a)^(5/2)+4/25*cos(b*x+a)/b^2/csc(b*x+a)^(3/2)+12/25*(sin(1
/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(cos(1/2*
a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b^2
```

## 3.358.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx = \frac{\left(-10+4\cos(a+bx)+2\cos(2(a+bx))+4\sqrt{2}\sqrt{\frac{1}{1+\cos(a+bx)}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},-\tan^2\left(\frac{1}{2}(a+b\right.\right.\right.}{25b^2\sqrt{\csc(a+bx)}}$$

input
```
Integrate[(x*Cos[a + b*x])/Csc[a + b*x]^(3/2),x]
```

output
```
((-10 + 4*Cos[a + b*x] + 2*Cos[2*(a + b*x)] + 4*Sqrt[2]*Sqrt[(1 + Cos[a +
b*x])^(-1)]*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[(a + b*x)/2]^2] + 10*b*x
*Sin[a + b*x] + 5*b*x*Sin[2*(a + b*x)])*Tan[(a + b*x)/2])/(25*b^2*Sqrt[Csc
[a + b*x]])
```

### 3.358.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {4701, 3042, 4256, 3042, 4258, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{4701}$$

$$\frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)}-\frac{2\int \frac{1}{\csc^{\frac{5}{2}}(a+bx)}dx}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)}-\frac{2\int \frac{1}{\csc(a+bx)^{5/2}}dx}{5b}$$

$$\Big\downarrow \text{4256}$$

$$\frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)}-\frac{2\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc(a+bx)}}dx-\frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}\right)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)}-\frac{2\left(\frac{3}{5}\int \frac{1}{\sqrt{\csc(a+bx)}}dx-\frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}\right)}{5b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)} - \frac{2\left(\frac{3}{5}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int\sqrt{\sin(a+bx)}dx - \frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}\right)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)} - \frac{2\left(\frac{3}{5}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int\sqrt{\sin(a+bx)}dx - \frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}\right)}{5b}$$

$$\Big\downarrow \text{3119}$$

$$\frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)} - \frac{2\left(\frac{6\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)|2\right)}{5b} - \frac{2\cos(a+bx)}{5b\csc^{\frac{3}{2}}(a+bx)}\right)}{5b}$$

input
```
Int[(x*Cos[a + b*x])/Csc[a + b*x]^(3/2),x]
```

output
```
(2*x)/(5*b*Csc[a + b*x]^(5/2)) - (2*((-2*Cos[a + b*x])/(5*b*Csc[a + b*x]^(
3/2)) + (6*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a
+ b*x]])/(5*b)))/(5*b)
```

### 3.358.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

3.358. $\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx$

rule 4701
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Csc[a + b*x^n]^(p
 - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ
[p, 1]
```

### 3.358.4 Maple [F]

$$\int \frac{x\cos\left(xb+a\right)}{\csc\left(xb+a\right)^{\frac{3}{2}}}dx$$

input
```
int(x*cos(b*x+a)/csc(b*x+a)^(3/2),x)
```

output
```
int(x*cos(b*x+a)/csc(b*x+a)^(3/2),x)
```

### 3.358.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)/csc(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

### 3.358.6 Sympy [F]

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\cos\left(a+bx\right)}{\csc^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

input
```
integrate(x*cos(b*x+a)/csc(b*x+a)**(3/2),x)
```

output
```
Integral(x*cos(a + b*x)/csc(a + b*x)**(3/2), x)
```

### 3.358.7 Maxima [F]

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\cos(bx+a)}{\csc(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*cos(b*x+a)/csc(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)/csc(b*x + a)^(3/2), x)
```

### 3.358.8 Giac [F]

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\cos(bx+a)}{\csc(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*cos(b*x+a)/csc(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)/csc(b*x + a)^(3/2), x)
```

### 3.358.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{x\cos(a+b\,x)}{\left(\frac{1}{\sin(a+b\,x)}\right)^{3/2}}\,dx$$

input
```
int((x*cos(a + b*x))/(1/sin(a + b*x))^(3/2),x)
```

output
```
int((x*cos(a + b*x))/(1/sin(a + b*x))^(3/2), x)
```

# 3.359 $\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx$


3.359.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2610
3.359.2 Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . 2610
3.359.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . 2611
3.359.4 Maple [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2613
3.359.5 Fricas [F(-2)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2613
3.359.6 Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2614
3.359.7 Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2614
3.359.8 Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2614
3.359.9 Mupad [F(-1)] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2615


## 3.359.1 Optimal result

Integrand size = 18, antiderivative size = 108

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx = \frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} + \frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)} + \frac{20\cos(a+bx)}{147b^2\sqrt{\csc(a+bx)}} - \frac{20\sqrt{\csc(a+bx)}\,\mathrm{EllipticF}\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right),2\right)\sqrt{\sin(a+bx)}}{147b^2}$$

output
```
2/7*x/b/csc(b*x+a)^(7/2)+4/49*cos(b*x+a)/b^2/csc(b*x+a)^(5/2)+20/147*cos(b
*x+a)/b^2/csc(b*x+a)^(1/2)+20/147*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(
1/2*a+1/4*Pi+1/2*b*x)*EllipticF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x
+a)^(1/2)*sin(b*x+a)^(1/2)/b^2
```

## 3.359.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.86

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx$$
$$= \frac{\sqrt{\csc(a+bx)}\Big(63bx-84bx\cos(2(a+bx))+21bx\cos(4(a+bx))+80\,\mathrm{EllipticF}\left(\frac{1}{4}(-2a+\pi-2bx),2\right)\sqrt{}}{588b^2}$$

input
```
Integrate[(x*Cos[a + b*x])/Csc[a + b*x]^(5/2),x]
```

output
```
(Sqrt[Csc[a + b*x]]*(63*b*x - 84*b*x*Cos[2*(a + b*x)] + 21*b*x*Cos[4*(a +
b*x)] + 80*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]] + 52*Sin
[2*(a + b*x)] - 6*Sin[4*(a + b*x)]))/(588*b^2)
```

## 3.359.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4701, 3042, 4256, 3042, 4256, 3042, 4258, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx$$

$$\Big\downarrow \ 4701$$

$$\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)}-\frac{2\int \frac{1}{\csc^{\frac{7}{2}}(a+bx)}dx}{7b}$$

$$\Big\downarrow \ 3042$$

$$\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)}-\frac{2\int \frac{1}{\csc(a+bx)^{7/2}}dx}{7b}$$

$$\Big\downarrow \ 4256$$

$$\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{5}{7}\int \frac{1}{\csc^{\frac{3}{2}}(a+bx)}dx-\frac{2\cos(a+bx)}{7b\csc^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

$$\Big\downarrow \ 3042$$

$$\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{5}{7}\int \frac{1}{\csc(a+bx)^{3/2}}dx-\frac{2\cos(a+bx)}{7b\csc^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

$$\Big\downarrow \ 4256$$

$$\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{5}{7}\left(\frac{1}{3}\int \sqrt{\csc(a+bx)}dx-\frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}\right)-\frac{2\cos(a+bx)}{7b\csc^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

$$\Big\downarrow \ 3042$$

$$\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{5}{7}\left(\frac{1}{3}\int\sqrt{\csc(a+bx)}dx-\frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}\right)-\frac{2\cos(a+bx)}{7b\csc^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

$$\Big\downarrow \text{4258}$$

$$\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int\frac{1}{\sqrt{\sin(a+bx)}}dx-\frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}\right)-\frac{2\cos(a+bx)}{7b\csc^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{5}{7}\left(\frac{1}{3}\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\int\frac{1}{\sqrt{\sin(a+bx)}}dx-\frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}\right)-\frac{2\cos(a+bx)}{7b\csc^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)}-\frac{2\left(\frac{5}{7}\left(\frac{2\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right),2\right)}{3b}-\frac{2\cos(a+bx)}{3b\sqrt{\csc(a+bx)}}\right)-\frac{2\cos(a+bx)}{7b\csc^{\frac{5}{2}}(a+bx)}\right)}{7b}$$

input
```
Int[(x*Cos[a + b*x])/Csc[a + b*x]^(5/2),x]
```

output
```
(2*x)/(7*b*Csc[a + b*x]^(7/2)) - (2*((-2*Cos[a + b*x])/(7*b*Csc[a + b*x]^(
5/2)) + (5*((-2*Cos[a + b*x])/(3*b*Sqrt[Csc[a + b*x]]) + (2*Sqrt[Csc[a + b
*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(3*b)))/7))/(7*b
)
```

### 3.359.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

---

3.359. $\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx$

rule 4256
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Simp[(n + 1)/(b^2*n)   Int[(b*Csc[c
+ d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*
n]
```

rule 4258
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n   Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
 EqQ[n^2, 1/4]
```

rule 4701
```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> Simp[(-x^(m - n + 1))*(Csc[a + b*x^n]^(p - 1)/(b*n*(p -
1))), x] + Simp[(m - n + 1)/(b*n*(p - 1))   Int[x^(m - n)*Csc[a + b*x^n]^(p
 - 1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ
[p, 1]
```

### 3.359.4 Maple [F]

$$\int \frac{x\cos(xb+a)}{\csc(xb+a)^{\frac{5}{2}}}dx$$

input
```
int(x*cos(b*x+a)/csc(b*x+a)^(5/2),x)
```

output
```
int(x*cos(b*x+a)/csc(b*x+a)^(5/2),x)
```

### 3.359.5 Fricas [F(-2)]

Exception generated.

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*cos(b*x+a)/csc(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

### 3.359.6 Sympy [F]

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\cos{(a+bx)}}{\csc^{\frac{5}{2}}{(a+bx)}}\,dx$$

input
```
integrate(x*cos(b*x+a)/csc(b*x+a)**(5/2),x)
```

output
```
Integral(x*cos(a + b*x)/csc(a + b*x)**(5/2), x)
```

### 3.359.7 Maxima [F]

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\cos{(bx+a)}}{\csc{(bx+a)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*cos(b*x+a)/csc(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x*cos(b*x + a)/csc(b*x + a)^(5/2), x)
```

### 3.359.8 Giac [F]

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\cos{(bx+a)}}{\csc{(bx+a)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*cos(b*x+a)/csc(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(x*cos(b*x + a)/csc(b*x + a)^(5/2), x)
```

### 3.359.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{x\ \cos\left(a+b\,x\right)}{{\left(\frac{1}{\sin(a+b\,x)}\right)}^{5/2}}\,dx$$

input
```
int((x*cos(a + b*x))/(1/sin(a + b*x))^(5/2),x)
```

output
```
int((x*cos(a + b*x))/(1/sin(a + b*x))^(5/2), x)
```

# 3.360 $\int x \csc(x) \sin(3x) \, dx$

3.360.1 Optimal result . . . 2616
3.360.2 Mathematica [A] (verified) . . . 2616
3.360.3 Rubi [A] (verified) . . . 2617
3.360.4 Maple [A] (verified) . . . 2618
3.360.5 Fricas [A] (verification not implemented) . . . 2618
3.360.6 Sympy [A] (verification not implemented) . . . 2618
3.360.7 Maxima [A] (verification not implemented) . . . 2619
3.360.8 Giac [A] (verification not implemented) . . . 2619
3.360.9 Mupad [B] (verification not implemented) . . . 2619

## 3.360.1 Optimal result

Integrand size = 8, antiderivative size = 31

$$\int x \csc(x) \sin(3x) \, dx = \frac{x^2}{2} + \frac{3\cos^2(x)}{4} + 2x\cos(x)\sin(x) - \frac{\sin^2(x)}{4}$$

output
```
1/2*x^2+3/4*cos(x)^2+2*x*cos(x)*sin(x)-1/4*sin(x)^2
```

## 3.360.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int x \csc(x) \sin(3x) \, dx = \frac{x^2}{2} + \frac{1}{2}\cos(2x) + x\sin(2x)$$

input
```
Integrate[x*Csc[x]*Sin[3*x],x]
```

output
```
x^2/2 + Cos[2*x]/2 + x*Sin[2*x]
```

### 3.360.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sin(3x)\csc(x)\,dx$$

$$\Big\downarrow \text{4931}$$

$$\int \left(3x\cos^2(x)-x\sin^2(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2}{2}-\frac{\sin^2(x)}{4}+\frac{3\cos^2(x)}{4}+2x\sin(x)\cos(x)$$

input
```
Int[x*Csc[x]*Sin[3*x],x]
```

output
```
x^2/2 + (3*Cos[x]^2)/4 + 2*x*Cos[x]*Sin[x] - Sin[x]^2/4
```

#### 3.360.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.360.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

| method | result | size |
|---|---|---|
| risch | $\frac{x^2}{2}+\frac{\cos(2x)}{2}+x\sin(2x)$ | 19 |
| default | $4x\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{3x^2}{2}-\sin(x)^2$ | 26 |

input
```
int(x*csc(x)*sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2+1/2*cos(2*x)+x*sin(2*x)
```

### 3.360.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.55

$$\int x\csc(x)\sin(3x)\,dx = 2\,x\cos(x)\sin(x)+\frac{1}{2}\,x^2+\cos(x)^2$$

input
```
integrate(x*csc(x)*sin(3*x),x, algorithm="fricas")
```

output
```
2*x*cos(x)*sin(x) + 1/2*x^2 + cos(x)^2
```

### 3.360.6 Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int x\csc(x)\sin(3x)\,dx = -\frac{x^2}{2}+x(x+\sin(2x))+\frac{\cos(2x)}{2}$$

input
```
integrate(x*csc(x)*sin(3*x),x)
```

output
```
-x**2/2 + x*(x + sin(2*x)) + cos(2*x)/2
```

### 3.360.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

$$\int x\csc(x)\sin(3x)\,dx = \frac{1}{2}\,x^2 + x\sin\left(2\,x\right) + \frac{1}{2}\,\cos\left(2\,x\right)$$

input
```
integrate(x*csc(x)*sin(3*x),x, algorithm="maxima")
```

output
```
1/2*x^2 + x*sin(2*x) + 1/2*cos(2*x)
```

### 3.360.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

$$\int x\csc(x)\sin(3x)\,dx = \frac{1}{2}\,x^2 + x\sin\left(2\,x\right) + \frac{1}{2}\,\cos\left(2\,x\right)$$

input
```
integrate(x*csc(x)*sin(3*x),x, algorithm="giac")
```

output
```
1/2*x^2 + x*sin(2*x) + 1/2*cos(2*x)
```

### 3.360.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.58

$$\int x\csc(x)\sin(3x)\,dx = \frac{\cos\left(2\,x\right)}{2} + x\,\sin\left(2\,x\right) + \frac{x^2}{2}$$

input
```
int((x*sin(3*x))/sin(x),x)
```

output
```
cos(2*x)/2 + x*sin(2*x) + x^2/2
```

# 3.361 $\int (c+dx)^4 \csc(x)\sin(3x)\,dx$

3.361.1 Optimal result . . . . . 2620
3.361.2 Mathematica [A] (verified) . . . . . 2620
3.361.3 Rubi [A] (verified) . . . . . 2621
3.361.4 Maple [A] (verified) . . . . . 2622
3.361.5 Fricas [A] (verification not implemented) . . . . . 2622
3.361.6 Sympy [A] (verification not implemented) . . . . . 2623
3.361.7 Maxima [A] (verification not implemented) . . . . . 2624
3.361.8 Giac [A] (verification not implemented) . . . . . 2624
3.361.9 Mupad [B] (verification not implemented) . . . . . 2625

## 3.361.1 Optimal result

Integrand size = 14, antiderivative size = 131

$$\int (c+dx)^4\csc(x)\sin(3x)\,dx=\frac{3d^4x}{2}-d(c+dx)^3+\frac{(c+dx)^5}{5d}-\frac{9}{2}d^3(c+dx)\cos^2(x)+3d(c+dx)^3\cos^2(x)+3d^4\cos(x)\sin(x)-6d^2(c+dx)^2\cos(x)\sin(x)+2(c+dx)^4\cos(x)\sin(x)+\frac{3}{2}d^3(c+dx)\sin^2(x)-d(c+dx)^3\sin^2(x)$$

output
```
3/2*d^4*x-d*(d*x+c)^3+1/5*(d*x+c)^5/d-9/2*d^3*(d*x+c)*cos(x)^2+3*d*(d*x+c)
^3*cos(x)^2+3*d^4*cos(x)*sin(x)-6*d^2*(d*x+c)^2*cos(x)*sin(x)+2*(d*x+c)^4*
cos(x)*sin(x)+3/2*d^3*(d*x+c)*sin(x)^2-d*(d*x+c)^3*sin(x)^2
```

## 3.361.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.18

$$\int (c+dx)^4\csc(x)\sin(3x)\,dx=c^4x+2c^3dx^2+2c^2d^2x^3+cd^3x^4+\frac{d^4x^5}{5}+d\left(2c^3+6c^2dx+d^3x\left(-3+2x^2\right)+3cd^2\left(-1+2x^2\right)\right)\cos(2x)+\frac{1}{2}\left(2c^4+8c^3dx+4cd^3x\left(-3+2x^2\right)+6c^2d^2\left(-1+2x^2\right)+d^4\left(3-6x^2+2x^4\right)\right)\sin(2x)$$

input
```
Integrate[(c + d*x)^4*Csc[x]*Sin[3*x],x]
```

output
```
c^4*x + 2*c^3*d*x^2 + 2*c^2*d^2*x^3 + c*d^3*x^4 + (d^4*x^5)/5 + d*(2*c^3 +
 6*c^2*d*x + d^3*x*(-3 + 2*x^2) + 3*c*d^2*(-1 + 2*x^2))*Cos[2*x] + ((2*c^4
 + 8*c^3*d*x + 4*c*d^3*x*(-3 + 2*x^2) + 6*c^2*d^2*(-1 + 2*x^2) + d^4*(3 -
6*x^2 + 2*x^4))*Sin[2*x])/2
```

## 3.361.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(3x)\csc(x)(c+dx)^4\,dx$$

$$\Big\downarrow \text{4931}$$

$$\int \left(3\cos^2(x)(c+dx)^4-\sin^2(x)(c+dx)^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{2}d^3\sin^2(x)(c+dx)-\frac{9}{2}d^3\cos^2(x)(c+dx)-6d^2\sin(x)\cos(x)(c+dx)^2+\frac{(c+dx)^5}{5d}-d(c+dx)^3-d\sin^2(x)(c+dx)^3+3d\cos^2(x)(c+dx)^3+2\sin(x)\cos(x)(c+dx)^4+\frac{3d^4x}{2}+3d^4\sin(x)\cos(x)$$

input
```
Int[(c + d*x)^4*Csc[x]*Sin[3*x],x]
```

output
```
(3*d^4*x)/2 - d*(c + d*x)^3 + (c + d*x)^5/(5*d) - (9*d^3*(c + d*x)*Cos[x]^
2)/2 + 3*d*(c + d*x)^3*Cos[x]^2 + 3*d^4*Cos[x]*Sin[x] - 6*d^2*(c + d*x)^2*
Cos[x]*Sin[x] + 2*(c + d*x)^4*Cos[x]*Sin[x] + (3*d^3*(c + d*x)*Sin[x]^2)/2
 - d*(c + d*x)^3*Sin[x]^2
```

### 3.361.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

## 3.361.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.33

| method | result |
|---|---|
| risch | $\frac{d^4x^5}{5}+c\,d^3x^4+2c^2d^2x^3+2c^3d\,x^2+c^4x+\frac{c^5}{5d}+d(2d^3x^3+6c\,d^2x^2+6c^2dx-3d^3x+2c^3-3c\,d^2)$ |
| default | $4d^4\left(x^4\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+x^3\cos(x)^2-3x^2\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{3x\cos(x)^2}{2}+\frac{3\cos(x)\sin(x)}{4}+\frac{3x}{4}+x^3\right.$ |

input
```
int((d*x+c)^4*csc(x)*sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/5*d^4*x^5+c*d^3*x^4+2*c^2*d^2*x^3+2*c^3*d*x^2+c^4*x+1/5/d*c^5+d*(2*d^3*x
^3+6*c*d^2*x^2+6*c^2*d*x-3*d^3*x+2*c^3-3*c*d^2)*cos(2*x)+1/2*(2*d^4*x^4+8*
c*d^3*x^3+12*c^2*d^2*x^2-6*d^4*x^2+8*c^3*d*x-12*c*d^3*x+2*c^4-6*c^2*d^2+3*
d^4)*sin(2*x)
```

## 3.361.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.53

$$\int (c+dx)^4\csc(x)\sin(3x)\,dx=\frac{1}{5}d^4x^5+cd^3x^4+2\left(c^2d^2-d^4\right)x^3+2\left(c^3d-3\,cd^3\right)x^2$$
$$+2\left(2\,d^4x^3+6\,cd^3x^2+2\,c^3d-3\,cd^3+3\left(2\,c^2d^2-d^4\right)x\right)\cos(x)^2$$
$$+\left(2\,d^4x^4+8\,cd^3x^3+2\,c^4-6\,c^2d^2+3\,d^4+6\left(2\,c^2d^2-d^4\right)x^2+4\left(2\,c^3d-3\,cd^3\right)x\right)\cos(x)\sin(x)$$
$$+\left(c^4-6\,c^2d^2+3\,d^4\right)x$$

input
```
integrate((d*x+c)^4*csc(x)*sin(3*x),x, algorithm="fricas")
```

output
```
1/5*d^4*x^5 + c*d^3*x^4 + 2*(c^2*d^2 - d^4)*x^3 + 2*(c^3*d - 3*c*d^3)*x^2
+ 2*(2*d^4*x^3 + 6*c*d^3*x^2 + 2*c^3*d - 3*c*d^3 + 3*(2*c^2*d^2 - d^4)*x)*
cos(x)^2 + (2*d^4*x^4 + 8*c*d^3*x^3 + 2*c^4 - 6*c^2*d^2 + 3*d^4 + 6*(2*c^2
*d^2 - d^4)*x^2 + 4*(2*c^3*d - 3*c*d^3)*x)*cos(x)*sin(x) + (c^4 - 6*c^2*d^
2 + 3*d^4)*x
```

## 3.361.6 Sympy [A] (verification not implemented)

Time = 14.92 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.70

$$\int (c+dx)^4 \csc(x)\sin(3x)\,dx = c^4(x+\sin(2x)) + 4c^3d\left(-\frac{x^2}{2} + x(x+\sin(2x)) + \frac{\cos(2x)}{2}\right) + 6c^2d^2\left(\frac{x^3}{3} + x^2(x+\sin(2x)) - 2x\left(\frac{x^2}{2} - \frac{\cos(2x)}{2}\right) - \frac{\sin(2x)}{2}\right) + 4cd^3\left(-\frac{x^4}{4} + x^3(x+\sin(2x)) - 3x^2\left(\frac{x^2}{2} - \frac{\cos(2x)}{2}\right) + 3x\left(\frac{x^3}{3} - \frac{\sin(2x)}{2}\right) - \frac{3\cos(2x)}{4}\right) + d^4\left(\frac{x^5}{5} + x^4(x+\sin(2x)) - 4x^3\left(\frac{x^2}{2} - \frac{\cos(2x)}{2}\right) + 6x^2\left(\frac{x^3}{3} - \frac{\sin(2x)}{2}\right) - 2x\left(\frac{x^4}{2} + \frac{3\cos(2x)}{2}\right) + \frac{3\sin(2x)}{2}\right)$$

input
```
integrate((d*x+c)**4*csc(x)*sin(3*x),x)
```

output
```
c**4*(x + sin(2*x)) + 4*c**3*d*(-x**2/2 + x*(x + sin(2*x)) + cos(2*x)/2) +
 6*c**2*d**2*(x**3/3 + x**2*(x + sin(2*x)) - 2*x*(x**2/2 - cos(2*x)/2) - s
in(2*x)/2) + 4*c*d**3*(-x**4/4 + x**3*(x + sin(2*x)) - 3*x**2*(x**2/2 - co
s(2*x)/2) + 3*x*(x**3/3 - sin(2*x)/2) - 3*cos(2*x)/4) + d**4*(x**5/5 + x**
4*(x + sin(2*x)) - 4*x**3*(x**2/2 - cos(2*x)/2) + 6*x**2*(x**3/3 - sin(2*x
)/2) - 2*x*(x**4/2 + 3*cos(2*x)/2) + 3*sin(2*x)/2)
```

### 3.361.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.11

$$\int (c+dx)^4 \csc(x)\sin(3x)\,dx$$
$$= 2\left(x^2+2x\sin(2x)+\cos(2x)\right)c^3d+\left(2x^3+6x\cos(2x)+3\left(2x^2-1\right)\sin(2x)\right)c^2d^2 + \left(x^4+3\left(2x^2-1\right)\cos(2x)+2\left(2x^3-3x\right)\sin(2x)\right)cd^3 + \frac{1}{10}\left(2x^5+10\left(2x^3-3x\right)\cos(2x)+5\left(2x^4-6x^2+3\right)\sin(2x)\right)d^4+c^4(x+\sin(2x))$$

input
```
integrate((d*x+c)^4*csc(x)*sin(3*x),x, algorithm="maxima")
```

output
```
2*(x^2 + 2*x*sin(2*x) + cos(2*x))*c^3*d + (2*x^3 + 6*x*cos(2*x) + 3*(2*x^2
 - 1)*sin(2*x))*c^2*d^2 + (x^4 + 3*(2*x^2 - 1)*cos(2*x) + 2*(2*x^3 - 3*x)*
sin(2*x))*c*d^3 + 1/10*(2*x^5 + 10*(2*x^3 - 3*x)*cos(2*x) + 5*(2*x^4 - 6*x
^2 + 3)*sin(2*x))*d^4 + c^4*(x + sin(2*x))
```

### 3.361.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.27

$$\int (c+dx)^4 \csc(x)\sin(3x)\,dx = \frac{1}{5}d^4x^5+cd^3x^4+2c^2d^2x^3+2c^3dx^2 + c^4x+\left(2d^4x^3+6cd^3x^2+6c^2d^2x-3d^4x+2c^3d-3cd^3\right)\cos(2x) + \frac{1}{2}\left(2d^4x^4+8cd^3x^3+12c^2d^2x^2-6d^4x^2+8c^3dx-12cd^3x+2c^4-6c^2d^2+3d^4\right)\sin(2x)$$

input
```
integrate((d*x+c)^4*csc(x)*sin(3*x),x, algorithm="giac")
```

output
```
1/5*d^4*x^5 + c*d^3*x^4 + 2*c^2*d^2*x^3 + 2*c^3*d*x^2 + c^4*x + (2*d^4*x^3
 + 6*c*d^3*x^2 + 6*c^2*d^2*x - 3*d^4*x + 2*c^3*d - 3*c*d^3)*cos(2*x) + 1/2
*(2*d^4*x^4 + 8*c*d^3*x^3 + 12*c^2*d^2*x^2 - 6*d^4*x^2 + 8*c^3*d*x - 12*c*
d^3*x + 2*c^4 - 6*c^2*d^2 + 3*d^4)*sin(2*x)
```

### 3.361.9 Mupad [B] (verification not implemented)

Time = 26.26 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.62

$$\int (c+dx)^4 \csc(x)\sin(3x)\,dx = c^4 \sin(2x) + \frac{3d^4 \sin(2x)}{2} + c^4 x + \frac{d^4 x^5}{5} - 3c^2 d^2 \sin(2x) + 2d^4 x^3 \cos(2x) - 3d^4 x^2 \sin(2x) + d^4 x^4 \sin(2x) + 2c^3 d x^2 + c d^3 x^4 + 2c^2 d^2 x^3 - 3c d^3 \cos(2x) + 2c^3 d \cos(2x) - 3d^4 x \cos(2x) + 6c^2 d^2 x^2 \sin(2x) - 6c d^3 x \sin(2x) + 4c^3 d x \sin(2x) + 6c^2 d^2 x \cos(2x) + 6c d^3 x^2 \cos(2x) + 4c d^3 x^3 \sin(2x)$$

input
```
int((sin(3*x)*(c + d*x)^4)/sin(x),x)
```

output
```
c^4*sin(2*x) + (3*d^4*sin(2*x))/2 + c^4*x + (d^4*x^5)/5 - 3*c^2*d^2*sin(2*
x) + 2*d^4*x^3*cos(2*x) - 3*d^4*x^2*sin(2*x) + d^4*x^4*sin(2*x) + 2*c^3*d*
x^2 + c*d^3*x^4 + 2*c^2*d^2*x^3 - 3*c*d^3*cos(2*x) + 2*c^3*d*cos(2*x) - 3*
d^4*x*cos(2*x) + 6*c^2*d^2*x^2*sin(2*x) - 6*c*d^3*x*sin(2*x) + 4*c^3*d*x*s
in(2*x) + 6*c^2*d^2*x*cos(2*x) + 6*c*d^3*x^2*cos(2*x) + 4*c*d^3*x^3*sin(2*
x)
```

# 3.362 $\int (c+dx)^3 \csc(x) \sin(3x)\, dx$

3.362.1 Optimal result . . . 2626
3.362.2 Mathematica [A] (verified) . . . 2626
3.362.3 Rubi [A] (verified) . . . 2627
3.362.4 Maple [A] (verified) . . . 2628
3.362.5 Fricas [A] (verification not implemented) . . . 2628
3.362.6 Sympy [A] (verification not implemented) . . . 2629
3.362.7 Maxima [A] (verification not implemented) . . . 2629
3.362.8 Giac [A] (verification not implemented) . . . 2630
3.362.9 Mupad [B] (verification not implemented) . . . 2630

## 3.362.1 Optimal result

Integrand size = 14, antiderivative size = 115

$$\int (c+dx)^3 \csc(x) \sin(3x)\, dx = -\frac{3}{2}cd^2x - \frac{3d^3x^2}{4} + \frac{(c+dx)^4}{4d} - \frac{9}{8}d^3\cos^2(x) + \frac{9}{4}d(c+dx)^2\cos^2(x) - 3d^2(c+dx)\cos(x)\sin(x) + 2(c+dx)^3\cos(x)\sin(x) + \frac{3}{8}d^3\sin^2(x) - \frac{3}{4}d(c+dx)^2\sin^2(x)$$

output
```
-3/2*c*d^2*x-3/4*d^3*x^2+1/4*(d*x+c)^4/d-9/8*d^3*cos(x)^2+9/4*d*(d*x+c)^2*
cos(x)^2-3*d^2*(d*x+c)*cos(x)*sin(x)+2*(d*x+c)^3*cos(x)*sin(x)+3/8*d^3*sin
(x)^2-3/4*d*(d*x+c)^2*sin(x)^2
```

## 3.362.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.95

$$\int (c+dx)^3 \csc(x) \sin(3x)\, dx = \frac{1}{4}\left(x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right) + 3d\left(2c^2+4cdx+d^2\left(-1+2x^2\right)\right)\cos(2x) + 2\left(2c^3+6c^2dx+d^3x\left(-3+2x^2\right)+3cd^2\left(-1+2x^2\right)\right)\sin(2x)\right)$$

input
```
Integrate[(c + d*x)^3*Csc[x]*Sin[3*x],x]
```

output
```
(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 3*d*(2*c^2 + 4*c*d*x + d^
2*(-1 + 2*x^2))*Cos[2*x] + 2*(2*c^3 + 6*c^2*d*x + d^3*x*(-3 + 2*x^2) + 3*c
*d^2*(-1 + 2*x^2))*Sin[2*x])/4
```

### 3.362.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(3x)\csc(x)(c+dx)^3\,dx$$

$$\Big\downarrow \text{4931}$$

$$\int \left(3\cos^2(x)(c+dx)^3-\sin^2(x)(c+dx)^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-3d^2\sin(x)\cos(x)(c+dx)+\frac{(c+dx)^4}{4d}-\frac{3}{4}d(c+dx)^2-\frac{3}{4}d\sin^2(x)(c+dx)^2+\frac{9}{4}d\cos^2(x)(c+dx)^2+2\sin(x)\cos(x)(c+dx)^3+\frac{3}{8}d^3\sin^2(x)-\frac{9}{8}d^3\cos^2(x)$$

input
```
Int[(c + d*x)^3*Csc[x]*Sin[3*x],x]
```

output
```
(-3*d*(c + d*x)^2)/4 + (c + d*x)^4/(4*d) - (9*d^3*Cos[x]^2)/8 + (9*d*(c +
d*x)^2*Cos[x]^2)/4 - 3*d^2*(c + d*x)*Cos[x]*Sin[x] + 2*(c + d*x)^3*Cos[x]*
Sin[x] + (3*d^3*Sin[x]^2)/8 - (3*d*(c + d*x)^2*Sin[x]^2)/4
```

#### 3.362.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

## 3.362.4 Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03

| method | result |
|---|---|
| risch | $\frac{d^3x^4}{4}+c\,d^2x^3+\frac{3c^2d\,x^2}{2}+c^3x+\frac{c^4}{4d}+\frac{3d(2x^2d^2+4cdx+2c^2-d^2)\cos(2x)}{4}+\frac{(2d^3x^3+6c\,d^2x^2+6c^2dx-3d^3x+2c^3-3c\,d^2)}{2}$ |
| default | $4d^3\left(x^3\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+\frac{3x^2\cos(x)^2}{4}-\frac{3x\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)}{2}+\frac{3x^2}{8}+\frac{3\sin(x)^2}{8}-\frac{3x^4}{8}\right)+12c\,d^2\Big(x^2\Big($ cos( |

input
```
int((d*x+c)^3*csc(x)*sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*d^3*x^4+c*d^2*x^3+3/2*c^2*d*x^2+c^3*x+1/4/d*c^4+3/4*d*(2*d^2*x^2+4*c*d
*x+2*c^2-d^2)*cos(2*x)+1/2*(2*d^3*x^3+6*c*d^2*x^2+6*c^2*d*x-3*d^3*x+2*c^3-
3*c*d^2)*sin(2*x)
```

## 3.362.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10

$$\begin{aligned}\int (c+dx)^3\csc(x)\sin(3x)\,dx &= \frac{1}{4}\,d^3x^4+cd^2x^3+\frac{3}{2}\left(c^2d-d^3\right)x^2+\frac{3}{2}\left(2\,d^3x^2+4\,cd^2x+2\,c^2d-d^3\right)\cos\left(x\right)^2\\&\quad+\left(2\,d^3x^3+6\,cd^2x^2+2\,c^3-3\,cd^2+3\left(2\,c^2d-d^3\right)x\right)\cos\left(x\right)\sin\left(x\right)+\left(c^3-3\,cd^2\right)x\end{aligned}$$

input
```
integrate((d*x+c)^3*csc(x)*sin(3*x),x, algorithm="fricas")
```

output
```
1/4*d^3*x^4 + c*d^2*x^3 + 3/2*(c^2*d - d^3)*x^2 + 3/2*(2*d^3*x^2 + 4*c*d^2
*x + 2*c^2*d - d^3)*cos(x)^2 + (2*d^3*x^3 + 6*c*d^2*x^2 + 2*c^3 - 3*c*d^2
+ 3*(2*c^2*d - d^3)*x)*cos(x)*sin(x) + (c^3 - 3*c*d^2)*x
```

### 3.362.6 Sympy [A] (verification not implemented)

Time = 7.78 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.21

$$\int (c+dx)^3 \csc(x)\sin(3x)\,dx = c^3(x+\sin(2x)) + 3c^2d\left(-\frac{x^2}{2} + x(x+\sin(2x)) + \frac{\cos(2x)}{2}\right) + 3cd^2\left(\frac{x^3}{3} + x^2(x+\sin(2x)) - 2x\left(\frac{x^2}{2} - \frac{\cos(2x)}{2}\right) - \frac{\sin(2x)}{2}\right) + d^3\left(-\frac{x^4}{4} + x^3(x+\sin(2x)) - 3x^2\left(\frac{x^2}{2} - \frac{\cos(2x)}{2}\right) + 3x\left(\frac{x^3}{3} - \frac{\sin(2x)}{2}\right) - \frac{3\cos(2x)}{4}\right)$$

input
```
integrate((d*x+c)**3*csc(x)*sin(3*x),x)
```

output
```
c**3*(x + sin(2*x)) + 3*c**2*d*(-x**2/2 + x*(x + sin(2*x)) + cos(2*x)/2) +
 3*c*d**2*(x**3/3 + x**2*(x + sin(2*x)) - 2*x*(x**2/2 - cos(2*x)/2) - sin(
2*x)/2) + d**3*(-x**4/4 + x**3*(x + sin(2*x)) - 3*x**2*(x**2/2 - cos(2*x)/
2) + 3*x*(x**3/3 - sin(2*x)/2) - 3*cos(2*x)/4)
```

### 3.362.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.88

$$\int (c+dx)^3 \csc(x)\sin(3x)\,dx = \frac{3}{2}\left(x^2 + 2\,x\sin(2\,x) + \cos(2\,x)\right)c^2d + \frac{1}{2}\left(2\,x^3 + 6\,x\cos(2\,x) + 3\left(2\,x^2 - 1\right)\sin(2\,x)\right)cd^2 + \frac{1}{4}\left(x^4 + 3\left(2\,x^2 - 1\right)\cos(2\,x) + 2\left(2\,x^3 - 3\,x\right)\sin(2\,x)\right)d^3 + c^3(x + \sin(2\,x))$$

input
```
integrate((d*x+c)^3*csc(x)*sin(3*x),x, algorithm="maxima")
```

output
```
3/2*(x^2 + 2*x*sin(2*x) + cos(2*x))*c^2*d + 1/2*(2*x^3 + 6*x*cos(2*x) + 3*
(2*x^2 - 1)*sin(2*x))*c*d^2 + 1/4*(x^4 + 3*(2*x^2 - 1)*cos(2*x) + 2*(2*x^3
 - 3*x)*sin(2*x))*d^3 + c^3*(x + sin(2*x))
```

### 3.362.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97

$$\int (c+dx)^3 \csc(x)\sin(3x)\,dx = \frac{1}{4}d^3x^4 + cd^2x^3 + \frac{3}{2}c^2dx^2 + c^3x + \frac{3}{4}\left(2\,d^3x^2 + 4\,cd^2x + 2\,c^2d - d^3\right)\cos\left(2\,x\right) + \frac{1}{2}\left(2\,d^3x^3 + 6\,cd^2x^2 + 6\,c^2dx - 3\,d^3x + 2\,c^3 - 3\,cd^2\right)\sin\left(2\,x\right)$$

input
```
integrate((d*x+c)^3*csc(x)*sin(3*x),x, algorithm="giac")
```

output
```
1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 + c^3*x + 3/4*(2*d^3*x^2 + 4*c*d^2
*x + 2*c^2*d - d^3)*cos(2*x) + 1/2*(2*d^3*x^3 + 6*c*d^2*x^2 + 6*c^2*d*x -
3*d^3*x + 2*c^3 - 3*c*d^2)*sin(2*x)
```

### 3.362.9 Mupad [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.18

$$\int (c+dx)^3 \csc(x)\sin(3x)\,dx = c^3\,\sin\left(2\,x\right) - \frac{3\,d^3\,\cos\left(2\,x\right)}{4} + c^3\,x + \frac{d^3\,x^4}{4} + \frac{3\,d^3\,x^2\,\cos\left(2\,x\right)}{2} + d^3\,x^3\,\sin\left(2\,x\right) + \frac{3\,c^2\,d\,x^2}{2} + c\,d^2\,x^3 + \frac{3\,c^2\,d\,\cos\left(2\,x\right)}{2} - \frac{3\,c\,d^2\,\sin\left(2\,x\right)}{2} - \frac{3\,d^3\,x\,\sin\left(2\,x\right)}{2} + 3\,c\,d^2\,x\,\cos\left(2\,x\right) + 3\,c^2\,d\,x\,\sin\left(2\,x\right) + 3\,c\,d^2\,x^2\,\sin\left(2\,x\right)$$

input
```
int((sin(3*x)*(c + d*x)^3)/sin(x),x)
```

output
```
c^3*sin(2*x) - (3*d^3*cos(2*x))/4 + c^3*x + (d^3*x^4)/4 + (3*d^3*x^2*cos(2
*x))/2 + d^3*x^3*sin(2*x) + (3*c^2*d*x^2)/2 + c*d^2*x^3 + (3*c^2*d*cos(2*x
))/2 - (3*c*d^2*sin(2*x))/2 - (3*d^3*x*sin(2*x))/2 + 3*c*d^2*x*cos(2*x) +
3*c^2*d*x*sin(2*x) + 3*c*d^2*x^2*sin(2*x)
```

# 3.363 $\int (c+dx)^2 \csc(x)\sin(3x)\,dx$

3.363.1 Optimal result . . . . . . . . . . 2631
3.363.2 Mathematica [A] (verified) . . . . . . . . . . 2631
3.363.3 Rubi [A] (verified) . . . . . . . . . . 2632
3.363.4 Maple [A] (verified) . . . . . . . . . . 2633
3.363.5 Fricas [A] (verification not implemented) . . . . . . . . . . 2633
3.363.6 Sympy [A] (verification not implemented) . . . . . . . . . . 2633
3.363.7 Maxima [A] (verification not implemented) . . . . . . . . . . 2634
3.363.8 Giac [A] (verification not implemented) . . . . . . . . . . 2634
3.363.9 Mupad [B] (verification not implemented) . . . . . . . . . . 2635

## 3.363.1 Optimal result

Integrand size = 14, antiderivative size = 73

$$\int (c+dx)^2\csc(x)\sin(3x)\,dx = -\frac{d^2x}{2}+\frac{(c+dx)^3}{3d}+\frac{3}{2}d(c+dx)\cos^2(x)-d^2\cos(x)\sin(x)+2(c+dx)^2\cos(x)\sin(x)-\frac{1}{2}d(c+dx)\sin^2(x)$$

output
```
-1/2*d^2*x+1/3*(d*x+c)^3/d+3/2*d*(d*x+c)*cos(x)^2-d^2*cos(x)*sin(x)+2*(d*x
+c)^2*cos(x)*sin(x)-1/2*d*(d*x+c)*sin(x)^2
```

## 3.363.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int (c+dx)^2\csc(x)\sin(3x)\,dx = c^2x+cdx^2+\frac{d^2x^3}{3}+d(c+dx)\cos(2x)+\left(2c^2+4cdx+d^2\left(-1+2x^2\right)\right)\cos(x)\sin(x)$$

input
```
Integrate[(c + d*x)^2*Csc[x]*Sin[3*x],x]
```

output
```
c^2*x + c*d*x^2 + (d^2*x^3)/3 + d*(c + d*x)*Cos[2*x] + (2*c^2 + 4*c*d*x +
d^2*(-1 + 2*x^2))*Cos[x]*Sin[x]
```

### 3.363.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(3x)\csc(x)(c+dx)^2\,dx$$

$$\Big\downarrow \text{4931}$$

$$\int \left(3\cos^2(x)(c+dx)^2-\sin^2(x)(c+dx)^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(c+dx)^3}{3d}-\frac{1}{2}d\sin^2(x)(c+dx)+\frac{3}{2}d\cos^2(x)(c+dx)+2\sin(x)\cos(x)(c+dx)^2-\frac{d^2x}{2}-d^2\sin(x)\cos(x)$$

input
```
Int[(c + d*x)^2*Csc[x]*Sin[3*x],x]
```

output
```
-1/2*(d^2*x) + (c + d*x)^3/(3*d) + (3*d*(c + d*x)*Cos[x]^2)/2 - d^2*Cos[x]
*Sin[x] + 2*(c + d*x)^2*Cos[x]*Sin[x] - (d*(c + d*x)*Sin[x]^2)/2
```

#### 3.363.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.363.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

| method | result |
|---|---|
| risch | $\frac{d^2x^3}{3}+dc\,x^2+c^2x+\frac{c^3}{3d}+d(dx+c)\cos(2x)+\frac{(2x^2d^2+4cdx+2c^2-d^2)\sin(2x)}{2}$ |
| default | $4d^2\left(x^2\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+\frac{x\cos(x)^2}{2}-\frac{\cos(x)\sin(x)}{4}-\frac{x}{4}-\frac{x^3}{3}\right)+8cd\left(x\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{x^2}{4}-\frac{\sin(x}{4}\right.$ |

input
```
int((d*x+c)^2*csc(x)*sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*d^2*x^3+d*c*x^2+c^2*x+1/3/d*c^3+d*(d*x+c)*cos(2*x)+1/2*(2*d^2*x^2+4*c*
d*x+2*c^2-d^2)*sin(2*x)
```

### 3.363.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.96

$$\int (c+dx)^2\csc(x)\sin(3x)\,dx=\frac{1}{3}d^2x^3+cdx^2+2\left(d^2x+cd\right)\cos(x)^2+\left(2\,d^2x^2+4\,cdx+2\,c^2-d^2\right)\cos(x)\sin(x)+\left(c^2-d^2\right)x$$

input
```
integrate((d*x+c)^2*csc(x)*sin(3*x),x, algorithm="fricas")
```

output
```
1/3*d^2*x^3 + c*d*x^2 + 2*(d^2*x + c*d)*cos(x)^2 + (2*d^2*x^2 + 4*c*d*x +
2*c^2 - d^2)*cos(x)*sin(x) + (c^2 - d^2)*x
```

### 3.363.6 Sympy [A] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int (c+dx)^2\csc(x)\sin(3x)\,dx=c^2(x+\sin(2x))+2cd\left(-\frac{x^2}{2}+x(x+\sin(2x))+\frac{\cos(2x)}{2}\right)+d^2\left(\frac{x^3}{3}+x^2(x+\sin(2x))-2x\left(\frac{x^2}{2}-\frac{\cos(2x)}{2}\right)-\frac{\sin(2x)}{2}\right)$$

input
```
integrate((d*x+c)**2*csc(x)*sin(3*x),x)
```

output
```
c**2*(x + sin(2*x)) + 2*c*d*(-x**2/2 + x*(x + sin(2*x)) + cos(2*x)/2) + d*
*2*(x**3/3 + x**2*(x + sin(2*x)) - 2*x*(x**2/2 - cos(2*x)/2) - sin(2*x)/2)
```

### 3.363.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82

$$\int (c+dx)^2\csc(x)\sin(3x)\,dx = \left(x^2+2\,x\sin\left(2\,x\right)+\cos\left(2\,x\right)\right)cd + \frac{1}{6}\left(2\,x^3+6\,x\cos\left(2\,x\right)+3\left(2\,x^2-1\right)\sin\left(2\,x\right)\right)d^2 + c^2(x+\sin\left(2\,x\right))$$

input
```
integrate((d*x+c)^2*csc(x)*sin(3*x),x, algorithm="maxima")
```

output
```
(x^2 + 2*x*sin(2*x) + cos(2*x))*c*d + 1/6*(2*x^3 + 6*x*cos(2*x) + 3*(2*x^2
 - 1)*sin(2*x))*d^2 + c^2*(x + sin(2*x))
```

### 3.363.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int (c+dx)^2\csc(x)\sin(3x)\,dx = \frac{1}{3}\,d^2x^3+cdx^2+c^2x+\left(d^2x+cd\right)\cos\left(2\,x\right) + \frac{1}{2}\left(2\,d^2x^2+4\,cdx+2\,c^2-d^2\right)\sin\left(2\,x\right)$$

input
```
integrate((d*x+c)^2*csc(x)*sin(3*x),x, algorithm="giac")
```

output
```
1/3*d^2*x^3 + c*d*x^2 + c^2*x + (d^2*x + c*d)*cos(2*x) + 1/2*(2*d^2*x^2 +
4*c*d*x + 2*c^2 - d^2)*sin(2*x)
```

### 3.363.9 Mupad [B] (verification not implemented)

Time = 25.72 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int (c+dx)^2 \csc(x)\sin(3x)\,dx = c^2 \sin(2x) - \frac{d^2 \sin(2x)}{2} + c^2 x + \frac{d^2 x^3}{3} + d^2 x^2 \sin(2x) + c\,d \cos(2x) + d^2 x \cos(2x) + c\,d\,x^2 + 2\,c\,d\,x \sin(2x)$$

input
```
int((sin(3*x)*(c + d*x)^2)/sin(x),x)
```

output
```
c^2*sin(2*x) - (d^2*sin(2*x))/2 + c^2*x + (d^2*x^3)/3 + d^2*x^2*sin(2*x) +
 c*d*cos(2*x) + d^2*x*cos(2*x) + c*d*x^2 + 2*c*d*x*sin(2*x)
```

# 3.364 $\int (c+dx)\csc(x)\sin(3x)\,dx$

3.364.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2636
3.364.2 Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . 2636
3.364.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2637
3.364.4 Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2638
3.364.5 Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . 2638
3.364.6 Sympy [A] (verification not implemented) . . . . . . . . . . . . . . . . . 2638
3.364.7 Maxima [A] (verification not implemented) . . . . . . . . . . . . . . . . 2639
3.364.8 Giac [A] (verification not implemented) . . . . . . . . . . . . . . . . . 2639
3.364.9 Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . 2639

## 3.364.1 Optimal result

Integrand size = 12, antiderivative size = 41

$$\int (c+dx)\csc(x)\sin(3x)\,dx = cx+\frac{dx^2}{2}+\frac{3}{4}d\cos^2(x)+2(c+dx)\cos(x)\sin(x)-\frac{1}{4}d\sin^2(x)$$

output
```
c*x+1/2*d*x^2+3/4*d*cos(x)^2+2*(d*x+c)*cos(x)*sin(x)-1/4*d*sin(x)^2
```

## 3.364.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int (c+dx)\csc(x)\sin(3x)\,dx = cx+\frac{dx^2}{2}+\frac{1}{2}d\cos(2x)+c\sin(2x)+dx\sin(2x)$$

input
```
Integrate[(c + d*x)*Csc[x]*Sin[3*x],x]
```

output
```
c*x + (d*x^2)/2 + (d*Cos[2*x])/2 + c*Sin[2*x] + d*x*Sin[2*x]
```

### 3.364.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sin(3x)\csc(x)(c+dx)\,dx$$

$$\Big\downarrow \text{4931}$$

$$\int \left(3\cos^2(x)(c+dx)-\sin^2(x)(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(c+dx)^2}{2d}+2\sin(x)\cos(x)(c+dx)-\frac{1}{4}d\sin^2(x)+\frac{3}{4}d\cos^2(x)$$

input
```
Int[(c + d*x)*Csc[x]*Sin[3*x],x]
```

output
```
(c + d*x)^2/(2*d) + (3*d*Cos[x]^2)/4 + 2*(c + d*x)*Cos[x]*Sin[x] - (d*Sin[
x]^2)/4
```

#### 3.364.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.364.4 Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

| method | result | size |
|---|---|---|
| risch | $\frac{d x^2}{2}+cx+\frac{d\cos(2x)}{2}+(dx+c)\sin(2x)$ | 28 |
| default | $4d\left(x\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{x^2}{4}-\frac{\sin(x)^2}{4}\right)+4c\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{d x^2}{2}-cx$ | 52 |

input
```
int((d*x+c)*csc(x)*sin(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*d*x^2+c*x+1/2*d*cos(2*x)+(d*x+c)*sin(2*x)
```

### 3.364.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int (c+dx)\csc(x)\sin(3x)\,dx=\frac{1}{2}\,dx^2+d\cos(x)^2+2\,(dx+c)\cos(x)\sin(x)+cx$$

input
```
integrate((d*x+c)*csc(x)*sin(3*x),x, algorithm="fricas")
```

output
```
1/2*d*x^2 + d*cos(x)^2 + 2*(d*x + c)*cos(x)*sin(x) + c*x
```

### 3.364.6 Sympy [A] (verification not implemented)

Time = 1.69 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.71

$$\int (c+dx)\csc(x)\sin(3x)\,dx=c(x+\sin(2x))+d\left(-\frac{x^2}{2}+x(x+\sin(2x))+\frac{\cos(2x)}{2}\right)$$

input
```
integrate((d*x+c)*csc(x)*sin(3*x),x)
```

output
```
c*(x + sin(2*x)) + d*(-x**2/2 + x*(x + sin(2*x)) + cos(2*x)/2)
```

### 3.364.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int (c+dx)\csc(x)\sin(3x)\,dx = \frac{1}{2}\left(x^2+2\,x\sin\left(2\,x\right)+\cos\left(2\,x\right)\right)d+c(x+\sin\left(2\,x\right))$$

input
```
integrate((d*x+c)*csc(x)*sin(3*x),x, algorithm="maxima")
```

output
```
1/2*(x^2 + 2*x*sin(2*x) + cos(2*x))*d + c*(x + sin(2*x))
```

### 3.364.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.66

$$\int (c+dx)\csc(x)\sin(3x)\,dx = \frac{1}{2}\,dx^2+cx+\frac{1}{2}\,d\cos\left(2\,x\right)+(dx+c)\sin\left(2\,x\right)$$

input
```
integrate((d*x+c)*csc(x)*sin(3*x),x, algorithm="giac")
```

output
```
1/2*d*x^2 + c*x + 1/2*d*cos(2*x) + (d*x + c)*sin(2*x)
```

### 3.364.9 Mupad [B] (verification not implemented)

Time = 26.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.73

$$\int (c+dx)\csc(x)\sin(3x)\,dx = c\,\sin\left(2\,x\right)+c\,x+\frac{d\,x^2}{2}+\frac{d\,\cos\left(2\,x\right)}{2}+d\,x\,\sin\left(2\,x\right)$$

input
```
int((sin(3*x)*(c + d*x))/sin(x),x)
```

output
```
c*sin(2*x) + c*x + (d*x^2)/2 + (d*cos(2*x))/2 + d*x*sin(2*x)
```

# 3.365 $\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx$

3.365.1 Optimal result . . . . . 2640
3.365.2 Mathematica [A] (verified) . . . . . 2640
3.365.3 Rubi [A] (verified) . . . . . 2641
3.365.4 Maple [A] (verified) . . . . . 2642
3.365.5 Fricas [A] (verification not implemented) . . . . . 2642
3.365.6 Sympy [F] . . . . . 2642
3.365.7 Maxima [C] (verification not implemented) . . . . . 2643
3.365.8 Giac [A] (verification not implemented) . . . . . 2643
3.365.9 Mupad [F(-1)] . . . . . 2643

## 3.365.1 Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx=\frac{2\cos\left(\frac{2c}{d}\right)\text{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d}+\frac{\log(c+dx)}{d}+\frac{2\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d}$$

output
```
2*Ci(2*c/d+2*x)*cos(2*c/d)/d+ln(d*x+c)/d+2*Si(2*c/d+2*x)*sin(2*c/d)/d
```

## 3.365.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx = \frac{2\cos\left(\frac{2c}{d}\right)\text{CosIntegral}\left(2\left(\frac{c}{d}+x\right)\right)+\log(c+dx)+2\sin\left(\frac{2c}{d}\right)\text{Si}\left(2\left(\frac{c}{d}+x\right)\right)}{d}$$

input
```
Integrate[(Csc[x]*Sin[3*x])/(c + d*x),x]
```

output
```
(2*Cos[(2*c)/d]*CosIntegral[2*(c/d + x)] + Log[c + d*x] + 2*Sin[(2*c)/d]*S
inIntegral[2*(c/d + x)])/d
```

### 3.365.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3x)\csc(x)}{c+dx}\,dx$$

$$\Big\downarrow \text{4931}$$

$$\int \left(\frac{3\cos^2(x)}{c+dx}-\frac{\sin^2(x)}{c+dx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\cos\left(\frac{2c}{d}\right)\text{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d}+\frac{2\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d}+\frac{\log(c+dx)}{d}$$

input
```
Int[(Csc[x]*Sin[3*x])/(c + d*x),x]
```

output
```
(2*Cos[(2*c)/d]*CosIntegral[(2*c)/d + 2*x])/d + Log[c + d*x]/d + (2*Sin[(2
*c)/d]*SinIntegral[(2*c)/d + 2*x])/d
```

#### 3.365.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.365.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

| method | result | size |
|---|---|---|
| default | $\frac{2\,\mathrm{Ci}\left(\frac{2c}{d}+2x\right)\cos\left(\frac{2c}{d}\right)}{d}+\frac{\ln(dx+c)}{d}+\frac{2\,\mathrm{Si}\left(\frac{2c}{d}+2x\right)\sin\left(\frac{2c}{d}\right)}{d}$ | 58 |
| risch | $\frac{\ln(dx+c)}{d}-\frac{\mathrm{e}^{\frac{2ic}{d}}\,\mathrm{Ei}_1\left(2ix+\frac{2ic}{d}\right)}{d}-\frac{\mathrm{e}^{-\frac{2ic}{d}}\,\mathrm{Ei}_1\left(-2ix-\frac{2ic}{d}\right)}{d}$ | 66 |

input
```
int(csc(x)*sin(3*x)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
2*Ci(2*c/d+2*x)*cos(2*c/d)/d+ln(d*x+c)/d+2*Si(2*c/d+2*x)*sin(2*c/d)/d
```

### 3.365.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx=\frac{2\,\cos\left(\frac{2\,c}{d}\right)\mathrm{Ci}\left(\frac{2\,(dx+c)}{d}\right)+2\,\sin\left(\frac{2\,c}{d}\right)\mathrm{Si}\left(\frac{2\,(dx+c)}{d}\right)+\log\left(dx+c\right)}{d}$$

input
```
integrate(csc(x)*sin(3*x)/(d*x+c),x, algorithm="fricas")
```

output
```
(2*cos(2*c/d)*cos_integral(2*(d*x + c)/d) + 2*sin(2*c/d)*sin_integral(2*(d
*x + c)/d) + log(d*x + c))/d
```

### 3.365.6 Sympy [F]

$$\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx=\int \frac{\sin\left(3x\right)\csc\left(x\right)}{c+dx}\,dx$$

input
```
integrate(csc(x)*sin(3*x)/(d*x+c),x)
```

output
```
Integral(sin(3*x)*csc(x)/(c + d*x), x)
```

### 3.365.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.70

$$\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx = -\frac{\left(E_1\left(\frac{2(-i\,dx-i\,c)}{d}\right)+E_1\left(-\frac{2(-i\,dx-i\,c)}{d}\right)\right)\cos\left(\frac{2c}{d}\right)-\left(i\,E_1\left(\frac{2(-i\,dx-i\,c)}{d}\right)-i\,E_1\left(-\frac{2(-i\,dx-i\,c)}{d}\right)\right)\sin\left(\frac{2c}{d}\right)-}{d}$$

input
```
integrate(csc(x)*sin(3*x)/(d*x+c),x, algorithm="maxima")
```

output
```
-((exp_integral_e(1, 2*(-I*d*x - I*c)/d) + exp_integral_e(1, -2*(-I*d*x -
I*c)/d))*cos(2*c/d) - (I*exp_integral_e(1, 2*(-I*d*x - I*c)/d) - I*exp_int
egral_e(1, -2*(-I*d*x - I*c)/d))*sin(2*c/d) - log(d*x + c))/d
```

### 3.365.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx = \frac{2\cos\left(\frac{2c}{d}\right)\operatorname{Ci}\left(\frac{2(dx+c)}{d}\right)+2\sin\left(\frac{2c}{d}\right)\operatorname{Si}\left(\frac{2(dx+c)}{d}\right)+\log\left(dx+c\right)}{d}$$

input
```
integrate(csc(x)*sin(3*x)/(d*x+c),x, algorithm="giac")
```

output
```
(2*cos(2*c/d)*cos_integral(2*(d*x + c)/d) + 2*sin(2*c/d)*sin_integral(2*(d
*x + c)/d) + log(d*x + c))/d
```

### 3.365.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx = \int \frac{\sin\left(3\,x\right)}{\sin\left(x\right)\,\left(c+d\,x\right)}\,dx$$

input
```
int(sin(3*x)/(sin(x)*(c + d*x)),x)
```

output
```
int(sin(3*x)/(sin(x)*(c + d*x)), x)
```

# 3.366 $\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx$


3.366.1 Optimal result . . . 2644
3.366.2 Mathematica [A] (verified) . . . 2644
3.366.3 Rubi [A] (verified) . . . 2645
3.366.4 Maple [A] (verified) . . . 2646
3.366.5 Fricas [A] (verification not implemented) . . . 2646
3.366.6 Sympy [F] . . . 2646
3.366.7 Maxima [C] (verification not implemented) . . . 2647
3.366.8 Giac [A] (verification not implemented) . . . 2647
3.366.9 Mupad [F(-1)] . . . 2648


## 3.366.1 Optimal result

Integrand size = 14, antiderivative size = 78

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx = -\frac{3\cos^2(x)}{d(c+dx)} + \frac{4\,\text{CosIntegral}\left(\frac{2c}{d}+2x\right)\sin\left(\frac{2c}{d}\right)}{d^2} + \frac{\sin^2(x)}{d(c+dx)} - \frac{4\cos\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d^2}$$

output
```
-3*cos(x)^2/d/(d*x+c)-4*cos(2*c/d)*Si(2*c/d+2*x)/d^2+4*Ci(2*c/d+2*x)*sin(2
*c/d)/d^2+sin(x)^2/d/(d*x+c)
```

## 3.366.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.78

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx = \frac{-\frac{d(1+2\cos(2x))}{c+dx} + 4\,\text{CosIntegral}\left(2\left(\frac{c}{d}+x\right)\right)\sin\left(\frac{2c}{d}\right) - 4\cos\left(\frac{2c}{d}\right)\text{Si}\left(2\left(\frac{c}{d}+x\right)\right)}{d^2}$$

input
```
Integrate[(Csc[x]*Sin[3*x])/(c + d*x)^2,x]
```

output
```
(-((d*(1 + 2*Cos[2*x]))/(c + d*x)) + 4*CosIntegral[2*(c/d + x)]*Sin[(2*c)/
d] - 4*Cos[(2*c)/d]*SinIntegral[2*(c/d + x)])/d^2
```

### 3.366.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3x)\csc(x)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{4931}$$

$$\int \left(\frac{3\cos^2(x)}{(c+dx)^2}-\frac{\sin^2(x)}{(c+dx)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4\sin\left(\frac{2c}{d}\right)\text{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d^2}-\frac{4\cos\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d^2}+\frac{\sin^2(x)}{d(c+dx)}-\frac{3\cos^2(x)}{d(c+dx)}$$

input
```
Int[(Csc[x]*Sin[3*x])/(c + d*x)^2,x]
```

output
```
(-3*Cos[x]^2)/(d*(c + d*x)) + (4*CosIntegral[(2*c)/d + 2*x]*Sin[(2*c)/d])/
d^2 + Sin[x]^2/(d*(c + d*x)) - (4*Cos[(2*c)/d]*SinIntegral[(2*c)/d + 2*x])
/d^2
```

#### 3.366.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.366.4 Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05

| method | result | size |
|---|---|---|
| default | $-\frac{2\cos(2x)}{(dx+c)d}-\frac{2\left(\frac{2\,\mathrm{Si}\left(\frac{2c}{d}+2x\right)\cos\left(\frac{2c}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(\frac{2c}{d}+2x\right)\sin\left(\frac{2c}{d}\right)}{d}\right)}{d}-\frac{1}{d(dx+c)}$ | 82 |
| risch | $-\frac{1}{d(dx+c)}+\frac{2i\,e^{\frac{2ic}{d}}\,\mathrm{Ei}_1\left(2ix+\frac{2ic}{d}\right)}{d^2}-\frac{2i\,e^{-\frac{2ic}{d}}\,\mathrm{Ei}_1\left(-2ix-\frac{2ic}{d}\right)}{d^2}-\frac{2i\cos(2x)}{d^2\left(ix+\frac{ic}{d}\right)}$ | 94 |

input
```
int(csc(x)*sin(3*x)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*cos(2*x)/(d*x+c)/d-2*(2*Si(2*c/d+2*x)*cos(2*c/d)/d-2*Ci(2*c/d+2*x)*sin(
2*c/d)/d)/d-1/d/(d*x+c)
```

### 3.366.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.97

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx$$
$$= -\frac{4\,d\cos\left(x\right)^2-4\,(dx+c)\,\mathrm{Ci}\left(\frac{2\,(dx+c)}{d}\right)\sin\left(\frac{2\,c}{d}\right)+4\,(dx+c)\cos\left(\frac{2\,c}{d}\right)\mathrm{Si}\left(\frac{2\,(dx+c)}{d}\right)-d}{d^3x+cd^2}$$

input
```
integrate(csc(x)*sin(3*x)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-(4*d*cos(x)^2 - 4*(d*x + c)*cos_integral(2*(d*x + c)/d)*sin(2*c/d) + 4*(d
*x + c)*cos(2*c/d)*sin_integral(2*(d*x + c)/d) - d)/(d^3*x + c*d^2)
```

### 3.366.6 Sympy [F]

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx = \int \frac{\sin\left(3x\right)\csc\left(x\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(csc(x)*sin(3*x)/(d*x+c)**2,x)
```

output
```
Integral(sin(3*x)*csc(x)/(c + d*x)**2, x)
```

### 3.366.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 330, normalized size of antiderivative = 4.23

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx = -\frac{\left(E_2\left(\frac{2(-i\,dx-i\,c)}{d}\right)+E_2\left(-\frac{2(-i\,dx-i\,c)}{d}\right)\right)\cos\left(\frac{2c}{d}\right)^3+\left(-i\,E_2\left(\frac{2(-i\,dx-i\,c)}{d}\right)+i\,E_2\left(-\frac{2(-i\,dx-i\,c)}{d}\right)\right)\sin\left(\frac{2c}{d}\right)^3}{}$$

input
```
integrate(csc(x)*sin(3*x)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/2*((exp_integral_e(2, 2*(-I*d*x - I*c)/d) + exp_integral_e(2, -2*(-I*d*
x - I*c)/d))*cos(2*c/d)^3 + (-I*exp_integral_e(2, 2*(-I*d*x - I*c)/d) + I*
exp_integral_e(2, -2*(-I*d*x - I*c)/d))*sin(2*c/d)^3 + ((exp_integral_e(2,
 2*(-I*d*x - I*c)/d) + exp_integral_e(2, -2*(-I*d*x - I*c)/d))*cos(2*c/d)
+ 2)*sin(2*c/d)^2 + (exp_integral_e(2, 2*(-I*d*x - I*c)/d) + exp_integral_
e(2, -2*(-I*d*x - I*c)/d))*cos(2*c/d) + 2*cos(2*c/d)^2 + ((-I*exp_integral
_e(2, 2*(-I*d*x - I*c)/d) + I*exp_integral_e(2, -2*(-I*d*x - I*c)/d))*cos(
2*c/d)^2 - I*exp_integral_e(2, 2*(-I*d*x - I*c)/d) + I*exp_integral_e(2, -
2*(-I*d*x - I*c)/d))*sin(2*c/d))/((cos(2*c/d)^2 + sin(2*c/d)^2)*d^2*x + (c
*cos(2*c/d)^2 + c*sin(2*c/d)^2)*d)
```

### 3.366.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.42

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx = \frac{4\,dx\,\mathrm{Ci}\left(\frac{2(dx+c)}{d}\right)\sin\left(\frac{2c}{d}\right)-4\,dx\cos\left(\frac{2c}{d}\right)\mathrm{Si}\left(\frac{2(dx+c)}{d}\right)+4\,c\,\mathrm{Ci}\left(\frac{2(dx+c)}{d}\right)\sin\left(\frac{2c}{d}\right)-4\,c\cos\left(\frac{2c}{d}\right)\mathrm{Si}\left(\frac{2(dx+c)}{d}\right)}{d^3x+cd^2}$$

input
```
integrate(csc(x)*sin(3*x)/(d*x+c)^2,x, algorithm="giac")
```

output
```
(4*d*x*cos_integral(2*(d*x + c)/d)*sin(2*c/d) - 4*d*x*cos(2*c/d)*sin_integ
ral(2*(d*x + c)/d) + 4*c*cos_integral(2*(d*x + c)/d)*sin(2*c/d) - 4*c*cos(
2*c/d)*sin_integral(2*(d*x + c)/d) - 2*d*cos(2*x) - d)/(d^3*x + c*d^2)
```

### 3.366.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx = \int \frac{\sin(3x)}{\sin(x)\,(c+d\,x)^2}\,dx$$

input
```
int(sin(3*x)/(sin(x)*(c + d*x)^2),x)
```

output
```
int(sin(3*x)/(sin(x)*(c + d*x)^2), x)
```

# 3.367 $\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx$


3.367.1 Optimal result . . . 2649
3.367.2 Mathematica [A] (verified) . . . 2649
3.367.3 Rubi [A] (verified) . . . 2650
3.367.4 Maple [A] (verified) . . . 2651
3.367.5 Fricas [A] (verification not implemented) . . . 2651
3.367.6 Sympy [F] . . . 2652
3.367.7 Maxima [C] (verification not implemented) . . . 2652
3.367.8 Giac [B] (verification not implemented) . . . 2653
3.367.9 Mupad [F(-1)] . . . 2653


## 3.367.1 Optimal result

Integrand size = 14, antiderivative size = 99

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx = -\frac{3\cos^2(x)}{2d(c+dx)^2} - \frac{4\cos\left(\frac{2c}{d}\right)\text{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d^3} + \frac{4\cos(x)\sin(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} - \frac{4\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d^3}$$

output
```
-4*Ci(2*c/d+2*x)*cos(2*c/d)/d^3-3/2*cos(x)^2/d/(d*x+c)^2-4*Si(2*c/d+2*x)*s
in(2*c/d)/d^3+4*cos(x)*sin(x)/d^2/(d*x+c)+1/2*sin(x)^2/d/(d*x+c)^2
```

## 3.367.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.78

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx = \frac{-8\cos\left(\frac{2c}{d}\right)\text{CosIntegral}\left(2\left(\frac{c}{d}+x\right)\right) + \frac{d(-d-2d\cos(2x)+4(c+dx)\sin(2x))}{(c+dx)^2} - 8\sin\left(\frac{2c}{d}\right)\text{Si}\left(2\left(\frac{c}{d}+x\right)\right)}{2d^3}$$

input
```
Integrate[(Csc[x]*Sin[3*x])/(c + d*x)^3,x]
```

output
```
(-8*Cos[(2*c)/d]*CosIntegral[2*(c/d + x)] + (d*(-d - 2*d*Cos[2*x] + 4*(c +
 d*x)*Sin[2*x]))/(c + d*x)^2 - 8*Sin[(2*c)/d]*SinIntegral[2*(c/d + x)])/(2
*d^3)
```

### 3.367.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3x)\csc(x)}{(c+dx)^3}\,dx$$

$$\Big\downarrow \ 4931$$

$$\int \left(\frac{3\cos^2(x)}{(c+dx)^3}-\frac{\sin^2(x)}{(c+dx)^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{4\cos\left(\frac{2c}{d}\right)\text{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d^3}-\frac{4\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d^3}+\frac{4\sin(x)\cos(x)}{d^2(c+dx)}+\frac{\sin^2(x)}{2d(c+dx)^2}-\frac{3\cos^2(x)}{2d(c+dx)^2}$$

input
```
Int[(Csc[x]*Sin[3*x])/(c + d*x)^3,x]
```

output
```
(-3*Cos[x]^2)/(2*d*(c + d*x)^2) - (4*Cos[(2*c)/d]*CosIntegral[(2*c)/d + 2*
x])/d^3 + (4*Cos[x]*Sin[x])/(d^2*(c + d*x)) + Sin[x]^2/(2*d*(c + d*x)^2) -
 (4*Sin[(2*c)/d]*SinIntegral[(2*c)/d + 2*x])/d^3
```

#### 3.367.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.367.4 Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.05

| method | result | size |
|---|---|---|
| default | $-\frac{\cos(2x)}{(dx+c)^2 d}-\frac{-\frac{2\sin(2x)}{(dx+c)d}+\frac{\frac{4\,\mathrm{Si}\left(\frac{2c}{d}+2x\right)\sin\left(\frac{2c}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(\frac{2c}{d}+2x\right)\cos\left(\frac{2c}{d}\right)}{d}}{d}}{d}-\frac{1}{2d(dx+c)^2}$ | 104 |
| risch | $-\frac{1}{2d(dx+c)^2}+\frac{2\,\mathrm{e}^{\frac{2ic}{d}}\,\mathrm{Ei}_1\left(2ix+\frac{2ic}{d}\right)}{d^3}+\frac{2\,\mathrm{e}^{-\frac{2ic}{d}}\,\mathrm{Ei}_1\left(-2ix-\frac{2ic}{d}\right)}{d^3}-\frac{\cos(2x)}{(dx+c)^2 d}+\frac{i(-4idx-4ic)\sin(2x)}{2(dx+c)^2 d^2}$ | 111 |

input
```
int(csc(x)*sin(3*x)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-cos(2*x)/(d*x+c)^2/d-(-2*sin(2*x)/(d*x+c)/d+2*(2*Si(2*c/d+2*x)*sin(2*c/d)
/d+2*Ci(2*c/d+2*x)*cos(2*c/d)/d)/d)/d-1/2/d/(d*x+c)^2
```

### 3.367.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.29

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx = -\frac{4\,d^2\cos\left(x\right)^2+8\left(d^2x^2+2\,cdx+c^2\right)\cos\left(\frac{2\,c}{d}\right)\mathrm{Ci}\left(\frac{2\,(dx+c)}{d}\right)-8\left(d^2x+cd\right)\cos\left(x\right)\sin\left(x\right)+8\left(d^2x^2+2\,cd\right.}{2\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

input
```
integrate(csc(x)*sin(3*x)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/2*(4*d^2*cos(x)^2 + 8*(d^2*x^2 + 2*c*d*x + c^2)*cos(2*c/d)*cos_integral
(2*(d*x + c)/d) - 8*(d^2*x + c*d)*cos(x)*sin(x) + 8*(d^2*x^2 + 2*c*d*x + c
^2)*sin(2*c/d)*sin_integral(2*(d*x + c)/d) - d^2)/(d^5*x^2 + 2*c*d^4*x + c
^2*d^3)
```

### 3.367.6 Sympy [F]

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx = \int \frac{\sin{(3x)}\csc{(x)}}{(c+dx)^3}\,dx$$

input
```
integrate(csc(x)*sin(3*x)/(d*x+c)**3,x)
```

output
```
Integral(sin(3*x)*csc(x)/(c + d*x)**3, x)
```

### 3.367.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 365, normalized size of antiderivative = 3.69

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx =$$
$$-\frac{\left(E_3\left(\frac{2(-i\,dx-i\,c)}{d}\right)+E_3\left(-\frac{2(-i\,dx-i\,c)}{d}\right)\right)\cos\left(\frac{2c}{d}\right)^3-\left(i\,E_3\left(\frac{2(-i\,dx-i\,c)}{d}\right)-i\,E_3\left(-\frac{2(-i\,dx-i\,c)}{d}\right)\right)\sin\left(\frac{2c}{d}\right)^3 -}{}$$

input
```
integrate(csc(x)*sin(3*x)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/2*((exp_integral_e(3, 2*(-I*d*x - I*c)/d) + exp_integral_e(3, -2*(-I*d*
x - I*c)/d))*cos(2*c/d)^3 - (I*exp_integral_e(3, 2*(-I*d*x - I*c)/d) - I*e
xp_integral_e(3, -2*(-I*d*x - I*c)/d))*sin(2*c/d)^3 + ((exp_integral_e(3,
2*(-I*d*x - I*c)/d) + exp_integral_e(3, -2*(-I*d*x - I*c)/d))*cos(2*c/d) +
 1)*sin(2*c/d)^2 + (exp_integral_e(3, 2*(-I*d*x - I*c)/d) + exp_integral_e
(3, -2*(-I*d*x - I*c)/d))*cos(2*c/d) + cos(2*c/d)^2 - ((I*exp_integral_e(3
, 2*(-I*d*x - I*c)/d) - I*exp_integral_e(3, -2*(-I*d*x - I*c)/d))*cos(2*c/
d)^2 + I*exp_integral_e(3, 2*(-I*d*x - I*c)/d) - I*exp_integral_e(3, -2*(-
I*d*x - I*c)/d))*sin(2*c/d))/((cos(2*c/d)^2 + sin(2*c/d)^2)*d^3*x^2 + 2*(c
*cos(2*c/d)^2 + c*sin(2*c/d)^2)*d^2*x + (c^2*cos(2*c/d)^2 + c^2*sin(2*c/d)
^2)*d)
```

### 3.367.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(95) = 190$.

Time = 0.27 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.03

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx = -\frac{8\,d^2x^2\cos\left(\frac{2\,c}{d}\right)\operatorname{Ci}\left(\frac{2\,(dx+c)}{d}\right)+8\,d^2x^2\sin\left(\frac{2\,c}{d}\right)\operatorname{Si}\left(\frac{2\,(dx+c)}{d}\right)+16\,cdx\cos\left(\frac{2\,c}{d}\right)\operatorname{Ci}\left(\frac{2\,(dx+c)}{d}\right)+16\,cdx\sin\left(\right.}{2}$$

input
```
integrate(csc(x)*sin(3*x)/(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/2*(8*d^2*x^2*cos(2*c/d)*cos_integral(2*(d*x + c)/d) + 8*d^2*x^2*sin(2*c
/d)*sin_integral(2*(d*x + c)/d) + 16*c*d*x*cos(2*c/d)*cos_integral(2*(d*x
+ c)/d) + 16*c*d*x*sin(2*c/d)*sin_integral(2*(d*x + c)/d) + 8*c^2*cos(2*c/
d)*cos_integral(2*(d*x + c)/d) - 4*d^2*x*sin(2*x) + 8*c^2*sin(2*c/d)*sin_i
ntegral(2*(d*x + c)/d) + 2*d^2*cos(2*x) - 4*c*d*sin(2*x) + d^2)/(d^5*x^2 +
 2*c*d^4*x + c^2*d^3)
```

### 3.367.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx = \int \frac{\sin(3\,x)}{\sin(x)\,\left(c+d\,x\right)^3}\,dx$$

input
```
int(sin(3*x)/(sin(x)*(c + d*x)^3),x)
```

output
```
int(sin(3*x)/(sin(x)*(c + d*x)^3), x)
```

# 3.368 $\int (c+dx)^4 \csc(a+bx)\sin(3a+3bx)\,dx$

3.368.1 Optimal result . . . 2654
3.368.2 Mathematica [A] (verified) . . . 2655
3.368.3 Rubi [A] (verified) . . . 2655
3.368.4 Maple [A] (verified) . . . 2656
3.368.5 Fricas [A] (verification not implemented) . . . 2657
3.368.6 Sympy [F(-1)] . . . 2657
3.368.7 Maxima [A] (verification not implemented) . . . 2657
3.368.8 Giac [B] (verification not implemented) . . . 2658
3.368.9 Mupad [B] (verification not implemented) . . . 2659

## 3.368.1 Optimal result

Integrand size = 23, antiderivative size = 198

$$\begin{aligned}\int (c+dx)^4 \csc(a+bx)\sin(3a+3bx)\,dx &= \frac{3d^4x}{2b^4}-\frac{d(c+dx)^3}{b^2}+\frac{(c+dx)^5}{5d}-\frac{9d^3(c+dx)\cos^2(a+bx)}{2b^4}\\&+\frac{3d(c+dx)^3\cos^2(a+bx)}{b^2}+\frac{3d^4\cos(a+bx)\sin(a+bx)}{b^5}\\&-\frac{6d^2(c+dx)^2\cos(a+bx)\sin(a+bx)}{b^3}+\frac{2(c+dx)^4\cos(a+bx)\sin(a+bx)}{b}\\&+\frac{3d^3(c+dx)\sin^2(a+bx)}{2b^4}-\frac{d(c+dx)^3\sin^2(a+bx)}{b^2}\end{aligned}$$

output
```
3/2*d^4*x/b^4-d*(d*x+c)^3/b^2+1/5*(d*x+c)^5/d-9/2*d^3*(d*x+c)*cos(b*x+a)^2
/b^4+3*d*(d*x+c)^3*cos(b*x+a)^2/b^2+3*d^4*cos(b*x+a)*sin(b*x+a)/b^5-6*d^2*
(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^3+2*(d*x+c)^4*cos(b*x+a)*sin(b*x+a)/b+3/
2*d^3*(d*x+c)*sin(b*x+a)^2/b^4-d*(d*x+c)^3*sin(b*x+a)^2/b^2
```

### 3.368.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.65

$$\int (c+dx)^4 \csc(a+bx)\sin(3a+3bx)\,dx$$
$$= c^4x+2c^3dx^2+2c^2d^2x^3+cd^3x^4+\frac{d^4x^5}{5}+\frac{d(c+dx)\left(-3d^2+2b^2(c+dx)^2\right)\cos(2(a+bx))}{b^4}$$
$$+\frac{\left(3d^4-6b^2d^2(c+dx)^2+2b^4(c+dx)^4\right)\sin(2(a+bx))}{2b^5}$$

input
```
Integrate[(c + d*x)^4*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
c^4*x + 2*c^3*d*x^2 + 2*c^2*d^2*x^3 + c*d^3*x^4 + (d^4*x^5)/5 + (d*(c + d*
x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])/b^4 + ((3*d^4 - 6*b^2*d^
2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Sin[2*(a + b*x)])/(2*b^5)
```

### 3.368.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sin(3a+3bx)\csc(a+bx)\,dx$$
$$\Big\downarrow \text{4931}$$
$$\int \left(3(c+dx)^4\cos^2(a+bx)-(c+dx)^4\sin^2(a+bx)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{3d^4\sin(a+bx)\cos(a+bx)}{b^5}+\frac{3d^3(c+dx)\sin^2(a+bx)}{2b^4}-\frac{9d^3(c+dx)\cos^2(a+bx)}{2b^4}-\frac{6d^2(c+dx)^2\sin(a+bx)\cos(a+bx)}{b^3}-\frac{d(c+dx)^3\sin^2(a+bx)}{b^2}+\frac{3d(c+dx)^3\cos^2(a+bx)}{b^2}+\frac{2(c+dx)^4\sin(a+bx)\cos(a+bx)}{b}+\frac{3d^4x}{2b^4}-\frac{d(c+dx)^3}{b^2}+\frac{(c+dx)^5}{5d}$$

input
```
Int[(c + d*x)^4*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
(3*d^4*x)/(2*b^4) - (d*(c + d*x)^3)/b^2 + (c + d*x)^5/(5*d) - (9*d^3*(c +
d*x)*Cos[a + b*x]^2)/(2*b^4) + (3*d*(c + d*x)^3*Cos[a + b*x]^2)/b^2 + (3*d
^4*Cos[a + b*x]*Sin[a + b*x])/b^5 - (6*d^2*(c + d*x)^2*Cos[a + b*x]*Sin[a
+ b*x])/b^3 + (2*(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x])/b + (3*d^3*(c + d*
x)*Sin[a + b*x]^2)/(2*b^4) - (d*(c + d*x)^3*Sin[a + b*x]^2)/b^2
```

### 3.368.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

## 3.368.4 Maple [A] (verified)

Time = 1.91 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.14

| method | result |
|---|---|
| risch | $\frac{d^4x^5}{5}+c\,d^3x^4+2c^2d^2x^3+2c^3d\,x^2+c^4x+\frac{c^5}{5d}+\frac{d\left(2b^2d^3x^3+6b^2c\,d^2x^2+6b^2c^2dx+2b^2c^3-3d^3x-3c\,d^2\right)\cos(2xb+2a)}{b^4}$ |
| default | Expression too large to display |

input
```
int((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output
```
1/5*d^4*x^5+c*d^3*x^4+2*c^2*d^2*x^3+2*c^3*d*x^2+c^4*x+1/5/d*c^5+1/b^4*d*(2
*b^2*d^3*x^3+6*b^2*c*d^2*x^2+6*b^2*c^2*d*x+2*b^2*c^3-3*d^3*x-3*c*d^2)*cos(
2*b*x+2*a)+1/2*(2*b^4*d^4*x^4+8*b^4*c*d^3*x^3+12*b^4*c^2*d^2*x^2+8*b^4*c^3
*d*x+2*b^4*c^4-6*b^2*d^4*x^2-12*b^2*c*d^3*x-6*b^2*c^2*d^2+3*d^4)/b^5*sin(2
*b*x+2*a)
```

### 3.368.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.43

$$\int (c+dx)^4 \csc(a+bx)\sin(3a+3bx)\,dx$$
$$= \frac{b^5d^4x^5 + 5\,b^5cd^3x^4 + 10\,(b^5c^2d^2 - b^3d^4)x^3 + 10\,(b^5c^3d - 3\,b^3cd^3)x^2 + 10\,(2\,b^3d^4x^3 + 6\,b^3cd^3x^2 + 2\,b^3c^3d -}{}$$

input
```
integrate((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

output
```
1/5*(b^5*d^4*x^5 + 5*b^5*c*d^3*x^4 + 10*(b^5*c^2*d^2 - b^3*d^4)*x^3 + 10*(
b^5*c^3*d - 3*b^3*c*d^3)*x^2 + 10*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3
*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^2 + 5*(2*b^
4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4
*c^2*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)*si
n(b*x + a) + 5*(b^5*c^4 - 6*b^3*c^2*d^2 + 3*b*d^4)*x)/b^5
```

### 3.368.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^4 \csc(a+bx)\sin(3a+3bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**4*csc(b*x+a)*sin(3*b*x+3*a),x)
```

output
```
Timed out
```

### 3.368.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.23

$$\int (c+dx)^4 \csc(a+bx)\sin(3a+3bx)\,dx$$
$$= \frac{(bx + \sin(2\,bx + 2\,a))c^4}{b} + \frac{2\,(b^2x^2 + 2\,bx\sin(2\,bx + 2\,a) + \cos(2\,bx + 2\,a))c^3d}{b^2}$$
$$+ \frac{(2\,b^3x^3 + 6\,bx\cos(2\,bx + 2\,a) + 3\,(2\,b^2x^2 - 1)\sin(2\,bx + 2\,a))c^2d^2}{b^3}$$
$$+ \frac{(b^4x^4 + 3\,(2\,b^2x^2 - 1)\cos(2\,bx + 2\,a) + 2\,(2\,b^3x^3 - 3\,bx)\sin(2\,bx + 2\,a))cd^3}{b^4}$$
$$+ \frac{(2\,b^5x^5 + 10\,(2\,b^3x^3 - 3\,bx)\cos(2\,bx + 2\,a) + 5\,(2\,b^4x^4 - 6\,b^2x^2 + 3)\sin(2\,bx + 2\,a))d^4}{10\,b^5}$$

input
```
integrate((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
(b*x + sin(2*b*x + 2*a))*c^4/b + 2*(b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + cos
(2*b*x + 2*a))*c^3*d/b^2 + (2*b^3*x^3 + 6*b*x*cos(2*b*x + 2*a) + 3*(2*b^2*
x^2 - 1)*sin(2*b*x + 2*a))*c^2*d^2/b^3 + (b^4*x^4 + 3*(2*b^2*x^2 - 1)*cos(
2*b*x + 2*a) + 2*(2*b^3*x^3 - 3*b*x)*sin(2*b*x + 2*a))*c*d^3/b^4 + 1/10*(2
*b^5*x^5 + 10*(2*b^3*x^3 - 3*b*x)*cos(2*b*x + 2*a) + 5*(2*b^4*x^4 - 6*b^2*
x^2 + 3)*sin(2*b*x + 2*a))*d^4/b^5
```

### 3.368.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1255 vs. $2(190) = 380$.

Time = 0.44 (sec) , antiderivative size = 1255, normalized size of antiderivative = 6.34

$$\int (c + dx)^4 \csc(a + bx) \sin(3a + 3bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
1/5*(10*b^4*c^4*cos(b*x + a)*sin(b*x + a) + 40*(b*x + a)*b^3*c^3*d*cos(b*x
 + a)*sin(b*x + a) - 40*a*b^3*c^3*d*cos(b*x + a)*sin(b*x + a) + 60*(b*x +
a)^2*b^2*c^2*d^2*cos(b*x + a)*sin(b*x + a) - 120*(b*x + a)*a*b^2*c^2*d^2*c
os(b*x + a)*sin(b*x + a) + 60*a^2*b^2*c^2*d^2*cos(b*x + a)*sin(b*x + a) +
40*(b*x + a)^3*b*c*d^3*cos(b*x + a)*sin(b*x + a) - 120*(b*x + a)^2*a*b*c*d
^3*cos(b*x + a)*sin(b*x + a) + 120*(b*x + a)*a^2*b*c*d^3*cos(b*x + a)*sin(
b*x + a) - 40*a^3*b*c*d^3*cos(b*x + a)*sin(b*x + a) + 10*(b*x + a)^4*d^4*c
os(b*x + a)*sin(b*x + a) - 40*(b*x + a)^3*a*d^4*cos(b*x + a)*sin(b*x + a)
+ 60*(b*x + a)^2*a^2*d^4*cos(b*x + a)*sin(b*x + a) - 40*(b*x + a)*a^3*d^4*
cos(b*x + a)*sin(b*x + a) + 10*a^4*d^4*cos(b*x + a)*sin(b*x + a) + 5*(b*x
+ a)*b^4*c^4 + 10*(b*x + a)^2*b^3*c^3*d - 20*(b*x + a)*a*b^3*c^3*d + 10*(b
*x + a)^3*b^2*c^2*d^2 - 30*(b*x + a)^2*a*b^2*c^2*d^2 + 30*(b*x + a)*a^2*b^
2*c^2*d^2 + 5*(b*x + a)^4*b*c*d^3 - 20*(b*x + a)^3*a*b*c*d^3 + 30*(b*x + a
)^2*a^2*b*c*d^3 - 20*(b*x + a)*a^3*b*c*d^3 + (b*x + a)^5*d^4 - 5*(b*x + a)
^4*a*d^4 + 10*(b*x + a)^3*a^2*d^4 - 10*(b*x + a)^2*a^3*d^4 + 5*(b*x + a)*a
^4*d^4 + 10*b^3*c^3*d*cos(b*x + a)^2 + 30*(b*x + a)*b^2*c^2*d^2*cos(b*x +
a)^2 - 30*a*b^2*c^2*d^2*cos(b*x + a)^2 + 30*(b*x + a)^2*b*c*d^3*cos(b*x +
a)^2 - 60*(b*x + a)*a*b*c*d^3*cos(b*x + a)^2 + 30*a^2*b*c*d^3*cos(b*x + a)
^2 + 10*(b*x + a)^3*d^4*cos(b*x + a)^2 - 30*(b*x + a)^2*a*d^4*cos(b*x + a)
^2 + 30*(b*x + a)*a^2*d^4*cos(b*x + a)^2 - 10*a^3*d^4*cos(b*x + a)^2 - ...
```

### 3.368.9 Mupad [B] (verification not implemented)

Time = 26.45 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.74

$$\int (c+dx)^4 \csc(a+bx)\sin(3a+3bx)\,dx$$
$$= \frac{\frac{3\,d^4\,\sin(2\,a+2\,b\,x)}{2} + b^5\,c^4\,x + b^4\,c^4\,\sin(2\,a+2\,b\,x) + \frac{b^5\,d^4\,x^5}{5} + 2\,b^3\,c^3\,d\,\cos(2\,a+2\,b\,x) + 2\,b^5\,c^3\,d\,x^2 + b^5\,c\,d^3\,x}{}$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x)^4)/sin(a + b*x),x)
```

output
```
((3*d^4*sin(2*a + 2*b*x))/2 + b^5*c^4*x + b^4*c^4*sin(2*a + 2*b*x) + (b^5*
d^4*x^5)/5 + 2*b^3*c^3*d*cos(2*a + 2*b*x) + 2*b^5*c^3*d*x^2 + b^5*c*d^3*x^
4 - 3*b^2*c^2*d^2*sin(2*a + 2*b*x) + 2*b^3*d^4*x^3*cos(2*a + 2*b*x) + 2*b^
5*c^2*d^2*x^3 - 3*b^2*d^4*x^2*sin(2*a + 2*b*x) + b^4*d^4*x^4*sin(2*a + 2*b
*x) - 3*b*c*d^3*cos(2*a + 2*b*x) - 3*b*d^4*x*cos(2*a + 2*b*x) + 6*b^4*c^2*
d^2*x^2*sin(2*a + 2*b*x) - 6*b^2*c*d^3*x*sin(2*a + 2*b*x) + 4*b^4*c^3*d*x*
sin(2*a + 2*b*x) + 6*b^3*c^2*d^2*x*cos(2*a + 2*b*x) + 6*b^3*c*d^3*x^2*cos(
2*a + 2*b*x) + 4*b^4*c*d^3*x^3*sin(2*a + 2*b*x))/b^5
```

# 3.369 $\int (c+dx)^3 \csc(a+bx)\sin(3a+3bx)\,dx$

3.369.1 Optimal result . . . . . 2660
3.369.2 Mathematica [A] (verified) . . . . . 2661
3.369.3 Rubi [A] (verified) . . . . . 2661
3.369.4 Maple [A] (verified) . . . . . 2662
3.369.5 Fricas [A] (verification not implemented) . . . . . 2663
3.369.6 Sympy [F] . . . . . 2663
3.369.7 Maxima [A] (verification not implemented) . . . . . 2663
3.369.8 Giac [B] (verification not implemented) . . . . . 2664
3.369.9 Mupad [B] (verification not implemented) . . . . . 2665

## 3.369.1 Optimal result

Integrand size = 23, antiderivative size = 171

$$\int (c+dx)^3 \csc(a+bx)\sin(3a+3bx)\,dx = -\frac{3cd^2x}{2b^2} - \frac{3d^3x^2}{4b^2} + \frac{(c+dx)^4}{4d} - \frac{9d^3\cos^2(a+bx)}{8b^4} + \frac{9d(c+dx)^2\cos^2(a+bx)}{4b^2} - \frac{3d^2(c+dx)\cos(a+bx)\sin(a+bx)}{b^3} + \frac{2(c+dx)^3\cos(a+bx)\sin(a+bx)}{b} + \frac{3d^3\sin^2(a+bx)}{8b^4} - \frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}$$

output
```
-3/2*c*d^2*x/b^2-3/4*d^3*x^2/b^2+1/4*(d*x+c)^4/d-9/8*d^3*cos(b*x+a)^2/b^4+
9/4*d*(d*x+c)^2*cos(b*x+a)^2/b^2-3*d^2*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^3+2
*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b+3/8*d^3*sin(b*x+a)^2/b^4-3/4*d*(d*x+c)^
2*sin(b*x+a)^2/b^2
```

### 3.369.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.61

$$\int (c+dx)^3 \csc(a+bx)\sin(3a+3bx)\,dx$$
$$= \frac{b^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)+3d(-d^2+2b^2(c+dx)^2)\cos(2(a+bx))+2b(c+dx)\left(-3d^2+2b^2(c+\right.}{4b^4}$$

input
```
Integrate[(c + d*x)^3*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
(b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 3*d*(-d^2 + 2*b^2*(c
+ d*x)^2)*Cos[2*(a + b*x)] + 2*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Si
n[2*(a + b*x)])/(4*b^4)
```

### 3.369.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin(3a+3bx)\csc(a+bx)\,dx$$
$$\Big\downarrow \text{4931}$$
$$\int \left(3(c+dx)^3\cos^2(a+bx)-(c+dx)^3\sin^2(a+bx)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{3d^3\sin^2(a+bx)}{8b^4}-\frac{9d^3\cos^2(a+bx)}{8b^4}-\frac{3d^2(c+dx)\sin(a+bx)\cos(a+bx)}{b^3}-\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}+\frac{9d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{2(c+dx)^3\sin(a+bx)\cos(a+bx)}{b}-\frac{3d(c+dx)^2}{4b^2}+\frac{(c+dx)^4}{4d}$$

input
```
Int[(c + d*x)^3*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
(-3*d*(c + d*x)^2)/(4*b^2) + (c + d*x)^4/(4*d) - (9*d^3*Cos[a + b*x]^2)/(8
*b^4) + (9*d*(c + d*x)^2*Cos[a + b*x]^2)/(4*b^2) - (3*d^2*(c + d*x)*Cos[a
+ b*x]*Sin[a + b*x])/b^3 + (2*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x])/b + (
3*d^3*Sin[a + b*x]^2)/(8*b^4) - (3*d*(c + d*x)^2*Sin[a + b*x]^2)/(4*b^2)
```

### 3.369.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

## 3.369.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.91

| method | result |
|---|---|
| risch | $\frac{d^{3} x^{4}}{4}+c\,d^{2} x^{3}+\frac{3 c^{2} d\,x^{2}}{2}+c^{3} x +\frac{c^{4}}{4 d}+\frac{3 d \left(2 x^{2} d^{2} b^{2}+4 b^{2} c d x +2 b^{2} c^{2}-d^{2}\right) \cos \left(2 x b +2 a \right)}{4 b^{4}}+\frac{\left(2 b^{2} d^{3} x^{3}+6 b^{2} c\,d^{2} x^{2}+6 b^{2} c^{2} d x +\right.}{2}$ |
| default | $-c^{3} x -\frac{d^{3} x^{4}}{4}+\frac{4 c^{3} \left(\frac{\cos \left(x b +a \right) \sin \left(x b +a \right)}{2}+\frac{x b}{2}+\frac{a}{2}\right)}{b}-c\,d^{2} x^{3}-\frac{3 c^{2} d\,x^{2}}{2}+\frac{4 d^{3} \left(\left(x b +a \right)^{3} \left(\frac{\cos \left(x b +a \right) \sin \left(x b +a \right)}{2}+\frac{x b}{2}+\frac{a}{2}\right)+\frac{3(x}{}\right.}{}$ |

input
```
int((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output
```
1/4*d^3*x^4+c*d^2*x^3+3/2*c^2*d*x^2+c^3*x+1/4/d*c^4+3/4*d*(2*b^2*d^2*x^2+4
*b^2*c*d*x+2*b^2*c^2-d^2)/b^4*cos(2*b*x+2*a)+1/2/b^3*(2*b^2*d^3*x^3+6*b^2*
c*d^2*x^2+6*b^2*c^2*d*x+2*b^2*c^3-3*d^3*x-3*c*d^2)*sin(2*b*x+2*a)
```

### 3.369.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.10

$$\int (c+dx)^3 \csc(a+bx) \sin(3a+3bx) \, dx = \frac{b^4d^3x^4 + 4\,b^4cd^2x^3 + 6\,(b^4c^2d - b^2d^3)x^2 + 6\,(2\,b^2d^3x^2 + 4\,b^2cd^2x + 2\,b^2c^2d - d^3)\cos{(bx+a)}^2 + 4\,(2\,b^3d^3x}{4\,b^4}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

output
```
1/4*(b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 6*(b^4*c^2*d - b^2*d^3)*x^2 + 6*(2*b^
2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)^2 + 4*(2*b^3*d
^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)
*x)*cos(b*x + a)*sin(b*x + a) + 4*(b^4*c^3 - 3*b^2*c*d^2)*x)/b^4
```

### 3.369.6 Sympy [F]

$$\int (c+dx)^3 \csc(a+bx) \sin(3a+3bx) \, dx = \int (c+dx)^3 \sin{(3a+3bx)} \csc{(a+bx)} \, dx$$

input
```
integrate((d*x+c)**3*csc(b*x+a)*sin(3*b*x+3*a),x)
```

output
```
Integral((c + d*x)**3*sin(3*a + 3*b*x)*csc(a + b*x), x)
```

### 3.369.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.01

$$\begin{aligned}\int (c+dx)^3 \csc(a+bx) \sin(3a+3bx) \, dx &= \frac{(bx + \sin{(2\,bx+2\,a)})c^3}{b} + \frac{3\,(b^2x^2 + 2\,bx\sin{(2\,bx+2\,a)} + \cos{(2\,bx+2\,a)})c^2d}{2\,b^2} \\ &+ \frac{(2\,b^3x^3 + 6\,bx\cos{(2\,bx+2\,a)} + 3\,(2\,b^2x^2-1)\sin{(2\,bx+2\,a)})cd^2}{2\,b^3} \\ &+ \frac{(b^4x^4 + 3\,(2\,b^2x^2-1)\cos{(2\,bx+2\,a)} + 2\,(2\,b^3x^3 - 3\,bx)\sin{(2\,bx+2\,a)})d^3}{4\,b^4}\end{aligned}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
(b*x + sin(2*b*x + 2*a))*c^3/b + 3/2*(b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + c
os(2*b*x + 2*a))*c^2*d/b^2 + 1/2*(2*b^3*x^3 + 6*b*x*cos(2*b*x + 2*a) + 3*(
2*b^2*x^2 - 1)*sin(2*b*x + 2*a))*c*d^2/b^3 + 1/4*(b^4*x^4 + 3*(2*b^2*x^2 -
 1)*cos(2*b*x + 2*a) + 2*(2*b^3*x^3 - 3*b*x)*sin(2*b*x + 2*a))*d^3/b^4
```

### 3.369.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(157) = 314$.

Time = 0.37 (sec) , antiderivative size = 682, normalized size of antiderivative = 3.99

$$\int (c+dx)^3 \csc(a+bx) \sin(3a+3bx) \, dx$$

$$= \frac{8\,b^3c^3\cos{(bx+a)}\sin{(bx+a)} + 24\,(bx+a)b^2c^2d\cos{(bx+a)}\sin{(bx+a)} - 24\,ab^2c^2d\cos{(bx+a)}\sin{(bx}{}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
1/4*(8*b^3*c^3*cos(b*x + a)*sin(b*x + a) + 24*(b*x + a)*b^2*c^2*d*cos(b*x
+ a)*sin(b*x + a) - 24*a*b^2*c^2*d*cos(b*x + a)*sin(b*x + a) + 24*(b*x + a
)^2*b*c*d^2*cos(b*x + a)*sin(b*x + a) - 48*(b*x + a)*a*b*c*d^2*cos(b*x + a
)*sin(b*x + a) + 24*a^2*b*c*d^2*cos(b*x + a)*sin(b*x + a) + 8*(b*x + a)^3*
d^3*cos(b*x + a)*sin(b*x + a) - 24*(b*x + a)^2*a*d^3*cos(b*x + a)*sin(b*x
+ a) + 24*(b*x + a)*a^2*d^3*cos(b*x + a)*sin(b*x + a) - 8*a^3*d^3*cos(b*x
+ a)*sin(b*x + a) + 4*(b*x + a)*b^3*c^3 + 6*(b*x + a)^2*b^2*c^2*d - 12*(b*
x + a)*a*b^2*c^2*d + 4*(b*x + a)^3*b*c*d^2 - 12*(b*x + a)^2*a*b*c*d^2 + 12
*(b*x + a)*a^2*b*c*d^2 + (b*x + a)^4*d^3 - 4*(b*x + a)^3*a*d^3 + 6*(b*x +
a)^2*a^2*d^3 - 4*(b*x + a)*a^3*d^3 + 6*b^2*c^2*d*cos(b*x + a)^2 + 12*(b*x
+ a)*b*c*d^2*cos(b*x + a)^2 - 12*a*b*c*d^2*cos(b*x + a)^2 + 6*(b*x + a)^2*
d^3*cos(b*x + a)^2 - 12*(b*x + a)*a*d^3*cos(b*x + a)^2 + 6*a^2*d^3*cos(b*x
 + a)^2 - 6*b^2*c^2*d*sin(b*x + a)^2 - 12*(b*x + a)*b*c*d^2*sin(b*x + a)^2
 + 12*a*b*c*d^2*sin(b*x + a)^2 - 6*(b*x + a)^2*d^3*sin(b*x + a)^2 + 12*(b*
x + a)*a*d^3*sin(b*x + a)^2 - 6*a^2*d^3*sin(b*x + a)^2 - 12*b*c*d^2*cos(b*
x + a)*sin(b*x + a) - 12*(b*x + a)*d^3*cos(b*x + a)*sin(b*x + a) + 12*a*d^
3*cos(b*x + a)*sin(b*x + a) - 3*d^3*cos(b*x + a)^2 + 3*d^3*sin(b*x + a)^2)
/b^4
```

### 3.369.9 Mupad [B] (verification not implemented)

Time = 26.39 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.26

$$\int (c+dx)^3\csc(a+bx)\sin(3a+3bx)\,dx = c^3\,x+\frac{d^3\,x^4}{4}+\frac{3\,c^2\,d\,x^2}{2}+c\,d^2\,x^3-\frac{3\,d^3\,\cos\left(2\,a+2\,b\,x\right)}{4\,b^4}+\frac{c^3\,\sin\left(2\,a+2\,b\,x\right)}{b}+\frac{3\,c^2\,d\,\cos\left(2\,a+2\,b\,x\right)}{2\,b^2}-\frac{3\,c\,d^2\,\sin\left(2\,a+2\,b\,x\right)}{2\,b^3}-\frac{3\,d^3\,x\,\sin\left(2\,a+2\,b\,x\right)}{2\,b^3}+\frac{3\,d^3\,x^2\,\cos\left(2\,a+2\,b\,x\right)}{2\,b^2}+\frac{d^3\,x^3\,\sin\left(2\,a+2\,b\,x\right)}{b}+\frac{3\,c\,d^2\,x\,\cos\left(2\,a+2\,b\,x\right)}{b^2}+\frac{3\,c^2\,d\,x\,\sin\left(2\,a+2\,b\,x\right)}{b}+\frac{3\,c\,d^2\,x^2\,\sin\left(2\,a+2\,b\,x\right)}{b}$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x)^3)/sin(a + b*x),x)
```

output
```
c^3*x + (d^3*x^4)/4 + (3*c^2*d*x^2)/2 + c*d^2*x^3 - (3*d^3*cos(2*a + 2*b*x
))/(4*b^4) + (c^3*sin(2*a + 2*b*x))/b + (3*c^2*d*cos(2*a + 2*b*x))/(2*b^2)
 - (3*c*d^2*sin(2*a + 2*b*x))/(2*b^3) - (3*d^3*x*sin(2*a + 2*b*x))/(2*b^3)
 + (3*d^3*x^2*cos(2*a + 2*b*x))/(2*b^2) + (d^3*x^3*sin(2*a + 2*b*x))/b + (
3*c*d^2*x*cos(2*a + 2*b*x))/b^2 + (3*c^2*d*x*sin(2*a + 2*b*x))/b + (3*c*d^
2*x^2*sin(2*a + 2*b*x))/b
```

# 3.370 $\int (c+dx)^2 \csc(a+bx) \sin(3a+3bx) \, dx$

3.370.1 Optimal result . . . . . 2666
3.370.2 Mathematica [A] (verified) . . . . . 2666
3.370.3 Rubi [A] (verified) . . . . . 2667
3.370.4 Maple [A] (verified) . . . . . 2668
3.370.5 Fricas [A] (verification not implemented) . . . . . 2668
3.370.6 Sympy [F] . . . . . 2669
3.370.7 Maxima [A] (verification not implemented) . . . . . 2669
3.370.8 Giac [B] (verification not implemented) . . . . . 2670
3.370.9 Mupad [B] (verification not implemented) . . . . . 2670

## 3.370.1 Optimal result

Integrand size = 23, antiderivative size = 112

$$\int (c+dx)^2 \csc(a+bx) \sin(3a+3bx) \, dx = -\frac{d^2x}{2b^2} + \frac{(c+dx)^3}{3d} + \frac{3d(c+dx)\cos^2(a+bx)}{2b^2} - \frac{d^2\cos(a+bx)\sin(a+bx)}{b^3} + \frac{2(c+dx)^2\cos(a+bx)\sin(a+bx)}{b} - \frac{d(c+dx)\sin^2(a+bx)}{2b^2}$$

output

```
-1/2*d^2*x/b^2+1/3*(d*x+c)^3/d+3/2*d*(d*x+c)*cos(b*x+a)^2/b^2-d^2*cos(b*x+
a)*sin(b*x+a)/b^3+2*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b-1/2*d*(d*x+c)*sin(b*
x+a)^2/b^2
```

## 3.370.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.65

$$\int (c+dx)^2 \csc(a+bx) \sin(3a+3bx) \, dx = c^2x + cdx^2 + \frac{d^2x^3}{3} + \frac{d(c+dx)\cos(2(a+bx))}{b^2} + \frac{(-d^2+2b^2(c+dx)^2)\sin(2(a+bx))}{2b^3}$$

input
```
Integrate[(c + d*x)^2*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
c^2*x + c*d*x^2 + (d^2*x^3)/3 + (d*(c + d*x)*Cos[2*(a + b*x)])/b^2 + ((-d^
2 + 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(2*b^3)
```

### 3.370.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin(3a+3bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{4931}$$

$$\int \left(3(c+dx)^2\cos^2(a+bx)-(c+dx)^2\sin^2(a+bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2\sin(a+bx)\cos(a+bx)}{b^3}-\frac{d(c+dx)\sin^2(a+bx)}{2b^2}+\frac{3d(c+dx)\cos^2(a+bx)}{2b^2}+\frac{2(c+dx)^2\sin(a+bx)\cos(a+bx)}{b}-\frac{d^2x}{2b^2}+\frac{(c+dx)^3}{3d}$$

input
```
Int[(c + d*x)^2*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
-1/2*(d^2*x)/b^2 + (c + d*x)^3/(3*d) + (3*d*(c + d*x)*Cos[a + b*x]^2)/(2*b
^2) - (d^2*Cos[a + b*x]*Sin[a + b*x])/b^3 + (2*(c + d*x)^2*Cos[a + b*x]*Si
n[a + b*x])/b - (d*(c + d*x)*Sin[a + b*x]^2)/(2*b^2)
```

### 3.370.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

## 3.370.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.85

| method | result |
|---|---|
| risch | $\frac{d^2x^3}{3}+dc\,x^2+c^2x+\frac{c^3}{3d}+\frac{d(dx+c)\cos(2xb+2a)}{b^2}+\frac{(2x^2d^2b^2+4b^2cdx+2b^2c^2-d^2)\sin(2xb+2a)}{2b^3}$ |
| default | $-c^2x-\frac{d^2x^3}{3}+\frac{4c^2\left(\frac{\cos(xb+a)\sin(xb+a)}{2}+\frac{xb}{2}+\frac{a}{2}\right)}{b}-dc\,x^2+\frac{4d^2\left((xb+a)^2\left(\frac{\cos(xb+a)\sin(xb+a)}{2}+\frac{xb}{2}+\frac{a}{2}\right)+\frac{(xb+a)\cos(xb+a)^2}{2}\right.}{}$ |

input
```
int((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output
```
1/3*d^2*x^3+d*c*x^2+c^2*x+1/3/d*c^3+1/b^2*d*(d*x+c)*cos(2*b*x+2*a)+1/2*(2*
b^2*d^2*x^2+4*b^2*c*d*x+2*b^2*c^2-d^2)/b^3*sin(2*b*x+2*a)
```

## 3.370.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.99

$$\int (c+dx)^2\csc(a+bx)\sin(3a+3bx)\,dx$$
$$=\frac{b^3d^2x^3+3\,b^3cdx^2+6\,(bd^2x+bcd)\cos{(bx+a)}^2+3\,(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2)\cos{(bx+a)}\sin{(bx}}{3\,b^3}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

output
```
1/3*(b^3*d^2*x^3 + 3*b^3*c*d*x^2 + 6*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + 3*
(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(b*x + a)*sin(b*x + a)
+ 3*(b^3*c^2 - b*d^2)*x)/b^3
```

### 3.370.6 Sympy [F]

$$\int (c+dx)^2 \csc(a+bx)\sin(3a+3bx)\,dx = \int (c+dx)^2 \sin(3a+3bx)\csc(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*csc(b*x+a)*sin(3*b*x+3*a),x)
```

output
```
Integral((c + d*x)**2*sin(3*a + 3*b*x)*csc(a + b*x), x)
```

### 3.370.7 Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.96

$$\int (c+dx)^2 \csc(a+bx)\sin(3a+3bx)\,dx = \frac{(bx+\sin(2\,bx+2\,a))c^2}{b} + \frac{(b^2x^2+2\,bx\sin(2\,bx+2\,a)+\cos(2\,bx+2\,a))cd}{b^2} + \frac{(2\,b^3x^3+6\,bx\cos(2\,bx+2\,a)+3\,(2\,b^2x^2-1)\sin(2\,bx+2\,a))d^2}{6\,b^3}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
(b*x + sin(2*b*x + 2*a))*c^2/b + (b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + cos(2
*b*x + 2*a))*c*d/b^2 + 1/6*(2*b^3*x^3 + 6*b*x*cos(2*b*x + 2*a) + 3*(2*b^2*
x^2 - 1)*sin(2*b*x + 2*a))*d^2/b^3
```

### 3.370.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(104) = 208$.

Time = 0.32 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.79

$$\int (c+dx)^2 \csc(a+bx) \sin(3a+3bx)\, dx$$
$$= \frac{6\, b^2 c^2 \cos{(bx+a)} \sin{(bx+a)} + 12\, (bx+a)bcd \cos{(bx+a)} \sin{(bx+a)} - 12\, abcd \cos{(bx+a)} \sin{(bx+a}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
1/3*(6*b^2*c^2*cos(b*x + a)*sin(b*x + a) + 12*(b*x + a)*b*c*d*cos(b*x + a)
*sin(b*x + a) - 12*a*b*c*d*cos(b*x + a)*sin(b*x + a) + 6*(b*x + a)^2*d^2*c
os(b*x + a)*sin(b*x + a) - 12*(b*x + a)*a*d^2*cos(b*x + a)*sin(b*x + a) +
6*a^2*d^2*cos(b*x + a)*sin(b*x + a) + 3*(b*x + a)*b^2*c^2 + 3*(b*x + a)^2*
b*c*d - 6*(b*x + a)*a*b*c*d + (b*x + a)^3*d^2 - 3*(b*x + a)^2*a*d^2 + 3*(b
*x + a)*a^2*d^2 + 3*b*c*d*cos(b*x + a)^2 + 3*(b*x + a)*d^2*cos(b*x + a)^2
- 3*a*d^2*cos(b*x + a)^2 - 3*b*c*d*sin(b*x + a)^2 - 3*(b*x + a)*d^2*sin(b*
x + a)^2 + 3*a*d^2*sin(b*x + a)^2 - 3*d^2*cos(b*x + a)*sin(b*x + a))/b^3
```

### 3.370.9 Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.08

$$\int (c+dx)^2 \csc(a+bx) \sin(3a+3bx)\, dx = c^2\, x + \frac{d^2\, x^3}{3} + \frac{c^2\, \sin{(2\, a + 2\, b\, x)}}{b} - \frac{d^2\, \sin{(2\, a + 2\, b\, x)}}{2\, b^3} + c\, d\, x^2 + \frac{d^2\, x\, \cos{(2\, a + 2\, b\, x)}}{b^2} + \frac{d^2\, x^2\, \sin{(2\, a + 2\, b\, x)}}{b} + \frac{c\, d\, \cos{(2\, a + 2\, b\, x)}}{b^2} + \frac{2\, c\, d\, x\, \sin{(2\, a + 2\, b\, x)}}{b}$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x)^2)/sin(a + b*x),x)
```

output
```
c^2*x + (d^2*x^3)/3 + (c^2*sin(2*a + 2*b*x))/b - (d^2*sin(2*a + 2*b*x))/(2
*b^3) + c*d*x^2 + (d^2*x*cos(2*a + 2*b*x))/b^2 + (d^2*x^2*sin(2*a + 2*b*x)
)/b + (c*d*cos(2*a + 2*b*x))/b^2 + (2*c*d*x*sin(2*a + 2*b*x))/b
```

# 3.371 $\int (c + dx) \csc(a + bx) \sin(3a + 3bx) \, dx$

3.371.1 Optimal result . . . 2671
3.371.2 Mathematica [A] (verified) . . . 2671
3.371.3 Rubi [A] (verified) . . . 2672
3.371.4 Maple [A] (verified) . . . 2673
3.371.5 Fricas [A] (verification not implemented) . . . 2673
3.371.6 Sympy [F] . . . 2673
3.371.7 Maxima [A] (verification not implemented) . . . 2674
3.371.8 Giac [A] (verification not implemented) . . . 2674
3.371.9 Mupad [B] (verification not implemented) . . . 2674

## 3.371.1 Optimal result

Integrand size = 21, antiderivative size = 66

$$\int (c + dx) \csc(a + bx) \sin(3a + 3bx) \, dx = cx + \frac{dx^2}{2} + \frac{3d \cos^2(a + bx)}{4b^2} + \frac{2(c + dx) \cos(a + bx) \sin(a + bx)}{b} - \frac{d \sin^2(a + bx)}{4b^2}$$

output
```
c*x+1/2*d*x^2+3/4*d*cos(b*x+a)^2/b^2+2*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b-1/4
*d*sin(b*x+a)^2/b^2
```

## 3.371.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.70

$$\int (c + dx) \csc(a + bx) \sin(3a + 3bx) \, dx = \frac{d \cos(2(a + bx)) + b(bx(2c + dx) + 2(c + dx) \sin(2(a + bx)))}{2b^2}$$

input
```
Integrate[(c + d*x)*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
(d*Cos[2*(a + b*x)] + b*(b*x*(2*c + d*x) + 2*(c + d*x)*Sin[2*(a + b*x)]))/
(2*b^2)
```

## 3.371.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin(3a+3bx)\csc(a+bx)\,dx$$

$$\Big\downarrow \text{4931}$$

$$\int \left(3(c+dx)\cos^2(a+bx)-(c+dx)\sin^2(a+bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\sin^2(a+bx)}{4b^2}+\frac{3d\cos^2(a+bx)}{4b^2}+\frac{2(c+dx)\sin(a+bx)\cos(a+bx)}{b}+\frac{(c+dx)^2}{2d}$$

input
```
Int[(c + d*x)*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
(c + d*x)^2/(2*d) + (3*d*Cos[a + b*x]^2)/(4*b^2) + (2*(c + d*x)*Cos[a + b*
x]*Sin[a + b*x])/b - (d*Sin[a + b*x]^2)/(4*b^2)
```

### 3.371.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.371.4 Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.67

| method | result |
|---|---|
| risch | $\frac{d\,x^{2}}{2}+cx+\frac{d\cos\left(2xb+2a\right)}{2b^{2}}+\frac{\left(dx+c\right)\sin\left(2xb+2a\right)}{b}$ |
| default | $-cx-\frac{d\,x^{2}}{2}+\frac{4c\left(\frac{\cos\left(xb+a\right)\sin\left(xb+a\right)}{2}+\frac{xb}{2}+\frac{a}{2}\right)}{b}+\frac{4d\left(\left(xb+a\right)\left(\frac{\cos\left(xb+a\right)\sin\left(xb+a\right)}{2}+\frac{xb}{2}+\frac{a}{2}\right)-\frac{\left(xb+a\right)^{2}}{4}-\frac{\sin\left(xb+a\right)^{2}}{4}-a\left(\frac{\cos(xb+a}{}\right.\right.}{b^{2}}$ |

input
```
int((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output
```
1/2*d*x^2+c*x+1/2*d/b^2*cos(2*b*x+2*a)+1/b*(d*x+c)*sin(2*b*x+2*a)
```

### 3.371.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.82

$$\int (c+dx)\csc(a+bx)\sin(3a+3bx)\,dx = \frac{b^{2}dx^{2}+2\,b^{2}cx+2\,d\cos\left(bx+a\right)^{2}+4\left(bdx+bc\right)\cos\left(bx+a\right)\sin\left(bx+a\right)}{2\,b^{2}}$$

input
```
integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

output
```
1/2*(b^2*d*x^2 + 2*b^2*c*x + 2*d*cos(b*x + a)^2 + 4*(b*d*x + b*c)*cos(b*x
+ a)*sin(b*x + a))/b^2
```

### 3.371.6 Sympy [F]

$$\int (c+dx)\csc(a+bx)\sin(3a+3bx)\,dx = \int (c+dx)\sin\left(3a+3bx\right)\csc\left(a+bx\right)dx$$

input
```
integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x)
```

output
```
Integral((c + d*x)*sin(3*a + 3*b*x)*csc(a + b*x), x)
```

### 3.371.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int (c+dx)\csc(a+bx)\sin(3a+3bx)\,dx = \frac{(bx+\sin(2\,bx+2\,a))c}{b} + \frac{(b^2x^2+2\,bx\sin(2\,bx+2\,a)+\cos(2\,bx+2\,a))d}{2\,b^2}$$

input
```
integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
(b*x + sin(2*b*x + 2*a))*c/b + 1/2*(b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + cos
(2*b*x + 2*a))*d/b^2
```

### 3.371.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.61

$$\int (c+dx)\csc(a+bx)\sin(3a+3bx)\,dx = \frac{4\,bc\cos(bx+a)\sin(bx+a)+4\,(bx+a)d\cos(bx+a)\sin(bx+a)-4\,ad\cos(bx+a)\sin(bx+a)+2\,(bx}{2\,b^2}$$

input
```
integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
1/2*(4*b*c*cos(b*x + a)*sin(b*x + a) + 4*(b*x + a)*d*cos(b*x + a)*sin(b*x
+ a) - 4*a*d*cos(b*x + a)*sin(b*x + a) + 2*(b*x + a)*b*c + (b*x + a)^2*d -
 2*(b*x + a)*a*d + d*cos(b*x + a)^2 - d*sin(b*x + a)^2)/b^2
```

### 3.371.9 Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int (c+dx)\csc(a+bx)\sin(3a+3bx)\,dx = c\,x+\frac{d\,x^2}{2}+\frac{\frac{d\,\cos(2\,a+2\,b\,x)}{2}+b\,(c\,\sin(2\,a+2\,b\,x)+d\,x\,\sin(2\,a+2\,b\,x))}{b^2}$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x))/sin(a + b*x),x)
```

output
```
c*x + (d*x^2)/2 + ((d*cos(2*a + 2*b*x))/2 + b*(c*sin(2*a + 2*b*x) + d*x*si
n(2*a + 2*b*x)))/b^2
```

---

3.371. $\int (c + dx) \csc(a + bx) \sin(3a + 3bx) \, dx$

# 3.372 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx$

3.372.1 Optimal result . . . 2676
3.372.2 Mathematica [A] (verified) . . . 2676
3.372.3 Rubi [A] (verified) . . . 2677
3.372.4 Maple [C] (verified) . . . 2678
3.372.5 Fricas [A] (verification not implemented) . . . 2678
3.372.6 Sympy [F] . . . 2679
3.372.7 Maxima [C] (verification not implemented) . . . 2679
3.372.8 Giac [A] (verification not implemented) . . . 2679
3.372.9 Mupad [F(-1)] . . . 2680

## 3.372.1 Optimal result

Integrand size = 23, antiderivative size = 71

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \frac{2\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d} + \frac{\log(c+dx)}{d} - \frac{2\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{d}$$

output
```
2*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d+ln(d*x+c)/d-2*Si(2*b*c/d+2*b*x)*sin
(2*a-2*b*c/d)/d
```

## 3.372.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \frac{2\cos\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)+\log(c+dx)-2\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2b(c+dx)}{d}\right)}{d}$$

input
```
Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]
```

output
```
(2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + Log[c + d*x] - 2*
Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d
```

### 3.372.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\csc(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{4931}$$

$$\int \left(\frac{3\cos^2(a+bx)}{c+dx}-\frac{\sin^2(a+bx)}{c+dx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}-\frac{2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{\log(c+dx)}{d}$$

input
```
Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]
```

output
```
(2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d + Log[c + d*x]/d
 - (2*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d
```

#### 3.372.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.372.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.49

| method | result |
|---|---|
| risch | $\frac{\ln(dx+c)}{d}-\frac{e^{-\frac{2i(ad-cb)}{d}}\operatorname{Ei}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{d}-\frac{e^{\frac{2i(ad-cb)}{d}}\operatorname{Ei}_1\left(-2ibx-2ia-\frac{2(-iad+icb)}{d}\right)}{d}$ |
| default | $-\frac{\ln(dx+c)}{d}+\frac{2\operatorname{Si}\left(2xb+2a+\frac{-2ad+2cb}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d}+\frac{2\operatorname{Ci}\left(2xb+2a+\frac{-2ad+2cb}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}+\frac{2\ln(-ad+cb+d(xb+}{d}$ |

input
```
int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
ln(d*x+c)/d-1/d*exp(-2*I*(a*d-b*c)/d)*Ei(1,2*I*b*x+2*I*a-2*I*(a*d-b*c)/d)-
1/d*exp(2*I*(a*d-b*c)/d)*Ei(1,-2*I*x*b-2*I*a-2*(-I*a*d+I*c*b)/d)
```

### 3.372.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \frac{2\cos\left(-\frac{2(bc-ad)}{d}\right)\operatorname{Ci}\left(\frac{2(bdx+bc)}{d}\right)-2\sin\left(-\frac{2(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{2(bdx+bc)}{d}\right)+\log(dx+c)}{d}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
```

output
```
(2*cos(-2*(b*c - a*d)/d)*cos_integral(2*(b*d*x + b*c)/d) - 2*sin(-2*(b*c -
 a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) + log(d*x + c))/d
```

### 3.372.6 Sympy [F]

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sin(3a+3bx)\csc(a+bx)}{c+dx}\,dx$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

output
```
Integral(sin(3*a + 3*b*x)*csc(a + b*x)/(c + d*x), x)
```

### 3.372.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.68

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx = -\frac{\left(E_1\left(\frac{2(-i\,bdx-i\,bc)}{d}\right)+E_1\left(-\frac{2(-i\,bdx-i\,bc)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)-\left(-i\,E_1\left(\frac{2(-i\,bdx-i\,bc)}{d}\right)+i\,E_1\left(-\frac{2(-i\,bdx-i\,bc)}{d}\right)\right.}{d}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")
```

output
```
-((exp_integral_e(1, 2*(-I*b*d*x - I*b*c)/d) + exp_integral_e(1, -2*(-I*b*
d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) - (-I*exp_integral_e(1, 2*(-I*b*d*x
 - I*b*c)/d) + I*exp_integral_e(1, -2*(-I*b*d*x - I*b*c)/d))*sin(-2*(b*c -
 a*d)/d) - log(d*x + c))/d
```

### 3.372.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.44

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \frac{2\,b\cos\left(-\frac{2(bc-ad)}{d}\right)\operatorname{Ci}\left(\frac{2(bc+(bx+a)d-ad)}{d}\right)+2\,b\sin\left(-\frac{2(bc-ad)}{d}\right)\operatorname{Si}\left(-\frac{2(bc+(bx+a)d-ad)}{d}\right)+b\log\left(bc+(bx+a\right.}{bd}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
```

output
```
(2*b*cos(-2*(b*c - a*d)/d)*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d) + 2
*b*sin(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x + a)*d - a*d)/d) + b*
log(b*c + (b*x + a)*d - a*d))/(b*d)
```

### 3.372.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sin(3\,a+3\,b\,x)}{\sin(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)),x)
```

output
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)), x)
```

# 3.373 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$


3.373.1 Optimal result . . . 2681
3.373.2 Mathematica [A] (verified) . . . 2681
3.373.3 Rubi [A] (verified) . . . 2682
3.373.4 Maple [C] (verified) . . . 2683
3.373.5 Fricas [A] (verification not implemented) . . . 2683
3.373.6 Sympy [F] . . . 2684
3.373.7 Maxima [C] (verification not implemented) . . . 2684
3.373.8 Giac [B] (verification not implemented) . . . 2684
3.373.9 Mupad [F(-1)] . . . 2685


## 3.373.1 Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = -\frac{3\cos^2(a+bx)}{d(c+dx)} - \frac{4b\,\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^2} + \frac{\sin^2(a+bx)}{d(c+dx)} - \frac{4b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}$$

output
```
-3*cos(b*x+a)^2/d/(d*x+c)-4*b*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^2-4*b*C
i(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^2+sin(b*x+a)^2/d/(d*x+c)
```

## 3.373.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = -\frac{\frac{d(1+2\cos(2(a+bx)))}{c+dx} + 4b\,\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right) + 4b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)}{d^2}$$

input
```
Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

output
```
-(((d*(1 + 2*Cos[2*(a + b*x)]))/(c + d*x) + 4*b*CosIntegral[(2*b*(c + d*x)
)/d]*Sin[2*a - (2*b*c)/d] + 4*b*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c +
 d*x))/d])/d^2)
```

### 3.373.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{4931}$$

$$\int \left(\frac{3\cos^2(a+bx)}{(c+dx)^2}-\frac{\sin^2(a+bx)}{(c+dx)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4b\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{4b\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{\sin^2(a+bx)}{d(c+dx)}-\frac{3\cos^2(a+bx)}{d(c+dx)}$$

input
```
Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

output
```
(-3*Cos[a + b*x]^2)/(d*(c + d*x)) - (4*b*CosIntegral[(2*b*c)/d + 2*b*x]*Si
n[2*a - (2*b*c)/d])/d^2 + Sin[a + b*x]^2/(d*(c + d*x)) - (4*b*Cos[2*a - (2
*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^2
```

#### 3.373.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.373.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.97 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.52

| method | result |
|---|---|
| risch | $-\frac{1}{d(dx+c)}+\frac{2ib\,\mathrm{e}^{-\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{d^2}-\frac{2ib\,\mathrm{e}^{\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(-2ibx-2ia-\frac{2(-iad+icb)}{d}\right)}{d^2}-\frac{(-2dxb-2cb)\cos(}{d(-dxb-cb)(}$ |
| default | $\frac{1}{d(dx+c)}+\frac{b^2\left(-\frac{2\cos(2xb+2a)}{(-ad+cb+d(xb+a))d}-\frac{2\left(\frac{2\,\mathrm{Si}\left(2xb+2a+\frac{-2ad+2cb}{d}\right)\cos\left(\frac{-2ad+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2xb+2a+\frac{-2ad+2cb}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d}\right)}{d}\right)-}{b}$ |

input
```
int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d/(d*x+c)+2*I*b/d^2*exp(-2*I*(a*d-b*c)/d)*Ei(1,2*I*b*x+2*I*a-2*I*(a*d-b
*c)/d)-2*I*b/d^2*exp(2*I*(a*d-b*c)/d)*Ei(1,-2*I*x*b-2*I*a-2*(-I*a*d+I*c*b)
/d)-1/d*(-2*b*d*x-2*b*c)/(-b*d*x-b*c)/(d*x+c)*cos(2*b*x+2*a)
```

### 3.373.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = -\frac{4\,d\cos\left(bx+a\right)^2+4\left(bdx+bc\right)\mathrm{Ci}\left(\frac{2\left(bdx+bc\right)}{d}\right)\sin\left(-\frac{2\left(bc-ad\right)}{d}\right)+4\left(bdx+bc\right)\cos\left(-\frac{2\left(bc-ad\right)}{d}\right)\mathrm{Si}\left(\frac{2\left(bdx+\right.}{d}\right.}{d^3x+cd^2}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-(4*d*cos(b*x + a)^2 + 4*(b*d*x + b*c)*cos_integral(2*(b*d*x + b*c)/d)*sin
(-2*(b*c - a*d)/d) + 4*(b*d*x + b*c)*cos(-2*(b*c - a*d)/d)*sin_integral(2*
(b*d*x + b*c)/d) - d)/(d^3*x + c*d^2)
```

### 3.373.6 Sympy [F]

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\sin{(3a+3bx)}\csc{(a+bx)}}{(c+dx)^2}\,dx$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**2,x)
```

output
```
Integral(sin(3*a + 3*b*x)*csc(a + b*x)/(c + d*x)**2, x)
```

### 3.373.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.18

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = -\frac{\left(E_2\left(\frac{2(-i\,bdx-i\,bc)}{d}\right)+E_2\left(-\frac{2(-i\,bdx-i\,bc)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)-\left(-i\,E_2\left(\frac{2(-i\,bdx-i\,bc)}{d}\right)+i\,E_2\left(-\frac{2(-i\,bdx-i\,bc)}{d}\right.\right.}{d^2x+cd}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-((exp_integral_e(2, 2*(-I*b*d*x - I*b*c)/d) + exp_integral_e(2, -2*(-I*b*
d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) - (-I*exp_integral_e(2, 2*(-I*b*d*x
 - I*b*c)/d) + I*exp_integral_e(2, -2*(-I*b*d*x - I*b*c)/d))*sin(-2*(b*c -
 a*d)/d) + 1)/(d^2*x + c*d)
```

### 3.373.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(102) = 204.

Time = 0.31 (sec) , antiderivative size = 308, normalized size of antiderivative = 3.02

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = -\frac{4\,b^3c\,\mathrm{Ci}\left(\frac{2(bc+(bx+a)d-ad)}{d}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)+4\,(bx+a)b^2d\,\mathrm{Ci}\left(\frac{2(bc+(bx+a)d-ad)}{d}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)-4\,ab^2}{}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
-(4*b^3*c*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d)*sin(-2*(b*c - a*d)/d
) + 4*(b*x + a)*b^2*d*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d)*sin(-2*(
b*c - a*d)/d) - 4*a*b^2*d*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d)*sin(
-2*(b*c - a*d)/d) - 4*b^3*c*cos(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (
b*x + a)*d - a*d)/d) - 4*(b*x + a)*b^2*d*cos(-2*(b*c - a*d)/d)*sin_integra
l(-2*(b*c + (b*x + a)*d - a*d)/d) + 4*a*b^2*d*cos(-2*(b*c - a*d)/d)*sin_in
tegral(-2*(b*c + (b*x + a)*d - a*d)/d) + 2*b^2*d*cos(2*b*x + 2*a) + b^2*d)
/((b*c*d^2 + (b*x + a)*d^3 - a*d^3)*b)
```

### 3.373.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\sin\left(3\,a+3\,b\,x\right)}{\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^2),x)
```

output
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^2), x)
```

# 3.374 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$

3.374.1 Optimal result . . . 2686
3.374.2 Mathematica [A] (verified) . . . 2686
3.374.3 Rubi [A] (verified) . . . 2687
3.374.4 Maple [A] (verified) . . . 2688
3.374.5 Fricas [A] (verification not implemented) . . . 2688
3.374.6 Sympy [F(-1)] . . . 2689
3.374.7 Maxima [C] (verification not implemented) . . . 2689
3.374.8 Giac [B] (verification not implemented) . . . 2690
3.374.9 Mupad [F(-1)] . . . 2690

## 3.374.1 Optimal result

Integrand size = 23, antiderivative size = 136

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = -\frac{3\cos^2(a+bx)}{2d(c+dx)^2} - \frac{4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3} + \frac{4b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{\sin^2(a+bx)}{2d(c+dx)^2} + \frac{4b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}$$

output
```
-4*b^2*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^3-3/2*cos(b*x+a)^2/d/(d*x+c)^2
+4*b^2*Si(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^3+4*b*cos(b*x+a)*sin(b*x+a)/d^
2/(d*x+c)+1/2*sin(b*x+a)^2/d/(d*x+c)^2
```

## 3.374.2 Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.76

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = -\frac{8b^2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2b(c+dx)}{d}\right) + \frac{d(d+2d\cos(2(a+bx))-4b(c+dx)\sin(2(a+bx)))}{(c+dx)^2} - 8b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\right.}{2d^3}$$

input
```
Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

output
```
-1/2*(8*b^2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + (d*(d +
2*d*Cos[2*(a + b*x)] - 4*b*(c + d*x)*Sin[2*(a + b*x)]))/(c + d*x)^2 - 8*b^
2*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d^3
```

### 3.374.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\csc(a+bx)}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{4931}$$

$$\int \left(\frac{3\cos^2(a+bx)}{(c+dx)^3}-\frac{\sin^2(a+bx)}{(c+dx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{4b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{4b\sin(a+bx)\cos(a+bx)}{d^2(c+dx)}+\frac{\sin^2(a+bx)}{2d(c+dx)^2}-\frac{3\cos^2(a+bx)}{2d(c+dx)^2}$$

input
```
Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

output
```
(-3*Cos[a + b*x]^2)/(2*d*(c + d*x)^2) - (4*b^2*Cos[2*a - (2*b*c)/d]*CosInt
egral[(2*b*c)/d + 2*b*x])/d^3 + (4*b*Cos[a + b*x]*Sin[a + b*x])/(d^2*(c +
d*x)) + Sin[a + b*x]^2/(2*d*(c + d*x)^2) + (4*b^2*Sin[2*a - (2*b*c)/d]*Sin
Integral[(2*b*c)/d + 2*b*x])/d^3
```

#### 3.374.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.374.4 Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.52

| method | result |
|---|---|
| default | $\frac{1}{2d(dx+c)^2}+\frac{b^3\left(-\frac{\cos(2xb+2a)}{(-ad+cb+d(xb+a))^2 d}-\frac{-\frac{2\sin(2xb+2a)}{(-ad+cb+d(xb+a))d}+\frac{\frac{4\,\mathrm{Si}\left(2xb+2a+\frac{-2ad+2cb}{d}\right)\sin\left(\frac{-2ad+2cb}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(2xb+2a+\frac{-2ad+2cb}{d}\right)\cdots}{d}}{d}}{d}\cdots\right.}{b}\cdots$ |
| risch | $-\frac{1}{2d(dx+c)^2}+\frac{2b^2 e^{-\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{d^3}+\frac{2b^2 e^{\frac{2i(ad-cb)}{d}}\,\mathrm{Ei}_1\left(-2ibx-2ia-\frac{2(-iad+icb)}{d}\right)}{d^3}+\frac{(-2b^2d^3x^2-4b\cdots}{2d^2(x^2d^2b^2\cdots}$ |

input
```
int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/d/(d*x+c)^2+4/b*(1/4*b^3*(-cos(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^2/d-(-2
*sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))/d+2*(2*Si(2*x*b+2*a+2*(-a*d+b*c)/d)*s
in(2*(-a*d+b*c)/d)/d+2*Ci(2*x*b+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)
/d)/d)-1/4*b^3/(-a*d+c*b+d*(b*x+a))^2/d)
```

### 3.374.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.34

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx =$$
$$-\frac{4\,d^2\cos\left(bx+a\right)^2+8\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)-8\left(bd^2x+bcd\right)\cos\left(bx+\cdots\right.}{2\left(d^5x^2+2\,cd^4x+c^2d^3\cdots\right.}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/2*(4*d^2*cos(b*x + a)^2 + 8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-
2*(b*c - a*d)/d)*cos_integral(2*(b*d*x + b*c)/d) - 8*(b*d^2*x + b*c*d)*cos
(b*x + a)*sin(b*x + a) - 8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-2*(b
*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) - d^2)/(d^5*x^2 + 2*c*d^4*x +
 c^2*d^3)
```

### 3.374.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**3,x)
```

output
```
Timed out
```

### 3.374.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.97

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = -\frac{2\left(E_3\left(\frac{2(-i\,bdx-i\,bc)}{d}\right)+E_3\left(-\frac{2(-i\,bdx-i\,bc)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+2\left(i\,E_3\left(\frac{2(-i\,bdx-i\,bc)}{d}\right)-i\,E_3\left(-\frac{2(-i\,bdx-i}{d}\right.\right.}{2\left(d^3x^2+2\,cd^2x+c^2d\right)}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/2*(2*(exp_integral_e(3, 2*(-I*b*d*x - I*b*c)/d) + exp_integral_e(3, -2*
(-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 2*(I*exp_integral_e(3, 2*(-
I*b*d*x - I*b*c)/d) - I*exp_integral_e(3, -2*(-I*b*d*x - I*b*c)/d))*sin(-2
*(b*c - a*d)/d) + 1)/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

### 3.374.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 704 vs. $2(132) = 264$.

Time = 0.34 (sec) , antiderivative size = 704, normalized size of antiderivative = 5.18

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx =$$
$$-\frac{8\,b^5c^2\cos\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Ci}\left(\frac{2\,(bc+(bx+a)d-ad)}{d}\right)+16\,(bx+a)b^4cd\cos\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Ci}\left(\frac{2\,(bc+(bx+a)d-ad)}{d}\right)-1}{}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
-1/2*(8*b^5*c^2*cos(-2*(b*c - a*d)/d)*cos_integral(2*(b*c + (b*x + a)*d -
a*d)/d) + 16*(b*x + a)*b^4*c*d*cos(-2*(b*c - a*d)/d)*cos_integral(2*(b*c +
 (b*x + a)*d - a*d)/d) - 16*a*b^4*c*d*cos(-2*(b*c - a*d)/d)*cos_integral(2
*(b*c + (b*x + a)*d - a*d)/d) + 8*(b*x + a)^2*b^3*d^2*cos(-2*(b*c - a*d)/d
)*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d) - 16*(b*x + a)*a*b^3*d^2*cos
(-2*(b*c - a*d)/d)*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d) + 8*a^2*b^3
*d^2*cos(-2*(b*c - a*d)/d)*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d) + 8
*b^5*c^2*sin(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x + a)*d - a*d)/d
) + 16*(b*x + a)*b^4*c*d*sin(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x
 + a)*d - a*d)/d) - 16*a*b^4*c*d*sin(-2*(b*c - a*d)/d)*sin_integral(-2*(b*
c + (b*x + a)*d - a*d)/d) + 8*(b*x + a)^2*b^3*d^2*sin(-2*(b*c - a*d)/d)*si
n_integral(-2*(b*c + (b*x + a)*d - a*d)/d) - 16*(b*x + a)*a*b^3*d^2*sin(-2
*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x + a)*d - a*d)/d) + 8*a^2*b^3*d
^2*sin(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x + a)*d - a*d)/d) - 4*
b^4*c*d*sin(2*b*x + 2*a) - 4*(b*x + a)*b^3*d^2*sin(2*b*x + 2*a) + 4*a*b^3*
d^2*sin(2*b*x + 2*a) + 2*b^3*d^2*cos(2*b*x + 2*a) + b^3*d^2)/((b^2*c^2*d^3
 + 2*(b*x + a)*b*c*d^4 - 2*a*b*c*d^4 + (b*x + a)^2*d^5 - 2*(b*x + a)*a*d^5
 + a^2*d^5)*b)
```

### 3.374.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\sin\left(3\,a+3\,b\,x\right)}{\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^3}\,dx$$

input
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^3),x)
```

output
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^3), x)
```

3.374. $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$

# 3.375 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx$


3.375.1 Optimal result . . . . . . . . . . 2692
3.375.2 Mathematica [A] (verified) . . . . . . . . . . 2693
3.375.3 Rubi [A] (verified) . . . . . . . . . . 2693
3.375.4 Maple [A] (verified) . . . . . . . . . . 2694
3.375.5 Fricas [A] (verification not implemented) . . . . . . . . . . 2695
3.375.6 Sympy [F(-1)] . . . . . . . . . . 2695
3.375.7 Maxima [C] (verification not implemented) . . . . . . . . . . 2696
3.375.8 Giac [B] (verification not implemented) . . . . . . . . . . 2696
3.375.9 Mupad [F(-1)] . . . . . . . . . . 2697


## 3.375.1 Optimal result

Integrand size = 23, antiderivative size = 205

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx = -\frac{2b^2}{3d^3(c+dx)} - \frac{\cos^2(a+bx)}{d(c+dx)^3} + \frac{2b^2\cos^2(a+bx)}{d^3(c+dx)} + \frac{8b^3\,\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{3d^4} + \frac{4b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2} + \frac{\sin^2(a+bx)}{3d(c+dx)^3} - \frac{2b^2\sin^2(a+bx)}{3d^3(c+dx)} + \frac{8b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}$$

output
```
-2/3*b^2/d^3/(d*x+c)-cos(b*x+a)^2/d/(d*x+c)^3+2*b^2*cos(b*x+a)^2/d^3/(d*x+
c)+8/3*b^3*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^4+8/3*b^3*Ci(2*b*c/d+2*b*x
)*sin(2*a-2*b*c/d)/d^4+4/3*b*cos(b*x+a)*sin(b*x+a)/d^2/(d*x+c)^2+1/3*sin(b
*x+a)^2/d/(d*x+c)^3-2/3*b^2*sin(b*x+a)^2/d^3/(d*x+c)
```

### 3.375.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.61

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx$$
$$= \frac{8b^3 \operatorname{CosIntegral}\left(\frac{2b(c+dx)}{d}\right)\sin\left(2a-\frac{2bc}{d}\right)+\frac{d\left(\left(-2d^2+4b^2(c+dx)^2\right)\cos(2(a+bx))+d(-d+2b(c+dx)\sin(2(a+bx)))\right)}{(c+dx)^3}+8b^3\cos}{3d^4}$$

input
```
Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^4,x]
```

output
```
(8*b^3*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (d*((-2*d^2 +
 4*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + d*(-d + 2*b*(c + d*x)*Sin[2*(a + b*
x)])))/(c + d*x)^3 + 8*b^3*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x)
)/d])/(3*d^4)
```

### 3.375.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\csc(a+bx)}{(c+dx)^4}\,dx$$
$$\Big\downarrow \; 4931$$
$$\int \left(\frac{3\cos^2(a+bx)}{(c+dx)^4}-\frac{\sin^2(a+bx)}{(c+dx)^4}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{8b^3\sin\left(2a-\frac{2bc}{d}\right)\operatorname{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{8b^3\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{2b^2\sin^2(a+bx)}{3d^3(c+dx)}+\frac{2b^2\cos^2(a+bx)}{d^3(c+dx)}+\frac{4b\sin(a+bx)\cos(a+bx)}{3d^2(c+dx)^2}+\frac{\sin^2(a+bx)}{3d(c+dx)^3}-\frac{\cos^2(a+bx)}{d(c+dx)^3}-\frac{2b^2}{3d^3(c+dx)}$$

input
```
Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^4,x]
```

output
```
(-2*b^2)/(3*d^3*(c + d*x)) - Cos[a + b*x]^2/(d*(c + d*x)^3) + (2*b^2*Cos[a
 + b*x]^2)/(d^3*(c + d*x)) + (8*b^3*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a
 - (2*b*c)/d])/(3*d^4) + (4*b*Cos[a + b*x]*Sin[a + b*x])/(3*d^2*(c + d*x)^
2) + Sin[a + b*x]^2/(3*d*(c + d*x)^3) - (2*b^2*Sin[a + b*x]^2)/(3*d^3*(c +
 d*x)) + (8*b^3*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(3*d^
4)
```

### 3.375.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

## 3.375.4 Maple [A] (verified)

Time = 3.63 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.19

| method | result |
|---|---|
| default | $\frac{1}{3d(dx+c)^3}+\frac{b^4\left(-\frac{2\cos(2xb+2a)}{3(-ad+cb+d(xb+a))^3d}-\frac{2\left(-\frac{\sin(2xb+2a)}{(-ad+cb+d(xb+a))^2d}+\frac{-\frac{2\cos(2xb+2a)}{(-ad+cb+d(xb+a))d}-\frac{2\left(\frac{2\operatorname{Si}\left(2xb+2a+\frac{-2ad+2cb}{d}\right)\cos\left(\frac{-2ad\cdots}{d}\right)}{d}\cdots\right.}{d}}{d}\cdots\right.}{3d}\cdots\right.}{b}$ |
| risch | $-\frac{1}{3d(dx+c)^3}-\frac{4ib^3\mathrm{e}^{-\frac{2i(ad-cb)}{d}}\operatorname{Ei}_1\left(2ibx+2ia-\frac{2i(ad-cb)}{d}\right)}{3d^4}+\frac{4ib^3\mathrm{e}^{\frac{2i(ad-cb)}{d}}\operatorname{Ei}_1\left(-2ibx-2ia-\frac{2(-iad+icb)}{d}\right)}{3d^4}-\frac{(-4b^5d^5x^5-2\cdots}{\cdots}$ |

input
```
int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/3/d/(d*x+c)^3+4/b*(1/4*b^4*(-2/3*cos(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^3/d
-2/3*(-sin(2*b*x+2*a)/(-a*d+c*b+d*(b*x+a))^2/d+(-2*cos(2*b*x+2*a)/(-a*d+c*
b+d*(b*x+a))/d-2*(2*Si(2*x*b+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*C
i(2*x*b+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d)/d)-1/6*b^4/(-a*d+
c*b+d*(b*x+a))^3/d)
```

### 3.375.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.40

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx =$$
$$-\frac{4\,b^2d^3x^2+8\,b^2cd^2x+4\,b^2c^2d-d^3-4\,(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-d^3)\cos{(bx+a)}^2-4\,(bd^3x+bcd}{}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/3*(4*b^2*d^3*x^2 + 8*b^2*c*d^2*x + 4*b^2*c^2*d - d^3 - 4*(2*b^2*d^3*x^2
 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)^2 - 4*(b*d^3*x + b*c*d^
2)*cos(b*x + a)*sin(b*x + a) - 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^
2*d*x + b^3*c^3)*cos_integral(2*(b*d*x + b*c)/d)*sin(-2*(b*c - a*d)/d) - 8
*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-2*(b*c - a
*d)/d)*sin_integral(2*(b*d*x + b*c)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5
*x + c^3*d^4)
```

### 3.375.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**4,x)
```

output
```
Timed out
```

### 3.375.7 Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.70

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx = -\frac{3\left(E_4\left(\frac{2(-i\,bdx-i\,bc)}{d}\right)+E_4\left(-\frac{2(-i\,bdx-i\,bc)}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+3\left(i\,E_4\left(\frac{2(-i\,bdx-i\,bc)}{d}\right)-i\,E_4\left(-\frac{2(-i\,bdx-i}{d}\right.\right.}{3\left(d^4x^3+3\,cd^3x^2+3\,c^2d^2x+c^3d\right)}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/3*(3*(exp_integral_e(4, 2*(-I*b*d*x - I*b*c)/d) + exp_integral_e(4, -2*
(-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 3*(I*exp_integral_e(4, 2*(-
I*b*d*x - I*b*c)/d) - I*exp_integral_e(4, -2*(-I*b*d*x - I*b*c)/d))*sin(-2
*(b*c - a*d)/d) + 1)/(d^4*x^3 + 3*c*d^3*x^2 + 3*c^2*d^2*x + c^3*d)
```

### 3.375.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1305 vs. 2(193) = 386.

Time = 0.38 (sec) , antiderivative size = 1305, normalized size of antiderivative = 6.37

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx = \text{Too large to display}$$

input
```
integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x, algorithm="giac")
```

output
```
1/3*(8*b^7*c^3*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d)*sin(-2*(b*c - a
*d)/d) + 24*(b*x + a)*b^6*c^2*d*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d
)*sin(-2*(b*c - a*d)/d) - 24*a*b^6*c^2*d*cos_integral(2*(b*c + (b*x + a)*d
 - a*d)/d)*sin(-2*(b*c - a*d)/d) + 24*(b*x + a)^2*b^5*c*d^2*cos_integral(2
*(b*c + (b*x + a)*d - a*d)/d)*sin(-2*(b*c - a*d)/d) - 48*(b*x + a)*a*b^5*c
*d^2*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d)*sin(-2*(b*c - a*d)/d) + 2
4*a^2*b^5*c*d^2*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d)*sin(-2*(b*c -
a*d)/d) + 8*(b*x + a)^3*b^4*d^3*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d
)*sin(-2*(b*c - a*d)/d) - 24*(b*x + a)^2*a*b^4*d^3*cos_integral(2*(b*c + (
b*x + a)*d - a*d)/d)*sin(-2*(b*c - a*d)/d) + 24*(b*x + a)*a^2*b^4*d^3*cos_
integral(2*(b*c + (b*x + a)*d - a*d)/d)*sin(-2*(b*c - a*d)/d) - 8*a^3*b^4*
d^3*cos_integral(2*(b*c + (b*x + a)*d - a*d)/d)*sin(-2*(b*c - a*d)/d) - 8*
b^7*c^3*cos(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x + a)*d - a*d)/d)
 - 24*(b*x + a)*b^6*c^2*d*cos(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*
x + a)*d - a*d)/d) + 24*a*b^6*c^2*d*cos(-2*(b*c - a*d)/d)*sin_integral(-2*
(b*c + (b*x + a)*d - a*d)/d) - 24*(b*x + a)^2*b^5*c*d^2*cos(-2*(b*c - a*d)
/d)*sin_integral(-2*(b*c + (b*x + a)*d - a*d)/d) + 48*(b*x + a)*a*b^5*c*d^
2*cos(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x + a)*d - a*d)/d) - 24*
a^2*b^5*c*d^2*cos(-2*(b*c - a*d)/d)*sin_integral(-2*(b*c + (b*x + a)*d - a
*d)/d) - 8*(b*x + a)^3*b^4*d^3*cos(-2*(b*c - a*d)/d)*sin_integral(-2*(b...
```

### 3.375.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx = \int \frac{\sin(3\,a+3\,b\,x)}{\sin(a+b\,x)\,(c+d\,x)^4}\,dx$$

input
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^4),x)
```

output
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^4), x)
```

# 3.376 $\int (c+dx)^3 \csc^2(a+bx)\sin(3a+3bx)\,dx$

3.376.1 Optimal result . . . 2698
3.376.2 Mathematica [A] (verified) . . . 2699
3.376.3 Rubi [A] (verified) . . . 2699
3.376.4 Maple [B] (verified) . . . 2701
3.376.5 Fricas [B] (verification not implemented) . . . 2701
3.376.6 Sympy [F(-1)] . . . 2702
3.376.7 Maxima [B] (verification not implemented) . . . 2703
3.376.8 Giac [F] . . . 2703
3.376.9 Mupad [F(-1)] . . . 2704

## 3.376.1 Optimal result

Integrand size = 25, antiderivative size = 255

$$\begin{aligned}\int (c+dx)^3 \csc^2(a+bx)\sin(3a+3bx)\,dx &= -\frac{6(c+dx)^3\operatorname{arctanh}\left(e^{i(a+bx)}\right)}{b} - \frac{24d^2(c+dx)\cos(a+bx)}{b^3} \\ &+ \frac{4(c+dx)^3\cos(a+bx)}{b} + \frac{9id(c+dx)^2\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} \\ &- \frac{9id(c+dx)^2\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} - \frac{18d^2(c+dx)\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3} \\ &+ \frac{18d^2(c+dx)\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3} - \frac{18id^3\operatorname{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4} \\ &+ \frac{18id^3\operatorname{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^4} + \frac{24d^3\sin(a+bx)}{b^4} - \frac{12d(c+dx)^2\sin(a+bx)}{b^2}\end{aligned}$$

output

```
-6*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b-24*d^2*(d*x+c)*cos(b*x+a)/b^3+4*(d*
x+c)^3*cos(b*x+a)/b+9*I*d*(d*x+c)^2*polylog(2,-exp(I*(b*x+a)))/b^2-9*I*d*(
d*x+c)^2*polylog(2,exp(I*(b*x+a)))/b^2-18*d^2*(d*x+c)*polylog(3,-exp(I*(b*
x+a)))/b^3+18*d^2*(d*x+c)*polylog(3,exp(I*(b*x+a)))/b^3-18*I*d^3*polylog(4
,-exp(I*(b*x+a)))/b^4+18*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+24*d^3*sin(b*
x+a)/b^4-12*d*(d*x+c)^2*sin(b*x+a)/b^2
```

### 3.376.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.80

$$\int (c+dx)^3 \csc^2(a+bx)\sin(3a+3bx)\,dx$$
$$= \frac{3(-2b^3(c+dx)^3\text{arctanh}(\cos(a+bx)+i\sin(a+bx))+3id(b^2(c+dx)^2\,\text{PolyLog}(2,-\cos(a+bx)-i\sin(}{}$$
$$+\frac{4\cos(bx)\,(b^3c^3\cos(a)-6bcd^2\cos(a)+3b^3c^2dx\cos(a)-6bd^3x\cos(a)+3b^3cd^2x^2\cos(a)+b^3d^3x^3\cos(a)}{b^4}$$
$$-\frac{4(3b^2c^2d\cos(a)-6d^3\cos(a)+6b^2cd^2x\cos(a)+3b^2d^3x^2\cos(a)+b^3c^3\sin(a)-6bcd^2\sin(a)+3b^3c^2dx\sin}{b^4}$$

input
```
Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

output
```
(3*(-2*b^3*(c + d*x)^3*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]] + (3*I)*d*(b
^2*(c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]] + (2*I)*b*d*(c +
 d*x)*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] - 2*d^2*PolyLog[4, -Cos[a
 + b*x] - I*Sin[a + b*x]]) - (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, Cos[a + b
*x] + I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, Cos[a + b*x] + I*Si
n[a + b*x]] - 2*d^2*PolyLog[4, Cos[a + b*x] + I*Sin[a + b*x]])))/b^4 + (4*
Cos[b*x]*(b^3*c^3*Cos[a] - 6*b*c*d^2*Cos[a] + 3*b^3*c^2*d*x*Cos[a] - 6*b*d
^3*x*Cos[a] + 3*b^3*c*d^2*x^2*Cos[a] + b^3*d^3*x^3*Cos[a] - 3*b^2*c^2*d*Si
n[a] + 6*d^3*Sin[a] - 6*b^2*c*d^2*x*Sin[a] - 3*b^2*d^3*x^2*Sin[a]))/b^4 -
(4*(3*b^2*c^2*d*Cos[a] - 6*d^3*Cos[a] + 6*b^2*c*d^2*x*Cos[a] + 3*b^2*d^3*x
^2*Cos[a] + b^3*c^3*Sin[a] - 6*b*c*d^2*Sin[a] + 3*b^3*c^2*d*x*Sin[a] - 6*b
*d^3*x*Sin[a] + 3*b^3*c*d^2*x^2*Sin[a] + b^3*d^3*x^3*Sin[a])*Sin[b*x])/b^4
```

### 3.376.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin(3a+3bx)\csc^2(a+bx)\,dx$$
$$\Big\downarrow \; 4931$$

$$\int \left(3(c+dx)^3\cos(a+bx)\cot(a+bx)-(c+dx)^3\sin(a+bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6(c+dx)^3\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{18id^3\,\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4}+\frac{18id^3\,\text{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^4}+\frac{24d^3\sin(a+bx)}{b^4}-\frac{18d^2(c+dx)\,\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{18d^2(c+dx)\,\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}-\frac{24d^2(c+dx)\cos(a+bx)}{b^3}+\frac{9id(c+dx)^2\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{9id(c+dx)^2\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{12d(c+dx)^2\sin(a+bx)}{b^2}+\frac{4(c+dx)^3\cos(a+bx)}{b}$$

input
```
Int[(c + d*x)^3*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

output
```
(-6*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b - (24*d^2*(c + d*x)*Cos[a + b*
x])/b^3 + (4*(c + d*x)^3*Cos[a + b*x])/b + ((9*I)*d*(c + d*x)^2*PolyLog[2,
 -E^(I*(a + b*x))])/b^2 - ((9*I)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))]
)/b^2 - (18*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (18*d^2*(c +
 d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((18*I)*d^3*PolyLog[4, -E^(I*(a +
 b*x))])/b^4 + ((18*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4 + (24*d^3*Sin[
a + b*x])/b^4 - (12*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

### 3.376.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.376.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 848 vs. $2(237) = 474$.

Time = 2.97 (sec) , antiderivative size = 849, normalized size of antiderivative = 3.33

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 849 |

input
```
int((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output
```
-6/b*c^3*arctanh(exp(I*(b*x+a)))-18/b^3*c*d^2*a^2*arctanh(exp(I*(b*x+a)))+
18/b^2*c^2*d*a*arctanh(exp(I*(b*x+a)))-9*I/b^2*c^2*d*polylog(2,exp(I*(b*x+
a)))+9*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))-9*I/b^2*d^3*polylog(2,exp(I*
(b*x+a)))*x^2+9*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+2*(d^3*x^3*b^3+3*
b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b*d^3*x+6*I*b^2*c*d^
2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))+2*(d^3*x^3*b^3+3*b
^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d^3*x-6*I*b^2*c*d^2
*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))+6/b^4*d^3*a^3*arct
anh(exp(I*(b*x+a)))-9/b^2*c^2*d*ln(exp(I*(b*x+a))+1)*a+9/b^3*d^2*c*ln(exp(
I*(b*x+a))+1)*a^2-3/b^4*d^3*ln(exp(I*(b*x+a))+1)*a^3-18*I*d^3*polylog(4,-e
xp(I*(b*x+a)))/b^4+9/b^2*d*c^2*ln(1-exp(I*(b*x+a)))*a-9/b^3*c*d^2*ln(1-exp
(I*(b*x+a)))*a^2+9/b*d*c^2*ln(1-exp(I*(b*x+a)))*x-9/b*d*c^2*ln(exp(I*(b*x+
a))+1)*x+9/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-9/b*c*d^2*ln(exp(I*(b*x+a))+1)
*x^2+3/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+18/b^3*d^3*polylog(3,exp(I*(b*x+a)
))*x-18/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+3/b*d^3*ln(1-exp(I*(b*x+a)))*
x^3-3/b*d^3*ln(exp(I*(b*x+a))+1)*x^3+18/b^3*c*d^2*polylog(3,exp(I*(b*x+a))
)-18/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))-18*I/b^2*d^2*c*polylog(2,exp(I*(
b*x+a)))*x+18*I/b^2*d^2*c*polylog(2,-exp(I*(b*x+a)))*x+18*I*d^3*polylog(4,
exp(I*(b*x+a)))/b^4
```

### 3.376.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 929 vs. $2(231) = 462$.

Time = 0.32 (sec) , antiderivative size = 929, normalized size of antiderivative = 3.64

$$\int (c+dx)^3 \csc^2(a+bx) \sin(3a+3bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

output
```
1/2*(18*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 18*I*d^3*polylog
(4, cos(b*x + a) - I*sin(b*x + a)) + 18*I*d^3*polylog(4, -cos(b*x + a) + I
*sin(b*x + a)) - 18*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + 8*(
b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d
^3)*x)*cos(b*x + a) - 9*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*di
log(cos(b*x + a) + I*sin(b*x + a)) - 9*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x -
 I*b^2*c^2*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) - 9*(I*b^2*d^3*x^2 + 2*
I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - 9*(-I
*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-cos(b*x + a) - I*sin(
b*x + a)) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*lo
g(cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + 3*(b^3*c
^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*
I*sin(b*x + a) + 1/2) + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + 3*(b^3*d^3*x^3 + 3
*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*
log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x +
 a) - I*sin(b*x + a) + 1) + 18*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a)
 + I*sin(b*x + a)) + 18*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) - I...
```

## 3.376.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^2(a+bx)\sin(3a+3bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**3*csc(b*x+a)**2*sin(3*b*x+3*a),x)
```

output
```
Timed out
```

### 3.376.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 606 vs. 2(231) = 462.

Time = 0.42 (sec) , antiderivative size = 606, normalized size of antiderivative = 2.38

$$\int (c+dx)^3 \csc^2(a+bx)\sin(3a+3bx)\,dx$$
$$= \frac{c^3\left(8\cos(bx+a) - 3\log\left(\cos(bx)^2 + 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 - 2\sin(bx)\sin(a) + \sin(a)\right.\right.}{2b}$$
$$- \frac{36i\,d^3\mathrm{Li}_4(-e^{(ibx+ia)}) - 36i\,d^3\mathrm{Li}_4(e^{(ibx+ia)}) - 6\left(-ib^3d^3x^3 - 3ib^3cd^2x^2 - 3ib^3c^2dx\right)\arctan(\sin(bx+a)}{}$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
1/2*c^3*(8*cos(b*x + a) - 3*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2
+ sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + 3*log(cos(b*x)^2 - 2*cos(b*
x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b - 1/2
*(36*I*d^3*polylog(4, -e^(I*b*x + I*a)) - 36*I*d^3*polylog(4, e^(I*b*x + I
*a)) - 6*(-I*b^3*d^3*x^3 - 3*I*b^3*c*d^2*x^2 - 3*I*b^3*c^2*d*x)*arctan2(si
n(b*x + a), cos(b*x + a) + 1) - 6*(-I*b^3*d^3*x^3 - 3*I*b^3*c*d^2*x^2 - 3*
I*b^3*c^2*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 8*(b^3*d^3*x^3 +
 3*b^3*c*d^2*x^2 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a) - 1
8*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(-e^(I*b*x + I*a))
- 18*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(e^(I*b*x + I*a
)) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x)*log(cos(b*x + a)^2
+ sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
+ 3*b^3*c^2*d*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1)
 + 36*(b*d^3*x + b*c*d^2)*polylog(3, -e^(I*b*x + I*a)) - 36*(b*d^3*x + b*c
*d^2)*polylog(3, e^(I*b*x + I*a)) + 24*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*
c^2*d - 2*d^3)*sin(b*x + a))/b^4
```

### 3.376.8 Giac [F]

$$\int (c+dx)^3 \csc^2(a+bx)\sin(3a+3bx)\,dx = \int (dx+c)^3 \csc(bx+a)^2 \sin(3\,bx+3\,a)\,dx$$

input
```
integrate((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*csc(b*x + a)^2*sin(3*b*x + 3*a), x)
```

### 3.376.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \csc^2(a+bx)\sin(3a+3bx)\,dx = \text{Hanged}$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x)^3)/sin(a + b*x)^2,x)
```

output
```
\text{Hanged}
```

# 3.377 $\int (c+dx)^2 \csc^2(a+bx)\sin(3a+3bx)\,dx$

3.377.1 Optimal result . . . 2705
3.377.2 Mathematica [A] (verified) . . . 2705
3.377.3 Rubi [A] (verified) . . . 2706
3.377.4 Maple [B] (verified) . . . 2707
3.377.5 Fricas [B] (verification not implemented) . . . 2708
3.377.6 Sympy [F(-1)] . . . 2708
3.377.7 Maxima [B] (verification not implemented) . . . 2709
3.377.8 Giac [F] . . . 2709
3.377.9 Mupad [F(-1)] . . . 2710

## 3.377.1 Optimal result

Integrand size = 25, antiderivative size = 172

$$\int (c+dx)^2 \csc^2(a+bx)\sin(3a+3bx)\,dx = -\frac{6(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{8d^2\cos(a+bx)}{b^3}+\frac{4(c+dx)^2\cos(a+bx)}{b}+\frac{6id(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{6id(c+dx)\operatorname{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{6d^2\operatorname{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{6d^2\operatorname{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}-\frac{8d(c+dx)\sin(a+bx)}{b^2}$$

output
```
-6*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b-8*d^2*cos(b*x+a)/b^3+4*(d*x+c)^2*co
s(b*x+a)/b+6*I*d*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^2-6*I*d*(d*x+c)*poly
log(2,exp(I*(b*x+a)))/b^2-6*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+6*d^2*polyl
og(3,exp(I*(b*x+a)))/b^3-8*d*(d*x+c)*sin(b*x+a)/b^2
```

## 3.377.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.30

$$\int (c+dx)^2 \csc^2(a+bx)\sin(3a+3bx)\,dx = \frac{3b^2(c+dx)^2\log\left(1-e^{i(a+bx)}\right)-3b^2(c+dx)^2\log\left(1+e^{i(a+bx)}\right)+6ibd(c+dx)\operatorname{PolyLog}\left(2,-e^{i(a+bx)}\right)-6i}{}$$

input
```
Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

output
```
(3*b^2*(c + d*x)^2*Log[1 - E^(I*(a + b*x))] - 3*b^2*(c + d*x)^2*Log[1 + E^
(I*(a + b*x))] + (6*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] - (6*I)*
b*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))] - 6*d^2*PolyLog[3, -E^(I*(a + b*
x))] + 6*d^2*PolyLog[3, E^(I*(a + b*x))] + 4*Cos[b*x]*((-2*d^2 + b^2*(c +
d*x)^2)*Cos[a] - 2*b*d*(c + d*x)*Sin[a]) - 4*(2*b*d*(c + d*x)*Cos[a] + (-2
*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x])/b^3
```

## 3.377.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin(3a+3bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{4931}$$

$$\int \left(3(c+dx)^2\cos(a+bx)\cot(a+bx)-(c+dx)^2\sin(a+bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6(c+dx)^2\text{arctanh}\left(e^{i(a+bx)}\right)}{b}-\frac{6d^2\text{ PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{6d^2\text{ PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}-\frac{8d^2\cos(a+bx)}{b^3}+\frac{6id(c+dx)\text{ PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{6id(c+dx)\text{ PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{8d(c+dx)\sin(a+bx)}{b^2}+\frac{4(c+dx)^2\cos(a+bx)}{b}$$

input
```
Int[(c + d*x)^2*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

output
```
(-6*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (8*d^2*Cos[a + b*x])/b^3 + (
4*(c + d*x)^2*Cos[a + b*x])/b + ((6*I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b
*x))])/b^2 - ((6*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 - (6*d^2*
PolyLog[3, -E^(I*(a + b*x))])/b^3 + (6*d^2*PolyLog[3, E^(I*(a + b*x))])/b^
3 - (8*d*(c + d*x)*Sin[a + b*x])/b^2
```

### 3.377.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

## 3.377.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(160) = 320$.

Time = 2.64 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.80

| method | result |
|---|---|
| risch | $\frac{2(x^2d^2b^2+2b^2cdx+2ib\,d^2x+b^2c^2+2ibcd-2d^2)\mathrm{e}^{i(xb+a)}}{b^3}+\frac{2(x^2d^2b^2+2b^2cdx-2ib\,d^2x+b^2c^2-2ibcd-2d^2)\mathrm{e}^{-i(xb+a)}}{b^3}-\frac{6d^2a^2\,\mathrm{arcta}}{}$ |

input
```
int((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output
```
2*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp(I*
(b*x+a))+2*(x^2*d^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*d)/b
^3*exp(-I*(b*x+a))-6/b^3*d^2*a^2*arctanh(exp(I*(b*x+a)))-3/b^3*d^2*ln(1-ex
p(I*(b*x+a)))*a^2+3/b^3*d^2*ln(exp(I*(b*x+a))+1)*a^2+3/b*d^2*ln(1-exp(I*(b
*x+a)))*x^2+6*d^2*polylog(3,exp(I*(b*x+a)))/b^3-3/b*d^2*ln(exp(I*(b*x+a))+
1)*x^2-6*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+6/b*d*c*ln(1-exp(I*(b*x+a)))*x
-6/b*d*c*ln(exp(I*(b*x+a))+1)*x-6*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))+12/b
^2*c*d*a*arctanh(exp(I*(b*x+a)))+6*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-
6*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x+6/b^2*d*c*ln(1-exp(I*(b*x+a)))*a-6
/b^2*c*d*ln(exp(I*(b*x+a))+1)*a+6*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-6/b
*c^2*arctanh(exp(I*(b*x+a)))
```

### 3.377.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(156) = 312$.

Time = 0.31 (sec) , antiderivative size = 566, normalized size of antiderivative = 3.29

$$\int (c+dx)^2 \csc^2(a+bx)\sin(3a+3bx)\,dx$$
$$= \frac{6\,d^2\text{polylog}(3, \cos(bx+a) + i\sin(bx+a)) + 6\,d^2\text{polylog}(3, \cos(bx+a) - i\sin(bx+a)) - 6\,d^2\text{polylog}(}{}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

output
```
1/2*(6*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*d^2*polylog(3, co
s(b*x + a) - I*sin(b*x + a)) - 6*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x
+ a)) - 6*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + 8*(b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a) - 6*(I*b*d^2*x + I*b*c*d)*di
log(cos(b*x + a) + I*sin(b*x + a)) - 6*(-I*b*d^2*x - I*b*c*d)*dilog(cos(b*
x + a) - I*sin(b*x + a)) - 6*(I*b*d^2*x + I*b*c*d)*dilog(-cos(b*x + a) + I
*sin(b*x + a)) - 6*(-I*b*d^2*x - I*b*c*d)*dilog(-cos(b*x + a) - I*sin(b*x
+ a)) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) + I*sin(b
*x + a) + 1) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) -
I*sin(b*x + a) + 1) + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x +
 a) + 1/2*I*sin(b*x + a) + 1/2) + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1
/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x
 + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 3*(b^2*d
^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x
+ a) + 1) - 16*(b*d^2*x + b*c*d)*sin(b*x + a))/b^3
```

### 3.377.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc^2(a+bx)\sin(3a+3bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2*csc(b*x+a)**2*sin(3*b*x+3*a),x)
```

output
```
Timed out
```

### 3.377.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 413 vs. 2(156) = 312.

Time = 0.42 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.40

$$\int (c+dx)^2\csc^2(a+bx)\sin(3a+3bx)\,dx$$
$$= \frac{c^2\left(8\cos(bx+a) - 3\log\left(\cos(bx)^2 + 2\cos(bx)\cos(a) + \cos(a)^2 + \sin(bx)^2 - 2\sin(bx)\sin(a) + \sin(a)\right.\right.}{2b}$$
$$- \frac{12d^2\mathrm{Li}_3(-e^{(ibx+ia)}) - 12d^2\mathrm{Li}_3(e^{(ibx+ia)}) - 6\left(-ib^2d^2x^2 - 2ib^2cdx\right)\arctan\left(\sin(bx+a),\cos(bx+a)+\right.}{}$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
1/2*c^2*(8*cos(b*x + a) - 3*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2
+ sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + 3*log(cos(b*x)^2 - 2*cos(b*
x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b - 1/2
*(12*d^2*polylog(3, -e^(I*b*x + I*a)) - 12*d^2*polylog(3, e^(I*b*x + I*a))
 - 6*(-I*b^2*d^2*x^2 - 2*I*b^2*c*d*x)*arctan2(sin(b*x + a), cos(b*x + a) +
 1) - 6*(-I*b^2*d^2*x^2 - 2*I*b^2*c*d*x)*arctan2(sin(b*x + a), -cos(b*x +
a) + 1) - 8*(b^2*d^2*x^2 + 2*b^2*c*d*x - 2*d^2)*cos(b*x + a) - 12*(I*b*d^2
*x + I*b*c*d)*dilog(-e^(I*b*x + I*a)) - 12*(-I*b*d^2*x - I*b*c*d)*dilog(e^
(I*b*x + I*a)) + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x)*log(cos(b*x + a)^2 + sin(b*
x + a)^2 + 2*cos(b*x + a) + 1) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x)*log(cos(b*x
 + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 16*(b*d^2*x + b*c*d)*sin(
b*x + a))/b^3
```

### 3.377.8 Giac [F]

$$\int (c+dx)^2\csc^2(a+bx)\sin(3a+3bx)\,dx = \int (dx+c)^2\csc(bx+a)^2\sin(3bx+3a)\,dx$$

input
```
integrate((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*csc(b*x + a)^2*sin(3*b*x + 3*a), x)
```

### 3.377.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \csc^2(a+bx) \sin(3a+3bx) \, dx = \text{Hanged}$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x)^2)/sin(a + b*x)^2,x)
```

output
```
\text{Hanged}
```

# 3.378 $\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx$

3.378.1 Optimal result . . . . . . . . . . 2711
3.378.2 Mathematica [A] (verified) . . . . . . . . . . 2711
3.378.3 Rubi [A] (verified) . . . . . . . . . . 2712
3.378.4 Maple [B] (verified) . . . . . . . . . . 2713
3.378.5 Fricas [B] (verification not implemented) . . . . . . . . . . 2714
3.378.6 Sympy [F(-1)] . . . . . . . . . . 2714
3.378.7 Maxima [F] . . . . . . . . . . 2715
3.378.8 Giac [F] . . . . . . . . . . 2715
3.378.9 Mupad [F(-1)] . . . . . . . . . . 2715

## 3.378.1 Optimal result

Integrand size = 23, antiderivative size = 95

$$\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx = -\frac{6(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b} + \frac{4(c+dx)\cos(a+bx)}{b} + \frac{3id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2} - \frac{3id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2} - \frac{4d\sin(a+bx)}{b^2}$$

output
```
-6*(d*x+c)*arctanh(exp(I*(b*x+a)))/b+4*(d*x+c)*cos(b*x+a)/b+3*I*d*polylog(
2,-exp(I*(b*x+a)))/b^2-3*I*d*polylog(2,exp(I*(b*x+a)))/b^2-4*d*sin(b*x+a)/
b^2
```

## 3.378.2 Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.80

$$\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx = \frac{4c\cos(a+bx)}{b} + \frac{4dx\cos(a+bx)}{b} - \frac{3c\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b} + \frac{3c\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b} - \frac{3ad\log\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)}{b^2} + \frac{3d\left((a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)+i\left(\text{PolyLog}\left(2,-e^{i(a+bx)}\right)-\text{PolyLog}\left(2,e^{i(a+bx)}\right)\right)\right)}{b^2} - \frac{4d\sin(a+bx)}{b^2}$$

input
```
Integrate[(c + d*x)*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

output
```
(4*c*Cos[a + b*x])/b + (4*d*x*Cos[a + b*x])/b - (3*c*Log[Cos[(a + b*x)/2]]
)/b + (3*c*Log[Sin[(a + b*x)/2]])/b - (3*a*d*Log[Tan[(a + b*x)/2]])/b^2 +
(3*d*((a + b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + I*
(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a + b*x))])))/b^2 - (4*d*
Sin[a + b*x])/b^2
```

### 3.378.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin(3a+3bx)\csc^2(a+bx)\,dx$$

$$\Big\downarrow \text{4931}$$

$$\int (3(c+dx)\cos(a+bx)\cot(a+bx)-(c+dx)\sin(a+bx))dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6(c+dx)\text{arctanh}\left(e^{i(a+bx)}\right)}{b}+\frac{3id\,\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{3id\,\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{4d\sin(a+bx)}{b^2}+\frac{4(c+dx)\cos(a+bx)}{b}$$

input
```
Int[(c + d*x)*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

output
```
(-6*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b + (4*(c + d*x)*Cos[a + b*x])/b +
 ((3*I)*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((3*I)*d*PolyLog[2, E^(I*(a
+ b*x))])/b^2 - (4*d*Sin[a + b*x])/b^2
```

#### 3.378.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.378.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(87) = 174$.

Time = 1.23 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.16

| method | result |
|---|---|
| risch | $\frac{2(dxb+cb+id)\mathrm{e}^{i(xb+a)}}{b^2}+\frac{2(dxb+cb-id)\mathrm{e}^{-i(xb+a)}}{b^2}-\frac{6c\,\mathrm{arctanh}\left(\mathrm{e}^{i(xb+a)}\right)}{b}+\frac{3d\ln\left(1-\mathrm{e}^{i(xb+a)}\right)x}{b}+\frac{3d\ln\left(1-\mathrm{e}^{i(xb+a)}\right)a}{b^2}-$ 3 |
| default | $-\frac{c\ln(\csc(xb+a)-\cot(xb+a))}{b}-\frac{d\left((xb+a)\ln\left(1-\mathrm{e}^{i(xb+a)}\right)-(xb+a)\ln\left(\mathrm{e}^{i(xb+a)}+1\right)+i\,\mathrm{dilog}\left(\mathrm{e}^{i(xb+a)}+1\right)-i\,\mathrm{dilog}\left(1-\mathrm{e}^{i(xb+a)}\right)-a\right.}{b^2}$ |

input
```
int((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output
```
2*(d*x*b+c*b+I*d)/b^2*exp(I*(b*x+a))+2*(d*x*b+c*b-I*d)/b^2*exp(-I*(b*x+a))
-6/b*c*arctanh(exp(I*(b*x+a)))+3/b*d*ln(1-exp(I*(b*x+a)))*x+3/b^2*d*ln(1-e
xp(I*(b*x+a)))*a-3*I*d*polylog(2,exp(I*(b*x+a)))/b^2-3/b*d*ln(exp(I*(b*x+a
))+1)*x-3/b^2*d*ln(exp(I*(b*x+a))+1)*a+3*I*d*polylog(2,-exp(I*(b*x+a)))/b^
2+6/b^2*d*a*arctanh(exp(I*(b*x+a)))
```

### 3.378.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(83) = 166$.

Time = 0.27 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.96

$$\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx$$
$$= \frac{8\,(bdx+bc)\cos{(bx+a)} - 3i\,d\mathrm{Li}_2(\cos{(bx+a)}+i\,\sin{(bx+a)}) + 3i\,d\mathrm{Li}_2(\cos{(bx+a)}-i\,\sin{(bx+a)}) -}{}$$

input
```
integrate((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

output
```
1/2*(8*(b*d*x + b*c)*cos(b*x + a) - 3*I*d*dilog(cos(b*x + a) + I*sin(b*x +
 a)) + 3*I*d*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*I*d*dilog(-cos(b*x +
 a) + I*sin(b*x + a)) + 3*I*d*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 3*(b
*d*x + b*c)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b*d*x + b*c)*log(c
os(b*x + a) - I*sin(b*x + a) + 1) + 3*(b*c - a*d)*log(-1/2*cos(b*x + a) +
1/2*I*sin(b*x + a) + 1/2) + 3*(b*c - a*d)*log(-1/2*cos(b*x + a) - 1/2*I*si
n(b*x + a) + 1/2) + 3*(b*d*x + a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1
) + 3*(b*d*x + a*d)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 8*d*sin(b*x
+ a))/b^2
```

### 3.378.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*csc(b*x+a)**2*sin(3*b*x+3*a),x)
```

output
```
Timed out
```

### 3.378.7 Maxima [F]

$$\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx=\int (dx+c)\csc\left(bx+a\right)^2\sin\left(3\,bx+3\,a\right)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
1/2*c*(8*cos(b*x + a) - 3*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 +
sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + 3*log(cos(b*x)^2 - 2*cos(b*x)
*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b + (4*b*
x*cos(b*x + a) + 3*b^2*integrate(x*sin(b*x + a)/(cos(b*x + a)^2 + sin(b*x
+ a)^2 + 2*cos(b*x + a) + 1), x) + 3*b^2*integrate(x*sin(b*x + a)/(cos(b*x
 + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1), x) - 4*sin(b*x + a))*d/b^2
```

### 3.378.8 Giac [F]

$$\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx=\int (dx+c)\csc\left(bx+a\right)^2\sin\left(3\,bx+3\,a\right)\,dx$$

input
```
integrate((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*csc(b*x + a)^2*sin(3*b*x + 3*a), x)
```

### 3.378.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx=\text{Hanged}$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x))/sin(a + b*x)^2,x)
```

output
```
\text{Hanged}
```

# 3.379 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$

3.379.1 Optimal result . . . . . . . . . . 2716
3.379.2 Mathematica [N/A] . . . . . . . . . . 2716
3.379.3 Rubi [N/A] . . . . . . . . . . 2717
3.379.4 Maple [N/A] (verified) . . . . . . . . . . 2718
3.379.5 Fricas [N/A] . . . . . . . . . . 2718
3.379.6 Sympy [N/A] . . . . . . . . . . 2718
3.379.7 Maxima [N/A] . . . . . . . . . . 2719
3.379.8 Giac [N/A] . . . . . . . . . . 2719
3.379.9 Mupad [N/A] . . . . . . . . . . 2720

## 3.379.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = -\frac{4\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d} - \frac{4\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d} + 3\text{Int}\left(\frac{\csc(a+bx)}{c+dx},x\right)$$

output
```
-4*cos(a-b*c/d)*Si(b*c/d+b*x)/d-4*Ci(b*c/d+b*x)*sin(a-b*c/d)/d+3*Unintegra
ble(csc(b*x+a)/(d*x+c),x)
```

## 3.379.2 Mathematica [N/A]

Not integrable

Time = 4.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

input
```
Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x),x]
```

output
```
Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x), x]
```

## 3.379.3 Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\csc^2(a+bx)}{c+dx}dx$$

$$\Big\downarrow \text{4931}$$

$$\int \left(\frac{3\cos(a+bx)\cot(a+bx)}{c+dx}-\frac{\sin(a+bx)}{c+dx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$3\int \frac{\csc(a+bx)}{c+dx}dx-\frac{4\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}-\frac{4\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

input
```
Int[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x),x]
```

output
```
$Aborted
```

### 3.379.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.379.4 Maple [N/A] (verified)

Not integrable

Time = 0.59 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\csc \left(xb+a\right)^{2} \sin \left(3xb+3a\right)}{dx+c}dx$$

input
```
int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
```

output
```
int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
```

### 3.379.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\csc \left(bx+a\right)^{2} \sin \left(3\,bx+3\,a\right)}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

### 3.379.6 Sympy [N/A]

Not integrable

Time = 125.86 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sin \left(3a+3bx\right) \csc^{2}\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(csc(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c),x)
```

output
```
Integral(sin(3*a + 3*b*x)*csc(a + b*x)**2/(c + d*x), x)
```

### 3.379.7 Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 230, normalized size of antiderivative = 9.20

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\csc\left(bx+a\right)^2 \sin\left(3\,bx+3\,a\right)}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")
```

output
```
-(2*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(1, -(I*b
*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - 3*d*integrate(sin(b*x + a)/((d*x +
 c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)*cos(b*x
+ a) + c), x) - 3*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d*
x + c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) - 2*(exp_i
ntegral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d
))*sin(-(b*c - a*d)/d))/d
```

### 3.379.8 Giac [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\csc\left(bx+a\right)^2 \sin\left(3\,bx+3\,a\right)}{dx+c}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

### 3.379.9 Mupad [N/A]

Not integrable

Time = 30.43 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sin(3\,a+3\,b\,x)}{\sin(a+b\,x)^2\,(c+d\,x)}\,dx$$

input
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)),x)
```

output
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)), x)
```

# 3.380 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$


3.380.1 Optimal result . . . 2721
3.380.2 Mathematica [N/A] . . . 2721
3.380.3 Rubi [N/A] . . . 2722
3.380.4 Maple [N/A] (verified) . . . 2723
3.380.5 Fricas [N/A] . . . 2723
3.380.6 Sympy [F(-1)] . . . 2723
3.380.7 Maxima [N/A] . . . 2724
3.380.8 Giac [N/A] . . . 2724
3.380.9 Mupad [N/A] . . . 2725


## 3.380.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = -\frac{4b\cos\left(a-\frac{bc}{d}\right)\operatorname{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{4\sin(a+bx)}{d(c+dx)}+\frac{4b\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+3\operatorname{Int}\left(\frac{\csc(a+bx)}{(c+dx)^2},x\right)$$

output
```
-4*b*Ci(b*c/d+b*x)*cos(a-b*c/d)/d^2+4*b*Si(b*c/d+b*x)*sin(a-b*c/d)/d^2+4*s
in(b*x+a)/d/(d*x+c)+3*Unintegrable(csc(b*x+a)/(d*x+c)^2,x)
```

## 3.380.2 Mathematica [N/A]

Not integrable

Time = 4.56 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2, x]
```

## 3.380.3 Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\csc^2(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{4931}$$

$$\int \left(\frac{3\cos(a+bx)\cot(a+bx)}{(c+dx)^2}-\frac{\sin(a+bx)}{(c+dx)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$3\int \frac{\csc(a+bx)}{(c+dx)^2}dx-\frac{4b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{4b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{4\sin(a+bx)}{d(c+dx)}$$

input
```
Int[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

### 3.380.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.380.4 Maple [N/A] (verified)

Not integrable

Time = 0.68 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\csc \left(x b+a\right)^{2} \sin \left(3 x b+3 a\right)}{\left(d x+c\right)^{2}}d x$$

input
```
int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

output
```
int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

### 3.380.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\csc \left(b x + a\right)^{2} \sin \left(3 \, b x + 3 \, a\right)}{{\left(d x + c\right)}^{2}} \,d x$$

input
```
integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.380.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**2,x)
```

output
```
Timed out
```

### 3.380.7 Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 342, normalized size of antiderivative = 13.68

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)^2\sin\left(3\,bx+3\,a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-(2*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(2, -(I*b
*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - 3*(d^2*x + c*d)*integrate(sin(b*x
+ a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*
x^2 + 2*c*d*x + c^2)*sin(b*x + a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*co
s(b*x + a)), x) - 3*(d^2*x + c*d)*integrate(sin(b*x + a)/(d^2*x^2 + 2*c*d*
x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2 + 2*c*d*x + c^2)*s
in(b*x + a)^2 + c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)), x) - 2*(e
xp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*
c)/d))*sin(-(b*c - a*d)/d))/(d^2*x + c*d)
```

### 3.380.8 Giac [N/A]

Not integrable

Time = 1.99 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\csc\left(bx+a\right)^2\sin\left(3\,bx+3\,a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^2, x)
```

### 3.380.9 Mupad [N/A]

Not integrable

Time = 33.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx=\int \frac{\sin\left(3\,a+3\,b\,x\right)}{{\sin\left(a+b\,x\right)}^2\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)^2),x)
```

output
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)^2), x)
```

# 3.381 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$


3.381.1 Optimal result . . . 2726
3.381.2 Mathematica [N/A] . . . 2726
3.381.3 Rubi [N/A] . . . 2727
3.381.4 Maple [N/A] (verified) . . . 2728
3.381.5 Fricas [N/A] . . . 2728
3.381.6 Sympy [F(-1)] . . . 2728
3.381.7 Maxima [N/A] . . . 2729
3.381.8 Giac [N/A] . . . 2729
3.381.9 Mupad [N/A] . . . 2730


## 3.381.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \frac{2b\cos(a+bx)}{d^2(c+dx)} + \frac{2b^2\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^3} + \frac{2\sin(a+bx)}{d(c+dx)^2} + \frac{2b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^3} + 3\text{Int}\left(\frac{\csc(a+bx)}{(c+dx)^3},x\right)$$

output
```
2*b*cos(b*x+a)/d^2/(d*x+c)+2*b^2*cos(a-b*c/d)*Si(b*c/d+b*x)/d^3+2*b^2*Ci(b
*c/d+b*x)*sin(a-b*c/d)/d^3+2*sin(b*x+a)/d/(d*x+c)^2+3*Unintegrable(csc(b*x
+a)/(d*x+c)^3,x)
```

## 3.381.2 Mathematica [N/A]

Not integrable

Time = 5.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

input
```
Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

output
```
Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3, x]
```

### 3.381.3 Rubi [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\csc^2(a+bx)}{(c+dx)^3}dx$$
$$\Big\downarrow \text{4931}$$
$$\int \left(\frac{3\cos(a+bx)\cot(a+bx)}{(c+dx)^3}-\frac{\sin(a+bx)}{(c+dx)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$3\int \frac{\csc(a+bx)}{(c+dx)^3}dx+\frac{2b^2\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^3}+\frac{2b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^3}+\frac{2b\cos(a+bx)}{d^2(c+dx)}+\frac{2\sin(a+bx)}{d(c+dx)^2}$$

input
```
Int[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

output
```
$Aborted
```

#### 3.381.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.381.4 Maple [N/A] (verified)

Not integrable

Time = 0.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\csc(xb+a)^2 \sin(3xb+3a)}{(dx+c)^3} dx$$

input
```
int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

output
```
int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

### 3.381.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\csc(bx+a)^2 \sin(3\,bx+3\,a)}{(dx+c)^3}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*
x + c^3), x)
```

### 3.381.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate(csc(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**3,x)
```

output
```
Timed out
```

### 3.381.7 Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 463, normalized size of antiderivative = 18.52

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\csc(bx+a)^2\sin(3\,bx+3\,a)}{(dx+c)^3}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-(2*(-I*exp_integral_e(3, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(3, -(I*b
*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - 3*(d^3*x^2 + 2*c*d^2*x + c^2*d)*in
tegrate(sin(b*x + a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3 + (d^3*x^3 +
 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*cos(b*x + a)^2 + (d^3*x^3 + 3*c*d^2*x^2 +
3*c^2*d*x + c^3)*sin(b*x + a)^2 + 2*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c
^3)*cos(b*x + a)), x) - 3*(d^3*x^2 + 2*c*d^2*x + c^2*d)*integrate(sin(b*x
+ a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3 + (d^3*x^3 + 3*c*d^2*x^2 + 3
*c^2*d*x + c^3)*cos(b*x + a)^2 + (d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)
*sin(b*x + a)^2 - 2*(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*cos(b*x + a)
), x) - 2*(exp_integral_e(3, (I*b*d*x + I*b*c)/d) + exp_integral_e(3, -(I*
b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

### 3.381.8 Giac [N/A]

Not integrable

Time = 4.52 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\csc(bx+a)^2\sin(3\,bx+3\,a)}{(dx+c)^3}\,dx$$

input
```
integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^3, x)
```

### 3.381.9 Mupad [N/A]

Not integrable

Time = 34.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx=\int \frac{\sin\left(3\,a+3\,b\,x\right)}{{\sin\left(a+b\,x\right)}^2\,{\left(c+d\,x\right)}^3} \,d x$$

input
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)^3),x)
```

output
```
int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)^3), x)
```

# 3.382 $\int (c+dx)^4 \sec(a+bx)\sin(3a+3bx)\,dx$


3.382.1 Optimal result . . . . . . . . . . 2731
3.382.2 Mathematica [B] (verified) . . . . . . . . . . 2732
3.382.3 Rubi [A] (verified) . . . . . . . . . . 2733
3.382.4 Maple [B] (verified) . . . . . . . . . . 2734
3.382.5 Fricas [B] (verification not implemented) . . . . . . . . . . 2735
3.382.6 Sympy [F(-1)] . . . . . . . . . . 2736
3.382.7 Maxima [B] (verification not implemented) . . . . . . . . . . 2737
3.382.8 Giac [F] . . . . . . . . . . 2737
3.382.9 Mupad [F(-1)] . . . . . . . . . . 2738


## 3.382.1 Optimal result

Integrand size = 23, antiderivative size = 299

$$\begin{aligned}\int (c+dx)^4\sec(a+bx)\sin(3a+3bx)\,dx =& \frac{6cd^3x}{b^3}+\frac{3d^4x^2}{b^3}-\frac{(c+dx)^4}{b}-\frac{i(c+dx)^5}{5d}\\&+\frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b}\\&-\frac{2id(c+dx)^3\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}\\&+\frac{3d^2(c+dx)^2\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3}\\&+\frac{3id^3(c+dx)\operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4}\\&-\frac{3d^4\operatorname{PolyLog}\left(5,-e^{2i(a+bx)}\right)}{2b^5}\\&-\frac{6d^3(c+dx)\cos(a+bx)\sin(a+bx)}{b^4}\\&+\frac{4d(c+dx)^3\cos(a+bx)\sin(a+bx)}{b^2}\\&+\frac{3d^4\sin^2(a+bx)}{b^5}-\frac{6d^2(c+dx)^2\sin^2(a+bx)}{b^3}\\&+\frac{2(c+dx)^4\sin^2(a+bx)}{b}\end{aligned}$$

output
```
6*c*d^3*x/b^3+3*d^4*x^2/b^3-(d*x+c)^4/b-1/5*I*(d*x+c)^5/d+(d*x+c)^4*ln(1+e
xp(2*I*(b*x+a)))/b-2*I*d*(d*x+c)^3*polylog(2,-exp(2*I*(b*x+a)))/b^2+3*d^2*
(d*x+c)^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+3*I*d^3*(d*x+c)*polylog(4,-exp(
2*I*(b*x+a)))/b^4-3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5-6*d^3*(d*x+c)*c
os(b*x+a)*sin(b*x+a)/b^4+4*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2+3*d^4*sin
(b*x+a)^2/b^5-6*d^2*(d*x+c)^2*sin(b*x+a)^2/b^3+2*(d*x+c)^4*sin(b*x+a)^2/b
```

### 3.382.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2529 vs. 2(299) = 598.

Time = 6.40 (sec) , antiderivative size = 2529, normalized size of antiderivative = 8.46

$$\int (c+dx)^4 \sec(a+bx) \sin(3a+3bx) \, dx = \text{Result too large to show}$$

input
```
Integrate[(c + d*x)^4*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
((I/2)*c^2*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*
I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))
] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^
3*E^(I*a)) + ((I/2)*c*d^3*E^(I*a)*((2*b^4*x^4)/E^((2*I)*a) - (4*I)*b^3*(1
+ E^((-2*I)*a))*x^3*Log[1 + E^((-2*I)*(a + b*x))] + 6*b^2*(1 + E^((-2*I)*a
))*x^2*PolyLog[2, -E^((-2*I)*(a + b*x))] - (6*I)*b*(1 + E^((-2*I)*a))*x*Po
lyLog[3, -E^((-2*I)*(a + b*x))] - 3*(1 + E^((-2*I)*a))*PolyLog[4, -E^((-2*
I)*(a + b*x))])*Sec[a])/b^4 + ((I/20)*d^4*E^(I*a)*((4*b^5*x^5)/E^((2*I)*a)
 - (10*I)*b^4*(1 + E^((-2*I)*a))*x^4*Log[1 + E^((-2*I)*(a + b*x))] + 20*b^
3*(1 + E^((-2*I)*a))*x^3*PolyLog[2, -E^((-2*I)*(a + b*x))] - (30*I)*b^2*(1
 + E^((-2*I)*a))*x^2*PolyLog[3, -E^((-2*I)*(a + b*x))] - 30*b*(1 + E^((-2*
I)*a))*x*PolyLog[4, -E^((-2*I)*(a + b*x))] + (15*I)*(1 + E^((-2*I)*a))*Pol
yLog[5, -E^((-2*I)*(a + b*x))])*Sec[a])/b^5 + (c^4*Sec[a]*(Cos[a]*Log[Cos[
a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) +
(2*c^3*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*
ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Lo
g[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[
a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[C
ot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin
[a]^2)]) + Sec[a]*(Cos[2*a + 2*b*x]/(40*b^5) - ((I/40)*Sin[2*a + 2*b*x]...
```

### 3.382.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4 \sin(3a+3bx)\sec(a+bx)\,dx$$

$$\Big\downarrow \text{4931}$$

$$\int \left(3(c+dx)^4\sin(a+bx)\cos(a+bx)-(c+dx)^4\sin^2(a+bx)\tan(a+bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3d^4\,\text{PolyLog}\left(5,-e^{2i(a+bx)}\right)}{2b^5}+\frac{3d^4\sin^2(a+bx)}{b^5}+\frac{3id^3(c+dx)\,\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4}-\frac{6d^3(c+dx)\sin(a+bx)\cos(a+bx)}{b^4}+\frac{3d^2(c+dx)^2\,\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3}-\frac{6d^2(c+dx)^2\sin^2(a+bx)}{b^3}-\frac{2id(c+dx)^3\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}+\frac{4d(c+dx)^3\sin(a+bx)\cos(a+bx)}{b^2}+\frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{2(c+dx)^4\sin^2(a+bx)}{b}+\frac{3d^2(c+dx)^2}{b^3}-\frac{(c+dx)^4}{b}-\frac{i(c+dx)^5}{5d}$$

input
```
Int[(c + d*x)^4*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
(3*d^2*(c + d*x)^2)/b^3 - (c + d*x)^4/b - ((I/5)*(c + d*x)^5)/d + ((c + d*
x)^4*Log[1 + E^((2*I)*(a + b*x))])/b - ((2*I)*d*(c + d*x)^3*PolyLog[2, -E^
((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x
))])/b^3 + ((3*I)*d^3*(c + d*x)*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 - (3
*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/(2*b^5) - (6*d^3*(c + d*x)*Cos[a +
b*x]*Sin[a + b*x])/b^4 + (4*d*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x])/b^2 +
 (3*d^4*Sin[a + b*x]^2)/b^5 - (6*d^2*(c + d*x)^2*Sin[a + b*x]^2)/b^3 + (2*
(c + d*x)^4*Sin[a + b*x]^2)/b
```

### 3.382.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

## 3.382.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 964 vs. $2(282) = 564$.

Time = 6.24 (sec) , antiderivative size = 965, normalized size of antiderivative = 3.23

| method | result |
|---|---|
| risch | $\frac{3d^2c^2 \operatorname{polylog}(3,-e^{2i(xb+a)})}{b^3}+\frac{3d^4 \operatorname{polylog}(3,-e^{2i(xb+a)})x^2}{b^3}+\frac{d^4 \ln(e^{2i(xb+a)}+1)x^4}{b}-id^3c\,x^4-2id^2c^2x^3-2id\,c^3x$ |

input
```
int((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output
```
-2/b*c^4*ln(exp(I*(b*x+a)))-1/5*I*d^4*x^5-1/4*(2*d^4*x^4*b^4-4*I*b^3*d^4*x
^3+8*b^4*c*d^3*x^3-12*I*b^3*c*d^3*x^2+12*b^4*c^2*d^2*x^2-12*I*b^3*c^2*d^2*
x+8*b^4*c^3*d*x-4*I*b^3*c^3*d+2*b^4*c^4-6*b^2*d^4*x^2+6*I*b*d^4*x-12*b^2*c
*d^3*x+6*I*b*c*d^3-6*b^2*c^2*d^2+3*d^4)/b^5*exp(-2*I*(b*x+a))-6*I/b^4*d^3*
c*a^4+3*I/b^4*d^3*c*polylog(4,-exp(2*I*(b*x+a)))+8*I/b^3*d^2*c^2*a^3+I*c^4
*x+1/5*I/d*c^5-1/4*(2*d^4*x^4*b^4+4*I*b^3*d^4*x^3+8*b^4*c*d^3*x^3+12*I*b^3
*c*d^3*x^2+12*b^4*c^2*d^2*x^2+12*I*b^3*c^2*d^2*x+8*b^4*c^3*d*x+4*I*b^3*c^3
*d+2*b^4*c^4-6*b^2*d^4*x^2-6*I*b*d^4*x-12*b^2*c*d^3*x-6*I*b*c*d^3-6*b^2*c^
2*d^2+3*d^4)/b^5*exp(2*I*(b*x+a))+6/b^3*d^3*c*polylog(3,-exp(2*I*(b*x+a)))
*x+4/b*d*c^3*ln(exp(2*I*(b*x+a))+1)*x+4/b*d^3*c*ln(exp(2*I*(b*x+a))+1)*x^3
+6/b*d^2*c^2*ln(exp(2*I*(b*x+a))+1)*x^2-12/b^3*a^2*d^2*c^2*ln(exp(I*(b*x+a
)))+8/b^2*a*d*c^3*ln(exp(I*(b*x+a)))+8/b^4*a^3*d^3*c*ln(exp(I*(b*x+a)))+2*
I/b^4*d^4*a^4*x+3*I/b^4*d^4*polylog(4,-exp(2*I*(b*x+a)))*x-2*I/b^2*d^4*pol
ylog(2,-exp(2*I*(b*x+a)))*x^3-4*I/b^2*d*c^3*a^2-2*I/b^2*d*c^3*polylog(2,-e
xp(2*I*(b*x+a)))-2/b^5*a^4*d^4*ln(exp(I*(b*x+a)))+3/b^3*d^2*c^2*polylog(3,
-exp(2*I*(b*x+a)))+3/b^3*d^4*polylog(3,-exp(2*I*(b*x+a)))*x^2+1/b*d^4*ln(e
xp(2*I*(b*x+a))+1)*x^4+8/5*I/b^5*d^4*a^5-I*d^3*c*x^4-2*I*d^2*c^2*x^3-2*I*d
*c^3*x^2+1/b*c^4*ln(exp(2*I*(b*x+a))+1)-3/2*d^4*polylog(5,-exp(2*I*(b*x+a)
))/b^5+12*I/b^2*d^2*c^2*a^2*x-8*I/b^3*d^3*c*a^3*x-8*I/b*d*c^3*x*a-6*I/b^2*
d^3*c*polylog(2,-exp(2*I*(b*x+a)))*x^2-6*I/b^2*d^2*c^2*polylog(2,-exp(2...
```

### 3.382.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1652 vs. $2(278) = 556$.

Time = 0.35 (sec) , antiderivative size = 1652, normalized size of antiderivative = 5.53

$$\int (c + dx)^4 \sec(a + bx) \sin(3a + 3bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

output
```
1/2*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 - 24*d^4*polylog(5, I*cos(b*x + a) +
sin(b*x + a)) - 24*d^4*polylog(5, I*cos(b*x + a) - sin(b*x + a)) - 24*d^4*
polylog(5, -I*cos(b*x + a) + sin(b*x + a)) - 24*d^4*polylog(5, -I*cos(b*x
+ a) - sin(b*x + a)) + 6*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 - 2*(2*b^4*d^4*x^4
+ 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 -
 b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^2 + 4*(2*b^3
*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 -
b*d^4)*x)*cos(b*x + a)*sin(b*x + a) + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x - 4*
(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2 - 3*I*b^3*c^2*d^2*x - I*b^3*c^3*d)*dil
og(I*cos(b*x + a) + sin(b*x + a)) - 4*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 +
 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 4
*(I*b^3*d^4*x^3 + 3*I*b^3*c*d^3*x^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dil
og(-I*cos(b*x + a) + sin(b*x + a)) - 4*(-I*b^3*d^4*x^3 - 3*I*b^3*c*d^3*x^2
 - 3*I*b^3*c^2*d^2*x - I*b^3*c^3*d)*dilog(-I*cos(b*x + a) - sin(b*x + a))
+ (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*
log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*
b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(cos(b*x + a) - I*sin(b*x + a) +
 I) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x +
 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(I*cos(b*
x + a) + sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2...
```

### 3.382.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^4 \sec(a+bx)\sin(3a+3bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**4*sec(b*x+a)*sin(3*b*x+3*a),x)
```

output
```
Timed out
```

### 3.382.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 610 vs. 2(278) = 556.

Time = 0.36 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.04

$$\int (c+dx)^4\sec(a+bx)\sin(3a+3bx)\,dx =$$
$$-\frac{c^4\left(2\cos\left(2\,bx+2\,a\right)-\log\left(\cos\left(2\,bx\right)^2+2\cos\left(2\,bx\right)\cos\left(2\,a\right)+\cos\left(2\,a\right)^2+\sin\left(2\,bx\right)^2-2\sin\left(2\,bx\right)\sin\ldots\right.\right.}{2\,b}$$
$$+\frac{-6i\,b^5d^4x^5-30i\,b^5cd^3x^4-60i\,b^5c^2d^2x^3-60i\,b^5c^3dx^2-90\,d^4\mathrm{Li}_5\left(-e^{(2i\,bx+2i\,a)}\right)-20\left(-3i\,b^4d^4x^4-8i\,b^4\ldots\right.}{}$$

input
```
integrate((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
-1/2*c^4*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) +
cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b + 1/30*
(-6*I*b^5*d^4*x^5 - 30*I*b^5*c*d^3*x^4 - 60*I*b^5*c^2*d^2*x^3 - 60*I*b^5*c
^3*d*x^2 - 90*d^4*polylog(5, -e^(2*I*b*x + 2*I*a)) - 20*(-3*I*b^4*d^4*x^4
- 8*I*b^4*c*d^3*x^3 - 9*I*b^4*c^2*d^2*x^2 - 6*I*b^4*c^3*d*x)*arctan2(sin(2
*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 15*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 -
 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d
- 3*b^2*c*d^3)*x)*cos(2*b*x + 2*a) - 60*(2*I*b^3*d^4*x^3 + 4*I*b^3*c*d^3*x
^2 + 3*I*b^3*c^2*d^2*x + I*b^3*c^3*d)*dilog(-e^(2*I*b*x + 2*I*a)) + 10*(3*
b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 9*b^4*c^2*d^2*x^2 + 6*b^4*c^3*d*x)*log(cos
(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - 60*(-3*I*
b*d^4*x - 2*I*b*c*d^3)*polylog(4, -e^(2*I*b*x + 2*I*a)) + 30*(6*b^2*d^4*x^
2 + 8*b^2*c*d^3*x + 3*b^2*c^2*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)) + 30*(
2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d
^2 - b*d^4)*x)*sin(2*b*x + 2*a))/b^5
```

### 3.382.8 Giac [F]

$$\int (c+dx)^4\sec(a+bx)\sin(3a+3bx)\,dx = \int (dx+c)^4\sec\left(bx+a\right)\sin\left(3\,bx+3\,a\right)\,dx$$

input
```
integrate((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^4*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

### 3.382.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^4 \sec(a+bx)\sin(3a+3bx)\,dx = \int \frac{\sin(3a+3bx)\,(c+dx)^4}{\cos(a+bx)}\,dx$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x)^4)/cos(a + b*x),x)
```

output
```
int((sin(3*a + 3*b*x)*(c + d*x)^4)/cos(a + b*x), x)
```

# 3.383 $\int (c+dx)^3 \sec(a+bx)\sin(3a+3bx)\,dx$

3.383.1 Optimal result . . . . . . . . . . 2739
3.383.2 Mathematica [B] (verified) . . . . . . . . . . 2740
3.383.3 Rubi [A] (verified) . . . . . . . . . . 2740
3.383.4 Maple [B] (verified) . . . . . . . . . . 2742
3.383.5 Fricas [B] (verification not implemented) . . . . . . . . . . 2742
3.383.6 Sympy [F(-1)] . . . . . . . . . . 2743
3.383.7 Maxima [B] (verification not implemented) . . . . . . . . . . 2744
3.383.8 Giac [F] . . . . . . . . . . 2744
3.383.9 Mupad [F(-1)] . . . . . . . . . . 2745

## 3.383.1 Optimal result

Integrand size = 23, antiderivative size = 242

$$\int (c+dx)^3 \sec(a+bx)\sin(3a+3bx)\,dx = \frac{3d^3x}{2b^3} - \frac{(c+dx)^3}{b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2} + \frac{3d^2(c+dx)\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3} + \frac{3id^3\operatorname{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4} - \frac{3d^3\cos(a+bx)\sin(a+bx)}{2b^4} + \frac{3d(c+dx)^2\cos(a+bx)\sin(a+bx)}{b^2} - \frac{3d^2(c+dx)\sin^2(a+bx)}{b^3} + \frac{2(c+dx)^3\sin^2(a+bx)}{b}$$

output
```
3/2*d^3*x/b^3-(d*x+c)^3/b-1/4*I*(d*x+c)^4/d+(d*x+c)^3*ln(1+exp(2*I*(b*x+a)
))/b-3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+a)))/b^2+3/2*d^2*(d*x+c)*po
lylog(3,-exp(2*I*(b*x+a)))/b^3+3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4-
3/2*d^3*cos(b*x+a)*sin(b*x+a)/b^4+3*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^2-
3*d^2*(d*x+c)*sin(b*x+a)^2/b^3+2*(d*x+c)^3*sin(b*x+a)^2/b
```

### 3.383.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1730 vs. $2(242) = 484$.

Time = 6.36 (sec) , antiderivative size = 1730, normalized size of antiderivative = 7.15

$$\int (c+dx)^3 \sec(a+bx)\sin(3a+3bx)\,dx = \text{Too large to display}$$

input
```
Integrate[(c + d*x)^3*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
((I/4)*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)
*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))]
- (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*
E^(I*a)) + ((I/8)*d^3*E^(I*a)*((2*b^4*x^4)/E^((2*I)*a) - (4*I)*b^3*(1 + E^
((-2*I)*a))*x^3*Log[1 + E^((-2*I)*(a + b*x))] + 6*b^2*(1 + E^((-2*I)*a))*x
^2*PolyLog[2, -E^((-2*I)*(a + b*x))] - (6*I)*b*(1 + E^((-2*I)*a))*x*PolyLo
g[3, -E^((-2*I)*(a + b*x))] - 3*(1 + E^((-2*I)*a))*PolyLog[4, -E^((-2*I)*(
a + b*x))])*Sec[a])/b^4 + (c^3*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]
*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (3*c^2*d*Csc[a]*((b^
2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi
*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x
- ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - Ar
cTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 +
 Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + Sec[a]*
(Cos[2*a + 2*b*x]/(16*b^4) - ((I/16)*Sin[2*a + 2*b*x])/b^4)*(-8*b^3*c^3*Co
s[a] + (12*I)*b^2*c^2*d*Cos[a] + 12*b*c*d^2*Cos[a] - (6*I)*d^3*Cos[a] - 24
*b^3*c^2*d*x*Cos[a] + (24*I)*b^2*c*d^2*x*Cos[a] + 12*b*d^3*x*Cos[a] - 24*b
^3*c*d^2*x^2*Cos[a] + (12*I)*b^2*d^3*x^2*Cos[a] - 8*b^3*d^3*x^3*Cos[a] - (
8*I)*b^4*c^3*x*Cos[a + 2*b*x] - (12*I)*b^4*c^2*d*x^2*Cos[a + 2*b*x] - (8*I
)*b^4*c*d^2*x^3*Cos[a + 2*b*x] - (2*I)*b^4*d^3*x^4*Cos[a + 2*b*x] + (8*...
```

### 3.383.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin(3a+3bx)\sec(a+bx)\,dx$$

$$\Big\downarrow \text{4931}$$

$$\int \left(3(c+dx)^3\sin(a+bx)\cos(a+bx)-(c+dx)^3\sin^2(a+bx)\tan(a+bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3id^3\,\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4}-\frac{3d^3\sin(a+bx)\cos(a+bx)}{2b^4}+\frac{3d^2(c+dx)\,\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{3d^2(c+dx)\sin^2(a+bx)}{b^3}-\frac{3id(c+dx)^2\,\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}+\frac{3d(c+dx)^2\sin(a+bx)\cos(a+bx)}{b^2}+\frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{2(c+dx)^3\sin^2(a+bx)}{b}+\frac{3d^3x}{2b^3}-\frac{(c+dx)^3}{b}-\frac{i(c+dx)^4}{4d}$$

input
```
Int[(c + d*x)^3*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
(3*d^3*x)/(2*b^3) - (c + d*x)^3/b - ((I/4)*(c + d*x)^4)/d + ((c + d*x)^3*L
og[1 + E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((
2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])
/(2*b^3) + (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 - (3*d^3*C
os[a + b*x]*Sin[a + b*x])/(2*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a +
b*x])/b^2 - (3*d^2*(c + d*x)*Sin[a + b*x]^2)/b^3 + (2*(c + d*x)^3*Sin[a +
b*x]^2)/b
```

### 3.383.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.383.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 647 vs. 2(219) = 438.

Time = 2.88 (sec) , antiderivative size = 648, normalized size of antiderivative = 2.68

| method | result |
|---|---|
| risch | $-\frac{6id\,c^2xa}{b}-\frac{3ic\,d^2\operatorname{polylog}\left(2,-\mathrm{e}^{2i(xb+a)}\right)x}{b^2}+\frac{6ic\,d^2a^2x}{b^2}+ic^3x+\frac{ic^4}{4d}+\frac{c^3\ln\left(\mathrm{e}^{2i(xb+a)}+1\right)}{b}-\frac{2c^3\ln\left(\mathrm{e}^{i(xb+a)}\right)}{b}-\frac{6c\,d^2}{}$ |

input
```
int((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output
```
-3/2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2-1/8*(4*d^3*x^3*b^3+6*I*b^2
*d^3*x^2+12*b^3*c*d^2*x^2+12*I*b^2*c*d^2*x+12*b^3*c^2*d*x+6*I*b^2*c^2*d+4*
b^3*c^3-6*b*d^3*x-3*I*d^3-6*c*d^2*b)/b^4*exp(2*I*(b*x+a))-3/2*I/b^2*c^2*d*
polylog(2,-exp(2*I*(b*x+a)))-1/8*(4*d^3*x^3*b^3-6*I*b^2*d^3*x^2+12*b^3*c*d
^2*x^2-12*I*b^2*c*d^2*x+12*b^3*c^2*d*x-6*I*b^2*c^2*d+4*b^3*c^3-6*b*d^3*x+3
*I*d^3-6*c*d^2*b)/b^4*exp(-2*I*(b*x+a))+3/2/b^3*d^3*polylog(3,-exp(2*I*(b*
x+a)))*x+3/2/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))+1/b*d^3*ln(exp(2*I*(b*
x+a))+1)*x^3+1/b*c^3*ln(exp(2*I*(b*x+a))+1)-6*I/b*d*c^2*x*a+6*I/b^2*c*d^2*
a^2*x-I*d^2*c*x^3-3/2*I*d*c^2*x^2-6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a)))+6/b^2
*c^2*d*a*ln(exp(I*(b*x+a)))-2/b*c^3*ln(exp(I*(b*x+a)))-1/4*I*d^3*x^4+3/b*c
*d^2*ln(exp(2*I*(b*x+a))+1)*x^2+3/b*c^2*d*ln(exp(2*I*(b*x+a))+1)*x-3*I/b^2
*d*c^2*a^2+4*I/b^3*c*d^2*a^3-2*I/b^3*d^3*a^3*x+3/4*I*d^3*polylog(4,-exp(2*
I*(b*x+a)))/b^4+2/b^4*d^3*a^3*ln(exp(I*(b*x+a)))-3/2*I/b^4*d^3*a^4+I*c^3*x
+1/4*I/d*c^4-3*I/b^2*d^2*c*polylog(2,-exp(2*I*(b*x+a)))*x
```

### 3.383.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1126 vs. 2(215) = 430.

Time = 0.34 (sec) , antiderivative size = 1126, normalized size of antiderivative = 4.65

$$\int (c+dx)^3\sec(a+bx)\sin(3a+3bx)\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

output
```
1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 6*I*d^3*polylog(4, I*cos(b*x + a) +
 sin(b*x + a)) + 6*I*d^3*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 6*I*d
^3*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(
b*x + a) - sin(b*x + a)) - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3
- 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2*b^2*d^3*x^2
 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 3*(2*b^3
*c^2*d - b*d^3)*x - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dil
og(I*cos(b*x + a) + sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I
*b^2*c^2*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2*I*
b^2*c*d^2*x + I*b^2*c^2*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - 3*(-I*b
^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*
x + a)) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x
+ a) + I*sin(b*x + a) + I) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^
3*d^3)*log(cos(b*x + a) - I*sin(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2
*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(
b*x + a) + sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*
d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) - sin(b*
x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b
^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*...
```

### 3.383.6 Sympy [F(-1)]

Timed out.

$$\int (c+dx)^3 \sec(a+bx)\sin(3a+3bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**3*sec(b*x+a)*sin(3*b*x+3*a),x)
```

output
```
Timed out
```

### 3.383.7 Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 444 vs. 2(215) = 430.

Time = 0.34 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.83

$$\int (c+dx)^3 \sec(a+bx)\sin(3a+3bx)\,dx = -\frac{c^3\left(2\cos(2bx+2a)-\log\left(\cos(2bx)^2+2\cos(2bx)\cos(2a)+\cos(2a)^2+\sin(2bx)^2-2\sin(2bx)\sin\right.\right.}{2b} + \frac{-3i b^4 d^3 x^4 - 12i b^4 c d^2 x^3 - 18i b^4 c^2 d x^2 + 12i d^3 \text{Li}_4(-e^{(2i bx+2i a)}) - 4(-4i b^3 d^3 x^3 - 9i b^3 c d^2 x^2 - 9i b^3 c^2 d}{}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
-1/2*c^3*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) +
cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b + 1/12*
(-3*I*b^4*d^3*x^4 - 12*I*b^4*c*d^2*x^3 - 18*I*b^4*c^2*d*x^2 + 12*I*d^3*pol
ylog(4, -e^(2*I*b*x + 2*I*a)) - 4*(-4*I*b^3*d^3*x^3 - 9*I*b^3*c*d^2*x^2 -
9*I*b^3*c^2*d*x)*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 6*(2*b^
3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(2
*b*x + 2*a) - 6*(4*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(
-e^(2*I*b*x + 2*I*a)) + 2*(4*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x
)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) +
6*(4*b*d^3*x + 3*b*c*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)) + 9*(2*b^2*d^3*
x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*sin(2*b*x + 2*a))/b^4
```

### 3.383.8 Giac [F]

$$\int (c+dx)^3 \sec(a+bx)\sin(3a+3bx)\,dx = \int (dx+c)^3 \sec(bx+a)\sin(3bx+3a)\,dx$$

input
```
integrate((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

### 3.383.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \sec(a+bx)\sin(3a+3bx)\,dx = \int \frac{\sin(3a+3bx)\,(c+dx)^3}{\cos(a+bx)}\,dx$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x)^3)/cos(a + b*x),x)
```

output
```
int((sin(3*a + 3*b*x)*(c + d*x)^3)/cos(a + b*x), x)
```

# 3.384 $\int (c+dx)^2 \sec(a+bx)\sin(3a+3bx)\,dx$


3.384.1 Optimal result . . . 2746
3.384.2 Mathematica [B] (verified) . . . 2747
3.384.3 Rubi [A] (verified) . . . 2748
3.384.4 Maple [B] (verified) . . . 2749
3.384.5 Fricas [B] (verification not implemented) . . . 2749
3.384.6 Sympy [F] . . . 2750
3.384.7 Maxima [A] (verification not implemented) . . . 2750
3.384.8 Giac [F] . . . 2751
3.384.9 Mupad [F(-1)] . . . 2751


## 3.384.1 Optimal result

Integrand size = 23, antiderivative size = 173

$$\int (c+dx)^2 \sec(a+bx)\sin(3a+3bx)\,dx = -\frac{2cdx}{b}-\frac{d^2x^2}{b}-\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{id(c+dx)\operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\operatorname{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{2d(c+dx)\cos(a+bx)\sin(a+bx)}{b^2}-\frac{d^2\sin^2(a+bx)}{b^3}+\frac{2(c+dx)^2\sin^2(a+bx)}{b}$$

output
```
-2*c*d*x/b-d^2*x^2/b-1/3*I*(d*x+c)^3/d+(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b-
I*d*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2+1/2*d^2*polylog(3,-exp(2*I*(b
*x+a)))/b^3+2*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2-d^2*sin(b*x+a)^2/b^3+2*(
d*x+c)^2*sin(b*x+a)^2/b
```

### 3.384.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 516 vs. $2(173) = 346$.

Time = 6.37 (sec) , antiderivative size = 516, normalized size of antiderivative = 2.98

$$\int (c+dx)^2 \sec(a+bx)\sin(3a+3bx)\,dx$$
$$= \frac{id^2e^{-ia}\left(2b^2x^2\left(2bx-3i(1+e^{2ia})\log\left(1+e^{-2i(a+bx)}\right)\right)+6b(1+e^{2ia})\,x\,\mathrm{PolyLog}\left(2,-e^{-2i(a+bx)}\right)-3i(1+e\right.}{12b^3}$$
$$+\frac{c^2\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))}{b\left(\cos^2(a)+\sin^2(a)\right)}$$
$$+\frac{cd\csc(a)\left(b^2e^{-i\arctan(\cot(a))}x^2-\frac{\cot(a)(ibx(-\pi-2\arctan(\cot(a)))-\pi\log(1+e^{-2ibx})-2(bx-\arctan(\cot(a)))\log(1-e^{2i(bx-\arctan(c}}{\sqrt{}}\right.}{b^2\sqrt{\csc^2(a)\left(\cos^2(a)+s\right.}}$$
$$-\frac{\cos(2bx)\left(2b^2c^2\cos(2a)-d^2\cos(2a)+4b^2cdx\cos(2a)+2b^2d^2x^2\cos(2a)-2bcd\sin(2a)-2bd^2x\sin(2a)\right.}{2b^3}$$
$$+\frac{(2bcd\cos(2a)+2bd^2x\cos(2a)+2b^2c^2\sin(2a)-d^2\sin(2a)+4b^2cdx\sin(2a)+2b^2d^2x^2\sin(2a))\sin(2bx)}{2b^3}$$
$$-\frac{1}{3}x\left(3c^2+3cdx+d^2x^2\right)\tan(a)$$

input
```
Integrate[(c + d*x)^2*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
((I/12)*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*
(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] -
 (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E
^(I*a)) + (c^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x
*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (c*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[C
ot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b
*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))]
+ Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*P
olyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/
(b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (Cos[2*b*x]*(2*b^2*c^2*Cos[2*
a] - d^2*Cos[2*a] + 4*b^2*c*d*x*Cos[2*a] + 2*b^2*d^2*x^2*Cos[2*a] - 2*b*c*
d*Sin[2*a] - 2*b*d^2*x*Sin[2*a]))/(2*b^3) + ((2*b*c*d*Cos[2*a] + 2*b*d^2*x
*Cos[2*a] + 2*b^2*c^2*Sin[2*a] - d^2*Sin[2*a] + 4*b^2*c*d*x*Sin[2*a] + 2*b
^2*d^2*x^2*Sin[2*a])*Sin[2*b*x])/(2*b^3) - (x*(3*c^2 + 3*c*d*x + d^2*x^2)*
Tan[a])/3
```

## 3.384.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin(3a+3bx)\sec(a+bx)\,dx$$

$$\Big\downarrow \text{4931}$$

$$\int \left(3(c+dx)^2\sin(a+bx)\cos(a+bx)-(c+dx)^2\sin^2(a+bx)\tan(a+bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{d^2\sin^2(a+bx)}{b^3}-\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}+\frac{2d(c+dx)\sin(a+bx)\cos(a+bx)}{b^2}+\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{2(c+dx)^2\sin^2(a+bx)}{b}-\frac{(c+dx)^2}{b}-\frac{i(c+dx)^3}{3d}$$

input
```
Int[(c + d*x)^2*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
-((c + d*x)^2/b) - ((I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[1 + E^((2*I)*(
a + b*x))])/b - (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 + (d^
2*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (2*d*(c + d*x)*Cos[a + b*x]*
Sin[a + b*x])/b^2 - (d^2*Sin[a + b*x]^2)/b^3 + (2*(c + d*x)^2*Sin[a + b*x]
^2)/b
```

### 3.384.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

## 3.384.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(161) = 322.

Time = 1.39 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.23

| method | result |
|---|---|
| risch | $ic^2x - \frac{id^2x^3}{3} - \frac{2idc\,a^2}{b^2} - \frac{4idcxa}{b} - \frac{(2x^2d^2b^2+4b^2cdx+2ib\,d^2x+2b^2c^2+2ibcd-d^2)\,e^{2i(xb+a)}}{4b^3} - \frac{(2x^2d^2b^2+4b^2cdx-2ib\,d^2x+}{4}$ |

input
```
int((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output
```
I*c^2*x-1/3*I*d^2*x^3-2*I/b^2*d*c*a^2-4*I/b*d*c*x*a-1/4*(2*x^2*d^2*b^2+2*I
*b*d^2*x+4*b^2*c*d*x+2*I*b*c*d+2*b^2*c^2-d^2)/b^3*exp(2*I*(b*x+a))-1/4*(2*
x^2*d^2*b^2-2*I*b*d^2*x+4*b^2*c*d*x-2*I*b*c*d+2*b^2*c^2-d^2)/b^3*exp(-2*I*
(b*x+a))-2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))+1/3*I/d*c^3+2/b*c*d*ln(exp(2*I*(
b*x+a))+1)*x-I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x+4/b^2*c*d*a*ln(exp(I
*(b*x+a)))-I/b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))+2*I/b^2*d^2*a^2*x-I*d*c*
x^2+4/3*I/b^3*d^2*a^3+1/b*d^2*ln(exp(2*I*(b*x+a))+1)*x^2+1/2*d^2*polylog(3
,-exp(2*I*(b*x+a)))/b^3+1/b*c^2*ln(exp(2*I*(b*x+a))+1)-2/b*c^2*ln(exp(I*(b
*x+a)))
```

## 3.384.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 681 vs. 2(158) = 316.

Time = 0.30 (sec) , antiderivative size = 681, normalized size of antiderivative = 3.94

$$\int (c+dx)^2\sec(a+bx)\sin(3a+3bx)\,dx$$
$$= \frac{2\,b^2d^2x^2 + 4\,b^2cdx - 2\,(2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 - d^2)\cos{(bx+a)}^2 + 2\,d^2\mathrm{polylog}(3, i\,\cos{(bx+a)} + \sin}{}$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

output
```
1/2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x - 2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*
c^2 - d^2)*cos(b*x + a)^2 + 2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)
) + 2*d^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*polylog(3, -I*
cos(b*x + a) + sin(b*x + a)) + 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x
+ a)) + 4*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) - 2*(-I*b*d^2*x - I*
b*c*d)*dilog(I*cos(b*x + a) + sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*dilo
g(I*cos(b*x + a) - sin(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*dilog(-I*cos(b*
x + a) + sin(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*dilog(-I*cos(b*x + a) -
sin(b*x + a)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) + I*sin(b
*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) - I*sin(b*
x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(
b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^
2*d^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x
 + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2
*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*x +
a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x +
a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x +
a) + I))/b^3
```

## 3.384.6 Sympy [F]

$$\int (c+dx)^2 \sec(a+bx)\sin(3a+3bx)\,dx=\int (c+dx)^2 \sin(3a+3bx)\sec(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*sec(b*x+a)*sin(3*b*x+3*a),x)
```

output
```
Integral((c + d*x)**2*sin(3*a + 3*b*x)*sec(a + b*x), x)
```

## 3.384.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.75

$$\int (c+dx)^2 \sec(a+bx)\sin(3a+3bx)\,dx=$$

$$-\frac{c^2\left(2\cos(2bx+2a)-\log\left(\cos(2bx)^2+2\cos(2bx)\cos(2a)+\cos(2a)^2+\sin(2bx)^2-2\sin(2bx)\sin\right.\right.}{2b}$$

$$+\frac{-2i\,b^3d^2x^3-6i\,b^3cdx^2+3\,d^2\mathrm{Li}_3\left(-e^{(2i\,bx+2i\,a)}\right)-6\left(-i\,b^2d^2x^2-2i\,b^2cdx\right)\arctan\left(\sin(2bx+2a),\cos(2\right.}{}$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
-1/2*c^2*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) +
cos(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b + 1/6*(
-2*I*b^3*d^2*x^3 - 6*I*b^3*c*d*x^2 + 3*d^2*polylog(3, -e^(2*I*b*x + 2*I*a)
) - 6*(-I*b^2*d^2*x^2 - 2*I*b^2*c*d*x)*arctan2(sin(2*b*x + 2*a), cos(2*b*x
 + 2*a) + 1) - 3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x - d^2)*cos(2*b*x + 2*a) - 6*
(I*b*d^2*x + I*b*c*d)*dilog(-e^(2*I*b*x + 2*I*a)) + 3*(b^2*d^2*x^2 + 2*b^2
*c*d*x)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) +
 1) + 6*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a))/b^3
```

### 3.384.8 Giac [F]

$$\int (c+dx)^2 \sec(a+bx)\sin(3a+3bx)\,dx = \int (dx+c)^2 \sec(bx+a)\sin(3\,bx+3\,a)\,dx$$

input
```
integrate((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

### 3.384.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \sec(a+bx)\sin(3a+3bx)\,dx = \int \frac{\sin(3\,a+3\,b\,x)\,(c+d\,x)^2}{\cos(a+b\,x)}\,dx$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x)^2)/cos(a + b*x),x)
```

output
```
int((sin(3*a + 3*b*x)*(c + d*x)^2)/cos(a + b*x), x)
```

# 3.385 $\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx$


3.385.1 Optimal result . . . 2752
3.385.2 Mathematica [B] (verified) . . . 2752
3.385.3 Rubi [A] (verified) . . . 2753
3.385.4 Maple [A] (verified) . . . 2754
3.385.5 Fricas [B] (verification not implemented) . . . 2755
3.385.6 Sympy [F] . . . 2755
3.385.7 Maxima [F] . . . 2756
3.385.8 Giac [F] . . . 2756
3.385.9 Mupad [F(-1)] . . . 2756


## 3.385.1 Optimal result

Integrand size = 21, antiderivative size = 107

$$\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx = -\frac{dx}{b}-\frac{i(c+dx)^2}{2d}+\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{id\,\mathrm{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}+\frac{d\cos(a+bx)\sin(a+bx)}{b^2}+\frac{2(c+dx)\sin^2(a+bx)}{b}$$

output
```
-d*x/b-1/2*I*(d*x+c)^2/d+(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b-1/2*I*d*polylog(
2,-exp(2*I*(b*x+a)))/b^2+d*cos(b*x+a)*sin(b*x+a)/b^2+2*(d*x+c)*sin(b*x+a)^
2/b
```

## 3.385.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 257 vs. 2(107) = 214.

Time = 3.79 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.40

$$\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx = \frac{c\log(\cos(a+bx))}{b}$$
$$+\frac{d\csc(a)\left(b^2e^{-i\arctan(\cot(a))}x^2 - \frac{\cot(a)(ibx(-\pi-2\arctan(\cot(a)))-\pi\log(1+e^{-2ibx})-2(bx-\arctan(\cot(a)))\log(1-e^{2i(bx-\arctan(co}{\sqrt{}}\right.}{2b^2\sqrt{\csc^2(a)\,(\cos^2(a)+s}}$$
$$-\frac{d\cos(2bx)(2bx\cos(2a)-\sin(2a))}{2b^2}+\frac{d(\cos(2a)+2bx\sin(2a))\sin(2bx)}{2b^2}$$
$$+\frac{2c\sin^2(a+bx)}{b}-\frac{1}{2}dx^2\tan(a)$$

input
```
Integrate[(c + d*x)*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
(c*Log[Cos[a + b*x]])/b + (d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot
[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x
- ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b
*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((
2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Cs
c[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (d*Cos[2*b*x]*(2*b*x*Cos[2*a] - Sin[2*a])
)/(2*b^2) + (d*(Cos[2*a] + 2*b*x*Sin[2*a])*Sin[2*b*x])/(2*b^2) + (2*c*Sin[
a + b*x]^2)/b - (d*x^2*Tan[a])/2
```

## 3.385.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin(3a+3bx)\sec(a+bx)\,dx$$

$$\Big\downarrow \text{4931}$$

$$\int \left(3(c+dx)\sin(a+bx)\cos(a+bx)-(c+dx)\sin^2(a+bx)\tan(a+bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{id \operatorname{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}+\frac{d\sin(a+bx)\cos(a+bx)}{b^2}+\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{2(c+dx)\sin^2(a+bx)}{b}-\frac{dx}{b}-\frac{i(c+dx)^2}{2d}$$

input
```
Int[(c + d*x)*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

output
```
-((d*x)/b) - ((I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 + E^((2*I)*(a + b*x)
)])/b - ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 + (d*Cos[a + b*x]*S
in[a + b*x])/b^2 + (2*(c + d*x)*Sin[a + b*x]^2)/b
```

### 3.385.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

## 3.385.4 Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.65

| method | result |
|---|---|
| risch | $-\frac{id\,x^2}{2}+icx-\frac{(2dxb+2cb+id)\mathrm{e}^{2i(xb+a)}}{4b^2}-\frac{(2dxb+2cb-id)\mathrm{e}^{-2i(xb+a)}}{4b^2}+\frac{c\ln\left(\mathrm{e}^{2i(xb+a)}+1\right)}{b}-\frac{2c\ln\left(\mathrm{e}^{i(xb+a)}\right)}{b}-\frac{2idxa}{b}-$ |

input
```
int((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*d*x^2+I*c*x-1/4*(2*d*x*b+I*d+2*c*b)/b^2*exp(2*I*(b*x+a))-1/4*(2*d*x
*b-I*d+2*c*b)/b^2*exp(-2*I*(b*x+a))+1/b*c*ln(exp(2*I*(b*x+a))+1)-2/b*c*ln(
exp(I*(b*x+a)))-2*I/b*d*x*a-I/b^2*d*a^2+1/b*d*ln(exp(2*I*(b*x+a))+1)*x-1/2
*I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2+2/b^2*d*a*ln(exp(I*(b*x+a)))
```

### 3.385.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(94) = 188$.

Time = 0.28 (sec) , antiderivative size = 340, normalized size of antiderivative = 3.18

$$\int (c + dx) \sec(a + bx) \sin(3a + 3bx) \, dx$$
$$= \frac{2\,bdx - 4\,(bdx + bc)\cos{(bx + a)}^2 + 2\,d\cos{(bx + a)}\sin{(bx + a)} + i\,d\mathrm{Li}_2(i\,\cos{(bx + a)} + \sin{(bx + a)}) - i}{}$$

input
```
integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

output
```
1/2*(2*b*d*x - 4*(b*d*x + b*c)*cos(b*x + a)^2 + 2*d*cos(b*x + a)*sin(b*x +
 a) + I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) - I*d*dilog(I*cos(b*x + a)
- sin(b*x + a)) - I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) + I*d*dilog(-I
*cos(b*x + a) - sin(b*x + a)) + (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x +
 a) + I) + (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) + (b*d*x + a
*d)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x + a*d)*log(I*cos(b*x +
 a) - sin(b*x + a) + 1) + (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x + a)
 + 1) + (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a*d
)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*c - a*d)*log(-cos(b*x + a)
- I*sin(b*x + a) + I))/b^2
```

### 3.385.6 Sympy [F]

$$\int (c + dx) \sec(a + bx) \sin(3a + 3bx) \, dx = \int (c + dx) \sin{(3a + 3bx)} \sec{(a + bx)} \, dx$$

input
```
integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x)
```

output
```
Integral((c + d*x)*sin(3*a + 3*b*x)*sec(a + b*x), x)
```

### 3.385.7 Maxima [F]

$$\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx = \int (dx+c)\sec(bx+a)\sin(3\,bx+3\,a)\,dx$$

input
```
integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
-1/2*c*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + co
s(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b - 1/2*(2*
b*x*cos(2*b*x + 2*a) + 4*b^2*integrate(x*sin(2*b*x + 2*a)/(cos(2*b*x + 2*a
)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1), x) - sin(2*b*x + 2*a))
*d/b^2
```

### 3.385.8 Giac [F]

$$\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx = \int (dx+c)\sec(bx+a)\sin(3\,bx+3\,a)\,dx$$

input
```
integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

### 3.385.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx = \int \frac{\sin(3\,a+3\,b\,x)\,(c+d\,x)}{\cos(a+b\,x)}\,dx$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x))/cos(a + b*x),x)
```

output
```
int((sin(3*a + 3*b*x)*(c + d*x))/cos(a + b*x), x)
```

# 3.386 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx$

3.386.1 Optimal result . . . . . . . . . . 2757
3.386.2 Mathematica [N/A] . . . . . . . . . . 2757
3.386.3 Rubi [N/A] . . . . . . . . . . 2758
3.386.4 Maple [N/A] (verified) . . . . . . . . . . 2759
3.386.5 Fricas [N/A] . . . . . . . . . . 2759
3.386.6 Sympy [F(-2)] . . . . . . . . . . 2759
3.386.7 Maxima [N/A] . . . . . . . . . . 2760
3.386.8 Giac [N/A] . . . . . . . . . . 2760
3.386.9 Mupad [N/A] . . . . . . . . . . 2760

## 3.386.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \frac{2\,\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d} + \frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d} - \text{Int}\left(\frac{\tan(a+bx)}{c+dx},x\right)$$

output
```
2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d+2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d
)/d-Unintegrable(tan(b*x+a)/(d*x+c),x)
```

## 3.386.2 Mathematica [N/A]

Not integrable

Time = 2.89 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

input
```
Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]
```

output
```
Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x), x]
```

## 3.386.3 Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\sec(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{4931}$$

$$\int \left(\frac{3\sin(a+bx)\cos(a+bx)}{c+dx}-\frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\int \frac{\tan(a+bx)}{c+dx}dx+\frac{2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d}$$

input
```
Int[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]
```

output
```
$Aborted
```

### 3.386.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.386.4 Maple [N/A] (verified)

Not integrable

Time = 0.54 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sec (xb+a)\sin (3xb+3a)}{dx+c}dx$$

input
```
int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

output
```
int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

### 3.386.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sec (bx+a)\sin (3\,bx+3\,a)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c), x)
```

### 3.386.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.386.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 182, normalized size of antiderivative = 7.91

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)\sin(3\,bx+3\,a)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")
```

output
```
-((-I*exp_integral_e(1, 2*(-I*b*d*x - I*b*c)/d) + I*exp_integral_e(1, -2*(
-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 2*d*integrate(sin(2*b*x + 2*
a)/((d*x + c)*cos(2*b*x + 2*a)^2 + (d*x + c)*sin(2*b*x + 2*a)^2 + d*x + 2*
(d*x + c)*cos(2*b*x + 2*a) + c), x) + (exp_integral_e(1, 2*(-I*b*d*x - I*b
*c)/d) + exp_integral_e(1, -2*(-I*b*d*x - I*b*c)/d))*sin(-2*(b*c - a*d)/d)
)/d
```

### 3.386.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)\sin(3\,bx+3\,a)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c), x)
```

### 3.386.9 Mupad [N/A]

Not integrable

Time = 29.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sin(3\,a+3\,b\,x)}{\cos(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)),x)
```

output
```
int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)), x)
```

3.386. $\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx$

# 3.387 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$

3.387.1 Optimal result . . . . . 2762
3.387.2 Mathematica [N/A] . . . . . 2762
3.387.3 Rubi [N/A] . . . . . 2763
3.387.4 Maple [N/A] (verified) . . . . . 2764
3.387.5 Fricas [N/A] . . . . . 2764
3.387.6 Sympy [F(-2)] . . . . . 2764
3.387.7 Maxima [N/A] . . . . . 2765
3.387.8 Giac [N/A] . . . . . 2765
3.387.9 Mupad [N/A] . . . . . 2766

## 3.387.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \frac{4b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2} - \frac{2\sin(2a+2bx)}{d(c+dx)} - \frac{4b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2} - \text{Int}\left(\frac{\tan(a+bx)}{(c+dx)^2},x\right)$$

output
```
4*b*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^2-4*b*Si(2*b*c/d+2*b*x)*sin(2*a-2
*b*c/d)/d^2-2*sin(2*b*x+2*a)/d/(d*x+c)-Unintegrable(tan(b*x+a)/(d*x+c)^2,x
)
```

## 3.387.2 Mathematica [N/A]

Not integrable

Time = 3.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2, x]
```

### 3.387.3 Rubi [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{4931}$$

$$\int \left(\frac{3\sin(a+bx)\cos(a+bx)}{(c+dx)^2}-\frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\int \frac{\tan(a+bx)}{(c+dx)^2}dx+\frac{4b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{4b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{2\sin(2a+2bx)}{d(c+dx)}$$

input
```
Int[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

#### 3.387.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.387.4 Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sec (xb+a)\sin (3xb+3a)}{(dx+c)^2}dx$$

input
```
int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

output
```
int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

### 3.387.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.57

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\sec (bx+a)\sin (3\,bx+3\,a)}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)*sin(3*b*x + 3*a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.387.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.387.7 Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 242, normalized size of antiderivative = 10.52

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx=\int \frac{\sec(bx+a)\sin(3\,bx+3\,a)}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-((-I*exp_integral_e(2, 2*(-I*b*d*x - I*b*c)/d) + I*exp_integral_e(2, -2*(
-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 2*(d^2*x + c*d)*integrate(si
n(2*b*x + 2*a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(2*b*x +
2*a)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(2*b*x + 2*a)^2 + c^2 + 2*(d^2*x^2 +
 2*c*d*x + c^2)*cos(2*b*x + 2*a)), x) + (exp_integral_e(2, 2*(-I*b*d*x - I
*b*c)/d) + exp_integral_e(2, -2*(-I*b*d*x - I*b*c)/d))*sin(-2*(b*c - a*d)/
d))/(d^2*x + c*d)
```

### 3.387.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx=\int \frac{\sec(bx+a)\sin(3\,bx+3\,a)}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c)^2, x)
```

### 3.387.9 Mupad [N/A]

Not integrable

Time = 30.66 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx=\int \frac{\sin\left(3\,a+3\,b\,x\right)}{\cos\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)^2),x)
```

output
```
int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)^2), x)
```

# 3.388 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$

3.388.1 Optimal result . . . 2767
3.388.2 Mathematica [N/A] . . . 2767
3.388.3 Rubi [N/A] . . . 2768
3.388.4 Maple [N/A] (verified) . . . 2769
3.388.5 Fricas [N/A] . . . 2769
3.388.6 Sympy [F(-1)] . . . 2770
3.388.7 Maxima [N/A] . . . 2770
3.388.8 Giac [N/A] . . . 2770
3.388.9 Mupad [N/A] . . . 2771

## 3.388.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = -\frac{2b\cos(2a+2bx)}{d^2(c+dx)} - \frac{4b^2\,\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^3} - \frac{\sin(2a+2bx)}{d(c+dx)^2} - \frac{4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3} - \text{Int}\left(\frac{\tan(a+bx)}{(c+dx)^3},x\right)$$

output
```
-2*b*cos(2*b*x+2*a)/d^2/(d*x+c)-4*b^2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d
^3-4*b^2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^3-sin(2*b*x+2*a)/d/(d*x+c)^2
-Unintegrable(tan(b*x+a)/(d*x+c)^3,x)
```

## 3.388.2 Mathematica [N/A]

Not integrable

Time = 4.68 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

input
```
Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

output
```
Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3, x]
```

### 3.388.3 Rubi [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\sec(a+bx)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{ 4931}$$

$$\int \left(\frac{3\sin(a+bx)\cos(a+bx)}{(c+dx)^3}-\frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\int \frac{\tan(a+bx)}{(c+dx)^3}dx-\frac{4b^2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{2b\cos(2a+2bx)}{d^2(c+dx)}-\frac{\sin(2a+2bx)}{d(c+dx)^2}$$

input
```
Int[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

output
```
$Aborted
```

#### 3.388.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.388.4 Maple [N/A] (verified)

Not integrable

Time = 0.50 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sec \left(x b +a \right) \sin \left(3 x b +3 a \right)}{\left(d x +c \right)^{3}}d x$$

input
```
int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

output
```
int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

### 3.388.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx=\int { \frac {\sec \left (b x + a\right ) \sin \left (3 \, b x + 3 \, a\right )}{{\left (d x + c\right )}^{3}} \,d x }$$

input
```
integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)*sin(3*b*x + 3*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
+ c^3), x)
```

### 3.388.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**3,x)
```

output
```
Timed out
```

### 3.388.7 Maxima [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 308, normalized size of antiderivative = 13.39

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\sec(bx+a)\sin(3\,bx+3\,a)}{(dx+c)^3}\,dx$$

input
```
integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-((-I*exp_integral_e(3, 2*(-I*b*d*x - I*b*c)/d) + I*exp_integral_e(3, -2*(
-I*b*d*x - I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 2*(d^3*x^2 + 2*c*d^2*x + c^2
*d)*integrate(sin(2*b*x + 2*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3 +
(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*cos(2*b*x + 2*a)^2 + (d^3*x^3 +
3*c*d^2*x^2 + 3*c^2*d*x + c^3)*sin(2*b*x + 2*a)^2 + 2*(d^3*x^3 + 3*c*d^2*x
^2 + 3*c^2*d*x + c^3)*cos(2*b*x + 2*a)), x) + (exp_integral_e(3, 2*(-I*b*d
*x - I*b*c)/d) + exp_integral_e(3, -2*(-I*b*d*x - I*b*c)/d))*sin(-2*(b*c -
 a*d)/d))/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

### 3.388.8 Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\sec(bx+a)\sin(3\,bx+3\,a)}{(dx+c)^3}\,dx$$

input
```
integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c)^3, x)
```

### 3.388.9 Mupad [N/A]

Not integrable

Time = 30.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\sin\left(3\,a+3\,b\,x\right)}{\cos\left(a+b\,x\right)\,{\left(c+d\,x\right)}^3}\,dx$$

input
```
int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)^3),x)
```

output
```
int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)^3), x)
```

# 3.389 $\int (c+dx)^3 \sec^2(a+bx) \sin(3a+3bx) \, dx$

3.389.1 Optimal result . . . 2772
3.389.2 Mathematica [B] (verified) . . . 2773
3.389.3 Rubi [A] (verified) . . . 2773
3.389.4 Maple [B] (verified) . . . 2775
3.389.5 Fricas [B] (verification not implemented) . . . 2775
3.389.6 Sympy [F(-2)] . . . 2776
3.389.7 Maxima [F] . . . 2777
3.389.8 Giac [F] . . . 2777
3.389.9 Mupad [F(-1)] . . . 2778

## 3.389.1 Optimal result

Integrand size = 25, antiderivative size = 230

$$\begin{aligned}\int (c+dx)^3 \sec^2(a+bx) \sin(3a+3bx) \, dx &= -\frac{6id(c+dx)^2 \arctan\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^2(c+dx)\cos(a+bx)}{b^3} - \frac{4(c+dx)^3\cos(a+bx)}{b} \\ &\quad + \frac{6id^2(c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} - \frac{6id^2(c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} \\ &\quad - \frac{6d^3\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4} + \frac{6d^3\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^4} \\ &\quad - \frac{(c+dx)^3\sec(a+bx)}{b} - \frac{24d^3\sin(a+bx)}{b^4} + \frac{12d(c+dx)^2\sin(a+bx)}{b^2}\end{aligned}$$

output
```
-6*I*d*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b^2+24*d^2*(d*x+c)*cos(b*x+a)/b^3-
4*(d*x+c)^3*cos(b*x+a)/b+6*I*d^2*(d*x+c)*polylog(2,-I*exp(I*(b*x+a)))/b^3-
6*I*d^2*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^3-6*d^3*polylog(3,-I*exp(I*(
b*x+a)))/b^4+6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4-(d*x+c)^3*sec(b*x+a)/b-
24*d^3*sin(b*x+a)/b^4+12*d*(d*x+c)^2*sin(b*x+a)/b^2
```

### 3.389.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 532 vs. 2(230) = 460.

Time = 1.71 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.31

$$\int (c+dx)^3 \sec^2(a+bx) \sin(3a+3bx)\, dx =$$
$$-\frac{\sec(a+bx)\left(3b^3c^3-12bcd^2+9b^3c^2dx-12bd^3x+9b^3cd^2x^2+3b^3d^3x^3+6ib^2c^2d\arctan\left(e^{i(a+bx)}\right)\cos(a$$

input
```
Integrate[(c + d*x)^3*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

output
```
-((Sec[a + b*x]*(3*b^3*c^3 - 12*b*c*d^2 + 9*b^3*c^2*d*x - 12*b*d^3*x + 9*b
^3*c*d^2*x^2 + 3*b^3*d^3*x^3 + (6*I)*b^2*c^2*d*ArcTan[E^(I*(a + b*x))]*Cos
[a + b*x] + 2*b^3*c^3*Cos[2*(a + b*x)] - 12*b*c*d^2*Cos[2*(a + b*x)] + 6*b
^3*c^2*d*x*Cos[2*(a + b*x)] - 12*b*d^3*x*Cos[2*(a + b*x)] + 6*b^3*c*d^2*x^
2*Cos[2*(a + b*x)] + 2*b^3*d^3*x^3*Cos[2*(a + b*x)] - 6*b^2*c*d^2*x*Cos[a
+ b*x]*Log[1 - I*E^(I*(a + b*x))] - 3*b^2*d^3*x^2*Cos[a + b*x]*Log[1 - I*E
^(I*(a + b*x))] + 6*b^2*c*d^2*x*Cos[a + b*x]*Log[1 + I*E^(I*(a + b*x))] +
3*b^2*d^3*x^2*Cos[a + b*x]*Log[1 + I*E^(I*(a + b*x))] - (6*I)*b*d^2*(c + d
*x)*Cos[a + b*x]*PolyLog[2, (-I)*E^(I*(a + b*x))] + (6*I)*b*d^2*(c + d*x)*
Cos[a + b*x]*PolyLog[2, I*E^(I*(a + b*x))] + 6*d^3*Cos[a + b*x]*PolyLog[3,
 (-I)*E^(I*(a + b*x))] - 6*d^3*Cos[a + b*x]*PolyLog[3, I*E^(I*(a + b*x))]
- 6*b^2*c^2*d*Sin[2*(a + b*x)] + 12*d^3*Sin[2*(a + b*x)] - 12*b^2*c*d^2*x*
Sin[2*(a + b*x)] - 6*b^2*d^3*x^2*Sin[2*(a + b*x)]))/b^4)
```

### 3.389.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \sin(3a+3bx) \sec^2(a+bx)\, dx$$
$$\Big\downarrow \text{ 4931}$$

$$\int \left(3(c+dx)^3\sin(a+bx)-(c+dx)^3\sin(a+bx)\tan^2(a+bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6id(c+dx)^2\arctan\left(e^{i(a+bx)}\right)}{b^2}-\frac{6d^3\operatorname{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4}+\frac{6d^3\operatorname{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^4}-\frac{24d^3\sin(a+bx)}{b^4}+\frac{6id^2(c+dx)\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}+\frac{24d^2(c+dx)\cos(a+bx)}{b^3}+\frac{12d(c+dx)^2\sin(a+bx)}{b^2}-\frac{4(c+dx)^3\cos(a+bx)}{b}-\frac{(c+dx)^3\sec(a+bx)}{b}$$

input
```
Int[(c + d*x)^3*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

output
```
((-6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 + (24*d^2*(c + d*x)*Cos
[a + b*x])/b^3 - (4*(c + d*x)^3*Cos[a + b*x])/b + ((6*I)*d^2*(c + d*x)*Pol
yLog[2, (-I)*E^(I*(a + b*x))])/b^3 - ((6*I)*d^2*(c + d*x)*PolyLog[2, I*E^(
I*(a + b*x))])/b^3 - (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 + (6*d^3
*PolyLog[3, I*E^(I*(a + b*x))])/b^4 - ((c + d*x)^3*Sec[a + b*x])/b - (24*d
^3*Sin[a + b*x])/b^4 + (12*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

### 3.389.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.389.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 676 vs. $2(213) = 426$.

Time = 8.57 (sec) , antiderivative size = 677, normalized size of antiderivative = 2.94

| method | result |
|---|---|
| risch | $-\frac{2\left(d^3x^3b^3+3b^3c\,d^2x^2+3ib^2d^3x^2+3b^3c^2dx+6ib^2c\,d^2x+b^3c^3+3ib^2c^2d-6b\,d^3x-6c\,d^2b-6id^3\right)e^{i(xb+a)}}{b^4}-\frac{2\left(d^3x^3b^3+3b^3c\,d^2x^2-3ib\right.}{}$ |

input
```
int((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output
```
-2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b*
d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))-
2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d
^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))-
2*exp(I*(b*x+a))*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(exp(2*I*(b*x+a))+1
)+3/b^4*d^3*a^2*ln(1+I*exp(I*(b*x+a)))-3/b^2*d^3*ln(1+I*exp(I*(b*x+a)))*x^
2+6*I/b^3*d^2*c*polylog(2,-I*exp(I*(b*x+a)))-6*I/b^3*d^3*polylog(2,I*exp(I
*(b*x+a)))*x+6/b^2*d^2*c*ln(1-I*exp(I*(b*x+a)))*x+3/b^2*d^3*ln(1-I*exp(I*(
b*x+a)))*x^2-6*I/b^2*d*c^2*arctan(exp(I*(b*x+a)))+6*d^3*polylog(3,I*exp(I*
(b*x+a)))/b^4-3/b^4*d^3*a^2*ln(1-I*exp(I*(b*x+a)))-6/b^3*d^2*c*ln(1+I*exp(
I*(b*x+a)))*a+6*I/b^3*d^3*polylog(2,-I*exp(I*(b*x+a)))*x-6*I/b^3*d^2*c*pol
ylog(2,I*exp(I*(b*x+a)))-6*I/b^4*d^3*a^2*arctan(exp(I*(b*x+a)))-6/b^2*d^2*
c*ln(1+I*exp(I*(b*x+a)))*x-6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4+6/b^3*d^
2*c*ln(1-I*exp(I*(b*x+a)))*a+12*I/b^3*d^2*c*a*arctan(exp(I*(b*x+a)))
```

### 3.389.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 896 vs. $2(204) = 408$.

Time = 0.30 (sec) , antiderivative size = 896, normalized size of antiderivative = 3.90

$$\int (c+dx)^3\sec^2(a+bx)\sin(3a+3bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

output
```
-1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*
cos(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a
)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3
, -I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b
*x + a) - sin(b*x + a)) + 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b
*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 6*(I*b*d^3*x + I*b*c*
d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)) + 6*(I*b*d^3*x + I*
b*c*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a)) + 6*(-I*b*d^3*x
 - I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + 6*(-I*b
*d^3*x - I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3
*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin
(b*x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(co
s(b*x + a) - I*sin(b*x + a) + I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*
c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(
b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*co
s(b*x + a) - sin(b*x + a) + 1) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*
d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b
^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*co
s(b*x + a) - sin(b*x + a) + 1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos
(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + 3*(b^2*c^2*d - 2*a*...
```

### 3.389.6 Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^3 \sec^2(a+bx)\sin(3a+3bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d*x+c)**3*sec(b*x+a)**2*sin(3*b*x+3*a),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.389.7 Maxima [F]

$$\int (c+dx)^3 \sec^2(a+bx)\sin(3a+3bx)\,dx = \int (dx+c)^3 \sec(bx+a)^2 \sin(3\,bx+3\,a)\,dx$$

input
```
integrate((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
-2*((cos(3*b*x + 3*a) + cos(b*x + a))*cos(4*b*x + 4*a) + (3*cos(2*b*x + 2*
a) + 1)*cos(3*b*x + 3*a) + 3*cos(2*b*x + 2*a)*cos(b*x + a) + (sin(3*b*x +
3*a) + sin(b*x + a))*sin(4*b*x + 4*a) + 3*sin(3*b*x + 3*a)*sin(2*b*x + 2*a
) + 3*sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))*c^3/(b*cos(3*b*x + 3*a
)^2 + 2*b*cos(3*b*x + 3*a)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x +
 3*a)^2 + 2*b*sin(3*b*x + 3*a)*sin(b*x + a) + b*sin(b*x + a)^2) - 3/2*(4*(
cos(a)^2 + sin(a)^2)*b*x*cos(b*x + a) + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x +
 2*a) + b*x*cos(b*x + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) +
b*x*sin(b*x + 2*a)*sin(a))*cos(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(
b*x + a))*cos(2*b*x + 3*a)^2 + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) + b
*x*cos(b*x + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(b
*x + 2*a)*sin(a))*sin(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))
*sin(2*b*x + 3*a)^2 + 4*((b*x*cos(2*b*x + 3*a) + b*x*cos(a) + sin(2*b*x +
3*a) + sin(a))*cos(3*b*x + 3*a)^2 + (b*x*cos(a) + sin(a))*cos(b*x + a)^2 +
 (b*x*cos(2*b*x + 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a))*sin(3*b*x
 + 3*a)^2 + (b*x*cos(a) + sin(a))*sin(b*x + a)^2 + 2*(b*x*cos(2*b*x + 3*a)
*cos(b*x + a) + (b*x*cos(a) + sin(a))*cos(b*x + a) + cos(b*x + a)*sin(2*b*
x + 3*a))*cos(3*b*x + 3*a) + (b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos
(2*b*x + 3*a) + 2*(b*x*cos(2*b*x + 3*a)*sin(b*x + a) + (b*x*cos(a) + sin(a
))*sin(b*x + a) + sin(2*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a) + (co...
```

### 3.389.8 Giac [F]

$$\int (c+dx)^3 \sec^2(a+bx)\sin(3a+3bx)\,dx = \int (dx+c)^3 \sec(bx+a)^2 \sin(3\,bx+3\,a)\,dx$$

input
```
integrate((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*sec(b*x + a)^2*sin(3*b*x + 3*a), x)
```

### 3.389.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \sec^2(a+bx) \sin(3a+3bx) \, dx = \text{Hanged}$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x)^3)/cos(a + b*x)^2,x)
```

output
```
\text{Hanged}
```

# 3.390 $\int (c+dx)^2 \sec^2(a+bx)\sin(3a+3bx)\,dx$

3.390.1 Optimal result . . . 2779
3.390.2 Mathematica [B] (verified) . . . 2780
3.390.3 Rubi [A] (verified) . . . 2780
3.390.4 Maple [B] (verified) . . . 2781
3.390.5 Fricas [B] (verification not implemented) . . . 2782
3.390.6 Sympy [F(-2)] . . . 2783
3.390.7 Maxima [F] . . . 2783
3.390.8 Giac [F] . . . 2784
3.390.9 Mupad [F(-1)] . . . 2784

## 3.390.1 Optimal result

Integrand size = 25, antiderivative size = 147

$$\int (c+dx)^2 \sec^2(a+bx)\sin(3a+3bx)\,dx = -\frac{4id(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b^2} + \frac{8d^2\cos(a+bx)}{b^3} - \frac{4(c+dx)^2\cos(a+bx)}{b} + \frac{2id^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3} - \frac{2id^2\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3} - \frac{(c+dx)^2\sec(a+bx)}{b} + \frac{8d(c+dx)\sin(a+bx)}{b^2}$$

output
```
-4*I*d*(d*x+c)*arctan(exp(I*(b*x+a)))/b^2+8*d^2*cos(b*x+a)/b^3-4*(d*x+c)^2
*cos(b*x+a)/b+2*I*d^2*polylog(2,-I*exp(I*(b*x+a)))/b^3-2*I*d^2*polylog(2,I
*exp(I*(b*x+a)))/b^3-(d*x+c)^2*sec(b*x+a)/b+8*d*(d*x+c)*sin(b*x+a)/b^2
```

### 3.390.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 364 vs. 2(147) = 294.

Time = 2.62 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.48

$$\int (c+dx)^2 \sec^2(a+bx) \sin(3a+3bx)\,dx$$

$$= \frac{4bcd\,\text{arctanh}\left(\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)\right)+2d^2\left(2\arctan(\cot(a))\text{arctanh}\left(\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)\right)-\frac{\csc(a)((}{}\right.}{}$$

input
```
Integrate[(c + d*x)^2*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

output
```
(4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + 2*d^2*(2*ArcTan[Cot[a]]*A
rcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - (Csc[a]*((b*x - ArcTan[Cot[a]])*(Lo
g[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcTan[Cot[a]]))
]) + I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - I*PolyLog[2, E^(I*(b*x
- ArcTan[Cot[a]]))]))/Sqrt[Csc[a]^2]) - b^2*(c + d*x)^2*Sec[a] - 4*Cos[b*x
]*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] - 2*b*d*(c + d*x)*Sin[a]) + 4*(2*b*d*
(c + d*x)*Cos[a] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x] - (b^2*(c +
 d*x)^2*Sin[(b*x)/2])/((Cos[a/2] - Sin[a/2])*(Cos[(a + b*x)/2] - Sin[(a +
b*x)/2])) + (b^2*(c + d*x)^2*Sin[(b*x)/2])/((Cos[a/2] + Sin[a/2])*(Cos[(a
+ b*x)/2] + Sin[(a + b*x)/2])))/b^3
```

### 3.390.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sin(3a+3bx) \sec^2(a+bx)\,dx$$

$$\Big\downarrow \; 4931$$

$$\int \left(3(c+dx)^2 \sin(a+bx)-(c+dx)^2 \sin(a+bx)\tan^2(a+bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4id(c+dx)\arctan\left(e^{i(a+bx)}\right)}{b^2}+\frac{2id^2\operatorname{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}-\frac{2id^2\operatorname{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}+\frac{8d^2\cos(a+bx)}{b^3}+\frac{8d(c+dx)\sin(a+bx)}{b^2}-\frac{4(c+dx)^2\cos(a+bx)}{b}-\frac{(c+dx)^2\sec(a+bx)}{b}$$

input
```
Int[(c + d*x)^2*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

output
```
((-4*I)*d*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 + (8*d^2*Cos[a + b*x])/b^
3 - (4*(c + d*x)^2*Cos[a + b*x])/b + ((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a +
b*x))])/b^3 - ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((c + d*x)^2
*Sec[a + b*x])/b + (8*d*(c + d*x)*Sin[a + b*x])/b^2
```

### 3.390.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

## 3.390.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(136) = 272.

Time = 4.24 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.27

| method | result |
|---|---|
| default | $-\frac{4c^2\cos(xb+a)}{b}-\frac{c^2}{b\cos(xb+a)}+\frac{4d^2\left(-(xb+a)^2\cos(xb+a)+2\cos(xb+a)+2(xb+a)\sin(xb+a)-2a(\sin(xb+a)-(xb+a)\cos(xb+\ldots\right.}{b^3}$ |
| risch | $-\frac{2\left(x^2d^2b^2+2b^2cdx+2ib\,d^2x+b^2c^2+2ibcd-2d^2\right)\mathrm{e}^{i(xb+a)}}{b^3}-\frac{2\left(x^2d^2b^2+2b^2cdx-2ib\,d^2x+b^2c^2-2ibcd-2d^2\right)\mathrm{e}^{-i(xb+a)}}{b^3}-\frac{2\,\mathrm{e}^{i(xb+a\ldots}}{b(\ldots}$ |

input
```
int((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output
```
-4*c^2*cos(b*x+a)/b-c^2/b/cos(b*x+a)+4*d^2/b^3*(-(b*x+a)^2*cos(b*x+a)+2*co
s(b*x+a)+2*(b*x+a)*sin(b*x+a)-2*a*(sin(b*x+a)-(b*x+a)*cos(b*x+a))-a^2*cos(
b*x+a))-d^2/b^3*((b*x+a)^2/cos(b*x+a)+2*(b*x+a)*ln(1+I*exp(I*(b*x+a)))-2*(
b*x+a)*ln(1-I*exp(I*(b*x+a)))-2*I*dilog(1+I*exp(I*(b*x+a)))+2*I*dilog(1-I*
exp(I*(b*x+a)))-2*a*((b*x+a)/cos(b*x+a)-ln(sec(b*x+a)+tan(b*x+a)))+a^2/cos
(b*x+a))+8*c*d/b^2*(sin(b*x+a)-(b*x+a)*cos(b*x+a)+cos(b*x+a)*a)-2*c*d/b/co
s(b*x+a)*x+2*c*d/b^2*ln(sec(b*x+a)+tan(b*x+a))
```

### 3.390.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(129) = 258.

Time = 0.28 (sec) , antiderivative size = 513, normalized size of antiderivative = 3.49

$$\int (c+dx)^2 \sec^2(a+bx) \sin(3a+3bx)\,dx =$$
$$-\frac{b^2d^2x^2 + 2\,b^2cdx + b^2c^2 + i\,d^2\cos(bx+a)\,\mathrm{Li}_2(i\,\cos(bx+a) + \sin(bx+a)) + i\,d^2\cos(bx+a)\,\mathrm{Li}_2(i\,\cos$$

input
```
integrate((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

output
```
-(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*cos(b*x + a)*dilog(I*cos(b*x
 + a) + sin(b*x + a)) + I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x
+ a)) - I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d^2*c
os(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 4*(b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - (b*c*d - a*d^2)*cos(b*x + a)*lo
g(cos(b*x + a) + I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(co
s(b*x + a) - I*sin(b*x + a) + I) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*co
s(b*x + a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(
b*x + a) - sin(b*x + a) + 1) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b
*x + a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*
x + a) - sin(b*x + a) + 1) - (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a
) + I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) -
 I*sin(b*x + a) + I) - 8*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3
*cos(b*x + a))
```

### 3.390.6 Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^2 \sec^2(a+bx) \sin(3a+3bx) \, dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d*x+c)**2*sec(b*x+a)**2*sin(3*b*x+3*a),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.390.7 Maxima [F]

$$\int (c+dx)^2 \sec^2(a+bx) \sin(3a+3bx) \, dx = \int (dx+c)^2 \sec(bx+a)^2 \sin(3bx+3a) \, dx$$

input
```
integrate((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
-2*((cos(3*b*x + 3*a) + cos(b*x + a))*cos(4*b*x + 4*a) + (3*cos(2*b*x + 2*
a) + 1)*cos(3*b*x + 3*a) + 3*cos(2*b*x + 2*a)*cos(b*x + a) + (sin(3*b*x +
3*a) + sin(b*x + a))*sin(4*b*x + 4*a) + 3*sin(3*b*x + 3*a)*sin(2*b*x + 2*a
) + 3*sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))*c^2/(b*cos(3*b*x + 3*a
)^2 + 2*b*cos(3*b*x + 3*a)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x +
 3*a)^2 + 2*b*sin(3*b*x + 3*a)*sin(b*x + a) + b*sin(b*x + a)^2) - (4*(cos(
a)^2 + sin(a)^2)*b*x*cos(b*x + a) + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a
) + b*x*cos(b*x + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*
sin(b*x + 2*a)*sin(a))*cos(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x
+ a))*cos(2*b*x + 3*a)^2 + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) + b*x*c
os(b*x + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(b*x +
 2*a)*sin(a))*sin(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))*sin
(2*b*x + 3*a)^2 + 4*((b*x*cos(2*b*x + 3*a) + b*x*cos(a) + sin(2*b*x + 3*a)
 + sin(a))*cos(3*b*x + 3*a)^2 + (b*x*cos(a) + sin(a))*cos(b*x + a)^2 + (b*
x*cos(2*b*x + 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a))*sin(3*b*x + 3
*a)^2 + (b*x*cos(a) + sin(a))*sin(b*x + a)^2 + 2*(b*x*cos(2*b*x + 3*a)*cos
(b*x + a) + (b*x*cos(a) + sin(a))*cos(b*x + a) + cos(b*x + a)*sin(2*b*x +
3*a))*cos(3*b*x + 3*a) + (b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos(2*b
*x + 3*a) + 2*(b*x*cos(2*b*x + 3*a)*sin(b*x + a) + (b*x*cos(a) + sin(a))*s
in(b*x + a) + sin(2*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a) + (cos(b*...
```

### 3.390.8 Giac [F]

$$\int (c+dx)^2 \sec^2(a+bx) \sin(3a+3bx)\,dx = \int (dx+c)^2 \sec(bx+a)^2 \sin(3\,bx+3\,a)\,dx$$

input
```
integrate((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*sec(b*x + a)^2*sin(3*b*x + 3*a), x)
```

### 3.390.9 Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \sec^2(a+bx) \sin(3a+3bx)\,dx = \text{Hanged}$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x)^2)/cos(a + b*x)^2,x)
```

output
```
\text{Hanged}
```

# 3.391 $\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx$

3.391.1 Optimal result . . . 2785
3.391.2 Mathematica [A] (verified) . . . 2785
3.391.3 Rubi [A] (verified) . . . 2786
3.391.4 Maple [A] (verified) . . . 2787
3.391.5 Fricas [A] (verification not implemented) . . . 2787
3.391.6 Sympy [F(-2)] . . . 2787
3.391.7 Maxima [B] (verification not implemented) . . . 2788
3.391.8 Giac [B] (verification not implemented) . . . 2788
3.391.9 Mupad [B] (verification not implemented) . . . 2789

## 3.391.1 Optimal result

Integrand size = 23, antiderivative size = 57

$$\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx=\frac{d\operatorname{arctanh}(\sin(a+bx))}{b^2}-\frac{4(c+dx)\cos(a+bx)}{b}-\frac{(c+dx)\sec(a+bx)}{b}+\frac{4d\sin(a+bx)}{b^2}$$

output
```
d*arctanh(sin(b*x+a))/b^2-4*(d*x+c)*cos(b*x+a)/b-(d*x+c)*sec(b*x+a)/b+4*d*
sin(b*x+a)/b^2
```

## 3.391.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.84

$$\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx=-\frac{\sec(a+bx)\left(3bc+3bdx+2b(c+dx)\cos(2(a+bx))+d\cos(a+bx)\left(\log\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+\right.\right.\right.\right.}{b^2}$$

input
```
Integrate[(c + d*x)*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

output
```
-((Sec[a + b*x]*(3*b*c + 3*b*d*x + 2*b*(c + d*x)*Cos[2*(a + b*x)] + d*Cos[
a + b*x]*(Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]] - Log[Cos[(a + b*x)/2]
+ Sin[(a + b*x)/2]]) - 2*d*Sin[2*(a + b*x)]))/b^2)
```

## 3.391.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sin(3a+3bx)\sec^2(a+bx)\,dx$$

$$\Big\downarrow \; 4931$$

$$\int (3(c+dx)\sin(a+bx)-(c+dx)\sin(a+bx)\tan^2(a+bx))\,dx$$

$$\Big\downarrow \; 2009$$

$$\frac{d\text{arctanh}(\sin(a+bx))}{b^2}+\frac{4d\sin(a+bx)}{b^2}-\frac{4(c+dx)\cos(a+bx)}{b}-\frac{(c+dx)\sec(a+bx)}{b}$$

input
```
Int[(c + d*x)*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

output
```
(d*ArcTanh[Sin[a + b*x]])/b^2 - (4*(c + d*x)*Cos[a + b*x])/b - ((c + d*x)*
Sec[a + b*x])/b + (4*d*Sin[a + b*x])/b^2
```

### 3.391.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.391.4 Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.68

| method | result |
|---|---|
| default | $-\frac{4c\cos(xb+a)}{b}-\frac{c}{b\cos(xb+a)}+\frac{4d(\sin(xb+a)-(xb+a)\cos(xb+a)+\cos(xb+a)a)}{b^2}-\frac{dx}{b\cos(xb+a)}+\frac{d\ln(\sec(xb+a)+\tan(xb+a$ |
| risch | $-\frac{2(dxb+cb+id)e^{i(xb+a)}}{b^2}-\frac{2(dxb+cb-id)e^{-i(xb+a)}}{b^2}-\frac{2\,e^{i(xb+a)}(dx+c)}{b(e^{2i(xb+a)}+1)}-\frac{d\ln(e^{i(xb+a)}-i)}{b^2}+\frac{d\ln(i+e^{i(xb+a)})}{b^2}$ |

input
```
int((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x,method=_RETURNVERBOSE)
```

output
```
-4*c*cos(b*x+a)/b-c/b/cos(b*x+a)+4*d/b^2*(sin(b*x+a)-(b*x+a)*cos(b*x+a)+co
s(b*x+a)*a)-d/b/cos(b*x+a)*x+d/b^2*ln(sec(b*x+a)+tan(b*x+a))
```

### 3.391.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.63

$$\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx =$$
$$-\frac{2\,bdx+8\,(bdx+bc)\cos(bx+a)^2-d\cos(bx+a)\log(\sin(bx+a)+1)+d\cos(bx+a)\log(-\sin(bx-}{2\,b^2\cos(bx+a)}$$

input
```
integrate((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

output
```
-1/2*(2*b*d*x + 8*(b*d*x + b*c)*cos(b*x + a)^2 - d*cos(b*x + a)*log(sin(b*
x + a) + 1) + d*cos(b*x + a)*log(-sin(b*x + a) + 1) - 8*d*cos(b*x + a)*sin
(b*x + a) + 2*b*c)/(b^2*cos(b*x + a))
```

### 3.391.6 Sympy [F(-2)]

Exception generated.

$$\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((d*x+c)*sec(b*x+a)**2*sin(3*b*x+3*a),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.391.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3330 vs. $2(57) = 114$.

Time = 0.39 (sec) , antiderivative size = 3330, normalized size of antiderivative = 58.42

$$\int (c + dx)\sec^2(a + bx)\sin(3a + 3bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

output
```
-2*((cos(3*b*x + 3*a) + cos(b*x + a))*cos(4*b*x + 4*a) + (3*cos(2*b*x + 2*
a) + 1)*cos(3*b*x + 3*a) + 3*cos(2*b*x + 2*a)*cos(b*x + a) + (sin(3*b*x +
3*a) + sin(b*x + a))*sin(4*b*x + 4*a) + 3*sin(3*b*x + 3*a)*sin(2*b*x + 2*a
) + 3*sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))*c/(b*cos(3*b*x + 3*a)^
2 + 2*b*cos(3*b*x + 3*a)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x + 3
*a)^2 + 2*b*sin(3*b*x + 3*a)*sin(b*x + a) + b*sin(b*x + a)^2) - 1/2*(4*(co
s(a)^2 + sin(a)^2)*b*x*cos(b*x + a) + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2
*a) + b*x*cos(b*x + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*
x*sin(b*x + 2*a)*sin(a))*cos(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*
x + a))*cos(2*b*x + 3*a)^2 + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) + b*x
*cos(b*x + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(b*x
 + 2*a)*sin(a))*sin(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))*s
in(2*b*x + 3*a)^2 + 4*((b*x*cos(2*b*x + 3*a) + b*x*cos(a) + sin(2*b*x + 3*
a) + sin(a))*cos(3*b*x + 3*a)^2 + (b*x*cos(a) + sin(a))*cos(b*x + a)^2 + (
b*x*cos(2*b*x + 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a))*sin(3*b*x +
 3*a)^2 + (b*x*cos(a) + sin(a))*sin(b*x + a)^2 + 2*(b*x*cos(2*b*x + 3*a)*c
os(b*x + a) + (b*x*cos(a) + sin(a))*cos(b*x + a) + cos(b*x + a)*sin(2*b*x
+ 3*a))*cos(3*b*x + 3*a) + (b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos(2
*b*x + 3*a) + 2*(b*x*cos(2*b*x + 3*a)*sin(b*x + a) + (b*x*cos(a) + sin(a))
*sin(b*x + a) + sin(2*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a) + (cos(...
```

### 3.391.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(57) = 114$.

Time = 0.37 (sec) , antiderivative size = 365, normalized size of antiderivative = 6.40

$$\int (c + dx)\sec^2(a + bx)\sin(3a + 3bx)\,dx$$

$$= \frac{10\,bc\tan\left(\frac{1}{2}\,bx + \frac{1}{2}\,a\right)^4 + 10\,(bx + a)d\tan\left(\frac{1}{2}\,bx + \frac{1}{2}\,a\right)^4 - 10\,ad\tan\left(\frac{1}{2}\,bx + \frac{1}{2}\,a\right)^4 + d\log\left(\frac{2\left(\tan\left(\frac{1}{2}\,bx + \frac{1}{2}\,a\right)^2 + \cdots}{\tan\left(\frac{1}{2}\,bx \cdots\right.}\right.}{}$$

input
```
integrate((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

output
```
1/2*(10*b*c*tan(1/2*b*x + 1/2*a)^4 + 10*(b*x + a)*d*tan(1/2*b*x + 1/2*a)^4
 - 10*a*d*tan(1/2*b*x + 1/2*a)^4 + d*log(2*(tan(1/2*b*x + 1/2*a)^2 + 2*tan
(1/2*b*x + 1/2*a) + 1)/(tan(1/2*b*x + 1/2*a)^2 + 1))*tan(1/2*b*x + 1/2*a)^
4 - d*log(2*(tan(1/2*b*x + 1/2*a)^2 - 2*tan(1/2*b*x + 1/2*a) + 1)/(tan(1/2
*b*x + 1/2*a)^2 + 1))*tan(1/2*b*x + 1/2*a)^4 - 12*b*c*tan(1/2*b*x + 1/2*a)
^2 - 12*(b*x + a)*d*tan(1/2*b*x + 1/2*a)^2 + 12*a*d*tan(1/2*b*x + 1/2*a)^2
 + 16*d*tan(1/2*b*x + 1/2*a)^3 + 10*b*c + 10*(b*x + a)*d - 10*a*d - d*log(
2*(tan(1/2*b*x + 1/2*a)^2 + 2*tan(1/2*b*x + 1/2*a) + 1)/(tan(1/2*b*x + 1/2
*a)^2 + 1)) + d*log(2*(tan(1/2*b*x + 1/2*a)^2 - 2*tan(1/2*b*x + 1/2*a) + 1
)/(tan(1/2*b*x + 1/2*a)^2 + 1)) - 16*d*tan(1/2*b*x + 1/2*a))/((b*tan(1/2*b
*x + 1/2*a)^4 - b)*b)
```

### 3.391.9 Mupad [B] (verification not implemented)

Time = 1.76 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.63

$$\begin{aligned}\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx &= \mathrm{e}^{-a\,1\mathrm{i}-b\,x\,1\mathrm{i}}\left(\frac{-2\,b\,c+d\,2\mathrm{i}}{b^2}-\frac{2\,d\,x}{b}\right)\\ &\quad-\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\left(\frac{2\,b\,c+d\,2\mathrm{i}}{b^2}+\frac{2\,d\,x}{b}\right)\\ &\quad-\frac{d\,\ln\left(\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}-\mathrm{i}\right)}{b^2}+\frac{d\,\ln\left(\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}+1\mathrm{i}\right)}{b^2}\\ &\quad-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,(c+d\,x)\,2\mathrm{i}}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}\end{aligned}$$

input
```
int((sin(3*a + 3*b*x)*(c + d*x))/cos(a + b*x)^2,x)
```

output
```
exp(- a*1i - b*x*1i)*((d*2i - 2*b*c)/b^2 - (2*d*x)/b) - exp(a*1i + b*x*1i)
*((d*2i + 2*b*c)/b^2 + (2*d*x)/b) - (d*log(exp(a*1i + b*x*1i) - 1i))/b^2 +
 (d*log(exp(a*1i + b*x*1i) + 1i))/b^2 - (exp(a*1i + b*x*1i)*(c + d*x)*2i)/
(b*(exp(a*2i + b*x*2i)*1i + 1i))
```

# 3.392 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$

3.392.1 Optimal result . . . 2790
3.392.2 Mathematica [N/A] . . . 2790
3.392.3 Rubi [N/A] . . . 2791
3.392.4 Maple [N/A] (verified) . . . 2792
3.392.5 Fricas [N/A] . . . 2792
3.392.6 Sympy [F(-2)] . . . 2792
3.392.7 Maxima [N/A] . . . 2793
3.392.8 Giac [N/A] . . . 2793
3.392.9 Mupad [N/A] . . . 2794

## 3.392.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \frac{4\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d} + \frac{4\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d} - \text{Int}\left(\frac{\sec(a+bx)\tan(a+bx)}{c+dx},x\right)$$

output
```
-CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)+4*cos(a-b*c/d)*Si(b*c/d+
b*x)/d+4*Ci(b*c/d+b*x)*sin(a-b*c/d)/d
```

## 3.392.2 Mathematica [N/A]

Not integrable

Time = 11.01 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

input
```
Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x),x]
```

output
```
Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x), x]
```

## 3.392.3 Rubi [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\sec^2(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \ 4931$$

$$\int \left(\frac{3\sin(a+bx)}{c+dx}-\frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}dx+\frac{4\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d}+\frac{4\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

input
```
Int[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x),x]
```

output
```
$Aborted
```

### 3.392.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.392.4 Maple [N/A] (verified)

Not integrable

Time = 0.61 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sec\left(xb+a\right)^{2} \sin\left(3xb+3a\right)}{dx+c}dx$$

input
```
int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
```

output
```
int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
```

### 3.392.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx=\int \frac{\sec\left(bx+a\right)^{2} \sin\left(3\,bx+3\,a\right)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

### 3.392.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx=\text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(sec(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.392.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 1275, normalized size of antiderivative = 51.00

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)^2\sin(3\,bx+3\,a)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")
```

output
```
2*(b*c*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(1, -(
I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(exp_integral_e(1, (I*b*d*x
 + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d
) + (b*c*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(1,
-(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(exp_integral_e(1, (I*b*d
*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)
/d) + (b*d*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(1
, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integral_e(1, (I*b
*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*
d)/d))*x)*cos(2*b*x + 2*a)^2 + (b*c*(-I*exp_integral_e(1, (I*b*d*x + I*b*c
)/d) + I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*
c*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x +
I*b*c)/d))*sin(-(b*c - a*d)/d) + (b*d*(-I*exp_integral_e(1, (I*b*d*x + I*b
*c)/d) + I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) -
b*d*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x
+ I*b*c)/d))*sin(-(b*c - a*d)/d))*x)*sin(2*b*x + 2*a)^2 - d*sin(2*b*x + 2*
a)*sin(b*x + a) + (b*d*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) + I*exp_
integral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integr
al_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*si
n(-(b*c - a*d)/d))*x + (2*b*c*(-I*exp_integral_e(1, (I*b*d*x + I*b*c)/d...
```

### 3.392.8 Giac [N/A]

Not integrable

Time = 1.90 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sec(bx+a)^2\sin(3\,bx+3\,a)}{dx+c}\,dx$$

input
```
integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

### 3.392.9 Mupad [N/A]

Not integrable

Time = 29.93 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \frac{\sin\left(3\,a+3\,b\,x\right)}{{\cos\left(a+b\,x\right)}^2\,\left(c+d\,x\right)}\,dx$$

input
```
int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)),x)
```

output
```
int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)), x)
```

# 3.393 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$

3.393.1 Optimal result . . . 2795
3.393.2 Mathematica [N/A] . . . 2795
3.393.3 Rubi [N/A] . . . 2796
3.393.4 Maple [N/A] (verified) . . . 2797
3.393.5 Fricas [N/A] . . . 2797
3.393.6 Sympy [F(-2)] . . . 2797
3.393.7 Maxima [N/A] . . . 2798
3.393.8 Giac [N/A] . . . 2798
3.393.9 Mupad [N/A] . . . 2799

## 3.393.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \frac{4b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2} - \frac{4\sin(a+bx)}{d(c+dx)} - \frac{4b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2} - \text{Int}\left(\frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2},x\right)$$

output
```
-CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)+4*b*Ci(b*c/d+b*x)*cos(
a-b*c/d)/d^2-4*b*Si(b*c/d+b*x)*sin(a-b*c/d)/d^2-4*sin(b*x+a)/d/(d*x+c)
```

## 3.393.2 Mathematica [N/A]

Not integrable

Time = 15.40 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

output
```
Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2, x]
```

### 3.393.3 Rubi [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{4931}$$

$$\int \left(\frac{3\sin(a+bx)}{(c+dx)^2}-\frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}dx+\frac{4b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{4b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{4\sin(a+bx)}{d(c+dx)}$$

input
```
Int[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

#### 3.393.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.393.4 Maple [N/A] (verified)

Not integrable

Time = 0.73 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sec(xb+a)^2 \sin(3xb+3a)}{(dx+c)^2}dx$$

input
```
int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

output
```
int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

### 3.393.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\sec(bx+a)^2 \sin(3\,bx+3\,a)}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

### 3.393.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate(sec(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.393.7 Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 1419, normalized size of antiderivative = 56.76

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\sec(bx+a)^2\sin(3\,bx+3\,a)}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
2*(b*c*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(2, -(
I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(exp_integral_e(2, (I*b*d*x
 + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d
) + (b*c*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(2,
-(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(exp_integral_e(2, (I*b*d
*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)
/d) + (b*d*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(2
, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integral_e(2, (I*b
*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*
d)/d))*x)*cos(2*b*x + 2*a)^2 + (b*c*(-I*exp_integral_e(2, (I*b*d*x + I*b*c
)/d) + I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*
c*(exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x +
I*b*c)/d))*sin(-(b*c - a*d)/d) + (b*d*(-I*exp_integral_e(2, (I*b*d*x + I*b
*c)/d) + I*exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) -
b*d*(exp_integral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x
+ I*b*c)/d))*sin(-(b*c - a*d)/d))*x)*sin(2*b*x + 2*a)^2 - d*sin(2*b*x + 2*
a)*sin(b*x + a) + (b*d*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) + I*exp_
integral_e(2, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integr
al_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*si
n(-(b*c - a*d)/d))*x + (2*b*c*(-I*exp_integral_e(2, (I*b*d*x + I*b*c)/d...
```

### 3.393.8 Giac [N/A]

Not integrable

Time = 4.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \frac{\sec(bx+a)^2\sin(3\,bx+3\,a)}{(dx+c)^2}\,dx$$

input
```
integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^2, x)
```

### 3.393.9 Mupad [N/A]

Not integrable

Time = 32.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx=\int \frac{\sin\left(3\,a+3\,b\,x\right)}{{\cos\left(a+b\,x\right)}^2\,{\left(c+d\,x\right)}^2}\,dx$$

input
```
int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)^2),x)
```

output
```
int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)^2), x)
```

# 3.394 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$

3.394.1 Optimal result . . . 2800
3.394.2 Mathematica [N/A] . . . 2800
3.394.3 Rubi [N/A] . . . 2801
3.394.4 Maple [N/A] (verified) . . . 2802
3.394.5 Fricas [N/A] . . . 2802
3.394.6 Sympy [F(-1)] . . . 2802
3.394.7 Maxima [N/A] . . . 2803
3.394.8 Giac [N/A] . . . 2803
3.394.9 Mupad [N/A] . . . 2804

## 3.394.1 Optimal result

Integrand size = 25, antiderivative size = 25

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = -\frac{2b\cos(a+bx)}{d^2(c+dx)} - \frac{2b^2\,\text{CosIntegral}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^3} - \frac{2\sin(a+bx)}{d(c+dx)^2} - \frac{2b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^3} - \text{Int}\left(\frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^3},x\right)$$

output
```
-CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^3,x)-2*b*cos(b*x+a)/d^2/(d*
x+c)-2*b^2*cos(a-b*c/d)*Si(b*c/d+b*x)/d^3-2*b^2*Ci(b*c/d+b*x)*sin(a-b*c/d)
/d^3-2*sin(b*x+a)/d/(d*x+c)^2
```

## 3.394.2 Mathematica [N/A]

Not integrable

Time = 17.36 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

input
```
Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

output
```
Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3, x]
```

## 3.394.3 Rubi [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sin(3a+3bx)\sec^2(a+bx)}{(c+dx)^3}\,dx$$
$$\Big\downarrow \text{4931}$$
$$\int \left(\frac{3\sin(a+bx)}{(c+dx)^3}-\frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^3}dx-\frac{2b^2\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{d^3}-\frac{2b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^3}-\frac{2b\cos(a+bx)}{d^2(c+dx)}-\frac{2\sin(a+bx)}{d(c+dx)^2}$$

input
```
Int[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

output
```
$Aborted
```

### 3.394.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.394.4 Maple [N/A] (verified)

Not integrable

Time = 0.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\sec \left(xb+a\right)^{2} \sin \left(3xb+3a\right)}{\left(dx+c\right)^{3}}dx$$

input
```
int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

output
```
int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

### 3.394.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.96

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\sec \left(bx+a\right)^{2} \sin \left(3\,bx+3\,a\right)}{\left(dx+c\right)^{3}}\,dx$$

input
```
integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
integral(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*
x + c^3), x)
```

### 3.394.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate(sec(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**3,x)
```

output
```
Timed out
```

### 3.394.7 Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 1563, normalized size of antiderivative = 62.52

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\sec(bx+a)^2\sin(3\,bx+3\,a)}{(dx+c)^3}\,dx$$

input
```
integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
2*(b*c*(-I*exp_integral_e(3, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(3, -(
I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(exp_integral_e(3, (I*b*d*x
 + I*b*c)/d) + exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d
) + (b*c*(-I*exp_integral_e(3, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(3,
-(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(exp_integral_e(3, (I*b*d
*x + I*b*c)/d) + exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)
/d) + (b*d*(-I*exp_integral_e(3, (I*b*d*x + I*b*c)/d) + I*exp_integral_e(3
, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integral_e(3, (I*b
*d*x + I*b*c)/d) + exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*
d)/d))*x)*cos(2*b*x + 2*a)^2 + (b*c*(-I*exp_integral_e(3, (I*b*d*x + I*b*c
)/d) + I*exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*
c*(exp_integral_e(3, (I*b*d*x + I*b*c)/d) + exp_integral_e(3, -(I*b*d*x +
I*b*c)/d))*sin(-(b*c - a*d)/d) + (b*d*(-I*exp_integral_e(3, (I*b*d*x + I*b
*c)/d) + I*exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) -
b*d*(exp_integral_e(3, (I*b*d*x + I*b*c)/d) + exp_integral_e(3, -(I*b*d*x
+ I*b*c)/d))*sin(-(b*c - a*d)/d))*x)*sin(2*b*x + 2*a)^2 - d*sin(2*b*x + 2*
a)*sin(b*x + a) + (b*d*(-I*exp_integral_e(3, (I*b*d*x + I*b*c)/d) + I*exp_
integral_e(3, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(exp_integr
al_e(3, (I*b*d*x + I*b*c)/d) + exp_integral_e(3, -(I*b*d*x + I*b*c)/d))*si
n(-(b*c - a*d)/d))*x + (2*b*c*(-I*exp_integral_e(3, (I*b*d*x + I*b*c)/d...
```

### 3.394.8 Giac [N/A]

Not integrable

Time = 7.78 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \frac{\sec(bx+a)^2\sin(3\,bx+3\,a)}{(dx+c)^3}\,dx$$

input
```
integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
integrate(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^3, x)
```

### 3.394.9 Mupad [N/A]

Not integrable

Time = 34.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx=\int \frac{\sin\left(3\,a+3\,b\,x\right)}{{\cos\left(a+b\,x\right)}^2\,{\left(c+d\,x\right)}^3}\,dx$$

input
```
int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)^3),x)
```

output
```
int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)^3), x)
```

# 3.395 $\int x\cos(2x)\sec(x)\,dx$

3.395.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2805
3.395.2 Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2805
3.395.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2806
3.395.4 Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2807
3.395.5 Fricas [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 2807
3.395.6 Sympy [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2808
3.395.7 Maxima [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2808
3.395.8 Giac [F] . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2808
3.395.9 Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 2809

## 3.395.1 Optimal result

Integrand size = 8, antiderivative size = 57

$$\int x\cos(2x)\sec(x)\,dx = 2ix\arctan\left(e^{ix}\right)+2\cos(x)-i\,\text{PolyLog}\left(2,-ie^{ix}\right)+i\,\text{PolyLog}\left(2,ie^{ix}\right)+2x\sin(x)$$

output
```
2*I*x*arctan(exp(I*x))+2*cos(x)-I*polylog(2,-I*exp(I*x))+I*polylog(2,I*exp
(I*x))+2*x*sin(x)
```

## 3.395.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.35

$$\int x\cos(2x)\sec(x)\,dx = 2\cos(x)-x\left(\log\left(1-ie^{ix}\right)-\log\left(1+ie^{ix}\right)\right)-i\left(\text{PolyLog}\left(2,-ie^{ix}\right)-\text{PolyLog}\left(2,ie^{ix}\right)\right)+2x\sin(x)$$

input
```
Integrate[x*Cos[2*x]*Sec[x],x]
```

output
```
2*Cos[x] - x*(Log[1 - I*E^(I*x)] - Log[1 + I*E^(I*x)]) - I*(PolyLog[2, (-I
)*E^(I*x)] - PolyLog[2, I*E^(I*x)]) + 2*x*Sin[x]
```

### 3.395.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos(2x)\sec(x)\,dx$$

$$\Big\downarrow \text{4931}$$

$$\int (x\cos(x)-x\sin(x)\tan(x))dx$$

$$\Big\downarrow \text{2009}$$

$$2ix\arctan\left(e^{ix}\right)-i\,\text{PolyLog}\left(2,-ie^{ix}\right)+i\,\text{PolyLog}\left(2,ie^{ix}\right)+2x\sin(x)+2\cos(x)$$

input
```
Int[x*Cos[2*x]*Sec[x],x]
```

output
```
(2*I)*x*ArcTan[E^(I*x)] + 2*Cos[x] - I*PolyLog[2, (-I)*E^(I*x)] + I*PolyLo
g[2, I*E^(I*x)] + 2*x*Sin[x]
```

#### 3.395.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

## 3.395.4 Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16

| method | result |
|---|---|
| default | $x\ln\left(1+i\,\mathrm{e}^{ix}\right)-x\ln\left(1-i\,\mathrm{e}^{ix}\right)-i\,\mathrm{dilog}\left(1+i\,\mathrm{e}^{ix}\right)+i\,\mathrm{dilog}\left(1-i\,\mathrm{e}^{ix}\right)+2\cos\left(x\right)+2x\sin\left(x\right)$ |
| risch | $-i(x+i)\,\mathrm{e}^{ix}+i(x-i)\,\mathrm{e}^{-ix}+x\ln\left(1+i\,\mathrm{e}^{ix}\right)-x\ln\left(1-i\,\mathrm{e}^{ix}\right)-i\,\mathrm{dilog}\left(1+i\,\mathrm{e}^{ix}\right)+i\,\mathrm{dilog}\left(1-i\mathrm{e}\right.$ |

input
```
int(x*cos(2*x)*sec(x),x,method=_RETURNVERBOSE)
```

output
```
x*ln(1+I*exp(I*x))-x*ln(1-I*exp(I*x))-I*dilog(1+I*exp(I*x))+I*dilog(1-I*ex
p(I*x))+2*cos(x)+2*x*sin(x)
```

## 3.395.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(36) = 72.

Time = 0.26 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.86

$$
\begin{aligned}
\int x\cos(2x)\sec(x)\,dx = {} & -\frac{1}{2}\,x\log\left(i\,\cos\left(x\right)+\sin\left(x\right)+1\right)+\frac{1}{2}\,x\log\left(i\,\cos\left(x\right)-\sin\left(x\right)+1\right) \\
& -\frac{1}{2}\,x\log\left(-i\,\cos\left(x\right)+\sin\left(x\right)+1\right) \\
& +\frac{1}{2}\,x\log\left(-i\,\cos\left(x\right)-\sin\left(x\right)+1\right)+2\,x\sin\left(x\right)+2\,\cos\left(x\right) \\
& +\frac{1}{2}i\,\mathrm{Li}_2(i\,\cos\left(x\right)+\sin\left(x\right))+\frac{1}{2}i\,\mathrm{Li}_2(i\,\cos\left(x\right)-\sin\left(x\right)) \\
& -\frac{1}{2}i\,\mathrm{Li}_2(-i\,\cos\left(x\right)+\sin\left(x\right))-\frac{1}{2}i\,\mathrm{Li}_2(-i\,\cos\left(x\right)-\sin\left(x\right))
\end{aligned}
$$

input
```
integrate(x*cos(2*x)*sec(x),x, algorithm="fricas")
```

output
```
-1/2*x*log(I*cos(x) + sin(x) + 1) + 1/2*x*log(I*cos(x) - sin(x) + 1) - 1/2
*x*log(-I*cos(x) + sin(x) + 1) + 1/2*x*log(-I*cos(x) - sin(x) + 1) + 2*x*s
in(x) + 2*cos(x) + 1/2*I*dilog(I*cos(x) + sin(x)) + 1/2*I*dilog(I*cos(x) -
 sin(x)) - 1/2*I*dilog(-I*cos(x) + sin(x)) - 1/2*I*dilog(-I*cos(x) - sin(x
))
```

### 3.395.6 Sympy [F]

$$\int x\cos(2x)\sec(x)\,dx = \int x\cos{(2x)}\sec{(x)}\,dx$$

input
```
integrate(x*cos(2*x)*sec(x),x)
```

output
```
Integral(x*cos(2*x)*sec(x), x)
```

### 3.395.7 Maxima [F]

$$\int x\cos(2x)\sec(x)\,dx = \int x\cos{(2\,x)}\sec{(x)}\,dx$$

input
```
integrate(x*cos(2*x)*sec(x),x, algorithm="maxima")
```

output
```
2*x*sin(x) + 2*cos(x) - 2*integrate((x*cos(2*x)*cos(x) + x*sin(2*x)*sin(x)
 + x*cos(x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1), x)
```

### 3.395.8 Giac [F]

$$\int x\cos(2x)\sec(x)\,dx = \int x\cos{(2\,x)}\sec{(x)}\,dx$$

input
```
integrate(x*cos(2*x)*sec(x),x, algorithm="giac")
```

output
```
integrate(x*cos(2*x)*sec(x), x)
```

### 3.395.9 Mupad [B] (verification not implemented)

Time = 26.55 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int x\cos(2x)\sec(x)\,dx = 2\,\cos\left(x\right)+2\,x\,\sin\left(x\right)-\mathrm{polylog}\left(2,-\mathrm{e}^{x\,1\mathrm{i}}\,1\mathrm{i}\right)\,1\mathrm{i} + \mathrm{polylog}\left(2,\mathrm{e}^{x\,1\mathrm{i}}\,1\mathrm{i}\right)\,1\mathrm{i}+x\,\mathrm{atan}\left(\mathrm{e}^{x\,1\mathrm{i}}\right)\,2\mathrm{i}$$

input
```
int((x*cos(2*x))/cos(x),x)
```

output
```
2*cos(x) - polylog(2, -exp(x*1i)*1i)*1i + polylog(2, exp(x*1i)*1i)*1i + x*
atan(exp(x*1i))*2i + 2*x*sin(x)
```

# 3.396 $\int x\cos(2x)\sec^2(x)\,dx$

3.396.1 Optimal result . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2810
3.396.2 Mathematica [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2810
3.396.3 Rubi [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2811
3.396.4 Maple [A] (verified) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2812
3.396.5 Fricas [A] (verification not implemented) . . . . . . . . . . . . . . . . . . . 2812
3.396.6 Sympy [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 2812
3.396.7 Maxima [B] (verification not implemented) . . . . . . . . . . . . . . . . . . 2813
3.396.8 Giac [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . . 2813
3.396.9 Mupad [B] (verification not implemented) . . . . . . . . . . . . . . . . . . . 2814

## 3.396.1 Optimal result

Integrand size = 10, antiderivative size = 14

$$\int x\cos(2x)\sec^2(x)\,dx = x^2 - \log(\cos(x)) - x\tan(x)$$

output
```
x^2-ln(cos(x))-x*tan(x)
```

## 3.396.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int x\cos(2x)\sec^2(x)\,dx = x^2 - \log(\cos(x)) - x\tan(x)$$

input
```
Integrate[x*Cos[2*x]*Sec[x]^2,x]
```

output
```
x^2 - Log[Cos[x]] - x*Tan[x]
```

### 3.396.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos(2x)\sec^2(x)\,dx$$

$$\Big\downarrow \; 4931$$

$$\int \left(x-x\tan^2(x)\right)dx$$

$$\Big\downarrow \; 2009$$

$$x^2-x\tan(x)-\log(\cos(x))$$

input
```
Int[x*Cos[2*x]*Sec[x]^2,x]
```

output
```
x^2 - Log[Cos[x]] - x*Tan[x]
```

#### 3.396.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.396.4 Maple [A] (verified)

Time = 3.40 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

| method | result | size |
|---|---|---|
| default | $x^2 - \ln(\cos(x)) - x\tan(x)$ | 15 |
| risch | $x^2 + 2ix - \frac{2ix}{e^{2ix}+1} - \ln(e^{2ix}+1)$ | 32 |

input
```
int(x*cos(2*x)*sec(x)^2,x,method=_RETURNVERBOSE)
```

output
```
x^2-ln(cos(x))-x*tan(x)
```

### 3.396.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.86

$$\int x\cos(2x)\sec^2(x)\,dx = \frac{x^2\cos(x) - \cos(x)\log(-\cos(x)) - x\sin(x)}{\cos(x)}$$

input
```
integrate(x*cos(2*x)*sec(x)^2,x, algorithm="fricas")
```

output
```
(x^2*cos(x) - cos(x)*log(-cos(x)) - x*sin(x))/cos(x)
```

### 3.396.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(12) = 24.

Time = 3.96 (sec) , antiderivative size = 144, normalized size of antiderivative = 10.29

$$\int x\cos(2x)\sec^2(x)\,dx = x^2 + \frac{2x\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1} - \frac{\log\left(\tan\left(\frac{x}{2}\right)-1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1} + \frac{\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{\tan^2\left(\frac{x}{2}\right)-1} - \frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1} + \frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{\tan^2\left(\frac{x}{2}\right)-1} + \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1} - \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)}{\tan^2\left(\frac{x}{2}\right)-1}$$

input
```
integrate(x*cos(2*x)*sec(x)**2,x)
```

output
```
x**2 + 2*x*tan(x/2)/(tan(x/2)**2 - 1) - log(tan(x/2) - 1)*tan(x/2)**2/(tan
(x/2)**2 - 1) + log(tan(x/2) - 1)/(tan(x/2)**2 - 1) - log(tan(x/2) + 1)*ta
n(x/2)**2/(tan(x/2)**2 - 1) + log(tan(x/2) + 1)/(tan(x/2)**2 - 1) + log(ta
n(x/2)**2 + 1)*tan(x/2)**2/(tan(x/2)**2 - 1) - log(tan(x/2)**2 + 1)/(tan(x
/2)**2 - 1)
```

### 3.396.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(14) = 28.

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 7.93

$$\int x\cos(2x)\sec^2(x)\,dx$$
$$= \frac{2\,x^2\cos\left(2\,x\right)^2 + 2\,x^2\sin\left(2\,x\right)^2 + 4\,x^2\cos\left(2\,x\right) + 2\,x^2 - \left(\cos\left(2\,x\right)^2 + \sin\left(2\,x\right)^2 + 2\,\cos\left(2\,x\right) + 1\right)\log\left(\cos\left(\right.\right.}{2\left(\cos\left(2\,x\right)^2 + \sin\left(2\,x\right)^2 + 2\,\cos\left(2\,x\right) + 1\right)}$$

input
```
integrate(x*cos(2*x)*sec(x)^2,x, algorithm="maxima")
```

output
```
1/2*(2*x^2*cos(2*x)^2 + 2*x^2*sin(2*x)^2 + 4*x^2*cos(2*x) + 2*x^2 - (cos(2
*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2
*x) + 1) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

### 3.396.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(14) = 28.

Time = 0.27 (sec) , antiderivative size = 118, normalized size of antiderivative = 8.43

$$\int x\cos(2x)\sec^2(x)\,dx$$
$$= \frac{2\,x^2\tan\left(\frac{1}{2}\,x\right)^2 - \log\left(\frac{4\left(\tan\left(\frac{1}{2}\,x\right)^4 - 2\,\tan\left(\frac{1}{2}\,x\right)^2 + 1\right)}{\tan\left(\frac{1}{2}\,x\right)^4 + 2\,\tan\left(\frac{1}{2}\,x\right)^2 + 1}\right)\tan\left(\frac{1}{2}\,x\right)^2 - 2\,x^2 + 4\,x\tan\left(\frac{1}{2}\,x\right) + \log\left(\frac{4\left(\tan\left(\frac{1}{2}\,x\right)^4 - 2\,\tan\left(\frac{1}{2}\right.\right.}{\tan\left(\frac{1}{2}\,x\right)^4 + 2\,\tan\left(\frac{1}{2}\right.}\right.}{2\left(\tan\left(\frac{1}{2}\,x\right)^2 - 1\right)}$$

input
```
integrate(x*cos(2*x)*sec(x)^2,x, algorithm="giac")
```

output
```
1/2*(2*x^2*tan(1/2*x)^2 - log(4*(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)/(tan(1
/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - 2*x^2 + 4*x*tan(1/2*x) + log
(4*(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)
))/(tan(1/2*x)^2 - 1)
```

### 3.396.9 Mupad [B] (verification not implemented)

Time = 28.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int x\cos(2x)\sec^2(x)\,dx = x^2 - \ln\left(e^{x\,2i} + 1\right) + x\,2i - \frac{x\,2i}{e^{x\,2i} + 1}$$

input
```
int((x*cos(2*x))/cos(x)^2,x)
```

output
```
x*2i - log(exp(x*2i) + 1) - (x*2i)/(exp(x*2i) + 1) + x^2
```

# 3.397 $\int x\cos(2x)\sec^3(x)\,dx$

3.397.1 Optimal result . . . 2815
3.397.2 Mathematica [B] (verified) . . . 2815
3.397.3 Rubi [A] (verified) . . . 2816
3.397.4 Maple [B] (verified) . . . 2817
3.397.5 Fricas [B] (verification not implemented) . . . 2817
3.397.6 Sympy [F] . . . 2818
3.397.7 Maxima [F] . . . 2818
3.397.8 Giac [F] . . . 2819
3.397.9 Mupad [B] (verification not implemented) . . . 2819

## 3.397.1 Optimal result

Integrand size = 10, antiderivative size = 67

$$\int x\cos(2x)\sec^3(x)\,dx = -3ix\arctan\left(e^{ix}\right)+\frac{3}{2}i\,\text{PolyLog}\left(2,-ie^{ix}\right)-\frac{3}{2}i\,\text{PolyLog}\left(2,ie^{ix}\right)+\frac{\sec(x)}{2}-\frac{1}{2}x\sec(x)\tan(x)$$

output
```
-3*I*x*arctan(exp(I*x))+3/2*I*polylog(2,-I*exp(I*x))-3/2*I*polylog(2,I*exp
(I*x))+1/2*sec(x)-1/2*x*sec(x)*tan(x)
```

## 3.397.2 Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 146 vs. 2(67) = 134.

Time = 0.23 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.18

$$\int x\cos(2x)\sec^3(x)\,dx = \frac{1}{4}\left(6x\log\left(1-ie^{ix}\right)-6x\log\left(1+ie^{ix}\right)+6i\,\text{PolyLog}\left(2,-ie^{ix}\right)-6i\,\text{PolyLog}\left(2,ie^{ix}\right)+\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}+\frac{x}{\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)^2}-\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}+\frac{x}{-1+\sin(x)}\right)$$

input
```
Integrate[x*Cos[2*x]*Sec[x]^3,x]
```

output
```
(6*x*Log[1 - I*E^(I*x)] - 6*x*Log[1 + I*E^(I*x)] + (6*I)*PolyLog[2, (-I)*E
^(I*x)] - (6*I)*PolyLog[2, I*E^(I*x)] + (2*Sin[x/2])/(Cos[x/2] - Sin[x/2])
 + x/(Cos[x/2] + Sin[x/2])^2 - (2*Sin[x/2])/(Cos[x/2] + Sin[x/2]) + x/(-1
+ Sin[x]))/4
```

### 3.397.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4931, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cos(2x)\sec^3(x)\,dx$$

$$\Big\downarrow \text{4931}$$

$$\int \left(x\sec(x)-x\tan^2(x)\sec(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-3ix\arctan\left(e^{ix}\right)+\frac{3}{2}i\,\text{PolyLog}\left(2,-ie^{ix}\right)-\frac{3}{2}i\,\text{PolyLog}\left(2,ie^{ix}\right)+\frac{\sec(x)}{2}-\frac{1}{2}x\tan(x)\sec(x)$$

input
```
Int[x*Cos[2*x]*Sec[x]^3,x]
```

output
```
(-3*I)*x*ArcTan[E^(I*x)] + ((3*I)/2)*PolyLog[2, (-I)*E^(I*x)] - ((3*I)/2)*
PolyLog[2, I*E^(I*x)] + Sec[x]/2 - (x*Sec[x]*Tan[x])/2
```

#### 3.397.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4931
```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

### 3.397.4 Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(48) = 96.

Time = 9.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.52

| method | result | size |
|---|---|---|
| risch | $\frac{i\,{\mathrm e}^{i x} \left({\mathrm e}^{2 i x} x -x -i\,{\mathrm e}^{2 i x}-i\right)}{\left({\mathrm e}^{2 i x}+1\right)^{2}}-\frac{3 x \ln \left(1+i\,{\mathrm e}^{i x}\right)}{2}+\frac{3 x \ln \left(1-i\,{\mathrm e}^{i x}\right)}{2}+\frac{3 i \operatorname{dilog}\left(1+i\,{\mathrm e}^{i x}\right)}{2}-\frac{3 i \operatorname{dilog}\left(1-i\,{\mathrm e}^{i x}\right)}{2}$ | 102 |

input
```
int(x*cos(2*x)*sec(x)^3,x,method=_RETURNVERBOSE)
```

output
```
I*exp(I*x)*(exp(I*x)^2*x-x-I*exp(I*x)^2-I)/(exp(I*x)^2+1)^2-3/2*x*ln(1+I*e
xp(I*x))+3/2*x*ln(1-I*exp(I*x))+3/2*I*dilog(1+I*exp(I*x))-3/2*I*dilog(1-I*
exp(I*x))
```

### 3.397.5 Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(38) = 76.

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.15

$$\int x\cos(2x)\sec^3(x)\,dx$$
$$= \frac{3\,x\cos\left(x\right)^{2}\log\left(i\,\cos\left(x\right)+\sin\left(x\right)+1\right)-3\,x\cos\left(x\right)^{2}\log\left(i\,\cos\left(x\right)-\sin\left(x\right)+1\right)+3\,x\cos\left(x\right)^{2}\log\left(-i\,\cos\right.}{}$$

input
```
integrate(x*cos(2*x)*sec(x)^3,x, algorithm="fricas")
```

output
```
1/4*(3*x*cos(x)^2*log(I*cos(x) + sin(x) + 1) - 3*x*cos(x)^2*log(I*cos(x) -
 sin(x) + 1) + 3*x*cos(x)^2*log(-I*cos(x) + sin(x) + 1) - 3*x*cos(x)^2*log
(-I*cos(x) - sin(x) + 1) - 3*I*cos(x)^2*dilog(I*cos(x) + sin(x)) - 3*I*cos
(x)^2*dilog(I*cos(x) - sin(x)) + 3*I*cos(x)^2*dilog(-I*cos(x) + sin(x)) +
3*I*cos(x)^2*dilog(-I*cos(x) - sin(x)) - 2*x*sin(x) + 2*cos(x))/cos(x)^2
```

### 3.397.6 Sympy [F]

$$\int x\cos(2x)\sec^3(x)\,dx = \int x\cos(2x)\sec^3(x)\,dx$$

input
```
integrate(x*cos(2*x)*sec(x)**3,x)
```

output
```
Integral(x*cos(2*x)*sec(x)**3, x)
```

### 3.397.7 Maxima [F]

$$\int x\cos(2x)\sec^3(x)\,dx = \int x\cos(2x)\sec(x)^3\,dx$$

input
```
integrate(x*cos(2*x)*sec(x)^3,x, algorithm="maxima")
```

output
```
-((x*sin(3*x) - x*sin(x) - cos(3*x) - cos(x))*cos(4*x) - (2*x*sin(2*x) + 2
*cos(2*x) + 1)*cos(3*x) - 2*(x*sin(x) + cos(x))*cos(2*x) - 3*(2*(2*cos(2*x
) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(
2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)*integrate((x*cos(2*x)*cos(x) + x*sin
(2*x)*sin(x) + x*cos(x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1), x) -
(x*cos(3*x) - x*cos(x) + sin(3*x) + sin(x))*sin(4*x) + (2*x*cos(2*x) + x -
 2*sin(2*x))*sin(3*x) + 2*(x*cos(x) - sin(x))*sin(2*x) - x*sin(x) - cos(x)
)/(2*(2*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 +
4*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)
```

### 3.397.8 Giac [F]

$$\int x\cos(2x)\sec^3(x)\,dx = \int x\cos(2\,x)\sec(x)^3\,dx$$

input
```
integrate(x*cos(2*x)*sec(x)^3,x, algorithm="giac")
```

output
```
integrate(x*cos(2*x)*sec(x)^3, x)
```

### 3.397.9 Mupad [B] (verification not implemented)

Time = 27.65 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94

$$\int x\cos(2x)\sec^3(x)\,dx = \frac{1}{2\,\cos(x)} + x\,\mathrm{atanh}\left(\mathrm{e}^{x\,1\mathrm{i}}\,1\mathrm{i}\right) - \frac{x\,\sin(x)}{2\cos(x)^2} + \frac{\mathrm{polylog}(2,-\mathrm{e}^{x\,1\mathrm{i}}\,1\mathrm{i})\,3\mathrm{i}}{2} - \frac{\mathrm{polylog}(2,\mathrm{e}^{x\,1\mathrm{i}}\,1\mathrm{i})\,3\mathrm{i}}{2} - x\,\mathrm{atan}\left(\mathrm{e}^{x\,1\mathrm{i}}\right)\,4\mathrm{i}$$

input
```
int((x*cos(2*x))/cos(x)^3,x)
```

output
```
(polylog(2, -exp(x*1i)*1i)*3i)/2 - (polylog(2, exp(x*1i)*1i)*3i)/2 + 1/(2*
cos(x)) - x*atan(exp(x*1i))*4i + x*atanh(exp(x*1i)*1i) - (x*sin(x))/(2*cos
(x)^2)
```

# CHAPTER 4

## Appendix

4.1 Listing of Grading functions . . . . . . . . . . . . . . . . . . . . . . . . . . . . 2820

## 4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

### 4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

### 4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```